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SELECTION OF MODELING MATERIALS AND ITS EFFECT ON THE MODELING GAS SELECTION

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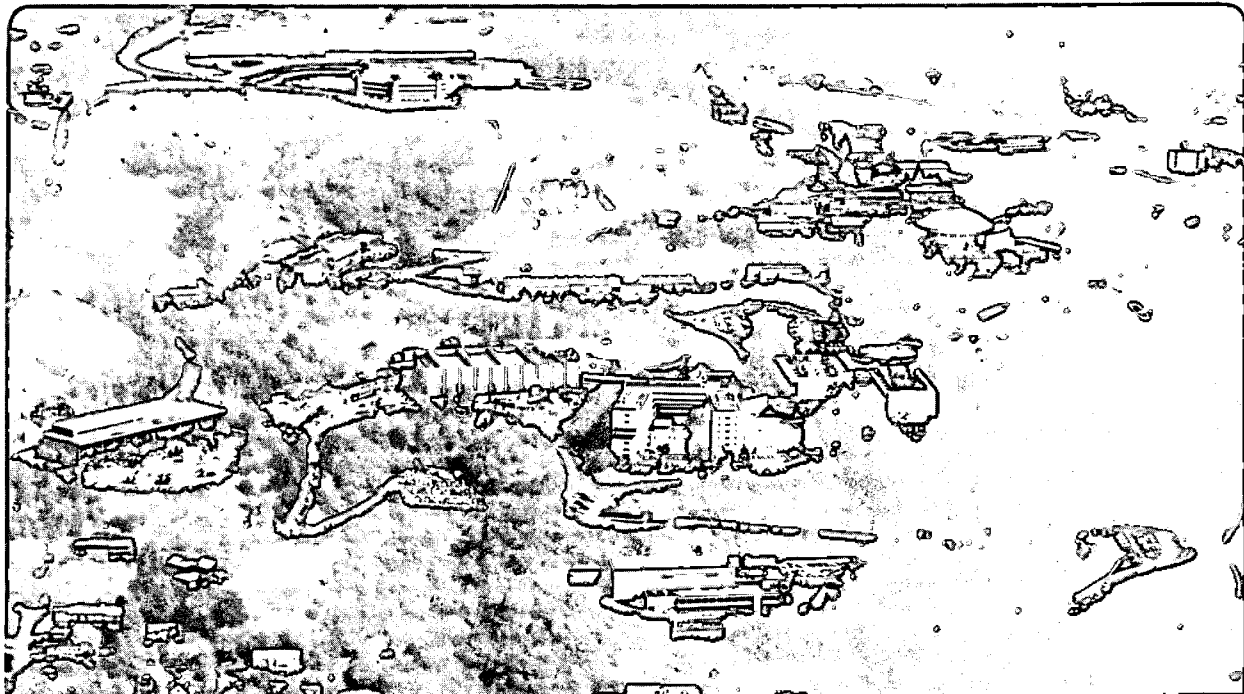
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PROGRAM - PROJECT - JOB ENERGY AND ENVIRONMENT		LOCATION Berkeley	DATE 14 April 1982
BUILDINGS: DYNAMIC SIMILITUDE PROJECT			
TITLE SELECTION OF MODELING MATERIALS AND ITS EFFECT ON THE MODELING GAS SELECTION			
This Engineering Note looks at the process of selecting modeling materials and the modeling gases together. The process of thermal modeling of buildings or parts of buildings using smaller-scale models requires careful selection of the atmosphere the scale model is to be tested in and the materials which must be used in the walls of the small-scale model. The basic theory of thermal scaling is described in <i>Reference 1</i> (LBL-13262). In previous Engineering Notes, I have shown that the properties of the atmosphere are quite variable. By mixing two or more gases, one can vary the thermal conductivity and volume specific heat of gas used for the modeling process.^{2,3} One can vary the thermal conductivity of the model wall but the volume specific heat of the wall material becomes fixed. This Engineering Note deals with the process of setting the gas thermal conductivity; then the properties of the wall materials can be fixed. The properties of the model walls are matched to one another and the properties of the walls are matched to the modeling gas for thermal similitude. 1. <u>Basic theory</u> For the derivation of the Dynamic Similitude theory, I refer the reader to <i>Reference 1</i> (LBL-13262) which deals with the modeling process. The equation numbers which are in parenthesis come from that report. The basic scaling relationships given in LBL-13262 can be stated as follows: $\ell_w = t^{-3} \kappa_w \quad (3.9) \quad -1-$ $\ell_f = t^{-5/2} \kappa_s \quad (3.10) \quad -2-$ $\tau = (\gamma_s P) \kappa_s t^{-13/2} \quad (3.13) \quad -3-$ $\delta = (\gamma_s P)^{-2} \kappa_s^{-1} t^{23/2} \quad (3.14) \quad -4-$ $\gamma_w \kappa_w = (\gamma_s P) \kappa_s t^{-1/2} \quad (3.15) \quad -5-$			

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where the following definitions apply:

$$\kappa_W = \frac{k_W'}{k_W} \quad (2.25) \quad -6-$$

$$\gamma_W = \frac{\rho_W' c_W'}{\rho_W c_W} \quad (3.7a) \quad -7-$$

$$\kappa_S = \frac{k_S'}{k_S} \quad (2.24) \quad -8-$$

$$\gamma_S = \frac{\rho_S' c_S'}{\rho_S c_S} \quad (3.7b) \quad -9-$$

$$\ell_W = \frac{L_W'}{L_W} \quad (2.23a) \quad -10-$$

$$\ell_f = \frac{L_f'}{L_f} \quad (2.23b) \quad -11-$$

$$\delta = \frac{\Delta T'}{\Delta T} \quad (2.22a) \quad -12-$$

$$\tau = \frac{\text{model time}}{\text{real time}} \quad -13-$$

based on the basic parameters of the gases

$$t = \frac{T_0'}{T_0} \quad (2.22b) \quad -14-$$

$$p = \frac{p_0'}{p_0} \quad (2.26) \quad -15-$$

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where the primed variable applies to the small-scale model and the unprimed variable applies to the full-scale model. A subscript of s applies to the gas; the subscript w applies to all the material. The basic parameters are:

- k = thermal conductivity ($\text{Wm}^{-1}\text{K}^{-1}$)
- ρ = density (kg m^{-3})
- C = specific heat per unit mass (in a gas $C = C_p$) ($\text{Jg}^{-1}\text{K}^{-1}$)
- L_w = thickness of the wall (m)
- L_f = other dimensions of the wall which affect convective heat transfer (m)
- T_o = baseline absolute temperature (K)
- P_o = baseline absolute pressure (atm)
- ΔT = temperature change (K)

Before proceeding it is useful to point out that the product of ρC is the specific heat per unit volume and the thermal diffusivity of the wall material is

$$\alpha = \frac{k}{\rho C} \quad -16-$$

Thus

$$a_w = \frac{\alpha'}{\alpha} = k_w^{-1} \gamma_w \quad -17-$$

Equations 1 through 5 relate the parameters of the walls and gas to one another. One can go one step farther and relate ℓ_f to ℓ_w . Let us assume that

$$\Delta = \frac{\ell_w}{\ell_f} \quad -18-$$

When perfect scaling occurs $\Delta = 1$. The factor Δ has a second-order effect on the dynamic similitude. In order to increase the latitude of conditions over which one might model, it might be useful to allow Δ to become a variable between 0.5 and say 2.5.

