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Skill Investment, Job Creation, and Labor Market Policy under Search Frictions

By

SHISHAM ADHIKARI
DISSERTATION

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Abstract

This dissertation studies skill investment, job creation, and labor market policy under search frictions. In each chapter, workers or firms make sunk investments in frictional labor markets, and private incentives diverge from the social value of those decisions. The central question across all three chapters is not whether to intervene, but where: which side of the matching process bears the relevant distortion, and what instruments are needed to correct it.

The green transition is often analyzed through the lens of technology adoption and capital reallocation, but its political economy turns on workers and jobs. Chapter 1 asks whether production subsidies alone are sufficient to implement the green transition, or whether worker-side support is also needed. I develop a two-sector Diamond–Mortensen–Pissarides model in which green workers invest in sector-specific human capital before matching with firms. Since training is sunk at the time of wage bargaining, workers capture only a fraction of its social return and underinvest relative to the planner. A production subsidy corrects the environmental externality and delivers the right number of green jobs, but does not implement the planner’s training choice; a worker-side training subsidy is also required to correct the skill margin. In a U.S. calibration, a production-only policy reaches the same 14 percent green employment benchmark as the coordinated policy, but with lower green human capital and lower output. The two policies are not substitutes: reaching the right employment level does not guarantee the right skill intensity.

Governments seeking to revitalize manufacturing often face a chicken-and-egg problem: firms hesitate to expand without skilled workers, while workers are reluctant to acquire those skills without job prospects. Chapter 2 asks whether public support should target firms, workers,

or both. It studies manufacturing development in a two-sector open economy with search frictions and endogenous job-specific capital. The key friction is capital hold-up: investment is sunk before wage bargaining, discouraging job creation most severely in manufacturing, where capital intensity is highest. The calibrated shortfall is about one percent of total employment, roughly one-fifth of the Latin America–East Asia industrial employment gap. Whether worker-side support is also needed depends on whether search externalities are efficiently internalized: when they are, an investment subsidy financed by a firm-level employment tax is sufficient; when they are not, workers’ sectoral choices are also distorted and targeted training support is needed alongside.

The first step in a worker’s career is often particularly hard: an entry-level job is required in order to get an entry-level job. Chapter 3 studies the school-to-work transition through this lens. Firms bear training costs when they hire inexperienced workers and therefore rank them below experienced applicants; entrants who remain unemployed too long face a persistent skill-loss shock, a cost borne by future employers rather than the firm that turned them away. Standard interventions such as anti-discrimination law and hiring subsidies improve welfare, but modestly. The largest gains come from subsidizing entrant hiring so heavily that firms actually prefer inexperienced workers over experienced ones. This reversal of the natural ranking generates a welfare gain of 1.51 percent and brings the economy close to the constrained-efficient allocation.

Together, the three chapters show that employment expansion is not a sufficient guide to policy design in frictional labor markets. Similar employment outcomes can mask distinct underlying distortions: worker underinvestment in green human capital, firm underinvestment in manufacturing capital, or the systematic disfavoring of inexperienced workers at labor market entry. Because these wedges operate on different margins of the matching process, a policy that raises employment on one margin need not restore efficiency on another. The relevant policy question is not simply how much employment to create, but which margin of matching and investment must be corrected to create it efficiently.

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~ *That is knowledge which liberates.* ~

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Chapter 1

Green Jobs Require Green Skills:

Why Production Subsidies Are Not Enough

Abstract

Large structural transitions require both firms and workers to act: firms must create jobs in expanding sectors, and workers must acquire the skills to move into them. Standard climate policy focuses on the firm side, but this chapter asks whether production support alone is sufficient for an efficient green transition, or whether worker-side support is also needed. I develop a two-sector Diamond–Mortensen–Pissarides model with sectoral choice and pre-match human-capital investment. Green workers acquire sector-specific skills before matching with firms; because training is sunk at the time of wage bargaining, workers do not internalize the full social return to their investment, even under the Hosios condition. A Pigouvian production subsidy corrects the environmental wedge and aligns vacancy creation for a given level of human capital, but it does not implement the planner’s training choice; exact decentralization also requires a worker-side training subsidy. In a calibration to the United States, a production-only policy reaches the same 14 percent green employment benchmark as the coordinated policy, but with lower green human capital and lower output. The break-even environmental value of green output for the coordi-

nated transition is 1.13 percent of the numeraire output unit. The results show that policy design matters beyond the scale of green employment: production support can create green jobs, but worker-side support determines the skill intensity and productivity of that expansion.

1.1 Introduction

Large structural transitions require firms to create jobs in expanding sectors and workers to move into those jobs. Policy often acts on these margins unevenly. Production incentives raise labor demand in an expanding sector, but workers must still decide whether to acquire the skills needed to enter it. This distinction arises in many recent reallocations, including advanced manufacturing, digital adoption, industrial policy, and the green transition.

The green transition is a leading example. Decarbonization requires cleaner technologies and capital, but it also requires labor to move into green production. Some of this movement is across sectors and regions; some is within occupations, as existing jobs add new tasks, certifications, and technical requirements. The political economy of climate policy depends in part on whether workers can make these transitions without prolonged unemployment or concentrated losses. Empirical work shows that environmental regulation and structural change can impose sizable and persistent adjustment costs on workers (Walker, 2011, 2013; Curtis, 2018; Gray et al., 2014). At the same time, evidence on green labor markets points to skill shortages and rising demand for green capabilities (International Labour Office, 2019; Bluedorn et al., 2023; International Energy Agency, 2023; OECD, 2023; Vona et al., 2018). Together, these facts suggest that green transition policy should be evaluated not only by how much green activity it creates, but also by whether workers can acquire the skills needed to staff that activity.

Most climate policy instruments operate on the production side. Carbon prices, clean production subsidies, tax credits, and investment incentives raise the return to clean activity and encourage firms to expand green production. These instruments are the natural correction for the environmental externality. They may be incomplete, however, when green production requires

worker-provided skills. If workers must acquire sector-specific human capital before matching with firms, they pay the investment cost before the match is formed and before wages are bargained. Once the return is split through bargaining, workers capture only part of the marginal return to their investment. Production support raises the value of green matches, but it does not by itself make workers internalize the full return to green-specific human capital.

This chapter asks whether production support alone can implement an efficient green labor transition, or whether worker-side support is also needed once green human capital is chosen before matching. The question is not whether to support green production at all. I take the environmental externality as the reason for expanding the green sector and study how labor-market frictions alter the policy package that delivers that expansion.

I develop a two-sector Diamond–Mortensen–Pissarides model with a green and a non-green sector. Workers first choose which sectoral labor market to enter. Green entrants then choose green-specific human capital before matching. Firms post vacancies in each sector, matches form through a standard matching function, and wages are set by Nash bargaining after the worker’s training is sunk. Green production generates an external social benefit that private agents do not internalize.

The model separates two policy margins. A green production subsidy raises the value of green matches and encourages vacancy creation and sectoral entry. A worker-side training subsidy acts directly on green workers’ pre-match human-capital choice. The two instruments correct different wedges: production support corrects the environmental wedge, while training support corrects the worker-side hold-up wedge.

I focus on the Hosios benchmark, in which the worker bargaining weight equals the matching elasticity. This shuts down the standard search externality and isolates the worker-side hold-up. Under Hosios, a Pigouvian production subsidy aligns the decentralized vacancy-creation and sectoral-allocation conditions with the planner’s conditions for a given level of green human capital, but it does not decentralize the planner’s training choice. The decentralized worker pays the full marginal cost of training but receives only the bargained share of the marginal return, whereas

the planner internalizes the full marginal value created when a trained green searcher finds a job. Exact decentralization therefore requires a worker-side training subsidy. It also requires a small entry offset that prevents the training subsidy from distorting the extensive sectoral-allocation margin. I treat this offset as a theoretical decentralization device rather than as a substantive policy proposal.

I quantify the model using U.S. data at an annual frequency. The baseline green employment share is 2.08 percent. For the main quantitative exercise, I use an external net-zero transition benchmark: WorkingNation’s Green Jobs Now project, based on MISI analysis, projects that jobs generated by the U.S. green economy could reach nearly 24 million by 2030, or about 14 percent of total U.S. jobs, under a net-zero transition path (WorkingNation and Management Information Services, Inc., 2022). I therefore set the externality parameter so that the planner selects a 14 percent green employment share. This is a benchmark transition scale for comparing policy instruments and for evaluating the environmental benefit required to justify the transition, not a model estimate of the optimal green employment share.

The quantitative results mirror the theory. Relative to the calibrated decentralized equilibrium, the planner raises the green employment share from 2.08 percent to 14 percent and raises green human capital, with little change in aggregate unemployment. The central policy comparison contrasts the coordinated policy that implements the planner allocation with a production-only policy chosen to reach the same 14 percent green employment share. Production support alone reaches the green employment target, but it does so with lower green human capital, lower green output per match, and lower total green output. Under the baseline conservative calibration, green output per match and total green output are each about 0.16 percent lower under production-only policy than under the coordinated policy. A final exercise evaluates the environmental value required for the coordinated transition to dominate the decentralized equilibrium. The break-even environmental benefit is 1.13 percent of the numeraire output unit.

The main lesson is that climate policy design matters, not only policy scale. Production support can expand green employment, but when green work requires costly pre-match skill invest-

ment, worker-side support changes the composition of that expansion. A production-only policy can deliver the same green headcount while falling short of the planner's skill intensity and productivity. The same logic applies more broadly to reallocations in which workers must make costly sector-specific investments before matching with expanding firms.

1.1.1 Relation to the Literature

This chapter relates first to the climate-macro literature on decarbonization and labor-market adjustment. A large body of work studies carbon pricing, clean subsidies, capital reallocation, and directed technical change as central forces in the green transition (Popp, 2002; Gerlagh, 2008; Acemoglu et al., 2012, 2016). These models clarify how environmental externalities distort production and innovation, but they typically treat labor reallocation and skill acquisition as frictionless relative to the technology and capital margins. A more recent literature incorporates unemployment, labor mobility, and adjustment costs more directly (Hafstead and Williams III, 2018; Hafstead et al., 2022; Castellanos and Heutel, 2024; Finkelstein Shapiro and Nuguer, 2024). Relative to that work, I isolate a policy-design margin: when green production requires worker-provided skills, production-side incentives and worker-side support act on different wedges. Labor-market policy is therefore not only a redistribution device that follows climate policy; it can be part of the efficiency problem itself.

Empirically, this chapter builds on evidence on worker adjustment, green jobs, and green skills. Studies of environmental regulation document that workers face sizable and persistent adjustment costs when policy reallocates activity across sectors and regions (Walker, 2011, 2013; Curtis, 2018; Gray et al., 2014). A separate literature documents that green jobs differ from non-green jobs in their task content, technical requirements, and human-capital intensity (Consoli et al., 2016; Vona et al., 2018; International Labour Office, 2019; Bluedorn et al., 2023; International Energy Agency, 2023; Tyros et al., 2023). These findings motivate the worker-side margin in the model. They also clarify what a production-only policy cannot deliver: even when it achieves the planner's green employment share, it need not achieve the planner's skill intensity of green

employment.

In the search-and-matching tradition, vacancy creation, unemployment, and wage bargaining interact in frictional labor markets (Diamond, 1982; Mortensen and Pissarides, 1994; Pissarides, 2000), and the Hosios condition gives the benchmark under which decentralized search decisions are constrained efficient (Hosios, 1990). I retain that environment and add a pre-match worker investment decision: green workers choose sector-specific human capital before matching, while wages are bargained after the investment is sunk. As a result, even when Hosios removes the standard search externality, the decentralized economy underinvests in green human capital. The inefficiency is not the usual vacancy-creation wedge but a worker-side hold-up wedge inside an otherwise standard search environment.

This wedge places this chapter in the literature on hold-up and match-specific investment under search frictions. Acemoglu and Shimer (1999) show how search frictions and bargaining distort ex-ante investment incentives when investment affects match surplus but is sunk before bargaining. Cardullo et al. (2015) study a two-sector search model in which sectoral differences in hold-up help explain cross-industry investment dynamics. Related work shows that search and bargaining frictions distort other match-related investments, including schooling choices (Flinn and Mullins, 2015), firm training incentives under wage compression (Acemoglu and Pischke, 1999a), and investments when match output depends on market tightness (Mangin and Julien, 2021). I apply the same logic to worker-provided green human capital: workers pay the training cost before matching, and the return is later split through wage bargaining.

The closest comparison is work on capital-side hold-up in sectoral reallocation. Adhikari and Guo (2026) study a two-sector search model in which firms make sunk capital investments before bargaining; because capital is sunk when wages are negotiated, firms do not capture the full marginal return to investment, distorting capital accumulation and sectoral allocation. I place the sunk investment on the opposite side of the match and keep the firm side close to a standard DMP vacancy-posting environment. The resulting policy lesson is different: even under Hosios, a production subsidy that corrects the green output externality does not implement the planner

allocation unless worker-side training incentives are also corrected.

Finally, this chapter relates to work on structural change and industrial policy. Models of structural transformation explain changes in sectoral shares through productivity differences, trade, and demand-side forces (Duarte and Restuccia, 2010; Uy et al., 2013; Huneeus and Rogerson, 2024). Industrial-policy models often emphasize coordination failures, increasing returns, or production-network spillovers as rationales for supporting particular sectors (Matsuyama, 1991; Liu, 2019). My focus is narrower: even in a constant-returns search environment, sectoral policy design changes once workers must make sunk skill investments before matching. Policies aimed at expanding a sector and policies aimed at workers are not substitutes; they correct different margins of the reallocation process.

The remainder of this chapter is organized as follows. Section 1.2 presents the model and characterizes the decentralized equilibrium, planner allocation, and policy implementation. Section 1.3 describes the calibration. Section 1.4 reports the quantitative results. The appendix provides derivations and robustness checks.

1.2 Model

This section develops a two-sector search model of labor reallocation during the green transition. The economy has a green sector, indexed by g , and a non-green sector, indexed by n . Workers choose a sectoral labor market before matching. Green entrants also choose sector-specific human capital before matching and before wage bargaining. Firms post vacancies in each sector, and matches form through a Diamond–Mortensen–Pissarides matching technology. The firm side is kept close to a standard DMP vacancy-posting environment, so the model does not study a separate firm-side sunk adoption or retooling hold-up. The focus is on how the green production externality interacts with worker-side pre-match human-capital hold-up.

Figure 1.1 motivates the worker-side friction. Projected green-skill demand rises faster than green-skill supply over the transition. The figure is not a calibration target; it motivates why the

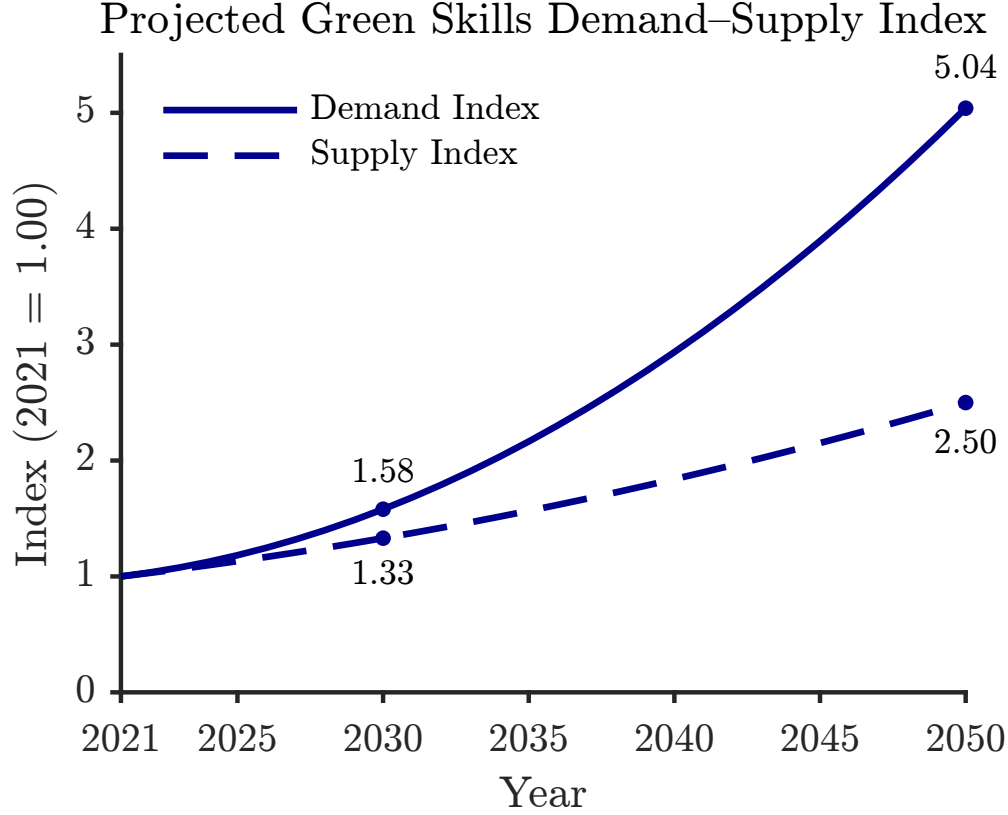


Figure 1.1. Projected green-skills demand and supply

Note: The figure plots indexes of projected green-skills demand and supply, both normalized to 2021. The gap between demand and supply widens over the horizon. *Source:* Author's calculations adapted from LinkedIn (2024).

green transition may require policies that address the worker training margin, not only policies that raise the value of green production.

1.2.1 Environment and Timing

Time is continuous and discounted at rate $r > 0$. A unit mass of risk-neutral workers and a continuum of risk-neutral firms populate the economy. Workers first choose a sectoral labor market and then search for jobs in that sector.

Let L_g and L_n denote the masses of workers attached to the green and non-green labor markets. The labor force is normalized to one,

$$L_g + L_n = 1, \tag{1.1}$$

and within each sector, unemployment u_j and employment n_j sum to the sectoral labor force,

$$L_j = u_j + n_j, \quad j \in \{g, n\}. \quad (1.2)$$

Entering the green sector requires a one-time reallocation cost with present value κ_g . I write this cost in flow-equivalent units as $r\kappa_g$, so that it can be compared with flow values in the sectoral-choice condition.¹ Entering the non-green sector is costless.² This cost captures search redirection, mobility, and other non-training costs associated with entering the green labor market.

After entering the green sector but before matching, green workers choose green-specific human capital $h_g \geq 0$. The choice captures training, certification, and other skill investments needed for green jobs. It requires a sunk one-time investment cost $C(h_g)$, incurred before matching. Assuming infinitely lived workers, I express this cost in flow-equivalent units as

$$\chi(h_g) \equiv rC(h_g), \quad (1.3)$$

with

$$\chi'(h_g) > 0, \quad \chi''(h_g) > 0. \quad (1.4)$$

The analytical model leaves χ unspecified; the quantitative section adopts a simple convex specification. The continuation value $J_g^U(h_g)$ is measured after h_g has been chosen, so the training cost does not enter the unemployment Bellman equation; it is subtracted in the worker's ex-ante training problem. Since h_g is chosen before matching and before wage bargaining, it is sunk at the time of bargaining.

¹In the quantitative model, I use a heterogeneous-entry-cost version of the sectoral-choice condition. Specifically, the homogeneous flow cost $r\kappa_g$ is replaced by the marginal entry cost $\bar{\kappa}_g L_g$, and aggregate entry costs in welfare are $K(L_g) = \bar{\kappa}_g L_g^2/2$. This modification preserves the analytical logic while allowing an interior green employment share.

²The zero cost of entering the non-green sector is a normalization. Equivalently, workers could pay a cost κ_n to enter the non-green sector and a larger cost $\kappa_g > \kappa_n$ to enter the green sector. In that case, the sectoral-choice condition depends on the relative reallocation cost $\kappa_g - \kappa_n$, and the results are unchanged.

Green output per filled job is $y_g(h_g)$, with

$$y'_g(h_g) > 0, \quad y''_g(h_g) < 0. \quad (1.5)$$

Non-green output per filled job is $y_n > 0$. Private agents value green output at $p_g y_g(h_g)$ and non-green output at $p_n y_n$. Green production also generates an external social benefit $\phi > 0$ that private agents do not internalize, so the planner values green output at $(p_g + \phi)y_g(h_g)$.

Figure 1.2 summarizes the sequence: workers first choose a sectoral labor market, green entrants choose h_g , firms post vacancies, matches form within each sector, and wages are bargained after h_g is sunk.

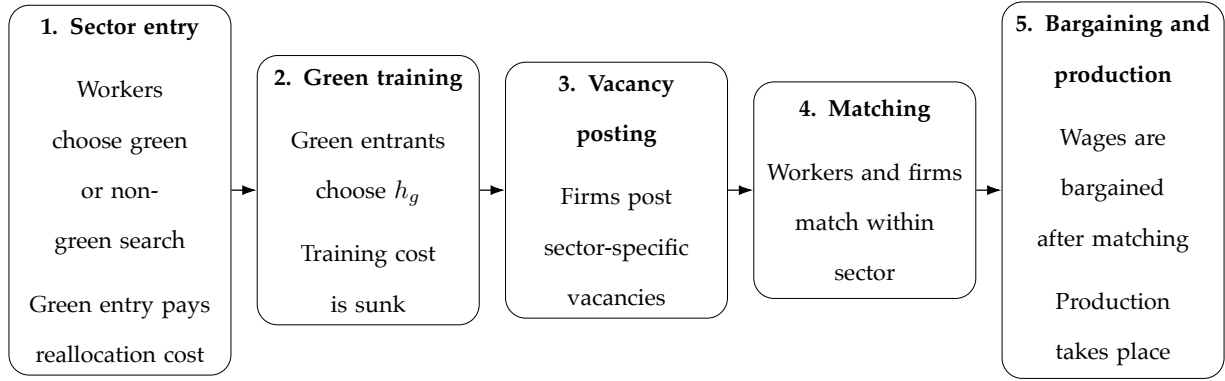


Figure 1.2. Timing of decisions

1.2.2 Labor Market and Production

Labor markets are segmented by sector. Figure 1.3 summarizes the stocks and flows. Workers enter either the green or the non-green labor market. Within each sector, unemployed workers find jobs at rate $f(\theta_j)$, and employed workers separate back into unemployment at rate λ .

In sector $j \in \{g, n\}$, unemployed workers u_j and vacancies v_j meet according to

$$m_j = M(u_j, v_j). \quad (1.6)$$

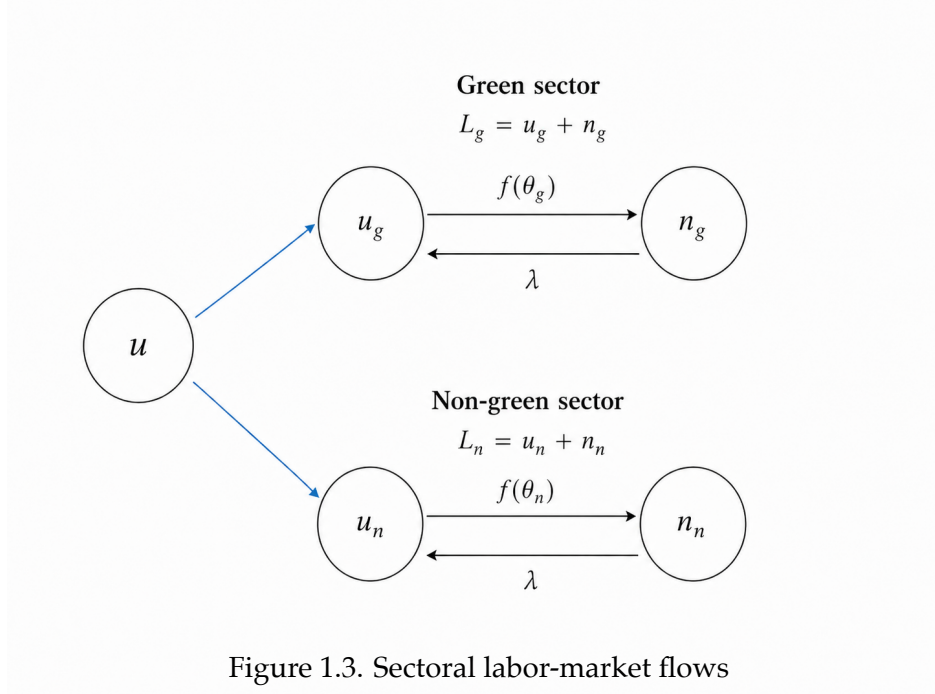


Figure 1.3. Sectoral labor-market flows

Note: Workers sort into the green and non-green labor markets. Within each sector $j \in \{g, n\}$, unemployed workers u_j become employed at job-finding rate $f(\theta_j)$, while employed workers n_j separate at rate λ . The sectoral labor force satisfies $L_j = u_j + n_j$.

I assume a constant-returns Cobb–Douglas matching function,

$$M(u_j, v_j) = A u_j^\eta v_j^{1-\eta}, \quad \eta \in (0, 1), \quad (1.7)$$

where η is the elasticity of matches with respect to unemployed searchers. Define market tightness as

$$\theta_j \equiv \frac{v_j}{u_j}. \quad (1.8)$$

The job-finding and vacancy-filling rates are

$$f(\theta_j) = \frac{M(u_j, v_j)}{u_j} = A \theta_j^{1-\eta}, \quad q(\theta_j) = \frac{M(u_j, v_j)}{v_j} = A \theta_j^{-\eta}. \quad (1.9)$$

Hence,

$$f(\theta_j) = \theta_j q(\theta_j). \quad (1.10)$$

Posting a vacancy costs $c > 0$ per unit of time, and matches separate at Poisson rate $\lambda > 0$. Employment in sector j then evolves as

$$\dot{n}_j = m_j - \lambda n_j, \quad j \in \{g, n\}. \quad (1.11)$$

Using $u_j = L_j - n_j$, $v_j = \theta_j u_j$, and $m_j = f(\theta_j)u_j$, the corresponding unemployment dynamics are

$$\dot{u}_j = \lambda(L_j - u_j) - f(\theta_j)u_j, \quad j \in \{g, n\}. \quad (1.12)$$

In steady state the inflow into employment equals the outflow,

$$\lambda n_j = f(\theta_j)u_j, \quad (1.13)$$

so that

$$u_j = \frac{\lambda}{\lambda + f(\theta_j)} L_j, \quad n_j = \frac{f(\theta_j)}{\lambda + f(\theta_j)} L_j. \quad (1.14)$$

The green employment share is

$$\ell_g = \frac{n_g}{n_g + n_n}. \quad (1.15)$$

Production in each sector is

$$Y_g = y_g(h_g)n_g, \quad (1.16)$$

$$Y_n = y_n n_n. \quad (1.17)$$

1.2.3 Decentralized Equilibrium

Let J_j^U and J_j^E denote a worker's values of unemployment and employment in sector j . Let J_j^V and J_j^F denote a firm's values of a vacancy and a filled job. Wages are denoted by w_j . In the green sector, values depend on h_g , but I suppress this dependence when it does not create ambiguity.

The Bellman equations in the green sector are:

$$rJ_g^U = z + f(\theta_g)(J_g^E - J_g^U), \quad (1.18)$$

$$rJ_g^E = w_g + \lambda(J_g^U - J_g^E), \quad (1.19)$$

$$rJ_g^V = -c + q(\theta_g)(J_g^F - J_g^V), \quad (1.20)$$

$$rJ_g^F = p_g y_g(h_g) - w_g + \lambda(J_g^V - J_g^F). \quad (1.21)$$

The non-green Bellman equations are analogous, with $p_g y_g(h_g)$ replaced by $p_n y_n$. The values J_g^U and J_g^E are continuation values, evaluated after green entry and after h_g has been chosen, so the reallocation and training costs do not appear inside the Bellman equations; they enter the ex-ante entry and training problems below.

After a worker and firm match, the wage is determined by Nash bargaining. Let $\xi \in (0, 1)$ denote the worker's bargaining weight. Define worker surplus, firm surplus, and total surplus in sector j as

$$W_j \equiv J_j^E - J_j^U, \quad F_j \equiv J_j^F - J_j^V, \quad S_j \equiv W_j + F_j. \quad (1.22)$$

Nash bargaining implies

$$W_j = \xi S_j, \quad F_j = (1 - \xi) S_j. \quad (1.23)$$

Free entry by firms implies

$$J_j^V = 0, \quad j \in \{g, n\}. \quad (1.24)$$

Green workers choose h_g before matching, taking θ_g as given. The green worker chooses h_g to maximize the value of entering green search net of the sunk training cost:

$$h_g^* = \arg \max_{h_g \geq 0} \{rJ_g^U(h_g) - \chi(h_g)\}. \quad (1.25)$$

For both sectors to be active, workers must be indifferent between entering the green and non-

green labor markets:

$$rJ_g^U - rJ_n^U = r\kappa_g + \chi(h_g). \quad (1.26)$$

This condition compares continuation values before the entry and training costs are paid. Since $J_g^U(h_g)$ is measured after h_g is chosen, $\chi(h_g)$ appears on the cost side of the ex-ante sectoral-choice condition rather than inside the Bellman equation.

A stationary decentralized equilibrium consists of sectoral labor allocations, market tightness levels, green human capital, wages, and value functions such that workers and firms satisfy the Bellman equations, wages satisfy Nash bargaining, firms satisfy free entry, green workers choose human capital according to (1.25), workers' sectoral choices satisfy (1.26), labor-market flows satisfy (1.14), and $L_g + L_n = 1$.

Characterization

I now characterize the stationary decentralized equilibrium. The derivation proceeds in four steps. First, I derive worker and firm surplus from the Bellman equations. Second, I use free entry to characterize equilibrium tightness. Third, I derive the green human-capital choice. Fourth, I derive the workers' sectoral-choice condition. The characterization assumes an interior solution for h_g and for sectoral choices. The relevant second-order conditions are satisfied under the maintained convexity of χ and the functional forms used in the quantitative exercises.

Worker surplus. Subtracting (1.18) from (1.19) gives

$$r(J_g^E - J_g^U) = w_g - z - (\lambda + f(\theta_g))(J_g^E - J_g^U). \quad (1.27)$$

Using $W_g = J_g^E - J_g^U$, this becomes

$$(r + \lambda + f(\theta_g))W_g = w_g - z. \quad (1.28)$$

Using Nash bargaining, $W_g = \xi S_g$, so

$$w_g - z = \xi(r + \lambda + f(\theta_g))S_g. \quad (1.29)$$

The analogous non-green expression is

$$w_n - z = \xi(r + \lambda + f(\theta_n))S_n. \quad (1.30)$$

Firm surplus. Using (1.21), the green firm surplus satisfies

$$rJ_g^F = p_g y_g(h_g) - w_g + \lambda(J_g^V - J_g^F). \quad (1.31)$$

Rearranging,

$$(r + \lambda)J_g^F = p_g y_g(h_g) - w_g + \lambda J_g^V. \quad (1.32)$$

Free entry implies $J_g^V = 0$. Therefore $F_g = J_g^F - J_g^V = J_g^F$, and

$$(r + \lambda)F_g = p_g y_g(h_g) - w_g. \quad (1.33)$$

Using Nash bargaining, $F_g = (1 - \xi)S_g$, so

$$p_g y_g(h_g) - w_g = (1 - \xi)(r + \lambda)S_g. \quad (1.34)$$

The analogous non-green expression is

$$p_n y_n - w_n = (1 - \xi)(r + \lambda)S_n. \quad (1.35)$$

Total match surplus. Adding (1.29) and (1.34) eliminates w_g and collects terms in S_g :

$$\begin{aligned}
p_g y_g(h_g) - z &= \xi(r + \lambda + f(\theta_g))S_g + (1 - \xi)(r + \lambda)S_g \\
&= (r + \lambda + \xi f(\theta_g))S_g.
\end{aligned} \tag{1.36}$$

Solving for green and non-green surplus,

$$S_g(h_g, \theta_g) = \frac{p_g y_g(h_g) - z}{r + \lambda + \xi f(\theta_g)}, \tag{1.37}$$

$$S_n(\theta_n) = \frac{p_n y_n - z}{r + \lambda + \xi f(\theta_n)}. \tag{1.38}$$

The term $\xi f(\theta_j)$ in the denominator captures the worker's option value of future job finding inside the bargained surplus.

Free entry and market tightness. Setting $J_j^V = 0$ in the vacancy Bellman equation gives

$$0 = -c + q(\theta_j)(J_j^F - J_j^V), \tag{1.39}$$

so that the per-period vacancy cost equals the expected gain from filling the vacancy,

$$c = q(\theta_j)F_j. \tag{1.40}$$

Substituting $F_j = (1 - \xi)S_j$ from Nash bargaining,

$$c = (1 - \xi)q(\theta_j)S_j. \tag{1.41}$$

Plugging in the surplus expressions (1.37) and (1.38) yields the sectoral free-entry conditions:

$$c = (1 - \xi)q(\theta_g) \frac{p_g y_g(h_g) - z}{r + \lambda + \xi f(\theta_g)}, \tag{1.42}$$

$$c = (1 - \xi)q(\theta_n) \frac{p_n y_n - z}{r + \lambda + \xi f(\theta_n)}. \tag{1.43}$$

Green human-capital choice. The worker's ex-ante training problem requires the flow value of being a green unemployed searcher. From the unemployment Bellman equation,

$$rJ_g^U = z + f(\theta_g)(J_g^E - J_g^U), \quad (1.44)$$

and substituting the Nash share $J_g^E - J_g^U = W_g = \xi S_g$,

$$rJ_g^U = z + \xi f(\theta_g) S_g(h_g, \theta_g). \quad (1.45)$$

Inserting (1.45) into the training problem (1.25), the worker chooses h_g to solve

$$\max_{h_g \geq 0} \{z + \xi f(\theta_g) S_g(h_g, \theta_g) - \chi(h_g)\}. \quad (1.46)$$

The first-order condition is

$$\chi'(h_g) = \xi f(\theta_g) \frac{\partial S_g(h_g, \theta_g)}{\partial h_g}, \quad (1.47)$$

and differentiating (1.37) with respect to h_g gives

$$\frac{\partial S_g(h_g, \theta_g)}{\partial h_g} = \frac{p_g y'_g(h_g)}{r + \lambda + \xi f(\theta_g)}. \quad (1.48)$$

Combining these two expressions yields the decentralized green human-capital condition,

$$\chi'(h_g) = \xi f(\theta_g) \frac{p_g y'_g(h_g)}{r + \lambda + \xi f(\theta_g)}. \quad (1.49)$$

The hold-up wedge is visible directly in (1.49): the worker pays the full marginal cost of training but receives only the bargaining share ξ of the marginal return when the green match forms.

Workers' sectoral choice. The flow value (1.45) extends symmetrically to either sector:

$$rJ_j^U = z + \xi f(\theta_j) S_j. \quad (1.50)$$

For both sectors to be active, workers must be indifferent between green and non-green search at the ex-ante margin:

$$rJ_g^U - rJ_n^U = r\kappa_g + \chi(h_g). \quad (1.51)$$

Substituting (1.50) on both sides, the common home-production term z cancels, and the indifference condition becomes

$$r\kappa_g + \chi(h_g) = \xi f(\theta_g)S_g(h_g, \theta_g) - \xi f(\theta_n)S_n(\theta_n). \quad (1.52)$$

Inserting the surplus expressions (1.37) and (1.38) gives the sectoral-choice condition in primitives:

$$r\kappa_g + \chi(h_g) = \xi f(\theta_g) \frac{p_g y_g(h_g) - z}{r + \lambda + \xi f(\theta_g)} - \xi f(\theta_n) \frac{p_n y_n - z}{r + \lambda + \xi f(\theta_n)}. \quad (1.53)$$

Proposition 1 (Decentralized equilibrium). *In an interior stationary decentralized equilibrium, the allocation $(h_g, \theta_g, \theta_n, L_g)$ is characterized by*

$$\{\text{Training choice}\} : \quad \chi'(h_g) = \xi f(\theta_g) \frac{p_g y'_g(h_g)}{r + \lambda + \xi f(\theta_g)}, \quad (1.54)$$

$$\{\text{Green tightness}\} : \quad c = (1 - \xi)q(\theta_g) \frac{p_g y_g(h_g) - z}{r + \lambda + \xi f(\theta_g)}, \quad (1.55)$$

$$\{\text{Non-green tightness}\} : \quad c = (1 - \xi)q(\theta_n) \frac{p_n y_n - z}{r + \lambda + \xi f(\theta_n)}, \quad (1.56)$$

$$\{\text{Sectoral choice}\} : \quad r\kappa_g + \chi(h_g) = \xi f(\theta_g) \frac{p_g y_g(h_g) - z}{r + \lambda + \xi f(\theta_g)} - \xi f(\theta_n) \frac{p_n y_n - z}{r + \lambda + \xi f(\theta_n)}. \quad (1.57)$$

Given L_g , employment and unemployment are determined by (1.14).

Proof. The Characterization subsection above derives the four conditions. The training condition (1.54) is the worker's first-order condition for h_g using (1.37). The tightness conditions (1.55)–(1.56) follow from free entry combined with (1.37)–(1.38). The sectoral-choice condition (1.57) follows from worker indifference at the ex-ante margin and substitution of the surplus expressions. $\square$

The characterization in Proposition 1 is the no-policy equilibrium. With a green production sub-

sidy τ_g , the policy-augmented version replaces $p_g y_g(h_g)$ with $(p_g + \tau_g) y_g(h_g)$ in the green tightness, training, and sectoral-choice conditions. With a worker-side training subsidy τ_h , the private cost of training becomes $(1 - \tau_h) \chi(h_g)$. The role of a green entry transfer is characterized below in Section 1.2.5.

1.2.4 Social Planner's Problem

I now characterize the constrained-efficient allocation. The planner chooses the allocation of workers across sectoral labor markets, market tightness in each sector, and green human capital. The planner faces the same matching technology and separation process as private agents, but internalizes the green production externality, the search externalities in matching, and the social return to green human capital.

For a fixed level of green human capital, the planner's flow payoff is

$$\begin{aligned} \mathcal{W} = & (p_g + \phi) y_g(h_g) n_g + p_n y_n n_n + z(u_g + u_n) \\ & - c(v_g + v_n) - r \kappa_g L_g - L_g \chi(h_g). \end{aligned} \quad (1.58)$$

The terms $r \kappa_g L_g$ and $L_g \chi(h_g)$ are the flow-equivalent costs of assigning workers to green search and equipping them with green-specific human capital. For a fixed h_g , the planner maximizes

$$\max_{\{\theta_g(t), \theta_n(t), L_g(t)\}_{t \geq 0}} \int_0^\infty e^{-rt} \mathcal{W}(t) dt \quad (1.59)$$

subject to, for each sector $j \in \{g, n\}$,

$$\dot{n}_j(t) = m_j(t) - \lambda n_j(t), \quad (1.60)$$

$$m_j(t) = f(\theta_j(t)) u_j(t), \quad (1.61)$$

$$u_j(t) = L_j(t) - n_j(t), \quad (1.62)$$

$$v_j(t) = \theta_j(t) u_j(t), \quad (1.63)$$

with

$$L_g(t) + L_n(t) = 1. \quad (1.64)$$

Because vacancies pay a flow posting cost rather than requiring sunk vacancy-creation investment, there is no separate law of motion for vacancy creation. The stock of vacancies is summarized by $v_j = \theta_j u_j$. Standard transversality conditions on the state variables $n_g(t)$ and $n_n(t)$ are assumed to hold as $t \rightarrow \infty$.

Green human capital is chosen before matching by workers already assigned to green search, so the planner's human-capital condition is a pre-match investment condition. A marginal increase in h_g changes neither the number of green searchers nor the number of vacancies; it raises the productivity of the match once a green searcher finds a job. The relevant marginal benefit of training is therefore the individual job-finding rate $f(\theta_g)$, not the congestion-adjusted marginal contribution of an additional searcher.

The full derivation is in Appendix A.1. For the tightness and sectoral-allocation margins, the Hamiltonian delivers the planner's shadow values of employment as

$$\mu_g = \frac{(p_g + \phi)y_g(h_g) - z}{r + \lambda + \eta f(\theta_g)}, \quad \mu_n = \frac{p_n y_n - z}{r + \lambda + \eta f(\theta_n)}. \quad (1.65)$$

The denominator contains $\eta f(\theta_j)$ because these margins change the measure of searchers or vacancies and therefore involve the matching externality. By contrast, the human-capital margin is intensive for workers already assigned to green search, so its first-order condition uses the individual transition rate $f(\theta_g)$.

Proposition 2 (Social planner). *In an interior steady-state planner allocation, the choices $(h_g, \theta_g, \theta_n, L_g)$ are characterized by*

$$\{\text{Green human capital}\} : \quad \chi'(h_g) = f(\theta_g) \frac{(p_g + \phi)y'_g(h_g)}{r + \lambda + f(\theta_g)}, \quad (1.66)$$

$$\{\text{Green tightness}\} : \quad c = (1 - \eta)q(\theta_g) \frac{(p_g + \phi)y_g(h_g) - z}{r + \lambda + \eta f(\theta_g)}, \quad (1.67)$$

$$\{\text{Non-green tightness}\} : \quad c = (1 - \eta)q(\theta_n) \frac{p_n y_n - z}{r + \lambda + \eta f(\theta_n)}, \quad (1.68)$$

$$\{\text{Sectoral allocation}\} : \quad r\kappa_g + \chi(h_g) = \eta f(\theta_g) \frac{(p_g + \phi)y_g(h_g) - z}{r + \lambda + \eta f(\theta_g)} - \eta f(\theta_n) \frac{p_n y_n - z}{r + \lambda + \eta f(\theta_n)}. \quad (1.69)$$

Given L_g , employment and unemployment are determined by (1.14).

