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Journal

Physics of Fluids, 32(8)

ISSN

1070-6631

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Publication Date

2020-08-01

DOI

10.1063/5.0015498

Peer reviewed

Pulsed-Jet Propulsion via Shape Deformation of an Axisymmetric Swimmer

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(Dated: 3 July 2020)

By using an axisymmetric immersed-boundary model, we numerically investigate the thrust generation of a deformable body via pulsed jetting. We focus on a single discharging process resulting from the deflation of the body as inspired by the jetting mechanism of cephalopods such as squids. We examine three jet velocity profiles, namely impulsive, half cosine, and cosine in the relatively low Reynolds number regime. For the impulsive profile, we demonstrate via wake visualization that the leading vortex ring does not pinch off from the trailing jet, although its circulation stops growing after a critical formation number (hereby the formation number is defined as the ratio between length and diameter of the jet plug) of 6~7. The exact value of the critical formation number depends on the jet velocity profile, suggesting that jet acceleration and viscous dissipation play significant roles in vortex ring evolution. In terms of thrust generation, our results indicate that besides the jet momentum flux, an important source of thrust generation is jet acceleration. The implication is that the jet velocity profile is a key factor in determining the propulsive performance.

I. INTRODUCTION

The locomotion mechanism of squids and other cephalopods in the jetting mode has recently attracted attention due to its high-speed capacity. In terms of the peak speed measured in body length per second (BL/s), squids can match the fastest fish in performance¹⁻³. Moreover, using jet propulsion some species of squids (e.g. *Ommastrephidae*) are able to propel themselves into the air and glide for a considerable distance before plunging back into water^{4,5}. The airborne speed of these animals approaches 50 BL/s (body length per second), surpassing the speed of other aquatic creatures⁶.

Indeed, squids have developed a unique locomotion system combining fin flapping and jet propulsion that works in a wide range of speed. Similar to the labriform swimming method utilized by species of fish with pectoral fins, the fins of squids are able to create thrusts or maneuvering forces through undulatory motions. Unlike fish fins, however, the squid fins has little supporting structure embedded so that they may not be able to work under high hydrodynamic loading. For that reason, it is likely that these fins are not effective in high-speed swimming when it is necessary to escape from predators or other danger. Instead, the animal relies on its mantle cavity, mantle wall, funnel tube, and associated muscles to achieve bursting motion in a jetting mode. To elaborate, by using two groups of muscles (radial and circular) as well as the elasticity of the mantle wall, the internal volume and the lateral dimension of the body vary over time⁷. The shrinking of the internal volume pushes water out of the funnel tube, which works as a nozzle, to create a jet. Meanwhile, the variation of the lateral dimension changes the added mass of the body, which contributes additional thrust as illustrated by recent studies⁸⁻¹⁰. These studies, however, do not include sophisticated modeling of flow inside the body and jet formation at the nozzle.

The investigation in squid swimming started with observations and measurements of live animals in field or laboratory environments^{2,3,7}. In laboratory experiments, the kinematics of the body (especially the fins) and the near-body flow fields in both the flapping and the jetting modes have been recorded¹¹⁻¹³. These studies illustrate the complicated nature of the underlying physics. Specifically, Digital Particle Image Velocimetry (DPIV) images demonstrate at least six distinctive vorticity patterns in the fin-activated mode. The jetting mode, likewise, consists of two sub-modes, namely Mode I and Mode II. In Mode I, each jetting creates a single dominant vortex ring. In Mode II, the wake includes a leading vortex ring followed by distributed vorticity along a track.

The existence of different wake modes in jetting is indeed consistent with experimental results by Gharib *et al.*¹⁴. In that experiment the formation of vortex rings by jet flow was found to be determined by a dimensionless parameter called *formation number*, which was defined as L/D , where L and D are the length and diameter of the jet plug, respectively. According to their observations, if the formation number is below a critical value the wake is dominated by a single vortex ring (*i.e.* Mode I). When the formation number is above the critical value a vortex pinch-off occurs in a phenomenon similar to Mode II. A theoretical explanation of this phenomenon was provided by Linden and Turner¹⁵. In that work they compared properties of the jet plug (volume, circulation, impulse, and kinetic energy) with those of a finite-core vortex ring. Their results indicate that the critical formation number corresponds roughly to the limiting case when these properties of the jet plug match those of a single vortex ring, *i.e.* the jet is completely rolled into a vortex ring with maximum strength. They further demonstrated that at this particular case the impulse created by the jet was maximized for given energy input. This finding is consistent with results that for bursting start of squid-inspired axisymmetric swimmers optimal speed is achieved close to the critical formation number¹⁶.

The aforementioned conclusions are obtained when the body is static or at the starting stage. There is little knowledge about the performance of the system with non-zero average

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forward speed (corresponding to the case of longer-distance swimming with repeated deflation-inflation cycles). Jiang and Grosenbaugh¹⁷ suggested that due to the ambient flow vorticity shedding from the body might affect the dynamics of the system significantly, to the extent that in many cases the effect of the critical formation number was not pronounced. Their study, however, was based on a rigid body so that the relevance to the actual animals or squid-inspired robots (both have deformable bodies) was not clear. In fact, according to our recent study periodic body deformations during the deflation-inflation cycles create synchronized vortex shedding from the body and the nozzle^{18,19}, which is different from the scenario considered by Jiang and Grosenbaugh¹⁷. The model we applied was based on a two-dimensional framework so that physical mechanisms of three-dimensional vortex dynamics, including the formation number effect, were not included.

To study the propulsion performance of a deformable body with a pressure chamber similar to the one possessed by a squid in jetting mode, we hereby develop an axisymmetric fluid dynamics model by using the immersed-boundary approach. In this model the body deformation and jet speed are both prescribed. Unlike our previous work¹⁶, where a potential-flow-based numerical model was applied, in the present study we take into account the effect of fluid viscosity by solving the axisymmetric Navier-Stokes equations. The concentration is on vortex ring formation and thrust generation capacity at various conditions in low Reynolds number environment.

The rest of the paper is organized as follows. We first describe the physical problem, including the geometry and kinematics of the swimmer with deformable body. This is followed by formulations of mathematical equations and numerical algorithm. The numerical results are then presented. Finally, conclusions are drawn.

II. PROBLEM DESCRIPTION

We consider a squid-like swimmer consisting of a deformable body with an empty chamber in it that imitates the mantle cavity. With prescribed deformation this swimmer can suck in or expel water through an opening that serves as an inlet or nozzle. In this particular study we concentrate on the deflation deformation, during which a jet flow is formed in the wake.

Figure 1a illustrates the geometry and body deformation of the squid-like swimmer. The body is modeled as a zero-thickness ellipsoidal shell with one end open to form a nozzle whose diameter is D , meanwhile the internal volume forms the pressure chamber. The position of the nozzle does not change. The contour length of the body, measured from the upper point of the nozzle to the lower point along the body, is $L = 10D$. With L and D being fixed, the body deformation is achieved by varying the eccentricity of the ellipsoid as $e = e(t)$ (a detailed description is included in our previous study¹⁶). For example, the increase of e decreases the internal volume $\forall(e)$ so that fluid is discharged out of the pressure chamber through the nozzle to create a jet flow. The

e_0	0.92	0.90	0.86	0.80	0.65
Γ_m	2.9	4.6	7.3	10.4	15.0

TABLE I. Maximum formation number Γ_m for various initial states e_0 with the fully deflated state fixed as $e_1 = 0.95$.

two extreme states, fully inflated and fully deflated, are represented by $e = e_0$ and $e = e_1$, respectively. Formation number associated with this deflation deformation is defined as $\Gamma = 4\Lambda/(\pi D^3)$, where $\Lambda = \forall(e_0) - \forall(e)$ is the fluid volume ejected out of the chamber. The maximum value of Γ , labelled as Γ_m , is reached at the end of body shrinking. Its exact value is $4(\forall(e_0) - \forall(e_1))/(\pi D^3)$.