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Equations 1 through 5 would take the following form if a variable Δ is allowed.

$$\ell_f = \kappa_S t^{-5/2} \quad -2-$$

$$\kappa_W = \Delta \kappa_S t^{1/2} \quad -19-$$

$$(\gamma_W \Delta) = (\gamma_S P) t^{-1} \quad -20-$$

$$\delta = (\gamma_S P)^{-2} \kappa_S^{-1} t^{23/2} \quad -21-$$

$$\tau = (\gamma_S P) \kappa_S t^{-13/2} \quad -22-$$

From equation 2, it is clear that ℓ_f is related to κ_S and t but no other gas properties. If one substitutes equation 2 into equations 19, 21 and 22 the following forms for κ_W , δ , and τ result:

$$\kappa_W = \Delta \ell_f t^3 \quad -23-$$

$$\delta = (\gamma_S P)^{-2} \ell_f^{-1} t^9 \quad -24-$$

and

$$\tau = (\gamma_S P) \ell_f t^{-4} \quad -25-$$

From 24 and 25 it is clear that there is antagonistic relationship between τ and δ which takes the following form:

$$\delta = \tau^{-2} \ell_f t \quad -26-$$

From 26 it is clear that the smaller the value of τ the larger δ is. From a practical standpoint, δ is limited to values of

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about 5 or less. As one shall see later in this report, the lower limit value for ℓ_f is about 0.10. From equation 26 one can see that the lower limit value for τ is around 0.14. (This value is higher than was desired by the authors of LBL-13262.) There is an alternative form for τ which is given as follows:

$$\tau = \ell_w^2 \kappa_w^{-1} \gamma_w \quad -27-$$

Equations 19 through 22 are expressed in terms of the properties of the gas. These equations can also be stated in terms of properties of the walls. This, in many cases, is a more useful form since the values of κ_w and γ_w are more fixed by material properties. Restating equations 19 through 22 and equation 2, the following set of equations results:

$$\ell_f = (\kappa_w \Delta^{-1}) t^{-3} \quad -28-$$

$$\kappa_s = (\kappa_w \Delta^{-1}) t^{-1/2} \quad -29-$$

$$(\gamma_s P) = (\Delta \gamma_w) t \quad -30-$$

$$\delta = (\Delta \gamma_w)^{-2} (\kappa_w \Delta^{-1})^{-1} t^{10} \quad -31-$$

$$\tau = (\Delta \gamma_w) (\kappa_w \Delta^{-1}) t^{-6} \quad -32-$$

For the sake of completeness one can restate equations 29, 31, 32, into a different form using equation 28. The new forms will be similar in structure to equations 23, 24 and 25. The following forms result:

$$\kappa_s = \ell_f t^{5/2} \quad -33-$$

$$\delta = (\Delta \gamma_w)^{-2} \ell_f^{-1} t^7 \quad -34-$$

$$\tau = (\Delta \gamma_w) \ell_f t^{-3} \quad -35-$$

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Equations 34 and 35 can be manipulated to yield equation 26.

The value of κ_s for the modeling gas can be made continuously variable from 0.21 to well over 2.2. One will note that γ_s is always used in conjunction with the pressure ratio P (see eqs. 20, 21, 22, 24 and 25). The value of $(\gamma_s P)$ is also continuously variable from 0 to rather high values (say 10). All that is required is a vacuum and pressure-tight vessel for the gas. The selection of a modeling gas is not the problem in setting up a modeling experiment.

The values of κ_w and γ_w are not continuously variable. They are a function of the thermal conductivity and volume specific heat of the walls to be modeled and the kinds of materials available for the scale model. There is far less control of κ_w and γ_w than there is for κ_s and $(\gamma_s P)$. The availability of suitable materials for the model walls has a strong effect on the values of t and δ which can be used. It is this limitation of suitable materials for the model walls which sets the lower limit values of l_f and τ to values which are higher than those espoused in *Reference 1*.

2. Characterization of building walls and materials used in building and model walls

The walls, roof, floors and ceiling of buildings are often composite structures consisting of a number of materials. These materials are often arranged so that the dense high specific heat per unit volume materials are sandwiched around low density low specific heat per unit volume materials, or no material at all. An example is the typical stud wall found in buildings all over the United States. One can model walls by looking at the materials which go into the wall (Note: my reference to walls or wall material refers to materials used in ceilings, roofs and floors as well) or one can look at the wall as an architect might using an average R value and an average specific heat per unit volume.

Values of density, thermal conductivity, specific heat per unit volume and upper limit operating temperature are given for various materials in TABLE I. The values given in TABLE I are approximate. One should have thermal conductivity data and specific heat data for the particular material which is being used. The upper limit temperature given is the upper limit operating temperature of these materials in an air atmosphere. For example, copper can be used in an inert gas to temperatures up to 1300K instead of 650K. One might be able to use

TABLE 1. The density, thermal conductivity, volume specific heat and upper operating temperature for a variety of materials which can be used in building or models of buildings^{4,5,6,7}

	density (kg m ⁻²)	thermal conductivity (Wm ⁻¹ K ⁻¹)	volume specific heat (Jm ⁻³ K ⁻¹)	upper limit temperature (K)
METALS				
Aluminum (6061)	2700	171	2.43×10^6	800
Steel (carbon)	7800	47	3.51×10^6	1650
Copper (OFHC)	8900	391	3.44×10^6	650
Lead	11300	34	1.44×10^6	450
BUILDING MATERIALS				
Light concrete	1400	0.44	1.23×10^6	1000
Heavy concrete	2250	1.10	2.16×10^6	1000
Light brick	1800	0.33	1.04×10^6	1000
Heavy brick	2000	1.22	2.07×10^6	1000
Dry adobe	1500	1.51	2.40×10^6	750
Glass	2400	0.60	1.80×10^6	1400
Gypsum board	1800	0.67	1.97×10^6	500
Asphalt shingle	1000	0.55	0.90×10^6	350
Plywood	800	0.14	1.40×10^6	500
Pine or fir	550	0.15	1.30×10^6	450
PLASTICS				
Hard rubber	1140	0.14	1.90×10^6	400
Nylon 6	1150	0.29	1.92×10^6	420
Unfilled epoxy	1160	0.35	2.10×10^6	450
Teflon	1200	0.30	$\sim 2.00 \times 10^6$	530
Phenolic resin	1320	0.26	1.85×10^6	420
NEMA G-10	1800	0.29	2.77×10^6	450
INSULATION MATERIAL				
Balsa wood	160	0.050	0.38×10^6	420
Glass-foam	135	0.050	0.10×10^6	750
Asbestos cotton	140	0.050	0.11×10^6	950
Corkboard	200	0.040	0.38×10^6	420
Fiberglass (TYP)	150	0.036	0.11×10^6	450
Styrofoam	100	0.027	0.17×10^6	400
Urethane foam	80	0.018	0.13×10^6	400

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wood and some of the plastics to temperatures up to 500K. The upper limit temperature for styrofoam and urethane foam is rather low if these materials are to be successfully used in a building model. A search for foam-type materials which will insulate at temperatures of 500K or more is worth pursuing.

An alternative way of determining the characteristics of a building wall is to use the R value method used by architects and building inspectors. A wall, a floor, a roof, a ceiling, a door, or a window structure is rated in terms of its resistance to heat (its R value). One can convert the R value of a wall to thermal conductivity (in SI units) by using the following relationship if the wall thickness L_w is known:

$$k \approx \frac{5.674 L_w}{(R - 0.8)} \quad -36-$$

where k is the wall average thermal conductivity in SI units ($\text{Wm}^{-1}\text{K}^{-1}$); L_w is the thickness of the wall in SI units; and R is the R value of the wall given in English units:

$$R \approx \frac{1}{U} \quad (\text{BTU}^{-1}\text{hr F ft}^2) \quad -37-$$

The 5.674 factor is the conversion from $\text{BTU hr}^{-1} \text{F}^{-1} \text{ft}^{-2}$ to $\text{Wm}^{-2} \text{K}^{-1}$. The 0.8 factor in the denominator is approximately the thermal resistance of the air boundary layers on either side of the wall.