Proof. See Appendix A.1. □

The distinction between (1.66) and (1.69) is important. A marginal increase in h_g by a worker already assigned to green search creates no matching congestion externality. It raises output only if that worker finds a job, which occurs at rate $f(\theta_g)$. By contrast, assigning an additional worker to green search changes the measure of green searchers and therefore affects other searchers through the matching technology. This is why the sectoral-allocation condition uses $\eta f(\theta_g)$, while the human-capital condition uses $f(\theta_g)$.

1.2.5 Efficiency

This section compares the decentralized equilibrium with the planner's allocation. I work at the Hosios benchmark,

$$\xi = \eta, \quad (1.70)$$

which shuts down the standard search externality and isolates the chapter's main mechanism: even when search externalities are removed, production support alone does not correct worker-side hold-up in green human capital.

Three potential wedges are present. First, private agents value green output at $p_g y_g(h_g)$, while the planner values it at $(p_g + \phi)y_g(h_g)$. Second, away from Hosios, bargaining weights do not coincide with matching elasticities. Third, green workers choose h_g before matching and wage bargaining; since this investment is sunk when wages are bargained, workers do not internalize the full social return to training.

A Pigouvian production subsidy

$$\tau_g = \phi \quad (1.71)$$

raises the private value of green output to its social value. Under Hosios, this aligns the decentralized tightness and sectoral-choice conditions with the planner's conditions for any given h_g . It does not follow that tightness and sectoral allocation are fully efficient: h_g affects green surplus, so any distortion in h_g propagates into θ_g and L_g . The point is that the first-order conditions for vacancy creation and entry are aligned at any given h_g ; the remaining primitive distortion is the worker's pre-match human-capital choice.

With $\tau_g = \phi$ and $\xi = \eta$, the decentralized training condition is

$$\chi'(h_g) = \xi f(\theta_g) \frac{(p_g + \phi)y'_g(h_g)}{r + \lambda + \xi f(\theta_g)}, \quad (1.72)$$

while the planner's condition is

$$\chi'(h_g) = f(\theta_g) \frac{(p_g + \phi)y'_g(h_g)}{r + \lambda + f(\theta_g)}. \quad (1.73)$$

Both conditions reflect search frictions: the return to training is realized only if a green searcher finds a job, and the match may later separate. The wedge between them is not that search frictions discount the return, but that bargaining further shrinks the worker's share of the marginal return. Concretely, for $\xi < 1$,

$$\frac{\xi f(\theta_g)}{r + \lambda + \xi f(\theta_g)} < \frac{f(\theta_g)}{r + \lambda + f(\theta_g)}.$$

Even under Hosios and a Pigouvian production subsidy, then, production support alone does not decentralize the planner's training choice. The worker internalizes only the bargained share of the marginal training return, while the planner internalizes the full social value generated once the green searcher finds a job.

Proposition 3 (Inefficiency under Hosios). *Suppose both sectors are active and $\xi = \eta$. Absent policy intervention, the decentralized allocation is inefficient because private agents do not internalize the green*

production externality and because green workers choose human capital before matching and bargaining. A Pigouvian production subsidy $\tau_g = \phi$ aligns the decentralized tightness and sectoral-choice conditions with the planner's conditions for a given h_g . However, the decentralized green human-capital choice remains inefficient. Hence production support alone does not decentralize the planner allocation.

Proof. Under $\xi = \eta$ and $\tau_g = \phi$, the policy-augmented free-entry and sectoral-choice conditions of Proposition 1 coincide with the planner's tightness and sectoral-allocation conditions (1.67)–(1.69) at any given h_g . The training condition is not corrected, however: comparing (1.72) and (1.73) and using $\xi < 1$,

$$\frac{\xi f(\theta_g)}{r + \lambda + \xi f(\theta_g)} < \frac{f(\theta_g)}{r + \lambda + f(\theta_g)},$$

so the private marginal benefit of training is strictly smaller than the social marginal benefit. The decentralized green human-capital choice is therefore inefficient even under Hosios and a Pigouvian production subsidy. $\square$

The planner allocation can be decentralized with two main instruments: a green production subsidy τ_g that corrects the environmental wedge, and a worker-side training subsidy τ_h that corrects the pre-match human-capital wedge.

With $\tau_g = \phi$, the decentralized training condition under a training subsidy is

$$(1 - \tau_h)\chi'(h_g) = \xi f(\theta_g) \frac{(p_g + \phi)y'_g(h_g)}{r + \lambda + \xi f(\theta_g)}. \quad (1.74)$$

Matching this with the planner's condition (1.73) pins down τ_h : the required subsidy satisfies

$$1 - \tau_h^* = \xi \frac{r + \lambda + f(\theta_g)}{r + \lambda + \xi f(\theta_g)}, \quad (1.75)$$

or equivalently,

$$\tau_h^* = 1 - \xi \frac{r + \lambda + f(\theta_g)}{r + \lambda + \xi f(\theta_g)}. \quad (1.76)$$

Under Hosios, (1.76) simplifies to

$$\tau_h^* = \frac{(1 - \xi)(r + \lambda)}{r + \lambda + \xi f(\theta_g)}, \quad (1.77)$$

evaluated at the planner allocation.

The training subsidy corrects the intensive human-capital margin, but it also lowers the total private cost of entering green search. To preserve the planner's extensive sectoral allocation, an offsetting entry transfer is needed. With an entry transfer s_g , the private sectoral-choice condition contains

$$r\kappa_g - s_g + (1 - \tau_h)\chi(h_g),$$

which matches the planner's resource cost $r\kappa_g + \chi(h_g)$ when

$$s_g^* = -\tau_h^*\chi(h_g). \quad (1.78)$$

Since $\tau_h^* > 0$ and $\chi(h_g) > 0$, the offset is negative: exact implementation requires a small entry fee that taxes back the entry-value effect of the training subsidy. This offset is a theoretical decentralization device that separates the intensive training margin from the extensive green-entry margin, not a substantive policy proposal.

Proposition 4 (Decentralizing the planner allocation). *Suppose both sectors are active and $\xi = \eta$. The constrained-efficient allocation can be decentralized by the policy package*

$$\tau_g^* = \phi, \quad \tau_h^* = \frac{(1 - \xi)(r + \lambda)}{r + \lambda + \xi f(\theta_g^{SP})}, \quad s_g^* = -\tau_h^*\chi(h_g^{SP}). \quad (1.79)$$

The production subsidy τ_g^ corrects the green production externality and aligns the tightness and sectoral-allocation conditions for a given h_g . The training subsidy τ_h^* corrects the worker-side human-capital wedge. The entry offset s_g^* preserves the planner's extensive sectoral allocation when the training subsidy changes the private cost of entering green search.*

Proof. With $\tau_g = \phi$, Proposition 3 aligns the policy-augmented tightness and sectoral-choice conditions with the planner's at any h_g . Equating the policy-augmented training condition (1.74) with the planner's training condition (1.73) yields (1.76), which under Hosios simplifies to (1.77). The training subsidy lowers the private cost of entering green search by $\tau_h^* \chi(h_g)$; for the private sectoral-choice condition to match the planner's resource cost $r\kappa_g + \chi(h_g)$, the entry transfer must satisfy $s_g^* = -\tau_h^* \chi(h_g)$. Evaluated at $(h_g^{SP}, \theta_g^{SP}, \theta_n^{SP}, L_g^{SP})$, all four decentralized first-order conditions coincide with the planner's. $\square$

Away from Hosios, bargaining shares no longer coincide with matching elasticities, and the standard search wedge reappears alongside the green externality and the worker-side training hold-up. Exact decentralization then also requires a vacancy-side correction, such as a vacancy subsidy or a bargaining-weight adjustment. Because this added wedge is the textbook Hosios inefficiency, the main analysis stays at the Hosios benchmark to isolate the worker-side mechanism; the non-Hosios case is taken up as a robustness check in Appendix A.2.1.

1.3 Calibration

I calibrate the model to the United States in 2022 at an annual frequency. The calibration disciplines the decentralized steady state before I turn to the planner allocation and the policy counterfactuals. Externally set parameters follow standard search-and-matching and macro-human-capital benchmarks; the remaining parameters are chosen jointly to match labor-market and green-sector moments. The baseline imposes the Hosios condition, $\xi = \eta$, so that the worker-side human-capital hold-up is not confounded with the standard search externality. The environmental benefit of green production, ϕ , is not calibrated from labor-market moments; it is varied in the quantitative exercises, and I compute the threshold value required for the green transition to be welfare-improving.

1.3.1 Functional Forms

The analytical model does not require specific functional forms for green output or training costs. For the quantitative exercises, I use the parsimonious specification

$$y_g(h_g) = \bar{y}_g h_g^\alpha, \quad \alpha = 0.8. \quad (1.80)$$

The curvature parameter α governs the elasticity of green output with respect to green-specific human capital. I set $\alpha = 0.8$ externally. Huggett et al. (2011) estimate a benchmark value of $\alpha = 0.70$ and consider sensitivity over the interval $[0.5, 0.9]$, so $\alpha = 0.8$ lies within that range while remaining close to a local-linear benchmark and still allowing diminishing returns. Robustness exercises consider alternative values of α , including the local-linear case $\alpha = 1$.

The training cost is specified as

$$\chi(h_g) = \frac{\chi_0}{2}(h_g - \underline{h})^2, \quad h_g \geq \underline{h}. \quad (1.81)$$

The shift parameter $\underline{h}$ lets the model discipline the marginal training condition and the average training-cost target separately. Economically, $\underline{h}$ can be read as a reduced-form measure of transferable baseline skills that workers bring into green jobs, with $h_g - \underline{h}$ the incremental green-specific skill gap. This reading is consistent with evidence that many workers in brown occupations carry skills that transfer to green occupations, while some groups require more reskilling, and that green jobs on average require more education, work experience, and job training than non-green jobs (Tyros et al., 2023; Consoli et al., 2016; Vona et al., 2015). I do not treat $\underline{h}$ as an independently measured object. Given the normalization $h_g^{DE} = 1$, the pair $(\chi_0, \underline{h})$ is pinned down jointly by the worker's baseline training first-order condition and the training-expenditure target described below.

For the extensive margin, the analytical model uses a representative green-entry cost to keep the equilibrium conditions transparent. In the quantitative model I replace this with a heteroge-

neous entry-cost schedule, which generates an interior green employment share. Workers differ in their flow-equivalent cost of entering green search,

$$\tilde{\kappa}_i \sim U[0, \bar{\kappa}_g]. \quad (1.82)$$

If a mass L_g of workers enters the green sector, the marginal entrant has cost

$$\tilde{\kappa}(L_g) = \bar{\kappa}_g L_g, \quad (1.83)$$

and the aggregate resource cost of green entry is

$$K(L_g) = \int_0^{L_g} \bar{\kappa}_g x \, dx = \frac{\bar{\kappa}_g}{2} L_g^2. \quad (1.84)$$

This delivers a smooth interior green employment share while keeping the interpretation of green entry costs as worker-level heterogeneity in reallocation costs.

The heterogeneous-entry-cost specification changes only the extensive-margin condition. Because $\tilde{\kappa}_i$ is in flow-equivalent units, the quantitative decentralized sectoral-choice condition replaces the homogeneous flow cost $r\kappa_g$ with the marginal cost $\bar{\kappa}_g L_g$:

$$\bar{\kappa}_g L_g + \chi(h_g) = \xi f(\theta_g) S_g(h_g, \theta_g) - \xi f(\theta_n) S_n(\theta_n). \quad (1.85)$$

Using the surplus expressions from the analytical model, this can be written as

$$\bar{\kappa}_g L_g + \chi(h_g) = \xi f(\theta_g) \frac{p_g y_g(h_g) - z}{r + \lambda + \xi f(\theta_g)} - \xi f(\theta_n) \frac{p_n y_n - z}{r + \lambda + \xi f(\theta_n)}. \quad (1.86)$$

Under policy, the corresponding sectoral-choice condition is

$$\bar{\kappa}_g L_g - s_g + (1 - \tau_h) \chi(h_g) = \xi f(\theta_g) \frac{(p_g + \tau_g) y_g(h_g) - z}{r + \lambda + \xi f(\theta_g)} - \xi f(\theta_n) \frac{p_n y_n - z}{r + \lambda + \xi f(\theta_n)}. \quad (1.87)$$

The quantitative planner condition is modified in the same way. The planner's sectoral-allocation condition replaces the homogeneous cost $r\kappa_g$ with the marginal resource cost $K'(L_g) = \bar{\kappa}_g L_g$:

$$\bar{\kappa}_g L_g + \chi(h_g) = \eta f(\theta_g) \frac{(p_g + \phi)y_g(h_g) - z}{r + \lambda + \eta f(\theta_g)} - \eta f(\theta_n) \frac{p_n y_n - z}{r + \lambda + \eta f(\theta_n)}. \quad (1.88)$$

Aggregate welfare uses the total resource cost $K(L_g)$, not the marginal cost. Thus, the quantitative welfare expression replaces $r\kappa_g L_g$ with $K(L_g) = \bar{\kappa}_g L_g^2/2$. The rest of the equilibrium system is unchanged.

1.3.2 Externally Set Parameters

Table 1.1 reports the externally set parameters.

Table 1.1. Externally Set Parameters

Parameter	Description	Value	Source / rationale
L	Total labor force	1	Normalization
$p_n y_n$	Non-green output per match	1	Numeraire
p_g	Private green output price	1	Normalization
h_g^{DE}	Baseline green human capital	1	Normalization
r	Annual discount rate	0.05	Standard annual macro value
η	Matching elasticity	0.5	Petrongolo and Pissarides (2001)
ξ	Worker bargaining weight	0.5	Hosios benchmark, $\xi = \eta$
z	Flow value of unemployment	0.4	Low outside-option benchmark, Shimer (2005)
α	Curvature of green output	0.8	Benchmark 0.70; sensitivity [0.5, 0.9], Huggett et al. (2011)
ϕ	External value of green output	–	Threshold and policy exercises

Note: The parameter ϕ is not fitted in the baseline calibration; it is varied in the planner and policy exercises.

The labor force is normalized to one and non-green output per match is the numeraire, $p_n y_n = 1$. I set $p_g = 1$, so that $\bar{y}_g$ captures relative green productivity, and normalize baseline green human capital to $h_g^{DE} = 1$. The annual discount rate is $r = 0.05$. The matching elasticity is $\eta = 0.5$, a standard value in the DMP literature (Petrongolo and Pissarides, 2001), and the worker

bargaining weight is set equal to it, $\xi = \eta$, to impose Hosios. The flow value of unemployment is $z = 0.4$, consistent with the low outside-option calibration used in standard search models (Shimer, 2005).

1.3.3 Internally Calibrated Parameters

Table 1.2 reports the internally calibrated parameters. The separation rate λ is taken from CPS employment-to-unemployment flows, converting the monthly transition probability into an annual continuous-time hazard (U.S. Bureau of Labor Statistics, 2024). Matching efficiency A is chosen so that the model matches aggregate unemployment given the observed tightness and separation rate, and the vacancy posting cost c is chosen to match aggregate labor-market tightness.

The productivity scale $\bar{y}_g$ is chosen to match a green/non-green wage premium of 1.02. This is a conservative 2% wage-premium target, in line with the estimates in Bergant et al. (2022).³

The training-cost parameters $(\chi_0, \underline{h})$ are identified jointly. At the baseline normalization $h_g^{DE} = 1$, the decentralized training first-order condition pins down the marginal cost of green-specific training:

$$\chi_0(1 - \underline{h}) = \xi f(\theta_g) \frac{p_g \alpha \bar{y}_g}{r + \lambda + \xi f(\theta_g)}. \quad (1.89)$$

The level of the training cost is disciplined using general employer training-expenditure data as a conservative proxy, not as a direct estimate of green-specific reskilling costs. Training Magazine reports average employer training spending of \$1,207 per learner in 2022 (Training Magazine, 2022). To keep the benchmark conservative, I target annual flow-equivalent training expenditures of \$1,000 per worker. Relative to the 2022 Social Security Administration national average wage index of \$63,795.13 (Social Security Administration, 2023), this gives

$$\frac{1000}{63795.13} = 0.0157,$$

³Other studies report larger premiums, about 4 percent in Bircan et al. (2023) and about 21 percent on average in Curtis and Marinescu (2023), with even higher premiums for jobs with lower educational requirements. I adopt the more conservative figure.

so I target

$$\frac{\chi(1)}{w_g} = 0.015. \quad (1.90)$$

Since $\chi(1) = rC(1)$, at $r = 0.05$ this corresponds to a present-value one-time training cost of

$$\frac{C(1)}{w_g} = \frac{0.015}{0.05} = 0.30.$$

I do not interpret this target as a direct measure of green-specific training costs. Rather, it uses general employer training expenditures to impose a conservative dollar benchmark on the worker-side investment margin. The green-specific reading is consistent with evidence that green occupations require more technical skills, preparation, and training than non-green occupations (Consoli et al., 2016; Vona et al., 2018, 2015; Tyros et al., 2023).

Finally, the green-entry-cost scale $\bar{\kappa}_g$ is chosen to match the baseline green employment share.

Table 1.2. Internally Calibrated Parameters and Implied Training Objects

Object	Description	Value	Target / equation
λ	Separation rate	0.1198	CPS $E \rightarrow U$ flow, annualized
A	Matching efficiency	2.2814	Aggregate unemployment
c	Vacancy posting cost	0.2898	Aggregate tightness
$\bar{y}_g$	Green productivity scale	1.0199	Green/non-green wage premium
$\bar{\kappa}_g$	Green-entry cost scale	0.1944	Green employment share
$\underline{h}$	Training-cost shift	0.9597	Training FOC and expenditure target
$\chi(1)$	Flow-equivalent training cost	0.01485	Training expenditure target
$\chi(1)/w_g$	Training cost relative to wage	0.0150	Training expenditure target

Note: λ , A , c , $\bar{y}_g$, and $\bar{\kappa}_g$ are calibrated parameters; the pair $(\chi_0, \underline{h})$ is identified jointly from the worker's baseline training first-order condition and the training-expenditure target, as described in the text. $\chi(1)$ and $\chi(1)/w_g$ are implied objects, not free parameters. All values are evaluated at the calibrated decentralized steady state with $h_g^{DE} = 1$.

1.3.4 Targeted Moments and Model Fit

Table 1.3 reports the targeted moments and their model counterparts. The unemployment rate is the 2022 annual average from the CPS (U.S. Bureau of Labor Statistics, 2023). Labor-market tightness is computed as JOLTS job openings divided by the CPS unemployment level (Federal

Reserve Bank of St. Louis, 2024; U.S. Bureau of Labor Statistics, 2023). The green employment share is constructed using USEER clean-energy employment, approximately 3.315 million workers in 2022, divided by 158.291 million employed workers in the CPS annual averages (Environmental Entrepreneurs, 2023; U.S. Department of Energy, 2023; U.S. Bureau of Labor Statistics, 2023). This yields a baseline green employment share of 0.02085.

Table 1.3. Targeted Moments

Moment	Data	Model
Aggregate unemployment rate	0.0370	0.0370
Aggregate labor-market tightness	1.8680	1.8680
Separation rate	0.1198	0.1198
Green employment share	0.02085	0.02085
Green/non-green wage premium	1.0200	1.0200
Training cost share, $\chi(1)/w_g$	0.0150	0.0150

Note: All moments are for the United States in 2022 at annual frequency. The aggregate unemployment rate and separation rate are CPS-based (U.S. Bureau of Labor Statistics, 2023, 2024); labor-market tightness is JOLTS job openings divided by the CPS unemployment level (Federal Reserve Bank of St. Louis, 2024; U.S. Bureau of Labor Statistics, 2023). The green employment share uses USEER clean-energy employment relative to CPS total employment (Environmental Entrepreneurs, 2023; U.S. Department of Energy, 2023; U.S. Bureau of Labor Statistics, 2023). The green/non-green wage premium target follows Bergant et al. (2022). The training-cost share is constructed from Training Magazine (2022) relative to the 2022 SSA national average wage index (Social Security Administration, 2023).

1.4 Quantitative Results

1.4.1 Decentralized Equilibrium versus Social Planner

Table 1.4 compares the calibrated decentralized equilibrium with the social planner’s allocation. The external value of green production is not pinned down by labor-market moments, so I discipline the benchmark planner allocation with an external transition target. WorkingNation’s Green Jobs Now project, based on MISI analysis, projects that jobs generated by the U.S. green economy

could reach nearly 24 million by 2030, or about 14 percent of total U.S. jobs, under a net-zero transition path (WorkingNation and Management Information Services, Inc., 2022). I therefore set $\phi = \phi^{14}$, with $\phi^{14} = 0.0238$ chosen so that the planner's green employment share equals 14 percent. This target defines a benchmark policy experiment; it is not a model estimate of the optimal green employment share.

Table 1.4. Decentralized Equilibrium and Social Planner Allocation

Moment	DE	SPP	Change
Green employment share, ℓ_g	2.08%	14.00%	+11.92 pp
Green human capital, h_g	1.000	1.003	+0.31%
Aggregate unemployment	3.70%	3.68%	-0.02 pp
Unemployment rate, green	3.64%	3.56%	-0.08 pp
Unemployment rate, non-green	3.70%	3.70%	-0.00 pp
Tightness, green θ_g	1.932	2.020	+0.088
Tightness, non-green θ_n	1.867	1.867	+0.000
Green output per match, $y_g(h_g)$	1.020	1.022	+0.25%
Aggregate private output	0.963	0.966	+0.29%
Aggregate social output	0.964	0.969	+0.58%

Note: The table compares the calibrated decentralized equilibrium (DE) with the social planner (SPP) allocation. The external value of green output is set to $\phi^{14} = 0.0238$, the value that makes the planner's green employment share equal to 14 percent. The baseline imposes the Hosios condition, $\xi = \eta = 0.5$. Aggregate private output is $Y_g + Y_n$; aggregate social output adds the external value of green production, $\phi y_g(h_g)n_g$. The 14 percent target is an external net-zero transition benchmark and is not interpreted as a model estimate of the optimal green employment share. Changes are reported in percentage points for shares and rates, and in percent for levels.

Relative to the decentralized equilibrium, the planner chooses a much larger green sector. The green employment share rises from 2.08 percent to 14.00 percent. This reflects the green production externality: private agents value green output at $p_g y_g(h_g)$, while the planner values it at $(p_g + \phi) y_g(h_g)$. Internalizing this social value raises the surplus from green matches and the marginal value of allocating workers to green production.

The planner also chooses higher green human capital, with h_g rising from 1.000 to 1.003. The increase is modest because h_g is chosen before matching, so its return is realized only when a

green searcher finds a job. The direction is nevertheless consistent with the worker-side hold-up mechanism: the worker chooses training before bargaining, while the planner internalizes the full social return generated by green-specific skills.

The non-green labor market barely moves. Non-green tightness remains at $\theta_n = 1.867$, and the non-green unemployment rate remains essentially 3.70 percent. Aggregate unemployment falls slightly, from 3.70 percent to 3.68 percent. The main planner effect is therefore not a change in aggregate unemployment but a reallocation toward green production, together with a small increase in green skill intensity. Aggregate private output rises from 0.963 to 0.966, while aggregate social output rises from 0.964 to 0.969.

1.4.2 Policy under Hosios

This section studies policy implementation under the Hosios benchmark, $\xi = \eta$. The goal is to separate the role of green production support from the role of worker-side training support. I keep $\phi = \phi^{14} = 0.0238$, the benchmark external value that makes the planner choose a 14 percent green employment share.

Implementing the Planner Allocation

Under Hosios, a Pigouvian production subsidy corrects the green production externality, but it does not by itself correct the worker-side human-capital margin. Proposition 4 shows that the planner allocation can be implemented by

$$\tau_g^* = \phi^{14}, \quad \tau_h^* = 1 - \xi \frac{r + \lambda + f(\theta_g^{SP})}{r + \lambda + \xi f(\theta_g^{SP})}, \quad s_g^* = -\tau_h^* \chi(h_g^{SP}).$$

Under Hosios, this training subsidy can equivalently be written as

$$\tau_h^* = \frac{(1 - \xi)(r + \lambda)}{r + \lambda + \xi f(\theta_g^{SP})}.$$

Under the baseline calibration, this gives

$$\tau_g^* = 0.0238, \quad \tau_h^* = 0.0474, \quad s_g^* = -0.0008.$$

The production subsidy raises the private value of green matches to their social value, and the training subsidy corrects the remaining pre-match human-capital wedge. Because the return to training is realized only when a green searcher finds a job, the exact value of τ_h^* depends on the job-finding rate $f(\theta_g^{SP})$. The entry offset $s_g^* < 0$ removes the extensive-margin effect of the training subsidy: without it, the training subsidy would also make green entry too attractive. The offset is very small in the baseline calibration, $s_g^* = -0.0008$. I therefore treat it as a theoretical decentralization device rather than as a policy proposal, and the quantitative comparison below focuses on the contrast between production support and worker-side training support.

By construction, the implementing package reproduces the planner allocation in Table 1.4: the green employment share is 14.00 percent, green human capital is $h_g = 1.003$, and green output per match is 1.022. Implementation therefore requires an expansion of green employment together with a modest increase in green-specific human capital.

Production-Only Policy with the Same Employment Target

I next consider a restricted policy that uses only production support. To make the comparison clean, I choose the production subsidy endogenously so that the restricted policy reaches the same green employment share as the planner allocation:

$$\ell_g = 14\%, \quad \tau_h = 0, \quad s_g = 0.$$

This exercise asks what production support misses conditional on achieving the same green employment target. The required production-only subsidy is

$$\tau_g^{PO} = 0.0238.$$

Table 1.5 reports the comparison. Production-only policy reaches the 14 percent green employment target, but it does not exactly implement the planner allocation. The remaining difference is on the human-capital margin: green human capital is $h_g = 1.003$ under the coordinated policy and $h_g = 1.001$ under production-only policy. The gap is quantitatively modest, but it goes in the direction predicted by the theory. Production support expands green employment, and worker-side support raises the skill intensity of green matches.

Table 1.5. Common-Target Policy Comparison under Hosios

Moment	Coordinated policy	Production-only target	Difference
Policy: production subsidy, τ_g	0.0238	0.0238	+0.0000
Policy: training subsidy, τ_h	0.0474	0.0000	−0.0474
Policy: entry offset, s_g	−0.0008	0.0000	+0.0008
Green employment share, ℓ_g	14.00%	14.00%	−0.00 pp
Green human capital, h_g	1.003	1.001	−0.20%
Green training gap, $h_g - \underline{h}$	0.043	0.041	−4.71%
Green tightness, θ_g	2.020	2.014	−0.27%
Green unemployment rate	3.56%	3.57%	+0.00 pp
Aggregate unemployment	3.68%	3.68%	+0.00 pp
Green output per match, $y_g(h_g)$	1.022	1.021	−0.16%
Green output, $G = y_g(h_g)n_g$	0.1379	0.1376	−0.16%
Aggregate private output	0.9662	0.9660	−0.02%
Aggregate social output at ϕ^{14}	0.9695	0.9692	−0.02%
Gross prod subsidy cost, % output	0.34%	0.34%	+0.00 pp

Note: Both policies are chosen to reach a 14 percent green employment share. The coordinated policy implements the planner allocation under Hosios; the production-only policy sets $\tau_h = s_g = 0$ and chooses τ_g to reach the same green employment target. The training subsidy is evaluated at the planner allocation using the pre-match training condition in Proposition 4. Differences in shares and rates are in percentage points; differences in levels (h_g , training gap, θ_g , output) are in percent, computed as the production-only allocation relative to the coordinated allocation, so negative values indicate that production-only delivers a lower value than the coordinated policy. The gross production subsidy cost is the resource cost of $\tau_g y_g(h_g)n_g$ expressed as a share of aggregate private output.

The mechanism is straightforward. Production support raises the value of green matches and can pull enough workers and firms into the green sector to reach the employment target. But workers still choose h_g before matching and bargaining, so production support alone does not align the pre-match training margin with the planner’s allocation. The coordinated policy

separates the extensive employment margin from the intensive training margin.

The output differences are correspondingly small but systematic. Even with the same green employment share, green output per match and total green output are each about 0.16 percent lower under production-only policy, and aggregate private and social output are each marginally lower. The result is not that production support fails to create green jobs; it does. The point is that production-only policy reaches the same green headcount with lower green skill intensity and lower green output than the coordinated policy.

The gaps in Table 1.5 are modest because the benchmark calibration is conservative along several dimensions. The green wage premium is set to 2 percent, the training-cost target uses general employer training expenditures rather than a larger green-specific reskilling cost, and the human-capital elasticity is close to a local-linear benchmark. I therefore interpret the baseline as a conservative reading of the worker-side mechanism. The robustness exercises in Appendix A.2 show that the sign of the effect is stable across alternative training costs, green wage premia, and human-capital curvature.

Appendix A.2.1 relaxes the Hosios restriction by setting $\xi < \eta$. The worker-side mechanism survives: without worker-side support, production policy still delivers lower green human capital than the package that implements the planner allocation. Away from Hosios, however, the training wedge coexists with the standard search wedge, so exact decentralization also requires a vacancy-side correction. I therefore treat the non-Hosios exercise as a robustness check rather than as the main quantitative comparison.

1.4.3 Break-even Environmental Benefit

The previous section shows that, under Hosios, a coordinated production and training policy implements the planner allocation. I now ask a simple cost-benefit question: how large must the environmental value of green output be for this coordinated transition to be welfare-improving relative to the calibrated decentralized equilibrium?

For any allocation A , let

$$G(A) \equiv y_g(h_g)n_g$$

denote green output. I evaluate welfare as

$$W(A; \phi) = Y(A) + \phi G(A) + zU(A) - cV(A) - K(L_g(A)) - L_g(A)\chi(h_g(A)). \quad (1.91)$$

Let A^{DE} denote the calibrated decentralized equilibrium and let A^{dual} denote the allocation implemented by the coordinated policy in Table 1.5. The welfare gain from the coordinated transition is

$$\Delta W(\phi) = W(A^{dual}; \phi) - W(A^{DE}; \phi). \quad (1.92)$$

Since the allocations are held fixed in this exercise, the welfare gain is linear in the environmental benefit:

$$\Delta W(\phi) = \Delta W(0) + \phi \Delta G, \quad \Delta G = G(A^{dual}) - G(A^{DE}). \quad (1.93)$$

The break-even environmental benefit is therefore

$$\phi^* = -\frac{\Delta W(0)}{\Delta G}. \quad (1.94)$$

This is the minimum social value of one unit of green output required for the coordinated transition to weakly dominate the decentralized equilibrium.

Figure 1.4 plots the net welfare gain from the coordinated policy as ϕ varies. The coordinated transition has a non-environmental resource cost of 0.0013, or 0.138% of baseline private output. It increases green output by 0.1174. The implied break-even value is

$$\phi^* = 0.0113.$$

Thus, the environmental value of one unit of green output must be at least 1.13 percent of the numeraire output unit for the coordinated transition to pass the welfare test. Equivalently, the

required environmental benefit at the cutoff is 0.138% of baseline output, exactly offsetting the resource cost of the transition.

The non-environmental cost reflects the resource cost of moving workers into green search and raising green-specific training. Higher private output partially offsets these costs, but at $\phi = 0$ the increase in entry and training costs dominates. The exercise therefore separates the labor-market resource cost of the transition from the environmental value needed to justify it.

At the benchmark value $\phi^{14} = 0.0238$ used above, the net welfare gain is positive. Expressed as a flow consumption-equivalent gain relative to baseline private output, the coordinated policy raises welfare by 0.152%. The required environmental benefit for the transition to be welfare-improving is therefore not large, but the welfare case for green reallocation does rest on the external value of green production.

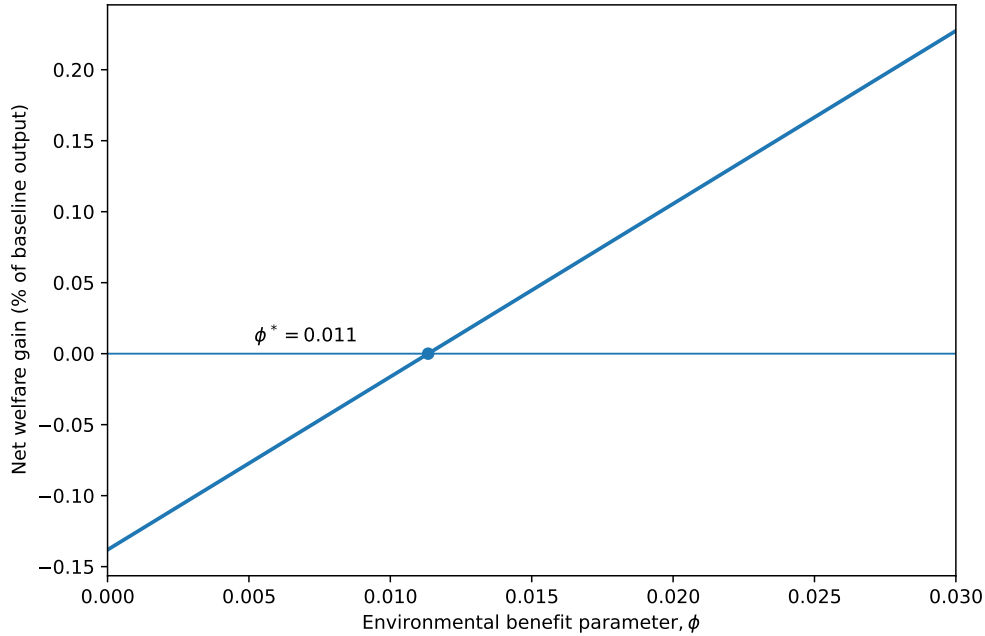


Figure 1.4. Break-even environmental benefit

Note: The vertical axis is the welfare gain $\Delta W(\phi)$ from the coordinated policy relative to the calibrated decentralized equilibrium, computed from (1.92). The horizontal axis is the environmental benefit parameter ϕ . The cutoff $\phi^* = 0.0113$ is the value at which the coordinated transition breaks even: welfare is below the decentralized benchmark for $\phi < \phi^*$ and above it for $\phi > \phi^*$. Allocations are held fixed while ϕ varies.

1.5 Conclusion

This chapter studies the green transition in a setting where expanding green production requires both green job creation and worker investment in green-specific skills. The model is a two-sector search model in which green workers choose human capital before matching, while wages are bargained after this investment is sunk. The resulting worker-side hold-up means that, even under Hosios, a production subsidy that corrects the environmental externality does not correct the pre-match training margin. A worker-side training subsidy is needed to implement the planner's green human-capital choice.

The quantitative results show why this distinction matters. In the benchmark calibration, a production-only policy can be chosen to reach the same 14 percent green employment target as the coordinated policy, but it reaches that target with lower green human capital, lower green output per match, and lower total green output. The differences are modest but systematic. The coordinated policy works because it separates the extensive employment margin from the intensive training margin: production support draws workers into green search, and training support raises the skill intensity of the resulting matches.

The broader implication is that labor-market policy is not merely a distributional complement to climate policy. When green production requires costly pre-match skill investment, production-side and worker-side support correct different distortions, and reaching the right employment level does not guarantee the right skill intensity. The same logic applies to other reallocations in which workers must make sector-specific investments before matching with firms in an expanding sector.

Two extensions are natural. First, the quantitative analysis compares steady states; studying the transition path would make it possible to trace how green employment, vacancy creation, training, unemployment, fiscal costs, and welfare evolve during the adjustment. Second, richer worker heterogeneity and alternative training technologies could sharpen the distributional and policy implications of worker-side support.

Chapter 2

Capital Holdup, Job Creation, and Skill Supply under Search Frictions

This chapter is coauthored with Si Guo. A version of this chapter is currently circulated as IMF Working Paper No. 26/93.¹

Abstract

There has been renewed interest in revitalizing manufacturing, yet policy often confronts a circular challenge: firms hesitate to expand because they cannot reliably find suitably skilled workers (e.g., STEM-trained), while workers are reluctant to acquire those skills when jobs remain limited. This raises a policy question: intervene at the firm margin or the worker margin, or both? We study this question by extending Acemoglu and Shimer (1999) to a two-sector open economy. The key friction is capital hold-up: firms invest upfront to create jobs, but sunk investment weakens their wage bargaining position, discouraging investment *ex ante*. Because manufacturing is more capital intensive, hold-up is more severe, leaving manufacturing employment inefficiently low. In the calibrated model, this inefficiency-induced industrial employment shortfall is about

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1 percent of total employment, roughly one-fifth of the LAC-East Asia gap. When the Hosios (1990) condition holds, the optimal policy can be solely on the firm side: an investment subsidy, or CIT cut, financed by an employment tax on firms. When the Hosios (1990) condition fails, an additional wedge distorts workers' sectoral choices, and targeted training subsidies become welfare-improving.

2.1 Introduction

The development of the manufacturing sector has received renewed attention in recent years. This renewed focus reflects several factors, including historically faster productivity growth in manufacturing, spillovers through input-output linkages, its relevance for national security, and its role in the trade balances and employment.²

Yet policymakers often face a chicken-and-egg dilemma in developing the manufacturing sector. Employers report persistent difficulty filling manufacturing vacancies, citing a shortage of workers with STEM training.³ At the same time, young people are less inclined to pursue STEM fields when manufacturing has a limited presence and offers few job opportunities.⁴ This raises two related questions. First, should the government play a role—through taxes, subsidies, or other policies—to support its expansion? Second, if so, should public support target firms to incentivize more STEM-related job creation, or should it focus on encouraging STEM-trained labor supply?

This chapter develops a Diamond–Mortensen–Pissarides (DMP) style search model with endogenous, job-specific capital in a two-sector small open economy to study these questions. Workers choose which sector to enter – manufacturing (m) or services (s) — subject to sector-specific training costs. Firms decide whether to create a vacancy and, if so, how much capital to invest.

²See, for example, Duarte and Restuccia (2010) on faster productivity growth in manufacturing; Liu (2019) and McNerney et al. (2022) on propagation through input-output linkages; Hicks (2022) on national security considerations; and Autor et al. (2013) and Rodrik (2016) on trade and employment implications.

³For example, in 2023, the Semiconductor Industry Association reported a shortage of technician, engineering and computer science workforce in the industry and advocated for expanding the pipeline of STEM graduates.

⁴For example, EU's Eurobarometer 55.2 survey shows 42.5% responders think that one of the main reasons for falling interests of youths in scientific studies and career is job and salary prospect.

The two sectors differ in capital intensity, with a higher capital share in manufacturing than in services.

The key inefficiency is the familiar capital holdup problem emphasized in Grou (1984) and Acemoglu and Shimer (1999). Capital is installed upfront, before wage bargaining. This weakens firms' bargaining positions and discourages investment. Because manufacturing is more capital intensive, the distortion is more severe there. The contribution of this chapter is to embed this same holdup problem in a two-sector setting with endogenous sectoral entry for both workers and firms.

Our first result is that in our two-sector environment, even when the Hosios (1990) condition is satisfied – so that search externalities are efficiently internalized – the decentralized equilibrium remains inefficient due to capital holdup. While the inefficiency from holdup is already well understood in a one-sector environment, what is new here is the implication for sectoral allocation of labor when there are two sectors. Because the manufacturing sector is more capital intensive, the holdup distortion depresses investment and job creation disproportionately in that sector. As a result, both the investment and sectoral composition of employment are inefficient in equilibrium. In contrast, conditional on efficient capital levels, workers' sectoral choice is efficient. This yields a sharp policy implication regarding the chicken-and-egg dilemma: when the Hosios condition holds, correcting underinvestment without subsidizing workers' training is sufficient to restore the planner's allocation. The associated optimal policy is fiscally neutral: an investment subsidy is fully financed by a lump-sum fee on firms in proportion to employment.

Our second result is that when the Hosios (1990) condition fails, an additional wedge distorts workers' sectoral choice, so that correcting underinvestment alone is no longer sufficient to replicate the planner's allocation. In particular, in the empirically relevant case where workers' bargaining power is smaller than the elasticity of matching with respect to unemployment, and the training cost of entering manufacturing is higher than services, it becomes harder for manufacturing workers to obtain sufficient surplus to offset their higher training costs. As a result, the optimal policy generally combines investment subsidies on firms with targeted training subsidies

on workers.

Finally, the open economy setting allows us to study how trade interacts with manufacturing development. Consider a country with comparative advantage in manufacturing, so that it exports goods and imports services: Greater openness to services imports lowers the relative prices of services and, in turn, raises the profitability and employment of manufacturing relative to services. In addition, when home production is denominated in either final consumption or services alone, unemployment declines, because the value of home production relative to manufacturing employment becomes lower.

2.1.1 Relations to the Literature

This chapter contributes to the macroeconomic literature on the sectoral shares of the economy. Past studies in the area of structural change, such as Duarte and Restuccia (2010), Uy et al. (2013), and Huneus and Rogerson (2024), examine how sectoral productivity differences and trade account for cross-country or -time variation in the shares of manufacturing and services. Because their goal is to explain these patterns positively, much of this literature relies on competitive environments in which the equilibrium is efficient, leaving limited scope for policy design. In contrast, we study manufacturing development in a frictional labor market where capital is subject to holdup and demonstrate the inefficiencies of the equilibrium, a rationale for policy intervention. A few recent exceptions incorporate search frictions into structural change models, for example, Feng et al. (2024). Relative to that work, our model features endogenous capital, trade openness, and an explicit focus on efficiency, allowing us to characterize whether policy should prioritize firm-side investment or worker-side training.

This chapter also complements the growing literature on the rationale and design of industrial policies to support certain industries, including the classic arguments based on coordination failure under increasing-return-to-scale (Matsuyama (1991)) and the role of industries within production networks (Liu (2019)). We emphasize a different mechanism – capital holdup interacting with search frictions and endogenous skill choice – in an otherwise standard constant-return-to-

scale environment. This channel links industrial policy directly to both job creation and skill acquisition, yielding sharp conditions under which investment subsidies alone are sufficient versus when training subsidies are warranted.

