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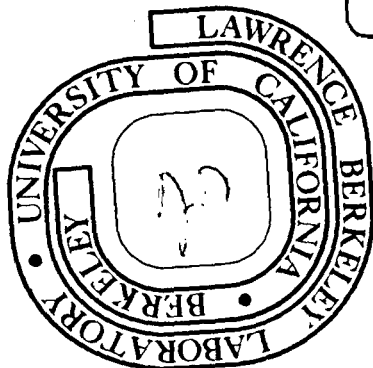TOROIDAL MAGNETIC-CUSP CONFINEMENT (TORMAC)

WITH INTERNAL CIRCULATION[*]

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ABSTRACT

Plasma confined in the Tormac configuration can be made to rotate about the minor axis so that guiding-center drifts do not carry the particles to the boundaries even in a purely toroidal field $B(R)$. It is shown that in this case the density $n(R)$ must decrease along the major radius R such that $Rn(R)/B(R) \approx$ const. When the motion is adiabatic and when centrifual effects can be neglected the solution $n(R)$ is found to be marginally MHD-stable for all values of β.

INTRODUCTION

Levine and coworkers have described the confinement of high-beta plasma in the Tormac device.[1-3] In this configuration the bulk of the plasma is contained within a toroidal volume of closed magnetic flux tubes (see Fig. 1). A thin boundary layer separates this space from an outer region, the so-called outer "sheath", which is characterized by strong poloidal field components produced by external coils in an annular cusp arrangement. Particles with guiding centers in the outer sheath are only mirror confined on the open field lines. The boundary layer can not measure more than a mean ion-cyclotron-orbit in thickness, because it is the region in which many ions with guiding centers on both open and closed flux surfaces coexist. While the structure and detailed properties of such a boundary layer and outer sheath have not yet been fully analyzed, it is certain that the major pressure gradients have to be concentrated there. The very large mass flow density resulting from the gyrating motion of the ions in this region may, at least in part, be suppressed by the automatic appearance of strong ambipolar electric fields. In that case all orbits are highly distorted in the layer and, in fact, a substantial portion of the confining current will be accounted for by E x B drift of the electrons in the boundary. In the following we shall assume that such thin layers exist and are stable.

GUIDING CENTER CIRCULATION

In the basic Tormac concept the internal field lines are concentric circles, i.e., no toroidal current is present so that there

is no magnetic rotational transform. It follows that, unless internal
electric fields modify the motion, guiding center drifts parallel to
the torus' major axis carry all particles to the plasma edge where
they have to be deflected and recirculated promptly along the boundary
if toroidal equilibrium is to be maintained. Obviously, this require-
ment must be incorporated in the prescription for the cusp-confinement
of plasma in Tormac. Unfortunately, the losses through the cusps pro-
duce an anisotropy in the velocity distribution of the ions in part
of the boundary layer which then is communicated to the interior
regions by the particle circulation mentioned above. This leads to
the unavoidable conclusion that either a magnetic rotational transform
or a net mass rotation is required in Tormac if the confinement is to
be better than that expected in a minimum-B stabilized mirror configu-
ration. The function of the rotation here is to force most particle
guiding centers to circulate inside the boundary. If such rotation
can be maintained, interior particles with velocity vectors in the cusp
loss-cones can reach the open field lines only by cross-field diffu-
sion, so that the expected confinement times are much improved over
that of ordinary open-ended magnetic configurations.[2]

Some internal circulation around the minor torus axis by $E \times B$
drifts may be expected to arise spontaneously because of initial pre-
ferential loss of particles of one sign or the other. Excess positive
charge in the interior of a Tormac plasma would give rise to electric
fields along the minor radius which here, instead of forcing ions out
along field lines, would enhance confinement by causing internal cir-
culation of guiding centers. If such spontaneous motion is insuffi-

cient for our purpose, rotation about the minor axis can

also be driven by application of a torque from the outside. For

instance, neutral-beam injection used to feed or heat a Tormac-con-

fined plasma may also be adding angular momentum either by accident

or by design. In short, we conclude that the plasma in Tormac is

likely to be in a state of rotation, and the description of the equi-

librium must take this mass motion into account, i.e., we are dealing

with a dynamic rather than a static equilibrium.