The spatially averaged speed of the jet flow at the nozzle is calculated as

$$V_j(t) = \frac{-4}{\pi D^2} \frac{d\forall}{dt} = \frac{d\Gamma}{dt} D. \quad (1)$$

We will simulate the pulsed jetting motion of the system through the deflation deformation, during which the body starts from its fully inflated state ($e = e_0$) and deflates until it reaches the fully deflated state ($e = e_1$). In the current study, the jet speed $V_j(t)$ is prescribed so that we are able to calculate the time evolution of the ellipsoidal chamber volume $\forall(e)$ through Eq. (1), from which its corresponding eccentricity $e(t)$ can be determined. A sequence of cases with different Γ_m will be examined as shown in Table I. Our simulations have been performed in a rectangular computational domain as sketched in Fig. 1b, whose dimension is $10L \times L$. The domain size is determined such that further size increase does not affect the results significantly. We consider the pulsed jetting in static fluid so that the incoming flow speed u_0 is set to be zero.

A jet-based Reynolds number is defined as

$$Re_j = \frac{\rho \tilde{V}_j D}{\mu}, \quad (2)$$

where $\tilde{V}_j$ stands for the maximum (or upper-bound) spatially averaged jet speed at the nozzle plane. ρ and μ are the density and dynamic viscosity of the fluid, respectively. Simulations in this paper (except for those in Appendix A) will be performed with $Re_j = 150$.

III. MATHEMATICAL FORMULATIONS AND NUMERICAL ALGORITHM

A. Governing equations

Defined within a cylindrical coordinate system $\mathbf{x}(x, r, \theta)$ (see Fig. 1a), the flow velocity $\mathbf{u} \equiv (u_x, u_r)$ is governed by the axisymmetric incompressible Navier-Stokes equations and the continuity equation. Let $\mathcal{H}, \mathcal{G}, \mathcal{D}, \mathcal{L}$ be the convective, gradient, divergence and Laplacian operators, we have

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathcal{H}(\mathbf{u}) = -\mathcal{G}(p) + \mu \mathcal{L}(\mathbf{u}) + \mathbf{f}, \quad (3)$$

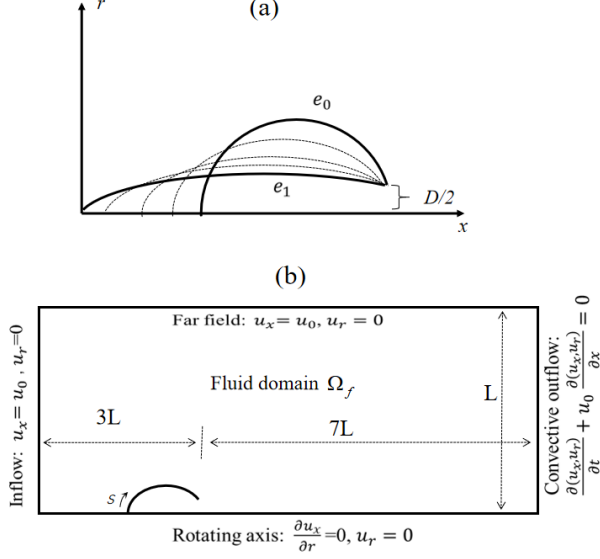


FIG. 1. (a) Trajectory of body deformation between e_0 and e_1 . (b) Computational domain and boundary conditions (not to scale).

and

$$\mathcal{D}(\mathbf{u}) = \frac{\partial u_x}{\partial x} + \frac{1}{r} \frac{\partial (ru_r)}{\partial r} = 0, \quad (4)$$

where p is the pressure. The operators specified above are defined as

$$\mathcal{H} = u_x \frac{\partial}{\partial x} + u_r \frac{\partial}{\partial r}, \quad (5)$$

$$\mathcal{L}_x = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r}, \quad (6)$$

$$\mathcal{L}_r = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2}. \quad (7)$$

In Eq. (3), $\mathbf{f} \equiv (f_x, f_r)$ represents the force density exerted by the immersed boundary on the surrounding fluid. This quantity is obtained through a feedback algorithm to enforce the no-slip and no-flux conditions at the fluid-solid interface^{18,20}. The feedback scheme is summarized as

$$\mathbf{U}(s, t) = \int_{\Omega_f} \mathbf{u}(\mathbf{x}, t) \delta(\mathbf{x} - \mathbf{X}(s, t)) d\mathbf{x}, \quad (8)$$

$$\mathbf{F}(s, t) = \alpha \int_0^t [\mathbf{U}(s, \tau) - \mathbf{V}(s, \tau)] d\tau + \beta [\mathbf{U}(s, t) - \mathbf{V}(s, t)], \quad (9)$$

$$\mathbf{f}(\mathbf{x}, t) = \int_{\Gamma_{ib}} \mathbf{F}(s, t) \delta(\mathbf{X}(s, t) - \mathbf{x}) ds, \quad (10)$$

in which s is a Lagrangian coordinate along the body Γ_{ib} , whose instantaneous position is $\mathbf{X}(s, t)$. The fluid velocity

$\mathbf{U}(s, t)$ at the position of the body is obtained via interpolation of fluid velocity over the whole domain Ω_f using a two dimensional Dirac delta function δ . $\mathbf{V}(s, t)$ is the body velocity, which is prescribed in this study. The difference between $\mathbf{U}$ and $\mathbf{V}$ is penalized using Eq. (9), in which α and β are negative parameters with sufficiently large absolute values so that $\mathbf{U}$ and $\mathbf{V}$ stay close to each other after iteration. Meanwhile, the interaction force $\mathbf{F}(s, t) \equiv (F_x, F_r)$ between fluid and the body is obtained using Eq. (9) and distributed to the fluid domain through the delta function, as shown in Eq. (10). Unless otherwise specified we choose $\alpha = -10^3 \rho \tilde{V}_j^2 / D$ and $\beta = -100 \rho \tilde{V}_j$. The exact values of α and β do not effect the simulation results significantly.

The performance of this propulsive system is characterized by the thrust coefficient C_T and the efficiency of impulse generation η . These quantities are defined as

$$C_T = \frac{F_T}{\rho \tilde{V}_j^2 D^2}, \quad (11)$$

and

$$\eta = \frac{I}{P_{in}}, \quad (12)$$

where F_T represents the thrust force and I is the overall impulse created by the pulsed jetting with time duration T_d . Hence

$$F_T = - \int_{\Gamma_{ib}} 2\pi r(s) F_x(s, t) ds, \quad (13)$$

$$I = \int_{T_d} F_T dt, \quad (14)$$

where the time integration is carried over the duration of the deflation T_d .

Meanwhile, P_{in} stands for the overall energy, which can be calculated as

$$P_{in} = \int_{T_d} P dt, \quad (15)$$

where

$$P = \int_{\Gamma_{ib}} 2\pi r(s) \mathbf{F} \cdot \mathbf{U} ds. \quad (16)$$

B. Numerical approach

Following previous studies^{19,21}, we employ a fractional-step algorithm to solve the dynamic equations numerically. A fully implicit time-advancement method (second-order Crank-Nicholson scheme) is used for temporal discretization, and a second-order central finite difference scheme is applied on staggered grid to calculate spatial derivatives. The discrete forms of Eqns. (3) and (4) are, respectively,

$$\rho \frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t} + \rho \frac{\mathcal{H}(\mathbf{u}^{n+1}) + \mathcal{H}(\mathbf{u}^n)}{2} = -\mathcal{G}(p^{n+1/2}) + \frac{\mu}{2} (\mathcal{L}(\mathbf{u}^{n+1}) + \mathcal{L}(\mathbf{u}^n)) + \mathbf{f}^n, \quad (17)$$

and

$$\mathcal{D}(\mathbf{u}^{n+1}) = 0, \quad (18)$$

where the superscript n denotes the n -th time step. Δt is the time step. The nonlinear convective term can be linearized as