From a labor-search perspective, our model builds on the holdup problem in wage bargaining highlighted in Acemoglu and Shimer (1999) but extends it to a two-sector, open-economy setting with endogenous education choices. Closely-related work, such as Cardullo et al. (2015), develops a two-sector closed-economy search model and shows how cross-industry differences in holdup help explain cross-industry investment growth. Our focus is on the inefficiencies implied by holdup and search frictions and how they map into policy prescriptions, and how they depend on whether the Hosios condition holds. Our analysis also relates to the classic efficiency results in search-and-matching models. In the standard one-sector DMP framework, deviations from the Hosios condition create a wedge between private incentives and the social contribution to match formation, typically showing up as inefficient vacancy creation and market tightness (Hosios (1990); Mortensen and Pissarides (1994); Pissarides (2000)). Related work shows that the same misalignment can also distort match-related investments, including schooling choices in Flinn and Mullins (2015) and firms' training incentives under wage compression in Acemoglu and Pischke (1999b), and that when match output depends on market tightness (Mangin and Julien (2021)). Our contribution is to show that in a two-sector environment with segmented labor markets and costly ex ante training, deviations from Hosios condition generate an additional distortion on the worker's sectoral choice margin. In equilibrium, workers' sectoral choice is pinned down by their privately captured share of match surplus, while the planner's condition depends on the matching elasticity. This creates a novel sectoral-choice wedge even conditional on efficient vacancy creation and efficient capital levels.

Finally, this chapter is related to the studies on the field-of-study choice and economic development, such as Maloney and Valencia Caicedo (2022) and Murphy et al. (1991). Maloney and Valencia Caicedo (2022) show that the density of engineers in 1900 helps explain today's income variation across U.S. counties. Their explanation is that engineers facilitate technology adoption.

Murphy et al. (1991) find that countries with more engineering and fewer law college graduates experienced faster income growth. They attribute the field-of-study choice to whether an economy encourages more entrepreneurship or rent-seeking. We complement these studies by embedding education choice in a frictional labor market where investment and job creation respond endogenously to policy and trade, and by studying how labor market frictions distort workers' field-of-study choices.

The rest of this chapter is organized as follows. Section 2.2 presents the stylized facts linking sectoral employment and the share of STEM graduates. Section 2.3 outlines the model and characterizes the decentralized equilibrium. Section 2.4 defines and characterizes the social planner's problem. Section 2.5 discusses the efficiency of decentralized equilibrium. Section 2.6 and 2.7 cover calibration and quantitative results. Section 2.8 concludes.

2.2 Stylized Facts

This section documents two stylized facts: (i) substantial cross-country variation in the employment share of the industrial sector, and (ii) substantial variation in the share of STEM graduates among all tertiary graduates. We then show that countries with higher STEM shares tend to have a larger industrial employment share.

Industrial employment. We measure sectoral employment using data from Groningen Growth and Development Centre (GGDC). We combine two sources. The GGDC Economic Transformation Database (ETD), released in 2021, provides annual sectoral employment and value added for 12 sectors from 1990 to 2018. However, the ETD mainly covers low- and middle-income economies, with limited coverage of high-income economies. To broaden country coverage, we merge the ETD sample with the GGDC 10-Sector Database released in 2014, which reports annual sectoral employment and value-added for most high-income economies and selected middle-income economies from 1950 to 2011. The combined sample includes 60 economies.

Following the structural change literature (e.g., Uy et al. (2013), Huneus and Rogerson

(2024)), we aggregate sectors into agriculture, industry, and services. The industrial sector includes mining, manufacturing, utilities, and construction, while the service sector comprise all remaining sectors outside agriculture and industry. For each country, we calculate sectoral employment shares using the most recent two years available: 2017-18 for economies covered by the ETD and 2010-11 for countries that appear only in the GGDC 10-Sector Database.

Industry employment shares vary widely across countries, consistent with previous studies (e.g., Rodrik (2016), Huneus and Rogerson (2024)). In our sample, the industrial employment share ranges from 4 percent in Mozambique to 33 percent in China. This heterogeneity persists even among economies with comparable income levels. For example, with the exception of Mexico, Latin American countries such as Peru, Chile, Colombia, and Brazil have industrial employment shares between 16 and 19 percent, while East Asian countries have noticeably higher shares – about 25 percent in Korea and 33 percent in China.

STEM graduate share. We measure field-of-study composition using data from the UNESCO Institute for Statistics (UIS), which reports the distribution of tertiary graduates across fields. UIS coverage begins in 2000, though for many middle- and low-income countries it starts only in the 2010s. We define the STEM share as the sum of graduates in natural sciences, mathematics and statistics, information and communication technologies, and engineering, manufacturing and construction programs. For countries with data available for multiple years, we use the earliest available year so that for most countries, STEM shares are measured before industrial employment shares. This yield a sample of 147 economies, with their STEM shares ranging from 2 percent in Namibia (2001) to 45 percent in Malaysia (2004). China and Japan are not in the UIS database, so for these two countries we supplement the data using statistics from the Chinese Ministry of Education’s Educational Statistics and the OECD Statistics.

2.2.1 Relationship between industrial employment and STEM share

Figure 2.1 shows that economies with higher industrial employment shares also tend to have higher STEM shares. To quantify this relationship, we estimate a cross-country OLS regression

$$S_i = \beta_{stem}STEM_i + \delta \cdot X_i + \epsilon_i \quad (2.1)$$

where S_i is the industrial employment share in country i , $STEM_i$ is the STEM graduate share, and X_i includes controls commonly used in the structural change literature: PPP-adjusted log GDP per capita, the agricultural employment share, and educational attainment. GDP per capita comes from Penn World Table 10.01. Educational attainment is measured as the share of adults aged 25 or older who have completed short-cycle tertiary education or higher, also from UNESCO-UIS.

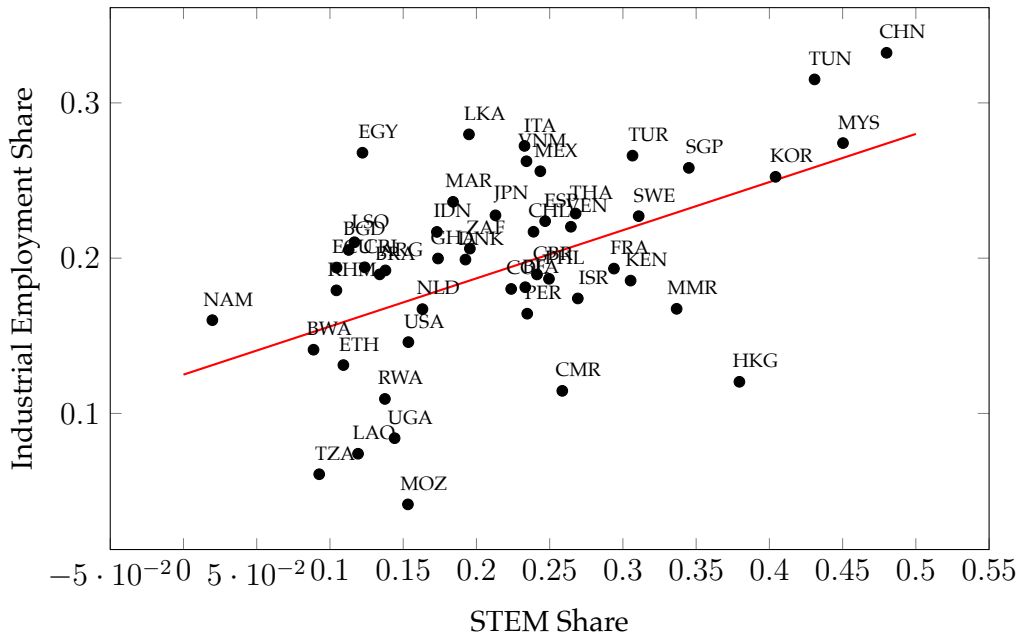


Figure 2.1. STEM share vs. industrial employment share.

Note: STEM share is calculated as the annual number of tertiary graduates in STEM fields divided by total number of tertiary graduates. Industrial employment share is calculated as the number of employees in the industry sector divided by total employment.

The estimates in Table 2.1 show a positive and statistically significant correlation between STEM graduate shares and industrial employment shares. In columns (1) and (3), β_{STEM_i} is sig-

nificant at the 1 percent level: a 1 percentage point increase in STEM share is associated with a 0.24-0.30 percentage point rise in industrial employment share. The coefficient on overall educational attainment is small and negative, suggesting that variation in the composition of tertiary graduates is more closely associated with industrial employment than educational attainment alone.

Table 2.1. Dependent Variable: Industrial Employment Share

	(1) All	(2) Sub Sample	(3) All
STEM share ($STEM_i$)	0.24*** (0.07)	0.19** (0.08)	0.30*** (0.08)
Edu. attainment	-0.001*** (0.00)	-0.000** (0.00)	-0.001 (0.00)
Agricultural emp. share	-0.20*** (0.04)	-0.18*** (0.05)	–
Log income	–	–	0.02*** (0.01)
Constant	0.23*** (0.03)	0.21*** (0.03)	-0.03 (0.07)
Observations	50	31	50
R ²	0.60	0.52	0.34

Note: The "Sub Sample" includes only economies with STEM share data available at least 5 years prior to the employment share data. Robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Correlation and causality. As will become clearer in the next section, our model is built on the premise that sectoral specialization and skill supply are jointly determined: countries with a larger industrial sector tend to attract more talent into STEM fields, which we use as a proxy for industrial sector-related skill supply, while a stronger STEM talent pipeline may support a larger industrial sector. Therefore, the purpose of Figure 2.1 and Equation (2.1) is to document this relationship as a correlation, rather than to establish a causality. That said, we take a modest step towards exploring the direction of the relationship by re-estimating Equation (2.1) using only the subsample of countries for which STEM share data are available at least 5 prior to the employment data. As presented in Column (2), the coefficient on $STEM_i$ remains significant at the 1 percent

level. While this exercise does not resolve concerns about omitted variables or endogeneity, it suggests that the positive association is not driven solely by contemporaneous correlation and provides suggestive timing evidence for the direction of the relationship.

2.3 Model

2.3.1 Environment and timing

We study a small open economy with two sectors: manufacturing (m) and services (s). Time is continuous and discounted at rate $r > 0$. A unit mass of risk-neutral workers and a continuum of risk-neutral firms populate the economy. Workers choose sector-specific educational tracks and then search for jobs in the corresponding sector. In each sector, jobs are formed with DMP-type search and matching frictions. Let L_m and L_s denote the masses of workers who have acquired the skills relevant for sectors m and s , respectively. Thus, L_i is the measure of the labor force in sector i . The total labor force in the two sectors is fixed at L : $L = L_m + L_s$, while the allocation across sectors is endogenous. Let unemployment within sector i be $u_i \equiv L_i - n_i$, where n_i stands for employment in sector i .

2.3.2 Preferences, prices, and trade

Employed workers consume a composite of manufacturing and service goods:⁵

$$c^E = (c_m^E)^\alpha (c_s^E)^{1-\alpha}. \quad (2.2)$$

Unemployed workers receive a flow payoff z , which can be interpreted as the value of non-marketable home production.

⁵Here we assume individual workers are grouped into households, and employed members within the same household can pool their consumption, so that every employed member's consumption level would be identical (c^E). This assumption helps us avoid distinguishing between the consumption of workers having a manufacturing jobs and of workers with services jobs.

Aggregation. Let n denote total manufacturing and service employment. Let C_i denote aggregate consumption of good i . Aggregate consumption satisfies

$$\begin{aligned} nc_m^E &\equiv C_m, \\ nc_s^E &\equiv C_s, \\ nc^E &= n(c_m^E)^\alpha (c_s^E)^{1-\alpha} \\ &= C_m^\alpha C_s^{1-\alpha}. \end{aligned}$$

Hence, we can define the aggregate final consumption as

$$C \equiv C_m^\alpha C_s^{1-\alpha}. \quad (2.3)$$

We therefore only need to keep track of aggregate consumption C_m , C_s and C in equilibrium. The unit price of final consumption is:

$$P = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} P_m^\alpha P_s^{1-\alpha}. \quad (2.4)$$

Manufacturing goods are fully tradable and are perfectly substitutes for foreign-produced manufacturing goods. Hence, the price of domestic manufacturing goods equals the world price : $P_m = p_m = p_m^F$. We set $p_m^F = 1$ as the numeraire. The service composite C_s is a CES aggregate of domestically produced C_s^H and imported services C_s^F with elasticity of substitution ν :

$$C_s = \left[\chi (C_s^H)^{\frac{\nu-1}{\nu}} + (1 - \chi) (C_s^F)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \quad \chi \in (0, 1). \quad (2.5)$$

where $1 - \chi$ governs the service-trade openness, and $\nu > 1$ implies that foreign and domestic services are gross substitutes. The import price of services is exogenous at p_s^F . The domestic

service price p_s is endogenous in equilibrium. The unit price of the service composite is:

$$P_s = \left[\chi p_s^{1-\nu} + (1 - \chi) (p_s^F)^{1-\nu} \right]^{\frac{1}{1-\nu}}, \quad (2.6)$$

Trade pattern. We assume that the home country has a comparative advantage in manufacturing relative to the rest of the world: $p_s/p_m > p_s^F/p_m^F = 1$. Therefore, the economy is a net exporter of manufacturing goods and a net importer of services. We assume balanced trade in every period.

Consumption demand. Let total nominal expenditure be $E \equiv PC$. Standard utility maximization implies the following consumption demands:

$$C_m = \frac{\alpha}{p_m} E, \quad (2.7)$$

$$C_s = \frac{1 - \alpha}{P_s} E, \quad (2.8)$$

$$C_s^H = \chi \left(\frac{p_s}{P_s} \right)^{-\nu} C_s, \quad (2.9)$$

$$C_s^F = (1 - \chi) \left(\frac{p_s^F}{P_s} \right)^{-\nu} C_s. \quad (2.10)$$

2.3.3 Labor market and production

Labor markets in manufacturing and services are segmented. Each sector $i \in \{m, s\}$ operates a labor market with DMP-style search and matching frictions. Each job consists of a one-vacancy-one-worker pair. Let u_i and v_i denote the masses of unemployed workers and unfilled vacancies in sector i . Matches are formed according to

$$m_i = M(u_i, v_i). \quad (2.11)$$

The matching function $M(\cdot, \cdot)$ is increasing, concave, and homogeneous of degree one. Assume

$$M(u_i, v_i) = u_i^\eta v_i^{1-\eta}, \quad (2.12)$$

where $\eta \in (0, 1)$ is the elasticity of matches with respect to unemployment. Define market tightness as

$$\theta_i \equiv v_i/u_i. \quad (2.13)$$

The vacancy–filling rate is then

$$q_i(\theta_i) \equiv M_i(u_i, v_i)/v_i, \quad (2.14)$$

and the job–finding rate is

$$\theta_i q_i(\theta_i) \equiv M_i(u_i, v_i)/u_i. \quad (2.15)$$

Job-specific capital and production. Creating a vacancy in sector i requires installing job-specific capital $k_i \geq 0$. Once chosen, the capital level cannot be changed until the capital depreciates and the vacancy is destroyed. Because capital is sunk at the time of wage bargaining, it generates a holdup problem. Once a vacancy is filled, the match produces output at rate $f_i(k_i)$ with $f'_i > 0$, $f''_i < 0$, and $f_i(0) = 0$. Total output in sector i is thus the product of per-match output and employment:

$$Y_i = f_i(k_i)n_i, \quad i \in \{m, s\}. \quad (2.16)$$

Following Acemoglu and Shimer (1999), we assume that the job-specific capital attached to any vacancy, filled or unfilled, is lost with exogenous Poisson probability s_i , at which point the match is also separated.⁶

⁶Allowing s_i to differ by sector helps reconcile two empirical regularities: (i) manufacturing and services exhibit similar or slightly lower *unemployment rates* in manufacturing, $\bar{u}_m \lesssim \bar{u}_s$, where $\bar{u}_i \equiv u_i/L_i$; (ii) manufacturing is more capital intensive. Higher k_m raises hold-up costs and would otherwise push $\bar{u}_m$ above $\bar{u}_s$; taking $s_m < s_s$ captures longer job durations in capital-intensive matches and restores consistency with the data without inflating education costs or wages.

Employment and unemployment dynamics. Employment evolves according to:

$$\dot{n}_i = m_i - s_i n_i, \quad i \in \{m, s\}. \quad (2.17)$$

Within each sector,

$$u_i = L_i - n_i, \quad (2.18)$$

$$v_i = \theta_i u_i. \quad (2.19)$$

Let x_i denote the flow of new vacancies created, vacancy dynamics thus satisfy

$$\dot{v}_i = x_i - s_i v_i - m_i. \quad (2.20)$$

Unemployment dynamics are given by

$$\dot{u}_i = s_i(L_i - u_i) - \theta_i q_i(\theta_i) u_i, \quad i \in \{m, s\},$$

In steady state, the unemployment rate $\bar{u}_i \equiv u_i/L_i$ is given by

$$u_i^{ss} = \frac{s_i}{s_i + \theta_i q_i(\theta_i)} L_i, \quad \bar{u}_i^{ss} = \frac{u_i^{ss}}{L_i} = \frac{s_i}{s_i + \theta_i q_i(\theta_i)}. \quad (2.21)$$

Education and workers' sectoral choices. Before entering the labor market, workers choose which sector-specific skills to acquire and this choice is permanent. Acquiring the skills for sector i requires a one-time training cost ϵ_i , measured in units of the final consumption good. We assume $\epsilon_m \geq \epsilon_s$, reflecting the idea that manufacturing-related skill training is typically more costly than service-related skill training.

2.3.4 Bellman equations and definition of decentralized equilibrium

Let J_i^U and $J_i^E(k_i)$ denote a worker's values of unemployment and employment in sector i , and let $J_i^V(k_i)$ and $J_i^F(k_i)$ denote a firm's values of a vacancy and a filled job. Here, $J_i^V(k_i)$ is the value of a vacancy before netting out the capital installation cost. A firm that creates a vacancy with capital k_i therefore earns a net value $J_i^V(k_i) - P_k k_i / P$ measured in units of the final consumption good. The corresponding Bellman equations are:

$$rJ_i^U = z + \theta_i q_i(\theta_i)(J_i^E(k_i^*) - J_i^U), \quad (2.22)$$

$$rJ_i^E(k_i) = \frac{w_i(k_i)}{P} + s_i(J_i^U - J_i^E(k_i)), \quad (2.23)$$

$$rJ_i^V(k_i) = q_i(\theta_i)(J_i^F(k_i) - J_i^V(k_i)) - s_i J_i^V(k_i), \quad (2.24)$$

$$rJ_i^F(k_i) = \frac{p_i f_i(k_i) - w_i(k_i)}{P} - s_i J_i^F(k_i), \quad (2.25)$$

$$\text{where } k_i^* = \arg \max_{k_i} \left\{ J_i^V(k_i) - \frac{P_k k_i}{P} \right\}. \quad (2.26)$$

Equation (2.22) gives the flow value of an unemployed worker in sector i : the unemployment income plus the expected payoff from finding a job. We model unemployment income as home production of z , measured in units of final consumption good. The expected gain from finding a job equals the job finding rate $\theta_i q_i(\theta_i)$ times the gain from employment, $J_i^E(k_i^*) - J_i^U$ where k_i^* denotes the equilibrium level of capital per worker in sector i . Equation (2.23) gives the flow value of an employed worker in sector i conditional on the job's capital level k_i , taking into account the real wage $w_i(k_i)/P$ and the separation rate s_i . Equation (2.24) gives the flow value of a vacancy with capital level k_i , which includes the expected gain from the vacancy being filled and the expected loss from the destruction of the vacancy. The level of k_i is optimally chosen by firms to maximize the value of the vacancy. Finally, (2.25) gives the flow value of a filled job with capital k_i : the firm receives real output $p_i f_i(k_i)/P$, pays labor $w_i(k_i)/P$, and accounts for the separation rate s_i .

Capital choice (holdup). The firm chooses the capital level when the vacancy is created. Once the vacancy is created, the capital cost becomes sunk. The firm therefore chooses capital to maximize its expected profit $J_i^V(k_i) - \frac{P_k k_i}{P}$.

Wage bargaining. After a worker is matched with a vacancy, the wage is determined through Nash bargaining, with the worker's bargaining weight given by $\beta \in (0, 1)$:

$$(1 - \beta)(J_i^E(k_i) - J_i^U) = \beta(J_i^F(k_i) - J_i^V). \quad (2.27)$$

Free entry condition. Free entry by firms implies that the expected equilibrium profit from creating a vacancy is zero:

$$J_i^V(k_i^*) = \frac{P_k k_i^*}{P}. \quad (2.28)$$

Workers' sectoral choice. In equilibrium, workers are indifferent between entering the manufacturing or service markets:

$$J_m^U - J_s^U = \epsilon_m - \epsilon_s \quad (2.29)$$

Definition of decentralized equilibrium. A stationary decentralized equilibrium (DE) consists of

- (i) Prices (p_s, P_s, P) ,
- (ii) Sectoral labor market objects $\{L_i, \theta_i, u_i, n_i, v_i, k_i, w_i\}_{i \in \{m, s\}}$, and
- (iii) Good market quantities $\{E, C_m, C_s, C_s^H, C_s^F, X_m, M_s\}$,

Such that:

1. Given prices (p_m, p_s, p_s^F) and total expenditure E , consumption demands satisfy (2.7) - (2.10).
2. Firms and workers' optimize in the labor market, so that the value functions satisfy (2.22)-(2.26) and wages $w_i(k_i)$ satisfies Nash bargaining (2.27).
3. Free entry of firms characterized by (2.28).

4. Workers' sectoral choices satisfy (2.29).
5. Total labor supply satisfies $L_s + L_m = L$.
6. Production satisfies $Y_i = f_i(k_i)n_i$.
7. Good markets are clear and trade is balanced, as in (2.30) - (2.34), and
8. Labor market flows satisfy (2.21).

2.3.5 Characterization of the Decentralized Equilibrium

We characterize the stationary decentralized equilibrium in two blocks.

- A *goods market block* maps (Y_m, Y_s, I) into the equilibrium domestic service price p_s consistent with household utility maximization, market clearing, and balanced trade. This mapping is independent of the structure of labor market.
- A *labor market block* maps p_s into labor market outcomes $(\theta_i, k_i, w_i, n_i, L_i)$, and therefore into (Y_m, Y_s, I) .

The decentralized equilibrium is a fixed point of these two mappings.

Goods market equilibrium block

This block derives the equilibrium domestic service price p_s , as well as consumption and trade flows, given foreign prices (p_m^F, p_s^F) , real output (Y_m, Y_s) , and nominal investment I .

Goods-market clearing and balanced trade. In the market for manufacturing goods, total output must equal the sum of domestic consumption, investment, and exports:

$$Y_m = C_m + I + X_m. \quad (2.30)$$

Total investment is given by

$$I = P_k(x_m k_m + x_s k_s), \quad (2.31)$$

where x_i denotes the flow of newly created vacancies in sector i .⁷

For services,

$$C_s^H = Y_s, \quad (2.32)$$

$$C_s^F = M_s. \quad (2.33)$$

Trade is balanced:

$$p_m X_m = p_s^F M_s. \quad (2.34)$$

Nominal GDP is defined as

$$Y^N = p_m Y_m + p_s Y_s. \quad (2.35)$$

Total nominal consumption expenditure

$$E \equiv PC \equiv p_m C_m + p_s C_s^H + p_s^F M_s \quad (2.36)$$

$$= p_m (Y_m - I) + p_s Y_s, \quad (2.37)$$

where the last equality uses (2.30) - (2.34).

Proposition 5. *Given exogenous prices p_m, p_s^F , output (Y_m, Y_s) , and investment I , the domestic service price $p_s(p_m, p_s^F, Y_m, Y_s, I)$ is determined by*

$$Y_s = \chi(1 - \alpha) (p_m (Y_m - I) + p_s Y_s) p_s^{-\nu} P_s^{\nu-1}, \quad (2.38)$$

where P_s is given by (2.6).

Given p_s , the final-good price P and nominal consumption E are determined by (2.4) and (2.37).

Consumption allocations are determined by (2.7)-(2.10). Exports X_m and imports M_s are pinned down by (2.33) and (2.30).

⁷At steady state, the number of vacancies created in each moment should equal the number of vacancies destroyed: $x_i = s_i(n_i + v_i)$.

Labor market equilibrium block

From (2.24) and (2.25), we have

$$J_i^F(k_i) = \frac{p_i f(k_i) - w_i(k_i)}{P(r + s_i)}, \quad (2.39)$$

$$J_i^V(k_i) = \frac{q_i(\theta_i)}{r + s_i + q_i(\theta_i)} J_i^F(k_i). \quad (2.40)$$

A firm chooses k_i to maximize its expected profit $J_i^V(k_i) - P_k k_i / P$. The first-order condition is therefore

$$P_k = \frac{q_i(\theta_i)}{r + s_i + q_i(\theta_i)} \frac{p_i f'(k_i) - w'_i(k_i)}{r + s_i}. \quad (2.41)$$

From (2.23) and (2.24), we have

$$\frac{p_i f(k_i) - w_i(k_i)}{P} = \frac{(1 - \beta_i)(r + s_i + q_i(\theta_i))}{r + s_i + (1 - \beta_i)q_i(\theta_i)} \left[\frac{p_i f(k_i)}{P} - r J_i^U \right]. \quad (2.42)$$

Substitute it into (2.41), we have the optimality condition for k_i :

$$P_k = \frac{q_i(\theta_i)(1 - \beta_i)}{r + s_i} \frac{p_i f'(k_i)}{r + s_i + (1 - \beta_i)q_i(\theta_i)}.$$

Total surplus $S_i(k_i)$: To build intuition, it is useful to define the total surplus of a match in in sector i as

$$S_i(k_i) \equiv (J_i^E(k_i) - J_i^U) + (J_i^F(k_i) - J_i^V(k_i)).$$

Nash bargaining in (2.27) then implies

$$J_i^E - J_i^U = \beta S_i, \quad J_i^F - J_i^V = (1 - \beta) S_i. \quad (2.43)$$

Using (2.22)–(2.25), we obtain:

$$r(J_i^E(k_i) - J_i^U) = \frac{w_i(k_i)}{P} - z - (s_i + \theta_i q(\theta_i))(J_i^E(k_i) - J_i^U), \quad (2.44)$$

$$r(J_i^F(k_i) - J_i^V(k_i)) = \frac{p_i f_i(k_i) - w_i(k_i)}{P} - (s_i + q(\theta_i))(J_i^F(k_i) - J_i^V(k_i)). \quad (2.45)$$

Adding these two equations and using (2.43), we get

$$S_i(k_i) = \frac{p_i f_i(k_i)/P - z}{D_i(\theta_i)}, \quad (2.46)$$

where

$$D_i(\theta_i) \equiv r + s_i + (1 - \beta) q_i(\theta_i) + \beta \theta_i q_i(\theta_i). \quad (2.47)$$

Thus, the total surplus of match in sector i equals the output flow net of home production, discounted by D_i . The discount term D_i reflects the time discount rate r , the separation rate s_i , and the adjustment due to the transitions between vacancies and matches and between unemployment and employment. Note that the sunk capital cost $P_k k_i$ does not enter $S_i(k_i)$ because capital investment happens before wage bargaining.

The other two key equilibrium conditions are the free-entry condition (2.28) and the workers' sectoral choice condition (2.29). Given total match surplus $S_i(k_i)$, the value of a vacancy $J_i^V(k_i)$ and the value of unemployment J_i^U follow directly from (2.22), (2.24) and Nash bargaining:

$$J_i^V(k_i) = \frac{q_i(\theta_i)(J_i^F(k_i) - J_i^V(k_i))}{r + s_i} = \frac{q_i(\theta_i)(1 - \beta)S_i(k_i)}{r + s_i}, \quad (2.48)$$

$$J_i^U = \frac{\theta_i q_i}{r}(J_i^E(k_i^*) - J_i^U) = \frac{\beta \theta_i q_i}{r} S_i(k_i^*). \quad (2.49)$$

Proposition 6 (Decentralized equilibrium). *In steady state, the decentralized equilibrium $(k_i^*, \theta_i^*, L_s^*)$ is characterized by the following conditions:*

$$\{\text{Capital choice}\} : P_k = \frac{q_i(\theta_i)(1 - \beta)}{r + s_i} \frac{p_i f'_i(k_i)}{r + s_i + (1 - \beta)q_i(\theta_i)}, \quad i \in \{m, s\}; \quad (2.50)$$

$$\{\text{Tightness}\} : \frac{P_k k_i}{P} = \frac{q_i(\theta_i)(1 - \beta)}{r + s_i} \frac{p_i f_i(k_i)/P - z}{D_i(\theta_i)}, \quad i \in \{m, s\}; \quad (2.51)$$

$$\{\text{Sectoral choice}\} : r(\epsilon_m - \epsilon_s) = \frac{\beta \theta_m q_m(\theta_m)}{D_m(\theta_m)} [p_m f_m(k_m)/P - z] - \frac{\beta \theta_s q_s(\theta_s)}{D_s(\theta_s)} [p_s f_s(k_s)/P - z], \quad (2.52)$$

Remarks. Equation (2.50) determines firms' optimal capital choices. The left-hand side is the rental cost of capital required to create or maintain a vacancy. The right-hand side captures the marginal life-time return from additional investment. The term $\frac{p_i f'_i(k_i)}{r + s_i + (1 - \beta)q_i(\theta_i)}$ captures the marginal increase in surplus from higher capital.⁸ This flow return is discounted by interest rate and separation rate, $(r + s_i)$, and is adjusted for the vacancy filling rate, $q_i(\theta_i)$. Importantly, the $(1 - \beta)$ term captures the *capital holdup problem* highlighted in Acemoglu and Shimer (1999) and Cardullo et al. (2015): firms internalize only a fraction of the match surplus when they choose k_i , leading to under-investment relative to the social planner's allocation which we characterize in the next section.

Equation (2.51) reflects the free-entry condition in equilibrium. A firm creates a vacancy at cost $P_k k_i/P$, and its return is $1 - \beta$ share of total surplus flow. This surplus flow is converted to a life-time value by discounting at $r + s_i$ and adjusting for the vacancy filling rate.

Equation (2.52) governs workers' sectoral choices. The left-hand side is the training cost differential, while the right-hand side captures the difference in workers' expected surpluses from entering each sector. The expected surplus from entering sector i is given by the worker's bargaining share β , times the job-finding rate $\theta_i q_i(\theta_i)$ and total surplus S_i .

⁸The denominator $r + s_i + (1 - \beta)q_i$ differs from D_i in (2.46) and (2.47). The derivation of (2.50) takes J_i^U as given because it is an equilibrium outcome. The derivation of (2.46) and (2.47) takes into account that a change k_i can also affect J_i^U in equilibrium.

2.4 Social Planner's (SP) Problem

Let $\mathcal{C}(Y_m, Y_s, I)$ denote the level of final consumption implied by the goods-market equilibrium for given output (Y_m, Y_s) and investment I , as characterized by Proposition 5. The social planner chooses $\{\theta_i(t), k_i(t), L_s(t)\}_{t \geq 0}$ to maximize the present discounted value of output net of capital and education costs:⁹

$$\max_{\{\theta_i, k_i, L_s\}} \int_0^\infty \left\{ \mathcal{C}(Y_m(t), Y_s(t), I(t)) + z(u_m(t) + u_s(t)) - r L_m(t) \epsilon_m - r L_s(t) \epsilon_s \right\} e^{-rt} dt, \quad (2.53)$$

subject to, for each sector $i \in \{s, m\}$,

$$\dot{n}_i(t) = m_i(t) - s_i n_i(t), \quad (2.54)$$

$$\dot{Y}_i(t) = -s_i Y_i(t) + m_i(t) f_i(k_i(t)), \quad (2.55)$$

$$u_i(t) = L_i(t) - n_i(t), \quad (2.56)$$

$$m_i(t) = q_i(\theta_i(t)) \theta_i(t) (L_i(t) - n_i(t)), \quad (2.57)$$

$$v_i(t) = \theta_i(t) (L_i(t) - n_i(t)), \quad (2.58)$$

$$L_m(t) = L - L_s(t). \quad (2.59)$$

Proposition 7 (Social Planner). *In steady state, the social planner's choices of k_i , θ_i and L_s are characterized by*

$$\{\text{Capital choice}\} : P_k = \frac{p_i f'(k_i)}{r + s_i} \frac{q_i(\theta_i)}{r + s_i + q_i(\theta_i)}, \quad i \in \{m, s\} \quad (2.60)$$

$$\{\text{Tightness}\} : \frac{P_k k_i}{P} = \frac{q_i(\theta_i) (1 - \eta) (p_i f(k_i)/P - z)}{(r + s_i) D_i(\eta)}, \quad i \in \{m, s\} \quad (2.61)$$

$$\{\text{Sectoral choice}\} : r(\epsilon_m - \epsilon_s) = \theta_m q_m(\theta_m) \eta \Phi_m - \theta_s q_s(\theta_s) \eta \Phi_s, \quad (2.62)$$

⁹Note that skill acquisition cost ϵ_i in the model is incurred once when workers choose a sector. In the planner's objective function, we can represent this one-time cost in an equivalent flow term $r\epsilon_i$ because $\int_0^\infty r e^{-rt} dt = 1$.

$$\text{where } \Phi_i = \frac{p_i f_i(k_i)/P - z}{D_i(\eta)}, \quad (2.63)$$

$$D_i(\eta) = r + s_i + (1 - \eta)q_i(\theta_i) + \eta\theta_i q_i(\theta_i). \quad (2.64)$$

$$\eta \equiv \frac{\partial \log M(u, v)}{\partial \log(u)}. \quad (2.65)$$

Remarks. Equation (2.60) characterizes the social planner's optimal choice of k_i . The right hand side is the marginal social return to capital: $p_i f'_i(k_i)$, the marginal product of capital, discounted by the interest rate and separation rate, and adjusted by the probability that the vacancy is filled, $\frac{q_i}{r+s_i+q_i}$. Equation (2.61) and (2.62) are the planner's counterparts to the decentralized equilibrium conditions (2.51) and (2.52), except that β is replaced with η , the elasticity of matches with respect to employment.

The planner internalizes how changes in labor market tightness affect match formation, so the relevant object is the elasticity η . In the decentralized equilibrium, however, the corresponding condition depends on workers' bargaining power β , which governs how surplus is privately shared. The Hosios condition, $\beta = \eta$, is therefore a natural benchmark: when it holds, the decentralized equilibrium coincides with the planner on search externality, but the capital holdup distortion remains.

2.5 Efficiency

To analyze the sources of inefficiency, we compare the optimality conditions of decentralized equilibrium (DE) with those of social planner's allocation (SP). Let $\{k_i^{DE}, \theta_i^{DE}, L_i^{DE}\}$ denote the allocation that satisfies DE's conditions (2.50)-(2.52) and $\{k_i^{SP}, \theta_i^{SP}, L_i^{SP}\}$ as bundle satisfying SP's conditions (2.60)-(2.62).

2.5.1 Hosios condition $\beta = \eta$

Proposition 8. *Under Hosios condition, $\beta = \eta$,*

(i) The decentralized equilibrium is constrained inefficient;

(ii) If $k_i = k_i^{SP}$ is fixed exogenously, then $\theta_i^{DE} = \theta_i^{SP}$ and $L_i^{DE} = L_i^{SP}$.

Remarks. Proposition 8 (i) follows directly from comparing the private and social marginal return to capital in (2.50) and (2.60). Define the capital wedge Ω_i^K as the ratio between the private and social marginal return to capital:

$$\begin{aligned}\Omega_i^K &\equiv \frac{\text{Marginal return to } k_i^{DE}}{\text{Marginal return to } k_i^{SP}} \\ &\equiv \frac{\frac{q_i(\theta_i)(1-\beta)}{r+s_i} \frac{p_i f'(k_i)}{r+s_i+(1-\beta)q_i(\theta_i)}}{\frac{p_i f'(k_i)}{r+s_i} \frac{q_i(\theta_i)}{r+s_i+q_i(\theta_i)}} = (1-\beta) \frac{r+s_i+q_i(\theta_i)}{r+s_i+(1-\beta)q_i(\theta_i)} < 1.\end{aligned}\quad (2.66)$$

Therefore, the private marginal return to capital in DE is smaller than the social marginal return to capital in SP. However, when $\beta = \eta$, (2.51)-(2.52) in DE are identical to (2.61)-(2.62), which leads to Proposition 8 (ii).

Proposition 9. Under Hosios condition $\beta = \eta$, a deficit-neutral policy consisting of a one-time investment subsidy τ_i^{K*} per unit of investment and an one-off lump-sum tax on firms τ_i^{T*} per vacancy implements the social planner's allocation:

$$\begin{aligned}\tau_i^{K*} &= 1 - \Omega_i^K = 1 - \frac{(1-\beta)(r+s_i+q_i(\theta_i^{SP}))}{r+s_i+(1-\beta)q_i(\theta_i^{SP})} > 0 \\ \tau_i^{T*} &= -\tau^{K*} P_k k_i^{SP} / P < 0\end{aligned}$$

The proof of Proposition 9 is available in Appendix B.1. Intuitively, the investment subsidy τ_i^{K*} corrects the inefficiency arising from capital hold-up and induces firms to choose higher k^i . This subsidy also raises firms' profits, encouraging (inefficiently) more firms to enter. Thus, a lump-sum tax on firm entry, τ_i^T , is needed to prevent inefficient over-entry. Another key implication is that once the capital level is efficient, there is no need to subsidize workers' trainings as long as Hosios condition holds, as higher per worker capital endogenously raises workers' incentives to enter the manufacturing sector.

What are the real-world counterparts of τ_i^K and τ_i^T ? The investment subsidy τ_i^K maps naturally into policies that reduce the net cost of investment, such as accelerated depreciation allowances, investment tax credits, or direct investment subsidies. The counterpart of the lump-sum tax τ_i^T is more subtle. It is best interpreted as a firm contribution that is proportional to employment but do not vary one-to-one with the total wage bill. Examples include a flat payroll tax, a flat social security contribution, or payroll taxes that apply only up to low wage threshold.

2.5.2 Empirical evidence: skill supply responds to skill demand

A key premise of Proposition 9 is that workers' educational choices respond to shifts in skill demand. To illustrate this mechanism empirically, we follow Han and Winters (2020) to use the response of petroleum-engineering enrollment to movements in real oil prices as evidence that skill supply adjusts to demand. This relationship has several advantages. First, real oil price can plausibly be treated as an exogenous shock to labor demand in the oil industry. When oil prices rise, an increase petroleum-engineering enrollment can be interpreted as a causal response to this demand shock. Second, unlike broad fields such as mathematics, petroleum engineering is narrowly defined and closely tied to the oil industry. This tighter mapping helps isolate the demand channel and reduces concerns that the observed enrollment response is driven by other contemporaneous shocks.

Our real oil price series is constructed from annual WTI price deflated by the U.S. CPI. Both series are obtained from FRED. There have been two real oil price booms since 1960: one beginning in 1973 during the energy crisis, and another starting in the mid-2000s.

We use the 2010-24 American Community Survey (ACS) to construct a cohort-level measure of the petroleum-engineering enrollment shares going back to 1960. The ACS is an annual national survey of the U.S. population. Relevant to our purpose, the survey asks respondents' demographic and socioeconomic characteristics, such as age, gender, income level, birth place, education level, and college major when applicable, along with sample weights. To construct the share of petroleum-engineering enrollment share by year, we follow Han and Winters (2020) and

extend their sample through the late 2010s to include the oil price boom of the 2000s. For each cohort year t between 1960 and 2019, we pool the 2010-24 ACS microdata and identify respondents who turned 18 years old in year t . We restrict the sample to individuals born in the largest three oil-producing states (Texas, North Dakota, and New Mexico), under the premise that they are more likely to observe oil industry labor demand conditions, and thus adjust their field-of-study choices when oil prices rise. We then compute the weighted number of respondents in cohort t who reported major is petroleum engineering. Dividing by the weighted sum of enrollment in the same cohort yields the petroleum-engineering share.

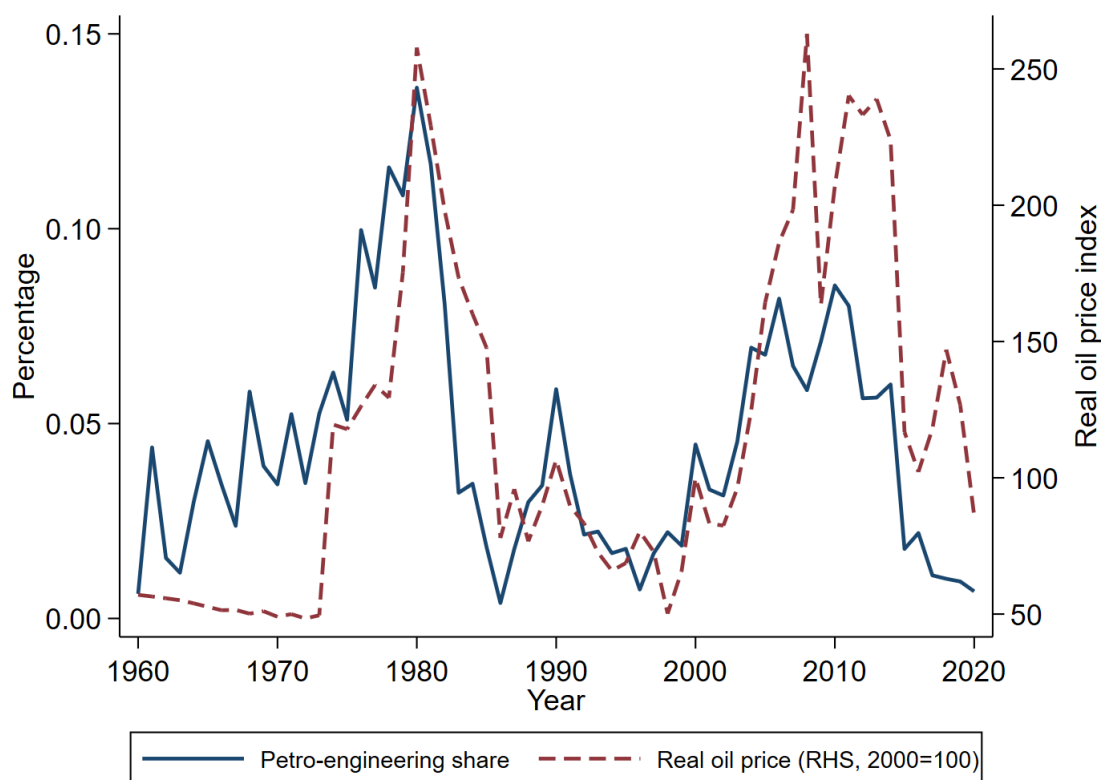


Figure 2.2. Real oil prices and petroleum engineering enrollment share.