The ρC product can be found by volume averaging the constituent component in the wall and applying the value of ρC given in TABLE 1. TABLE 2 gives one the thermal conductivity k and ρC product of twenty types of wall, roofs, ceiling, floors, doors and windows used in the United States. Figure 1, a map of the United States, shows the recommended (building code) R values for walls, ceilings and floors in new construction in various parts of the United States. This figure comes from *HOME RETROFIT MANUAL* published by the Community Services Administration and the National Bureau of Standards.⁷

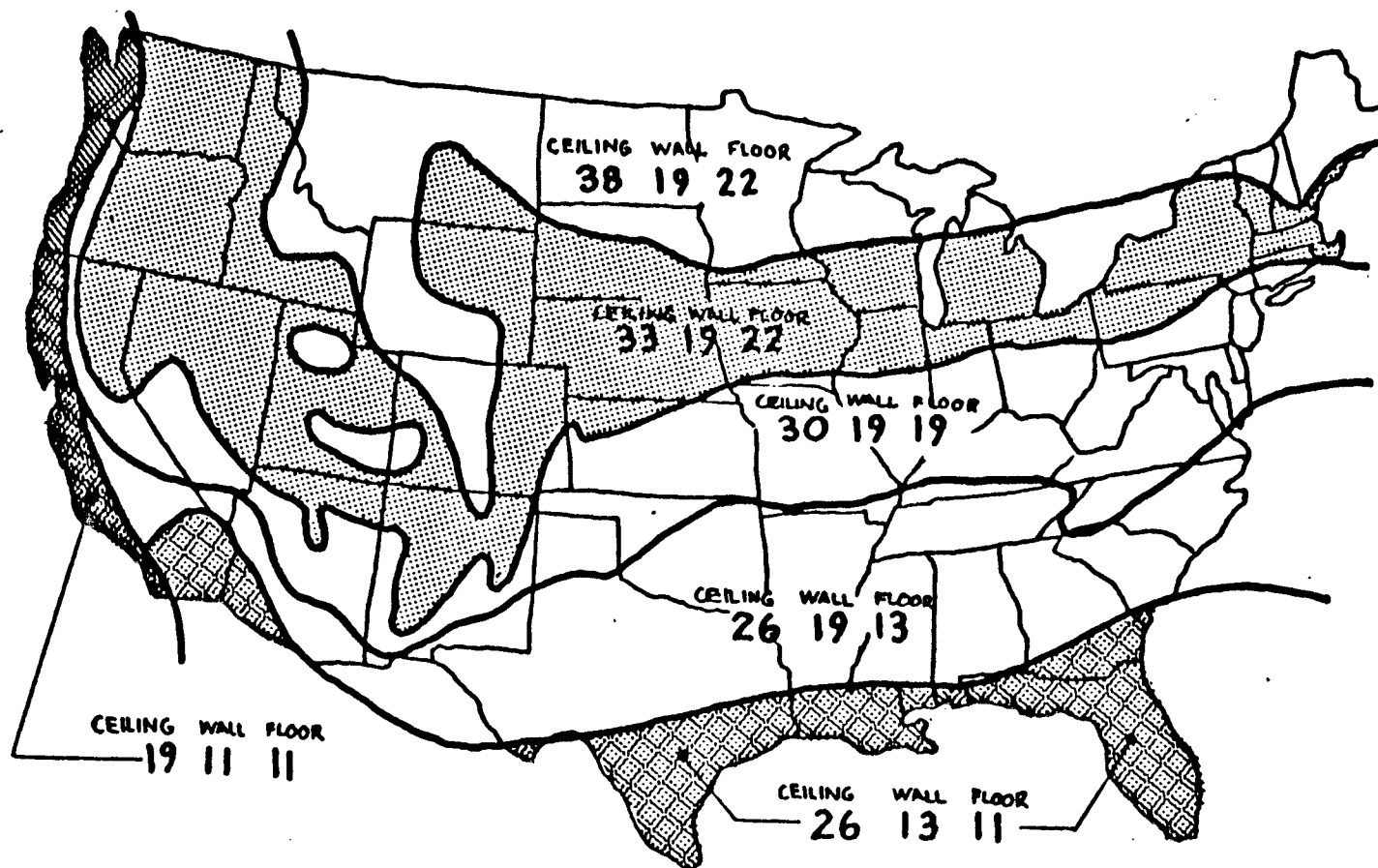


FIGURE 1. Recommended R values for ceilings, walls and floor for New Homes in various parts of the United States. (This map comes from the *HOME RETROFIT MANUAL*.⁷)

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The following cases describe various types of construction used in American buildings. These cases numbered 1 through 20 describe the cases given in TABLE 2. (NOTE: most of these cases come from the *HOME RETROFIT MANUAL*.⁷)

Uninsulated walls

Case 1 2x4-inch studs, 16 inches O.C., with lapped wood siding and 0.5-inch asphalt on the outside and 0.5-inch gypsum board inside ($R = 4.5$)

Case 2 2 x 4-inch studs, 16 inches O.C., with 4-inch face brick and 0.5-inch asphalt sheathing outside and 0.5-inch gypsum board inside ($R = 4.2$)

Case 3 4-inch heavy face brick, 2-inch air space, and 4-inch heavy brick on the inside ($R = 2.7$)

Case 4 Typical west coast interior wall 0.5 gypsum board, 2 x 4-inch studs, 16 inches O.C., with 0.5-inch gypsum board on other side ($R \approx 3.0$)

Insulated Walls

Case 5 Case 1 with space between the studs filled with fiberglass ($R = 14$)

Case 6 Case 1 with space between the studs filled with urethane foam ($R = 26$)

Case 7 Case 2 with space between the studs filled with fiberglass ($R = 13.6$)

Case 8 Case 3 with airspace filled with fiberglass ($R = 9$)

Case 9 Case 3 with airspace filled with urethane foam ($R = 18.7$)

Case 10 Polar wall 0.5-inch wood, 4-inch urethane foam and 0.5-inch gypsum board ($R = 42$)

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Ceilings and Roofs

- Case 11 0.375-inch asphalt shingle, 0.625-inch plywood roof, 2 x 4 studs 16 inches O.C., and 0.5-inch gypsum board (R = 4.5)
- Case 12 Case 11 except space is filled with fiberglass (R = 14)
- Case 13 Case 11 except space is filled with urethane foam (R = 26)
- Case 14 Standard west coast ceiling with 2 x 4-inch studs, 16 inches O.C., with 0.5-inch gypsum board (R = 1.6)
- Case 15 Standard west coast ceiling with 6 inches of fiberglass on the ceiling (R = 20.6)

Floors

- Case 16 2 x 10-inch joists, 16 inches O.C., with hardwood and wood subfloor, 1-inch thick, and an 0.5-inch gypsum board ceiling below (R ≈ 5)
- Case 17 Case 16 except the space between the joists is filled with fiberglass (R ≈ 31)

Doors and Windows

- Case 18 1.75-inch thick solid-core door (R ≈ 2.8)
- Case 19 Single glazed window (R ≈ 0.86)
- Case 20 Double-glazed window (R ≈ 1.4)

In TABLE 2, one can find that the walls range in thermal conductivity from masonry walls with a $k = 0.361 \text{ Wm}^{-1}\text{K}^{-1}$ (Case 2 which is still found in the eastern United States) to well-insulated frame house walls $k = 0.029 \text{ Wm}^{-1}\text{K}^{-1}$ (Case 6 which might be used in new construction in the midwest) and polar-type wall where $k = 0.017 \text{ Wm}^{-1}\text{K}^{-1}$ (Case 10 for modular buildings built in Northern Canada or Alaska). One can also see that insulation has a marked effect on the thermal conductivity of ceilings and floors. From TABLE 2, one can see the range of values of k (from 0.017 to 0.72 $\text{Wm}^{-1}\text{K}^{-1}$) and ρC (from 3.4×10^5 to $17 \times 10^5 \text{ Jm}^{-3}\text{K}^{-1}$) used in house construction in the United States which might be thermally modeled by the dynamic similitude process.

