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An Empirical Study of Capital Asset Pricing Model
and Fama-French Three-Factor Model

A thesis submitted in partial satisfaction
of the requirements for the degree
Master of Science in Statistics

by

Soo Woo Choi

2017

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ABSTRACT OF THE THESIS

An Empirical Study of Capital Asset Pricing Model
and Fama-French Three-Factor Model

by

Soo Woo Choi

Master of Science in Statistics

University of California, Los Angeles, 2017

Professor Nicolas Christou, Co-Chair

Professor Yingnian Wu, Co-Chair

The thesis tests performances of Capital Asset Pricing Model and Fama-French Three-Factor Model. Through an empirical study on the US stocks from January 2000 to August 2017, the thesis demonstrates that Fama-French Three-Factor model performs better than Capital Asset Pricing Model.

The thesis of Soo Woo Choi is approved.

Jingyi Jessica Li

Yingnian Wu, Committee Co-Chair

Nicolas Christou, Committee Co-Chair

University of California, Los Angeles

2017

*To my parents . . .
who have given me endless support throughout my life. . .*

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CHAPTER 1

Introduction

Asset pricing models attempt to define the relationship between return and risk. The capital asset pricing model (CAPM), developed in early 1960s by William Sharpe, Jack Treynor, John Lintner and Jan Mossin, is one of the original models explaining risk-return relationship in the financial market. Though widely known and extensive usage, empirical tests on CAPM have been poor, and many have tried to discover other factors which are not included in CAPM.

Eugene Fama and Kenneth French introduced three-factor model which is one of the well-known alternative to CAPM. They argued small-cap stocks and value stocks (stocks with high book-to-market ratios) tend to outperform large-cap stocks and growth stocks (stocks with low book-to-market ratios), and added size factor and value factor to market factor in CAPM. Basically, Fama-French Three-Factor model can be thought of as the extension of CAPM which has two additional factors. They argued that their three-factor model predicts stock prices more accurately than CAPM.

The main objective of this thesis is to compare the performance of these two models, Capital Asset Pricing Model and Fama-French Three-Factor model, using US stock market from January 2000 to August 2017.

The sequence of the thesis is as follows. After this introductory Chapter 1, Chapter 2 defines basic financial concepts. In Chapter 3 Modern portfolio theory by Markowitz in which CAPM was built on will be thoroughly reviewed. CAPM and Fama-French Three-Factor Model will be thoroughly discussed in Chapter 4 and in Chapter 5, respectively. Chapter 6 states the methodology and data employed in this study, and conducts an empirical study comparing the performances of CAPM and Fama-French Three-Factor Model. Chapter 7

concludes the thesis by summarizing the findings, and discusses the future works.

CHAPTER 2

Theoretical Framework

Before explaining CAPM and Fama-French three-factor model, I will first present the theoretical framework which is needed for understanding the empirical part of the thesis.

2.1 Definition

2.1.1 Return of a Stock and its Statistics

Suppose that $P_{i,t}$, and $P_{i,t-1}$ are stock prices at time t and $t - 1$ respectively. Then,

- The return at time t of the stock i is

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}$$

- The mean and the variance of the returns of stock i are:

$$\bar{R}_i = \frac{1}{n} \sum_{t=1}^n R_{i,t}, \quad \hat{\sigma}_i^2 = \frac{1}{n-1} \sum_{t=1}^n (R_{i,t} - \bar{R}_i)^2$$

- The covariance between returns of stock i and stock j is:

$$cov(R_i, R_j) = \hat{\sigma}_{ij} = \frac{1}{n-1} \sum_{t=1}^n (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j)$$

- The correlation coefficient $\hat{\rho}$ between stock i and stock j is :

$$\hat{\rho}_{ij} = \frac{cov(R_i, R_j)}{\hat{\sigma}_i \hat{\sigma}_j}$$

2.1.2 Risk of a Stock

In finance, risk is the chance that actual return of investment will be different from the expected return, and it can be thought as volatility of expected returns. It has been known that there is a positive relationship between risk and return: if investors are willing to accept more risk, they should expect higher returns.

There are several risk measures. One of the most easy and common risk measure is the standard deviation.

$$\hat{\sigma}_i = \sqrt{\hat{var}(R_i)} = \left[\frac{1}{n-1} \sum_{t=1}^n (R_{i,t} - \bar{R}_i)^2 \right]^{\frac{1}{2}}$$

The higher standard deviation means that it has greater risk.

When measuring risk, we need to take a closer look at beta coefficient, $\beta_{i,m}$. In CAPM it is essential to know what portion is associated with market (systematic Risk) and what portion is associated with the firm itself (unsystematic Risk). While unsystematic risk can be eliminated by diversification, systematic risk remains even after diversification. Therefore $(\beta_{i,m})$ represents risk after diversification, and it measures volatility of a stock or a portfolio in comparison to market.

When comparing performances, we can examine Sharpe Ratio, which is defined as a ratio of the excess return divided by its risk:

$$Sharpe\ ratio = \frac{\bar{R}_i - R_f}{\hat{\sigma}_i}$$

where R_f is risk-free rate of return. This measure helps compare performances of investments by adjusting for their risk.

2.1.3 Portfolio Return and Variance

A portfolio is a grouping of financial assets. The return on a portfolio of stocks is a weighted average of the returns on the individual stocks, and the expected return on a portfolio of stocks is a weighted average of the expected returns on the individual stocks.