Note: The figure plots the petroleum engineering major share among cohorts born in Texas, North Dakota, and New Mexico (left axis) against the annual real oil price (2000=100, right axis). Cohorts are indexed by the year individuals turned age 18.

Figure 2.2 plots the real oil price (right axis, dashed line) together with the petroleum-engineering enrollment share for cohorts born in Texas, North Dakota, and New Mexico (left

axis, solid line). A clear co-movement emerges: petroleum-engineering enrollment rose during the two major oil price booms in the 1970s and mid-2000s, and declined when oil prices fell. This pattern was more pronounced during the first boom, but the response remains evident in the 2000s episode. Overall, the figure provides suggestive evidence that educational choices respond to changes in industry labor demand.

2.5.3 Inefficiency when Hosios condition fails

The empirically relevant case: $\eta > \beta$

The preceding subsection focused on the Hosios benchmark $\beta = \eta$, under which the decentralized equilibrium internalizes the standard search externality. Empirically, however, a relevant case is that workers' bargaining power β is lower than the matching elasticity η , that is, $\eta > \beta$.

The matching elasticity with respect to unemployment, η , is relatively well identified. Petrongolo and Pissarides (2001) survey the empirical literature and report estimates typically in the range of 0.5 to 0.7. More recent studies, such as (Borowczyk-Martins et al., 2013), are broadly consistent with this range. In contrast, estimates of workers' bargaining power, β , are more dispersed but generally suggest values below η . Hagedorn and Manovskii (2008) estimate $\beta \approx 0.05$ in their baseline specification, though this rises to about 0.46 when capital is incorporated into the model. Flinn (2006) estimates bargaining power in the range of 0.3-0.4. More recent evidence points to a secular decline in workers' bargaining power: Azar et al. (2022) document falling labor shares and attribute this partly to declining worker bargaining power, consistent with lower unionization and greater labor market concentration.

Proposition 10. *When $\eta > \beta$ and $\epsilon_m > \epsilon_s$, the decentralized equilibrium is inefficient even if the capital is fixed exogenously at the social planner's level, $k_i = k_i^{SP}$.*

The gap between the tightness conditions in the decentralized equilibrium, equation(2.51), and in the social planner's allocation, equation (2.61), reflects the familiar inefficiency that arises when Hosios condition fails.

What is new in our two-sector environment is an additional wedge in workers' sectoral choices. Substitute the planner's tightness condition (2.61) into (2.62) and the decentralized equilibrium's tightness condition (2.51) into (2.52), we have

$$r(\epsilon_m - \epsilon_s) = \frac{\eta}{1 - \eta} [\theta_m P_k k_m (r + s_m) - \theta_s P_k k_s (r + s_s)], \quad (2.67)$$

$$r(\epsilon_m - \epsilon_s) = \frac{\beta}{1 - \beta} [\theta_m P_k k_m (r + s_m) - \theta_s P_k k_s (r + s_s)]. \quad (2.68)$$

Even holding θ_i and k_i fixed, it is immediate that when $\eta > \beta$ and $\epsilon_m > \epsilon_s$, the right-hand side of (2.67) exceeds that of (2.68). Intuitively, the right hand side of (2.68) represents the flow surplus gained by an unemployed worker who enters manufacturing rather than services. The term $P_k k_i (r + s_i)$ is the user cost of firm investment, and multiplying by θ_i scales this cost by the number of vacancies per unemployed worker. Finally, the factor $\frac{\beta}{1 - \beta}$ converts the firm's flow surplus into worker's surplus via Nash bargaining.

Because $\epsilon_m > \epsilon_s$, a worker must obtain more surplus in manufacturing than in services to compensate for the higher educational cost. When $\beta < \eta$, the decentralized equilibrium understates the social return to shifting workers toward manufacturing.

Proposition 11. *Under $\beta < \eta$, the social planner's allocation can be implemented using three policy instruments $\{\tau_i^{K**}, \tau_i^{T**}, \tau_i^{L**}\}$: a sector-specific investment subsidy, a sector-specific lump-sum tax on firms, and a training subsidy for manufacturing workers. Specifically,*

$$\tau_i^{K**} = 1 - \frac{(1 - \beta)(r + s_i + q_i)}{r + s_i + (1 - \beta)q_i} > 0, \quad (2.69)$$

$$\tau_i^{T**} = \frac{q_i(1 - \eta)}{(r + s_i)} \Phi_i - \tau_i^{K**} P_k k_i / P - \frac{q_i(1 - \beta)[p_i f_i(k_i)/P - z]}{(r + s_i)D_i(\beta)}, \quad (2.70)$$

$$\tau_m^{L**} = r(\epsilon_m - \epsilon_s) - [\theta_m q_m \eta \Phi_m - \theta_s q_s \eta \Phi_s] \quad (2.71)$$

$$- [\theta_m q_m \beta \frac{p_m f_m(k_m)/P - z}{D_m(\beta)} - \theta_s q_s \beta \frac{p_s f_s(k_s)/P - z}{D_s(\beta)}], \quad (2.72)$$

$$\tau_s^{L**} = 0, \quad (2.73)$$

where $\{\theta_i, k_i, q_i, L_i\} = \{\theta_i^{SP}, k_i^{SP}, q_i^{SP}, L_i^{SP}\}$, and

$$D_i(\beta) = r + s_i + (1 - \beta)q_i + \beta\theta_i q_i,$$

$$\Phi_i = \frac{p_i f_i(k_i)/P - z}{D_i(\eta)}.$$

2.6 Calibration

We calibrate the benchmark model at an annual frequency to Chile. Several parameters are set exogenously based on direct empirical counterparts or standard values in the literature. While we target Chilean moments whenever possible, for some parameters we rely on U.S. evidence because of data limitations.

We set the elasticity of substitution between domestic and foreign services $\nu = 2$, within the range commonly used in applied trade models, for example, in Dimaranan et al. (2006). This implies that domestic and foreign services are gross substitutes. We normalize the foreign prices p_m and p_s^F to one. We set the total labor force to one and choose the sum of manufacturing and service labor forces $L = 0.94$, so that the implied agricultural labor force – treated as exogenous – is 6 percent of the total labor force. This targets Chile’s observed agricultural employment share, which is about 6% in 2024. We assume the production functions are in Cobb-Douglas form: $f_i(k_i) = A_i k_i^{\xi_i}$. We normalize the service sector productivity A_s to one. We set ξ_m and ξ_s , to be 0.65 and 0.3, respectively. These values are consistent with the evidence that capital’s income share is substantially higher in industry than in services (0.63 vs. 0.34 in 2022, based on Comisión Nacional de Productividad (CNEP) data). Because sectoral factor-income shares are measured with error, we treat these numbers as informative benchmarks rather than exact targets.

For labor market parameters, we normalize the matching efficiency to one, following standard practice (Mortensen and Pissarides 1994; Petrongolo and Pissarides 2001). We set the elasticity of the matches with respect to unemployment to $\eta = 0.5$, the midpoint of the empirical estimates surveyed by Petrongolo and Pissarides (2001), and in line with standard calibrations (e.g., Hall and Milgrom 2008; Menzio et al. 2016). In the benchmark calibration, we set workers’ bargaining power to $\beta = \eta$, so that the standard matching externality is shut down. We will report

the results for the case $\beta < \eta$.

Table 2.2. Exogenously Set Parameters

Parameter	Description	Value	Source
ν	CES aggregator coefficient	2	Dimaranan et al. (2006)
m	Matching efficiency	1	Normalization
η	Matching elasticity	0.5	Petrongolo and Pissarides (2001)
β	Worker bargaining weight	0.5	Hosios condition
A_s	Services technology	1	Normalization
p_m	m goods price	1	Normalization
p_s^F	Foreign service goods price	1	Normalization
L	sum of L_m and L_s	0.94	Agriculture labor force ~ 0.06
ζ_m	Capital share in manufacturing	0.65	CNEP
ζ_s	Capital share in services	0.3	CNEP

Endogenously calibrated parameters. The remaining parameters are calibrated to match moments in the decentralized equilibrium. Their values are reported in Table 2.3. We choose the preference weight on manufacturing goods, α , so that $L_m = 21.7\%$ in equilibrium, matching Chile’s industrial employment share in 2024. The separation rates, s_m and s_s , are pinned down by sectoral unemployment rates. Specifically, we target a 6.9% unemployment rate in each sector, which equals the average unemployment rate in Chile during 2015-19.¹⁰ The time discount rate r is pinned down by the aggregate capital-to-output ratio $\frac{P_k K}{Y^N}$. In the model, total capital is $K \equiv \sum_i k_i(n_i + v_i)$, and nominal output $Y^N \equiv p_m Y_m + p_s Y_s$. We set r to 0.06, which implies a capital-to-output ratio $K/Y^N = 2.9$. This matches Chile’s non-mining, non-agriculture capital-to-output ratio in 2022 (2.9).¹¹ and is also consistent with the range of estimates in Feenstra et al. (2015) for a broader set of countries.¹²

We calibrate the relative productivity, A_m/A_s , to match the sectoral wage ratio w_m/w_s , which

¹⁰Arroyo et al. (2024) documents that unemployment rates for workers who chose STEM majors are similar to those in non-STEM tracks.

¹¹Chile’s mining sector is capital intensive and accounts for about 10–12 percent of aggregate output. We therefore target the capital-to-output ratio excluding mining to be more representative of other small open economies. Including mining, Chile’s aggregate capital-to-output ratio is about 3.3 in 2022.

¹²See Appendix C in Feenstra et al. (2015).

Elvery and Dunn (2021) estimate to be 4-14 percent for the United States. Finally, we calibrate the educational cost differential, $\epsilon_m - \epsilon_s$, to 70 percent of w_m based on Altonji and Zimmerman (2018), who document training cost by major. Using their estimates, we compare the average training costs in STEM and non-STEM fields, and obtain a difference equivalent to 70-75 percent of the per worker income. The parameter χ governs the equilibrium service import-to-GDP ratio. Because of the balanced trade assumption, the closest data counterpart is goods trade surplus-to-GDP ratio. We choose $1 - \chi = 0.0044$ so that the goods trade surplus is about 5.5 percent of GDP, matching Chile's average trade surplus in 2023-24. Finally, we set unemployment income to $z = 2.5$ which implies a home production-to-wage ratio of 0.30 in equilibrium, within the range of 0.25-0.35 estimated by House et al. (2008).

Table 2.3. Internally Calibrated Parameters.

Parameter	Description	Value	Data Moment
α	Preference weight on m goods	0.174	Industrial employment share
s_m	Separation rate in m	0.048	Unemployment rate in m
s_s	Separation rate in s	0.115	Unemployment rate in s
r	Discount rate (annual)	0.06	Capital-output ratio
$1 - \chi$	Service import weight	0.0044	Goods trade balance-to-GDP
A_m/A_s	relative productivity	2.0	sectoral wage ratio
$\epsilon_m - \epsilon_s$	Education cost differential	6.0	Δ training cost-to-labor income
z	Home production utility	2.5	Home production-to-wage

Table 2.4. Matching the calibration targets and non-targeted moments.

Moment	Target	Model
Share of industrial labor, L_m	21.7%	21.7%
Unemployment rate (m)	6.9%	6.9%
Unemployment rate (s)	6.9%	6.9%
Capital-to-output ratio	2.9	2.9
Goods trade surplus-to-GDP	5.4%	5.4%
Wage ratio w_m/w_s	1.09	1.09
Training cost differential-to-wage	0.72	0.71
Home production-to-wage	0.30	0.29

2.7 Quantitative Results

2.7.1 Decentralized equilibrium versus social planner

Table 2.5 compares the decentralized equilibrium (DE) with the social planner's allocation (SP). Relative to the decentralized equilibrium, the planner chooses a higher capital–output ratio, $K/Y = 3.3$ instead of 2.9. This reflects the holdup inefficiency in the decentralized economy: because wages are bargained after capital is sunk, firms internalize only a fraction of the marginal match surplus when choosing vacancy capital, which leads to underinvestment.

Capital deepening raises output per worker in both sectors, with a larger increase in manufacturing because production there is more capital intensive. Relative to the decentralized equilibrium, real labor productivity, $f_m(k_m)$, is 13.7 percent higher, compared with 10.0 percent in services. Total output also rises more in manufacturing than services, by 17.9 percent versus 7.4 percent.

The social planner also chooses a higher industrial labor force share than the DE: L_m rises by 1.0 percentage point. Intuitively, because manufacturing is more capital intensive, correcting the holdup wedge raises match surplus in manufacturing more than in services. This increases the marginal gain from relocating one more worker from services to manufacturing. The open-economy goods block dampens the relative-price adjustment, so the reallocation primarily through labor quantities rather than relative prices.

Unemployment rates are **higher under the social planner** in both sectors. Intuitively, the planner chooses higher capital per worker, k_i , than in the decentralized equilibrium. Because vacancy creation requires installing job-specific capital, a higher k_i raises the cost of vacancy creation. The planner hence chooses lower market tightness, $\theta_i^{SP} < \theta_i^{DE}$, which reduces job-finding rates and increases unemployment rates. Finally, real output is about 11.3% higher in planner's allocation, driven almost entirely by capital deepening and the resulting increase in labor productivity.

Table 2.5. DE and SP Allocation under Endogenous Capital Level

Moments	DE	SP	Δ
Labor force share (m), L_m	21.7%	22.7%	+1.0 pp
Employment share (m), $n_m/(n + L_a)$	21.6%	22.7%	+1.1 pp
Unemployment rate (m), u_m/L_m	6.9%	7.5%	+0.6 pp
Unemployment rate (s), u_s/L_s	6.9%	7.9%	+1.0 pp
Tightness, θ_m	0.42	0.35	-0.07
Tightness, θ_s	2.40	1.77	-0.63
Capital per worker (m), k_m	841	1024	+21.8%
Capital per worker (s), k_s	84	116	+37.5%
Capital-to-output ratio, K/Y	2.9	3.3	+0.4
Total capital, K	242	309	+27.5%
Relative prices, p_s/p_m	20.1	20.1	-0.1%
Real labor productivity, $f_m(k_m)$	79.6	90.5	+13.7%
Real labor productivity, $f_s(k_s)$	3.8	4.2	+10.0%
Output (m), Y_m	32.2	38.0	+17.9%
Output (s), Y_s	2.5	2.7	+7.4%
Real output, Y	11.8	13.1	+11.5%

2.7.2 Policy when $\beta = \eta$

Policies that implement the planner's allocation.

Under $\beta = \eta$, the decentralized equilibrium is inefficient only because vacancy capital is sunk before bargaining, which generates the holdup wedge. Proposition 9 shows that the planner's allocation can be implemented by a subsidy τ_i^{K*} on newly created vacancies, together with an off-setting lump sum tax τ^{T*i} that preserves the free entry condition. Under the baseline calibration, this yields $\tau_m^{K*} = 5.6\%$ and $\tau_s^{K*} = 15.7\%$. Because the subsidy τ_i^{K*} applies the flow of newly created vacancies, the aggregate fiscal cost of the investment subsidy measured in units of final consumption unit, is

$$\text{Sub}_K = \sum_i \tau_i^{K*} \frac{P_k k_i x_i}{P} = \sum_i \tau_i^{K*} \frac{P_k k_i s_i (n_i + v_i)}{P}. \quad (2.74)$$

where the second equality uses the steady-state condition $x_i = s_i(n_i + v_i)$. Under the baseline

calibration, this subsidy costs 2.4% of GDP per year. It is fully financed by the lump sum tax τ^{T*} .

Investment subsidy only.

In practice, the government may not be able to levy employment-based lump sum taxes due to political constraints, and sector-specific investment subsidies may be difficult to administer. Table 2.6 therefore reports equilibrium outcomes under three policy configurations. DE1 implements the planner's allocation. DE2 sets $\tau^T = 0$ while keeping sector-specific investment subsidies. DE3 imposes a sectoral uniform subsidy and again set $\tau_i^T = 0$.

Labor market outcomes. Removing the lump-sum tax in DE2 and DE3 induces over-entry. Market tightness, θ_i , rises in both sectors, and vacancy-filling rates, q_i , fall. The higher tightness increases job-finding rates and lowers unemployment. The industrial labor force share rises further in DE2 relative to DE1, reflecting stronger vacancy creation in manufacturing when the lump sum tax is absent.

Capital, output, and welfare. Relative to DE1, DE2 and DE3 feature slightly lower capital-to-output ratios. Intuitively, when lump sum tax is absent, firms have stronger incentives to create vacancies than to deepen capital. Output differences are modest: Y_m is slightly higher in DE2 while Y_s is slightly lower. Welfare in DE2 slightly lower than that in DE1 because the lower unemployment also means less home production.

Fiscal cost. DE1 is fiscally neutral, whereas DE2 and DE3 require net fiscal costs of 2.4 and 2.1 percent of GDP, respectively.

Table 2.6. Equilibrium Allocations under Taxes and Subsidies when $\beta = \eta$

Moment	DE1	DE2	DE3
Description	Implements SP	Only τ^K	Only uniform τ^K
τ_i^K	$\tau_i^K = \tau_i^{K*}$	$\tau_i^K = \tau_i^{K*}$	$\tau_i^K = \tau_m^{K*}$
τ_i^T	$\tau_i^T = \tau_i^{T*}$	0	0
Industrial labor force, L_m	22.7%	23.2%	22.7%
Industrial unemploy. rate, u_m/L_m	7.5%	6.8%	6.8%
Services unemploy. rate, u_s/L_s	7.9%	6.8%	6.8%
Capital-to-output ratio, K/Y	3.3	3.3	3.1
Tightness, θ_m	0.35	0.44	0.43
Tightness, θ_s	1.77	2.52	2.49
Real output, Y_m	38.0	38.3	37.4
Real output, Y_s	2.7	2.7	2.64
Welfare, W	4.18	4.15	4.09
Annual fiscal cost (% of GDP)	0	2.4	2.1

2.7.3 Optimal taxes and subsidies when $\beta < \eta$

We now relax the Hosios condition and consider the empirically relevant case in which workers' bargaining power, β , is lower than the matching elasticity, η . We set $\beta = 0.3$ while holding all other parameters, including $\eta = 0.5$, at their benchmark values. This departure introduces the standard search externality, so that vacancy creation is no longer efficient. It also generates the additional wedge in workers' sectoral choices that is central to our model: because workers capture too small a share of match surplus, their sectoral choices remain distorted even after the vacancy-creation externality is corrected.

To illustrate this point, Table 2.7 reports the equilibrium outcomes under three policy settings. First, DE-A is the decentralized equilibrium under $(\beta, \eta) = (0.3, 0.5)$ with no policy instruments. Second, DE-B implements the social planner's allocation using the full policy package: sector-specific investment subsidies and lump sum taxes, $\{\tau_i^{K**}, \tau_i^{T**}\}_{i \in \{m,s\}}$, together with a manufacturing training subsidy τ_m^{L**} .¹³ Third, DE-C applies the same firm-side policies but sets $\tau_m^L = 0$. This last case isolates the additional sectoral choice wedge: if correcting firm-side distortions were

¹³Note that $\tau_s^{L**} = 0$ in Proposition 11.

sufficient, DE-C would coincide with the planner's allocation.

Table 2.7. Equilibrium allocations, $\beta < \eta$

Moment	DE	DE-A	DE-B	DE-C
Description	Benchmark	DE ($\beta < \eta$)	Implements SP	No training subsidy
β	0.5	0.3	0.3	0.3
η	0.5	0.5	0.5	0.5
τ_i^K	0	0	$\tau_i^K = \tau_i^{K**}$	$\tau_i^K = \tau_i^{K**}$
τ_i^T	0	0	$\tau_i^T = \tau_i^{T**}$	$\tau_i^T = \tau_i^{T**}$
τ_m^L	0	0	$\tau_m^L = \tau_m^{L**}$	0
Industry labor force, L_m	21.7%	21.9%	22.7%	20.3%
Industry unemp. rate, u_m/L_m	6.9%	4.6%	7.5%	7.1%
Services unemp. rate, u_s/L_s	6.9%	4.8%	7.9%	8.4%
Capital-to-output ratio, K/Y	2.9	3.0	3.3	3.3
Tightness, θ_m	0.42	0.98	0.35	0.40
Tightness, θ_s	2.40	5.76	1.77	1.59
Real output, Y_m	32.2	32.7	38.0	33.8
Real output, Y_s	2.5	2.6	2.7	2.7
Real consumption C	3.8	3.8	4.1	3.9
Welfare, W	3.9	3.8	4.2	4.0

Relative to the benchmark decentralized equilibrium with $\beta = \eta = 0.5$ (column DE), lowering workers' bargaining power to $\beta = 0.3$ in DE-A raises firms' incentives to create vacancies, resulting in substantially higher market tightness and lower unemployment in both sectors. Manufacturing output increases modestly, while service output changes little. Despite these gains, welfare falls slightly as the labor market becomes excessively tight, allocating too many resources to vacancy creation and reducing the time spent on home production.

The comparison between DE-B and DE-C highlights the additional wedge in sectoral choice. By construction, DE-B coincides with the planner's allocation. In contrast, when the training subsidy is shut down, the labor force share in manufacturing falls sharply in DE-C relative to DE-B. The mechanism is precisely the sectoral wedge implied by $\beta < \eta$: even after firm-side distortions are corrected, workers capture too small a share of surplus to justify the move into manufacturing. A targeted subsidy to manufacturing skill acquisition therefore plays a distinct role in correcting

this wedge.

2.7.4 Trade openness and sectoral employment

In our model, greater service trade openness, captured by a higher value of $1 - \chi$, raises the demand for imported services. Under balanced trade, higher service imports require higher manufacturing exports, which raises labor demand in manufacturing and shifts workers towards that sector. This mechanism is present even in a frictionless small open economy.

Search frictions, however, affect the adjustment margin. As service trade openness rises, imported services account for a larger share of the service consumption basket. Because imported services price is cheaper than domestically produced services, service price index P_s falls. This lowers the relative prices of final consumption compared with both manufacturing and services, P/p_m and P/p_s , and reduces the value of home production z relative to wages in both sectors and thereby lowering equilibrium unemployment.

To illustrate the impact of greater service trade openness, we increase the service-import weight in the service basket from $1 - \chi$ to $1 - \chi'$, where $1 - \chi' = 3(1 - \chi)$. Several changes to the equilibrium follow. First, the service import-to-GDP ratio increases from 5.4 percent to 14.0 percent. Second, because trade is balanced, manufacturing exports also rise, raising manufacturing labor force share from 22.7 percent to 28.9 percent. Third, unemployment falls in both sectors, with the aggregate rate dropping from 6.9 percent to 6.7 percent. This decline is largely driven by the decline of service price P_s and final good consumption P relative to p_m , which reduces the value of home production relative to wages.

2.8 Conclusion

This chapter studies the chicken-and-egg problem in manufacturing development: should policy prioritize boosting investment or closing sector-specific skill gaps? We develop a two-sector small open economy model with DMP-style search and matching frictions to address this ques-

tion. Our key mechanism is job-specific capital chosen before wage bargaining, which creates a capital holdup distortion. The policy implication depends on the Hosios condition. When the Hosios condition holds, correcting capital underinvestment alone is sufficient to replicate the planner's allocation. The optimal policy consists of an investment subsidy, fully financed by lump sum tax on firms. When the Hosios condition fails, an additional wedge distorts workers' sectoral choices beyond the standard search externality, implying a role for targeted training subsidies alongside investment incentives.

Chapter 3

How much work experience do you need to get your first job? The macroeconomic implications of bias against labor market entrants

This chapter is coauthored with Athanasios Geromichalos, Ateş Gürsoy, and Ioannis Kospentaris. A version of this chapter is published in the Review of Economic Dynamics.¹

Abstract

The first step in a worker's career is often particularly hard. Many firms seeking workers require experience in a related field, creating a vicious circle in which an entry-level job is required in order to get an entry-level job. As a result, entrant workers have lower job-finding rates and longer unemployment durations than unemployed workers with prior job-search experience. To study the welfare implications of these observations, we consider a version of the Diamond–Mortensen–Pissarides model in which firms that match with entrant workers incur training costs. Firms are therefore biased against entrants, who remain unemployed for longer periods and become

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exposed to a persistent skill-loss shock. We use a calibrated version of the model to quantitatively assess the effectiveness of four government interventions whose common goal is to reduce bias against entrant workers. We find that the most effective intervention is a subsidy that induces firms to rank entrants higher than experienced workers, bringing the economy close to the constrained-efficient allocation.

3.1 Introduction

The first step in a worker's career is often particularly hard. Entrant workers have much lower job-finding rates than the unemployed who have looked for a job in the past (Figure 3.1). Moreover, positions targeted to workers who have just entered the labor market often still require prior relevant experience (see Section 3.2.1 for a discussion and an illustrative example from online job postings). So a vicious circle is created, whereby an entry level job is required in order to get an entry level job. These observations raise a number of important questions. First, why would firms choose to exclude from the applicant pool workers who are inexperienced, but may turn out to be extremely able? Second, what are the aggregate welfare implications of this bias against inexperienced workers, given that, by definition, *all* workers in the economy start their careers as inexperienced? This last question becomes especially important once we consider the recent literature in labor economics arguing that market conditions during the start of a worker's career have long lasting effects (Von Wachter, 2020). Finally, one wonders whether there is room for welfare improving government interventions and what form these interventions should take.

To study these questions, we augment the classic Diamond-Mortensen-Pissarides (DMP) model in several directions. First, we assume that firms that hire entrant/ inexperienced workers have to incur training expenses. This creates bias against these workers that we capture with a Petrongolo and Pissarides (2001) matching function with ranking. Second, we assume that entrants who stay unemployed for an extended time period are more likely to suffer skill loss. The combination of these two channels creates novel welfare trade offs, since the inherent bias against

entrant workers increases their unemployment duration, which, in turn, tends to lower their productivity. In this environment, an obvious market failure arises. Firms that hire inexperienced workers provide a *benefit to society* by helping these workers stay unemployed for a shorter period of time, thus reducing their exposure to the skill loss shock. (A shock which leaves a permanent “scar”, thus affecting the entrant workers’ productivity when they are hired by other firms later in their career.) However, firms cannot fully internalize this societal contribution, and ultimately choose to discriminate against entrant workers, causing a social welfare loss.

Given this market failure, there is obvious scope for government intervention. Since the root of the inefficiency is the inability of firms to fully recoup the training costs, which results in hiring bias against entrants, we consider government interventions whose common goal is to alleviate the bias and reduce the inexperienced workers’ exposure to the skill loss shock. We use a calibrated version of the model to quantitatively study the effectiveness of three government interventions. The first intervention, which we dub “unbiased matching”, bans discrimination against inexperienced workers by law. In the second intervention (“government subsidies”), the government raises taxes to subsidize firms that hire inexperienced workers. Finally, the third intervention (“internships”) also bans discrimination but additionally explores the possibility that the compensation of entrant workers is determined exogenously by the government.

We find that all three government interventions improve aggregate welfare. This is true even though the aggregate unemployment rate is typically higher under the policy interventions than in the benchmark economy with ranking. To explain the economics behind this result, let us begin with the first intervention, i.e., unbiased matching. Without the ability to discriminate against entrants, firms are effectively forced to incur larger training expenses. As a result, firm entry is discouraged and equilibrium unemployment is higher compared to the baseline economy. Despite this unintended consequence on aggregate unemployment, entrant workers have shorter unemployment spells and are less likely to suffer skill loss. The productivity gains from the latter channel are so large that aggregate welfare increases by 0.58%. The economics behind the second intervention, i.e., government subsidies, is similar, but the welfare increase is smaller (around

0.52%) because the tax needed to finance training subsidies reduces match surplus and further distorts entry.

The third intervention, i.e., internships, achieves all the benefits of the other two, since it also involves unbiased matching, but it suffers less from the downside of discouraged entry. With internships, entrant workers' wages are exogenous and treated as parameters whose level varies. For high enough wages, entrant workers are compensated almost as much as they would under Nash bargaining, and welfare levels are similar to those of Intervention 1. On the other extreme, if entrant wages are too low, firms realize they can hire these workers almost for free, which leads to inefficiently large levels of training and vacancy creation costs. In total, aggregate welfare has an inverse-U shape with respect to the entrants wage level, and it achieves its maximum (0.62% greater than the baseline) when entrants' wages are at intermediate levels. That is, a carefully designed internship scenario achieves the highest welfare among these interventions.

The common thread among the interventions considered so far is that they raise aggregate productivity by abolishing bias against entrant workers and, as a result, lowering their exposure to skill depreciation. In light of this finding, we consider an additional, fourth, intervention, in which the government "goes all the way" subsidizing the hiring of entrant workers so heavily that firms actually prefer to match with these workers rather than the experienced ones. The results of this intervention are striking: it improves aggregate welfare by 1.51%, an improvement almost three times larger than that in the previous interventions. This intervention has the largest impact among all four interventions because it directly confronts the problem of inexperienced workers spending a lot of time in unemployment upon entry.² Finally, we examine how close this fourth intervention can bring the economy to its efficient level by comparing it with the allocation of a social planner who also favors entrant workers. The comparison reveals that the fourth intervention brings the economy arbitrarily close to the constrained efficient outcome. The social planner implements a Hosios (1990)-type condition for our environment; however, it turns out that im-

² It should be pointed out that programs that target entrant workers are particularly popular in Europe. The Youth Employment Initiative, a large EU-sponsored program, directly finances young workers' apprenticeships, traineeships, job placements, and further education within 4 months of leaving school or becoming unemployed. Our model predicts that interventions of this kind generate sizable welfare gains.

plementing the “correct” ranking (i.e., favoring entrant workers) is an order of magnitude more important than fine-tuning the level of vacancy creation.

A crucial assumption in our framework is that firms devote resources to train entrant workers. These resources capture the fact that the experienced workers have to take time away from production to teach the necessary traits to the entrants. Examples of these traits include the ability to work in teams, follow instructions, understand and complete a task, or how to network. There is a plethora of recent empirical papers documenting the importance of firm-provided training in the labor market. Ma et al. (2022), in a cross-country study, document that firm-provided training is a key determinant of workers’ human capital. (See Herkenhoff et al. (2024) for a complementary interpretation of the workers’ human capital component.) Faccini and Yashiv (2022), using German and Swiss data, estimate that training costs (in the form of opportunity costs incurred by managers and coworkers to make new hires as productive as experienced workers) are the dominant source of all hiring costs. Bertheau et al. (2023), using linked survey-administrative data from Denmark, find that one-third of employers consider the time to train new recruits as a major obstacle.

Moreover, there is a literature that highlights the empirical relevance of the externality identified in this paper, namely, the idea that firms underhire inexperienced workers, since they are not fully compensated for the social benefit of the training they provide (Becker, 2009; Acemoglu, 1997; Acemoglu and Pischke, 1999a).³ The literature sometimes refers to this as the “future employer externality”; see Acemoglu (1997) and Lentz and Roys (2024). An important empirical finding in this line of work is that firms provide general training which is not fully offset by lower wages; see Acemoglu and Pischke (1998), Loewenstein and Spletzer (1998), and Autor (2001). Pallais (2014) aptly surveys this literature and concludes that “...neither the theoretical nor the empirical literature shows that firms recoup the full value of their training investments resulting in their providing the optimal level of training” (p. 3568).

³ Moen and Rosén (2004) and Lentz and Roys (2024) study environments with on-the-job search and examine whether the ability of workers to move to competing firms distorts training incentives. The literature has also studied the provision of firm-specific human capital (see Lentz and Roys 2024 for details), but given our research question we focus on general skills that are transferable across firms.

Another important assumption in our model is that entrant workers' skills depreciate in their first unemployment spell. There is compelling empirical evidence documenting that skill loss during the early unemployment spells has persistent negative effects on a worker's career. Arellano-Bover (2022) shows that early career unemployment shocks have negative effects on measured cognitive skills several decades later. Similarly, Dinerstein et al. (2022), using quasi-experimental variation in unemployment duration at the beginning of teachers' careers in Greece, document strong negative effects of the length of unemployment on teachers' performance measured by students' test scores. More generally, there is a large empirical literature, which began with Kahn (2010) and Oreopoulos et al. (2012), that highlights the persistence of the effects of labor market conditions upon entry for young workers on multiple outcomes later in their careers.⁴

To conclude, we should highlight that it is the *interaction* of firm provided training with skill loss during early unemployment that creates novel implications in our paper. As we explained above, each one of these mechanisms on its own is well-studied and its implications are well-understood. In our model, however, the combination of the two elements creates a novel intertemporal trade-off: to save on training costs, firms hire fewer entrant workers today, which, in turn, lengthens the entrants' unemployment duration and results in lower future aggregate productivity. To the best of our knowledge, we are the first to study the theoretical and quantitative implications of this trade-off.

The rest of the paper is organized as follows. Section 3.2 describes the model, including a discussion of the matching protocol, and Section 3.3 analyzes the baseline model with matching bias against inexperienced workers. Section 3.4 considers three government interventions intended to improve welfare and Section 3.5 describes the calibration strategy. Section 3.6 introduces the fourth intervention, discusses the quantitative effects of all policy interventions, and solves the social planner's problem, while Section 3.7 concludes. Appendix C.1 contains proofs of all theoretical results and explores several extensions of the baseline model, including non-permanent skill loss and heterogeneous bargaining powers for entrant and experienced workers.

⁴ For references with structural models of skill loss in unemployment, see Pissarides (1992), Ljungqvist and Sargent (1998), Coles and Masters (2000), Ortego-Marti (2016), Flemming (2020), and Kospentaris (2021), among others.

3.2 The Model

We consider an extension of the Diamond-Mortensen-Pissarides framework. Time is continuous with an infinite horizon, and all agents discount future at rate r . The labor force is normalized to 1. Workers exit the labor market (retire) at Poisson rate δ , and each retired worker is replaced by a new entrant who enters the labor market as unemployed. The retirement of workers and their subsequent replacement by an entrant is crucial in our model because these new entrants will be a special category, and the length of time they spend as unemployed will have long-term consequences. There is a large mass of *ex ante* homogeneous firms who can enter the labor market with one vacancy. As is standard, the measure of active firms in equilibrium is determined by free entry.

Firms that decide to enter the labor market and search for workers must pay a flow recruiting cost c . Existing jobs are terminated at the job destruction rate λ . Generally, job matches produce an amount p of the numeraire good, but this productivity will be affected by the worker's specific type. Firms that have hired entrant (inexperienced) workers must pay a flow training cost κ until the match dissolves. This is our way of modeling the real-world observation that firms are often biased against workers who do not have any working experience. Maybe these new entrants were brilliant students, but they still need training to become *productive workers*. Thus, a possible interpretation is that κ is the time and effort other colleagues must spend with inexperienced workers "showing them the ropes". After losing their first job, workers become automatically experienced and firms that will hire them in the future will not have to pay the κ cost ever again.⁵ (See Appendix Section C.4.2 for a version of the model where workers can gain experience while

⁵ Our "training cost" story is not the only way to capture the bias against inexperienced workers. Another possibility is that employers have asymmetric information about workers' quality. While this is certainly plausible, we think that training is more relevant for our purposes. First, general skills training is an issue that concerns exclusively inexperienced workers, while asymmetric information plagues almost equally experienced and inexperienced workers. Second, a common feature of real-world labor markets is the extensive use of probationary / trial periods during which newly recruited employees' performance is assessed. Since the typical length of such trial periods is as short as 3 months, the cost of hiring a potential "lemon" is not high enough to justify the significant job-finding differences we see in the data. Third, training leads to a simple and tractable model. Finally, as we document in Section 3.1, it is extremely relevant and easier to quantify, as there is a large literature studying training costs in the labor market.

they are still on their first job.)

Since the effective productivity of an inexperienced worker is lower than that of an experienced worker (with κ representing the differential), firms are biased against inexperienced workers, and we will capture this by using the matching process of Petrongolo and Pissarides (2001). As a result, entrant workers will tend to stay in the unemployment pool for longer periods of time, which is exactly what we see in the data (Figure 3.1). Another crucial and empirically relevant assumption we make is that entrants who stay unemployed for a prolonged period of time are at risk of suffering skill loss. We assume that this skill loss is permanent and refer to this phenomenon as the “scar”. The skill loss of entrant workers takes place stochastically, at Poisson rate γ ; when that shock hits an inexperienced worker her productivity declines by an amount $\tilde{\kappa}$, and that skill/productivity loss follows that specific worker for the rest of her life. The literature review in Section 3.1 outlines the large body of work providing support for the permanent nature of skill loss caused by prolonged early career unemployment. However, in Appendix Section C.4.1, we explore a version of the model where the skill loss is not permanent.⁶

To fix ideas and offer the reader a mnemonic rule that will help them comprehend the notation that follows, we now provide a description of the various worker types. We will refer to new entrants who just replaced a retired worker as type-0 workers (i.e., they have “0 working experience”.) Type-0 workers enter the market as unemployed. If they find their first job quickly, they will become employed type-0 workers, and, as discussed, their effective productivity will be $p - \kappa$. After leaving their first job, type-0 workers permanently become type-1 workers, and their productivity in any future job will be equal to p . However, if type-0 workers stay unemployed for a long period of time, they are more likely to be hit by the skill-loss shock; if this happens (before they found their first job), they will turn into type- $\tilde{0}$ (unemployed) workers. When these

⁶ We assume that unemployment spells after the first one have no effect on workers’ skills. We do so because the impact of unemployment on the skills of experienced workers is well-studied and its inclusion would only make the model unnecessarily complicated. More importantly, there is evidence that skill loss may be of limited importance in older ages. For instance, Cohen et al. (2023) find no indication for a decline in skills over the unemployment spell in the overall population in Germany and for the older workers in the US. In the authors’ own words, “This suggests that the negative consequences of unemployment might be a more relevant concern *at younger ages*” (p. 5, emphasis added).

types find their first job, they will become type- $\tilde{0}$ employed workers, and their productivity will be equal to $p - \kappa - \tilde{\kappa}$, since these are inexperienced workers who need training (hence the $-\kappa$), and they have suffered skill loss (hence the $-\tilde{\kappa}$). Since skill loss is permanent, when type- $\tilde{0}$ workers find and, eventually, lose their first job, they will turn into type- $\tilde{1}$ workers. This means that at any future job their productivity will be equal to $p - \tilde{\kappa}$. (These workers are now experienced, but the skill-loss scar remains.)

To sum up, at any point in time there are $2^3 = 8$ types of workers. First, workers can be unemployed or employed. Next, they can be inexperienced or experienced, where an experienced worker is one who has held (and lost) at least one job in the past. Finally, workers can be scarred or not scarred, depending on whether they got hit by the skill loss shock when they first entered the labor market. Notation-wise, the number 0 (1) will denote an inexperienced (experienced) worker, and the “tilde” symbol will denote variables related to a worker who suffered skill loss during youth. For example, $\tilde{f}_0$ is the *job-finding rate* of an inexperienced worker who suffered skill loss, $\tilde{f}_1$ is the *job-finding rate* of an experienced worker who suffered skill loss at youth, w_0 is the *wage* of an inexperienced worker who has not suffered skill loss, u_1 is the measure of experienced (non-scarred) *unemployed* workers, $\tilde{e}_1$ is the measure of scarred experienced *employed* workers, and so on.

The matching process we adopt is a generalization of the Blanchard and Diamond (1994) *matching with ranking* proposed by Petrongolo and Pissarides (2001). This specification allows us to capture the fact that firms are biased against inexperienced and scarred workers, as these workers are less productive. (See Section 3.2.1 for a more detailed discussion of this modeling choice.) This matching function exhibits bias against certain types of unemployed workers who are considered less desirable. The main and very simple idea of this matching technology is that the more productive types of workers get matched first, without being crowded out by the inferior types. When that “first round” of matching has concluded, the less desirable types of workers get a chance to match with firms. Note that in Petrongolo and Pissarides (2001) there are only two types of unemployed workers, while in our paper there are four (type 0, $\tilde{0}$, 1, and $\tilde{1}$). However,

their simple idea that more desirable workers match first is easily carried over to our analysis.

The firms' ranking of workers is based on workers' productivity, which, in turn, is affected by their experience and whether they are scarred. Type-1 workers are the most productive and, hence, the most desirable. It is also quite clear that type- $\tilde{0}$ workers, who are inexperienced and scarred, are the least desirable. In principle, it is not obvious whether firms would prefer workers of type 0 or workers of type $\tilde{1}$, because their ranking depends on the magnitude of training costs (κ) versus the skill loss ($\tilde{\kappa}$). Since our calibration in Section 3.5 implies that $\tilde{\kappa} < \kappa$, we assume that firms prefer workers of type $\tilde{1}$ to type 0. In sum, firms rank workers in the following order: type 1 $\succ$ type $\tilde{1}$ $\succ$ type 0 $\succ$ type $\tilde{0}$.