$$\mathcal{N}(\mathbf{u}^{n+1}) = \frac{1}{2} [\mathcal{H}(\mathbf{u}^{n+1}) + \mathcal{H}(\mathbf{u}^n)], \quad (19)$$

where $\mathcal{N}$ is a linear convective operator. Specifically, the linearized convective term in axial and radial directions are given as

$$\begin{aligned} \mathcal{N}(u_x^{n+1}) &= \frac{1}{2} \left\{ \frac{\partial(u_x^{n+1}u_x^{n+1})}{\partial x} + \frac{\partial(u_x^{n+1}u_r^{n+1})}{\partial r} + \frac{u_x^{n+1}u_r^{n+1}}{r} \right. \\ &\quad \left. + \frac{\partial(u_x^n u_x^n)}{\partial x} + \frac{\partial(u_x^n u_r^n)}{\partial r} + \frac{u_x^n u_r^n}{r} \right\} \\ &= \frac{1}{2} \left\{ 2 \frac{\partial(u_x^n u_x^{n+1})}{\partial x} + \frac{\partial(u_x^n u_r^{n+1} + u_x^{n+1} u_r^n)}{\partial r} \right. \\ &\quad \left. + \frac{u_x^n u_r^{n+1} + u_x^{n+1} u_r^n}{r} \right\}, \end{aligned} \quad (20)$$

and

$$\begin{aligned} \mathcal{N}(u_r^{n+1}) &= \frac{1}{2} \left\{ \frac{\partial(u_x^{n+1}u_r^{n+1})}{\partial x} + \frac{\partial(u_r^{n+1}u_r^{n+1})}{\partial r} + \frac{u_r^{n+1}u_r^{n+1}}{r} \right. \\ &\quad \left. + \frac{\partial(u_x^n u_r^n)}{\partial x} + \frac{\partial(u_r^n u_r^n)}{\partial r} + \frac{u_r^n u_r^n}{r} \right\} \\ &= \frac{1}{2} \left\{ \frac{\partial(u_x^n u_r^{n+1} + u_x^{n+1} u_r^n)}{\partial x} + 2 \frac{\partial(u_r^n u_r^{n+1})}{\partial r} \right. \\ &\quad \left. + 2 \frac{u_r^n u_r^{n+1}}{r} \right\}, \end{aligned} \quad (21)$$

in which the linearization scheme yields $u_x^{n+1}u_r^{n+1} = u_x^n u_r^n + O(\Delta t^2)$ and $u_x^{n+1}u_r^{n+1} = u_x^n u_r^n + O(\Delta t^2)$.

However, directly solving Eqns. (17) and (18) is computationally expensive due to multiple large sparse matrices involved as well as the coupling between velocity and pressure. For numerical efficiency we employ LU decomposition together with approximate factorization to decouple those two variables (see Kim and Sung²¹ for more details). To summarize, the discretization of these equations is conducted by the following steps:

$$\frac{1}{\Delta t} [\rho \mathbf{I} + \Delta t (\rho \mathcal{N} - \frac{\mu}{2} \mathcal{L})] \mathbf{u}^* = \frac{\rho}{\Delta t} \mathbf{u}^n - \mathcal{G}(p^{n-1/2}) + \frac{\mu}{2} \mathcal{L}(\mathbf{u}^n) + \mathbf{f}^n, \quad (22)$$

$$\Delta t \mathcal{D}(\delta p) = \rho \mathcal{D}(\mathbf{u}^*), \quad (23)$$

$$\mathbf{u}^{n+1} = \mathbf{u}^* - \frac{\Delta t \mathcal{G}(\delta p)}{\rho}, \quad (24)$$

and

$$p^{n+1/2} = p^{n-1/2} + \delta p, \quad (25)$$

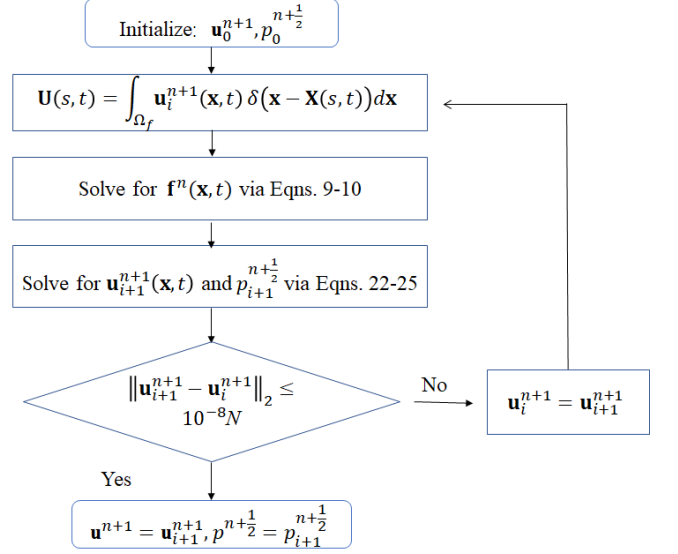


FIG. 2. Feedback scheme to update the velocity and pressure fields. i denotes the iteration index, N is the number of grid cells in the domain.

where $\mathbf{u}^*$ is an intermediate value of $\mathbf{u}^{n+1}$ and $\mathbf{I}$ is the identity matrix. With the velocity and pressure distribution at time step n determined, $\mathbf{u}^{n+1}$ and $p^{n+1/2}$ can be updated by the feedback scheme as shown in Fig. 2.

The numerical setup together with the boundary conditions are displayed in Fig. 1b. The computational domain is discretized with staggered grids in both longitudinal and transverse directions. The finest grid $\Delta x = \Delta r$ is used in the near-body region, then the mesh size gradually increases towards the far field. Uniform distributed Lagrangian grid ($\Delta s = 0.0167D$) is applied to discretize the body shell.

The effectiveness of this axisymmetric fluid solver has been verified through the simulation of flow around a fixed sphere under inflow-based Reynolds number less than 210 (beyond which the flow wake is not axisymmetric). Our numerical predictions match well with previous experiments, as shown in Appendix A.

IV. RESULTS

We start by employing an impulsive jet velocity profile, in which the time history of the spatially averaged jet speed at the nozzle is

$$V_j = \tilde{V}_j \left\{ 0.5 \tanh \left[\frac{t - 5T_0}{0.1(T - 5T_0)} \right] + 0.5 \right\}, t \in [0, T] \quad (26)$$

where $T_0 = D/\tilde{V}_j$ denotes the reference time by using $\tilde{V}_j$ and D as the reference velocity and length, respectively. The duration T of the whole procedure is determined by the equation

$$\int_0^T V_j dt = \Gamma_m D. \quad (27)$$

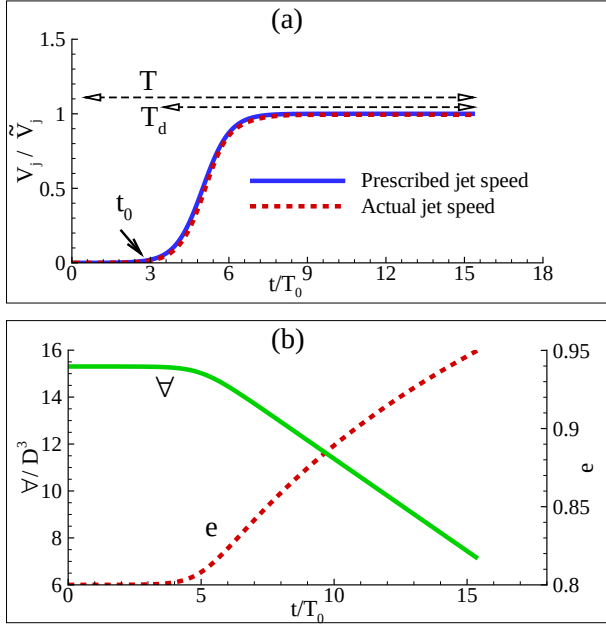


FIG. 3. (a) Comparison of the prescribed jet speed and actual speed. (b) Time histories of eccentricity e and internal volume $\tilde{V}$ during the jetting, $\Gamma_m = 10.4$, $\Delta x = \Delta r = 0.02D$ and $\Delta t = 5 \times 10^{-4}T_0$.