Suppose x_i is the fraction of the portfolio invested in stock i . Then,

- The return on a portfolio consisting of N stocks is

$$R_p = \sum_{i=1}^N x_i \bar{R}_i$$

- The expected return on a portfolio consisting of N stocks is:

$$\hat{E}(R_p) = \hat{E}\left(\sum_{i=1}^N x_i R_i\right) = \sum_{i=1}^N x_i \bar{R}_i$$

- The variance of the portfolio is:

$$\hat{var}(R_p) = \hat{var}\left(\sum_{i=1}^N x_i R_i\right) = \sum_{i=1}^N \sum_{j=1}^N x_i x_j \hat{\sigma}_{ij} = \sum_{i=1}^N \sum_{j=1}^N x_i x_j \hat{\rho}_{ij} \hat{\sigma}_i \hat{\sigma}_j$$

2.1.4 Risk-Free Rate of Return

Risk-free rate of return, denoted by R_f , is the return that investors can earn with no risk. U.S. Treasury bills are often used as a proxy for the risk-free rate. I will use the interest rate on the 1 month U.S. Treasury bill as the risk-free rate for models on U.S. financial market.

2.1.5 Market Return

Market return, denoted by R_m , is the return rate of the market portfolio. Theoretically market portfolio is defined as a bundle of investments which include every type of asset available in the financial market, with each asset weighted in proportion to its total market value. A market index, such as the Dow Jones Industrial Average and *S&P500* is often used as the proxy market portfolio, and I will use market return obtained directly from Kenneth Frenchs website.

CHAPTER 3

Modern Portfolio Theory

CAPM builds on Modern Portfolio Theory (MPT) by Harry Markowitz (1952), which made him won a Nobel prize in 1990. Therefore, in order to understand the theory of CAPM, it is essential to know MPT. Thus, I will first review MPT before explaining CAPM.

MPT states that it is possible to construct Efficient Frontier which represents the set of all portfolios of which expected returns ($\hat{E}(R_p)$) reach the maximum given a certain level of risk ($\hat{var}(R_p)$).

3.1 Assumptions of Modern Portfolio Theory

Markowitz Efficient Frontier Theory relies on the following assumptions:

- The market is efficient which means that a stock price accurately reflects its value.
- Investors are risk-averse and rational: they are evaluating portfolios solely on expected return and standard deviation of return.
- There are no financial transactions.
- Investors hold investments for the same one-period of time.
- Asset returns are distributed by the normal distribution.

3.2 Efficient Frontier

Suppose there are n risky assets, and let $\mathbf{X} = (x_1, \dots, x_n)^T$, where x_i denotes the proportion of wealth invested in asset i , and $\sum_{i=1}^n x_i = 1$. If we plot every possible asset combination of n assets in risk-return framework, then the collection of all such possible portfolios defines a region and the boundary of the cloud of points is called portfolio possibilities curve. Efficient Frontier is the upper half of the portfolio possibilities curve.

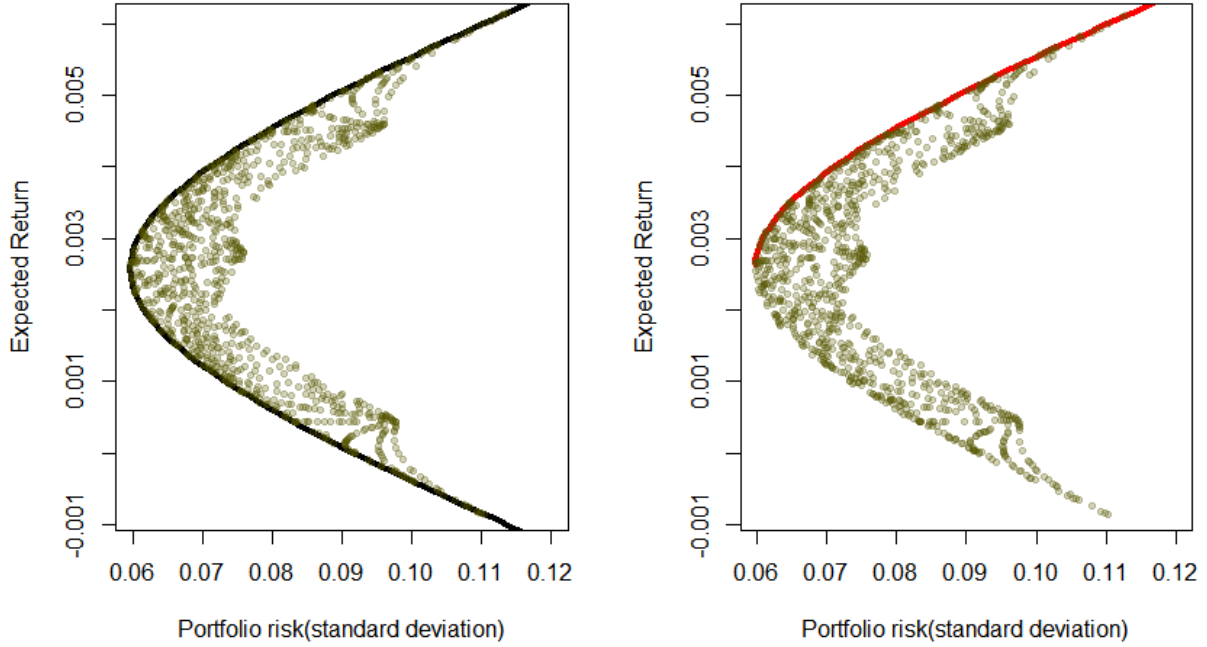


Figure 3.1: Portfolio Possibilities Curve (left) and Efficient Frontier (right)

Figure 3.1 plots the portfolio possibilities curve and the efficient frontier. On both plots, green points represent all the possible portfolios using n assets; the black curve in the left plot represents the portfolio possibilities curve, and the red curve in the right plot represents the efficient frontier.