It turns out that the job-finding rates of the various worker types, implied by the Petrongolo and Pissarides (2001) matching process, can be conveniently expressed as functions of the *queue lengths* of the various types. Thus, we define

$$b_1 \equiv \frac{u_1}{v}; \quad \tilde{b}_1 \equiv \frac{\tilde{u}_1}{v}; \quad b_0 \equiv \frac{u_0}{v}; \quad \tilde{b}_0 \equiv \frac{\tilde{u}_0}{v}, \quad (3.1)$$

where v is the measure of vacancies in the economy. Notice that the each queue length is simply the unemployment-vacancy ratio for that particular worker type (which is the inverse of the market tightness, typically used in the baseline DMP model). Extending the Petrongolo and Pissarides (2001) methodology under a Cobb-Douglas specification in our framework, implies the following job-finding rates for each worker type:⁷

$$f_1 = \frac{m(u_1, v)}{u_1} = b_1^{\alpha-1},$$

$$\tilde{f}_1 = \frac{m(u_1 + \tilde{u}_1, v) - m(u_1, v)}{\tilde{u}_1} = \frac{(b_1 + \tilde{b}_1)^\alpha - b_1^\alpha}{\tilde{b}_1},$$

⁷ Notice how the "ranking" manifests itself in the various job-finding rates. The job-finding rate of the most desirable workers, type 1, is a function only of the queue length for that particular type. But take the next most desirable group, type- $\tilde{1}$ workers: this type's job-finding rate is a function of their own queue length, $\tilde{b}_1$, as well as the queue length of the types that are "above" them in the ranking, b_1 . Intuitively, type- $\tilde{1}$ workers are crowded out by each other and by type-1 workers. Similarly, the least desirable type- $\tilde{0}$ workers are crowded out by all other types, including their own.

$$f_0 = \frac{m(u_1 + \tilde{u}_1 + u_0, v) - m(u_1 + \tilde{u}_1, v)}{u_0} = \frac{(b_1 + \tilde{b}_1 + b_0)^\alpha - (b_1 + \tilde{b}_1)^\alpha}{b_0},$$

$$\tilde{f}_0 = \frac{m(u_1 + \tilde{u}_1 + u_0 + \tilde{u}_0, v) - m(u_1 + \tilde{u}_1 + u_0, v)}{\tilde{u}_0} = \frac{(b_1 + \tilde{b}_1 + b_0 + \tilde{b}_0)^\alpha - (b_1 + \tilde{b}_1 + b_0)^\alpha}{\tilde{b}_0}.$$

The details of these derivations have been relegated to Appendix Section C.1 (where we also report the rates at which firms meet the various worker types).

We close the model with a few standard assumptions. After the matching has concluded and firms have met the various types of workers, the two parties negotiate over the wage using Nash Bargaining. We let $\eta \in [0, 1]$ denote the bargaining power of the worker. All unemployed workers enjoy a benefit z , which we think of as the utility of leisure and/or the value of home production. Notice that, with the exception of productivity and the consequent differences in job-finding rates, all the other parameters of the model $(\eta, z, \lambda, r, \delta)$ are independent of the worker type. This is intentional since we want the results to be driven *only* by differences in the workers' experience and whether they suffered skill loss during their youth, which is the focus of our paper. (That said, in Appendix Section C.4.3 we explore a version of the model where experienced and inexperienced workers have different bargaining powers.)

3.2.1 Discussion of the Matching Process

Given the description of the environment so far, it is obvious that the core of the model consists of homogeneous firms searching for workers of different types. (These types depend on the worker's history, including the length of their unemployment spell during youth and whether they previously held a job.) Deviating from the standard DMP model with homogeneous workers means that we must take a stance on how firms meet with these different types. A natural first attempt may be to consider a model of competitive search, since these models were developed to deal with one-sided market heterogeneity (e.g., see Montgomery 1991). However, it turns out that this modeling choice would not be a good fit for our research question. Instead, we choose to work with the "matching with ranking" model of Petrongolo and Pissarides (2001). The rest of this section

aims to justify this modeling choice.

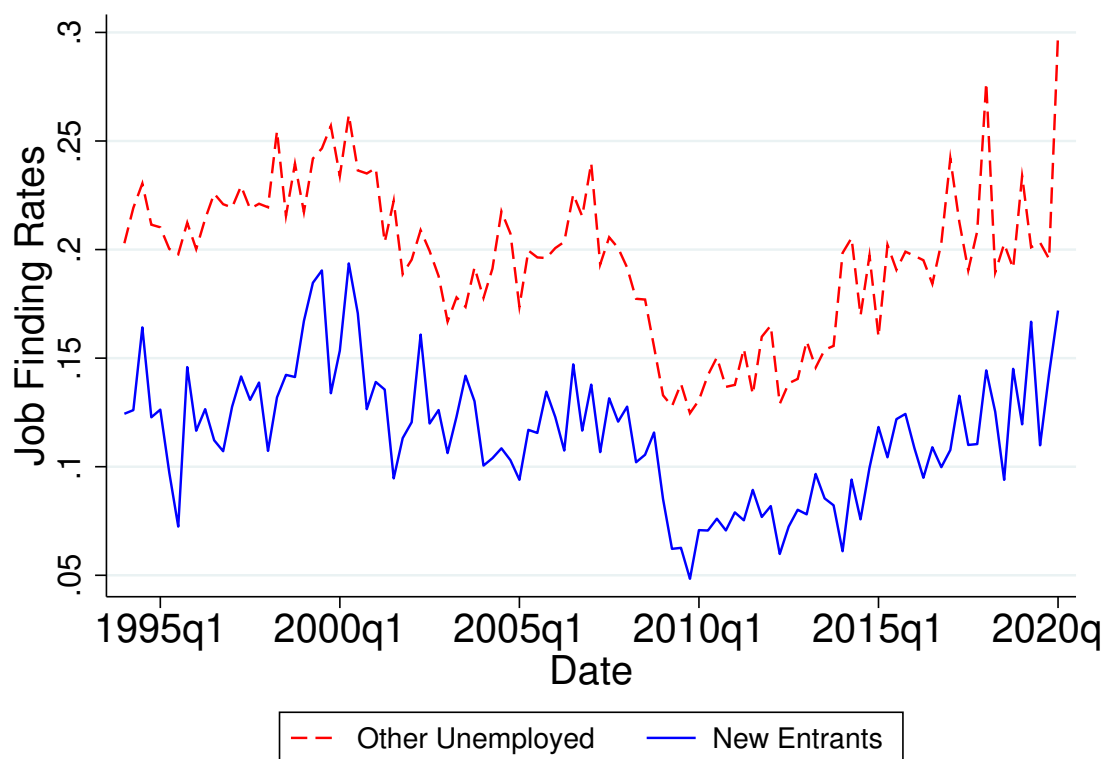


Figure 3.1. Job-finding rates for unemployed workers 16-24 years old.

Note: New entrants are workers who are looking for a job for the first time, while the other unemployed category includes those who have looked for and/or held a job in the past. Calculations based on monthly data of the Current Population Survey for 1994 to 2020.

As can be seen in Figure 3.1, the job-finding rates of entrants are much lower than the job-finding rates of experienced unemployed workers between 16 and 24 years old.⁸ The matching with ranking model of Petrongolo and Pissarides (2001) provides a natural and tractable way to deliver job-finding rates consistent with this fact. Moreover, there is extensive evidence indicating that many firms rank workers without previous experience lower than experienced applicants. Recent analyses of online job postings, for example, suggest that many positions labeled as entry-level still ask for prior relevant work experience.

⁸ The difference is almost the same for workers between 16 and 64 years old as well. However, almost 85% of entrants is between 16 and 24, hence we present the data for this age group.

Why does the Petrongolo and Pissarides (2001) matching with ranking capture the key aspects of the data when other candidates fail to do so? The Petrongolo and Pissarides (2001) specification allows us to model firms' ranking of worker types according to their *current* productivity. In contrast, the alternative matching specifications determine firms' ranking of worker types according to the surplus generated by the workers' improved *future* career prospects. Specifically, the alternative specifications allow firms to grasp an unrealistically large fraction of the future surplus generated due to the higher present job-finding rate of entrant workers. This has two problematic implications: first, it leads to counterfactual job arrival rates and unemployment durations, since it predicts that entrants have higher job-finding rates than experienced workers. Second, there is a large empirical literature (summarized by Pallais 2014) arguing that firms do not recoup the full value of their training investments, which contradicts the implication that firms grasp their fair share of the workers' future surplus. Our "matching with ranking" protocol, under the additional assumption that the ranking is determined by *current* worker productivity, is the only specification that allows the model to be consistent with all these aspects of the data.

There are several features of the actual labor matching process that could explain why firms rank workers according to their current productivity. Ranking according to current productivity is optimal in an environment where firms cannot get their hands on the surplus produced by workers later in their career. This could be because of legal reasons, e.g., minimum wage laws, or practical reasons, e.g., often, workers' compensation is based on heuristic criteria, such as, "pay workers according to their performance".⁹ Details aside, if firms cannot capture a fraction of the future surplus that they help create by offering entrant workers their first job, they end up optimally ranking workers based on the current surplus, i.e., their productivity. We think of our "matching with ranking" specification as a *shortcut* that stands in for the aforementioned frictions, and delivers empirically relevant theoretical predictions in a tractable way.

⁹ It is well-documented that a large fraction of U.S. jobs explicitly pay workers based on their productivity using bonus pay, commissions, or piece-rate contracts; see Lazear (2000) and Lemieux et al. (2009).

3.3 Analysis of the Model

3.3.1 Beveridge curves

We begin the analysis of the model with the derivation of the Beveridge curves. It is useful to inspect Figure 3.2, which illustrates the worker flows in and out of every state. While at first glance the figure may look complicated (workers could be in one of eight possible states), the logic is simple.

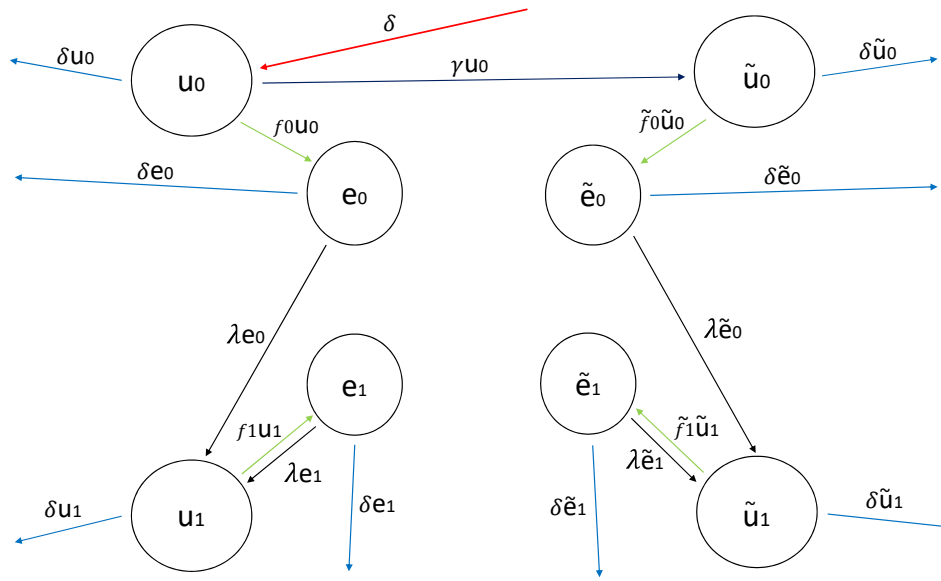


Figure 3.2. Worker flows in and out of the various states.

New entrants come into the labor market as type-0 unemployed workers at rate δ ; this is indicated by the red arrow at the top of the graph. Then, at any state of the world, workers could get hit by the retirement shock and exit the labor market; these are the light blue arrows pointing away from the eight “bubbles” representing the various states. There are also four black arrows starting from employment and pointing to unemployment bubbles; clearly, these are flows initiated by the job destruction shock. Notice that job destruction always leads to the bubble of “experienced

unemployed”, which may be scarred ($\tilde{u}_1$) or not scarred (u_1). Green arrows indicate workers who found a job and are moving from unemployment to employment. As discussed, the rate at which this transition takes place is different for each worker type and governed by the Petrongolo and Pissarides (2001) matching process. Finally, the dark blue arrow starting at u_0 and pointing to $\tilde{u}_0$ captures the crucial group of workers who stayed in the “inexperienced unemployed” pool for too long and got hit by the skill loss shock.

Equating the flows in and out of each state, and after some algebra, we can show that the steady state measure of workers in the various states are as follows:

$$\begin{aligned}
u_0 &= \frac{\delta}{\delta + \gamma + f_0}, \\
\tilde{u}_0 &= \frac{\gamma}{\delta + \tilde{f}_0} \cdot \frac{\delta}{\delta + \gamma + f_0}, \\
e_0 &= \frac{f_0}{\delta + \lambda} \cdot \frac{\delta}{\delta + \gamma + f_0}, \\
\tilde{e}_0 &= \frac{\tilde{f}_0}{\delta + \lambda} \cdot \frac{\gamma}{\delta + \tilde{f}_0} \cdot \frac{\delta}{\delta + \gamma + f_0}, \\
u_1 &= \frac{\lambda f_0}{(\gamma + \delta + f_0)(\delta + \lambda + f_1)}, \\
e_1 &= \frac{f_1}{\delta + \lambda} \cdot \frac{\lambda f_0}{(\gamma + \delta + f_0)(\delta + \lambda + f_1)}, \\
\tilde{u}_1 &= \frac{\lambda \gamma \tilde{f}_0}{(\delta + \tilde{f}_0)(\gamma + \delta + f_0)(\delta + \lambda + \tilde{f}_1)}, \\
\tilde{e}_1 &= \frac{\tilde{f}_1 \lambda \gamma \tilde{f}_0}{(\delta + \lambda)(\delta + \tilde{f}_0)(\gamma + \delta + f_0)(\delta + \lambda + \tilde{f}_1)}.
\end{aligned}$$

3.3.2 Value Functions

We move to the steady state value functions and start with the firms. We let q denote the arrival rate of workers to firms and follow the usual notation, e.g., $\tilde{q}_0$ stands for the arrival rate of an inexperienced worker who has suffered skill loss. The various q 's are derived in Appendix Section

C.1. The value function of a vacant firm is given by

$$rV = -c + q_0(J_0 - V) + \tilde{q}_0(\tilde{J}_0 - V) + q_1(J_1 - V) + \tilde{q}_1(\tilde{J}_1 - V).$$

Of course, free entry implies that in equilibrium we must have $V = 0$, therefore, we can state the free entry condition as

$$c = q_0J_0 + \tilde{q}_0\tilde{J}_0 + q_1J_1 + \tilde{q}_1\tilde{J}_1. \quad (3.2)$$

We also have four value functions for productive firms in the various states, i.e., for firms who matched with the four different types of workers (type 0, $\tilde{0}$, 1, and $\tilde{1}$). These are given as follows:

$$rJ_0 = p - \kappa - w_0 - \lambda J_0 - \delta J_0, \quad (3.3)$$

$$r\tilde{J}_0 = p - \kappa - \tilde{\kappa} - \tilde{w}_0 - \lambda\tilde{J}_0 - \delta\tilde{J}_0, \quad (3.4)$$

$$rJ_1 = p - w_1 - \lambda J_1 - \delta J_1, \quad (3.5)$$

$$r\tilde{J}_1 = p - \tilde{\kappa} - \tilde{w}_1 - \lambda\tilde{J}_1 - \delta\tilde{J}_1. \quad (3.6)$$

Next, consider the value functions of workers in the various states. Let U (W) denote the value function of an unemployed (employed) worker. The remaining notation is standard. (For example, $\tilde{W}_1$ is the value function of a worker who is employed, has had some work experience, but was hit by the skill loss shock during her youth.) The value functions for unemployed workers in the various states are given by:

$$rU_0 = z + f_0(W_0 - U_0) + \gamma(\tilde{U}_0 - U_0) - \delta U_0, \quad (3.7)$$

$$r\tilde{U}_0 = z + \tilde{f}_0(\tilde{W}_0 - \tilde{U}_0) - \delta\tilde{U}_0, \quad (3.8)$$

$$rU_1 = z + f_1(W_1 - U_1) - \delta U_1, \quad (3.9)$$

$$r\tilde{U}_1 = z + \tilde{f}_1(\tilde{W}_1 - \tilde{U}_1) - \delta\tilde{U}_1. \quad (3.10)$$

The value functions for employed workers in the various states are given by:

$$rW_0 = w_0 + \lambda(U_1 - W_0) - \delta W_0, \quad (3.11)$$

$$r\tilde{W}_0 = \tilde{w}_0 + \lambda(\tilde{U}_1 - \tilde{W}_0) - \delta \tilde{W}_0, \quad (3.12)$$

$$rW_1 = w_1 + \lambda(U_1 - W_1) - \delta W_1, \quad (3.13)$$

$$r\tilde{W}_1 = \tilde{w}_1 + \lambda(\tilde{U}_1 - \tilde{W}_1) - \delta \tilde{W}_1. \quad (3.14)$$

Notice that inexperienced workers who lose their first job now move to the pool of experienced unemployed workers. (That is precisely why the terms U_1 and $\tilde{U}_1$ appear on the right-hand side of equations 3.11 and 3.12).

Having described the value functions of all economic agents in detail, we are now ready to study the bargaining problems in the various types of meetings.

3.3.3 Bargaining problems

Bargaining in a type-1 meeting

We begin with the description of the terms of trade in a meeting between a firm and an unemployed worker of type-1, which, as we shall see, is the simplest case. Solving the standard Nash bargaining problem implies that the following condition must be satisfied:

$$(1 - \eta)(W_1 - U_1) = \eta J_1. \quad (3.15)$$

This condition states that each party will enjoy a fraction of the total surplus of the match, and that fraction will be equal to her bargaining power. Replacing the value functions W_1 and J_1 from (3.13) and (3.5), respectively, allows us to write the wage of a type-1 worker as $w_1 = \eta p + (1 - \eta)(r + \delta)U_1$. Substituting U_1 from equation (3.9) into this expression yields

$$w_1 = \eta p + (1 - \eta)z + (1 - \eta)f_1(W_1 - U_1) = \eta p + (1 - \eta)z + \eta f_1 J_1,$$

where the second equality follows from (3.15). Substituting J_1 one more time from equation (3.5), and solving for w_1 , delivers the final version of our “wage curve” for type-1 workers:

$$w_1 = \frac{\eta p(r + \lambda + \delta + f_1) + (1 - \eta)z(r + \lambda + \delta)}{r + \lambda + \delta + \eta f_1}. \quad (3.16)$$

Clearly, this is a relationship between the wage for type-1 workers and their job arrival rate, which, in turn, depends on firm entry and market tightness.

Bargaining in a type- $\tilde{1}$ meeting

Next, consider the bargaining problem between a firm and a worker who is experienced but suffered skill loss during her youth. Once again, we must have:

$$(1 - \eta)(\tilde{W}_1 - \tilde{U}_1) = \eta \tilde{J}_1.$$

As in the case of type-1 workers, we can replace the functions $\tilde{W}_1$ and $\tilde{J}_1$ from (3.14) and (3.6) into the last expression. Following identical steps, and after some algebra, one can easily derive the wage curve for type- $\tilde{1}$ workers:

$$\tilde{w}_1 = \frac{\eta(p - \tilde{\kappa})(r + \lambda + \delta + \tilde{f}_1) + (1 - \eta)z(r + \lambda + \delta)}{r + \lambda + \delta + \eta \tilde{f}_1}. \quad (3.17)$$

Once again, we obtain a relationship between the wage for type- $\tilde{1}$ workers and their job arrival rate, which, in turn, depends on firm entry and market tightness.

Bargaining in type- $\tilde{0}$ meeting

We now move to the bargaining problem between a firm and an inexperienced worker who suffered skill loss. In this case the surplus sharing rule is given by

$$(1 - \eta)(\tilde{W}_0 - \tilde{U}_0) = \eta \tilde{J}_0.$$

As is standard, we first replace the value functions $\tilde{W}_0$ and $\tilde{J}_0$ from (3.12) and (3.4), respectively, which allows us to write the wage of a type- $\tilde{0}$ worker as

$$\tilde{w}_0 = \eta(p - \kappa - \tilde{\kappa}) - \lambda(1 - \eta)(\tilde{U}_1 - \tilde{U}_0) + (1 - \eta)(r + \delta)\tilde{U}_0. \quad (3.18)$$

Unlike the previous cases, where the wage depended on the value function of unemployment for that specific type (only), here $\tilde{w}_0$ depends on both $\tilde{U}_0$ and $\tilde{U}_1$, and specifically on their difference $\tilde{U}_1 - \tilde{U}_0$. To deal with this, subtract (3.8) from (3.10) to obtain

$$\tilde{U}_1 - \tilde{U}_0 = \frac{\tilde{f}_1(\tilde{W}_1 - \tilde{U}_1) - \tilde{f}_0(\tilde{W}_0 - \tilde{U}_0)}{r + \delta}. \quad (3.19)$$

To obtain a useful expression for the term $\tilde{W}_1 - \tilde{U}_1$, that now appears in (3.19), subtract (3.10) from (3.14), to get

$$\tilde{W}_1 - \tilde{U}_1 = \frac{\tilde{w}_1 - z}{r + \delta + \lambda + \tilde{f}_1}. \quad (3.20)$$

Substitute equation (3.20) into (3.19), and the resulting outcome into equation (3.18), and, after some algebra, one can arrive at the wage curve for the type- $\tilde{0}$ worker, specifically:

$$\begin{aligned} \tilde{w}_0 = \frac{1}{r + \delta + \eta\tilde{f}_0} & \left[\eta(p - \kappa - \tilde{\kappa})(r + \delta + \tilde{f}_0) + \right. \\ & \left. + \frac{(r + \delta + \lambda)(r + \delta + \tilde{f}_1)}{r + \delta + \lambda + \tilde{f}_1} (1 - \eta)z - \frac{\lambda(1 - \eta)\tilde{f}_1}{r + \delta + \lambda + \tilde{f}_1} \tilde{w}_1 \right]. \end{aligned} \quad (3.21)$$

Inspection of the last wage curve reveals that, in this case, the wage for type- $\tilde{0}$ workers is not only a function of this type's job arrival rate (as was the case for type-1 and type- $\tilde{1}$ workers). The wage $\tilde{w}_0$ also depends on the wage that type- $\tilde{1}$ workers make. The intuition is clear. When a type- $\tilde{0}$ worker meets a firm, working for that firm is the step that will allow her to move out of the “inexperienced” state, and earn the wage $\tilde{w}_1$ for the rest of her life. This is precisely, why the term $\tilde{w}_1$ enters equation (3.21) with a minus: a higher (future) wage $\tilde{w}_1$ induces the type- $\tilde{0}$ worker to be more eager to accept a lower (current) wage $\tilde{w}_0$, since that lower wage comes together with

the opportunity of abandoning the bad “inexperienced” stage once and for all.

Bargaining in type-0 meeting

The last type of meeting is the one between a firm and an unskilled worker who has not yet been hit by the skill loss shock. The surplus sharing rule is given by

$$(1 - \eta)(W_0 - U_0) = \eta J_0.$$

Following standard steps, substitute W_0 and J_0 from (3.11) and (3.3), respectively, to write the wage of a type-0 worker as

$$w_0 = \eta(p - \kappa) - (1 - \eta)\lambda(U_1 - U_0) + (1 - \eta)(r + \delta)U_0.$$

Just like in the case of type- $\tilde{0}$ workers (and unlike the cases of type-1 and type- $\tilde{1}$ workers), the wage for the types under consideration (also) depends on the differential term $U_1 - U_0$. Since the steps for deriving the final version of the wage curve are virtually identical to the case of type- $\tilde{0}$ workers presented above, we will skip the details and move directly to the final formula:

$$\begin{aligned} w_0 = & \frac{r + \delta + \gamma + f_0}{r + \delta + \gamma + \eta f_0} \eta(p - \kappa) + \frac{(r + \lambda + \delta)(r + f_1 + \delta)(r + \gamma + \delta)}{(r + \delta)(r + \delta + \gamma + \eta f_0)(r + \delta + \lambda + f_1)} (1 - \eta)z + \\ & + \frac{\gamma \tilde{f}_0 \eta(p - \kappa - \tilde{\kappa} - \tilde{w}_0)}{(r + \delta)(r + \delta + \gamma + \eta f_0)} - \frac{(1 - \eta)\lambda f_1(r + \gamma + \delta)w_1}{(r + \delta)(r + \delta + \gamma + \eta f_0)(r + \delta + \lambda + f_1)}. \end{aligned} \quad (3.22)$$

The wage curve for type-0 workers admits an interpretation that is similar to the one following the wage curve for type- $\tilde{0}$ workers, i.e., equation (3.21). The wage that type-0 workers are willing to accept is not just a function of their job arrival rate: w_0 also depends on w_1 and $\tilde{w}_0$. Why w_0 depends on w_1 should now be obvious, given our earlier discussion: the type-0 worker realizes that if she agrees to work for that firm, she will be able to move out of the inexperienced state and earn the wage w_1 henceforth. Why does w_0 also depend on $\tilde{w}_0$? When the type-0 worker agrees to work for the firm with which she has matched, she realizes that she will never again be subject

to the skill loss shock. Thus, what the worker *would* have made if she turned down the firm's offer and continued searching (and being subject to the skill loss shock), i.e., the wage $\tilde{w}_0$, enters the currently negotiated wage through her outside option.

3.3.4 Definition of Steady State Equilibrium

We conclude this section with a formal definition of equilibrium.

Definition 1. A steady state equilibrium in our model is a list of wages for the four types of workers $(w_0, \tilde{w}_0, w_1, \tilde{w}_1)$, a measure of vacant firms v , and measures of unemployed and employed workers in the various states $(u_0, \tilde{u}_0, u_1, \tilde{u}_1, e_0, \tilde{e}_0, e_1, \tilde{e}_1)$, satisfying the free entry condition (3.2), the four wage curves (3.16), (3.17), (3.21), and (3.22), and the eight Beveridge curves reported at the end of Section 3.3.1.¹⁰

3.4 Government Interventions

The discussion so far reveals that our model is characterized by a prominent market failure. Firms that hire entrant workers provide a *public good* to society by transforming them into experienced, and therefore (more) productive workers. However, firms cannot internalize the societal benefits of this public service and choose to discriminate against inexperienced workers. This bias increases the inexperienced workers' unemployment duration, which, in turn, raises their exposure to skill loss, a skill loss which permanently scars the workers, thus affecting their productivity when they are hired by other firms later in their lifetime.

It is obvious that in this environment there is scope for government intervention. Since the root of the inefficiency lies in the firms' decision to underhire inexperienced workers (see also Pallais (2014) and the references therein), we consider three possible government interventions

¹⁰ Implicit in this definition are the queue lengths $(b_0, \tilde{b}_0, b_1, \tilde{b}_1)$, defined in equation (3.1) as functions of the various unemployment measures and v . In turn, these queue lengths are used to determine the various job-finding rates $(f_0, \tilde{f}_0, f_1, \tilde{f}_1)$, which appear in the Beveridge curves, as well as the firms' worker-finding rates $(q_0, \tilde{q}_0, q_1, \tilde{q}_1)$ (described in Appendix Section C.1), which appear in the job creation curve.

whose common goal is to alleviate bias and reduce the exposure of entrant workers to the skill loss shock, with the intention of increasing welfare. We start with a short description of the idea behind each of these three interventions, and then we analyze them in detail in the rest of this section. A *vis-à-vis* comparison of the effectiveness of each intervention will follow in Section 3.6.

Intervention 1: “Unbiased matching”. We dub the first intervention “unbiased matching”, as it describes the case where the government makes it *illegal* for firms to discriminate against any group of workers. Even though one could argue that this intervention is somewhat unrealistic (firms have the right to not hire less productive workers), it is still interesting, from a theoretical point of view, to study a benchmark model without biased matching, which has been a staple of the analysis so far, and appears to be the root of the ongoing inefficiency. Comparing the baseline model with the economy under Intervention 1 can tell us how much welfare can improve if the firms’ bias against inexperienced workers were eradicated. In technical terms, the unbiased matching version of the model simply replaces the Petrongolo and Pissarides (2001) matching process with a standard Pissarides (2000) matching function in which all workers meet firms at the same rate.

Intervention 2: “Government subsidies”. Our second intervention is one where the government raises funds to subsidize firms who hire less productive workers. Subsidies are designed so that firms are effectively *indifferent* among the various types of workers, so that they *choose* to not discriminate against any type of workers. Thus, one can think of Intervention 2 as a market-based way to achieve what Intervention 1 achieves by law, which is arguably the more empirically relevant approach.¹¹

Intervention 3: “Internships”. The third intervention explores the possibility that the wage of entrant workers is not determined endogenously in the model, but chosen exogenously by the government. We dub this intervention “internships” because it assumes that inexperienced workers are not compensated based on their true productivity, but they are treated as trainees whose

¹¹ Despite this similarity, the two interventions do not lead to the same equilibrium outcomes and are not equally effective at improving welfare. The key reason is that Intervention 2 must make use of distortionary taxes to eradicate bias. For details, see Section 3.6.

salary is predetermined and exogenous.¹² We think this is a crucial element of an internship, although we realize that other important elements of real-world internships are absent from our model. In any case, the term “internship” here is just a tag that will summarize Intervention 3. This intervention allows firms to employ inexperienced workers at lower salaries, effectively lowering the bias against these workers and mitigating the inefficiency described in the beginning of this section.

3.4.1 Intervention 1: Unbiased matching

Beveridge Curves

Consider first the Beveridge curves for this version of our model. When it is illegal for firms to discriminate against certain types of workers, all workers match with firms at the same rate. All the Beveridge curves reported at the end of Section 3.3.1 are still valid, but the various matching rates are now equal. That is, $f_0 = \tilde{f}_0 = f_1 = \tilde{f}_1 = f$, where the common arrival rate f is now given by a standard (unbiased) matching process, i.e.,

$$f = \frac{m(u, v)}{u}, \quad (3.23)$$

with u representing the total mass of unemployed workers (of all types).

Value Functions

Next, we move to the value functions under the regime of Intervention 1, starting with the firms. With unbiased matching, the probability that a firm meets a worker of a specific type depends only on that type’s relative representation in the pool of unemployed. Letting q denote the arrival rate of a(ny) worker to the typical firm, i.e., $q = m(u, v)/v$, the value function of a vacant firm is

¹² The fact that inexperienced workers are treated differently than the experienced ones also creates a connection with the “dual labor markets” policies, which specify that some workers are hired under temporary or fixed-term contracts. See Bentolila et al. (2020) and Cahuc (2024).

given by

$$rV = -c + q \left[\frac{u_0}{u}(J_0 - V) + \frac{\tilde{u}_0}{u}(\tilde{J}_0 - V) + \frac{u_1}{u}(J_1 - V) + \frac{\tilde{u}_1}{u}(\tilde{J}_1 - V) \right].$$

As is standard, free entry implies that in equilibrium $V = 0$, therefore, we can state the free entry condition as

$$c = \frac{q}{u} \left(u_0 J_0 + \tilde{u}_0 \tilde{J}_0 + u_1 J_1 + \tilde{u}_1 \tilde{J}_1 \right). \quad (3.24)$$

Even though the process through which firms meet workers is different compared to the baseline model of Section 3.3, once a firm has met a specific type of worker, the value functions for productive firms in the various states remain the same, i.e., they are still given by equations (3.3)-(3.6).

Moving on to the workers, the value functions for unemployed workers in the various states are given by:

$$rU_0 = z + f(W_0 - U_0) + \gamma(\tilde{U}_0 - U_0) - \delta U_0, \quad (3.25)$$

$$r\tilde{U}_0 = z + f(\tilde{W}_0 - \tilde{U}_0) - \delta \tilde{U}_0, \quad (3.26)$$

$$rU_1 = z + f(W_1 - U_1) - \delta U_1, \quad (3.27)$$

$$r\tilde{U}_1 = z + f(\tilde{W}_1 - \tilde{U}_1) - \delta \tilde{U}_1. \quad (3.28)$$

Notice that these expressions are almost identical to the value functions (3.7)-(3.10) reported in Section 3.3.2, with the only difference being that the various arrival rates of that section have now been replaced by the common rate f , defined in equation (3.23).

The last set of value functions for this model specification concerns employed workers. Since employed workers have already matched with a firm, the different matching process assumed in this section will not affect their employment value functions, which are still given by equations (3.11)-(3.14) in Section 3.3.2.

Bargaining problems

As the discussion so far reveals, the only parts of the analysis that are affected by the adoption of the new (unbiased) matching process are those that take place *before* a firm and a worker match. Consequently, all the derivations of the wage curves in Section 3.3.3 remain valid, with the only difference being that the various arrival rates will now be replaced by the common arrival rate f . This observation also sheds some light on the economic insights of this intervention. The government, under Intervention 1, does not intervene in the labor market to change the way in which firms and workers produce or negotiate over the wages. It only intervenes by stating that discriminating against any type of worker, at the recruiting stage, is illegal. By doing this, the government ensures that *all* types of workers have the same matching rate, which, in turn, ensures that entrant/inexperienced workers will not stay unemployed for a prolonged period of time, thus risking a skill loss that will scar them for the rest of their career. We will discuss the effectiveness of this intervention, and how it compares to the alternatives, in Section 3.6.

Given that the derivations of the wage curves in Section 3.3.3 remain unaltered, we will not repeat them here, and we will only report the wage curves for the four types of workers, reminding the reader that they are identical to ones reported in equations (3.16), (3.17), (3.21), and (3.22), once one has replaced the various f 's with the common arrival rate f defined in equation (3.23). More precisely, we have:

$$w_1 = \frac{\eta p(r + \delta + \lambda + f) + (1 - \eta)z(r + \delta + \lambda)}{r + \delta + \lambda + \eta f}, \quad (3.29)$$

$$\tilde{w}_1 = \frac{\eta(p - \tilde{\kappa})(r + \delta + \lambda + f) + (1 - \eta)z(r + \delta + \lambda)}{r + \delta + \lambda + \eta f}, \quad (3.30)$$

$$\tilde{w}_0 = \frac{r + \delta + f}{r + \delta + \eta f} \left[\eta(p - \kappa - \tilde{\kappa}) + \frac{r + \delta + \lambda}{r + \delta + \lambda + f}(1 - \eta)z - \frac{\lambda(1 - \eta)f}{(r + \delta + \lambda + f)(r + \delta + f)}\tilde{w}_1 \right], \quad (3.31)$$

$$\begin{aligned} w_0 = & \frac{r + \delta + \gamma + f}{r + \delta + \gamma + \eta f} \eta(p - \kappa) + \frac{(r + \delta + \lambda)(r + \delta + f)(r + \delta + \gamma)}{(r + \delta)(r + \delta + \gamma + \eta f)(r + \delta + \lambda + f)}(1 - \eta)z \\ & + \frac{\gamma f \eta(p - \kappa - \tilde{\kappa} - \tilde{w}_0)}{(r + \delta)(r + \delta + \gamma + \eta f)} - \frac{(1 - \eta)\lambda f(r + \delta + \gamma)w_1}{(r + \delta)(r + \delta + \gamma + \eta f)(r + \delta + \lambda + f)}. \end{aligned} \quad (3.32)$$

Definition 2. A steady state equilibrium, under Intervention 1, is a list of wages for the four types of workers $(w_0, \tilde{w}_0, w_1, \tilde{w}_1)$, a measure of vacancies v , and measures of unemployed and employed workers in the various states $(u_0, \tilde{u}_0, u_1, \tilde{u}_1, e_0, \tilde{e}_0, e_1, \tilde{e}_1)$, satisfying the free entry condition (3.24), the four wage curves (3.29), (3.30), (3.31), and (3.32), and eight Beveridge curves, which are the ones reported at the end of Section 3.3.1, after one replaces the various f 's with the common arrival rate f defined in equation (3.23).

3.4.2 Intervention 2: Government subsidies

Unlike Intervention 1, where the government could impose unbiased matching by law, here the government raises funds to subsidize firms who hire less productive workers. Specifically, a firm who hires a type-0 worker will receive a (flow) subsidy σ_0 , a firm who hires a type- $\tilde{0}$ worker will receive a subsidy $\tilde{\sigma}_0$, and a firm who hires a type- $\tilde{1}$ worker will receive a subsidy $\tilde{\sigma}_1$. To raise funding for these subsidies, every active firm pays a flat (lump-sum) tax equal to τ . The aforementioned subsidies are designed so that firms are *effectively indifferent* among the various types of workers. Thus, one can think of Intervention 2 as a market-based (as opposed to legislative) way of achieving unbiased matching.

Beveridge Curves

Even though under Intervention 2 this happens for different reasons (subsidies rather than anti-discriminating laws), the end result is that firms are indifferent among the various types of workers. This simply means that all workers face identical job-finding rates, and, consequently, all the Beveridge curves remain the same as the ones described in Section 3.4.1. This, in turn, means that relevant Beveridge curves are the ones reported at the end of Section 3.3.1, but with $f_0 = \tilde{f}_0 = f_1 = \tilde{f}_1 = f$ (where f was defined in equation (3.23)).

Value Functions

Next, we move to the value functions under the regime of Intervention 2, starting with the firms. Using our standard notation, the value functions of firms who have matched with the various

types of workers are given by

$$rJ_0 = p - \kappa - w_0 + \sigma_0 - \tau - \lambda J_0 - \delta J_0, \quad (3.33)$$

$$r\tilde{J}_0 = p - \kappa - \tilde{\kappa} - \tilde{w}_0 + \tilde{\sigma}_0 - \tau - \lambda\tilde{J}_0 - \delta\tilde{J}_0, \quad (3.34)$$

$$rJ_1 = p - \tau - w_1 - \lambda J_1 - \delta J_1, \quad (3.35)$$

$$r\tilde{J}_1 = p - \tilde{\kappa} - \tilde{w}_1 + \tilde{\sigma}_1 - \tau - \lambda\tilde{J}_1 - \delta\tilde{J}_1. \quad (3.36)$$

However, recall that here the various subsidies are designed to make firms indifferent among the various types of workers. This implies that $J_0 = \tilde{J}_0 = J_1 = \tilde{J}_1 = J$. This greatly simplifies the value function of a vacant firm, which is now given by

$$rV = -c + q(J - V),$$

where, as before, $q = m(u, v)/v$, is the worker-finding rate of the typical firm. Free entry implies that in equilibrium we must have

$$c = qJ. \quad (3.37)$$

Since all types of workers under Intervention 2 have identical job-finding rates (albeit for different reasons), the value functions for unemployed workers reported in Section 3.4.1, i.e., equations (3.25)-(3.28), remain valid. The value functions for employed workers are also identical to the ones reported in Section 3.4.1, which, in turn, are identical to the value functions given by equations (3.11)-(3.14) in Section 3.3.2.

Before we move on to the bargaining problems, we present an auxiliary result that will significantly simplify our task.

Lemma 1. *Under Intervention 2 all workers types receive the same wage, i.e., $w_0 = \tilde{w}_0 = w_1 = \tilde{w}_1 = w$.*

Proof. See Appendix Section C.1. □

The proof of the lemma has been relegated to the appendix, but the statement is quite in-

tuitive. Since the government's intervention makes firms indifferent among the various types of workers, it turns out that all the workers will make the same wage in equilibrium. It should also be obvious why Lemma 1 simplifies the analysis: under Intervention 2, we will only have to solve one bargaining problem, instead of four.

Bargaining problem(s)

Since the various J_i terms are all equal to each other, and since the bargaining problem in each type of meeting would imply $(1 - \eta)(W_i - U_i) = \eta J_i$, we must have that the various $W_i - U_i$ terms are also equal to each other. Thus, instead of solving four distinct bargaining problems, in this specification of the model we only need to solve one bargaining problem. As one can see in detail in the proof of Lemma 1, the various $W_i - U_i$ terms are all equal to

$$W - U = \frac{w - z}{r + \lambda + \delta + f},$$

where w is the common wage established in Lemma 1. As for the J term, we can simply replace it from the free entry condition, i.e., equation (3.37). Then, the standard bargaining protocol, prescribing that $(1 - \eta)(W - U) = \eta J$, here implies that

$$w = z + \frac{\eta}{1 - \eta} \frac{c(r + \delta + \lambda + f)}{q}, \quad (3.38)$$

which is our (unique) wage curve under Intervention 2.

The size of the tax and the various subsidies

Before we proceed to the definition of equilibrium under Intervention 2, we must characterize the size of the various subsidies and the flat tax. Exploiting equations (3.33)-(3.36), and keeping in mind that $J_0 = \tilde{J}_0 = J_1 = \tilde{J}_1 = J$, we can deduce that

$$p - \kappa - w_0 - \tau + \sigma_0 = p - \kappa - \tilde{\kappa} - \tilde{w}_0 - \tau + \tilde{\sigma}_0 = p - \tau - w_1 = p - \tilde{\kappa} - \tau - \tilde{w}_1 + \tilde{\sigma}_1.$$

But since Lemma 1 has established that all the wages must be equal (and since all firms pay the same flat tax τ), we have:

$$\sigma_0 = \kappa; \quad \tilde{\sigma}_0 = \kappa + \tilde{\kappa}; \quad \tilde{\sigma}_1 = \tilde{\kappa}. \quad (3.39)$$

Again, this is intuitive. The only way in which the government can make firms indifferent among the various types of workers is by fully covering the “cost” associated with hiring a less productive type (i.e., anyone other than type-1).