Figure 3a displays the time history of the prescribed jet velocity specified in Eq. (26) when $\Gamma_m = 10.4$. As shown in this figure, the system undergoes three distinctive phases during the simulation, starting (quasi-resting) phase, acceleration phase, and steady jetting phase. In the quasi-resting phase the jet speed is negligibly small, which is necessary to mitigate the initial effect. Strictly speaking, only the acceleration and steady jetting phases are considered as the jetting process. Thus we define the time instant t_0 when $V_j = 0.02\tilde{V}_j$ as the starting point of the pulsed jet so that the duration of the jetting process T_d is calculated as $T - t_0$. Figure 3b presents the time evolution of the body eccentricity e and the internal volume $\tilde{V}$ for the same case.

Indeed, the actual jet velocity (average axial velocity at the nozzle plane) might be slightly different from the prescribed one due to the finite accuracy of achieving the prescribed body shape and calculating the internal flow field numerically. We calculate the actual jet velocity by using $4 \int_A u_x dS / (\pi D^2)$ (A stands for the nozzle plane and u_x is the longitudinal component of the jet speed) and compare with the desired velocity described in Eq. (26). As presented in Fig. 3a, the good agreement of prescribed and calculated jet speed profiles verifies the robustness of our numerical feedback algorithm in imposing the moving boundary conditions.

Based on a sensitivity test (Appendix B), simulations hereafter will be conducted using $\Delta x = \Delta r = 0.02D$ and $\Delta t = 5 \times 10^{-4}T_0$.

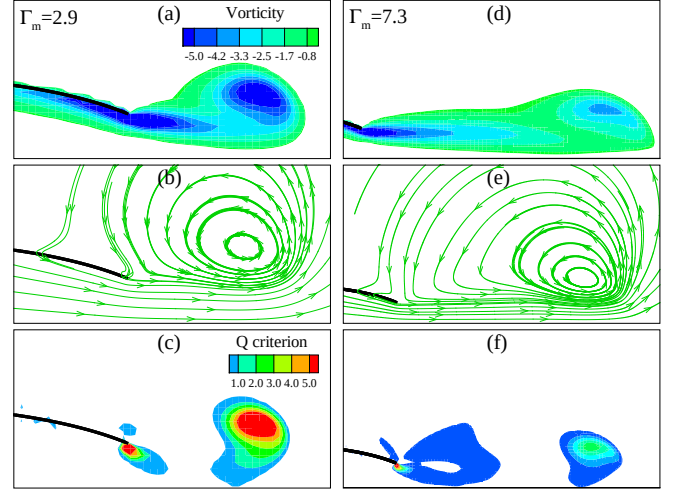


FIG. 4. Vorticity visualization (a,d), streamline (b,e) and Q criterion distribution (c,f) of the wake at $\Gamma_m = 2.9$ and 7.3 . The vorticity is normalized by $\tilde{V}_j/D$. These snapshots are captured at the end of the deflation phase.

A. Wake evolution and vortex ring formation

As discovered in previous studies¹⁴, the formation number Γ is closely related to the evolution of the wake created by the jet. Specifically, before Γ reaches a threshold value (the critical formation number; the value was found to be around 4¹⁴), the leading vortex ring is continuously increased in circulation. Beyond the critical value, the growth of the leading vortex ring will stop since it pinches off from the rest of the wake so that vorticity shed from the inner boundary layer of the chamber is no longer entrained into it. The exact value of the critical Formation number may vary depending on different jet velocity profiles²² and velocity distribution at the nozzle²³. It is important to note that the jet-based Reynolds number Re_j in most studies is above 2000. In lower Reynolds number environment, Palacios-Morales and Zenit²⁴ found that the leading vortex hardly pinched off from its trailing flow and the leading ring circulation stopped growing when the formation number is $4 \sim 6$. This study concentrates on constant jet speed, while the effect of jet profiles under these conditions was not explored.

To identify vorticity structures we visualize the wake through vorticity contour, stream function, and the Q criterion²⁴. In the Q criterion method the value of Q (second invariant of the velocity gradient tensor $\nabla \mathbf{u}$) represents the difference in magnitude between the rotation rate and the strain rate so that a region with positive values of Q is recognized as a vortex^{25,26}. The circulation of the vortex ring is obtained by integrating the vorticity over the closed region of the vortex identified with the Q criterion. The circulation C is computed using

$$C = \int_{Q>0} \omega dS, \quad (28)$$

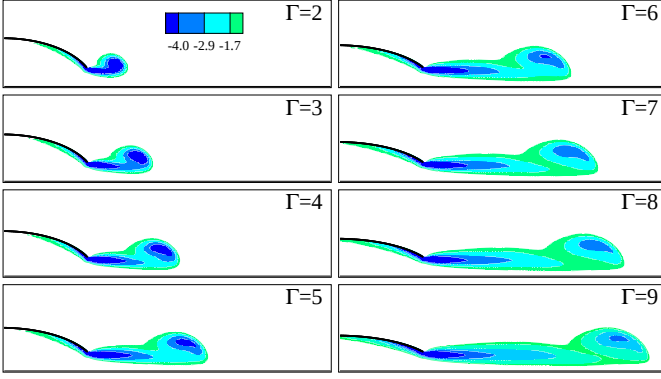


FIG. 5. Wake evolution at $\Gamma_m = 10.4$ visualized *via* vorticity.

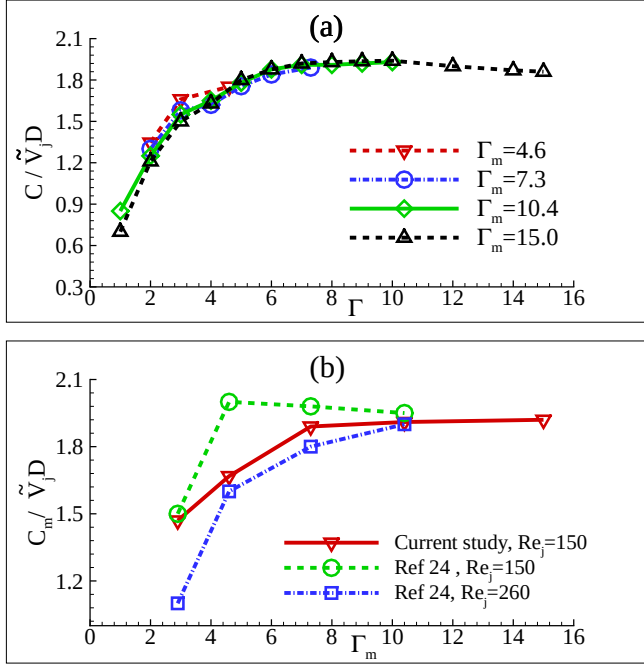


FIG. 6. (a) Vortex ring circulation C as a function of Γ , (b) peak value of vortex ring circulation C_m as a function of Γ_m . Results from a cylindrical piston experiment by Palacios-Morales and Zenit²⁴ are presented for comparison.

where w is the azimuthal vorticity

$$w = \frac{\partial u_x}{\partial r} - \frac{\partial u_r}{\partial x}. \quad (29)$$

Using the aforementioned impulsive jet velocity profile, we investigate the vortex ring generation process during the deflation deformation. Figures 4a and d present the flow field at the end of the deflation for the cases with $\Gamma_m = 2.9$ and $\Gamma_m = 7.3$, respectively. The corresponding flow fields visualized *via* streamlines (Figs. 4b and e) and Q criterion (Figs. 4c and f) are also displayed. These figures show that in the case of $\Gamma_m = 2.9$ most of the ejected fluid is entrained into a single vortex ring. When Γ_m is 7.3, on the other hand, the wake consists of a leading vortex followed by a trailing flow. The

behavior here is somewhat different from previous observations that show additional vortices being formed in the trailing flow¹⁴. The difference is mostly attributed to the Reynolds number effect. Indeed, these results are consistent with experimental data at similar range of Reynolds numbers²⁴.