As can be seen from Figure 3.1, portfolios along Efficient Frontier represent portfolios offering the lowest risk for a given level of return. Conversely, the portfolio lying on the

efficient frontier represents the portfolio offering the best possible return for a given level of portfolio risk. Portfolios that below the curve are inefficient since they either carry too much risk relative to the return or too little return relative to the risk.

3.3 Mathematical Formulation of Efficient Frontier

In order to obtain efficient portfolios mathematically, we need to minimize the risk for a given rate of return. The objective function is following:

$$\min \frac{1}{2} \hat{\sigma}_p^2$$

subject to

$$\begin{aligned} \sum_{i=1}^N x_i &= 1 \\ R_p &= \sum_{i=1}^N x_i \bar{R}_i \\ \hat{\sigma}_p^2 &= \sum_{i=1}^N \sum_{j=1}^N x_i x_j \hat{\rho}_{ij} \hat{\sigma}_i \hat{\sigma}_j \end{aligned}$$

In order to solve this problem, I will apply the method of Lagrange multipliers to the convex optimization subject to linear constraints:

$$\min \{Q = \mathbf{X}^T \Sigma \mathbf{X} - 2\lambda_1 (\mathbf{X}^T \mathbf{R} - R_p) - 2\lambda_2 (\mathbf{1}^T \mathbf{X} - 1)\},$$

where $\mathbf{X}$ is the vector of weights, Σ is the covariance matrix of the portfolio, and λ_1 and λ_2 are Lagrangian multipliers. It can be solved by taking the derivatives with respect to $\mathbf{X}^T$, λ_1 , λ_2 , and set the derivatives equal to zero.

3.4 Capital Market Line

James Tobin (1958) contributed to MPT by adding leverage to Efficient Frontier. Tobin argued that by adding a risk-free asset, investors can construct a optimal portfolio which outperforms portfolios on Efficient Frontier.

When risk-free asset is added, only the tangent portfolio G matters among portfolios on Efficient Frontier. The tangent portfolio G is the portfolio with the highest Sharpe ratio on Efficient Frontier. Given the identical risk-free rate, every investor will choose the identical tangent portfolio G. Therefore, the tangent Portfolio G becomes the Market portfolio. The only difference among investors will be choices of proportions invested into risk-free asset. For example, one may invest all his wealth in the Market portfolio, and the other may invest half of his wealth in the Market portfolio and other half in the risk free asset.

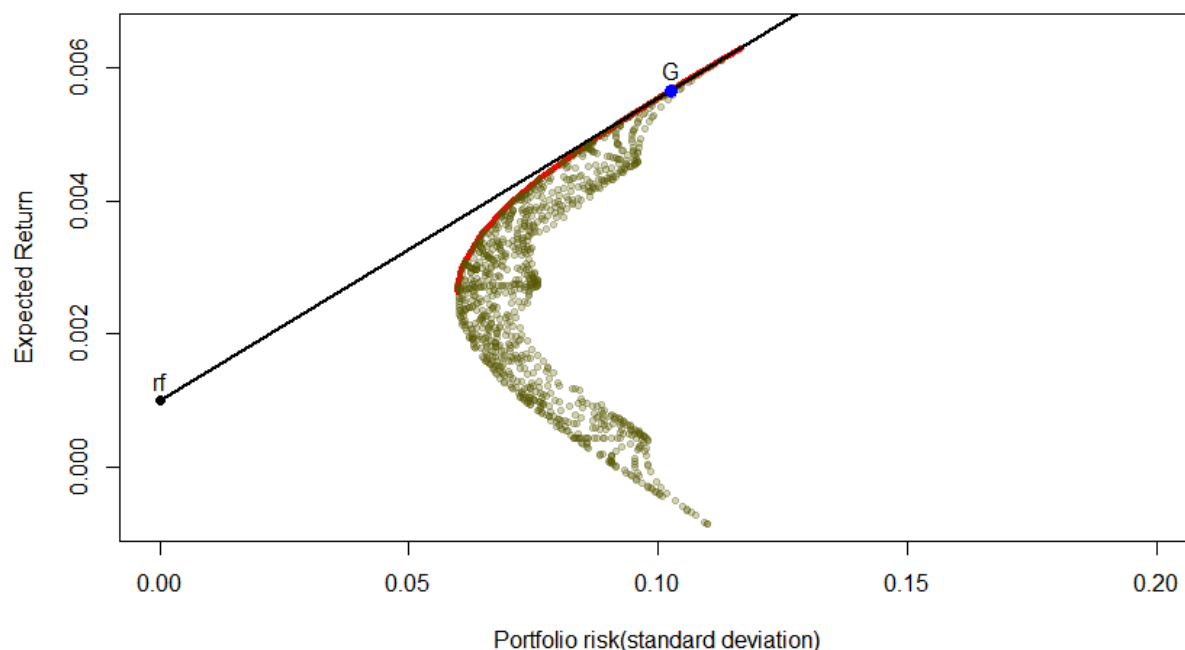


Figure 3.2: Capital Market Line

In Figure 3.2, the black bold line indicates Capital Market Line (CML). CML is a line connecting a risk-free rate of return and the Market portfolio. Every investor will choose an optimal portfolio located on CML since CML is generating highest excess return from every risk level taken among all portfolios.

3.5 Mathematical Formulation of CML

Mathematically, the optimal portfolio can be found by the following method:

1. Compute the excess return vector $\mathbf{R} = (\bar{R}_1 - R_f, \dots, \bar{R}_n - R_f)$.
2. Compute the variance-covariance matrix of the return of the N stocks $\hat{\Sigma} = (\hat{\sigma}_{ij})_{N \times N}$
3. Compute $\mathbf{Z} = \hat{\Sigma}^{-1} \mathbf{R}$
4. Compute $x_k = \frac{z_k}{\sum_{i=1}^N z_i}$ for $k = 1, \dots, N$, then $x_1, \dots, x_n$ shows the composition of the tangency portfolio.
5. Investors can invest all their wealth in the tangency portfolio or invest some of their wealth in the tangency portfolio and the remaining in R_f , or borrow money to invest in the tangency portfolio

CHAPTER 4

Capital Asset Pricing Model

Based on the pioneering work of Markowitz (1952) and Tobin (1958), Sharpe (1964), Lintner (1965) and Mossin (1966) introduced the Capital Asset Pricing Model (CAPM). CAPM classifies risk into two main categories: unsystematic risk and systematic risk.