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TABLE 2. Thickness, thermal conductivity and specific heat for composite walls, roofs, floors, doors and windows

Case No.	Thickness (mm)	Thermal conductivity ($\text{Wm}^{-1}\text{K}^{-1}$)	Volume specific heat ($\text{Jm}^{-3}\text{K}^{-1}$)
1	127	0.195	5.3×10^5
2	216	0.361	12.1×10^5
3	254	0.307	11.5×10^5
4	114	0.295	5.5×10^5
5	127	0.055	6.0×10^5
6	127	0.029	6.1×10^5
7	216	0.097	12.8×10^5
8	254	0.176	16.7×10^5
9	254	0.080	16.8×10^5
10	127	0.017	4.3×10^5
11	127	0.194	5.5×10^5
12	127	0.055	6.2×10^5
13	127	0.029	6.3×10^5
14	102	0.721	3.9×10^5
15	165	0.047	3.5×10^5
16	292	0.394	3.4×10^5
17	292	0.054	4.2×10^5
18	44.5	0.126	14.0×10^5
19	6.4*	0.6*	18.0×10^5
20	19.1**	0.180**	12.0×10^5

* 0.25-inch-thick glass; $k = 0.6 \text{ Wm}^{-1}\text{K}^{-1}$

** 2 x 0.25-inch-thick glass with 0.25-inch air space

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In order to model the walls, ceilings and floors given in TABLE 2, one can use a combination of plastic or metals with a central core which is made from an insulator. (As an example, the model walls can be made from NEMA G-10 sheets with styrofoam sandwiched between them.) In addition highly insulated walls can be modeled with walls in the model which have a Δ greater than one. The importance of Δ and other factors such as δ and τ is given in the next section. Section 4 of this report presents a procedure for selecting the model wall materials and the modeling gas.

3. The relative importance of various factors on the modeling process

This section gives the basis for selection of model wall materials, the factor Δ for the walls, and the gas used in the modeling process. The following factors are looked at in terms of their importance in the modeling process: 1) t the base line temperature ratio, 2) P the base line pressure ratio, 3) Δ the ratio of the wall thickness sealing factor ℓ_w to the wall length scaling factor ℓ_f , 4) δ the temperature change ratio, and 5) τ the ratio of modeling time to real time.

a) The effect of t on the modeling process

By equation 14 the base line temperature of the model T_0' can be related to the base line temperature T_0 of the building to be modeled. T_0 is usually about 20°C (293.2K). Equation 2 equates t to κ_s (the ratio of the modeling gas thermal conductivity to the thermal conductivity of air) and ℓ_f the model scaling parameters. The lower limit for κ_s is 0.21 which is Xenon. κ_s can be varied from 0.21 to say 2 almost at will. The value of t cannot be varied at will. If a t of 1.35 is used, the base line temperature of the model T_0' would be 395.8K. A number of attractive materials which could be used in model walls have upper temperature limits of 400 - 420K. δ is highly dependent on t ; if t is high, δ is also large.

It is desirable to keep t less than 1.35. Even if the model gas has the minimum κ_s , the minimum practical scaling factor ℓ_f is about 0.1. We shall see later that one can model a variety of structures at one-tenth scale. However, one-eighth or one-fifth scale is better. It is always desirable to keep t small if possible.

b) The effect of P on the modeling process

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From equations 20, 21, 22, 24 and 25 one can see that P is always used in conjunction with γ_s . The $\gamma_s P$ product is set by Δ , t and γ_w . It is clear that testing should be done in a chamber so that P can be set. The minimum value for γ_s is about 0.7. γ_w is often around 1.5 to 2.0 and the maximum value of t should be 1.35. As a result the maximum pressure ratio a test chamber should be designed for is around 6 atm. If one-quarter or one-fifth scale models are tested, there is more latitude in γ_s . As a result, test pressure can be lower. Provided one has designed the proper test vessel, P is not an important scaling factor.

c) The effect of Δ on the modeling process

The relationship of Δ to κ_s , κ_w and t is given in equations 19 and 29. The relationship between Δ , κ_w , ℓ_f and t is given by equation 23. Perfect modeling occurs when $\Delta = 1$. When Δ is greater than one, the modeling is imperfect. However, the error is not too important because heat flow through a wall is for the most part one-dimensional.

Typical wall material in the model might have κ_w from 0.2 to 0.8 depending on the wall being modeled. A poorly insulated wall can be modeled using wall material with a low thermal conductivity thus creating a low value of κ_w . If the wall to be modeled is very well insulated, it becomes very difficult to find a model wall material that permits κ_w to be much less than one. One can make up for this deficiency by making Δ greater than one. Equation 23 shows that one is more likely to have Δ greater than one the smaller the scale factor ℓ_f is.

d) The effect of δ on the modeling process

δ can be treated purely as a dependent variable which results from design decisions during the modeling process. The value of δ is affected greatly by the selection of t , the modeling base temperature ratio. δ has a large effect on the peak temperature the models see during thermal modeling. This peak temperature can be calculated using the following expressions:

$$T_{\max} = T_0 t + \Delta T_{\max} \delta \quad -38-$$

where $T_{\max}$ is the maximum temperature in the model during thermal tests; T_0 is the base temperature of the full-scale building; $\Delta T_{\max}$ is the maximum positive temperature excursion seen by the full-scale building; t is the base temperature ratio defined in equation 14; and δ is the temperature excursion ratio defined by equation 12.

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Equations 38 and 24 can be rearranged using Equations 2 and 24 to yield the following forms:

$$T_{\max} = T_0 \left[\kappa_S^{0.4} \ell_f^{-0.4} \right] + \Delta T_{\max} \left[(\gamma_S P)^{-2} \ell_f^{-4.6} \kappa_S^{3.6} \right] \quad -39-$$

and

$$\delta = (\gamma_S P)^{-2} \ell_f^{-4.6} \kappa_S^{3.6} \quad -40-$$

It is clear from equations 39 and 40 that for a given value of ℓ_f , one must try to minimize κ_S in order to minimize $T_{\max}$ and δ .

e) The effect of τ on the modeling process

In *Reference 1* Richardson and Berman¹ stated that it was desirable to minimize τ . This should be re-examined in terms of the entire modeling process. τ , in my opinion, is in reality not an important parameter. In most scale models where ℓ_f is less than one τ will also be less than one. The values of τ will in many cases be of the same order as ℓ_f . The time required to measure the response of the model to various thermal stimuli will be small compared to the time involved in building the model and setting up the experiment.

The mini-computer permits many data points to be taken simultaneously without human assistance. It can be argued that τ can be as large as one without grossly affecting the experiment.