For further insight, Fig. 5 displays snapshots of the flow field at various time instants throughout the jetting process for the case of $\Gamma_m = 10.4$. It shows that the jet flow continuously feeds the leading vortex ring until around $\Gamma = 4$, when the leading vortex ring detaches from the trailing flow. As mentioned earlier, in this low Reynolds number regime the trailing flow is not sufficiently strong to induce a secondary vortex ring. Instead, it remains connected with the nozzle as well as the leading vortex ring. The consequence is that after $\Gamma = 4$ the trailing flow is still able to feed the leading vortex ring. Although complete disconnection between these two flow structures (the leading vortex ring and its trailing flow) may never occur²⁷, a clearer separation is observed at $\Gamma = 6 \sim 7$. After that, the strength of the leading ring stops growing and decays slowly due to viscous dissipation. This is clearly shown in Fig. 6a, where the circulation of the leading vortex ring C is plotted as a function of Γ .

As displayed in Fig. 6a, the normalized circulation of the vortex ring reaches a peak value around 2.0 when $\Gamma = 6 \sim 7$. This implies that after the apparent separation of the vortex ring from the trailing flow, it is still fed by the jet. The supply rate, however, decreases gradually so that the growth rate of the circulation curve declines prior to the plateau. At $\Gamma = 6 \sim 7$ the supply rate is balanced by the rate of dissipation and the leading ring stops growing. This coincides with a clearer spatial isolation of the leading vortex ring (see Fig. 5).

Figure 6b displays the peak circulation of leading vortex ring C_m as a function of Γ_m . For comparison, results from a cylindrical piston experiment are plotted in the same figure as well. Even though the maximum value of C_m we obtain is in good agreement with the reported value, there is still certain discrepancy in terms of the critical formation number (stroke ratio). This is likely to be caused by differences in details of the jet profile. First, a constant piston velocity profile without an acceleration phase was applied in the experiment, while our system undergoes an accelerating stage that accounts for about 25% of the whole deformation time. The viscous dissipation is more significant in our model so that higher formation number is needed to attain the saturated state. Second, the difference in nozzle geometry (conical versus tubular) may also cause some difference in vortex ring formation (see Appendix C).

B. Thrust generation

Figure 7a presents the instantaneous thrust coefficient C_T at different values of Γ_m . In the same figure, we also plot the thrust coefficient C_{Tj} attributed to the momentum flux out of the nozzle. Here C_{Tj} is calculated as

$$C_{Tj} = F_j / (\rho \tilde{V}_j^2 D^2), \quad (30)$$

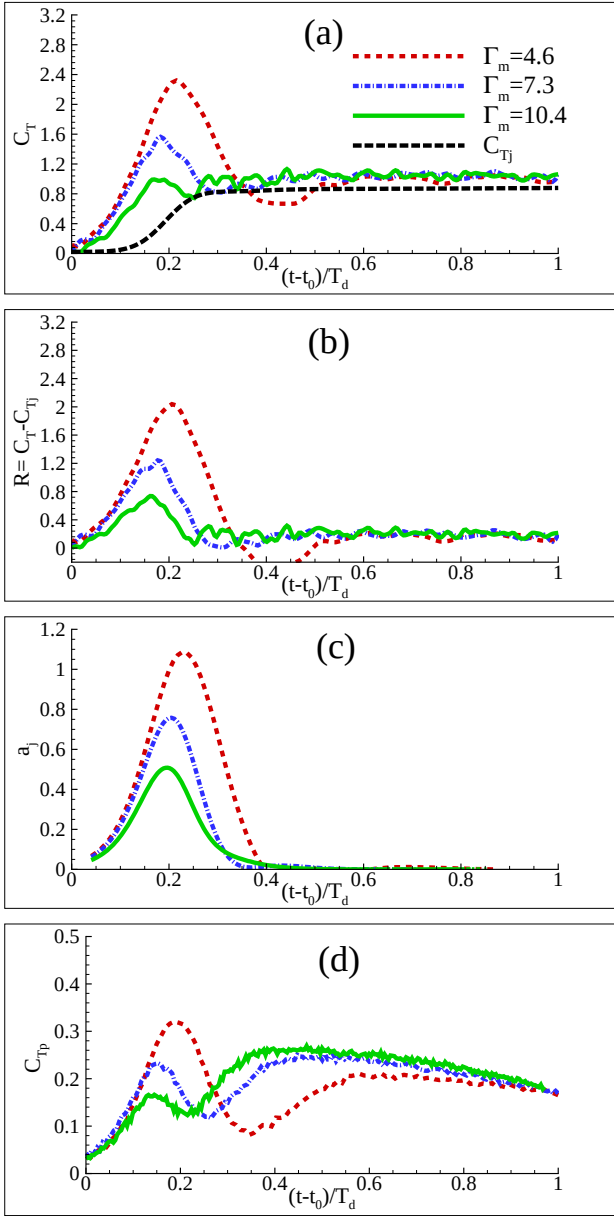


FIG. 7. Time histories of (a) C_T and C_{Tj} , (b) $C_T - C_{Tj}$, (c) jet acceleration a_j , and (d) thrust due to excessive pressure at exit plane C_{Tp} .

where

$$F_j = \int_A \rho u_x^2 dS. \quad (31)$$

According to Fig. 7a, after the jet speed V_j reaches its steady value the main source of the thrust C_T is the momentum flux, i.e. $C_T \sim C_{Tj}$. However, in the initial stage the overall thrust generation is much larger than the contribution from the momentum flux (which is small during the acceleration procedure), as clearly shown in Fig. 7a. It suggests that there exists other thrust sources that dominate the thrust generation during the unsteady jet process. The unknown contribution $R = C_T - C_{Tj}$, presented in Fig. 7b, turns out to be roughly

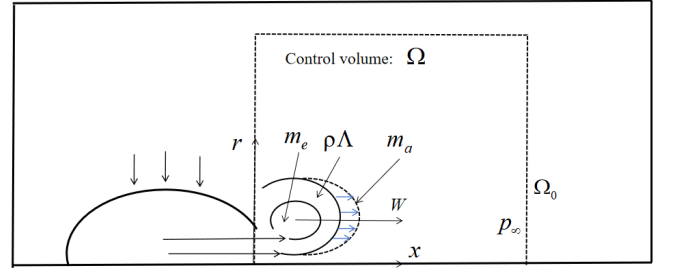


FIG. 8. Illustration of the axisymmetric control volume and the vortex ring structure.

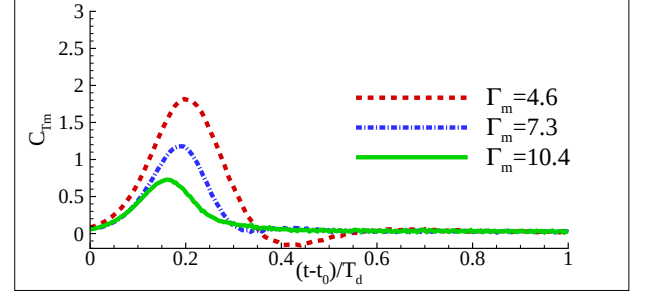


FIG. 9. Time histories of C_{Tm} .

proportional to the normalized jet acceleration $a_j = \frac{D}{\tilde{V}_j^2} \frac{dV_j}{dt}$ (see Fig 7c).

