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ABSTRACT

Highway automation entails the application of control, sensing and communication technologies to road vehicles, with the objective of improving highway performance. It has been envisioned that automation could increase highway capacity by a factor of three. To attain this capacity, it will be important to minimize the amount **of** lane-changing and optimally assign vehicles to lanes. This paper develops and applies a linear programming based lane assignment model. The highway system is modeled as a multi-commodity network, where the commodities represent trip destinations (i.e., exit ramps on highways). ***An*** unusual feature of the model is that capacities are defined by bundle constraints, which are functions of the flow entering, leaving, continuing and passing through lanes in each highway segment. The objective is to maximize total flow, subject to a fixed origiddestinationpattern, expressed on a proportional basis. The model **is** tested for highways with up to 80 segments, 20 destinations and ***5*** lanes, and parametric analyses are provided with respect to the time-space requirement for lane-changes, number of lanes, number of segments and origiddestinationpattern.

Keywords: **Automated Highway Systems**

Lane Assignment

Link Layer

EXECUTIVE SUMMARY

This project is directed at developing capacity estimates for future automated highways systems. The focus is on estimating the effects of lane changes on capacity. To estimate these effects, a model is developed to optimize the assignment of vehicles to lanes on an origiddestination basis, with the objective of maximizing total highway capacity. The model relies on a linear program optimization algorithm. Linear programs have been applied in many contexts, such as production planning and network routing. A linear program is defined by a set **of** constraints, decision variables, and **an** objective hnction. The linear program ***algorithm*** determines the values of the decision variables that maximize (or minimize) the objective hnction, while satisfying the constraints.

In this paper there are five types of constraints: (1) flow must be conserved throughout the network, **(2)** capacities of lanes/segments cannot be exceeded, **(3)** ramp capacities cannot be exceeded, **(4)** pre-set origin destination patterns must be obeyed, and ***(5)*** all decision variables must be non-negative. The decision variables represent network flows on a destination basis (i.e., exit ramps on highways). ***An*** unusual feature of the model is that lane/segment capacities are hnctions of four flow variables: flow entering, leaving, continuing and passing through the given lanes in the given highway segments.

The paper documents the theory behind the model and performs parametric analyses to determine how origin-destination patterns and lane-change characteristics affect capacity. The software is documented in a separate working paper entitled "LANE-OPT Version 1.0, User's Manual" (PATH working paper **96-02)**

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1. INTRODUCTION

Highway automation entails the application of control, sensing and communication technologies to road vehicles, with the objective of improving highway performance (e.g., capacity and safety). For instance, some predictions forecast that automation could increase highway capacity by a factor of three, and that capacity increases of this magnitude would greatly reduce highway congestion.

By definition, automation demands that some or all of the functions performed by drivers be transferred to computer control, either on the vehicle or at a central location. At a minimum, this would likely include braking, steering and throttle control. It might also eventually include control of higher level functions, such as lane selection and/or route selection. Control of lane selection may prove to be especially important. In order to attain high capacity, Automated-Highway-Systems (AHS) must greatly reduce the average separation between vehicles. As a consequence, execution of lane changes could become much more difficult, both due to the reduced size and number of gaps, and due to the imposition of time/space consuming maneuvers to free-up gaps for lane changes.

Control of lane changes is especially important for AHS concepts that rely on short-headway platoons. Within a platoon of vehicles, the separation may be as small as one meter, whereas between platoons, the separation may be 100 meters or more. As a consequence, the likelihood of platoons colliding with each other is small, and the likelihood of a ***severe*** collision *within* a platoon is also small. On the other hand, the "overhead" associated with lane changes could be quite high. Whenever a lane change is needed, the platoon may have to break apart, freeing up space for the vehicle to safely move between lanes. The time and space requirement for such a maneuver creates a strong incentive for minimizing lane changes.

By contrast, on a conventional highway, drivers have freedom to decide when to change lanes, and which lane to travel in. Drivers select lanes based on a variety of criteria, such as their comfort level when driving at a particular speed, the length of their trip, and their aggressiveness in avoiding delay. While it is conceivable that optimized lane assignment could improve the performance of conventional highways, there is no practical mechanism for implementing such an assignment. An AHS, on the other hand, provides both a mechanism for assigning traffic, and a direct incentive for doing ***so*** (the relatively large lane-change "overhead").

This paper develops and applies a linear programming based model for assigning traffic to lanes on **an** automated highway system. The highway system is modeled as a multi-commodity network, where the commodities are differentiated by trip destinations (i.e., exit ramps on highways). An unusual feature of the model is that capacities are defined by bundle constraints, which are functions of the flow entering, leaving, continuing and passing through highway segments. The objective is to maximize total flow, subject to a fixed origiddestination pattern, expressed on a proportional basis. The current research utilizes a static formulation. Future research will extend the model **to** dynamic networks.

The remainder of the paper is divided into four sections. First, literature on AHS capacity analysis is reviewed. Next, the linear programming formulation is provided. This is followed by a series of analyses showing how the highway configuration affects capacity. Finally, conclusions are provided.

2. LITERATURE REVIEW

The literature on AHS has expanded greatly in recent years due to two programs funded by the United States government: (1) AHS Precursor System Analysis program, and **(2)** National Automated Highway System Consortium (NAHSC). Unfortunately, for the most part, findings under these programs only appear in report format. Furthermore, most of these reports focus on system integration and institutional issues, rather than questions of capacity. The most relevant papers resulting from the federal effort present alternative system concepts, and dimensions for defining alternative system concepts.

These include Hall **(1995)**, Stevens **(1993)** and Tsao, Hall and Shladover **(1993).** In these papers, system concepts are defined by such elements as: **(1)** platooning strategy, **(2)** existence of barriers separating lanes, **(3)** mixing of vehicle classes and types, **(4)** distribution of intelligence, and ***(5)*** deployment strategy.

The earliest systematic study of automated highway capacity appears to be the paper by Rumsey and Powner **(1974)**, which examined a moving-cell operating concept. Recently, however, the interest in automated highways has focused more on the platooning concept, as introduced by Shladover **(1979).** Shladover developed capacity estimates based on a variety **of safety** criteria, in which the objective was to prevent severe collisions. In a related paper, Tsao and Hall **(1994)** compare the platooning concept to a "non-platooning" concept (i.e., vehicles do not travel in clusters), and conclude that platooning leads to more fiequent small collisions, but less frequent severe collisions. Neither paper analyzed the effects of lane changes. At a more detailed level, vehicle control rules have been investigated, to determine the effects of vehicle performance characteristics on lane-following behavior (Ioannou and Xu **(1994)** and Hedrick et **al (1994)** for example).

The capacity of automated highways with platooning and lane changing has been investigated by Rao et **al (1993)**, Rao and Varaiya **(1993,1994)** and Tsao et **al** (1993). All of these utilize the SmartPath simulator developed by **Eskafi** and Varaiya **(1992).** SmartPath is microscopic, and models the system down to the level of exchange of messages between vehicles. Tsao et al also includes stochastic/analytical models to represent the time to execute a lane change maneuver.

The model that follows is most closely related to a paper by Hall **(1995)**, in which **a** stationary/analytical model was developed for throughput analysis. Hall analyzed capacity as a hnction of the lane-change overhead, which was measured as the time-space requirement for a lane-change. One of the findings was that for large overhead values, adding lanes does not necessarily add to highway capacity, because the entrance lane becomes saturated. The concept of space-occupancy has also been incorporated in Broucke and Varaiya (1995), in which control activity plans (covering velocity and assignment of maneuvers) are developed for a single lane highway. Also related is the paper by Medanic *et* al (1994), which develops path based lane assignment strategies. Medanic is similar to our paper in the respect that the problem **is** to assign traffic to lanes with a bottleneck related objective. However, Medanic et al's solution is heuristic.