- Unsystematic Risk, also known as company specific risk, is risk specific to a particular stock or industry. CAPM believes that this type of risk can be eliminated by diversification.
- Systemic Risk, also known as market risk, is risk which affects the overall stock market. This type of risk is not specific to a particular company or industry, and it cannot be eliminated by diversification.

CAPM believes that all investors need to be compensated in the form of extra return, for taking systematic risk. CAPM assumes that unsystematic risk does not need to be compensated since it can be eliminated through diversification. Therefore, according to CAPM an asset's expected return is equal to sum of the risk-free rate R_f and a risk premium, which is $\beta_{i,m}$ times $(E(R_m) - R_f)$. $\beta_{i,m}$ is a measure of systematic risk of an asset. A stock with high beta indicates that the asset is greatly affected by changes in the market. A stock with low beta indicates that the asset is not much affected by changes in the market.

4.1 Assumptions of CAPM

CAPM has been based on Markowitz's MPT. It basically shares all the assumptions held by Markowitz model with additional two assumptions. The two additional assumptions are

marked in bold.

- The market is efficient which means that a stock price accurately reflects its value.
- Investors are risk-averse and rational: they are evaluating portfolios solely on expected return and standard deviation of return.
- There are no financial transactions.
- Investors hold investments for the same one-period of time.
- Asset returns are distributed by the normal distribution.
- **Investors have homogeneous expectations about asset returns.**
- **Investors may borrow or lend unlimitedly at the risk free rate (r_f).**

4.2 Formula of CAPM

The formula of the CAPM model is:

$$E(R_i) = R_f + \beta_{i,m}[E(R_m) - R_f], \text{ , where } i = 1, \dots, N$$

where the market beta is defined as below:

$$\beta_{i,m} = \frac{\sigma_{i,m}}{\sigma_m^2}$$

Here, $E(R_i)$ is expected return on an asset i . $\beta_{i,m}$ is the market beta of the asset i . $\sigma_{i,m}$ is covariance between the market and an asset i . σ_m^2 is variance of the market. R_f is the risk-free rate of return. R_m is the market return.

4.3 Security Market Line

Security market line (SML) is a graphical representation of CAPM. Figure 4.1 shows the SML. As can be seen from the figure, the x-axis represents systematic risk, β , the y-axis represents the expected return, and the intercept represents risk-free rate of return. This indicates that the expected rate of return is a function of β which represents systematic risk.

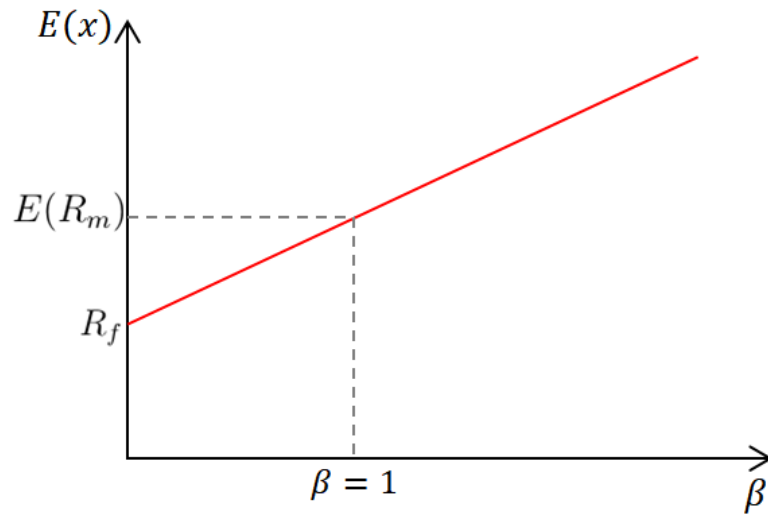


Figure 4.1: Security Market Line

CHAPTER 5

Fama-French Three-Factor Model

Since CAPM was introduced, CAPM has been criticized in many studies: many studies test the validity of CAPM and raised doubts, arguing that CAPM performed poorly in explaining stock prices. Many researchers attempted to find other factors which the CAPM did not consider, such as company size, earnings-to-price ratio, and book-to-market ratio. In 1977, Basu found that average returns on stocks with high earnings/price ratio are too high, given their beta and average returns on stocks with low earnings/price ratio are too low, given their beta. This finding was one of the first severe blows to CAPM since in CAPM the beta coefficient is one and only variable which can predict future returns. Empirical evidence against CAPM continued. In 1981, Banz argued that return of stock with low market capitalization is higher than return of stock with large market capitalization. In 1984 Rosenberg, Reid and Lanstein showed that average returns on stocks with low book-to-market ratio is higher than average returns on stocks with high book-to-market ratio. In 1988, Bhandari pointed out that firms with high leverage have higher average returns than firms with low leverage, given their beta.

In 1992 Fama and French jointly tested the roles of aforementioned variables - β , size, earnings-to-price ratio, leverage, and book-to-market ratio - in predicting future stock returns. They argued that the relation between beta coefficient and return is not reliable, and confirmed that the aforementioned variables affect future returns of stocks. They also argued that size and book-to-market ratio can absorb the apparent roles of leverage and E/P in predicting future returns. As a result, they expanded the CAPM by adding two factors, the value effect and the size effect, and proposed three factor asset pricing model.