The last item we need to specify is the flat tax rate. A balanced government budget constraint implies that

$$\tau = \frac{\kappa e_0 + (\kappa + \tilde{\kappa}) \tilde{e}_0 + \tilde{\kappa} \tilde{e}_1}{e_0 + \tilde{e}_0 + e_1 + \tilde{e}_1}. \quad (3.40)$$

Definition 3. A steady state equilibrium, under Intervention 2, is a (common) wage, w , for all types of workers, a list of subsidies $(\sigma_0, \tilde{\sigma}_0, \tilde{\sigma}_1)$, a flat tax, τ , paid by all active firms, a measure of vacancies v , and measures of unemployed and employed workers in the various states $(u_0, \tilde{u}_0, u_1, \tilde{u}_1, e_0, \tilde{e}_0, e_1, \tilde{e}_1)$. The three types of subsidies are described in equation (3.39), and the tax satisfies equation (3.40). The remaining equilibrium variables satisfy the free entry condition (3.37), the wage curve (3.38), and the eight Beveridge curves reported at the end of Section 3.3.1, after one replaces the various f ’s with the common arrival rate f defined in equation (3.23).

3.4.3 Intervention 3: Internships

Our third intervention is the one dubbed “internships”. Here, we explore the possibility that the wage of entrant/inexperienced workers is not determined endogenously in the model (i.e., by Nash bargaining), but it is chosen exogenously by the government. Let us denote the wages of type-0 and type- $\tilde{0}$ workers by w_0 and $\tilde{w}_0$, respectively. We treat them as exogenous parameters, and in Section 3.6 we discuss how changes in these two terms affect equilibrium welfare. All experienced workers (of type-1 and type- $\tilde{1}$) will continue to earn wages determined by Nash bargaining.

A question that arises is whether firms still have an incentive to discriminate against certain

types of workers. Think, for example, of type-0 workers. These workers must be trained by the firms (their productivity is $p - \kappa$), but their wage is given exogenously by w_0 , which could be very low (perhaps even zero). Whether firms would prefer to hire a type-1 to a type-0 worker depends on the size of κ and w_0 . In fact, as long as κ is not too large, the government could always choose w_0 to be low enough, so that firms prefer to match with a type-0 worker and discriminate against type-1 workers. Since, for now, we have decided to treat w_0 and $\tilde{w}_0$ as parameters whose value can change (thus tilting the firms' preferences towards the various worker types), we will strive for the maximum degree of flexibility, by assuming that firms *do not discriminate* against any type of worker.¹³ This assumption also allows a direct comparison of Intervention 3 with Interventions 1 and 2.

Beveridge Curves

Given our modeling choice to assume no bias in matching, all types of workers face identical job-finding rates, and the Beveridge curves remain the same as in Sections 3.4.1 and 3.4.2. This, in turn, means that relevant Beveridge curves are the ones reported at the end of Section 3.3.1, but with $f_0 = \tilde{f}_0 = f_1 = \tilde{f}_1 = f$ (where f was defined in equation 3.23).

Value Functions

Next, we move to the value functions under the regime of Intervention 3, starting with the firms. Since we assume unbiased matching, the probability that a firm meets a worker of a specific type depends only on that type's relative representation in the pool of unemployed. Letting q denote the arrival rate of a(ny) worker to the typical firm, as we did under Interventions 1 and 2, the value function of a vacant firm is given by

$$rV = -c + q \left[\frac{u_0}{u}(J_0 - V) + \frac{\tilde{u}_0}{u}(\tilde{J}_0 - V) + \frac{u_1}{u}(J_1 - V) + \frac{\tilde{u}_1}{u}(\tilde{J}_1 - V) \right].$$

¹³ Our framework would allow us to explore many different versions, including the less standard case where firms discriminate against type-1 workers; this would be relevant if inexperienced workers are not too unproductive, and they are very cheap. Intervention 4, which we study in Section 3.6.4, is in that spirit.

Free entry implies that $V = 0$, therefore, we can state the free entry condition as

$$c = \frac{q}{u} \left(u_0 J_0 + \tilde{u}_0 \tilde{J}_0 + u_1 J_1 + \tilde{u}_1 \tilde{J}_1 \right). \quad (3.41)$$

The value functions for productive firms in the various states are still given by equations (3.3)-(3.6), but there is an important difference. In Section 3.3, equations (3.3) and (3.4), the terms w_0 and $\tilde{w}_0$ represented endogenous variables; here the value functions appear identical, but these terms represent exogenous policy parameters.

Moving on to the workers, the value functions for unemployed workers in the various states are still given by equation (3.25)-(3.28) in Section 3.4.1, and the value functions for employed workers in the various states are still described by equations (3.11)-(3.14) in Section 3.3. Again, it is useful to point out that the only (conceptual) difference is that the terms w_0 and $\tilde{w}_0$ appearing in equations (3.11) and (3.12) are endogenous variables, while here these same terms are understood to be exogenous policy parameters.

Bargaining problems

As we have explained, under Intervention 3 we only need to solve the bargaining problem in the two types of meetings with experienced workers. Consider first a meeting between a firm and a type-1 worker. As is standard, the Nash protocol requires $(1 - \eta)(W_1 - U_1) = \eta J_1$. Replacing W_1 from (3.13) and J_1 from (3.5), and following the standard steps, we conclude that the wage curve for type-1 workers, under Intervention 3, is given by

$$w_1 = \frac{\eta p(r + \delta + \lambda + f) + (1 - \eta)z(r + \delta + \lambda)}{r + \delta + \lambda + \eta f}. \quad (3.42)$$

Next, consider a meeting between a firm and a type- $\tilde{1}$ worker. Once again, the Nash bargaining protocol requires $(1 - \eta)(\tilde{W}_1 - \tilde{U}_1) = \eta \tilde{J}_1$. As in the case of type-1 workers, we can replace the functions $\tilde{W}_1$ and $\tilde{J}_1$ from (3.14) and (3.6) into the surplus splitting rule. Following identical steps, and after some algebra, one can easily derive the wage curve for type- $\tilde{1}$ workers, under

Intervention 3:

$$\tilde{w}_1 = \frac{\eta(p - \tilde{\kappa})(r + \delta + \lambda + f) + (1 - \eta)z(r + \delta + \lambda)}{r + \delta + \lambda + \eta f}. \quad (3.43)$$

Notice that, the two wage curves are identical expect for the fact that they adjust for the worker's productivity, i.e., p versus $p - \tilde{\kappa}$.

Definition 4. A steady state equilibrium, under Intervention 3, consists of two wages for experienced workers $(w_1, \tilde{w}_1)$, a measure of vacancies v , and measures of unemployed and employed workers in the various states $(u_0, \tilde{u}_0, u_1, \tilde{u}_1, e_0, \tilde{e}_0, e_1, \tilde{e}_1)$, satisfying the free entry condition (3.41), the two wage curves (3.42) and (3.43), and eight Beveridge curves, which are the ones reported at the end of Section 3.3.1, after one replaces the various f 's with the common arrival rate f defined in equation (3.23).

3.5 Calibration

We calibrate the benchmark model with ranking at a monthly frequency. Several parameters are set exogenously to their direct empirical counterparts or by following the literature. We normalize the match output p to 1 and set the discount rate r to 0.0042, consistent with an annual interest rate of 5%. We set the elasticity of the aggregate matching function with respect to unemployment α to 0.5, the midpoint of the estimates reported in Petrongolo and Pissarides (2001) (following, e.g. Menzio et al. 2016, among many others). Following Shimer (2005), the workers' bargaining weight η is also set equal to the elasticity of the aggregate matching function.

Finally, we set the skill loss shock intensity γ to $1/6$, which implies that an unemployed entrant spends on average six months in unemployment before their skills depreciate. We chose the six months interval for two reasons, one conventional and one substantial: first, the definition of "long-term unemployment" according to the Bureau of Labor Statistics is consecutive unemployment of 27 weeks and over. Second, and more substantial, it is well known from the duration dependence literature that the job-finding probability strongly decreases for the first six months

in unemployment and flattens out afterwards (see Jarosch and Pilossoph 2019 and Kospentaris 2021 among many others). Hence, based on the job-finding duration profile, it seems that the six months threshold is a discrete event for the transition from short- to long-term unemployment and we treat it as such in our calibration.

Table 3.1. Exogenously Set Parameters.

Parameter	Description	Value	Source
r	Discount Rate	0.0042	Annual Interest Rate of 5%
p	Match Output	1	Normalization
α	Matching Function Elasticity	0.5	Menzio et al. (2016)
η	Worker Bargaining Power	0.5	Shimer (2005)
γ	Skill Loss Intensity	1/6	Duration Dependence Literature

The remaining six parameters are calibrated through the model and their values are reported in Table 3.2. The vacancy creation cost c , the worker exit/entry rate δ , and the separation rate λ are chosen to make the model consistent with the following labor market moments, respectively: i) the aggregate unemployment rate $(u_0 + \tilde{u}_0 + u_1 + \tilde{u}_1)$, ii) the fraction of entrants in the unemployment pool $((u_0 + \tilde{u}_0)/(u_0 + \tilde{u}_0 + u_1 + \tilde{u}_1))$, and iii) the fraction of long-term unemployed among entrants $(\tilde{u}_0/(u_0 + \tilde{u}_0))$. Next, we follow Hall and Milgrom (2008) and set the opportunity cost of employment z to 71% of average worker productivity. Regarding the size of skill loss $\tilde{\kappa}$, we employ the estimates of Ortego-Marti (2016, 2017) which imply a monthly 1.22% drop in worker wages while the worker is unemployed.

Table 3.2. Internally Calibrated Parameters.

Parameter	Description	Value
c	Vacancy Cost	0.4877
δ	Worker Exit Rate	0.0023
λ	Separation Rate	0.0499
z	Unemployment Value	0.6535
$\tilde{\kappa}$	Skill Loss Scar	0.0588
κ	Training Cost	0.1590

Finally, to discipline the training cost parameter κ we use numbers reported by training professionals for US businesses. The specialist publication *Training Magazine* asks businesses from several industries about their expenses devoted to employee training and reports the results in their Training Industry reports.¹⁴ The annual training expenses per employee were \$ 1,075 in 2017 and reached \$ 1,207 in 2022. To be conservative, we chose κ to match annual training expenses of \$ 1,000 per employee, which is 0.75% of the US GDP. This means that total training expenses are of a similar order of magnitude as total vacancy creation costs which are usually estimated to be 1-2% of GDP (see, e.g., Michaillat and Saez 2021). Using a different calibration strategy, Masui (2023) also estimates training costs to be close to vacancy creation costs, which provides a sanity check for our strategy. As can be seen in Table 3.3, the model exactly matches the calibration target (the difference between model-implied and data moments is in the order of 10^{-8}).

Table 3.3. Matching the Calibration Targets.

Target	Data	Model
Unemployment Rate	5.8%	5.8%
Unemployed Entrants/Unemployed	9%	9%
Long-term Unemployed Entrants/ Unemployed Entrants	28%	28%
Value of Non-Employment/ Average Productivity	71%	71%
Wage Loss for Six Months Unemployment	7.1%	7.1%
Training Expenses/ GDP	0.75%	0.75%

We also report various untargeted moments to support the model's external validity. To begin with, the unemployment rates of the different worker types predicted by the model are: $u_0 = 10.7\%$, $\tilde{u}_0 = 10.9\%$, $u_1 = 4.1\%$, and $\tilde{u}_1 = 9.2\%$. Taking the youth unemployment rate as a proxy for the the unemployment rate of inexperienced workers, the model does quite well: the corresponding long run average in the data is 12% (World Development Indicators by the World Bank, 1991-2023).¹⁵ Next, using the outgoing rotation groups in the CPS, we look at the

¹⁴ Available here: <https://trainingmag.com/2022-training-industry-report/>.

¹⁵ Unfortunately, the short panel dimension of the CPS does not allow us to compute the appropriate statistics for the unemployment rate of each worker type. Specifically, we cannot distinguish between the unemployed of

wage differences between the two groups: newly hired experienced workers have on average 12% greater hourly wages compared to the newly hired entrants. Our model can explain why inexperienced workers have lower wages, although it slightly overestimates the differential which is 35%. Specifically, the model does quite well estimating the wage differential between workers of types $\tilde{1}$ and $\tilde{0}$ (21%) but overpredicts the differential between types 1 and 0 (48%); see Table 3.4. This is intuitive because in the model workers of type 0 are willing to accept very low wages to move out of state 0 as quickly as possible (see Section 3.6 for additional details). In the real world, however, legal restrictions, such as minimum wage laws, may prevent the wages of entrants to fall below some threshold even if workers were willing to accept such wages.

3.6 Quantitative Results

In this section, we present the quantitative effects of the government interventions analyzed in Section 3.4. We focus on the main aggregate variables of interest: i) the aggregate output minus total vacancy costs (Y , which is also our main measure of welfare, since agents are risk neutral), ii) the aggregate unemployment rate (u), iii) the job-finding rate of each worker type ($f_1, \tilde{f}_1, f_0, \tilde{f}_0$), and iv) the wage of each worker type ($w_1, \tilde{w}_1, w_0, \tilde{w}_0$). The quantitative results are presented in Table 3.5 as percentage differences from the baseline ranking economy and analyzed in the rest of this section.

Table 3.4. Baseline Economy: Matching with Ranking.

Y	u	f_1	$\tilde{f}_1$	f_0	$\tilde{f}_0$	w_1	$\tilde{w}_1$	w_0	$\tilde{w}_0$
0.9379	5.8%	1.21	0.51	0.44	0.42	0.9852	0.9152	0.6620	0.7535

types 1 and $\tilde{1}$ and it is impossible to compute the masses of employed workers of different types. Regarding the unemployed, we do not have information on the first unemployment spell for experienced workers who are currently unemployed. Hence, we do not know whether an unemployed worker is type 1 or type $\tilde{1}$ in the data. Regarding the employed, the CPS does not keep track of the unemployment history of employed workers who entered the survey as employed. Hence, we cannot distinguish between different groups of employed workers according to their unemployment duration before finding their current job unless we observe them finding a job during their CPS tenure (which constitutes a very small fraction of employed workers).

Before analyzing the results of each government intervention, we summarize the results of the baseline economy in Table 3.4 (i.e., the model of Section 3.2 evaluated at the calibrated parameters of Section 3.5). A few features of the benchmark economy are worth noting. First, the order of the job-finding rates follows the order of the productivity exhibited by the different worker types ($p > p - \tilde{\kappa} > p - \kappa > p - \kappa - \tilde{\kappa}$). There is, however, a discrete jump between the job-finding rate of type-1 workers and the other unemployed, which are relatively close in magnitude. That is, firms strongly prefer to match with experienced workers without a scar than with the other unemployed types. Second, entrant workers of type 0 have particularly low wages compared to the other types. Strikingly, they receive lower wages than their scarred counterparts of type $\tilde{0}$ who also have lower productivity. This is consistent with the intuition we gave in Section 3.3.3 that entrant workers are eager to leave their current state and, as a result, willing to accept very low wages to incentivize firms to hire them. This force is stronger for type-0 workers because they have more to gain by leaving the entry state as soon as possible to enjoy a career as non-scarred experienced workers, while for type- $\tilde{0}$ workers it is too late.

Table 3.5. Quantitative Effects of Interventions and the Constrained Optimum.

Variable	Unbiased	Subsidies	Internships	Type-0 Bias	Planner
Y	0.58%	0.52%	0.62%	1.51%	1.52%
u	2.65%	9.96%	-9.53%	7.81%	6.75%
f_1	-31.78%	-36.60%	-22.01%	-37.57%	-36.92%
$\tilde{f}_1$	60.96%	49.59%	83.99%	-23.24%	-22.43%
f_0	88.75%	75.41%	115.77%	374.99%	379.29%
$\tilde{f}_0$	93.27%	79.62%	120.94%	-8.26%	-7.28%
w_1	-0.62%	-2.46%	-0.38%	-1.52%	-
$\tilde{w}_1$	0.94%	4.50%	1.16%	-1.42%	-
w_0	14.43%	45.17%	-2.93%	49.65%	-
$\tilde{w}_0$	1.67%	27.53%	-13.28%	-1.05%	-

Note: Each number is the percentage difference between the value of the variable in the equilibrium with a particular intervention versus the value of the variable in the baseline economy of matching with ranking.

3.6.1 Unbiased Matching

In this scenario, it is illegal for firms to rank workers of different types. Essentially, there is a common job-finding rate for all workers, given by a standard Cobb-Douglas matching function (equation 3.23). Intuitively, this intervention forces firms to incur larger training expenses, since they have no way of discriminating against entrants. As a result, entry drops and the aggregate unemployment rate is almost 3% higher than the baseline economy with ranking (second column of Table 3.5). At the same time, this intervention features a substantially higher welfare level than the ranking economy. The reason is the sizable improvement in the job-finding prospects of entrant workers: f_0 and $\tilde{f}_0$ are 89% and 93% higher than their baseline levels, respectively. This means that entrant workers spend less time in unemployment and thus suffer less from lower skill loss compared to the baseline model. This trumps the productivity losses due to larger training costs and results in a 0.58% increase in aggregate welfare. Finally, the wages of type-0 entrants also improve considerably making them the big winners of this intervention.¹⁶

3.6.2 Government Subsidies

As explained in Section 3.4, this intervention is the market-oriented way of implementing what the ranking ban achieves through legislation: unbiased matching. To incentivize firms to not rank workers in hiring, the government subsidizes training and skill loss costs (see equation 3.39) and balances its budget with a uniform lump-sum tax τ equal to 1.95% of average match productivity. Given the similarity of outcomes between Interventions 1 and 2, the economics behind the two reforms is also similar: incentivizing firms to hire entrants increases training costs, lowers entry but saves on skill loss and raises aggregate welfare (third column of Table 3.5). The fact that the government takes away part of the surplus in tax revenue explains the relatively smaller effect on welfare and the job-finding rates of entrants compared to Intervention 1. It is of interest, how-

¹⁶ In terms of levels, wages follow the order of worker productivity ($w_1 > \tilde{w}_1 > w_0 > \tilde{w}_1$) but recall that the wage of type-0 workers is by far the lowest in the ranking economy. Hence, in percentage terms, workers of type 0 see the biggest percentage improvement in wages.

ever, that the wage effects of the two interventions differ: the common wage of Intervention 2 results in larger gains for workers of types $\tilde{1}$, 0, and $\tilde{0}$, as well as larger losses for type 1. In other words, there is a trade-off in terms of job-finding rates and wages which depends on whether unbiased matching is achieved through legislation (with smaller wage but larger job-finding effects) or market-based incentives (with larger wage but smaller job-finding effects).

3.6.3 Internships

The third intervention we look at is “internships”: entrant workers’ wages are exogenously fixed and discrimination against entrants is not allowed.¹⁷ To ease the presentation, we fix $\tilde{w}_0 = z$ and show how welfare changes for various values of w_0 (the results in the fourth column of Table 3.5 are for the value of w_0 that maximizes aggregate welfare). None of the economics of the intervention rests on this assumption though; that is, fixing $\tilde{w}_0$ at some other level or varying both $\tilde{w}_0$ and w_0 would deliver the same insights.

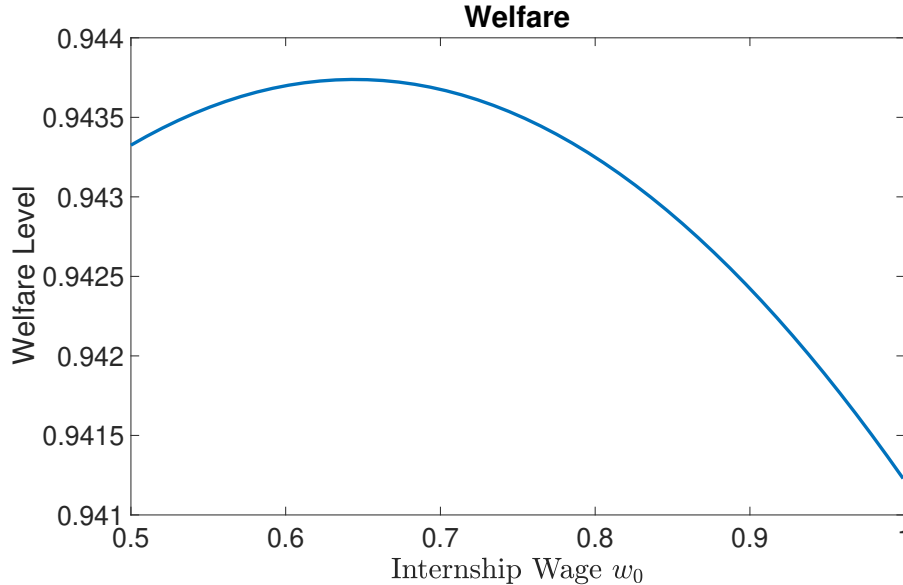


Figure 3.3. Aggregate Welfare for Various Levels of Entrant Wages.

Figure 3.3 graphs the aggregate welfare for different values of w_0 . As the entrants’ wage

¹⁷ As explained in Section 3.4.3, there are several reasons behind this choice. Most importantly, it is the most transparent formulation and allows a direct comparison with the first two interventions. It should be noted, however, that our model can easily handle internships with endogenous ranking.

increases, aggregate welfare initially increases but after some critical value decreases, yielding an inverse-U shape. For relatively large values of w_0 , the intervention yields familiar insights: entrants are expensive, firms are discouraged to enter to avoid training costs, and wage increases lead to larger skill loss and lower aggregate welfare. For relatively low values of w_0 , however, the intervention creates the opposite problem: entrants are too cheap and entry is inefficiently high. As a result, when w_0 increases, aggregate welfare also increases as the economy saves in vacancy creation and training costs (this is the well-known reasoning of Hosios 1990). In total, there is a welfare maximizing level of w_0 , which is 3% lower than the corresponding level in the baseline economy.¹⁸

The most important takeaway is that a carefully designed “internship” achieves the highest welfare level among all three interventions because it delivers the largest increase in the job-finding rate of type-0 workers. At the same time, every worker type gains from internships other than the experienced type-1 workers. All in all, these results offer a rationalization of internships in terms of raising aggregate welfare, which may partially explain the popularity of actual internship positions found in modern labor markets.

3.6.4 A fourth intervention

The discussion so far conveys that government interventions that eradicate bias against entrant workers can achieve significant welfare improvements through the reduced unemployment duration of these workers, which lowers their exposure to skill depreciation shocks and raises aggregate productivity. In light of this finding, one wonders about the effects of an intervention that “goes all the way” and actually induces firms to be biased *in favor* of entrant workers. A simple way to achieve this is to consider a government that subsidizes type-0 workers generously enough to tilt the firms’ ranking in favor of type-0 over type-1 workers. In this environment,

¹⁸ In the discussion of Interventions 1 and 2, we highlighted that equilibrium welfare is higher despite the fact that the aggregate unemployment rate is higher than the benchmark economy. This result is not universally true under Intervention 3: unemployment in the model with ranking is higher than in the model with Internships if and only if w_0 is lower than 75% of p .

matching is still characterized by the Petrongolo and Pissarides (2001) specification, but, due to the government's decision to (heavily) subsidize type-0 workers, that worker type is now ranked number one in the firms' preferences. Specifically, we consider a government that provides a subsidy $\sigma_0 = \kappa(1 + \rho)$ to every firm that hires a type-0 worker.¹⁹ We treat ρ as a parameter that captures the government's generosity, but, importantly, any $\rho > 0$ implies that firms now rank type-0 workers first. Thus, the firms' ranking of workers is: type 0 $\succ$ type 1 $\succ$ type $\tilde{1}$ $\succ$ type $\tilde{0}$. The arrival rates, value functions, and various equilibrium conditions under Intervention 4 are described in Appendix Section C.2 for the interested reader. We dub this intervention "type-0 bias" and present its effects in the fifth column of Table 3.5.²⁰

The results of this intervention are striking: it improves aggregate welfare by 1.51%, an improvement almost three times larger than the one achieved with the previous interventions. The reason for this sizable improvement is of course the treatment of type-0 workers, whose job-finding rate increases by an order of magnitude. On the contrary, all other worker types lose from the intervention, since both their job-finding rates and wages substantially decrease, while aggregate unemployment also increases. The success of Intervention 4 lies on the fact that it directly confronts the problem of type-0 workers spending a lot of time in unemployment upon entry. As a result, it has the largest impact on the economy's aggregate productivity among all four interventions.

It is worth mentioning that even though programs of this kind are not popular in the US, they are actually quite common in Europe. The Youth Employment Initiative, for example, is one of the main financial resources of the European Union to support young people who are not in education or employment. The total budget of the Youth Employment Initiative was €8.9 billion for the period 2014-2020 to directly finance young workers' apprenticeships, traineeships, job placements, and further education within 4 months of leaving school or becoming unemployed.²¹

¹⁹ Since the subsidy pays for itself, we assume that every firm who is actively producing must pay a flat tax, so $\tau = \kappa(1 + \rho)e_0/(e_0 + \tilde{e}_0 + e_1 + \tilde{e}_1)$, which is the direct analogue of equation (3.40).

²⁰ For this exercise we have set $\rho = 5\%$ because it is small enough to not distort entry incentives too much but still sizable to incentivize firms to hire type-0 workers.

²¹ The details of the program are available here: <https://ec.europa.eu/social/main.jsp?langId=en&catId=89&n>

Moreover, there are several vocational and training programs directed to inexperienced workers in several European countries: the German *Duale Ausbildung*, for example, combines on-the-job training with classroom education at vocational schools. In similar spirit, it is quite common to combine apprenticeships with vocational education and training programs. As an example, the French government has supported the increase of apprenticeship contracts, from 283,500 in 2015 to 809,000 contracts in 2022.²² Based on our results, interventions of this kind are expected to have large welfare gains, even though they may raise the unemployment rates of other worker groups.

Having shown that Intervention 4 (type-0 bias) yields sizable welfare gains, it is natural to ask how close it brings the economy to its efficient level. As a benchmark of efficiency, we consider the problem of a social planner who chooses the aggregate measure of vacancies, while being constrained by the matching technology, which in our case is the Petrongolo and Pissarides (2001) matching with ranking. Importantly, however, the ranking the planner is subject to ought to be *aligned* with the ranking described under Intervention 4, since this is the spirit of our exercise. The details of the social planner’s problem and its solution are presented in Appendix Section C.3. Here we illustrate the main results.

The quantitative implications of the planner’s allocation are shown in the last column of Table 3.5. The main takeaway from these results is that the planner implements what Intervention 4 does: raises the job-finding rate of type-0 workers by an order of magnitude to save their skills from depreciation. To make this happen, the planner has to give priority to entrants compared to the other unemployed, something Intervention 4 achieves by subsidizing firms to rank type-0 workers above the other unemployed. As a result, the job-finding rates of the other worker types are substantially lower in the constrained efficient allocation than in the baseline economy. Table 3.5 demonstrates that Intervention 4 brings the economy particularly close to the constrained efficient outcome. Intuitively, the social planner implements a Hosios-type condition for our environment; however, it becomes apparent that implementing the “correct” ranking is an order of

ewsId=9793&furtherNews=yes.

²² More details can be found here: <https://www.centre-inffo.fr/content/uploads/2023/03/fpc-en-france-2023-a4-en-march-2023.pdf>.

magnitude more important than fine-tuning the level of vacancy creation.

3.7 Conclusion

We develop a DMP model where entrant workers must be trained by firms, and if they stay unemployed for a prolonged period of time they are subject to a permanent “scar” in their productivity. Depending on their history (how much time they spent in unemployment during their first unemployment spell and whether they previously held a job), unemployed workers attain one of four types that differ in productivity, and homogeneous firms search for these different worker types. We adopt the Petrongolo and Pissarides (2001) “matching with ranking” process, assuming that firms exhibit bias against the workers with the lowest productivity, i.e., entrants. The adoption of this matching specification allows our model to capture the stylized fact that motivates this paper: entrant workers have lower job-finding rates and longer unemployment spells.

In our model, firms that hire entrant workers provide a *public good* by reducing these workers’ unemployment spells, thus mitigating their exposure to the skill loss shock and reducing the mass of “scarred” workers in the unemployment pool (and the probability that other firms will match with them in the future). However, firms cannot fully internalize this societal contribution, and ultimately discriminate against entrant workers, causing a social welfare loss. Given this market failure, there is obvious scope for government intervention, aimed to alleviating bias against entrant workers. In a calibrated version of the model, we quantify the effectiveness of three government interventions: “unbiased matching”, “government subsidies”, and “internships”. We find that all three interventions improve aggregate welfare, even though the aggregate unemployment rate is typically higher under the interventions than in the benchmark economy with ranking/bias. The key behind this result is that all government interventions effectively induce firms to incur larger training expenses, thus discouraging entry and increasing unemployment. Despite this unintended consequence on aggregate unemployment, entrant workers have shorter unemployment spells and, as a result, are less likely to suffer skill loss. The productivity gains

from the latter channel are so large that aggregate welfare ultimately increases.

Taking this reasoning to its extreme, we consider a fourth intervention in which the government subsidizes the hiring of entrant workers so heavily that firms actually rank these workers higher than the experienced ones. This intervention delivers a substantial improvement in aggregate welfare (roughly three times larger than the other three interventions) because it directly confronts the problem of inexperienced workers spending a lot of time in unemployment upon entry. Hence, our analysis predicts that programs that promote the training and job-placement of young workers, such as the Youth Employment Initiative in the EU, generate sizable welfare gains. Finally, we examine how close this fourth intervention can bring the economy to its efficient level, by comparing the equilibrium welfare with the welfare achieved in an appropriately chosen social planner's problem. We find that the fourth intervention brings the economy arbitrarily close to the constrained efficient outcome.

Chapter A

Appendix for Chapter One

A.1 Derivation of the Social Planner Conditions

This appendix derives Proposition 2. The derivation proceeds in four steps. First, I derive the planner's tightness condition. Second, I derive the shadow value of employment. Third, I derive the sectoral-allocation condition. Fourth, I derive the green human-capital condition as a pre-match investment condition.

For notational compactness, define

$$a_g(h_g) \equiv (p_g + \phi)y_g(h_g), \quad a_n \equiv p_n y_n,$$

and use the identities

$$u_j = L_j - n_j, \quad v_j = \theta_j u_j, \quad m_j = f(\theta_j)u_j, \quad L_n = 1 - L_g.$$

Let μ_j denote the current-value shadow value of employment in sector j .

The Hamiltonian characterizes tightness and sectoral allocation for a given level of green human capital. Because h_g is chosen before matching by green searchers, its optimality condition is derived separately as a pre-match investment problem rather than as a contemporaneous control

in the matching block. Holding h_g fixed, the current-value Hamiltonian is

$$\begin{aligned}\mathcal{H} = & a_g(h_g)n_g + a_n n_n + z(u_g + u_n) - c\theta_g u_g - c\theta_n u_n \\ & - r\kappa_g L_g - L_g \chi(h_g) \\ & + \mu_g [f(\theta_g)u_g - \lambda n_g] + \mu_n [f(\theta_n)u_n - \lambda n_n].\end{aligned}\tag{A.1.1}$$

Planner tightness. The first-order condition with respect to θ_j is

$$\frac{\partial \mathcal{H}}{\partial \theta_j} = -cu_j + \mu_j f'(\theta_j)u_j = 0.\tag{A.1.2}$$

For Cobb–Douglas matching,

$$f'(\theta_j) = (1 - \eta)A\theta_j^{-\eta} = (1 - \eta)q(\theta_j).\tag{A.1.3}$$

Therefore,

$$c = (1 - \eta)q(\theta_j)\mu_j.\tag{A.1.4}$$

Shadow value of employment. The current-value costate equation is

$$\dot{\mu}_j = r\mu_j - \frac{\partial \mathcal{H}}{\partial n_j}.\tag{A.1.5}$$

Because $u_j = L_j - n_j$,

$$\frac{\partial \mathcal{H}}{\partial n_j} = a_j - z + c\theta_j - \mu_j f(\theta_j) - \lambda\mu_j.\tag{A.1.6}$$

Hence,

$$\dot{\mu}_j = r\mu_j - [a_j - z + c\theta_j - \mu_j f(\theta_j) - \lambda\mu_j].\tag{A.1.7}$$

In steady state, $\dot{\mu}_j = 0$, so

$$[r + \lambda + f(\theta_j)]\mu_j = a_j - z + c\theta_j.\tag{A.1.8}$$

Using (A.1.4),

$$c\theta_j = (1 - \eta)q(\theta_j)\theta_j\mu_j = (1 - \eta)f(\theta_j)\mu_j.$$

Substituting into (A.1.8),

$$[r + \lambda + f(\theta_j)]\mu_j = a_j - z + (1 - \eta)f(\theta_j)\mu_j, \quad (\text{A.1.9})$$

$$[r + \lambda + \eta f(\theta_j)]\mu_j = a_j - z. \quad (\text{A.1.10})$$

Therefore,

$$\mu_j = \frac{a_j - z}{r + \lambda + \eta f(\theta_j)}. \quad (\text{A.1.11})$$

Thus,

$$\mu_g = \frac{(p_g + \phi)y_g(h_g) - z}{r + \lambda + \eta f(\theta_g)}, \quad (\text{A.1.12})$$

$$\mu_n = \frac{p_n y_n - z}{r + \lambda + \eta f(\theta_n)}. \quad (\text{A.1.13})$$

Substituting these shadow values into (A.1.4) gives the planner's tightness conditions:

$$c = (1 - \eta)q(\theta_g) \frac{(p_g + \phi)y_g(h_g) - z}{r + \lambda + \eta f(\theta_g)}, \quad (\text{A.1.14})$$

$$c = (1 - \eta)q(\theta_n) \frac{p_n y_n - z}{r + \lambda + \eta f(\theta_n)}. \quad (\text{A.1.15})$$

Sectoral allocation. Since $L_n = 1 - L_g$, an increase in L_g raises u_g and lowers u_n , holding employment states fixed. The first-order condition for L_g is

$$\begin{aligned} 0 = \frac{\partial \mathcal{H}}{\partial L_g} &= [z - c\theta_g + \mu_g f(\theta_g)] - [z - c\theta_n + \mu_n f(\theta_n)] \\ &\quad - r\kappa_g - \chi(h_g). \end{aligned} \quad (\text{A.1.16})$$

Canceling the z terms gives

$$r\kappa_g + \chi(h_g) = \mu_g f(\theta_g) - c\theta_g - [\mu_n f(\theta_n) - c\theta_n]. \quad (\text{A.1.17})$$

Using $c\theta_j = (1 - \eta)f(\theta_j)\mu_j$,

$$\begin{aligned} r\kappa_g + \chi(h_g) &= \mu_g f(\theta_g) - (1 - \eta)f(\theta_g)\mu_g \\ &\quad - [\mu_n f(\theta_n) - (1 - \eta)f(\theta_n)\mu_n] \end{aligned} \quad (\text{A.1.18})$$

$$= \eta f(\theta_g)\mu_g - \eta f(\theta_n)\mu_n. \quad (\text{A.1.19})$$

Substituting the expressions for μ_g and μ_n gives

$$r\kappa_g + \chi(h_g) = \eta f(\theta_g) \frac{(p_g + \phi)y_g(h_g) - z}{r + \lambda + \eta f(\theta_g)} - \eta f(\theta_n) \frac{p_n y_n - z}{r + \lambda + \eta f(\theta_n)}. \quad (\text{A.1.20})$$

Green human-capital choice. The human-capital condition is a pre-match investment condition rather than a contemporaneous derivative of aggregate flow welfare. Green human capital is chosen before matching by workers assigned to green search, and a marginal increase in h_g does not change the number of green searchers, the number of vacancies, or the matching congestion externality. It raises the productivity of the match only if the worker finds a green job.

The relevant object is therefore the ex-ante social value of green search. Rearranging the costate equation (A.1.8) before imposing the tightness condition gives

$$\mu_g^{raw}(h_g, \theta_g) = \frac{(p_g + \phi)y_g(h_g) - z + c\theta_g}{r + \lambda + f(\theta_g)}. \quad (\text{A.1.21})$$

For a worker assigned to green search, the planner's ex-ante value of training is

$$f(\theta_g)\mu_g^{raw}(h_g, \theta_g) - \chi(h_g). \quad (\text{A.1.22})$$

By the envelope theorem, the derivative of this optimized value with respect to h_g is taken holding

the optimized θ_g fixed, so the relevant marginal benefit uses (A.1.21) rather than the shadow value after substituting the tightness condition. With θ_g held fixed, the term $c\theta_g$ is fixed and the direct marginal effect operates only through $y_g(h_g)$.

The planner solves

$$h_g = \arg \max_{\tilde{h} \geq 0} \left\{ f(\theta_g) \mu_g^{raw}(\tilde{h}, \theta_g) - \chi(\tilde{h}) \right\}. \quad (\text{A.1.23})$$

The first-order condition is

$$\chi'(h_g) = f(\theta_g) \frac{\partial \mu_g^{raw}(h_g, \theta_g)}{\partial h_g}. \quad (\text{A.1.24})$$

Using (A.1.21),

$$\frac{\partial \mu_g^{raw}(h_g, \theta_g)}{\partial h_g} = \frac{(p_g + \phi) y'_g(h_g)}{r + \lambda + f(\theta_g)}. \quad (\text{A.1.25})$$

Therefore,

$$\chi'(h_g) = f(\theta_g) \frac{(p_g + \phi) y'_g(h_g)}{r + \lambda + f(\theta_g)}. \quad (\text{A.1.26})$$

This condition differs from the sectoral-allocation condition because the two margins are different. Assigning an additional worker to green search changes the measure of searchers and creates a matching externality, which is why $\eta f(\theta_g)$ appears in the sectoral-allocation condition. Increasing h_g for a worker already assigned to green search is an intensive investment margin and does not change matching congestion. Hence the planner's human-capital condition uses the individual job-finding rate $f(\theta_g)$.

This completes the derivation of Proposition 2.

A.2 Robustness of the Policy Mechanism

This appendix checks whether the common-target policy result is sensitive to the baseline calibration. For each robustness exercise, I recalibrate the decentralized economy to the same targeted moments, solve for the planner allocation associated with a 14 percent green employment share,

and compare the coordinated policy with a production-only policy chosen to reach the same green employment share.

Table A.1 reports the results. The key objects are the gaps in green human capital and green output between the coordinated policy and the production-only policy. Across all exercises, the production-only policy reaches the same green employment target with lower green human capital and lower green output. In the baseline, the coordinated policy delivers a green human-capital gap of 0.2041% and a green output gap of 0.1639%. These gaps vary with the curvature of the production function: a lower $\alpha = 0.6$ widens the human-capital gap to 0.2681% while compressing the output gap to 0.1614%, consistent with the approximation $G\text{-gap} \approx \alpha \times h\text{-gap}$ near $h_g = 1$. The magnitudes are modest across all exercises, but the direction of the mechanism is stable: production support expands green employment, and worker-side training support raises the skill intensity of green matches.

Table A.1. Robustness: Common-Target Policy Comparison

Scenario	ϕ^{14}	τ_g^{PO}	τ_h^{dual}	$h_g^{dual}/h_g^{PO} - 1$	$G^{dual}/G^{PO} - 1$
Baseline	0.0238	0.0238	0.0474	0.2041%	0.1639%
$\alpha = 0.6$	0.0238	0.0238	0.0474	0.2681%	0.1614%
$\alpha = 1.0$	0.0238	0.0238	0.0474	0.1647%	0.1654%
Training cost = 1% of wage	0.0529	0.0529	0.0464	0.1373%	0.1103%
Green wage premium = 3%	0.0775	0.0775	0.0452	0.2044%	0.1641%
Green wage premium = 4%	0.1298	0.1298	0.0433	0.2048%	0.1644%

Note: Each row recalibrates the decentralized economy and repeats the common-target policy comparison. Both policies are chosen to reach a 14 percent green employment share. The coordinated (dual) policy implements the planner allocation under Hosios; the production-only policy sets $\tau_h = s_g = 0$ and chooses τ_g^{PO} to reach the same employment target. $G = y_g(h_g)n_g$ denotes green output. The training subsidy τ_h^{dual} is evaluated at the planner allocation using the pre-match training condition in Proposition 4. The last two columns report percentage differences computed as the coordinated allocation relative to the production-only allocation; positive values indicate that the coordinated policy delivers higher h_g or higher green output G than the production-only policy at the same employment target.

A.2.1 Robustness Away from Hosios

The baseline imposes the Hosios condition, $\xi = \eta$, to isolate the worker-side human-capital hold-up from the standard search externality. This appendix relaxes that restriction by setting

$$\eta = 0.5, \quad \xi = 0.3.$$

Table A.2. Robustness Away from Hosios

	Hosios DE	Non-Hosios DE	Full SPP implementation	Production only
η	0.5	0.5	0.5	0.5
ξ	0.5	0.3	0.3	0.3
Production subsidy, τ_g	0	0	0.0238	0.0238
Training subsidy, τ_h	0	0	0.1040	0
Entry transfer, s_g	0	0	-0.00025	0
Vacancy-cost multiplier, green	1.000	1.000	2.195	1.000
Vacancy-cost multiplier, non-green	1.000	1.000	2.190	1.000
Green labor force, L_g	2.08%	2.02%	13.98%	13.84%
Green employment share, ℓ_g	2.08%	2.02%	13.99%	13.85%
Green human capital, h_g	1.0000	0.9996	1.0031	1.0007
Green training gap, $h_g - \underline{h}$	0.0403	0.0399	0.0434	0.0410
Green unemployment rate	3.64%	2.42%	3.56%	2.38%
Non-green unemployment rate	3.70%	2.47%	3.70%	2.47%
Aggregate unemployment	3.70%	2.46%	3.68%	2.45%
Tightness, green θ_g	1.932	4.465	2.020	4.656
Tightness, non-green θ_n	1.867	4.316	1.867	4.316
Green output per match, $y_g(h_g)$	1.0199	1.0196	1.0224	1.0204
Green output, $G = y_g(h_g)n_g$	0.0204	0.0201	0.1378	0.1379
Aggregate private output	0.9634	0.9757	0.9662	0.9782
Aggregate social output at ϕ^{14}	0.9639	0.9762	0.9695	0.9815
Resource welfare, W	0.9583	0.9549	0.9598	0.9563
Gross production subsidy cost, % output	0.00%	0.00%	0.339%	0.335%

Note: The Hosios decentralized equilibrium sets $\xi = \eta = 0.5$. The non-Hosios exercises set $\eta = 0.5$ and $\xi = 0.3$. The full SPP implementation column uses the policy instruments required to decentralize the planner allocation away from Hosios, including the vacancy-side correction. The production-only column applies the same production subsidy as the full implementation column but sets $\tau_h = s_g = 0$ and does not apply the vacancy-side correction. The vacancy-cost multipliers are private vacancy-side policy wedges that scale the cost of vacancy posting facing firms; they are not changes in social resource costs, and aggregate welfare in the resource-welfare row continues to use the social vacancy cost $c(v_g + v_n)$.