Another related area of research is the traffic assignment literature (reviewed in Ran and Boyce (1994) and Sheffi (1985)). The objective of the traffic assignment problem is to predict driver route choices over congested roadway networks. These models are used for the purpose of transportation planning. Differences between traffic assignment and AHS lane assignment include: (1) in traffic assignment, the path is defined by a route choice rather than lane choices, **(2)** in traffic assignment, the objective function is based on a user-optimal criterion, and **(3)** in traffic assignment, the objective is a convex function of flow. In the model that follows, the objective function will be linear, because the objective is to maximize total network flow. The underlying assumption is that traffic speeds are unaffected by flows **(up** to capacity) on **an** AHS, due to automated controls. Furthermore, the intended application of the model is capacity analysis, rather than traditional transportation planning. By maximizing flow, an estimate is obtained for total network capacity.

3. MODEL FORMULATION

The AHS consists of a set of highway segments and sets **of** on-ramps and off-ramps. Each segment contains one or more automated lanes, which are always situated on the left-side of the highway. The *AHS* may also have manual lanes, which are always situated on the right-side of the highway. The number of lanes can vary from segment to segment. Lane drops and lane additions are assumed to occur on the right-side of the highway (the model is easily generalizable to more complicated structures).

3.1 Network Representation

As shown in Figure 1 the highway is represented by a flow network. Highway segments are indexed by location and defined by segment type (on-ramp, off-ramp, or neither), length of the segment, number of lanes, and ramp capacity (for segments containing ramps). Nodes are assigned to the end of each lane in each segment, as well as to the start of each on-ramp and end of each off-ramp. On-ramp nodes are source nodes without entering arcs, and off-rampnodes are sink nodes without out-going arcs. Source nodes are also placed at the start of each lane in the first segment, and a "super-sink" node is placed after the last highway segment to absorb all continuing highway traffic. Source nodes at the start of the highway pre-assign entering traffic to a specific lane, whereas the "super-sink" node allows the assignment of continuing traffic to be optimized among lanes (however, in experiments that follow, the highway was assumed to start with zero traffic).

Each arc represents a vehicle trajectory through a highway segment. **A** trajectory may entail staying in a lane, transitioning fiom one lane to another, transitioning from a lane to an off-ramp, or transitioning fiom an on-ramp to a lane. In each case, the arc **is** defined by a segment **(s)**, an initial lane (i), and an ending lane **(j)**. Hence, an arc (i,j,s) represents vehicles that begin segment **s** in lane i and end segment **s** in lane j. For any segment, the graph **is** completely connected, meaning that vehicles are allowed to transition between any pair of lanes within the segment. The exceptions are on-ramp nodes and off-ramp nodes, which represent sources and sinks. *As* in Figure 1, arcs that are incident on these nodes only **flow** in one direction.

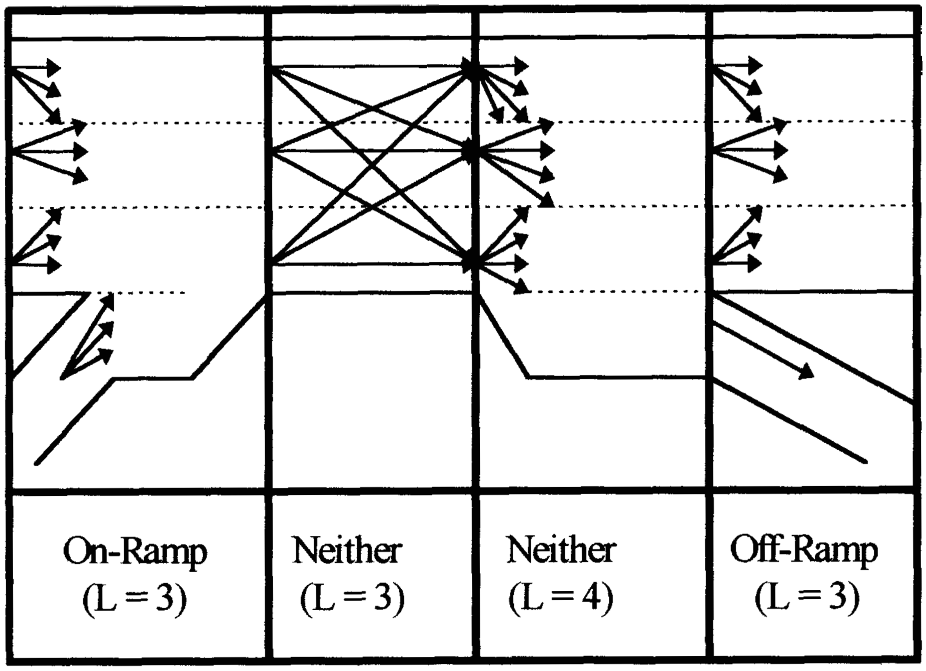


Figure 1: Highway Segments

Network flows are multi-commodity, where a commodity represents a destination off-ramp. Hence, the formulation optimizes flow by destination, but does not distinguish between origins. The decision variables are as follows:

$F_{i,j}^{s,d}$ = the flow in segment s to destination d, starting in lane i and ending in lane j.

Lanes are numbered from left to right, with the left-most lane numbered 1. Lane drops and additions are assumed to occur on the right-side of the highway, meaning that continuing lanes retain their numbering sequence from segment to segment. On-ramp and off-ramp nodes are designated as lane "0". Segments are numbered from upstream **to** downstream, beginning with number 1 (representing the start of the first segment). The super-sink node is designated as segment S+2. On-ramps and off-ramps are numbered by segment. For example, d = 8 would represent an off-ramp node situated at the start of segment 8.

The linear program has five types of constraints, four of which are presented here: (1) flow conservation, (2) lane/segment capacity, (3) ramp capacity, and (4) non-negativity. A fifth constraint, presented in the following section, defines the objective function in terms of a fixed origin-destination pattern

With the exception of source and sink nodes, flow is conserved by equating the sum of flows on in-coming arcs to the sum of flows on out-going arcs plus flow originating or terminating at that point:

$$\sum_{k=1}^{L_{s+1}} F_{j,k}^{s,d} - \sum_{i=1}^{L_{s-1}} F_{i,j}^{s-1,d} = b_j \quad \forall d \in D, s \in S, j = 1,\cdots,L_s \qquad (1)$$

where

$$b_j = \begin{cases} \sum_{i=1}^{L_s} F_{i,j}^{s,d} & if\ s = d, j = 0 \\ \sum_{i=1}^{L_s} F_{j,k}^{s,d} & if\ s = o, j = 0 \\ 0 & otherwise \end{cases}$$

D = set of destinations,

o = element of the set of origins O, and

L_s = The number of lanes in segment s.