Through control volume analysis, it has been suggested that the overall impulse injected into the wake by a pulsed jet depends not only on the flux term but also on the excessive pressure effect at the nozzle exit plane²⁸. Following this method, we create an axisymmetric control volume (Ω) surrounding the entire jet flow as demonstrated in Fig. 8. Ω_0 stands for the boundary of the control volume. Applying conservation of momentum yields

$$\rho \frac{\partial}{\partial t} \int_{\Omega} \mathbf{u} dV + \rho \int_{\Omega_0} \mathbf{u}(\mathbf{u} \cdot \mathbf{n}) dS + \int_{\Omega_0} p \mathbf{n} dS = 0, \quad (32)$$

where $\mathbf{n}$ is the unit normal vector pointing outwards. Since the flow flux and hydrodynamic pressure on boundaries $\Omega_0 - A$ is negligible compared with those on the nozzle exit A , Eq. (32) can be simplified in the horizontal direction as

$$\rho \frac{\partial}{\partial t} \int_{\Omega} u_x dV = \rho \int_A u_x^2 dS + \int_A \Delta p dS = F_j + F_p, \quad (33)$$

where

$$F_p = \int_A \Delta p dS, \quad (34)$$

$\Delta p = p - p_{\infty}$ is the excessive pressure at the nozzle exit. Equation (33) indicates that the rate of momentum change in the control volume is approximately a combination of F_j and F_p (whose normalized form is $C_{Tp} = F_p / (\rho \tilde{V}_j^2 D^2)$). Considering the wake structure itself, during the formation of the leading vortex ring the overall horizontal momentum of the wake

M_w is dominated by the momentum of a solitary vortex ring, which is simplified as²⁸

$$M_w \approx (\rho\Lambda + m_e + m_a)W, \quad (35)$$

where Λ is the volume of ejected fluid that is entrained into the vortex ring. m_e and m_a are the ambient fluid entrained into the vortex ring and the added mass surrounding the ring bubble, respectively. W represents the propagating velocity of the vortex ring core, which is proportional to the jet velocity V_j ²⁹. The rate of momentum change is then written as

$$\begin{aligned} \frac{dM_w}{dt} &\approx \rho \frac{d\Lambda}{dt} W + \frac{d(m_e + m_a)}{dt} W + (\rho\Lambda + m_e + m_a) \frac{dW}{dt} \\ &\approx F_j + F_p. \end{aligned} \quad (36)$$

Given that $\frac{d\Lambda}{dt} = AV_j$, we have

$$\begin{aligned} F_j + F_p &\approx \rho AV_j W + \frac{d(m_e + m_a)}{dt} W \\ &\quad + (\rho\Lambda + m_e + m_a) \frac{dW}{dt}. \end{aligned} \quad (37)$$

Note that F_j originates from the momentum flux (the first term in the RHS of Eq. (37)), so that the pressure effect F_p roughly accounts for the accumulation rate of $m_e + m_a$ (the second term) and the acceleration of the fluid mass associated with the vortex ring (the third term).

Time histories of C_{Tp} are presented in Fig. 7d, showing that the pressure contribution is highly associated with the jet acceleration a_j . However, quantitatively C_{Tp} is much smaller than R . This is caused by the fact that this control volume analysis does not account for the fluid motion inside the pressure chamber, which also contributes to fluid dynamic force on the solid body when the internal fluid is accelerated. In fact, the force we obtain here ($F_j + F_p$) is applied on a system including the solid body and the fluid inside it. Based on conservation of momentum the force acting on the internal fluid equals its rate of momentum change dM/dt , where M is the momentum of the internal fluid in the horizontal direction. Without considering the added mass effect the force on the solid body is $F_j + F_p + dM/dt$. For convenience, dM/dt is labelled as F_m , whose non-dimensional form is $C_{Tm} = F_p/(\rho\tilde{V}_j^2 D^2)$. Figure 9 presents the time histories of C_{Tm} in various conditions. Not surprisingly, C_{Tm} roughly bridges the gap between C_T and $C_{Tj} + C_{Tp}$. Indeed, we find that C_{Tm} dominates the thrust generation during the unsteady jetting while C_{Tj} accounts for the majority of thrust in steady jetting. The effect of pressure contribution C_{Tp} is relatively small in current conditions. Incidentally, it is observed that the internal momentum increment is mainly attributed to the flow acceleration even though the internal volume is shrinking. As a result, C_{Tm} is more or less proportional to jet acceleration a_j , as shown in Figs. 9 and 7(c).

Figure 10a presents the power expenditure in the corresponding cases. It is found that the generation of steady jet that induces constant thrust requires constant energy cost. However, additional power is needed in unsteady jet to accelerate the internal fluid and the ambient fluid from rest. It

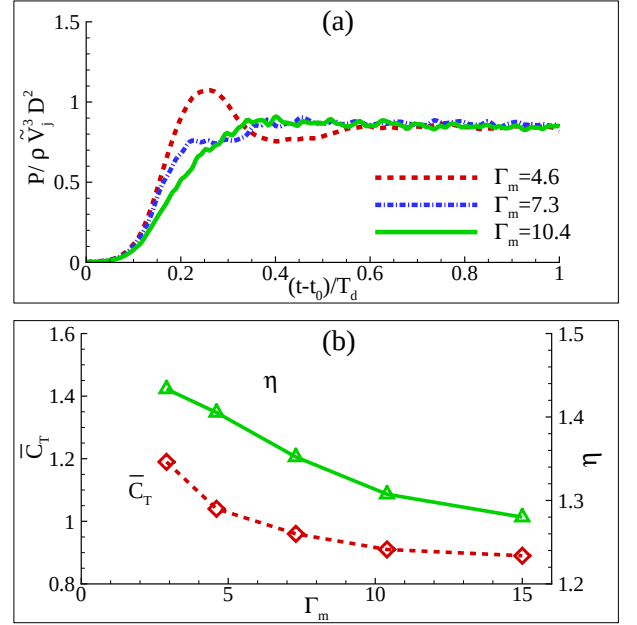


FIG. 10. (a) Time histories of power expenditure.(b) Mean thrust coefficient $\bar{C}_T$ and efficiency η as functions of Γ_m .

is thus reasonable that power expenditure displays the same trend as thrust generation. The dependencies of average thrust coefficient $\bar{C}_T$ and efficiency of impulse generation η on Γ_m are shown in Fig. 10b. Lower values of Γ_m tend to yield higher thrust coefficients and higher efficiencies of impulse generation. For higher values of Γ_m the effect of acceleration is smaller (see Fig. 7) so that the curves of $\bar{C}_T$ and η tend to flatten in the high Γ_m region.

C. Effect of jet velocity profile

Our simulations demonstrate that jet acceleration, which is closely associated with the jet velocity profile, plays a significant role in the wake evolution and the dynamic performance of the system. To further explore this effect we consider two additional jet velocity profiles with time histories of the jet speed specified as

$$V_j = 0.5\tilde{V}_j [1 - \cos(\pi t/T_d)], \quad (38)$$

$$V_j = 0.5\tilde{V}_j [1 - \cos(2\pi t/T_d)]. \quad (39)$$

Hereafter these are referred to as the half-cosine and cosine profiles, respectively (see Fig. 11).

Figure 12 presents time histories of the leading vortex ring circulation. For the half-cosine velocity distribution, in which the jet is accelerated throughout the entire deflation process, the circulation of vortex ring keeps growing till the end of jetting because of the continuous vorticity feeding from the jet. Without exception, lower Γ_m yields larger circulation for a fixed Γ since the same amount of fluid mass is ejected

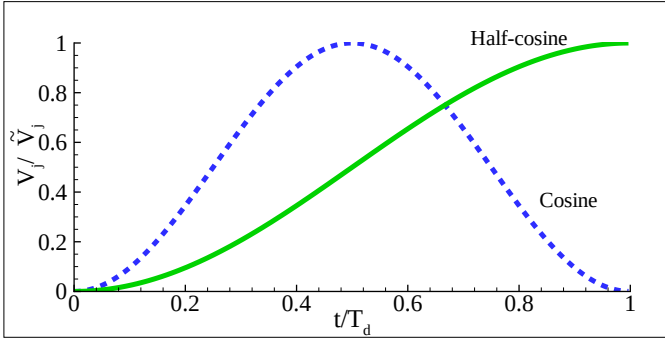


FIG. 11. Jet velocity profiles for two additional acceleration conditions.

with higher speed. For the same reason, the maximum values of circulation in these cases are lower than that with the impulse velocity profile. With the cosine profile, the circulation reaches a peak after an initial increase. The peak occurs approximately at $\Gamma = 0.75 \sim 0.85\Gamma_m$, when the jet speed V_j slows down to $0.3 \sim 0.5\tilde{V}_j$. Afterwards the vorticity supply from the jet is unable to compensate for the viscous dissipation due to the decreasing jet speed and the increasing distance between the ring and the nozzle, so that the circulation of the leading vortex ring is gradually reduced.