CONDITION FOR THE STATIONARY STATE

In the basic Tormac the internal plasma is assumed to be uniform

in temperature and density so that the internal magnetic field would

have to be force-free. The simplest case is the one with $\nabla \times \vec{B} = 0$ so

that the field lines are circles, as mentioned above, and $BR = const$

if R denotes the distance from the major axis. This would be the state

of magnetostatic equilibrium with minimal free energy. However, such

an ideal uniform-pressure condition cannot be maintained in the pres-

ence of even the slightest guiding-center rotation around the minor axis

because it would lead to a finite divergence in the mass flow. The con-

dition that must be satisfied in the presence of any rotation is readily

derived using a purely macroscopic two-fluid description.

In the stationary state we must have $\nabla \cdot n\vec{u} = 0$ for both ions and

electrons, as well as $\nabla \times \vec{E} = 0$. In this paper we restrict ourselves

to the simple case without toroidal current, i.e., $\vec{B} = B(R,z)\hat{\theta}$, so that

the parallel flow $nu_{\parallel}$ does not enter the problem. The mean perpendicu-

lar flow velocities $\vec{u}_{\perp}$ in the steady state can be analyzed in terms of

components due to pressure gradients, inertial effects, cross-field

diffusion caused by friction between species or by anomalous dissipa-
tion, and electric fields.[4] Under conditions of interest only the last
of these, $\vec{u}_E \equiv c\vec{E} \times \vec{B}/B^2$, needs to be considered. Pressure gradients
cause no divergence of the flow in hydromagnetic equilibrium, while
diffusive transport may be assumed small, and the inertial drift is
negligible compared to u_E when the rotation is slow compared to gyro-
frequencies. Thus the divergence condition becomes $(\nabla \ln n) \cdot \vec{E} \times \vec{B}/B^2 = \vec{E} \cdot \nabla \times (\vec{B}/B^2)$, which is satisfied for all $\vec{E}$ if

$$\nabla \times \vec{B} = \vec{B} \times \nabla \ln (n/B^2) \qquad (1)$$

If $\vec{B}$ is purely toroidal, Eq. (1) can be integrated at once, with
the result

$$nR/n_0 R_0 = B/B_0 \qquad (2)$$

where $n_0 = n(R_0, z_0)$ and $B_0 = B(R_0, z_0)$ at an arbitrary point (R_0, z_0)
inside the plasma. This relation merely states formally the obvious
fact that the number of particles in a flux tube must remain constant
during the motion under our conditions when cross-field diffusion is
negligible. This means, of course, that $n(R,z) = $ const is not an
acceptable solution if $\vec{E} \times \vec{B} \neq 0$ anywhere in this torus. To find the
specific required dependence of n (or B) on R and z we must use the
equations for the dynamic equilibrium to obtain a second and independ-
ent relation between these variables.

DYNAMIC EQUILIBRIUM

Condition (2) must hold for both ions and electrons. It is there-
fore now convenient to sum over species and to describe the equilibrium

for the plasma as a single fluid. Since we are anticipating effective isolation from the open field lines, use of a scalar pressure $p = p_i + p_e$ should be adequate. The dynamic equilibrium then requires that $(n_i m_i + n_e m_e)(\vec{u} \cdot \nabla)\vec{u} + \nabla p = (\nabla \times \vec{B}) \times \vec{B}/4\pi$. Neglecting $n_e m_e$ on the left and using the adiabatic approximation $p = p_0(n/n_0)^\gamma$ with $n = n_i$ to substitute for p, and Eqs. (1) and (2) to eliminate $\vec{B}$, this requirement can be written in the form

$$n_0 m_i (\vec{u} \cdot \nabla)\vec{u} + \frac{B_0^2}{8\pi} \nabla \left(\frac{\gamma \beta_0}{\gamma - 1} \frac{n^{\gamma-1}}{n_0^{\gamma-1}} + \frac{2nR^2}{n_0 R_0^2} \right) = 0 \qquad (3)$$

where we have introduced $\beta_0 \equiv 8\pi p_0 / B_0^2$ for convenience. Equation (3) determines $n(R,z)$ in dynamic equilibrium when $\vec{u}(R,z)$ is given.