- The value effect indicates the tendency that value stocks (stocks with high book value-

to-market value ratios) outperform the market, and growth stocks (stocks with low book-to market ratios) underperform the market.

- The size effect indicates the tendency that stocks with low market capitalization outperform stocks with high market capitalization.

5.1 Formula of French-French Three-Factor Model

The formula of the French Three-Factor model is following:

$$E(R_i) - R_f = \beta_{i,m}[E(R_m) - R_f] + \beta_{i,HML}F_{HML} + \beta_{i,SMB}F_{SMB}$$

HML (High minus Low) accounts for value premium, and it can be calculated by $F_{HML} = (E[R_{value}] - E[R_{growth}])$, where R_{value} denotes the return of value stocks, and R_{growth} denotes the return of growth stocks. SMB (Small minus Big) accounts for size premium, and can be calculated by $F_{SMB} = (E[R_{small}] - E[R_{big}])$, where R_{small} denotes the return of stocks with low market capitalization, and R_{big} denotes the return of stocks with high market capitalization.

5.2 CAPM vs. Fama-French Three-Factor Model

After Fama and French published their paper in 1992, many studies empirically tested Fama-French Three-Factor Model and CAPM, and compared their performances.

There are several studies which present empirical evidence in support of Fama-French Three-Factor model. For example, Homsud, Wasunsakul, Phuangnark and Joongpong (2009) tested the validity of Fama-French Three-Factor model by examining Thailand stock market from July 2002 to May 2007. They added size factor and value factor into CAPM, and concluded that portfolio returns are better explained when size and value factors are added into CAPM.

Belen Blanco (2012) and Bahtnagar and Ramlogan (2012) also concluded that Fama French Three-Factor model outperforms CAPM by using the American NYSE market from

July 1926 to January 2006, and by using the United Kingdom Market from April 2000 to June 2007, respectively.

In contrast, several studies present different results. Daniel and Titman (1997) tested the Fama and French model on NYSE, AMEX and NASDAQ from 1973 to 1993, and argued that there were no size effect and value effect when they controlled for firm characteristics. Malin and Veeraraghavan (2004) tested the robustness of Fama and French model in Germany, France and UK, and found growth stocks yielding higher returns than value stocks which is contrary to Fama-French Three-Factor model.

CHAPTER 6

Empirical Studies

6.1 Aim of the study

I would like to compare the performance of two most well-known asset pricing models, CAPM and Fama-French three-factor model using US stock data.

Model	Description
CAPM	$E(R_i) - R_f = \beta_{i,m}[E(R_m) - R_f]$
Fama-French three-factor	$E(R_i) - R_f = \beta_{i,m}[E(R_m) - R_f] + \beta_{i,HML}F_{HML} + \beta_{i,SMB}F_{SMB}$

Table 6.1: Asset Pricing Models

6.2 Data

This study is based on US stock market from January 2000 to August 2017. I have worked with monthly returns on 25 portfolios. Historical monthly Fama-French three-factor ($R_m - R_f$, SMB, HML) data as well as historical monthly return rates for the 25 portfolios are obtained from Kenneth French's website.

I divided the data into 2 categories: training set and test set. The training set covers January 2000 to August 2016, and the test set covers September 2016 to August 2017. I used the training set to do statistical analysis and to generate coefficients and evaluate their significance. Then I used the test set to compare the predictive power of both models using the generated coefficients.

According to Kenneth French's website, NYSE, AMEX, and NASDAQ stocks are divided

into five groups by value, as measured by book equity to market equity (BE/ME), and at the same time, NYSE, AMEX and NASDAQ stocks are divided into five groups by size, as measured by market equity (ME). Based on the above data division, the 25 portfolios are formed by the intersections of 5 portfolios formed on size and 5 portfolios formed on book-to-market ratio. Table 6.2 shows the summary of average returns on the 25 value-weighted portfolios from January 2000 to August 2016.

		Value(BE/ME)				
		5(big)	4	3	2	1(small)
Size(ME)	5(high)	0.9089	0.8775	1.1742	1.0968	1.1393
	4	0.9832	1.2113	1.0342	1.2839	1.3592
	3	0.8475	1.1158	1.1068	1.4140	1.3288
	2	0.9939	1.1057	1.1481	1.2556	1.3051
	1(low)	1.0240	1.0929	0.8911	1.1543	0.6229

Table 6.2: Average Monthly Percent Returns for Portfolios Formed on Size and BE/ME

The Fama-French factors, $R_m - R_f$, SMB and HML, are constructed by 6 portfolios formed on size and book-to-market. Similar to the method to construct 25 portfolios, NYSE, AMEX, and NASDAQ stocks are divided into 3 groups according to BE/ME ratio, and at the same time, NYSE, AMEX and NASDAQ stocks are divided into 2 groups by size. The 6 Portfolios are constructed by the intersections of 2 portfolios formed on size, and 3 portfolios by BE/ME ratio. Table 6.3 shows how the 6 portfolios are constructed.