Capacity constraints bound the flow within each lane/segment. These are bundle constraints, which account for straight traffic, as well as traffic that enters, exits and passes through each lane within each segment. These four types of flow are defined as follows:

$$F^{s}_{stay\,in\,j} = \sum_{d \in D} F^{s,d}_{j,j} \tag{2}$$

$$F^{s}_{enter\,j} = \sum_{d \in D} \sum_{i=1}^{L_s} F^{s,d}_{i,j} \tag{3}$$

$$F^{s}_{exit\,j} = \sum_{d \in D} \sum_{k=1}^{L_s} F^{s,d}_{j,k} \tag{4}$$

$$F^{s}_{pass\,through\,j} = \sum_{d \in D} \sum_{k>j} \sum_{i<j} F^{s,d}_{i,k} + \sum_{d \in D} \sum_{k<j} \sum_{i>j} F^{s,d}_{i,k} \tag{5}$$

The total workload is a linear combination of the flow variables in equations **2** - 5, which cannot exceed the capacity of the lane/segment:

$$\alpha F^{s}_{stay\,in\,j} + \beta F^{s}_{enter\,j} + \eta F^{s}_{exit\,j} + \gamma F^{s}_{pass\,through\,j} \le W^{s}_{j} \tag{6}$$

The workload parameters $(\alpha, \beta, \eta, \gamma)$ are all measured in units of work-time per vehicle. The left-hand side of Eq. 6 represents the total time to perform all work in lane **j** of segment **s** as a ratio to time available. If all parameters are measured in the same time unit, the upper bound on **Eq.** 6 would be **1.** In our implementation, flow is measured in vehicles per hour, whereas $(\alpha, \beta, \eta, \gamma)$ are measured in seconds per vehicle. Hence, **W** equals the number of seconds in an hour **(3600).**

$(\alpha, \beta, \eta, \gamma)$ depend on the length of the segment, as well as the workload associated with lane-changes and straight movements. Let:

c_{in} = workload for entering a lane (time-distance)

c_{out} = workload for exiting a lane (time-distance)

c_{str} = workload for continuing straight within a lane (time)

l = length of the segment (distance)

The parameters $(\alpha, \beta, \eta, \gamma)$ are defined by the following:

(1) **A** vehicle that continues straight in a lane imposes a workload of c_{str} on that lane.

(2) **A** vehicle that enters a lane imposes a workload of (1/2) c_{str}, plus a workload of c_{in}/l to account for turn movements into the lane (dividing by l normalizes the turn workload in units of time).

(3) **A** vehicle that exits a lane imposes a workload of (1/2) c_{str}, plus a workload of c_{out}/l, to account for the turn movement out from the lane.

(4) **A** vehicle that passes through a lane imposes no "straight" workload, but imposes a workload of $(c_{in} + c_{out})/l$ to account for turns into and out-from the lane.

Clearly, the assumptions are only approximate, especially for large l where turn-movements might not be uniformly distributed over the length of the segment.

The assumptions are summarized below.

Parameter	Parameter Formula
α	c_{str}
β	$c_{in}/l + c_{str}/2$
η	$c_{out}/l + c_{str}/2$
γ	$(c_{in} + c_{out})/l$
W_j^s	3600 seconds/hour

The third type of constraint is that ramps are limited in how much flow they can accommodate, or by a pre-selected metering rate:

$$\sum_{d \in D} F_{0,j}^{o,d} \le R^{o} = \text{on-ramp capacity (origin)} \qquad (7)$$

$$\sum_{o \in O} F_{j,0}^{o,d} \le R^{d} = \text{off-ramp capacity (destination)} \qquad (8)$$

The fourth constraint is that all flows must be non-negative:

$$F_{i,j}^{s,d} \ge 0 \qquad (9)$$

3.2 Objective Functions and Origin/Destination Patterns

The objective is to maximize the total flow, where the total flow is defined as the sum of the flows fiom each origin to each destination:

$$\max \sum_{o \in O} \sum_{d \in D} \sum_{1 \le j \le L_o} F_{0,j}^{o,d} = \mathit{Total\ Flow} \qquad (10)$$

Total flow is bounded by the fifth constraint type, which defines a fixed origin-destination pattern on a proportional basis:

$$\sum_{1 \le j \le L_o} F_{0,j}^{o,d} = p_{o,d} * \mathit{Total\ Flow} \quad \forall o \in O, d \in D \qquad (11)$$

where

O is the set of origins,

D is the set of destinations, and

$p_{o,d}$ is the fixed proportion of total flow that must travel fiom origin o to destination d.

$p_{o,d}$ are user-defined parameters, which depend on vehicle travel patterns for the highway under investigation. The underlying philosophy is that the linear-program determines the maximum highway capacity, under the condition that no origiddestination pair is preferred over any other. Hence, the formulation does not allow the proportions to be violated, even when some origin-destination pairs have surplus capacity.

After performing tests, we discovered that some types of highways possess multiple optimal solutions. This is especially prevalent on highways that contain more lanes than can be reasonably utilized. That is, when total flow is dictated by on-ramp capacity, off-ramp capacity, or lane-change workload in the right lane, the **L.P.** produces multiple optimal solutions. Some of these solutions are less desirable than others, because they produced unusual flow paths with excessive lane-changes. In order to produce a more desirable solution, the objective hnction was modified as follows:

$$\max\left(p_{o,d} * \textit{Total Flow}\right) + \varepsilon\left[\sum_{1\le j\le Ls} W_j^s - \left(\alpha F^s_{stay\,in\,j} + \beta F^s_{enter\,j} + \eta F^s_{exit\,j} + \gamma F^s_{pass\,through\,j}\right)\right] \quad (12)$$

where ε is a small-valued multiplier. The term within the brackets represents the total surplus capacity among **all** lanes and segments. Hence, the objective hnction maximizes total flow plus ε multiplied by the surplus capacity. When ε is suitably low, the modified objective hnction produces the same total flow as the original. However, it tends to produce solutions that require fewer lane-changes and consolidates most **of** the traffic in the right-most lanes.

3.3 Accuracy of Discretization

The use of discrete segments to represent a highway is only approximate in that lane changes can occur anywhere along the segment length and not just at a point. The accuracy of the approximation depends in part on the degree to which traffic accumulates or dissipates over the segment's length. Whenever flow is non-stationary within **a** segment, the rate of lane-changes is also non-stationary within the segment. For instance, if a lane accumulates traffic, the rate at which lane-changes can be admitted declines non-linearly over the segment.

The accuracy of the discretization was investigated by examining an extreme condition: the build-up of traffic within a single lane (designated as lane **2)** that begins with zero flow. Lane **2** receives flow **from** Lane 1 through lane-changes. No lane changes occur from Lane **2** to Lane 1, and the highway has just two lanes. The following model represents the build-up of traffic for segments of varying length . Let:

$f_i(z)$	=	flow in lane i at location z
$f_{12}(z)$	=	flux at location z
	=	rate at which lane changes occur from lane **1** to lane **2,** per unit length of highway, at location z
α	=	time occupancy of longitudinal flow
c_{in}	=	time-space occupancy of flow entering lane 2.

Then:

$$f_2(z+l) \approx f_2(z) + f_{12}(z)l, \tag{13}$$

for a segment of length l. The total work-load within any segment is defined by the flux and flow, along with α and c_{in}. Allowing for the flow to vary over the segment, the flux into lane **2** is constrained as follows:

$$c_{in} f_{12}(z) + (\alpha/2)[f_2(z) + f_2(z+l)] \leq 1 \tag{14}$$

$$f_{12}(z) < \frac{1 - (\alpha/2)[f_2(z) + f_2(z+l)]}{c_{in}} \tag{15}$$

The accuracy of **Eqs. 13-15** depend on the segment length. Averaging the flow between the start and end of the segment improves accuracy (relative to a single point estimate), and is consistent with the L.P. formulation provided earlier.