τ is a dependent variable. It can be calculated using equations 13, 22, 25, 32 and 35. As stated previously and illustrated by equation 26, there is an antagonistic relationship between τ and δ . The value of δ affects the material selection during modeling. τ does not affect this material selection. Unlike Richardson and Berman¹ I advocate allowing τ to assume whatever value results from playing around with the other parameters.

Using equations 25 and 2 one can come up with a form for τ which is similar to equation 40. This equation takes the following form:

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$$\tau = (\gamma_s P) \ell_f^{2.6} \kappa_s^{-1.6}$$

-41-

From equation 41, it is clear that reducing κ_s will increase τ for a given value of ℓ_f . This is a small price to pay for reducing δ and $T_{\max}$. (Note equations 31 and 32 present τ and δ in terms of wall parameters. The values of τ and δ gotten from wall parameters must be the same as those gotten from gas parameters, equations 21, 22, 24, 25, 40 and 41, or the model is not valid.)

4. The process of selection of modeling parameters and the selection of wall materials and modeling gas

The procedure for selecting the model parameters starts with the selection of ℓ_f . In general ℓ_f should be greater than 0.1. The larger ℓ_f is the better in terms of the assembly of the model and in terms of having enough flexibility to model a wide variety of conditions which may coexist within the model. When ℓ_f is smaller than 0.1, the value of δ becomes large and it becomes increasingly difficult to find wall materials which have the desired properties and yet withstand the elevated temperature.

Rule 1: Keep ℓ_f at 0.1 or above. The larger ℓ_f is the more flexible the modeling process becomes.

Once ℓ_f has been selected a trade off between κ_s and t can be made using equations 2 or 33. Remember κ_s can be as low as 0.21 and remember the upper limit for t is around 1.35. There is a relationship between κ_s and κ_w as given by equations 19 or 28. The relationship between κ_w and ℓ_f is given by equation 23 or 28. It is, in general, better to make Δ greater than one than to make κ_s or t too large.

Rule 2: Minimize κ_s in order to minimize t and δ . It may be better to make κ_s larger than the minimum value in order to have larger available values of γ_s .

Rule 3: It is better to make Δ greater than one than it is to make κ_s too large.

Determine the thermal conductivity of the full-scale wall by using equation 36 or TABLES 1 and 2. Once the thermal conductivity of the full-scale wall or walls has been determined, κ_w and Δ can be

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juggled so that a κ_W can be obtained that is in the range of material which can be used in a model. Each wall may have a different κ_W and Δ as long as equation 19 is satisfied. The product of $\Delta \gamma_W$ must be the same for all walls so that equation 20 is satisfied for all walls. The use of two different materials in the model wall plus a variation of Δ should permit both equations 19 and 20 to be satisfied simultaneously.

Rule 4: In a system of multiple types of walls the product of $\kappa_W \Delta^{-1}$ must be the same for all walls.

Rule 5: In a system of multiple types of walls the product of $\gamma_W \Delta$ must be the same for all walls.

Rule 6: The $\gamma_W \Delta$ product for the walls should be set according to the maximum model temperature criteria which can be calculated using equations 39 and 20. (Note: other equations can be used as well.)

The process of setting $\gamma_W \Delta$ from $T_{\max}$ (the maximum allowable temperature in the model), T_0 (the mean temperature at full scale), and T (the maximum temperature variation at full scale) can be given as follows:

$$(\gamma_W \Delta) = \left[\frac{\ell_f^{-1} t^7}{\delta} \right]^{1/2} = \left[\frac{\ell_f^{-3.8} \kappa_S^{2.8}}{\delta} \right]^{1/2} \quad -42-$$

where

$$\delta = \frac{T_{\max} - T_0 t}{\Delta T} \quad -43-$$

and where ℓ_f is the overall model scaling factor, t is the ratio of mean temperatures between the model and the full-scale structure, κ_S is the ratio thermal conductivities of the model gas to air.

A wall can be modeled by a composite structure made from two materials. One material should have a low thermal conductivity such as styrofoam or urethane foam. The other material can have a much higher thermal conductivity. The thermal conductivity required for the model wall should lie between the two. Both Rule 4 and Rule 5 must be satisfied simultaneously for each type of wall in the model. Three factors

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can be used in insuring simultaneous application of Rules 4 and 5. They are: x_1 the fraction of the high thermal conductivity material; x_2 the fraction of the low thermal conductivity material; and Δ the ratio of ℓ_w over ℓ_f (the relative thickness of the wall model factor compared to the overall model factor).

The following equations solved simultaneously will yield x_1 , x_2 and Δ :

$$\Delta \left[x_1 (\rho C)_1 + x_2 (\rho C)_2 \right] = (\Delta \gamma_w) (\rho C)_0 \quad -44-$$

$$x_1 = \frac{R_2 - R_0}{R_2 - R_1} \quad -45$$

$$x_2 = \frac{R_0 - R_1}{R_2 - R_1} \quad -46-$$

where

$$R_1 = \frac{1}{k_1} \quad -47-$$

$$R_2 = \frac{1}{k_2} \quad -48-$$

$$R_0 = \frac{1}{\Delta \kappa_s t^{1/2} k_0} \quad -49-$$

where k_0 is the thermal conductivity of the full-scale wall; k_1 is the thermal conductivity of the higher thermal conductivity material in the model wall; and k_2 is the thermal conductivity of the lower thermal conductivity material in the model wall. $(\rho C)_0$ is the volume specific heat of the full-scale wall; $(\rho C)_1$ is the volume specific heat of constituent 1 of the model wall; and $(\rho C)_2$ is the volume specific heat of constituent 2 of the model wall. κ_s and t are previously defined. If the materials for the model walls have been properly selected, x_1 and x_2 will both be less than 1 and Δ will not be too large (in most cases Δ should be less than 2.5).

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Once Δ , x_1 and x_2 have been calculated, one can calculate κ_W and γ_W for the model walls. Then one can check to see that the $(\gamma_W \Delta)$ and $(\kappa_W \Delta^{-1})$ products are the same for all walls. The method of calculating κ_W and γ_W for the wall is as follows:

$$\kappa_W = \frac{\left[\frac{x_1}{k_1} + \frac{x_2}{k_2} \right]^{-1}}{k_0} \quad -50-$$

and

$$\gamma_W = \frac{x_1 (\rho C)_1 + x_2 (\rho C)_2}{(\rho C)_0} \quad -51-$$

where k_0 , k_1 , k_2 , x_1 , x_2 , $(\rho C)_0$, $(\rho C)_1$, and $(\rho C)_2$ have been defined previously.

The selection of a modeling gas is determined by κ_S and the $(\gamma_S P)$ product. The $(\gamma_S P)$ production is determined by the selected value of $(\gamma_W \Delta)$ for the walls. In general the lower $\gamma_W \Delta$ the lower $\gamma_S P$ is. For all practical purposes κ_S is bounded by 0.21 as a lower limit (this is the κ_S for Xenon) and a bit over 5 as an upper limit (this is the κ_S for helium). In all practical cases κ_S is less than 0.6. $(\gamma_S P)$ is determined by the γ_S of the gas and the pressure rating of the test vessel. In general, the theory works best if simple gases are used. Simple gases by their very nature have low values of γ_S .

Rule 7: The vessel for testing the model should have a high pressure rating. The Dynamic Similitude theory works best with gases which have a low value of γ_S .