Under conditions of slow (very subsonic) rotation, such that $nm_i \left[(\vec{u} \cdot \nabla)\vec{u} \right]_R \ll \partial p / \partial R$, the R-component of (3) can be integrated by neglecting the inertial term, and we obtain for any fixed $z = z_0$

$$\frac{n}{n_0} \left[\frac{n^{\gamma-2}}{n_0^{\gamma-2}} + \frac{2(\gamma - 1)}{\gamma \beta_0} \frac{R^2}{R_0^2} \right] = 1 + \frac{2(\gamma - 1)}{\gamma \beta_0} \qquad (4)$$

By the same argument it follows from the z-component of (3) that $\partial \ln n / \partial z$ is insignificant when $8\pi n_0 m_i \left[(\vec{u} \cdot \nabla)\vec{u} \right]_z / B_0^2$ is very small, i.e., $n \approx n(R)$.

Of special interest is the high-beta situation, when $\beta_0 \gg 2(\gamma - 1)R^2/\gamma R_0^2$, so that the term in R^2 in Eq. (3) or (4) can also be neglected as a first approximation. In that case we see that $n \approx n_0$ everywhere which means that, because of the constancy of nR/B, $\vec{B}(R) \approx (B_0 R/R_0)\hat{\theta}$. In other words, in the high-β limit the field is

modified so that the volume of a toroidal flux tube remains constant as it changes its major radius R. It should be noted, also, that $n \approx$ const does not mean that the plasma is current-free. On the contrary, the equilibrium here requires a uniform internal current density large enough to balance the finite residual pressure gradient that is consistent with the continuity requirement. The situation is sketched schematically in Fig. 2. As an aside we point out that in a toroidal magnetic field which <u>increases</u> with major radius R the mean guiding center drifts are reduced because the effects of the curvature and of $\nabla|B|$ oppose each other. Thus $\beta_0 > 1$ introduces an additional advantage to our Tormac confinement.

In the opposite limit, for very low values of β_0, Eq. (4) tells us that $n(R) = n_0 R_0^2/R^2$ so that $B(R) \propto R^{-1}$, as expected when plasma currents are negligible.

STABILITY

The density distribution $n(R)$, according to Eq. (4), decreases with increasing R, and so does the pressure p. The configuration has much in common with that of the diffuse linear pinch where we have a distributed current density in the z-direction only, and $\vec{j} \times \vec{B}$ balances a cylindrically symmetric radial pressure gradient. Analysis based on the energy principle requires $-R\partial p/p\partial R \leq 4\gamma/(2 + \gamma\beta)$ for stability at all values of R for such systems.[5] Using the adiabatic approximation we find the equality is satisfied, i.e., the stability is neutral for the distribution $n(R)$ given by (4). This is not surprising since the slow rotation about the minor axis envisioned in this discussion represents nothing but a pure interchange process which we have prescribed

to remain in hydromagnetic equilibrium.

The centrifugal effects which were neglected in going from Eq. (3) to Eq. (4) may then be expected to render the system unstable. On the other hand, any dissipative process, e.g., as caused by heat flow, viscosity, or electrical resistivity, may well have a stabilizing effect. These matters remain to be explored. We conclude tentatively that slow mass rotation about the minor axis is probably permissible in Tormac without causing difficulties.