If lane changes occur at the maximum rate:

$$f_2(z + lz) = f_2(z) + \frac{\delta\{1-(\alpha/2)[f_2(z)+f_2(z+l)]\}}{c_{in}}, \quad (16)$$

To facilitate analysis, **Eq. (16)** can be normalized relative to the ratio c_{in}/α:

$$l = k(c_{in}/\alpha) \quad (17)$$

$$z = z'(c_{in}/a) \quad (18)$$

$$f_2[(z'+k)(c_{in}/\alpha)] = \frac{(1-k/2)}{(1+k/2)} f_2[z'(c_{in}/\alpha)] + \frac{(k/\alpha)}{(1+k/2)} \quad (19)$$

To place k in context, if a lane-change consumes 500 meter-seconds, and the maximum sustainable capacity is **7200** vehicleshour (i.e., $a = .5$ seconds), then a k value of **.75** translates to a segment length of **.75 km**. **Eq. (19)** can be further normalized by dividing through by:

$$f_{max} = 1/\alpha, \quad (20)$$

where f_{max} represents the maximum sustainable flow on the lane (in the absence of lane changes). Dividing **Eq. (19)** by f_{max} yields:

$$\frac{f_2[(z' + k)(c_{in}/\alpha)]}{f_{max}} = \frac{(1-k/2)}{(1+k/2)} \frac{f_2[z'(c_{in}/\alpha)]}{f_{max}} + \frac{k}{(1+k/2)} \quad (21)$$

or

$$C[(z' + k)(c_{in}/\alpha)] = \frac{(1-k/2)}{(1+k/2)} C[z'(c_{in}/\alpha)] + \frac{k}{(1+k/2)} \quad (22)$$

where:

$C(w)$ = proportion of sustainable capacity utilized at location w.
= $f_2(w)/f_{max}$

To assess the accuracy of discrete segments, C(w) was calculated for a range of k values. **As** k approaches zero, the model is effectively continuous. Larger values of k represent discrete segments and are approximate. Results are shown in Figure 2. **As** can be seen, values of k as large as 1.0 (**i**.e., $c_{in}/\alpha = l$) accurately represent traffic growth

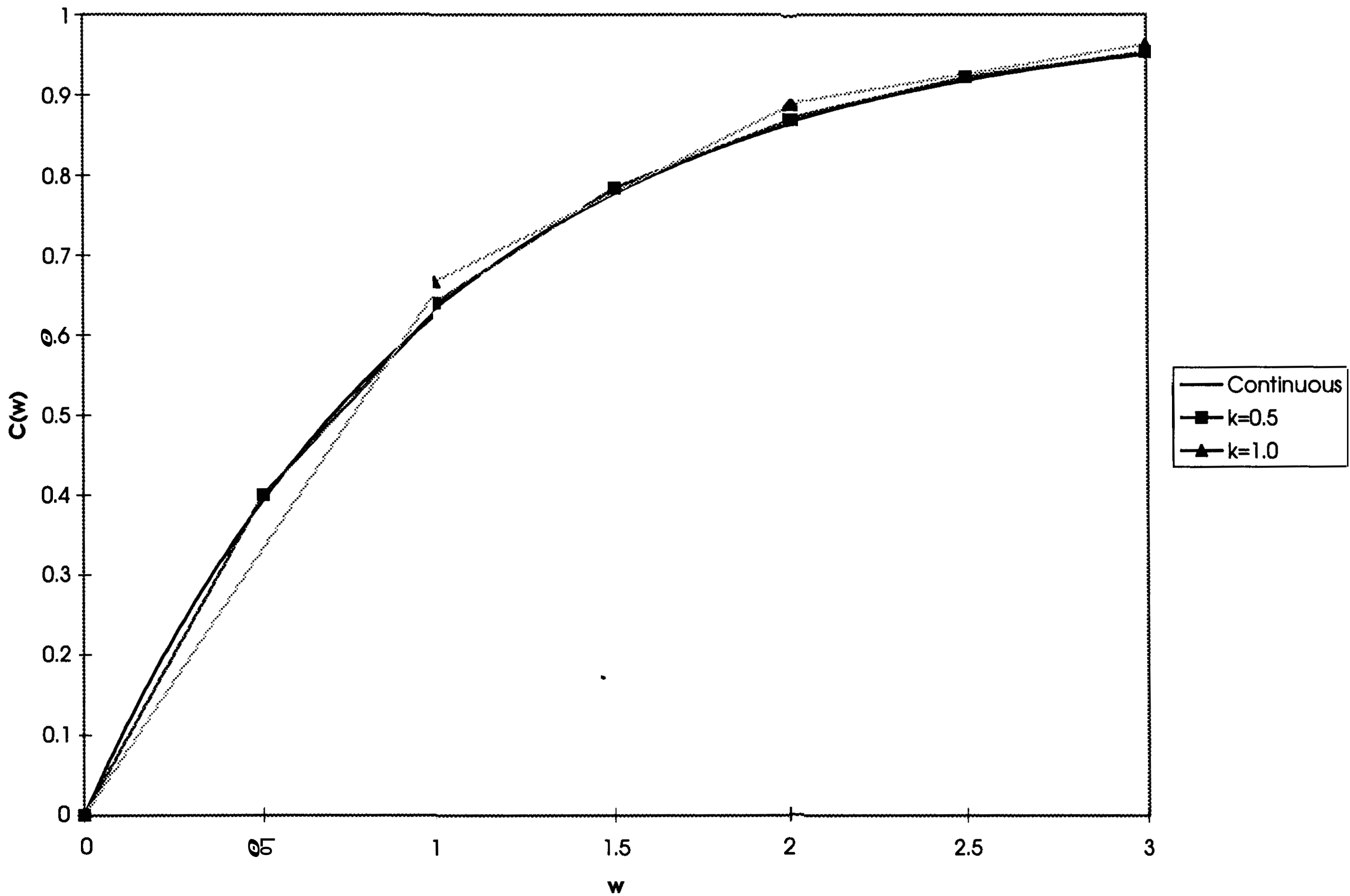


Figure 2 : Accuracy of Discretization

(within ***6%).*** Errors are self-regulating. That is, when traffic is overestimated for a given z', the rate of lane changes is adjusted downward, causing the error to dissipate. It is unlikely that segments need to be any smaller than this value to be accurate. They can likely be much larger, as the situation examined is extreme.

Segments are also approximate in that the workload associated with a lane change is, in reality, not isolated to individual segments. In this sense, longer segments provide greater accuracy as there is **a** lower likelihood that a maneuver will cross over segments.

Let:

l = length of segment

τ = time to execute lane-change

v = velocity

p = proportion of lane-change work that is completed in a segment.

Assuming, as an approximation, that lane-changes are initiated uniformly over the segment, then:

$$p = (l-\tau v)/l + (1/2)(\tau v/l) = 1 - (1/2)(\tau v/l) \qquad (23)$$

Interpreting the equation, if l is one kilometer, v is 100 km/hr, and a lane-change takes 20 seconds to execute, then p = .83. Because cross-overs occur both into and out-from each segment, accuracy at this level should be sufficient, at least for research purposes.

Based on these results, it appears that the linear-programming formulation would have to be modified for large values of τ. In such a modification, the workload associated with a particular segment would depend, in part, on lane-changes initiated in prior segments. Such a change is relatively straight-forward. However, the model of Eqs. 1-10 should be reasonably accurate for smaller values **of** τ (under 20 seconds).