If one has a pressure vessel for testing the model, one can set γ_S to 0.7. This permits the Nobel gases to be used as model gases. A mixture of Xenon and Argon is the technical choice as long as one has a vessel which will withstand pressures up to say 6 atmospheres. The Nobel gases fit the theory very well. The temperature relationship of thermal conductivity, viscosity and specific heat is very close to that assumed for the theory of Richardson and Berman.¹

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If one has a test vessel which is pressure limited, the value of γ_S must be higher than 0.7. When one desires a larger γ_S one can mix the two Nobel gases with a third gas such as one of the refrigerants. Gases such as R-11, R113, R114 and R115 might be attractive as the third component to increase γ_S . (κ_S will be above the minimum value if a third component is added.) Gases such as SO_2 , CS_2 , CH_3Br , CH_3I and SF_6 may be of interest as well. (Note CS_2 , CH_3Br and CH_3I have a high boiling point and they are poisonous. Proper care should be exercised when using these gases and other gases such as SO_2 .)

5. The scaling of solar flux, heat input and infiltration

There is one matter which has not been discussed. This is the matter of thermal sources and sinks. Heat enters or leaves a structure through three mechanisms which are: (a) radiative flux (solar flux is an example), (b) a point source heater or cooler, and (c) air infiltration into or out of the building. Let us define the following:

$$\sigma = S'/S \qquad -52-$$

$$\eta = H'/H \qquad -53-$$

and

$$\dot{n} = \dot{N}'/\dot{N} \qquad -54-$$

where S = the solar flux or thermal radiation into or out of the building (Wm^{-2})

H = heat rate for a point source heater or cooler (W)

$\dot{N}$ = fluid volume changes per unit time

The non-primed values for the full-scale building and the primed values apply to the model.

The modeling laws require the following values for σ , η , and $\dot{n}$:

$$\sigma = \delta t^3 \qquad (2.35) \qquad -55-$$

$$\eta = \delta \ell_f^2 t^3 \qquad (2.36) \qquad -56-$$

and

$$\dot{n} = (\gamma_S P)^{-1} \ell_f^{-2} t^3 \qquad -57-$$

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NOTE: Equation 57 is similar to (2.37) in *Reference 1*. I think (2.37) is in error because γ_s must always be multiplied by P to get the correct volume specific heat ratio (which is dependent on ρ'/ρ which is in turn dependent on P_0'/P_0).

When equations 55 and 56 are stated in terms of the gas parameters using 24, the following forms result:

$$\sigma = (\gamma_s P)^{-2} \ell_f^{-1} t^{12} \quad -58-$$

and

$$\eta = (\gamma_s P)^{-2} \ell_f t^{12} \quad -59-$$

When equations 55, 56 and 57 are stated in terms of the wall parameters using 34, the following forms result:

$$\sigma = (\Delta\gamma_w)^{-2} \ell_f^{-1} t^{10} \quad -60-$$

$$\eta = (\Delta\gamma_w)^{-2} \ell_f t^{10} \quad -61-$$

and

$$\dot{n} = (\Delta\gamma_w)^{-1} \ell_f^{-1} t^2 \quad -62-$$

As a final statement, one can also put σ and η in terms of τ using 55, 56 and 24. These forms can be stated as follows:

$$\sigma = \tau^{-2} \ell_f t^4 \quad -63-$$

and

$$\eta = \tau^{-2} \ell_f^3 t^4 \quad -64-$$

It is interesting to note that solar flux through a window can be modeled by a non-convective heating radiant heat source which fits the scaling laws given by 55, 58, 60 or 63. A space heater can be modeled by a simple electrical resistance heater where the heat input obeys the scaling laws given by 56, 59, 61 or 64. The modeling of infiltration is more difficult because one can't just scale the cracks in a house. Infiltration would have to be modeled using

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holes of set diameter which controls the infiltration gas flow through the model.

6. An example of the scaling process on a passive solar structure

In order to demonstrate the process of modeling, I have chosen a passive solar structure.

The structure shown in Figure 2 is typical of what might be used in a passive solar system. The structure to be modeled consists of two rooms; one collects the solar energy, the other is a living space. The structure shown in Figure 2 is about 8m wide and 5m high. The length is not important. (In fact, none of the dimensions is important to the modeling process except when one considers the vessel which might contain the model.) It is assumed there is only solar flux through the window on the right. It is also assumed there is no infiltration of internal space heating.

Three types of walls are considered. Wall Number 1 is 0.6m thick and made from heavy concrete. This wall is part of the dividing structure between the two rooms and it is the floor of the solar greenhouse section. This wall has a high specific heat per unit volume and a high thermal conductivity. Wall Number 2 is the main insulating wall, roof and floor for the structure; it is represented by Case 5 in TABLE 2. This wall has a low thermal conductivity and a low specific heat per unit of volume. Wall #3 is a typical interior wall for a west coast home and is represented by Case 4 in TABLE 2. This wall has an intermediate thermal conductivity and a low specific heat per unit volume. The characteristics of the three walls are as follows:

WALL #1: Heavy concrete wall ~2-feet thick (see TABLE 1)

$$\begin{aligned}\rho &= 2250 \text{ kg m}^{-3} \\ \rho C &= 2.16 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1} \\ k &= 1.10 \text{ Wm}^{-1}\text{K}^{-1} \\ L_w &= 0.6\text{m}\end{aligned}$$