		Value(BE/ME)		
		High	Neutral	Low
Size(ME)	Small	Portfolio 1(P1)	Portfolio 2(P2)	Portfolio 3(P3)
	Big	Portfolio 4(P4)	Portfolio 5(P5)	Portfolio 6(P6)

Table 6.3: Size and Value Sorting

For the $R_m - R_f$ factor which indicates the average excess return on the market, Fama and French subtract the one-month Treasury bill rate from the value-weighted return on

all NYSE, AMEX, and NASDAQ stocks (from CRSP). For SMB (Small minus Big), they calculate the average return on the three small portfolios minus the average return on three big portfolios.

$$SMB = \frac{1}{3} (P1 + P2 + P3) - \frac{1}{3} (P4 + P5 + P6)$$

For HML (High minus Low), they calculate the average return on two value portfolios minus the average return on two growth portfolios.

$$HML = \frac{1}{2} (P1 + P4) - \frac{1}{2} (P3 + P6)$$

	Mean	Maximum	Minimum	Standard Deviation
Mkt-Rf	0.3453	11.3500	-17.2300	4.4832
SMB	0.3071	21.7100	-16.8800	3.3856
HML	0.3398	12.9000	-11.1000	3.1870
Rf	0.1377	0.5600	0.0000	0.1641
Number of Observation: 92				
Correlation between SMB and HML: -0.2972				

Table 6.4: Summary Statistics for Fama-French Three-Factors

More detailed description of data can be found in Kenneth French's website. Table 6.4 shows the summary statistics for Fama-French 3 factors. As seen in the table, SMB has a positive average return of 0.307% over the sample period. This indicates stocks with low market capitalization outperform stocks with high market capitalization, confirming the size effect suggested by Fama and French. HML also has a positive average return of 0.339% and this also confirms the value effect that stocks with high BE/ME ratio outperform stocks with low BE/ME ratio. Correlation between SMB and HML is -0.297%, and this low value indicates Size factor is largely independent from Value factor. This fact confirms that these two factors explain fluctuation in stock returns along different dimensions.

6.3 Testing Models

In order to test CAPM and Fama-French Three-Factor Model, I run the both CAPM and the Fama-French model described in 6.3.1 and 6.3.2 respectively. Both models are all linear regressions, and I used ordinary least square method to estimate coefficients of the both models with the training set and implemented the program R.

After running both regressions, I compared the predictive power of both models. In order to do so, for each model, I used the generated coefficients to predict on the testing set and compared the mean squared errors and correlation.

6.3.1 CAPM Regression

The CAPM can be tested by running the following linear regression:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,m}(R_{m,t} - R_{f,t}) + \epsilon_{i,t}$$

$R_{i,t}$ is the return on asset i for period t , and here asset i is one of the 25 portfolios formed on size and book-to-market. $R_{f,t}$ is one-month Treasury bill rate for period t , and $\epsilon_{i,t}$ represents unsystematic risk of investing in asset i for period t . Importantly, α_i is the estimated abnormal return on asset i : alpha calculates the difference between the expected excess return and the actual return. Therefore, in order for CAPM to be hold, the expected value of α_i needs to be zero.

6.3.2 Fama-French Three-Factor Model Regression

The Fama-French Three-Factor Model can be tested by running the following regression:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,m}(R_{m,t} - R_{f,t}) + \beta_{i,HML}HML + \beta_{i,SMB}SMB + \epsilon_{i,t}$$

Again, α_i is the estimated expected excess return on asset i which is not captured by $\beta_{i,m}(R_{m,t} - R_{f,t}) + \beta_{i,HML}HML + \beta_{i,SMB}SMB$. Therefore, in order for Fama-French Three-Factor Model to be hold, the expected value of α_i needs to be zero. Table 6.5 summarizes the variables used in CAPM and Fama-French Three-Factor Model.

Variables	Description
R_{it}	Return on asset i for period t
R_{ft}	Risk-free rate of return for period t
$R_{m,t}$	Return on market portfolio for period t
SMB_t	Size premium for period t
HML_t	Value premium for period t
$\epsilon_{i,t}$	unsystematic risk on asset i for period t
α_i	Abnormal return on asset i

Table 6.5: Descriptions of Variables in Asset Pricing Models

6.4 Findings and Interpretation

Regression analysis, using the method of ordinary least squares, has been done to test both CAPM and Fama-French Three-Factor Model. I used the program R. CAPM argues that the expected value of returns can be fully explained by its risk premium. Thus, if CAPM is correct, the beta coefficient $\beta_{i,m}$ should explain all the expected value of returns, and the alpha coefficient α_i for all portfolios should be zero. This approach is also valid for Fama-French Three-Factor Model as well. If the Fama-French Three-Factor is correct, $\beta_{i,m}$, $\beta_{i,HML}$ and $\beta_{i,SMB}$ should explain all the expected value of returns, and the alpha coefficient α_i for all portfolios should be zero.

Table 6.6 summarizes the regression estimates for CAPM along with its corresponding P values. I added an asterisk to a P value which is less than 5%. It shows that 10 of the 25 alpha coefficient α_i are significantly different than zero at the 5% level. This indicates among the 25 portfolios 10 portfolios are not fully explained by the beta coefficient $\beta_{i,m}$ contained in the CAPM, and suggests that CAPM does not perform well to predict risk premium. When we look at the beta coefficient $\beta_{i,m}$ in CAPM, the value is close to one and significant at the 5% level for all the 25 portfolios.

CAPM: $R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,m}(R_{m,t} - R_{f,t}) + \epsilon_{i,t}$										
	α_i					p value of α_i				
	BE/ME Quantiles					BE/ME Quantiles				
Size	5(high)	4	3	2	1(low)	5(high)	4	3	2	1(low)
5(big)	-0.001	-0.003	0.001	0.000	-0.002	0.634	0.235	0.421	0.711	*0.025
4	0.001	0.004	0.002	0.004	0.001	0.762	*0.030	0.255	*0.012	0.625
3	0.006	0.005	0.004	0.004	-0.001	*0.024	*0.023	*0.020	*0.012	0.634
2	0.003	0.004	0.005	0.004	-0.001	0.227	0.081	*0.022	0.072	0.807
1(small)	0.006	0.005	0.003	0.003	-0.005	*0.029	*0.042	0.279	0.418	0.191
	$\beta_{i,m}$					p value of $\beta_{i,m}$				
	BE/ME Quantiles					BE/ME Quantiles				
5(big)	1.148	0.957	0.810	0.864	0.924	*0.000	*0.000	*0.000	*0.000	*0.000
4	1.171	1.002	1.036	1.019	1.223	*0.000	*0.000	*0.000	*0.000	*0.000
3	1.114	0.994	1.038	1.076	1.278	*0.000	*0.000	*0.000	*0.000	*0.000
2	1.217	1.036	1.042	1.122	1.346	*0.000	*0.000	*0.000	*0.000	*0.000
1(small)	1.096	1.005	1.075	1.264	1.474	*0.000	*0.000	*0.000	*0.000	*0.000