3.4 Coding and Solution

The LP was solved using the CPLEX Linear Programming Solver. **A** front-end program was written in C that creates a MPS formatted file describing the mathematical program. This MPS file is then read by CPLEX and optimized, using its optimize or command. CPLEX writes the solution to a text file. **A** final C program reads the text file, and creates tables showing total flow within each segment/lane, aggregated across all destinations, and disaggregated by destination. Tables are also produced to show surplus capacity by lane and segment, and **left** and right lane-changes for each lane in each segment. Finally, these data are read into excel spreadsheets to produce graphical output.

4. MODEL ANALYSIS

Experiments were conducted to measure how total flow is affected by the following model features:

- Origin-destination patterns
- Lane change coefficient
- Number of lanes
- Length of highway (measured in segments).

Four different origin-destination matrices were examined. The *equalized* matrix assigns an identical proportion to each on-ramp/off-ramp pair, independent of the distance separating the ramps (with mean trip lengths shown below). The matrix also yields zero

Highway Length	Equalized OD	Irregular OD
16	7.00	6.00
32	12.33	11.00
48	17.66	16.26
64	23.01	21.57

traffic at the start and end of the highway. **As** a consequence of these assumptions, the greatest on-flow occurs at the first on-ramp and the greatest off-flow occurs at the final off-ramp. The second and third matrices assume an ***exponential*** trip length distribution (rounded-off to discrete values) with equalized on-ramp flows. One matrix uses a mean trip length of 16 segments, and the other uses a mean of 32 segments. The highway begins with zero traffic, but ends with an amount of traffic equalling the residual off-ramp flow. That is, if the trip's destination is beyond the end of the highway, that traffic is assumed to remain in one **of** the lanes, rather than exiting at the final off-ramp. The final pattern is called ***irregular.*** **This** pattern introduces additional weaving in a middle segment of the highway by tripling the off-ramp percentages and tripling the on-ramp percentages for the center-most ramps. Otherwise, proportions are equalized.

In all experiments, highway structure exhibited the same pattern as Figure 1. **As** shown, the highway is divided into 4-segment blocks, with the pattern: on-ramp, neither, neither (adding a lane), and off-ramp (dropping a lane). Hence, the number of on-ramps equals the number of off-ramps, which are both **1/4** the number of segments. Furthermore, in all experiments, $a = .5$ **s**. (i.e., f_{max} = 7200 vehicleshour) and ramp capacities are all set at 7200 vehicles/hour.

Experiments were executed to measure the effects of the following features on total network flow:

1. Number of automated lanes (ranging from **2** to ***5; 0*** manual) and lane change coefficient ($c_{in} = c_{out}$ ranging from **100** to **3000** meter-seconds), for all four origin-destination patterns and a constant highway length **(48** segments).
2. Number of automated lanes (ranging from **1** to **3)** and lane change coefficient ($c_{in} = c_{out}$ ranging from **100** to **3000** meter-seconds), providing two manual lanes, for the exponential origin-destination matrices and a constant highway length **(48** segments).
3. Highway length **(16** to **64** segments) and number of automated lanes **(2** to ***5; 0*** manual) for the equalized and exponential **(μ=32)** matrices and a constant $c_{in} = c_{out}$ (500 meter-seconds).
4. Cross comparsion of the exponential origin-destination matrices for **2** to ***5*** automated (0 manual) lanes and $c_{in} = c_{out}$ ranging from **100** to **3000** meter-seconds.
5. Provision of two manual lanes in addition to autoamted lanes, in which $f_{max} = 2400$ vehicles per hour and $c_{in} = c_{out} = 300$ meter-seconds on the manual lanes (providing smaller longitudinal capacity but greater lateral capacity, in most instances).

Experiments were also conducted to examine characteristics of the flow patterns, using the **E** optimized objective function. The purpose of these experiments was to ascertain whether the solution exhibited a regular structure, which might be exploited through new problem formulations, or perhaps decomposition based algorithms. Alternative formulations may prove to be important in future research addressing problems with time-varying traffic patterns. Decomposition and simplification are not essential to the current line of research, as the static formulations are readily solvable in their current form. **This** is reflected in the table below, which gives **CPU** times on a UNIX based **SPARC** station with 128 **MB** of memory.

Segments	Lanes	Decision Variables	Constraints	CPU Time (sec)
16	3	406	181	0.27
32	3	1456	583	1.91
48	3	3283	1205	13.07
64	3	5428	2011	26.80
64	4	8434	2371	126.62
64	5	14709	3302	236.50

4.1 Experimental Results

Figures **3** to ***6*** provide results fi-om the first set of experiments, in which $c_{in} = c_{out}$ varies from 100 to **3000** meter-seconds (48 segment highway). **As** shown, total-flow is a non-increasing hnction of $c_{in} = c_{out}$. Further, the slope of the total flow fbnction generally declines at a decreasing rate as $c_{in} = c_{out}$ increases. ***An*** exception is that when $c_{in} = c_{out}$ and the number of lanes is large, total flow can be a constant value. This occurs when the highway is constrainted by the capacity of on and off ramps. More importantly, as $c_{in} = c_{out}$ grows large, the total flow hnctions for **3-5** lanes converge toward the total