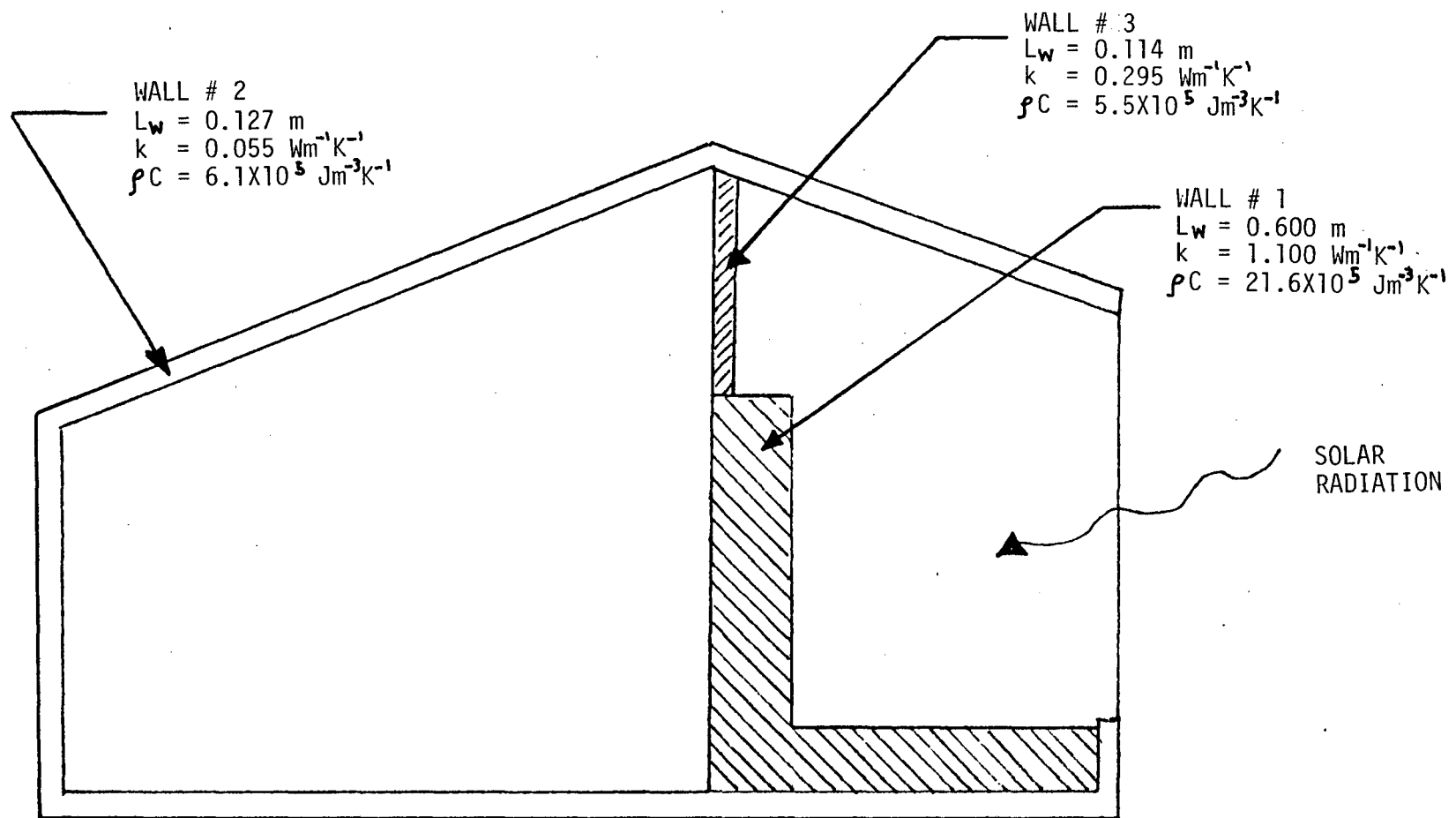


FIGURE 1. A two-room Passive Solar Structure with three different types of walls

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WALL #2: Case 5 wall R-14 wall 5 inches thick (see TABLE 2)

$$\begin{aligned}\rho &= 430 \text{ kg m}^{-3} \\ \rho C &= 0.60 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1} \\ k &= 0.055 \text{ Wm}^{-1}\text{K}^{-1} \\ L_w &= 0.127\text{m}\end{aligned}$$

WALL #3: Case 4 wall R-3 wall 4.5 inches thick (see TABLE 2)

$$\begin{aligned}\rho &= 450 \text{ kg m}^{-3} \\ \rho C &= 0.55 \times 10^5 \text{ Jm}^{-3}\text{K}^{-1} \\ k &= 0.295 \text{ Wm}^{-1}\text{K}^{-1} \\ L_w &= 0.114\text{m}\end{aligned}$$

The passive solar structure is assumed to be in air at a pressure of 1 atm. The average temperature assumed is the desired temperature of the room: 20°C (293.2K). The assumed allowable temperature swing of the concrete is 20°C so that its peak temperature is 40°C. Outside the house the temperature could be as low as 0°C so that the plus and minus swing is 20K degrees.

Let us go through the modeling process in a step-by-step fashion.

STEP #1 Determine the overall scale factor for the model

The scale factor is an arbitrary decision. Since there is a wide variety of wall properties, it is useful to make ℓ_f larger than 0.1 so that there is flexibility in the modeling process. In this case ℓ_f was arbitrarily set to 0.167.

STEP #2 Choose a value for κ_s ; set t and select the gases

The selection of κ_s is arbitrary. It is desirable that t be near one. κ_s was arbitrarily set to 0.25. Using equation 2, t becomes 1.1751. Since κ_s is low, one of the gas components will be Xenon. Therefore I arbitrarily use an Argon-Xenon gas mixture.

$$\begin{aligned}\ell_f &= 0.167 \\ \kappa_s &= 0.25 \\ t &= 1.1751 \\ \text{gas mixture:} & \quad \text{Xe} = 0.916 \\ & \quad \text{Ar} = 0.084 \\ \gamma_s &= 0.705\end{aligned}$$

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STEP #3 Set the maximum temperature in the system

This decision is arbitrarily made based on materials one might want to use in the model. Since urethane foam or styrofoam are attractive materials to use, the upper limit temperature $T_{\max}$ was set to 400K (see TABLE 1).

$T_{\max} = 400K \quad (126.8C)$
 $T_0 = 293.2K \quad (20.0C)$
 $\Delta T = 20K$

Using equations 43, 12 and 14

$\delta = 2.775$
 $T_0' = 344.5K \quad (77.3C)$
 $\Delta T' = 55.5K$

STEP #4 Set $\gamma_S P$ and $\gamma_W \Delta$ for the model gas and the model walls

From equations 20, 21 and 42 one can set the values of $\gamma_S P$ and $\gamma_W \Delta$ when one knows ℓ_f , κ_S and δ or κ_S , t and δ . Using these equations

$$(\gamma_S P) = [\kappa_S^{-1} t^{23/2} \delta^{-1}]^{1/2}$$

and

$$(\gamma_W \Delta) = (\gamma_S P) t^{-1}$$

one finds that

$(\gamma_S P) = 3.0362$
 $P = 4.3067$
 $P_0' = 4.3067 \text{ atm}$

and

$$(\gamma_W \Delta) = 2.5838$$

for all three wall types.

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STEP #5 Set the value of $\kappa_W \Delta^{-1}$ for the model walls

The value $\kappa_W \Delta^{-1}$ can be found using either equation 19 or 23 as long as one knows either κ_S and t or ℓ_f and t (from equation 2 also ℓ_f and κ_S)

$$\kappa_W \Delta^{-1} = 0.271$$

STEP #6 Selection of the materials for use in the walls of the model

Once $\Delta \gamma_W$ and $\kappa_W \Delta^{-1}$ are given for the walls (they are the same for all of the walls), one must choose the model wall materials judiciously in order to have a meaningful solution to the simultaneous equations 44, 45 and 46. If the solution is correct physically, x_1 and x_2 will be less than 1, and the sum of x_1 and x_2 will equal 1. Also required is a positive value of Δ . $(\rho C)_0$ and κ_0 are given for each of the three walls to be modeled. In order for equations 44, 45 and 46 to work, one of the values of $(\rho C)_1$ or $(\rho C)_2$ should be greater $(\rho C)_0$. The other value should be less $(\rho C)_0$. (This is not an absolutely necessary requirement but in most cases one gets a better result.) In addition $R_2 > R_0 > R_1$ if a correct solution is to be obtained. Let us look at the materials which may be used for the models of the three walls.

WALL #1: Concrete

$$(\rho C)_0 = 2.16 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1}, \kappa_0 = 1.1 \text{ Wm}^{-1}\text{K}^{-1}$$

After some searching the following materials were selected for this wall:

$$\text{Material 1: copper, } (\rho C)_1 = 3.44 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1} \\ k_1 = 391 \text{ Wm}^{-1}\text{K}^{-1}, R_1 = 0.0026 \text{ mKW}^{-1}$$

$$\text{Material 2: hard rubber, } (\rho C)_2 = 1.90 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1} \\ k_2 = 0.14 \text{ Wm}^{-1}\text{K}^{-1}, R_2 = 7.08 \text{ mKW}^{-1}$$

Material 1 could also be steel, Material 2 could be Balsa wood, epoxy, NEMA G-10, or foam insulation.

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WALL #2: Case 5, 5-inch-thick R-14 insulated house wall $(\rho C)_0 = 0.60 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1}$ and $k_0 = 0.055 \text{ Wm}^{-1}\text{K}^{-1}$

Since the product of $\kappa_{st}^{1/2}k_0$ is less than the thermal conductivity of the best foam insulation Δ must be greater than 1. Material 1 can be any plastic such as NEMA G-10. Material 2 must be a low conductivity foam insulation. Urethane foam is the best but other foams can be used.