This case is empirically relevant because workers may receive less than the matching-elasticity share of match surplus. When $\xi < \eta$, the standard search wedge reappears: workers

receive too small a share of surplus relative to their contribution to matching, and firms therefore overpost vacancies.

Table A.2 reports four allocations. The first column is the baseline Hosios decentralized equilibrium. The second lowers the worker bargaining weight to $\xi = 0.3$ without policy. The third implements the planner allocation using the full policy package required away from Hosios: a production subsidy, a training subsidy, an entry offset, and vacancy-side corrections. The fourth applies the same production subsidy as the implementation column but removes worker-side training support, the entry offset, and the vacancy-side correction.

The non-Hosios case introduces an additional source of inefficiency, which is visible in the decentralized non-Hosios column: green tightness rises from 1.932 under Hosios to 4.465, and aggregate unemployment falls from 3.70% to 2.46%. The lower unemployment is not efficient, because it is achieved through excessive vacancy creation. Resource welfare falls accordingly, from 0.9583 to 0.9549.

The full implementation column shows that decentralizing the planner allocation away from Hosios requires more than production and training subsidies. The vacancy-cost multipliers raise the effective cost of vacancy posting and bring tightness back toward the planner allocation. This correction is absent in the production-only column, where tightness remains very high.

The comparison between the full implementation and production-only columns shows that the worker-side mechanism survives away from Hosios. With the same production subsidy but no worker-side support, the economy nearly reaches the planner's green employment scale, but green human capital is lower, $h_g = 1.0007$ rather than 1.0031, and the green training gap is smaller, 0.0410 rather than 0.0434. Production support expands green employment, and worker-side support raises the skill intensity of green matches.

Relaxing Hosios therefore reinforces rather than overturns the main result. Exact implementation now requires correcting three margins: a production subsidy for the green externality, a worker-side training subsidy for the pre-match training hold-up, and a vacancy-side correction for inefficient vacancy creation.

Chapter B

Appendix for Chapter Two

B.1 Decentralized equilibrium with policy instruments

This appendix derives the decentralized equilibrium (DE) with policy instruments: τ_i^K for investment subsidy, τ_i^L for education subsidy, and τ_i^T for lump-sum transfers. Starting from the Bellman equations:

$$rJ_i^U = \theta_i q_i(\theta_i)(J_i^E(k_i^*) - J_i^U) + \tau_i^L + z, \quad (\text{B.1.1})$$

$$rJ_i^E(k_i) = w_i(k_i)/P + s_i(J_i^U - J_i^E(k_i)) + \tau_i^L, \quad (\text{B.1.2})$$

$$rJ_i^V(k_i) = q_i(\theta_i)(J_i^F(k_i) - J_i^V(k_i)) - s_i J_i^V(k_i), \quad (\text{B.1.3})$$

$$rJ_i^F(k_i) = p_i f(k_i)/P - w_i(k_i)/P - s_i J_i^F(k_i). \quad (\text{B.1.4})$$

$$k_i^* = \operatorname{argmax}_{k_i} J_i^V(k_i) \quad (\text{B.1.5})$$

The equilibrium free-entry condition implies

$$\tau_i^T + J_i^V(k_i) = (1 - \tau_i^K)P_k k_i/P \quad (\text{B.1.6})$$

For each vacancy, it is subsidized with a one-off investment credit $\tau_i^K P_k k_i$. When it opens

the vacancy, it also receives a lump-sum transfer τ_i^{T1} . For each worker who chooses into sector i , it receives a subsidy flow τ_i . The total subsidy cost is financed by a lump sum tax T . The government's net fiscal transfer is:

$$T = \sum_i \tau_i^K P_k k_i s_i (n_i + v_i) / P + \sum_i \tau_i^T s_i (n_i + v_i) + \sum_i \tau_i^L L_i \quad (\text{B.1.7})$$

From (B.1.3) and (B.1.4), we have

$$J_i^F(k_i) = \frac{p_i f(k_i) - w_i(k_i)}{P(r + s_i)} \quad (\text{B.1.8})$$

$$\begin{aligned} J_i^V(k_i) &= \frac{q_i(\theta_i) J_i^F(k_i)}{r + s_i + q_i(\theta_i)} \\ &= \frac{q_i(\theta_i)}{r + s_i + q_i(\theta_i)} \frac{p_i f(k_i) - w_i(k_i)}{P(r + s_i)} \end{aligned} \quad (\text{B.1.9})$$

$$J_i^F(k_i) - J_i^V(k_i) = \frac{p_i f(k_i) - w_i(k_i)}{P(r + s_i + q_i(\theta_i))} \quad (\text{B.1.10})$$

From (B.1.9) and the standard first order condition $J_i^{V'}(k_i) = (1 - \tau_i^K) \frac{P_k}{P}$, we have

$$(1 - \tau_i^K) P_k = \frac{q_i(\theta_i)}{r + s_i + q_i(\theta_i)} \frac{p_i f'(k_i) - w'_i(k_i)}{r + s_i}. \quad (\text{B.1.11})$$

Next, we solve for $p_i f(k_i) - w_i(k_i)$, taking J_i^U as given. Subtracting (B.1.3) from (B.1.4), we have

$$\begin{aligned} [p_i f(k_i) - w_i(k_i)] / P &= (r + s_i + q_i(\theta_i)) (J_i^F(k_i) - J_i^V(k_i)) \\ &= (r + s_i + q_i(\theta_i)) (1 - \beta_i) S_i(k_i) \end{aligned} \quad (\text{B.1.12})$$

¹When $\tau_i^T < 0$, this is a lump-sum tax on firms.

From (B.1.2), we have

$$\begin{aligned} w_i(k_i)/P - rJ_i^U + \tau_i^L &= (r + s_i)(J_i^E(k_i) - J_i^U) \\ &= (r + s_i)\beta_i S_i(k_i) \end{aligned} \quad (\text{B.1.13})$$

Adding (B.1.12) and (B.1.13), we have

$$p_i f(k_i)/P - rJ_i^U + \tau_i^L = [r + s_i + (1 - \beta_i)q_i(\theta_i)] S_i(k_i), \quad (\text{B.1.14})$$

Hence the total surplus from a match with capital k_i is

$$S_i(k_i) = \frac{p_i f(k_i)/P - rJ_i^U + \tau_i^L}{r + s_i + (1 - \beta_i)q_i(\theta_i)}, \quad (\text{B.1.15})$$

Substitute (B.1.14) and (B.1.15) into (B.1.12),

$$[p_i f(k_i) - w_i(k_i)]/P = \frac{(1 - \beta_i)(r + s_i + q_i(\theta_i))}{r + s_i + (1 - \beta_i)q_i(\theta_i)} [p_i f(k_i)/P - rJ_i^U + \tau_i^L] \quad (\text{B.1.16})$$

Take the derivative with to k_i and plug into (B.1.11), we have the optimality condition for k_i with policy instruments:

$$(1 - \tau_i^K)P_k = \frac{q_i(\theta_i)(1 - \beta_i)}{r + s_i} \frac{p_i f'(k_i)}{r + s_i + (1 - \beta_i)q_i(\theta_i)} \quad (\text{B.1.17})$$

Next we derive the free-entry condition. From the Bellman equations, we easily have

$$r(J_i^E(k_i) - J_i^U) = w_i(k_i)/P - z - (s_i + \theta_i q_i(\theta_i))(J_i^E(k_i) - J_i^U), \quad (\text{B.1.18})$$

$$r(J_i^F(k_i) - J_i^V(k_i)) = [p_i f(k_i) - w_i(k_i)]/P - (s_i + q_i(\theta_i))(J_i^F(k_i) - J_i^V(k_i)). \quad (\text{B.1.19})$$

Substitute the Nash bargaining outcome $J_i^E(k_i) - J_i^U = \beta_i S_i(k_i)$ and $J_i^F(k_i) - J_i^V(k_i) =$

$(1 - \beta_i)S_i(k_i)$, we have

$$S_i(k_i) = [p_i f(k_i)/P - z]/D_i \quad (\text{B.1.20})$$

where $D_i = r + s_i + \beta_i \theta_i q_i(\theta_i) + (1 - \beta_i)q_i(\theta_i)$ as defined in (2.47).

From (B.1.3), we have at the steady state,

$$\begin{aligned} J_i^V(k_i) &= \frac{q_i(\theta_i)}{r + s_i} (J_i^F(k_i) - J_i^V(k_i)) \\ &= \frac{q_i(\theta_i)}{r + s_i} (1 - \beta_i) S_i(k_i) \\ &= q_i(\theta_i) (1 - \beta_i) \frac{p_i f(k_i)/P - z}{D_i (r + s_i)}. \end{aligned} \quad (\text{B.1.21})$$

This yields the tightness condition

$$(1 - \tau_i^K) \frac{P_k k_i}{P} = q_i(\theta_i) (1 - \beta_i) \frac{p_i f(k_i)/P - z}{D_i (r + s_i)} + \tau_i^T \quad (\text{B.1.22})$$

Finally, we can derive the equilibrium condition for sectoral choice. From (B.1.1) and the Nash bargaining outcome $J_i^E(k_i) - J_i^U = \beta_i S_i(k_i)$, we have

$$\begin{aligned} r J_i^U &= \theta_i q_i(\theta_i) \beta_i S_i(k_i) + \tau_i^L \\ &= \frac{\theta_i q_i(\theta_i) \beta_i [p_i f(k_i)/P - z]}{D_i} + \tau_i^L. \end{aligned}$$

So the optimal sectoral choice condition becomes

$$r(\epsilon_m - \epsilon_s) = \frac{\theta_m q_m(\theta_m) \beta_m [p_m f_m(k_m)/P - z]}{D_m} - \frac{\theta_s q_s(\theta_s) \beta_s [p_s f_s(k_s)/P - z]}{D_s} + (\tau_m^L - \tau_s^L). \quad (\text{B.1.23})$$

B.2 Proof of Proposition 7

Proof. The current-value Hamiltonian associated with this problem is²

$$H = p_m Y_m / P + p_s Y_s / P - \sum_i m_i(t) P_k k_i(t) / P - \sum_i (r + s_i) v_i(t) P_k k_i(t) / P \quad (\text{B.2.1})$$

$$\begin{aligned} & - r L_s(t) \epsilon_s - r (1 - L_a - L_s(t)) \epsilon_m \\ & + \sum_i \lambda_i^n(t) (m_i(t) - s_i n_i(t)) + \sum_i \lambda_i^Y(t) (m_i(t) f_i(k_i(t)) - s_i Y_i(t)) \\ & + z(u_m(t) + u_s(t)) \end{aligned} \quad (\text{B.2.2})$$

State variables: Y_i, n_i , controls: k_i, θ_i, L_s

Note that $u_i = L_i - n_i$, $v_i = \theta_i u_i = \theta_i (L_i - n_i)$, $m_i = \theta_i q_i u_i = \theta_i q_i (L_i - n_i)$.

$$v_{\theta_i} = u_i,$$

$$q'(\theta_i) = -\eta \frac{q_i}{\theta_i},$$

$$m_{\theta_i} = q_i u_i + \theta_i u_i q'(\theta_i) = u_i q_i (1 + \frac{\theta_i q'_i}{q_i}) = (1 - \eta) u_i q_i.$$

The first order conditions with respect to k_i is:

$$H_{k_i} = 0 : [m_i + (r + s_i) v_i] P_k / P = \lambda_i^Y m_i f'_i(k_i) \quad (\text{B.2.3})$$

$$(\text{B.2.4})$$

The co-state condition for Y_i :

$$\dot{\lambda}_i^Y = r \lambda_i^Y - H_{Y_i} \quad (\text{B.2.5})$$

$$H_{Y_i} = p_i / P - \lambda_i^Y s_i. \quad (\text{B.2.6})$$

²Before deriving the first order conditions, we applied two mathematically equivalent transformations to the objective function. First, following Acemoglu and Shimer (1999), we rewrite the capital cost $I(t)$ into $\sum_i m_i(t) P_k k_i(t) / P - \sum_i (r + s_i) v_i(t) P_k k_i(t) / P$ in the integral. Second, we substitute $p_i = \frac{\partial C}{\partial m}$. We can treat p_i “as if” it were exogenous without taking its derivative with respect to control variables due to envelope theorem.

Therefore, at steady state,

$$(r + s_i)\lambda_i^Y = \frac{p_i}{P} \quad (\text{B.2.7})$$

Plug (B.2.7) into (B.2.3), and make use of $m_i + (r + s_i)v_i = m_i(1 + \frac{r+s_i}{q_i})$, we have the optimal condition for the choice of k_i :

$$P_k = \frac{q_i p_i f'_i(k_i)}{(r + s_i + q_i)(r + s_i)} \quad (\text{B.2.8})$$

FOC wrt θ_i .

$$H_{\theta_i} = m_{\theta_i}[-P_k k_i/P + \lambda_i^n + \lambda_i^Y f_i(k_i)] - (r + s_i)v_{\theta_i} P_k k_i/P = 0 \quad (\text{B.2.9})$$

Denote $-P_k k_i/P + \lambda_i^n + \lambda_i^Y f_i(k_i) = \Phi_i$, the equation above can be simplified to

$$(1 - \eta)q_i \Phi_i = (r + s_i)P_k k_i/P \quad (\text{B.2.10})$$

Co-state condition for n_i : at the steady state,

$$r\lambda_i^n = H_{n_i} = m_{n_i}[-P_k k_i/P + \lambda_i^n + \lambda_i^Y f_i(k_i)] - (r + s_i)v_{n_i} P_k k_i/P - s_i \lambda_i^n - z \quad (\text{B.2.11})$$

$$= -\theta_i q_i \Phi_i + \theta_i (r + s_i) P_k k_i/P - s_i \lambda_i^n - z \quad (\text{B.2.12})$$

where the last equality made use of $m_{n_i} = -\theta_i q_i$ and $v_{n_i} = -\theta_i$.

Hence

$$(r + s_i)\lambda_i^n = -z - \theta_i q_i \Phi_i + (r + s_i)P_k k_i/P \quad (\text{B.2.13})$$

$$= -z - \theta_i q_i \Phi_i + \theta_i (1 - \eta)q_i \Phi_i \quad (\text{B.2.14})$$

$$= -z - \eta \theta_i q_i \Phi_i \quad (\text{B.2.15})$$

where the second equality made use of (B.2.10).

Use (B.2.15) to eliminate λ_i^n and plug into the definition of Φ_i and make use of (B.2.10), we have

$$\Phi_i = \frac{p_i f_i(k_i)/P - z}{D_i} \quad (\text{B.2.16})$$

Plug it into (B.2.10) we have the tightness condition:

$$\frac{P_k k_i}{P} = (1 - \eta) q_i \frac{p_i f_i(k_i)/P - z}{(r + s_i) D_i} \quad (\text{B.2.17})$$

FOC with respect to L_m

$$r \Delta \epsilon = \theta_m q_m \Phi_m - \theta_s q_s \Phi_s - \theta_m (r + s_m) P_k k_m / P + \theta_s (r + s_s) P_k k_s / P \quad (\text{B.2.18})$$

where the equations above made use of

$$\frac{\partial m_m}{\partial L_m} = \theta_m q_m \quad (\text{B.2.19})$$

$$\frac{\partial m_s}{\partial L_m} = -\theta_s q_s \quad (\text{B.2.20})$$

$$\frac{\partial v_m}{\partial L_m} = \theta_m \quad (\text{B.2.21})$$

$$\frac{\partial v_s}{\partial L_m} = -\theta_s \quad (\text{B.2.22})$$

$$(\text{B.2.23})$$

Substitute (B.2.10) into (B.2.18), we get the optimality condition for sectoral choice

$$\eta \theta_m q_m \Phi_m - \eta \theta_s q_s \Phi_s = r \Delta \epsilon \quad (\text{B.2.24})$$

Chapter C

Appendix for Chapter Three

C.1 Proofs of Results

Proof. Derivation of the job-finding rates for each type of workers.

Using the formula for a CRS Cobb-Douglas matching function and the usual notations for the variables, the job-finding rate for a worker of type 1 is given by:

$$f_1 = \frac{m(u_1, v)}{u_1} = \frac{u_1^\alpha v^{1-\alpha}}{u_1^\alpha u_1^{1-\alpha}} = b_1^{\alpha-1}.$$

Now, for the type- $\tilde{1}$ workers, one can use the Petrongolo and Pissarides (2001) logic that type-1 workers “move first”, and, only then, type- $\tilde{1}$ workers get a chance to match. The authors of that paper capture this idea by subtracting the matches of only type-1 workers (the “first movers”)

from the total matches of type-1 and type- $\tilde{1}$ workers, i.e.,

$$\begin{aligned}
\tilde{f}_1 &= \frac{m(u_1 + \tilde{u}_1, v) - m(u_1, v)}{\tilde{u}_1} \\
&= \frac{(u_1 + \tilde{u}_1)^\alpha v^{1-\alpha}}{\tilde{u}_1} - \frac{u_1^\alpha v^{1-\alpha}}{\tilde{u}_1} \\
&= \left(\frac{u_1 + \tilde{u}_1}{v} \right)^\alpha \cdot \frac{v}{\tilde{u}_1} - \left(\frac{u_1}{v} \right)^\alpha \cdot \frac{v}{\tilde{u}_1} \\
&= \frac{(b_1 + \tilde{b}_1)^\alpha - b_1^\alpha}{\tilde{b}_1}.
\end{aligned}$$

The derivations for the job-finding rates of type-0 and type- $\tilde{0}$ workers follow similar steps, hence, they are omitted. But the logic of matching with ranking based on Petrongolo and Pissarides (2001) is exactly the same, adjusted for a richer set of types.

Based on the Petrongolo and Pissarides (2001) method, and following similar steps, we can also derive the various worker-finding rates for firms. (These are the q expressions that appear in the job creation curves.) Specifically, we have:

$$\begin{aligned}
q_1 &= b_1^\alpha, \\
\tilde{q}_1 &= (b_1 + \tilde{b}_1)^\alpha - b_1^\alpha, \\
q_0 &= (b_1 + \tilde{b}_1 + b_0)^\alpha - (b_1 + \tilde{b}_1)^\alpha, \\
\tilde{q}_0 &= (b_1 + b_0 + \tilde{b}_1 + \tilde{b}_0)^\alpha - (b_1 + b_0 + \tilde{b}_1)^\alpha.
\end{aligned}$$

□

Proof. Proof of Lemma 1. Since the various J_i terms are all equal to each other, and since the bargaining problem in each type of meeting will imply $(1 - \eta)(W_i - U_i) = \eta J_i$, we must have that the various $W_i - U_i$ terms are also equal to each other. Let us subtract U_1 , given by equation (27), from W_1 , given by equation (13); this will give us

$$W_1 - U_1 = \frac{w_1 - z}{r + \lambda + \delta + f}. \quad (\text{A.1})$$

Similarly, let us subtract $\tilde{U}_1$, given by equation (28), from $\tilde{W}_1$, given by equation (14); this will give us

$$\tilde{W}_1 - \tilde{U}_1 = \frac{\tilde{w}_1 - z}{r + \lambda + \delta + f}. \quad (\text{A.2})$$

Direct comparison of equations (A.1) and (A.2), implies that $w_1 = \tilde{w}_1$.

Next, subtract $\tilde{U}_0$, given by equation (28), from $\tilde{W}_0$, given by equation (12), to obtain

$$(r + \delta + f)(\tilde{W}_0 - \tilde{U}_0) = \tilde{w}_0 - z + \lambda(\tilde{U}_1 - \tilde{W}_1). \quad (\text{A.3})$$

To obtain a useful expression for the term $\tilde{U}_1 - \tilde{W}_1$ in the last equation, subtract $\tilde{W}_0$, given by equation (12), from $\tilde{U}_1$, given by equation (28). This will give us

$$(r + \delta + \lambda)(\tilde{U}_1 - \tilde{W}_0) = z - \tilde{w}_0 + f(\tilde{W}_1 - \tilde{U}_1),$$

which we can now use to replace the term $\tilde{U}_1 - \tilde{W}_0$ in equation (A.3). After this substitution, one should find that

$$(r + \delta + f)(\tilde{W}_0 - \tilde{U}_0) = (\tilde{w}_0 - z) \left(1 - \frac{\lambda}{r + \delta + \lambda} \right) + \frac{\lambda f}{r + \delta + \lambda}(\tilde{W}_1 - \tilde{U}_1).$$

But since we have already established that $(\tilde{W}_0 - \tilde{U}_0) = (\tilde{W}_1 - \tilde{U}_1)$, the last equation can be re-written as

$$\left(r + \delta + f - \frac{\lambda f}{r + \delta + \lambda} \right) (\tilde{W}_0 - \tilde{U}_0) = \tilde{w}_0 - z + \frac{r + \delta}{r + \delta + \lambda},$$

which, after some algebra, simplifies to

$$\tilde{W}_0 - \tilde{U}_0 = \frac{\tilde{w}_0 - z}{r + \delta + \lambda + f}. \quad (\text{A.4})$$

Direct comparison of equations (A.1), (A.2), and (A.4), implies that $w_1 = \tilde{w}_1 = \tilde{w}_0$.

The last thing is to show that the remaining wage w_0 is also equal to the wages of the other three types. To that end, start by subtracting subtract U_0 , given by equation (25), from W_0 , given

by equation (11), to obtain

$$(r + \delta + f)(W_0 - U_0) = w_0 - z + \lambda(U_1 - W_0) + \gamma(U_0 - \tilde{U}_0). \quad (\text{A.5})$$

Again, we can find more useful expressions for the terms $U_1 - W_0$ and $U_0 - \tilde{U}_0$ in the last equation.

Subtracting W_0 , given by equation (11), from U_1 , given by equation (27), we get

$$(r + \delta + \lambda)(U_1 - W_0) = z - w_0 + f(W_1 - U_1).$$

As for the term $U_0 - \tilde{U}_0$, it is easy to show that

$$(r + \delta + \gamma)(U_0 - \tilde{U}_0) = 0 \implies U_0 = \tilde{U}_0.$$

Using these two facts back into equation (A.5) allows to write

$$(r + \delta + f)(W_0 - U_0) = w_0 - z + \frac{\lambda}{r + \delta + \lambda} [z - w_0 + f(W_1 - U_1)].$$

But since we have established that $(W_0 - U_0) = (W_1 - U_1)$, the last equation can be re-written as

$$\left(r + \delta + f - \frac{\lambda f}{r + \delta + \lambda} \right) (W_0 - U_0) = (w_0 - z) \cdot \frac{r + \delta}{r + \delta + \lambda},$$

which, after some algebra, simplifies to

$$W_0 - U_0 = \frac{w_0 - z}{r + \delta + \lambda + f}. \quad (\text{A.6})$$

Comparison of equations (A.1), (A.2), (A.4), and (A.6) implies that $w_1 = \tilde{w}_1 = \tilde{w}_0 = w_0$, which concludes the proof. $\square$

C.2 The details of Intervention 4

This appendix contains the details of the analysis under “Intervention 4”, i.e., a government subsidy *only* to type-0 workers, which is generous enough to make them the preferred types for firms.

Arrival rates and Beveridge curves

As discussed in Chapter 3.4, we will still have a Petrongolo and Pissarides (2001) matching process, but the ranking will now favor type-0 workers. (The relative ranking of the remaining types will stay unchanged.) Recalling the definition of the various queue lengths from equation (3.1), one can easily show that the new job-finding rates for the four worker types are as follows:

$$\begin{aligned} f_0 &= \frac{m(u_0, v)}{u_0} = b_0^{\alpha-1}, \\ f_1 &= \frac{m(u_0 + u_1, v) - m(u_0, v)}{u_1} = \frac{(b_0 + b_1)^\alpha - b_0^\alpha}{b_1}, \\ \tilde{f}_1 &= \frac{m(u_0 + u_1 + \tilde{u}_1, v) - m(u_0 + u_1, v)}{\tilde{u}_1} = \frac{(b_0 + b_1 + \tilde{b}_1)^\alpha - (b_0 + b_1)^\alpha}{\tilde{b}_1}, \\ \tilde{f}_0 &= \frac{m(u_0 + u_1 + \tilde{u}_1 + \tilde{u}_0, v) - m(u_0 + u_1 + \tilde{u}_1, v)}{\tilde{u}_0} = \frac{(b_0 + b_1 + \tilde{b}_1 + \tilde{b}_0)^\alpha - (b_0 + b_1 + \tilde{b}_1)^\alpha}{\tilde{b}_0}. \end{aligned}$$

The formulas admit an interpretation similar to the one provided in Chapter 3.2. Although the job-finding rates have changed to reflect the new ranking of workers, the formulas describing the Beveridge curves in Section 3.3.1 are still valid.

Next, following similar steps, we can derive the various worker-finding rates for firms. These are given by:

$$\begin{aligned} q_0 &= b_0^\alpha, \\ q_1 &= (b_0 + b_1)^\alpha - b_0^\alpha, \\ \tilde{q}_1 &= (b_0 + b_1 + \tilde{b}_1)^\alpha - (b_0 + b_1)^\alpha, \\ \tilde{q}_0 &= (b_0 + b_1 + \tilde{b}_1 + \tilde{b}_0)^\alpha - (b_0 + b_1 + \tilde{b}_1)^\alpha. \end{aligned}$$

Value functions and free entry

Since in equilibrium we must have $V = 0$, we can state the free entry condition as

$$c = q_0 J_0 + \tilde{q}_0 \tilde{J}_0 + q_1 J_1 + \tilde{q}_1 \tilde{J}_1, \quad (\text{A.1})$$

where the value functions for productive firms in the various states are as follows:

$$\begin{aligned} rJ_0 &= p + \kappa\rho - \tau - w_0 - \lambda J_0 - \delta J_0, \\ r\tilde{J}_0 &= p - \kappa - \tilde{\kappa} - \tau - \tilde{w}_0 - \lambda\tilde{J}_0 - \delta\tilde{J}_0, \\ rJ_1 &= p - \tau - w_1 - \lambda J_1 - \delta J_1, \\ r\tilde{J}_1 &= p - \tilde{\kappa} - \tau - \tilde{w}_1 - \lambda\tilde{J}_1 - \delta\tilde{J}_1. \end{aligned}$$

Clearly, the difference of these value functions with the ones provided in Section 3.3 is that firms who hire type-0 workers are better off, due to the government subsidy (equal to $\sigma_0 = \kappa(1 + \rho)$), and that all active firms pay the flat tax τ .¹

The value functions of unemployed workers in the various states are still given by equations (3.7)–(3.10), even though the respective job-finding rates have now changed. Similarly, the value functions of employed workers in the various states are still described by equations (3.11)–(3.14).

Bargaining and wage curves

Deriving the various wage curves follows identical steps to the analysis in Section 3.3.3 of Chapter 3.3. We will therefore skip the details and proceed to the final formulas. The careful reader will observe that the following wage curves look identical to the ones provided in Section 3.3.3, after

¹ Which, in equilibrium, will be equal to $\tau = \kappa(1 + \rho)e_0/e$, in order to keep the government budget constraint balanced.

adjusting the formulas to reflect the new productivities.² More precisely, we have:

$$\begin{aligned}
w_1 &= \frac{\eta(p - \tau)(r + \delta + \lambda + f_1) + (1 - \eta)z(r + \delta + \lambda)}{r + \delta + \lambda + \eta f_1}, \\
\tilde{w}_1 &= \frac{\eta(p - \tilde{\kappa} - \tau)(r + \delta + \lambda + \tilde{f}_1) + (1 - \eta)z(r + \delta + \lambda)}{r + \delta + \lambda + \eta \tilde{f}_1}, \\
\tilde{w}_0 &= \frac{1}{r + \delta + \eta \tilde{f}_0} \left[\eta(p - \kappa - \tilde{\kappa} - \tau)(r + \delta + \tilde{f}_0) \right. \\
&\quad \left. + \frac{(r + \delta + \lambda)(r + \delta + \tilde{f}_1)}{r + \delta + \lambda + \tilde{f}_1} (1 - \eta)z - \frac{\lambda(1 - \eta)\tilde{f}_1}{r + \delta + \lambda + \tilde{f}_1} \tilde{w}_1 \right], \\
w_0 &= \frac{r + \delta + \gamma + f_0}{r + \delta + \gamma + \eta f_0} \eta(p + \kappa\rho - \tau) + \frac{(r + \delta + \lambda)(r + \delta + f_1)(r + \delta + \gamma)}{(r + \delta)(r + \delta + \gamma + \eta f_0)(r + \delta + \lambda + f_1)} (1 - \eta)z \\
&\quad + \frac{\gamma \tilde{f}_0 \eta(p - \kappa - \tilde{\kappa} - \tau - \tilde{w}_0)}{(r + \delta)(r + \delta + \gamma + \eta f_0)} - \frac{(1 - \eta)\lambda f_1(r + \delta + \gamma)w_1}{(r + \delta)(r + \delta + \gamma + \eta f_0)(r + \delta + \lambda + f_1)}.
\end{aligned}$$

Definition 5. A steady state equilibrium under Intervention 4 is a list of wages for the four types of workers $(w_0, \tilde{w}_0, w_1, \tilde{w}_1)$, a measure of vacant firms v , and measures of unemployed and employed workers in the various states $(u_0, \tilde{u}_0, u_1, \tilde{u}_1, e_0, \tilde{e}_0, e_1, \tilde{e}_1)$, satisfying the free entry condition (A.1), the four wage curves described in this section, and the eight Beveridge curves reported at the end of Section 3.3.1. Implicit in this definition is the flat tax paid by all active firms, given by $\tau = \kappa(1 + \rho)e_0/e$.

C.3 The social planner's problem under matching with ranking and type-0 bias

In Section 3.6.4, we showed that Intervention 4 (type-0 bias) yields sizable welfare gains. A natural question to ask is how close that intervention brings the economy to its efficient level. However, given the non-standard features of our model, characterizing this “efficient level” is not obvious.

² Namely, the fact that all firms must now pay a tax, and that firms that match with type-0 workers receive a subsidy which makes the pre-tax effective productivity of these workers equal to $p + \kappa\rho$.

The most straightforward way to describe the efficient allocation is to repeat the textbook exercise (i.e., Chapter 8 of Pissarides 2000), where the social planner chooses the aggregate measure of vacancies, while being constrained by the matching technology, which in our case is the Petrongolo and Pissarides (2001) matching with ranking. Importantly, however, the ranking the planner is subject to ought to be *aligned* with the ranking described under Intervention 4, since this is the spirit of our exercise.³ The social planner's problem and its solution are formally presented in the following proposition.

Proposition 12. *The social planner solves the following problem:*

$$\max_v \int_0^\infty e^{-rt} [e_0(p - k) + e_1p + \tilde{e}_1(p - \tilde{k}) + \tilde{e}_0(p - k - \tilde{k}) + (u_0 + u_1 + \tilde{u}_1 + \tilde{u}_0)z - cv] dt,$$

subject to

$$\begin{aligned} \dot{u}_0 &= \delta - u_0(\delta + \gamma + f_0), & \dot{e}_0 &= f_0u_0 - (\delta + \lambda)e_0, \\ \dot{u}_1 &= \lambda(e_0 + e_1) - (\delta + f_1)u_1, & \dot{e}_1 &= f_1u_1 - (\delta + \lambda)e_1, \\ \dot{\tilde{u}}_1 &= \lambda(\tilde{e}_0 + \tilde{e}_1) - (\delta + \tilde{f}_1)\tilde{u}_1, & \dot{\tilde{e}}_0 &= \tilde{f}_0\tilde{u}_0 - (\delta + \lambda)\tilde{e}_0, \\ \dot{\tilde{u}}_0 &= \gamma u_0 - (\delta + \tilde{f}_0)\tilde{u}_0, & \dot{\tilde{e}}_1 &= \tilde{f}_1\tilde{u}_1 - (\delta + \lambda)\tilde{e}_1, \end{aligned}$$

where

$$\begin{aligned} f_0 &= b_0^{a-1}, \\ f_1 &= \frac{(b_0 + b_1)^a - b_0^a}{b_1}, \\ \tilde{f}_1 &= \frac{(b_0 + b_1 + \tilde{b}_1)^a - (b_0 + b_1)^a}{\tilde{b}_1}, \\ \tilde{f}_0 &= \frac{(b_0 + b_1 + \tilde{b}_1 + \tilde{b}_1)^a - (b_0 + b_1 + \tilde{b}_1)^a}{\tilde{b}_0}. \end{aligned}$$

³ Studying the problem of a planner who is subject to a different ranking of worker types (say, the one described in the baseline model) would imply that the planner does not have the tools to address the main inefficiency in the model, i.e., the fact that firms do not internalize the social benefit of hiring inexperienced workers. Since the goal of this section is to see how close Intervention 4 brings the economy to the constrained efficient allocation, making sure that the two specifications share the same ranking is the only meaningful way to carry out this comparison.

The steady state solution to the planner's problem is summarized by unemployment levels for each type, $(u_0, u_1, \tilde{u}_0, \tilde{u}_1)$, employment levels for each type, $(e_0, e_1, \tilde{e}_0, \tilde{e}_1)$, and a measure of vacancies, v . The planner's solution is uniquely determined by the eight Beveridge curves reported at the end of Section 3.3.1, where the arrival rates are stated above, and an equation that governs the optimal level of vacancies which is given below:

$$f_0 u_0 (\mu_{e_0} - \mu_{u_0}) + \tilde{f}_0 \tilde{u}_0 (\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0}) + f_1 u_1 (\mu_{e_1} - \mu_{u_1}) + \tilde{f}_1 \tilde{u}_1 (\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1}) = \frac{vc}{1-a} e^{-rt}. \quad (\text{A.1})$$

The various μ terms are the co-state variables in the social planner's problem, and the differentials $(\mu_{e_0} - \mu_{u_0})$, $(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0})$, $(\mu_{e_1} - \mu_{u_1})$, and $(\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1})$ are described in detail in the proof.

Proof of Proposition 12. The Hamiltonian function for the planner's problem is the following:

$$\begin{aligned} \mathcal{H} = & e^{-rt} (e_1 p + e_0 (p - \kappa) + \tilde{e}_1 (p - \tilde{\kappa}) + \tilde{e}_0 (p - \tilde{\kappa} - \kappa) + (u_1 + u_0 + \tilde{u}_1 + \tilde{u}_0) z - cv) \\ & + \mu_{u_0} (\delta - u_0 (\delta + \gamma + f_0)) + \mu_{\tilde{u}_0} (\gamma u_0 - \tilde{u}_0 (\delta + \tilde{f}_0)) + \mu_{u_1} (\lambda (e_0 + e_1) - u_1 (\delta + f_1)) \\ & + \mu_{\tilde{u}_1} (\lambda (\tilde{e}_0 + \tilde{e}_1) - \tilde{u}_1 (\tilde{f}_1 + \delta)) + \mu_{e_0} (u_0 f_0 - e_0 (\delta + \lambda)) + \mu_{\tilde{e}_0} (\tilde{f}_0 \tilde{u}_0 - (\delta + \lambda) \tilde{e}_0) \\ & + \mu_{e_1} (f_1 u_1 - (\lambda + \delta) e_1) + \mu_{\tilde{e}_1} (\tilde{f}_1 \tilde{u}_1 - \tilde{e}_1 (\lambda + \delta)). \end{aligned}$$

It will be useful to define some objects to make the notation simpler.

$$g_1 \equiv b_0^{a-1},$$

$$g_2 \equiv (b_0 + b_1)^{a-1},$$

$$g_3 \equiv (b_0 + b_1 + b_2)^{a-1},$$

$$g_4 \equiv (b_0 + b_1 + b_2 + b_3)^{a-1},$$

where $b_0 = \frac{u_0}{v}$, $b_1 = \frac{u_1}{v}$, $b_2 = \frac{\tilde{u}_1}{v}$, and, $b_3 = \frac{\tilde{u}_0}{v}$.

We get the first order-conditions from the above Hamiltonian:

$$\begin{aligned} \{v\} : & f_0 u_0 (\mu_{e_0} - \mu_{u_0}) + \tilde{f}_0 \tilde{u}_0 (\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0}) + f_1 u_1 (\mu_{e_1} - \mu_{u_1}) + \tilde{f}_1 \tilde{u}_1 (\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1}) \\ & = \frac{vC}{1-a} e^{-rt}, \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \{u_0\} : & z e^{-rt} + a(\mu_{e_0} - \mu_{u_0}) f_0 + a(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0})(g_4 - g_3) + a(\mu_{e_1} - \mu_{u_1})(g_2 - g_1) \\ & + a(\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1})(g_3 - g_2) + \gamma \mu_{\tilde{u}_0} - (\delta + \gamma) \mu_0 = -\dot{\mu}_{u_0}, \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \{u_1\} : & z e^{-rt} + a(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0})(g_4 - g_3) + a(\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1})(g_3 - g_2) + a(\mu_{e_1} - \mu_{u_1}) g_2 \\ & - \delta \mu_{u_1} = -\dot{\mu}_{u_1}, \end{aligned} \quad (\text{A.4})$$

$$\{\tilde{u}_1\} : z e^{-rt} + a(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0})(g_4 - g_3) + a(\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1}) g_3 - \delta \mu_{\tilde{u}_1} = -\dot{\mu}_{\tilde{u}_1}, \quad (\text{A.5})$$

$$\{\tilde{u}_0\} : z e^{-rt} + a(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0}) g_4 - \delta \mu_{\tilde{u}_0} = -\dot{\mu}_{\tilde{u}_0}, \quad (\text{A.6})$$

$$\{e_0\} : (p - \kappa) e^{-rt} + \mu_{u_1} - \mu_{e_0} (\lambda + \delta) = -\dot{\mu}_{e_0}, \quad (\text{A.7})$$

$$\{e_1\} : p e^{-rt} + \lambda \mu_{u_1} - \mu_{e_1} (\lambda + \delta) = -\dot{\mu}_{e_1}, \quad (\text{A.8})$$

$$\{\tilde{e}_1\} : (p - \tilde{\kappa}) e^{-rt} + \mu_{\tilde{u}_1} \lambda - \mu_{\tilde{e}_1} (\lambda + \delta) = -\dot{\mu}_{\tilde{e}_1}, \quad (\text{A.9})$$

$$\{\tilde{e}_0\} : (p - \kappa - \tilde{\kappa}) e^{-rt} + \mu_{\tilde{u}_1} \lambda - \mu_{\tilde{e}_0} (\lambda + \delta) = -\dot{\mu}_{\tilde{e}_0}. \quad (\text{A.10})$$

Subtracting (A.2) from (A.9) gives

$$e^{-rt} (p - \tilde{\kappa} - z) - a(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0})(g_4 - g_3) = (r + \delta + \lambda + a g_3)(\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1}). \quad (\text{A.11})$$

Similarly, subtracting (A.6) from (A.10) yields

$$e^{-rt} (p - \kappa - \tilde{\kappa} - z) + \lambda(\mu_{\tilde{u}_1} - \mu_{\tilde{e}_0}) = (r + \delta + a g_4)(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0}). \quad (\text{A.12})$$

To get an expression for $(\mu_{\tilde{u}_1} - \mu_{\tilde{e}_0})$, subtract (A.5) from (A.10) to obtain

$$\begin{aligned} (r + \delta + \lambda)(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_1}) = & e^{-rt} (p - \kappa - \tilde{\kappa} - z) - a(\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1}) g_3 \\ & - a(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0})(g_4 - g_3). \end{aligned} \quad (\text{A.13})$$

Next, using (A.13) in (A.12), we get

$$(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0}) = \left(r + \delta + ag_4 \frac{r + \delta}{r + \delta + \lambda} + \frac{a\lambda}{r + \delta + \lambda} g_3 \right)^{-1} \left(e^{-rt} (p - \kappa - \tilde{\kappa} - z) \frac{r + \delta}{r + \delta + \lambda} + \frac{\lambda}{r + \delta + \lambda} ag_3 (\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1}) \right). \quad (\text{A.14})$$

Note that equations (A.14) and (A.11) consist of only two unknowns, namely, $(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0})$ and $(\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1})$. Solving for these expressions, we get

$$(\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1}) = \frac{e^{-rt}}{r + \delta + \lambda + ag_3} \left((p - \tilde{\kappa} - z) - a(g_4 - g_3) \frac{(r + \delta)(p - \kappa - \tilde{\kappa} - z) + \frac{a\lambda g_3}{r + \delta + \lambda + ag_3} (p - \tilde{\kappa} - z)}{(r + \delta + \lambda + ag_4)(r + \delta + \frac{a\lambda g_3}{r + \delta + \lambda + ag_3})} \right) \quad (\text{A.15})$$

and

$$(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0}) = e^{-rt} \frac{(r + \delta)(p - \kappa - \tilde{\kappa} - z) + \frac{a\lambda g_3}{r + \delta + \lambda + ag_3} (p - \tilde{\kappa} - z)}{(r + \delta + \lambda + ag_4)(r + \delta + \frac{a\lambda g_3}{r + \delta + \lambda + ag_3})}. \quad (\text{A.16})$$

Next, we subtract (A.4) from (A.8) and, rearranging, we get a formula for $(\mu_{e_1} - \mu_{u_1})$:

$$(\mu_{e_1} - \mu_{u_1}) = \frac{1}{r + \delta + \lambda + ag_2} \left(e^{-rt} (p - z) - a(g_3 - g_2)(\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1}) - a(g_4 - g_3)(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0}) \right). \quad (\text{A.17})$$

Note that above expression is a closed-form solution given equations (A.16) and (A.15), thus there is no reason to plug them in here. Now, we only need to solve for $(\mu_{e_0} - \mu_{u_0})$. Towards that, by subtracting (A.3) from (A.7) we have

$$\begin{aligned} \lambda(\mu_{e_0} - \mu_{u_1}) = & e^{-rt} (p - \kappa - z) + \gamma(\mu_{u_0} - \mu_{\tilde{u}_0}) - (r + \delta + af_0)(\mu_{e_0} - \mu_{u_0}) \\ & - a(g_2 - g_1)(\mu_{e_1} - \mu_{u_1}) - a(g_3 - g_2)(\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1}) \\ & - a(g_4 - g_3)(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0}). \end{aligned} \quad (\text{A.18})$$

Now, derive $(\mu_{e_0} - \mu_{u_1})$ by subtracting (A.4) from (A.7) and derive $(\mu_{u_0} - \mu_{\tilde{u}_0})$ by subtracting (A.6) from (A.3). Plugging both solutions in (A.18) results in the following expression for $(\mu_{e_0} -$

μ_{u_0}):

$$\begin{aligned}
(\mu_{e_0} - \mu_{u_0}) = & \frac{r + \delta + \gamma}{(af_0 + r + \delta + \gamma)(r + \delta)} \left[\frac{r + \delta}{r + \delta + \lambda} (p - \kappa - z) e^{-rt} \right. \\
& + (\mu_{e_1} - \mu_{u_1}) \left(\frac{\lambda}{r + \delta + \lambda} ag_2 - a(g_2 - g_1) \frac{r + \delta}{r + \delta + \lambda} \right) \\
& + a(g_3 - g_2)(\mu_{\tilde{e}_1} - \mu_{\tilde{u}_1}) \left(\frac{\lambda}{r + \delta + \lambda} - \frac{r + \delta}{r + \delta + \gamma} \right) \\
& + a(g_4 - g_3)(\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0}) \left(\frac{\lambda}{r + \delta + \lambda} - \frac{r + \delta}{r + \delta + \gamma} \right) \\
& \left. - \frac{a\gamma}{r + \delta + \gamma} g_4 (\mu_{\tilde{e}_0} - \mu_{\tilde{u}_0}) \right]. \tag{A.19}
\end{aligned}$$

Using (A.19), (A.17), (A.16) and (A.15) altogether in (A.2), we get an equation that governs the optimal level of vacancies only including parameters of the model and other equilibrium variables. We do not state it here. Moreover, since we are interested in the steady state solution of planner's problem, the eight Beveridge curves at the end of Section 3.3.1 must hold as well. Thereby, we end up with nine equations in nine variables which uniquely solves for the planner's problem. $\square$

Inspection of Proposition 12 reveals that the social planner chooses the number of vacancies to maximize social welfare, defined as production appropriately weighed by the measure of each employed type, plus enjoyment of leisure by all unemployed types, net of vacancy costs. The first eight constraints indicate that the planner must respect the laws of motion of the measures of the various worker types. The four latter constraints ensure that the job-finding rates of the four unemployed types are governed by the Petrongolo and Pissarides (2001) matching process, where, however, the ranking has been tweaked to give priority to type-0 workers (thus matching the ranking of Intervention 4). A natural interpretation of equation (A.1) follows the equalization of marginal social cost with marginal social benefit of vacancy creation. The right hand-side of equation (A.1) captures the effective discounted cost of new vacancy creation. The left hand-side of this equation illustrates the social gain an additional vacancy generates by assisting each worker type to transition from unemployment to employment.