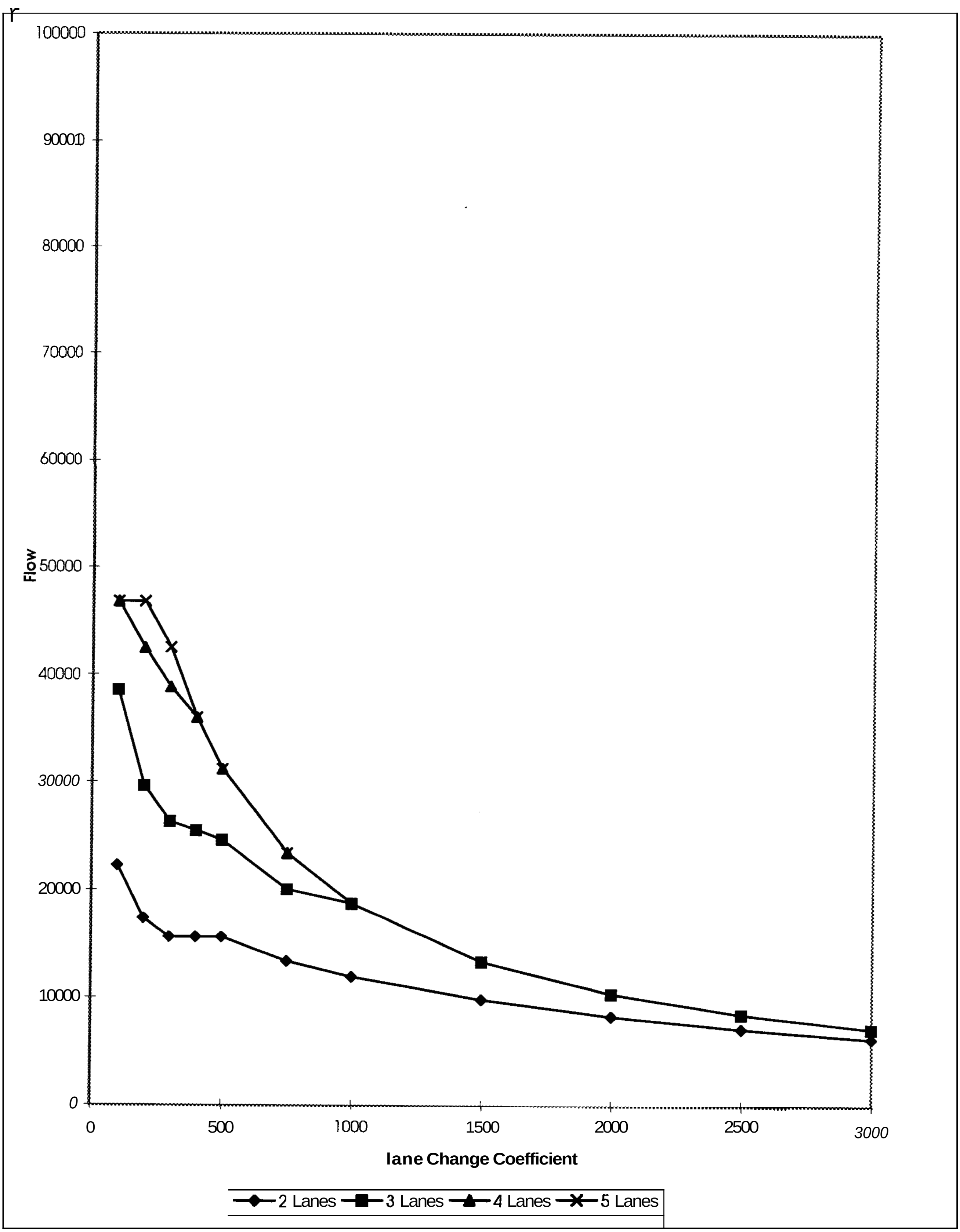


Figure 3: Total Flow across 48 Segment Highway with Equalized OD

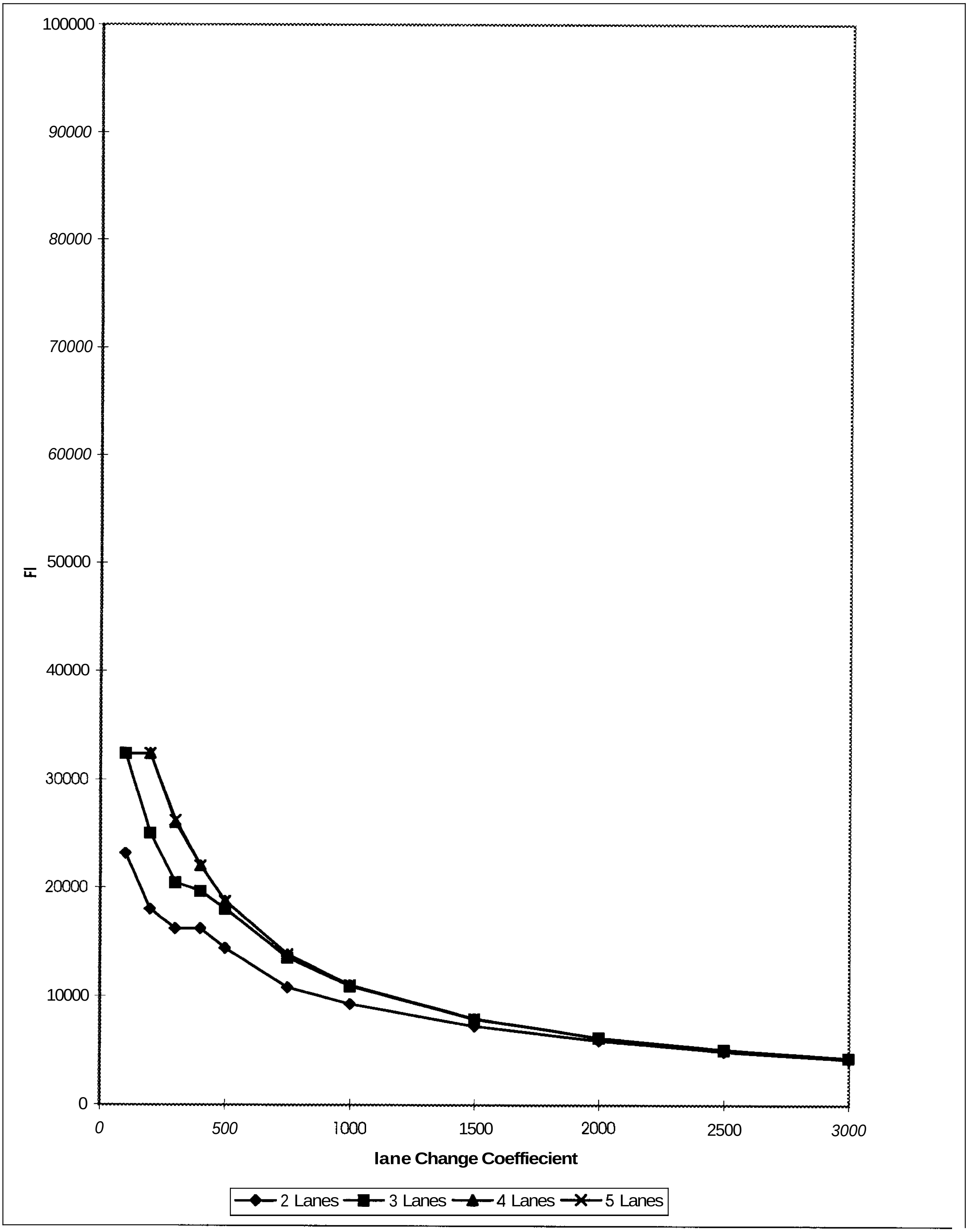


Figure 4: Total Flow across 48 Segment Highway with Irregular OD

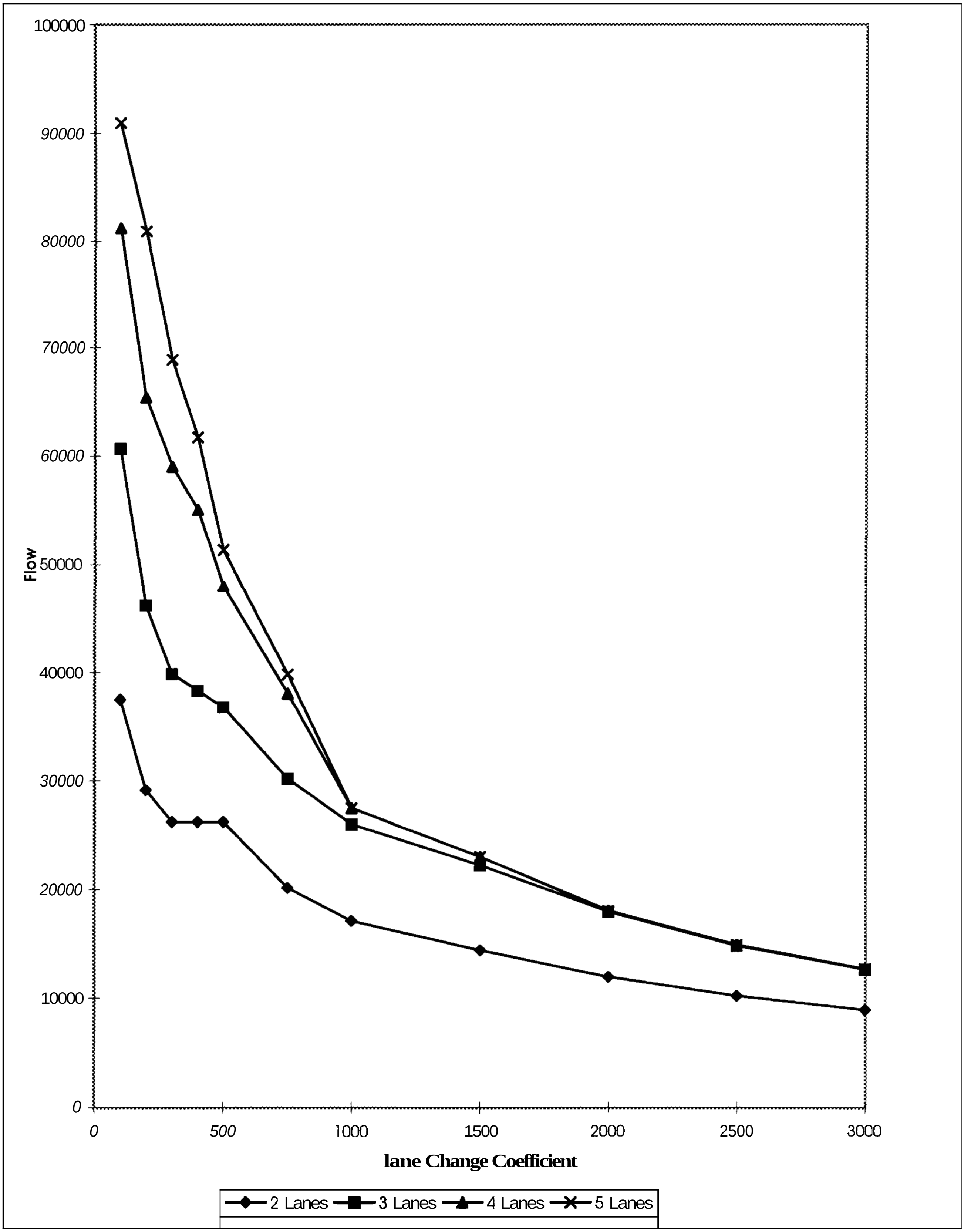


Figure 5: Total Flow across a 48 Segment Highway with an Exponential-16 OD

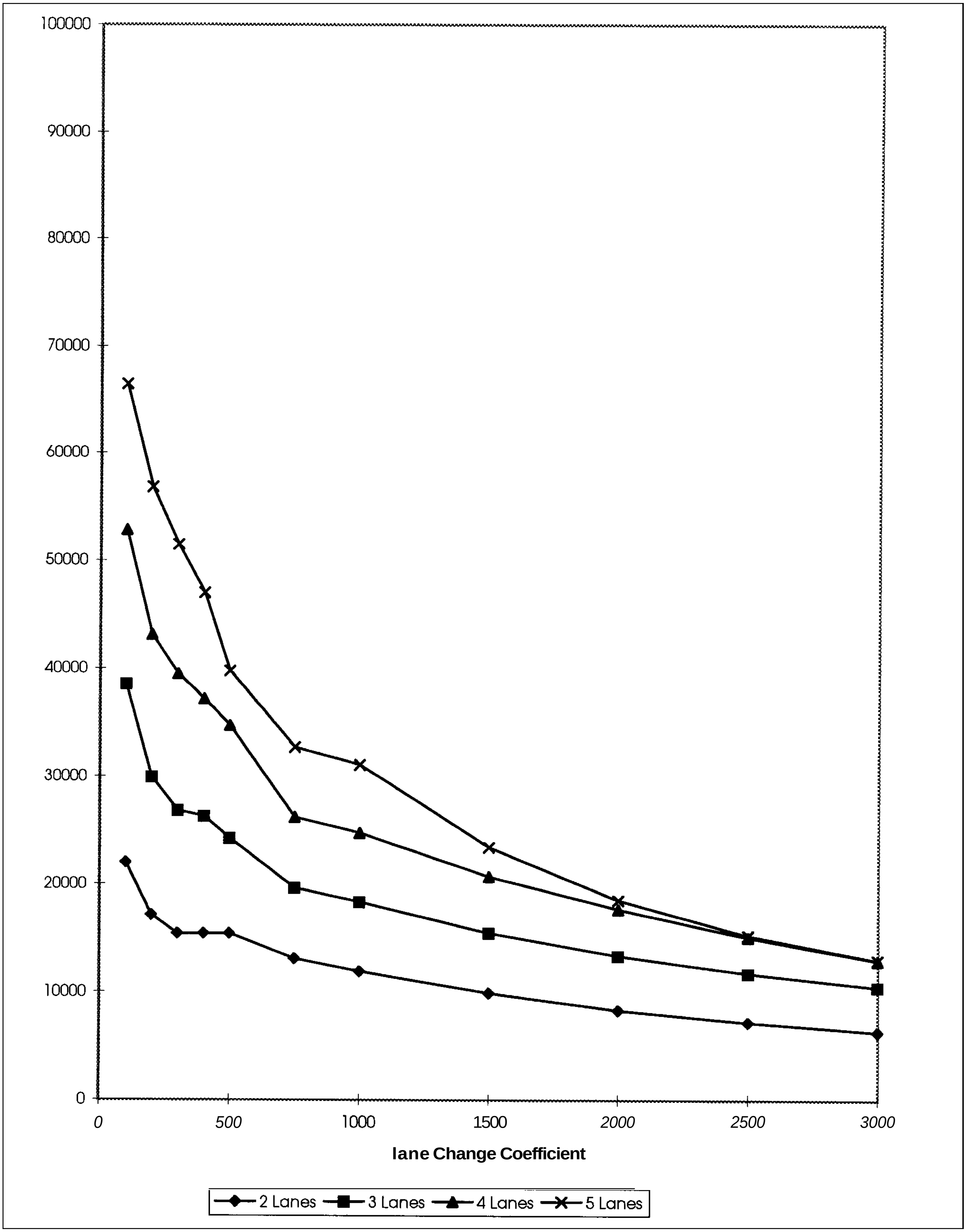


Figure 6: Total Flow across 48 Segment Highway with Exponential-32 OD

flow hnction for **2** lanes. This illustrates that when the lane-change coefficient is large, the highway may be constrained by the capacity of the right-most lane to accommodate lane changes. Providing more than 2 lanes then provides no additional capacity.

Comparison of Figures 3-6 also illustrates how the origin-destination pattern affects total flow. ***As*** the average trip length increases, total flow generally declines, as the flow produce a greater "straight workload". This is further illustrated in Figure **7,** which compares the two exponential distributions. However, for large values of $c_{in} = c_{out}$, it is possible for the curves to cross (this is something of an end-effect, as long trip lengths result in more traffic remaining on the highway at its end, and fewer lane changes). Furthermore, evenly spacing **on** and off ramp flows is also beneficial. This can be seen by comparing the exponential cases to the irregular and equalized cases. The irregular case performs the worst, as a large proportion of the on and off traffic occurs in the middle of the highway, where lane flow is also large.

Figures **8** and 9 analyze the effect of highway length on total flow. For an exponential origin-destination matrix ($\mu = 32$), total flow is approximately a linear hnction **of** the highway length. This is because longer highways provide greater on and off ramp capacity, as well as greater lane-change capacity. For the equalized pattern, the relationship is more complicated. Increasing highway length provides greater capacity, but it also results in longer average trip length. The result, for the 2-lane highway, is a reduction in total flow when highway length increases from 32 to 64 segments. However, when the number of lanes is larger (3-5), total flow is not reduced, as there is excess capacity in the left-side lanes.