If we wish to restrict the displacement of a drift orbit along the z-direction to the order of a few gyroradii and at the same time introduce negligible centrifugal effects, then $r_c/R \leq u/v_{th} \ll (r_c/R)^{1/2}$, where v_{th} is the mean thermal speed, and r_c denotes the radius of curvature of the rotating flow, i.e., r_c is of the order the plasma minor radius, so that the centrifigual acceleration is $|(\vec{u}\cdot\nabla)\vec{u}| \approx u^2/r_c$. Evidently this condition can be readily satisfied provided the aspect ratio R/r_c is kept fairly large, e.g., as indicated in Fig. 1.
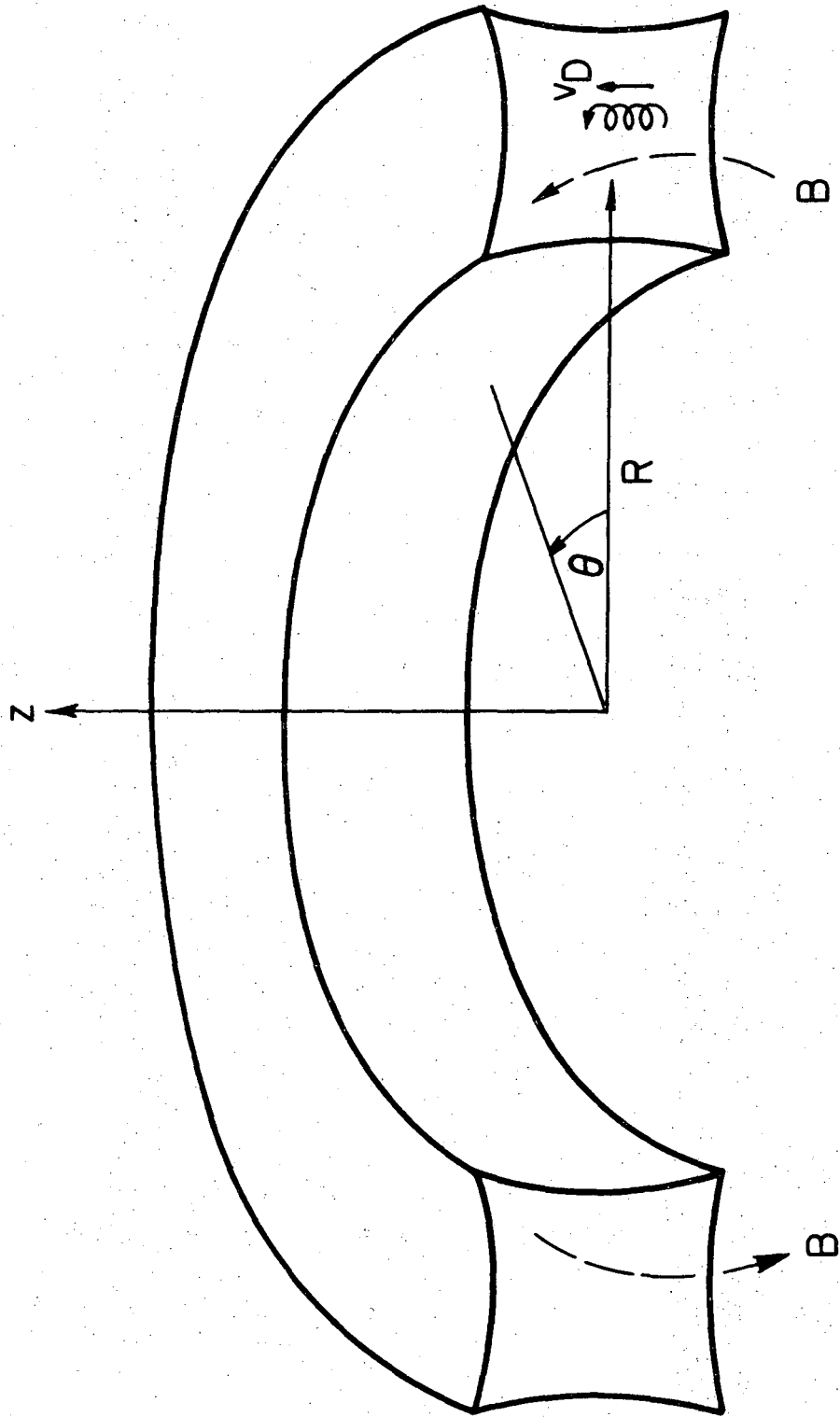
FOOTNOTE AND REFERENCES