Table 6.6: Regression Estimates for CAPM along with P Values

Table 6.7 summarizes the regression estimates for Fama-French Three-Factor model along with its corresponding P values. It shows that only 3 out of the 25 alpha coefficient α_i are significantly different than zero at the 5% level. This indicates almost all the 25 portfolios are explained by the $\beta_{i,m}$, $\beta_{i,HML}$ and $\beta_{i,SMB}$, leaving no abnormal returns to the portfolios. This finding suggests that when strictly comparing the alpha coefficient α_i , Fama-French Three-Factor Model does perform better than CAPM.

In addition, $\beta_{i,m}$, $\beta_{i,HML}$ and $\beta_{i,SMB}$ support Fama-French Three-Factor Model: all the 25 $\beta_{i,m}$ are significantly different from zero, and almost all $\beta_{i,SMB}$ and $\beta_{i,HML}$ (24 out of 25) are significantly different from zero. It is worth to note that $\beta_{i,SMB}$ decrease monotonically from small-size to big-size quantiles for each BE/ME quantiles. In addition, $\beta_{i,HML}$ increase monotonically from the lowest BE/ME quantile to the highest BE/ME quantile for each

$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,m}(R_{m,t} - R_{f,t}) + \beta_{i,HML}HML + \beta_{i,SMB}SMB + \epsilon_{i,t}$										
	α_i					P value of α_i				
	BE/ME Quantiles					BE/ME Quantiles				
Size	5(high)	4	3	2	1(low)	5(high)	4	3	2	1(low)
5(big)	-0.004	-0.005	0.000	0.000	0.000	0.086	*0.001	0.780	0.749	0.424
4	-0.002	0.002	0.000	0.002	0.000	0.178	0.241	0.902	0.079	0.433
3	0.002	0.001	0.001	0.002	-0.002	0.315	0.288	0.228	0.106	0.177
2	-0.002	0.000	0.001	0.001	-0.002	*0.035	0.774	0.284	0.331	0.137
1(small)	0.001	0.001	-0.001	-0.001	-0.007	0.425	0.324	0.378	0.714	*0.001
	$\beta_{i,m}$					P value of $\beta_{i,m}$				
	BE/ME Quantiles					BE/ME Quantiles				
Size	5(high)	4	3	2	1(low)	5(high)	4	3	2	1(low)
5(big)	1.222	1.042	0.868	0.916	0.957	*0.000	*0.000	*0.000	*0.000	*0.000
4	1.186	0.996	1.042	1.011	1.111	*0.000	*0.000	*0.000	*0.000	*0.000
3	1.072	0.961	1.002	1.010	1.111	*0.000	*0.000	*0.000	*0.000	*0.000
2	1.077	0.928	0.930	0.968	0.967	*0.000	*0.000	*0.000	*0.000	*0.000
1(small)	0.938	0.817	0.890	0.996	1.204	*0.000	*0.000	*0.000	*0.000	*0.000
	$\beta_{i,SMB}$					P value of $\beta_{i,SMB}$				
	BE/ME Quantiles					BE/ME Quantiles				
Size	5(high)	4	3	2	1(low)	5(high)	4	3	2	1(low)
5(big)	-0.197	-0.268	-0.201	-0.224	-0.237	*0.004	*0.000	*0.000	*0.000	*0.000
4	0.114	0.156	0.083	0.111	0.496	*0.046	*0.001	*0.096	*0.008	*0.000
3	0.418	0.321	0.290	0.405	0.760	*0.000	*0.000	*0.000	*0.000	*0.000
2	0.934	0.704	0.690	0.842	0.967	*0.000	*0.000	*0.000	*0.000	*0.000
1(small)	0.980	1.080	1.023	1.375	1.312	*0.000	*0.000	*0.000	*0.000	*0.000

Table 6.7: Regression Estimates for Fama-French Three-Factor along with P Values

$R_{i,t} - R_{f,t} = \alpha_i + \beta_{i,m}(R_{m,t} - R_{f,t}) + \beta_{i,HML}HML + \beta_{i,SMB}SMB + \epsilon_{i,t}$										
	$\beta_{i,HML}$					P value of $\beta_{i,HML}$				
	BE/ME Quantiles					BE/ME Quantiles				
Size	5(high)	4	3	2	1(low)	5(high)	4	3	2	1(low)
5(big)	0.782	0.719	0.416	0.192	-0.290	*0.000	*0.000	*0.000	*0.000	*0.000
4	0.821	0.551	0.498	0.305	-0.327	*0.000	*0.000	*0.000	*0.000	*0.000
3	0.850	0.650	0.437	0.267	-0.401	*0.000	*0.000	*0.000	*0.000	*0.000
2	0.921	0.620	0.494	0.231	-0.275	*0.000	*0.000	*0.000	*0.000	*0.000
1(small)	0.706	0.497	0.330	0.021	-0.314	*0.000	*0.000	*0.000	0.653	*0.000

Table 6.8: Regression Estimates for Fama-French Model along with P Values (continued)

Size quantiles. This is consistent with Fama-French finding small-size companies tend to outperform large-size companies and value companies tend to outperform growth companies.