Material 1: NEMA G-10, $(\rho C)_1 = 2.77 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1}$
 $k_1 = 0.29 \text{ Wm}^{-1}\text{K}^{-1}$ $R_1 = 3.45 \text{ mKW}^{-1}$

Material 2: styrofoam, $(\rho C)_2 = 0.17 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1}$
 $k_2 = 0.027 \text{ Wm}^{-1}\text{K}^{-1}$ $R_2 = 37.04 \text{ mKW}^{-1}$

Material 1 could be hard rubber, any plastic or even an insulator which has a ρC substantially over $0.6 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1}$.

WALL #3: Case 4, interior wall, R-3, 4.5-inches thick.
 $(\rho C)_0 = 0.55 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1}$ and $k_0 = 0.295 \text{ Wm}^{-1}\text{K}^{-1}$

In this case the product of $\kappa_{st}^{1/2}k_0$ is more than the thermal conductivity of all of the insulating materials given in TABLE 1. The same materials as in the previous case can be used.

Material 1: NEMA G-10, $(\rho C)_1 = 2.77 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1}$
 $k_1 = 0.29 \text{ Wm}^{-1}\text{K}^{-1}$, $R_1 = 3.45 \text{ mKW}^{-1}$

Material 2: styrofoam, $(\rho C)_2 = 0.17 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1}$
 $k_2 = 0.027 \text{ Wm}^{-1}\text{K}^{-1}$, $R_2 = 37.04 \text{ mKW}^{-1}$

STEP #7 Calculate Δ , x_1 and x_2 for each wall and reselect the wall materials if this proves to be necessary.

The selection of wall materials in the previous case created a design which will work. Δ , x_1 and x_2 can be solved using equations 44, 45 and 46. The value of ℓ_w , the scaling factor for wall thickness, can be found using equation 18 or 19. The value of κ_w is calculated using equation 1 or 19 when ℓ_w , Δ , t or κ_s are given. The solutions for the three walls are as follows:

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WALL #1: copper plus hard rubber

$$\begin{aligned}
 x_1 &= 0.744 \text{ copper} \\
 x_2 &= 0.256 \text{ hard rubber} \\
 \Delta &= 1.8326 \\
 k_W' &= 0.547 \text{ Wm}^{-1}\text{K}^{-1} \\
 \kappa_W &= 0.497 \\
 \kappa_W \Delta^{-1} &= 0.271 \text{ QED} \\
 (\rho C)_W &= 3.046 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1} \\
 \gamma_W &= 1.4102 \\
 \gamma_W \Delta &= 2.5843 \text{ QED} \\
 \ell_W &= 0.3060
 \end{aligned}$$

WALL #2: NEMA G-10 plus styrofoam

$$\begin{aligned}
 x_1 &= 0.203 \text{ NEMA G-10} \\
 x_2 &= 0.797 \text{ styrofoam} \\
 \Delta &= 2.2204 \\
 k_W' &= 0.0331 \text{ Wm}^{-1}\text{K}^{-1} \\
 \kappa_W &= 0.602 \\
 \kappa_W \Delta^{-1} &= 0.271 \text{ QED} \\
 (\rho C)_W &= 0.698 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1} \\
 \gamma_W &= 1.1633 \\
 \gamma_W \Delta &= 2.5831 \text{ QED} \\
 \ell_W &= 0.3708
 \end{aligned}$$

WALL #3: NEMA G-10 plus styrofoam

$$\begin{aligned}
 x_1 &= 0.629 \text{ NEMA G-10} \\
 x_2 &= 0.371 \text{ styrofoam} \\
 \Delta &= 0.7867 \\
 k_W' &= 0.0629 \text{ Wm}^{-1}\text{K}^{-1} \\
 \kappa_W &= 0.2130 \\
 \kappa_W \Delta^{-1} &= 0.271 \text{ QED} \\
 (\rho C)_W &= 1.8054 \times 10^6 \text{ Jm}^{-3}\text{K}^{-1} \\
 \gamma_W &= 3.2825 \\
 \gamma_W \Delta &= 2.5824 \text{ QED} \\
 \ell_W &= 0.1314
 \end{aligned}$$

STEP 8: Once the walls have been calculated and it has been established that $\kappa_W \Delta^{-1}$ and $\gamma_W \Delta$ are the same for all walls, one can calculate δ and τ using $(\Delta \gamma_W)$, ℓ_f and t from equations 34 and 35. If $(\gamma_S P)$ is used to calculate δ and τ , use equations 24 and 25. δ should agree with δ calculated in STEP 3.

$$\begin{aligned}
 \delta &= 2.7752 \text{ QED} \\
 \tau &= 0.2659
 \end{aligned}$$

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STEP 9: Model the solar heat flux through the window. Model infiltration and non-solar space heating. Since no space heating and infiltration are assumed, only solar flux comes into play. Using 55

$$\sigma = 4.5024$$

So if the solar flux on the window when the sun is shining is

$$S = 600 \text{ Wm}^{-2}$$

then one can estimate S' :

$$S' = 2701 \text{ Wm}^{-2}$$

STEP 10: Set the timing of the outside temperature and the solar flux. This is all done using τ . The timing of the model would be cyclical based on a model day of 6 hours and 23 minutes. The temperature of the outside walls of the full-scale structure might vary from 0°C to 50°C depending on the time of day. The outside wall temperature of the scale model would vary from 289.0K (15.8C) to 316.7K (43.6C) in the same time pattern as the full-scale house multiplied by τ . Since it is desirable to understand the behaviour of a passive solar structure over long periods of time, data-taking on the model would take place in a 2-4-day period, which corresponds to a full-scale structure operating over a period of 1 to 2 weeks.

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The ten steps are complete. The three types of walls match each other and the modeling gas. Dynamic thermal similitude has been achieved. The model is not perfect because Δ is not one for each of the walls. It is expected that little error will result from this. This simple case demonstrates that well-insulated wall will be modeled by walls with Δ greater than 1 when ℓ_f is small. The model also demonstrates the very dense high-volume specific heat wall will also be modeled so that Δ is greater than 1. Ordinary interior walls where k and ρc are moderate in value will have values of Δ near 1. One can select other materials if one wants values of Δ closer to 1. (In the case of WALL #2, the use of urethane foam instead of styrofoam results in a value of Δ about 1.75. In the case of WALL #3, the use of nylon or hard rubber instead of NEMA G-10 will result in a Δ closer to 1.) The case given here illustrates that a complex thermal structure can be modeled using the Richardson-Berman theory and its ramifications.

7. Concluding Comments

This Engineering Note demonstrates that the process of selecting the model wall material goes hand in hand with selecting the modeling gas. The gas should be selected on the basis of its κ_s . The gas pressure can be varied in order to get the proper value of $\gamma_s P$ for matching with the walls. In general one wants a value of κ_s which is almost as low as ℓ_f . (Mixtures of Nobel gases such as Xenon and Argon are attractive despite low values of γ_s .) The test vessel should be designed for service at pressure up to 6 atm.

The selection of wall materials is a function of the maximum allowable temperature in the model; hence δ . The value of δ in turn determines what the value of $(\gamma_s P)$ and $(\gamma_w \Delta)$ is. The value of $(\gamma_w \Delta)$ and $\kappa_w \Delta^{-1}$ must be the same for all walls in the model. (Note: a wall is defined as a ceiling, a roof, a floor, etc.) It is desirable to have Δ near one but the error of Δ not being one is not serious. It can be argued the τ is not an important parameter because the time of testing is usually small compared to building and instrumenting the model (in most cases τ is somewhat larger than ℓ_f , say a factor of 1.5 larger). The example given in this report shows that modeling can be done of structures which have a large variation in wall material properties.

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