As explained in Section 3.6.4 (see the discussion of Table 3.5), the social planner's solution is

similar in spirit to Intervention 4: it raises the job-finding rate of type-0 workers by an order of magnitude to save their skills from depreciation. To make this happen, the planner gives priority to entrants compared to the other unemployed, something Intervention 4 achieves by subsidizing firms to rank type-0 workers above the other unemployed. As a result, the job-finding rates of the other worker types are substantially lower in the constrained efficient allocation than in the baseline economy. Table 3.5 demonstrates that Intervention 4 brings the economy particularly close to the constrained efficient outcome. Intuitively, the social planner implements a Hosios-type condition for our environment; however, it becomes apparent that implementing the “correct” ranking is an order of magnitude more important than fine-tuning the level of vacancy creation.

C.4 Extensions of the baseline model

In the main text of Chapter 3.2, we provide justification for some of the key assumptions of the model. In the baseline model, we strive for tractability and choose to employ a relatively simple model that captures the salient features of the question we are investigating. In this appendix section, we relax some of the stark assumptions one at a time, and we show that all the main results go through under the more general or complicated environments. Specifically, we consider three extensions. In the first, workers who have been hit by the skill-loss shock in youth can stochastically regain their skills later in their career. In the second, workers can become experienced while they still hold their first job (instead of having to wait until they lose their first job). In the third extension, we consider the possibility of experienced and inexperienced workers having different bargaining powers.

C.4.1 Extension 1: Regain skill later in career

The first assumption of the baseline model that we relax here is that the skill loss that takes place during a worker’s youth lasts until that worker retires. Of course, there is strong empirical evidence (reported in the literature review in Section 1) documenting that skill loss during the early

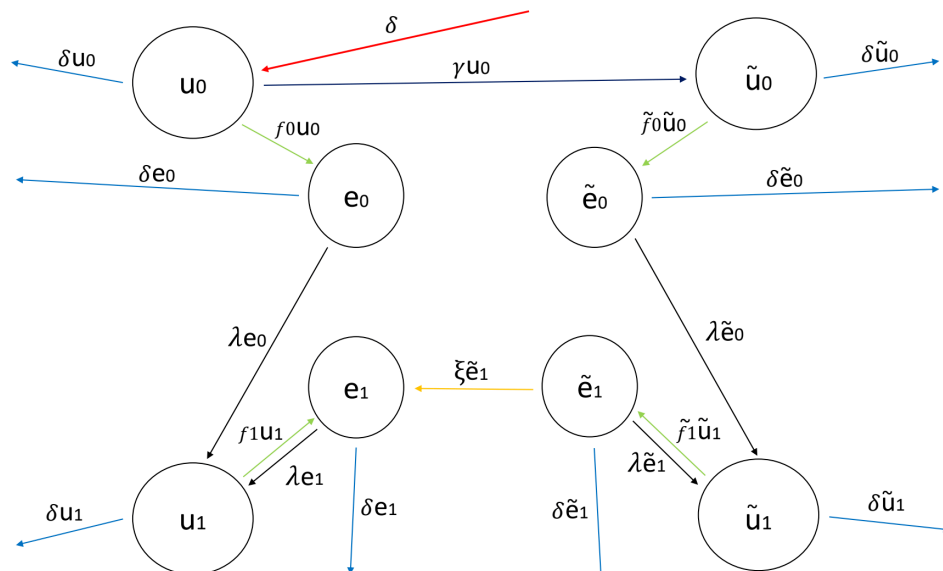


Figure C.1. Worker flows in and out of the various states when regaining skills is possible.

unemployment spells has persistent negative effects on a worker's career. But here we will adopt a milder version of this assumption, namely, that a worker who suffered skill loss during youth can stochastically regain her skill later in her career. Naturally, we assume that this "regaining skill shock" can happen only while a worker is employed. In terms of the notation developed in Chapter 3.2, what is new here is that type- $\tilde{1}$ workers may get hit by a Poisson shock ξ and become type-1 workers. Figure C.1 illustrates the worker flows in and out of every state, and it is useful for deriving the Beveridge curves. The figure is identical to the baseline worker-flow diagram in Figure 3.2, with the exception of the new (yellow) arrow pointing from the $\tilde{e}_1$ bubble towards the e_1 bubble, indicating the newly introduced possibility of workers regaining their skills later in life.

Beveridge curves

With the visual help of Figure C.1, we can equate the flows in and out of each state, and after some algebra, we can show that the steady state measure of workers in the various states are as follows:

$$\begin{aligned}
 u_0 &= \frac{\delta}{\delta + \gamma + f_0}, \\
 \tilde{u}_0 &= \frac{\gamma}{\delta + \tilde{f}_0} \cdot \frac{\delta}{\delta + \gamma + f_0}, \\
 e_0 &= \frac{f_0}{\delta + \lambda} \cdot \frac{\delta}{\delta + \gamma + f_0}, \\
 \tilde{e}_0 &= \frac{\tilde{f}_0}{\delta + \lambda} \cdot \frac{\gamma}{\delta + \tilde{f}_0} \cdot \frac{\delta}{\delta + \gamma + f_0}, \\
 \tilde{u}_1 &= \frac{\lambda(\delta + \lambda + \xi)\tilde{f}_0\gamma\delta}{[\delta(\delta + \lambda + \xi) + \tilde{f}_1(\delta + \xi)](\delta + \lambda)(\delta + \tilde{f}_0)(\delta + \gamma + f_0)}, \\
 \tilde{e}_1 &= \frac{\tilde{f}_1\lambda\tilde{f}_0\gamma\delta}{[\delta(\delta + \lambda + \xi) + \tilde{f}_1(\delta + \xi)](\delta + \lambda)(\delta + \tilde{f}_0)(\delta + \gamma + f_0)}, \\
 u_1 &= \frac{\lambda}{(\gamma + \delta + f_0)(\delta + \lambda + f_1)} \left[f_0 + \xi \frac{\lambda\gamma\tilde{f}_0\tilde{f}_1}{[\delta(\delta + \lambda + \xi) + \tilde{f}_1(\delta + \xi)](\delta + \lambda)(\delta + \tilde{f}_0)} \right], \\
 e_1 &= \frac{1}{(\delta + \lambda)(\delta + \lambda + f_1)} \left[\frac{f_0f_1\lambda}{\delta + \gamma + f_0} + \xi \frac{(\delta + \lambda)(\delta + f_1)}{\delta} \tilde{e}_1 \right].
 \end{aligned}$$

Value Functions

We now move to the value functions starting with firms. While we repeat all value functions for the reader's convenience, it is important to highlight that, under this extension, the only firm value function that differs from the baseline model in Chapter 3.3 is that of a firm employing a type- $\tilde{1}$ worker. Now, the value function of a vacant firm is given by:

$$rV = -c + q_0(J_0 - V) + \tilde{q}_0(\tilde{J}_0 - V) + q_1(J_1 - V) + \tilde{q}_1(\tilde{J}_1 - V).$$

Free entry implies that in equilibrium we must have $V = 0$, therefore, we can state the free entry condition as

$$c = q_0 J_0 + \tilde{q}_0 \tilde{J}_0 + q_1 J_1 + \tilde{q}_1 \tilde{J}_1. \quad (\text{A.1})$$

We also have four value functions for productive firms in the various states, i.e., for firms that matched with the four different types of workers (type 0, $\tilde{0}$, 1, and $\tilde{1}$). These are given as follows:

$$\begin{aligned} rJ_0 &= p - \kappa - w_0 - \lambda J_0 - \delta J_0, \\ r\tilde{J}_0 &= p - \kappa - \tilde{\kappa} - \tilde{w}_0 - \lambda \tilde{J}_0 - \delta \tilde{J}_0, \\ rJ_1 &= p - w_1 - \lambda J_1 - \delta J_1, \\ r\tilde{J}_1 &= p - \tilde{\kappa} - \tilde{w}_1 - \lambda \tilde{J}_1 - \delta \tilde{J}_1 + \xi(J_1 - \tilde{J}_1). \end{aligned}$$

As explained, all the functions are the same as before, except the last one.

We now move to the value functions for unemployed workers. All these functions are the same as in the baseline model:

$$\begin{aligned} rU_0 &= z + f_0(W_0 - U_0) + \gamma(\tilde{U}_0 - U_0) - \delta U_0, \\ r\tilde{U}_0 &= z + \tilde{f}_0(\tilde{W}_0 - \tilde{U}_0) - \delta \tilde{U}_0, \\ rU_1 &= z + f_1(W_1 - U_1) - \delta U_1, \\ r\tilde{U}_1 &= z + \tilde{f}_1(\tilde{W}_1 - \tilde{U}_1) - \delta \tilde{U}_1. \end{aligned}$$

The value functions for employed workers in the various states are given by:

$$\begin{aligned} rW_0 &= w_0 + \lambda(U_1 - W_0) - \delta W_0, \\ r\tilde{W}_0 &= \tilde{w}_0 + \lambda(\tilde{U}_1 - \tilde{W}_0) - \delta \tilde{W}_0, \\ rW_1 &= w_1 + \lambda(U_1 - W_1) - \delta W_1, \\ r\tilde{W}_1 &= \tilde{w}_1 + \lambda(\tilde{U}_1 - \tilde{W}_1) - \delta \tilde{W}_1 + \xi(W_1 - \tilde{W}_1). \end{aligned}$$

Notice that all these equations are the same as in the main text of Chapter 3.3, except the last one, where the presence of the term multiplied by ξ indicates the possibility that a type- $\tilde{1}$ worker can regain her skills later in her career.

Wage Curves

The next task is to solve the bargaining problems in the various types of meetings and obtain the various wage curves. Following similar steps as in Section 3.3.3, we arrive at the following wage equations:

Type 0 : $(1 - \eta)(W_0 - U_0) = \eta J_0$

$$w_0 = \frac{r + \delta + \gamma + f_0}{r + \delta + \gamma + \eta f_0} \eta (p - \kappa) + \frac{(r + \lambda + \delta)(r + f_1 + \delta)(r + \gamma + \delta)}{(r + \delta)(r + \delta + \gamma + \eta f_0)(r + \delta + \lambda + f_1)} (1 - \eta) z + \frac{\gamma \tilde{f}_0 \eta (p - \kappa - \tilde{\kappa} - \tilde{w}_0)}{(r + \delta)(r + \delta + \gamma + \eta f_0)} - \frac{(1 - \eta) \lambda f_1 (r + \gamma + \delta) w_1}{(r + \delta)(r + \delta + \gamma + \eta f_0)(r + \delta + \lambda + f_1)}. \quad (\text{A.2})$$

Type $\tilde{0}$: $(1 - \eta)(\tilde{W}_0 - \tilde{U}_0) = \eta \tilde{J}_0$

$$\tilde{w}_0 = \frac{1}{r + \delta + \eta \tilde{f}_0} \left\{ (r + \delta + \tilde{f}_0) \eta (p - \kappa - \tilde{\kappa}) + (r + \delta) (1 - \eta) z - \frac{\lambda \eta \tilde{f}_1}{r + \delta + \lambda + \xi} (p - \tilde{\kappa} - \tilde{w}_1) - \xi \frac{\lambda \tilde{f}_1 \eta}{(r + \delta + \lambda)(r + \delta + \lambda + \xi)} (p - w_1) \right\}. \quad (\text{A.3})$$

Type 1: $(1 - \eta)(W_1 - U_1) = \eta J_1$

$$w_1 = \frac{\eta p (r + \lambda + \delta + f_1) + (1 - \eta) z (r + \lambda + \delta)}{r + \lambda + \delta + \eta f_1}. \quad (\text{A.4})$$

Type $\tilde{1}$: $(1 - \eta)(\tilde{W}_1 - \tilde{U}_1) = \eta \tilde{J}_1$

$$\begin{aligned} \tilde{w}_1 = & \left\{ [(r + \delta)(r + \delta + \lambda + \xi + \tilde{f}_1) + \xi \tilde{f}_1] \eta (p - \tilde{\kappa}) + (1 - \eta) z (r + \delta)(r + \lambda + \delta + \xi) \right. \\ & \left. + \xi \frac{\tilde{f}_1(r + \delta + \xi) - f_1(r + \delta + \lambda + \xi)}{r + \delta + \lambda} \eta (p - w_1) \right\} \\ & \frac{1}{(r + \delta)(r + \delta + \lambda + \xi) + \eta \tilde{f}_1(r + \delta + \xi)}. \end{aligned} \quad (\text{A.5})$$

Definition of Steady State Equilibrium

Definition 6. A steady state equilibrium in the model where type- $\tilde{1}$ workers stochastically regain skills later in their career is a list of wages for the four types of workers $(w_0, \tilde{w}_0, w_1, \tilde{w}_1)$, a measure of vacant firms v , and measures of unemployed and employed workers in the various states $(u_0, \tilde{u}_0, u_1, \tilde{u}_1, e_0, \tilde{e}_0, e_1, \tilde{e}_1)$, satisfying the free entry condition (A.1), the four wage curves (A.2), (A.3), (A.4), and (A.5), and the eight Beveridge curves reported earlier in this subsection.

Quantitative Results

As we report in Section 1, the losses from early unemployment spells are large and persist for several years later in a worker's career; see Von Wachter (2020) for a review of the empirical literature. This motivated us to assume that the scarring effects of the skill shock γ last for the whole career of a worker in the baseline model. Indeed, Rothstein (2023) reports that the employment effects of high unemployment rates upon labor market entry are *permanent*: cohorts that face high unemployment when they enter the labor market have lower employment rates throughout their careers. However, it is useful to understand the quantitative impact of this assumption for the model implications. To do so, we present the results of the Extension 1 model for various levels of the skill regaining parameter ξ .

Table C.1. Comparison of the Baseline Model with Extension 1 for various levels of ξ .

Variable	Baseline	Large ξ	Medium ξ	Small ξ
Y	0.9379	0.9501	0.9486	0.9466
u	5.80%	5.98%	5.91%	5.84%
f_1	1.21	0.90	0.94	0.99
$\tilde{f}_1$	0.51	0.44	0.45	0.47
f_0	0.43	0.42	0.43	0.43
$\tilde{f}_0$	0.42	0.41	0.42	0.42
w_1	0.9852	0.9808	0.9814	0.9823
$\tilde{w}_1$	0.9152	0.9006	0.9055	0.9090
w_0	0.6620	0.7443	0.7254	0.7059
$\tilde{w}_0$	0.7535	0.7475	0.7492	0.7508

Note: Large ξ implies an average employment duration of five years, medium ξ of ten years, and small ξ of twenty years until the worker regains their skill.

Table C.1 summarizes the values of the main endogenous variables for the model of Extension 1, as well as the Baseline Model for comparison purposes. We show the results for three levels of the skill regaining shock: $\xi = 1/(12 * 5)$ ("large ξ "), $\xi = 1/(12 * 10)$ ("medium ξ "), and $\xi = 1/(12 * 20)$ ("small ξ "). These values of ξ imply different lengths of employment duration needed for the average worker's productivity to return to its initial level of p . The large ξ implies an average employment duration of five years, the medium ξ of ten years, and the small ξ of twenty years for skills to be regained. Through the lens of the empirical literature, these three values are quite conservative: all studies surveyed by Von Wachter (2020) report losses that persist for fifteen years after the worker's labor market entry, while Arellano-Bover (2022) reports effects on workers' skills that can be attributed to unemployment durations forty years earlier in a worker's career. As ξ becomes smaller, Extension 1 approaches the Baseline Model (the two are equivalent for $\xi = 0$). Moreover, all variables move in the expected direction, approaching their baseline levels as the value of ξ diminishes.

The impact of the value of ξ on the economy's welfare is in the order 0.9 - 1.3%. In general, the magnitude of the effects for all variables is in the same order of magnitude as the policy interventions studied in Chapter 3.6. This is natural: the main economic trade-off in Chapter 3 lies

in the skill loss of entrant workers while in unemployment. Extension 1 makes this channel less severe by directly moving workers from the $\tilde{e}_1$ to the e_1 state. The policy interventions studied in Chapter 3.6 lower the impact of the skill loss channel by raising the job-finding rate of entrant workers. Thus, the quantitative impact of a larger ξ is similar to increases in f_0 in the baseline model. Indeed, one could interpret the model of Extension 1 as the outcome of an active labor market policy (Heckman et al., 1999) targeted to employed workers who have been through long unemployment spells early in their career.

C.4.2 Extension 2: Gain experience while on first job

The second assumption of the baseline model that we relax in this appendix is that young workers gain experience (only) after they lose their very first job. Specifically, we will adopt the milder assumption that workers who are employed at their first job can get hit by a shock and stochastically acquire experience while they are still holding that job. Workers who lose their first job before getting hit by that shock still become experienced, just like in the baseline model. Importantly, this new “gain experience while on the first job shock” can hit workers who did not suffer skill loss during youth, as well as those who did suffer such skill loss. (We assume that the arrival rate of the shock is the same for both of these types.) In terms of the notation developed in Chapter 3.2, what is new here is that type-0 and type- $\tilde{0}$ workers may get hit by the Poisson shock ζ and become type-1 and type- $\tilde{1}$ workers, respectively. Figure C.2 illustrates the worker flows in and out of every state in this new environment, and it is useful for deriving the Beveridge curves. The figure is identical to the baseline worker-flow diagram in Figure 3.2, with the exception of the two new (orange) arrows pointing from the e_0 towards the e_1 bubble, and from the $\tilde{e}_0$ towards the $\tilde{e}_1$ bubble, indicating the newly introduced possibility of young workers gaining experience while they are still on their very first job.

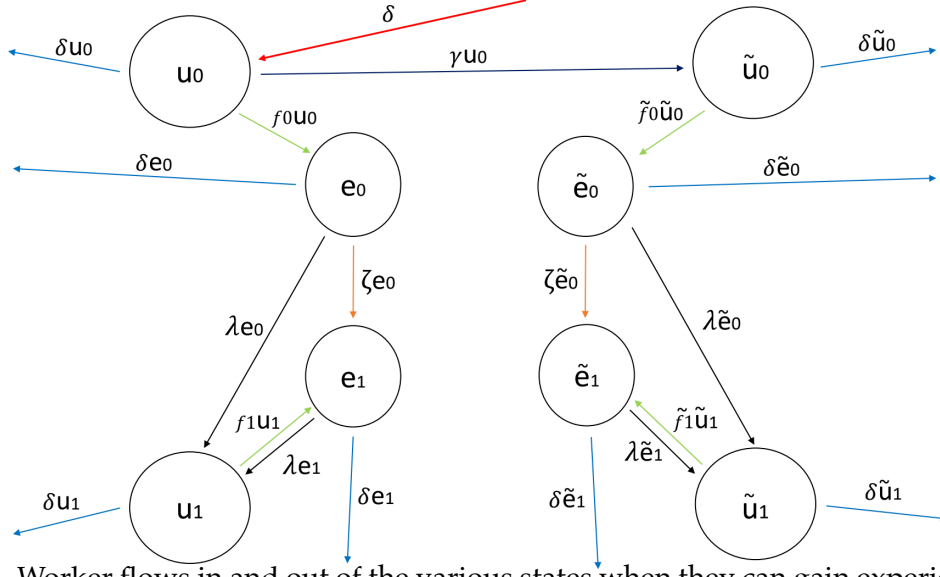


Figure C.2. Worker flows in and out of the various states when they can gain experience while on their first job.

Beveridge curves

With the visual help of Figure C.2, we can equate the flows in and out of each state, and after some algebra, we can show that the steady state measure of workers in the various states are as follows:

$$\begin{aligned}
 u_0 &= \frac{\delta}{\delta + \gamma + f_0}, \\
 \tilde{u}_0 &= \frac{\gamma}{\delta + \tilde{f}_0} \cdot \frac{\delta}{\delta + \gamma + f_0}, \\
 e_0 &= \frac{f_0}{\delta + \lambda + \zeta} \cdot \frac{\delta}{\delta + \gamma + f_0}, \\
 \tilde{e}_0 &= \frac{\tilde{f}_0}{\delta + \lambda + \zeta} \cdot \frac{\gamma}{\delta + \tilde{f}_0} \cdot \frac{\delta}{\delta + \gamma + f_0}, \\
 \tilde{u}_1 &= \frac{\lambda \tilde{f}_0 \gamma}{(\delta + \lambda + \tilde{f}_1)(\delta + \tilde{f}_0)(\delta + f_0 + \gamma)}, \\
 \tilde{e}_1 &= \frac{\tilde{f}_0 \gamma}{(\delta + \lambda)(\delta + f_0 + \gamma)(\delta + \tilde{f}_0)} \left[\frac{\lambda \tilde{f}_1}{\delta + \lambda + \tilde{f}_1} + \frac{\zeta \delta}{\delta + \lambda + \zeta} \right], \\
 u_1 &= \frac{\lambda}{\delta + f_1 + \lambda} \frac{f_0}{\delta + f_0 + \gamma}, \\
 e_1 &= \frac{f_0}{(\delta + \lambda)(\delta + f_0 + \gamma)} \left[\frac{\lambda f_1}{\delta + \lambda + f_1} + \frac{\zeta \delta}{\delta + \lambda + \zeta} \right].
 \end{aligned}$$

Value Functions

We now move to the value functions starting with firms. The value function of a vacant firm is given by:

$$rV = -c + q_0(J_0 - V) + \tilde{q}_0(\tilde{J}_0 - V) + q_1(J_1 - V) + \tilde{q}_1(\tilde{J}_1 - V).$$

Free entry implies that in equilibrium we must have $V = 0$, therefore, we can state the free entry condition as

$$c = q_0J_0 + \tilde{q}_0\tilde{J}_0 + q_1J_1 + \tilde{q}_1\tilde{J}_1.$$

We also have four value functions for productive firms in the various states, i.e., for firms that matched with the four different types of workers (type 0, $\tilde{0}$, 1, and $\tilde{1}$). These value functions are as follows:

$$rJ_1 = p - w_1 - \lambda J_1 - \delta J_1,$$

$$r\tilde{J}_1 = p - \tilde{\kappa} - \tilde{w}_1 - \lambda\tilde{J}_1 - \delta\tilde{J}_1,$$

$$rJ_0 = p - \kappa - w_0 - \lambda J_0 - \delta J_0 + \zeta(J_1 - J_0),$$

$$r\tilde{J}_0 = p - \kappa - \tilde{\kappa} - \tilde{w}_0 - \lambda\tilde{J}_0 - \delta\tilde{J}_0 + \zeta(\tilde{J}_1 - \tilde{J}_0).$$

It is straightforward to notice that the first two are identical to the ones reported in the baseline model in Chapter 3.3, but the last two value functions are different, as is evident from the presence of the new shock ζ .

Next, consider the value functions of workers in the various states. The value functions for unemployed workers in the various states are identical to the ones reported in the main text of

Chapter 3.3 and are given by:

$$\begin{aligned}
rU_0 &= z + f_0(W_0 - U_0) + \gamma(\tilde{U}_0 - U_0) - \delta U_0, \\
r\tilde{U}_0 &= z + \tilde{f}_0(\tilde{W}_0 - \tilde{U}_0) - \delta \tilde{U}_0, \\
rU_1 &= z + f_1(W_1 - U_1) - \delta U_1, \\
r\tilde{U}_1 &= z + \tilde{f}_1(\tilde{W}_1 - \tilde{U}_1) - \delta \tilde{U}_1.
\end{aligned}$$

For employed workers in the various states, we have:

$$\begin{aligned}
rW_0 &= w_0 + \lambda(U_1 - W_0) + \zeta(W_1 - W_0) - \delta W_0, \\
r\tilde{W}_0 &= \tilde{w}_0 + \lambda(\tilde{U}_1 - \tilde{W}_0) + \zeta(\tilde{W}_1 - \tilde{W}_0) - \delta \tilde{W}_0, \\
rW_1 &= w_1 + \lambda(U_1 - W_1) - \delta W_1, \\
r\tilde{W}_1 &= \tilde{w}_1 + \lambda(\tilde{U}_1 - \tilde{W}_1) - \delta \tilde{W}_1.
\end{aligned}$$

Again, it is obvious that the last two value functions are as in the baseline model in Chapter 3.3. However, the first two are different, with the term multiplied by ζ representing the possibility that inexperienced employed workers (with or without skill loss) can become experienced without having to go through unemployment first.

Wage Curves

The next task is to solve the bargaining problems in the various types of meetings and obtain the various wage curves. Following similar steps as in Section 3.3.3, we arrive at the following wage equations:

Type 1: $(1 - \eta)(W_1 - U_1) = \eta J_1$

$$w_1 = \frac{\eta p(r + \lambda + \delta + f_1) + (1 - \eta)z(r + \lambda + \delta)}{r + \lambda + \delta + \eta f_1}. \quad (\text{A.6})$$

Type $\tilde{1}$: $(1 - \eta)(\tilde{W}_1 - \tilde{U}_1) = \eta\tilde{J}_1$

$$\tilde{w}_1 = \frac{\eta(p - \tilde{\kappa})(r + \lambda + \delta + \tilde{f}_1) + (1 - \eta)z(r + \lambda + \delta)}{r + \lambda + \delta + \eta\tilde{f}_1}. \quad (\text{A.7})$$

Type $\tilde{0}$: $(1 - \eta)(\tilde{W}_0 - \tilde{U}_0) = \eta\tilde{J}_0$

$$\begin{aligned} \tilde{w}_0 = & \frac{1}{r + \delta + \eta\tilde{f}_0} \left\{ (r + \delta + \tilde{f}_0)\eta(p - \kappa - \tilde{\kappa}) + (1 - \eta)z \left(r + \delta + \frac{(\lambda + \zeta)\tilde{f}_1}{r + \lambda + \delta + \tilde{f}_1} \right) \right. \\ & \left. + \frac{\eta\tilde{f}_0\zeta(p - \tilde{\kappa})}{r + \lambda + \delta} - \tilde{w}_1 \left(\frac{\eta\tilde{f}_0\zeta}{r + \lambda + \delta} + \frac{(1 - \eta)(\lambda + \zeta)\tilde{f}_1}{r + \lambda + \delta + \tilde{f}_1} \right) \right\}. \end{aligned} \quad (\text{A.8})$$

Type 0 : $(1 - \eta)(W_0 - U_0) = \eta J_0$

$$\begin{aligned} w_0 = & \frac{1}{r + \delta + \gamma + \eta f_0} \left\{ (r + \delta + \gamma + f_0)\eta(p - \kappa) \right. \\ & + \frac{(1 - \eta)z(r + \delta + \gamma)}{r + \delta} \\ & \quad \times \frac{(r + \delta + f_1)(r + \delta + \lambda) + \zeta f_1}{r + \lambda + \delta + f_1} \\ & + \eta f_0 \zeta \frac{p - w_1}{r + \delta + \lambda} + \frac{\eta \tilde{f}_0 \gamma}{r + \delta} (p - \kappa - \tilde{\kappa} - \tilde{w}_0) \\ & + \frac{\gamma \tilde{f}_0 \eta}{r + \delta} \zeta (p - \tilde{\kappa} - \tilde{w}_1) \\ & - \frac{(1 - \eta)(\lambda + \zeta)(r + \delta + \gamma)}{r + \delta} f_1 \\ & \left. \times \frac{w_1}{r + \lambda + \delta + f_1} \right\}. \end{aligned} \quad (\text{A.9})$$

Definition of Steady State Equilibrium

Definition 7. A steady state equilibrium in the model where agents can gain experience while on their first job is a list of wages for the four types of workers $(w_0, \tilde{w}_0, w_1, \tilde{w}_1)$, a measure

of vacant firms v , and measures of unemployed and employed workers in the various states $(u_0, \tilde{u}_0, u_1, \tilde{u}_1, e_0, \tilde{e}_0, e_1, \tilde{e}_1)$, satisfying the free entry condition, the four wage curves (A.6), (A.7), (A.8), and (A.9), and the eight Beveridge curves reported earlier in this subsection.

Quantitative Results

Table C.2 summarizes the values of the main endogenous variables for the model of Extension 2, as well as the Baseline Model for comparison purposes. We show the results for three levels of the experience gain shock: $\zeta = 1/12$ ("large ζ "), $\zeta = 1/15$ ("medium ζ "), and $\zeta = 1/18$ ("small ζ "). These values of ζ imply different lengths for the duration of an entrant's first employment spell until the entrant becomes an experienced worker. The large ζ implies an average employment duration of 12 months, the medium ζ of 15 months, and the small ζ of 18 months for the training costs to disappear. In the model, the average job lasts for $1/\lambda \approx 20$ months. As ζ becomes smaller, Extension 2 approaches the Baseline Model (the two are equivalent for $\zeta = 0$). Moreover, all variables move in the expected direction, approaching their baseline levels as ζ becomes smaller.

The impact of the value of ζ on the economy's welfare is in the order 0.4 - 0.5%, roughly half of the impact of Extension 1. In general, larger values of ζ raise the average productivity of the unemployment pool and, as a result, firm entry and workers' job-finding rates. Interestingly, the wages of type-0 workers plummet: employment in state e_0 is even more beneficial in the model of Extension 2, since workers can move to e_1 without an intervening unemployment spell. As a result, entrant workers are willing to work for next to nothing just to improve their career prospects. We should highlight that this result of Extension 2 has a flavor similar to the Internships intervention discussed in Chapter 3.6. Moreover, the quantitative effects of Extension 2 are roughly in the same order of magnitude as the Internships scenario (which raised welfare by 0.62%).

Table C.2. Comparison of the Baseline Model with Extension 2 for various levels of ζ .

Variable	Baseline	Large ζ	Medium ζ	Small ζ
Y	0.9379	0.9429	0.9424	0.9420
u	5.80%	4.98%	5.08%	5.14%
f_1	1.21	1.37	1.34	1.33
$\tilde{f}_1$	0.51	0.59	0.58	0.57
f_0	0.43	0.51	0.50	0.49
$\tilde{f}_0$	0.42	0.50	0.49	0.48
w_1	0.9852	0.9867	0.9866	0.9865
$\tilde{w}_1$	0.9152	0.9178	0.9055	0.9176
w_0	0.6620	0.0094	0.1170	0.2069
$\tilde{w}_0$	0.7535	0.7502	0.7517	0.7522

Note: Large ζ implies an average employment duration of twelve months, medium ζ of fifteen months, and small ζ of eighteen months in their first job until an entrant worker becomes experienced.

C.4.3 Extension 3: Different bargaining powers based on experience

As we explain in Section 3.2, with the exception of productivity and the consequent differences in job-finding rates, all the other parameters of the model (including η , the bargaining power of the worker) are independent of the worker's type, and this is intentional. The goal of this modeling choice is to deliver results that are driven *only* by differences in workers' experience, and whether they suffered skill loss during their youth, which is the focus of the chapter. That said, it is not unreasonable to assume that workers of different (age and) experience may have different bargaining abilities. Therefore, in this appendix subsection, we consider yet another extension of the baseline model, namely, one where experienced and inexperienced workers have different bargaining powers. Using our standard notation, we will let η_0 denote the bargaining power of inexperienced workers (types 0 and $\tilde{0}$), and η_1 the bargaining power of experienced workers (types 1 and $\tilde{1}$). (We assume that whether workers suffered skill loss or not during youth is irrelevant for their bargaining power; only experience matters.)

Moving on to the analysis of the model, it is important to note that all Beveridge curves and all value functions remain unchanged compared to the baseline model. Only the structure of the bargaining problems in different types of meetings is modified, and this affects the derivation of

the wage curves as shown below.

Bargaining problems

Bargaining in a type-1 meeting

The standard Nash bargaining condition is:

$$(1 - \eta_1)(W_1 - U_1) = \eta_1 J_1.$$

Replacing W_1 and J_1 in the above, the type-1 wage can be written as

$$w_1 = \eta_1 p + (1 - \eta_1) z + \eta_1 f_1 J_1.$$

Further substitution and rearrangement deliver the wage curve:

$$w_1 = \frac{\eta_1 p (r + \lambda + \delta + f_1) + (1 - \eta_1) z (r + \lambda + \delta)}{r + \lambda + \delta + \eta_1 f_1}. \quad (\text{A.10})$$

Bargaining in a type- $\tilde{1}$ meeting

The standard Nash bargaining condition is:

$$(1 - \eta_1)(\tilde{W}_1 - \tilde{U}_1) = \eta_1 \tilde{J}_1.$$

By substituting the relevant value functions and rearranging, the wage curve for type- $\tilde{1}$ workers is:

$$\tilde{w}_1 = \frac{\eta_1 (p - \tilde{\kappa})(r + \lambda + \delta + \tilde{f}_1) + (1 - \eta_1) z (r + \lambda + \delta)}{r + \lambda + \delta + \eta_1 \tilde{f}_1}. \quad (\text{A.11})$$

Bargaining in a type- $\tilde{0}$ meeting

The standard Nash condition is:

$$(1 - \eta_0)(\tilde{W}_0 - \tilde{U}_0) = \eta_0 \tilde{J}_0,$$

which leads to the following wage curve for type-0 workers:

$$\begin{aligned} \tilde{w}_0 = \frac{1}{r + \delta + \eta_0 \tilde{f}_0} & \left[\eta_0 (p - \kappa - \tilde{\kappa}) (r + \delta + \tilde{f}_0) \right. \\ & + \frac{(r + \delta + \lambda)(r + \delta + \tilde{f}_1)}{r + \delta + \lambda + \tilde{f}_1} (1 - \eta_0) z \\ & \left. - \frac{\lambda (1 - \eta_0) \tilde{f}_1}{r + \delta + \lambda + \tilde{f}_1} \tilde{w}_1 \right]. \end{aligned} \quad (\text{A.12})$$

Bargaining in a type-0 meeting

The standard Nash condition here is:

$$(1 - \eta_0)(W_0 - U_0) = \eta_0 J_0,$$

which yields the wage curve for type-0 workers:

$$\begin{aligned} w_0 = \frac{r + \delta + \gamma + f_0}{r + \delta + \gamma + \eta_0 f_0} & \eta_0 (p - \kappa) \\ & + \frac{(r + \lambda + \delta)(r + f_1 + \delta)(r + \gamma + \delta)}{(r + \delta)(r + \delta + \gamma + \eta_0 f_0)(r + \delta + \lambda + f_1)} (1 - \eta_0) z \\ & + \frac{\gamma \tilde{f}_0 \eta_0 (p - \kappa - \tilde{\kappa} - \tilde{w}_0)}{(r + \delta)(r + \delta + \gamma + \eta_0 f_0)} \\ & - \frac{(1 - \eta_0) \lambda f_1 (r + \gamma + \delta) w_1}{(r + \delta)(r + \delta + \gamma + \eta_0 f_0)(r + \delta + \lambda + f_1)}. \end{aligned} \quad (\text{A.13})$$

Definition of Steady State Equilibrium

A steady state equilibrium for this version of the model is a list of wages $(w_0, \tilde{w}_0, w_1, \tilde{w}_1)$, a measure of vacant firms v , and measures of unemployed and employed workers in all states,

$(u_0, \tilde{u}_0, u_1, \tilde{u}_1, e_0, \tilde{e}_0, e_1, \tilde{e}_1)$, satisfying the free-entry condition, the four wage curves (A.10), (A.11), (A.12), and (A.13), and the standard Beveridge curves (i.e., the ones described in Section 3.3.1).

Quantitative Results

Table C.3 summarizes the values of the main endogenous variables for the model of Extension 3, as well as the Baseline Model for comparison purposes. We show the results for two combinations of bargaining power parameters: i) $\eta_0 = \alpha/2$, $\eta_1 = \alpha$ and ii) $\eta_0 = \alpha$, $\eta_1 = \alpha/2$ (as a reminder, $\eta_0 = \eta_1 = \alpha$ in the baseline model). The first combination corresponds to a scenario in which entrants have lower bargaining power than experienced workers, while the second combination to the opposite case. In general, the empirical evidence on the bargaining power over a worker's life-cycle is not conclusive. As one can infer from the work of Cordoba et al. (2023), there are factors that point to a lower bargaining power early (e.g., lower human capital) or later (e.g., retirement) in a worker's career. Hence, we show the model implications for both scenarios (interpreting entrants as younger and the experienced ones as older workers).

The first implication of Table C.3 is that the impact of changes in workers' bargaining power has very modest effects on welfare. The overall effects are an order of magnitude smaller than the effects predicted by the models of Extensions 1 and 2. In general, smaller values of η 's raise the fraction of the surplus received by the firms and, as a result, firm entry and workers' job-finding rates. Similar to Extension 2, the wages of type-0 workers decrease following the exact same reasoning. Comparing the different bargaining power combinations, welfare's behavior is non-linear: it increases when η_0 decreases but it actually decreases when η_1 decreases. The economics here follows the well-known reasoning of Hosios (1990), according to which unemployment may be inefficiently low if workers' bargaining power is below some threshold value, because of the large costs of vacancy creation due to inefficiently high firm entry.

Table C.3. Comparison of the Baseline Model with Extension 3 for various levels of η_0 and η_1 .

Variable	Baseline	$\eta_0 = \alpha/2, \eta_1 = \alpha$	$\eta_0 = \alpha, \eta_1 = \alpha/2$
Y	0.9379	0.9385	0.9364
u	5.80%	4.91%	3.74%
f_1	1.21	1.38	1.73
$\tilde{f}_1$	0.51	0.60	0.78
f_0	0.43	0.52	0.69
$\tilde{f}_0$	0.42	0.51	0.67
w_1	0.9852	0.9869	0.9701
$\tilde{w}_1$	0.9152	0.9184	0.8927
w_0	0.6620	0.4870	0.7069
$\tilde{w}_0$	0.7535	0.7097	0.7648

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