Another way to evaluate performances of models is by examining adjusted coefficient goodness of fit (adjusted R^2). This metrics indicates what percentage of variation in returns are explained by the model. Table 6.9 shows the R^2 adjusted statistics of CAPM and Fama-French Three-Factor model.

While the adjusted R^2 of CAPM range between 57% and 91% for the 25 portfolios, the adjusted R^2 of Fama-French Three-Factor model range between 79% and 96% for the 25 portfolios. More importantly, adjusted R^2 increases significantly for all the 25 portfolios, as we switch from CAPM to Fama-French Three-Factor model. This result suggests that Fama-French Three-Factor model has better explanatory power than CAPM when explaining variations in stocks' returns.

Adjusted R^2 of CAPM					
	BE/ME Quantiles				
Size	5(high)	4	3	2	1(low)
5(big)	0.619	0.631	0.707	0.841	0.913
4	0.685	0.749	0.751	0.834	0.804
3	0.666	0.714	0.803	0.822	0.727
2	0.667	0.719	0.728	0.722	0.694
1(small)	0.641	0.589	0.652	0.569	0.587
Adjusted R^2 of Fama-French Three-Factor					
	BE/ME Quantiles				
Size	5(high)	4	3	2	1(low)
5(big)	0.794	0.878	0.850	0.906	0.968
4	0.843	0.853	0.831	0.867	0.929
3	0.854	0.862	0.879	0.885	0.941
2	0.953	0.938	0.912	0.921	0.931
1(small)	0.938	0.932	0.941	0.926	0.883

Table 6.9: R squared Comparison (CAPM and Fama-French Three-Factor Model)

6.4.1 Predictive Power Comparison

In order to compare the predictive ability of CAPM and the Fama-French three-factor model, I compared their mean squared error and their correlation between actual value and its predicted value.

For comparing their mean squared error, I used training set to generate the coefficients of both models, and then made prediction on test set using the generated coefficients, and tracked the errors of all prediction. Table 6.10 notes the prediction errors of the CAPM and Fama-French Three-Factor model.

Fama-French Three-Factor Model not only had a lower mean squared error than CAPM, but also had a narrower range of error. This result indicates that Fama-French Three-Factor

Error Summary	CAPM	Fama-French Three-Factor Model
Sum of Squared Error	0.0121845	0.002150228
Min of Squared Error	0.000002517	0.0000001293
Max of Squared Error	0.006708	0.0007514
Mean Squared Error	0.001015	0.0001792
Range of Squared Error	0.006705151	0.0007152246

Table 6.10: Predictive Power Comparison (MSE)

Model has a better predictive ability than CAPM, in terms of calculating errors.

In order to compare their correlation between the actual value and the predicted value, I calculated the correlation between the actual return on test set and the predicted return on test set using the generated coefficients. Table 6.11 notes CAPM and Fama-French Three-Factor Model's correlation between actual returns and predicted returns.

Correlation between the Actual Returns and the Predicted Returns	
CAPM	0.696348
Fama-French Three-Factor Model	0.963373

Table 6.11: Predictive Power Comparison (Correlation)

As can be seen, Fama-French Three-Factor Model had a higher correlation between predicted returns and actual returns than CAPM, double-confirming that Fama-French Three-Factor Model has a better predictive ability than CAPM.

By comparing predictability of both models using MSE and correlation, we can conclude that the additional risk factors in the Fama-French Model, SMB and HML, have a positive effect on forecasting future return.

CHAPTER 7

Concluding Remarks

7.1 Conclusion

In this thesis, I test both Capital Asset Pricing Model and Fama-French Three-Factor Model on US stock market. The test period covers January 2000 to August 2017.

The main objective of this thesis is to compare performances of CAPM and Fama-French Three-Factor model in estimating stocks expected returns. In order to do so, I perform OLS regression analysis on both models and compare α_i and adjusted R^2 .

In CAPM, 10 of the 25 α_i , α_i are non-zero and statistically significant, and the adjusted R^2 of CAPM range between 57% and 91% for the 25 portfolios. In Fama-French Three-Factor model only 3 out of 25 of the alpha coefficient (α_i) are significantly different from zero and the adjusted R^2 of Fama-French Three-Factor model range between 79% and 96% for the 25 portfolios. Moreover, the adjusted R^2 increases significantly for all the 25 portfolios as we switch from CAPM to Fama-French Three-Factor model. This comparison of α_i and adjusted R^2 suggests Fama-French Three-Factor model outperforms CAPM in explaining US stock market returns since $\alpha_i = 0$ indicates the model explain the excess stock's return leaving no abnormal return and the adjusted R^2 indicates goodness of fit.

The thesis also compares the predictive power of CAPM and Fama-French Three-Factor model in forecasting stocks' returns. In order to do so, for each model, I use the early data which covers from January 2000 to August 2016 to generate coefficients of both models and use the generated coefficients to predict on the late data which covers September 2016 to August 2017. Then I calculated error of each prediction, and compare the mean squared errors and the range of errors of both models. I also compared their correlation between

the actual returns and predicted returns on late data. The result indicates that Fama-French Three-Factor Model had a lower mean squared error and higher correlation between actual and predicted value, suggesting that Fama-French Three-Factor Model has a better predictive ability than CAPM.

We can conclude from this study that the additional risk factors in Fama-French Three-Factor model not only help explain the excess return of all the 25 portfolios but also have a positive effect on forecasting future return.

7.2 Future Research Topics

In this study, we have not examined whether additional risk factors can explain stocks' excess returns. We can begin with Fama-French Five-Factor model, and evaluate the two additional risk factors - Profitability and Investment factors. And then, we can further expand our study by finding other variables which may have some effect on explaining stocks' excess returns.

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