An important factor that determines the value of C at any formation number Γ is the time history of the jet speed before that instant. The differences among the impulsive jet profiles with different values of Γ_m are much smaller than those among the cosine (or half-cosine) jet profiles since a significant portion of an impulsive profile is constant. This explains why the C versus Γ curves in the impulsive case are closer to each other than those in the cosine and half-cosine cases.

The thrust coefficient C_T of these two velocity profiles are displayed in Fig. 13. In the half-cosine profile accelerating jet speed leads to increasing C_T (Fig. 13a). In the cosine profile positive thrust is created during the accelerating period, whereas during the deceleration period the thrust generation could be negative (Fig. 13b). This is again attributed to the effect of jet acceleration. The thrust generation becomes negative once the acceleration effect (which is negative) overwhelms the effect of the momentum flux of the jet (which remains positive throughout the jetting procedure).

V. CONCLUSIONS

By using an axisymmetric immersed-boundary algorithm, we numerically investigated the pulsed jetting process of an ellipsoidal swimmer through body deflation achieved by prescribing the body eccentricity. A feedback scheme is employed to calculate the interaction between the body and its fluidic environment. Systematic simulations have been performed to study the vortex ring formation and thrust generation of low Reynolds number pulsed jetting with three jet speed profiles, impulsive, half-cosine, and cosine.

Our results show that during the impulsive jetting, the vor-

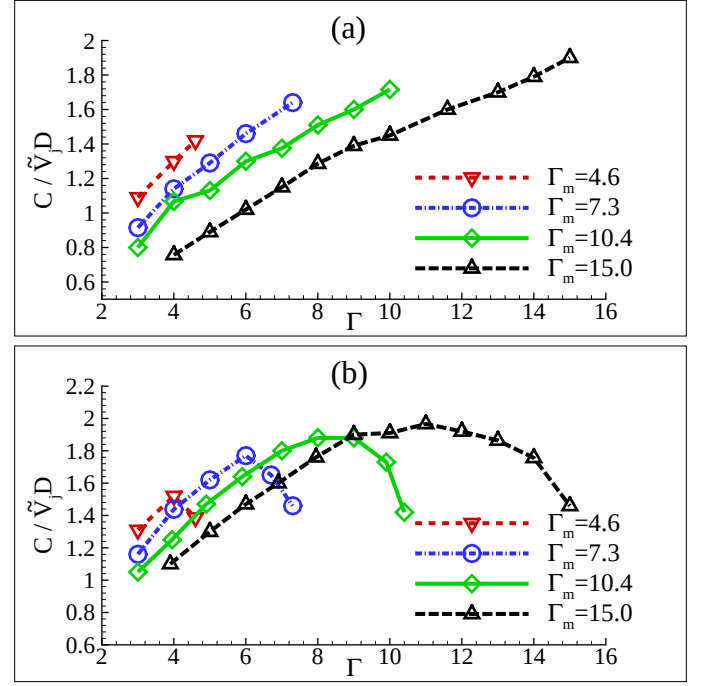


FIG. 12. Vortex ring circulation C as functions of Γ , (a) half-cosin profile; (b) cosine profile.

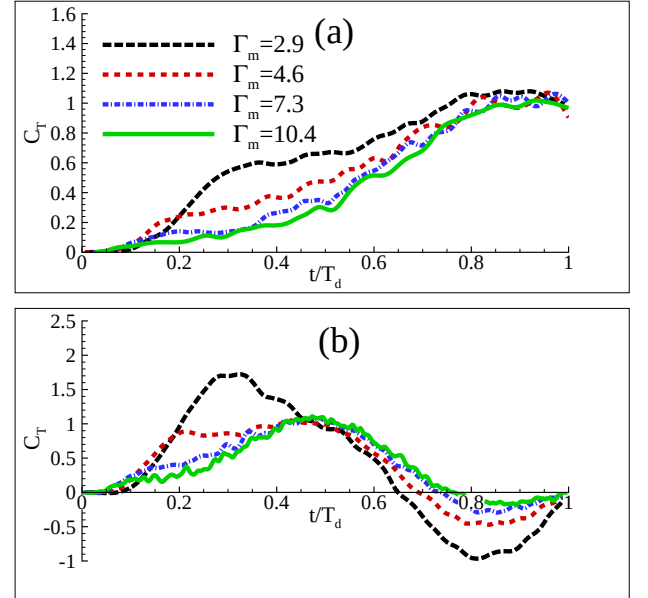


FIG. 13. Time histories of thrust coefficient C_T for half-cosine profile (a) and cosine profile (b).

tex ring attains its saturated condition (maximum circulation) at formation number $\Gamma = 6 \sim 7$, after which the circulation slowly drops as the ring propagates downstream due to viscous dissipation. However, the vortex ring does not pinch off completely from its trailing jet throughout the jetting process, neither are there any secondary vortex rings formed in the trailing flow. It suggests that the pulsed jetting in low

Reynolds number is a smooth vortex ring generation process, which is different from cases in high Reynolds numbers. Besides, our results indicate that factors such as jet acceleration and viscous dissipation will have great impact on the vortex ring evolution. For example, during the half-cosine profile jetting the vortex ring keeps on growing as jet speed increases so that no saturated state is observed. For the cosine profile where the jet speed undergoes an initial rise then decreases to zero at the end of deflation, the circulation of the vortex ring undergoes the same trend because viscous dissipation will be significant when jet speed is sufficiently small.

Based on control volume analysis, we identify at least three distinctive sources that contribute to thrust generation on the solid body, nozzle momentum flux depending on jet speed, jet acceleration-related excessive pressure, and acceleration of the internal fluid in the pressure chamber. Different jet velocity profiles lead to different thrust generation patterns. The tremendous jet acceleration in the initial stage of the impulsive jetting creates large excessive pressure and internal flow acceleration during the accelerating phase, while most of the thrust is originated from the jet flux during the steady jetting stage in which jet acceleration is almost zero. In the cosine profile, it is observed that the deceleration of jet might create negative thrust due to the negative contribution of pressure and internal flow acceleration.

Appendix A VALIDATION

To validate the accuracy of the axisymmetric fluid solver and the feedback algorithm, they are used to simulate the canonical problem of a uniform flow around a stationary sphere. The numerical results are then compared with benchmark results from previous studies. There are different types of wake patterns behind the sphere depending on the Reynolds number Re (based on upcoming flow velocity u_0 and sphere diameter d). The flow remains unseparated when Re is smaller than 25. As Re increases the flow starts to separate from the surface of the sphere, forming an axisymmetric wake with a closed recirculating vortex ring behind. The axisymmetry and stability of the flow will then be disturbed as Re rises beyond 210. We perform simulations within the range of Reynolds number between 25 and 210, within which the axisymmetric flow pattern dominates. Simulations are performed using uniform grid $\Delta x = \Delta r = \Delta s = 0.02d$ and time step $\Delta t = 0.001d/u_0$. The feedback parameters α and β are chosen to be $-400\frac{\rho u_0^2}{d}$ and $-20\rho u_0$, respectively.