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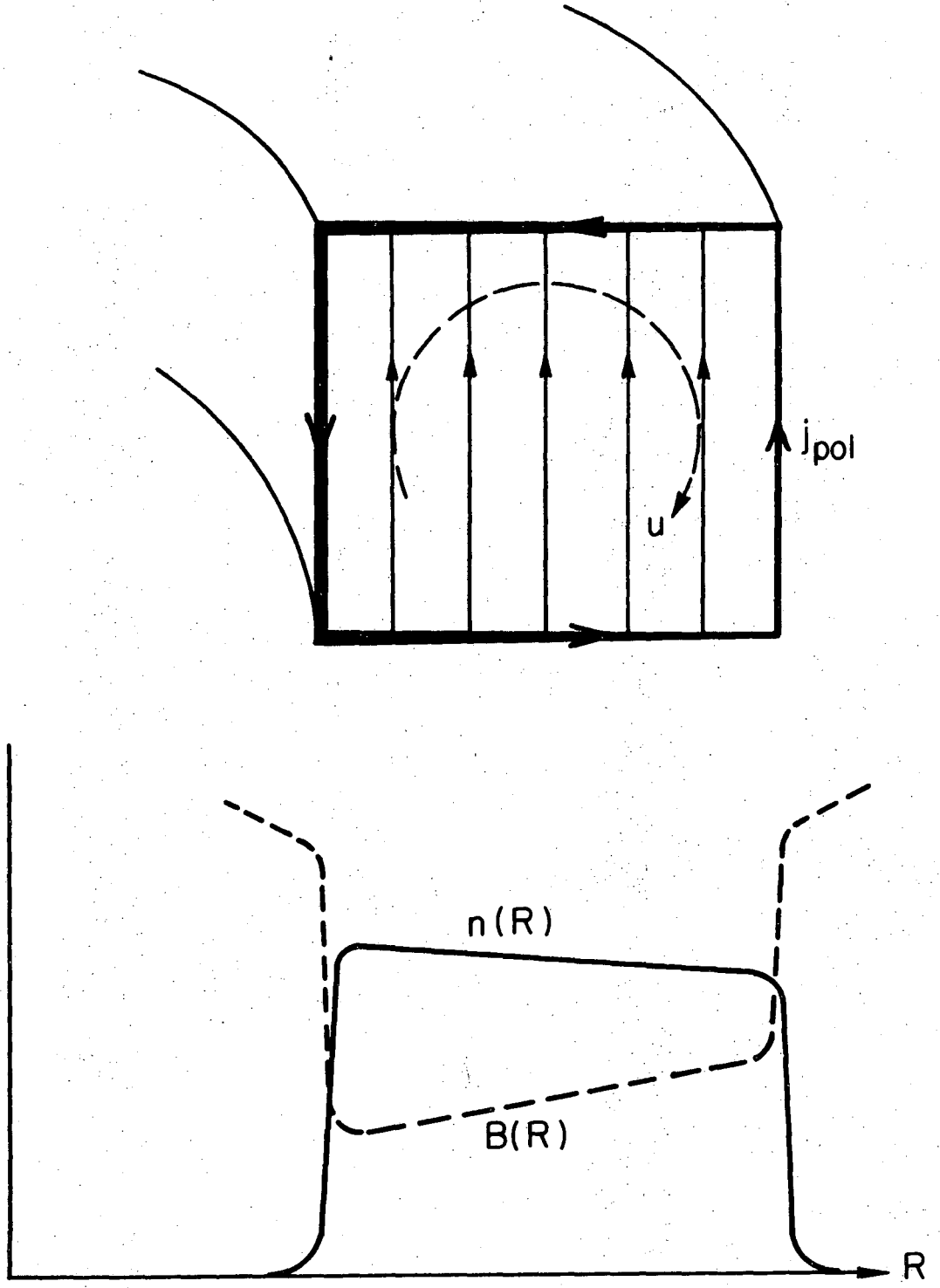5. N. A. Krall and A. W. Trivelpiece, Principles of Plasma Physics (McGraw-Hill, New York, 1973), p. 262.

FIGURE CAPTIONS

Fig. 1. The Tormac configuration, showing a typical ion-guiding center drift in the interior.

Fig. 2. At high values of β, $n(R)$ decreases only slightly with R while $B(R)$ increases, and a nearly uniform z-current passes through the interior.

Fig. 1.

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j_{pol}

u

$n(R)$

$B(R)$

R

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Fig. 2.