Figure 14 shows comparisons between the present results and the data from previous studies^{30–34} in terms of the drag coefficient C_d , the separation angle θ , and the length of the standing eddy L_e . The drag coefficient is obtained by $C_d = F_d / (0.5\rho u_0^2 A_d)$, where F_d is the drag force on the body, and $A_d = \pi d^2/4$ is the projected area. The definitions of θ and L_e are provided in the insets in Figs. 14c and d. It is clearly seen that the results of the current model are in good agreement with the previous data.

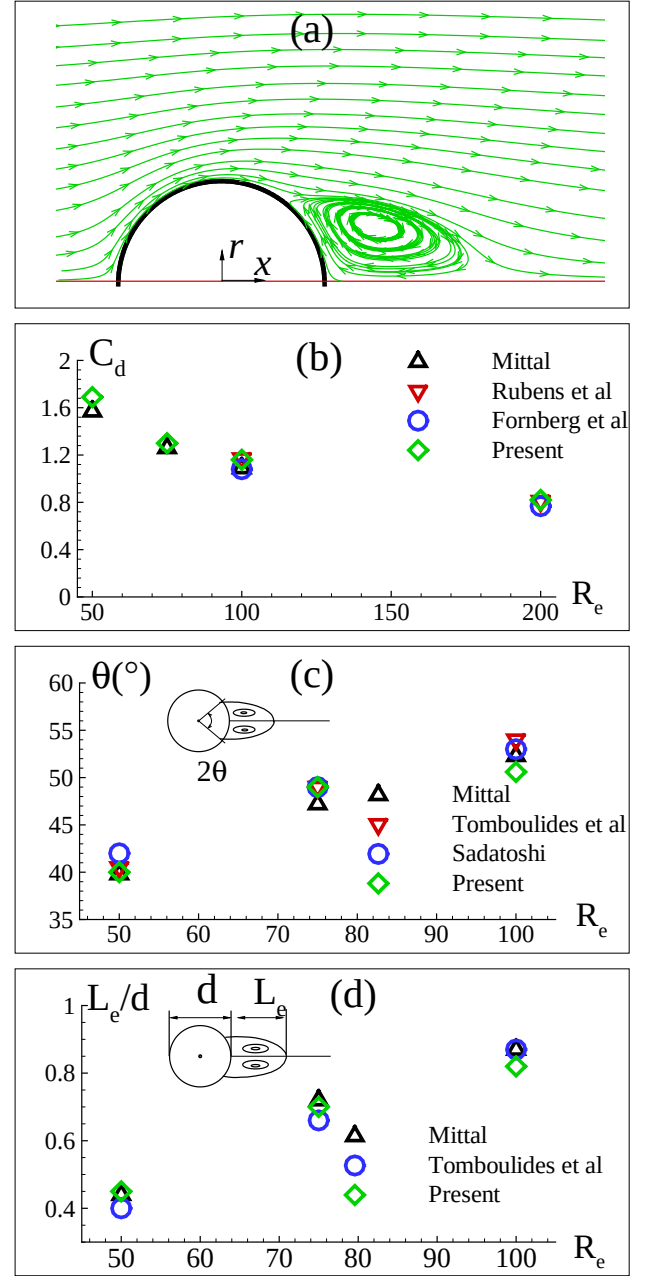


FIG. 14. Numerical simulations of flow past a sphere with diameter d and comparisons with previous results: (a) flow pattern (visualized via streamlines) at $Re = 100$, (b) the drag coefficient, (c) the separation angle, and (d) length of the standing eddy.

Appendix B GRID SENSITIVITY

To find proper numerical parameters, a grid independence study has been performed as shown in Fig. 15a, where Δx is the size of the fluidic grid in the neighborhood of the body. The effect of time step Δt is also studied (see Fig. 15b). These tests show that the results are not sensitive to the grid size and the time step if they are sufficiently small.

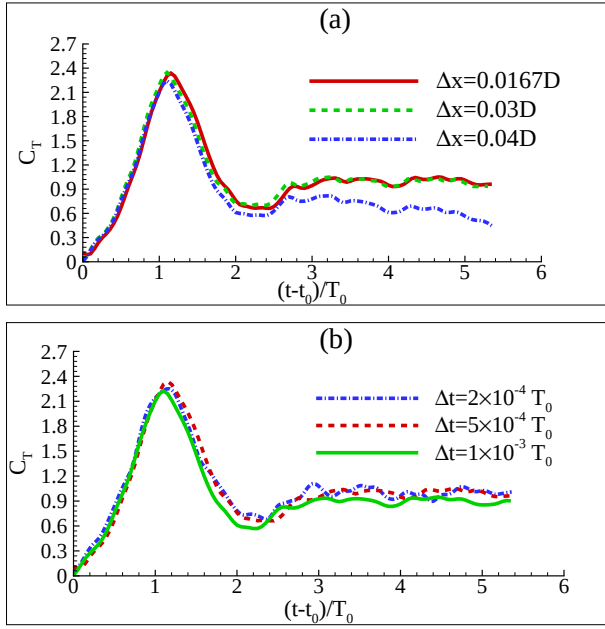


FIG. 15. (a) Sensitivity tests for fluidic grid size when $\Delta r = \Delta x$ and $\Delta t = 5 \times 10^{-4} T_0$, (b) sensitivity tests for time step when $\Delta x = \Delta r = 0.02D$. The impulsive jet velocity profile (Eq. (26)) is used with $\Gamma_m = 4.6$.

Appendix C AXIAL AND RADIAL VELOCITIES AT THE NOZZLE

It is important to note that our model features a conical nozzle (rather than the tubular nozzle used in some previous studies), through which the flow is ejected with converging streamlines. For this reason, there is significant radial component in the jet velocity as shown in Fig. 16. Not surprisingly, the radial velocity tends to be reduced as Γ increases because the conical nozzle is getting flattened as the body shrinks. According to Kreig and Mohseni³⁵, the radial velocity may play a role in the dynamics of this system. It may contribute to discrepancies between our results and previous studies with tubular nozzles.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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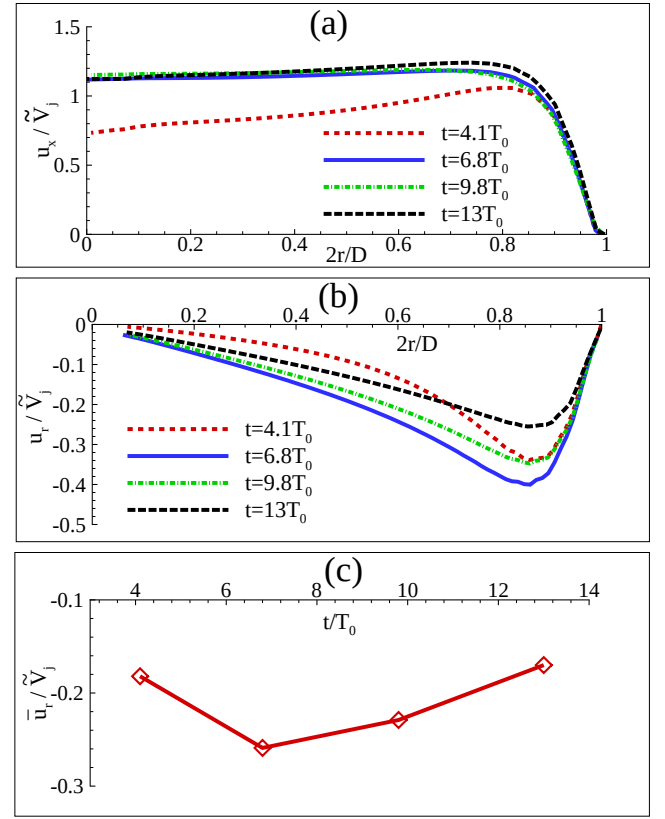


FIG. 16. (a) Axial velocity and (b) radial velocity distributions at the exit plane. (c) The averaged radial velocity along the r axis $\bar{u}_r$ at the exit plane in various time instants. The impulsive jet velocity profile (Eq. (26)) is used with $\Gamma_m = 10.4$.

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