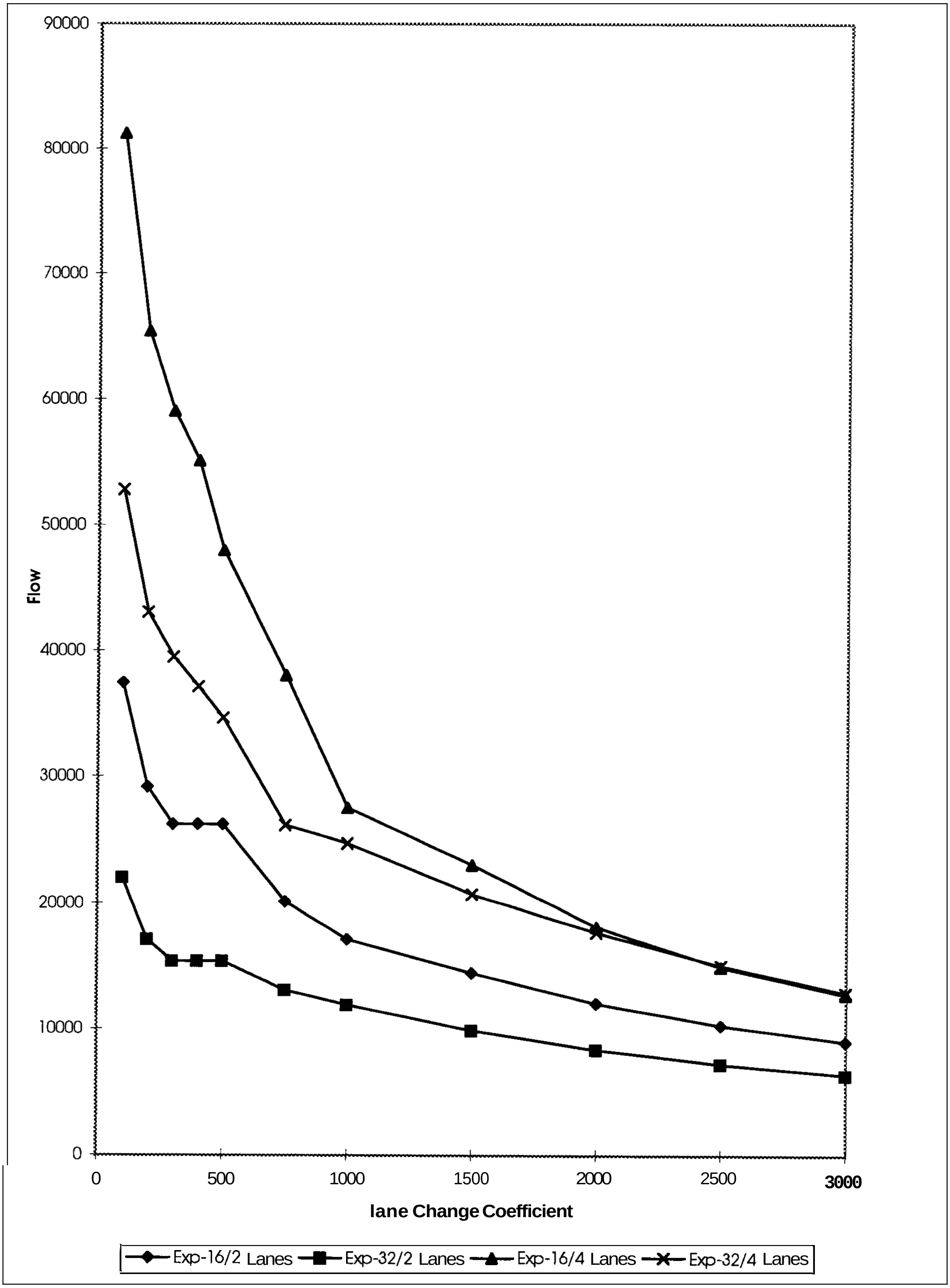


Figure 7: Total Flow versus Lane Change Coefficient for Exponential Cases (48 Segments)

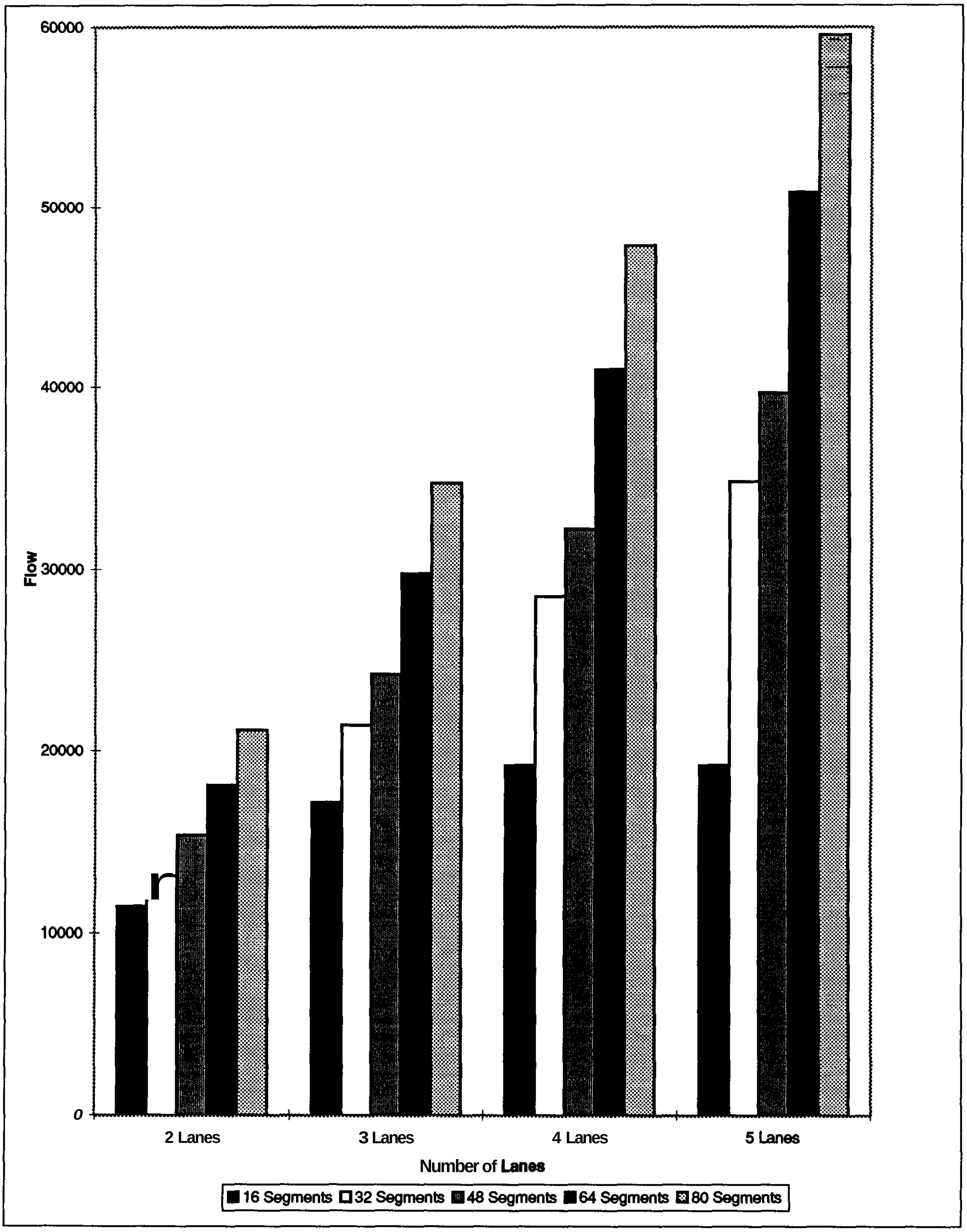


Figure 8: Total Flow versus Highway Length (Exponential432 OD)

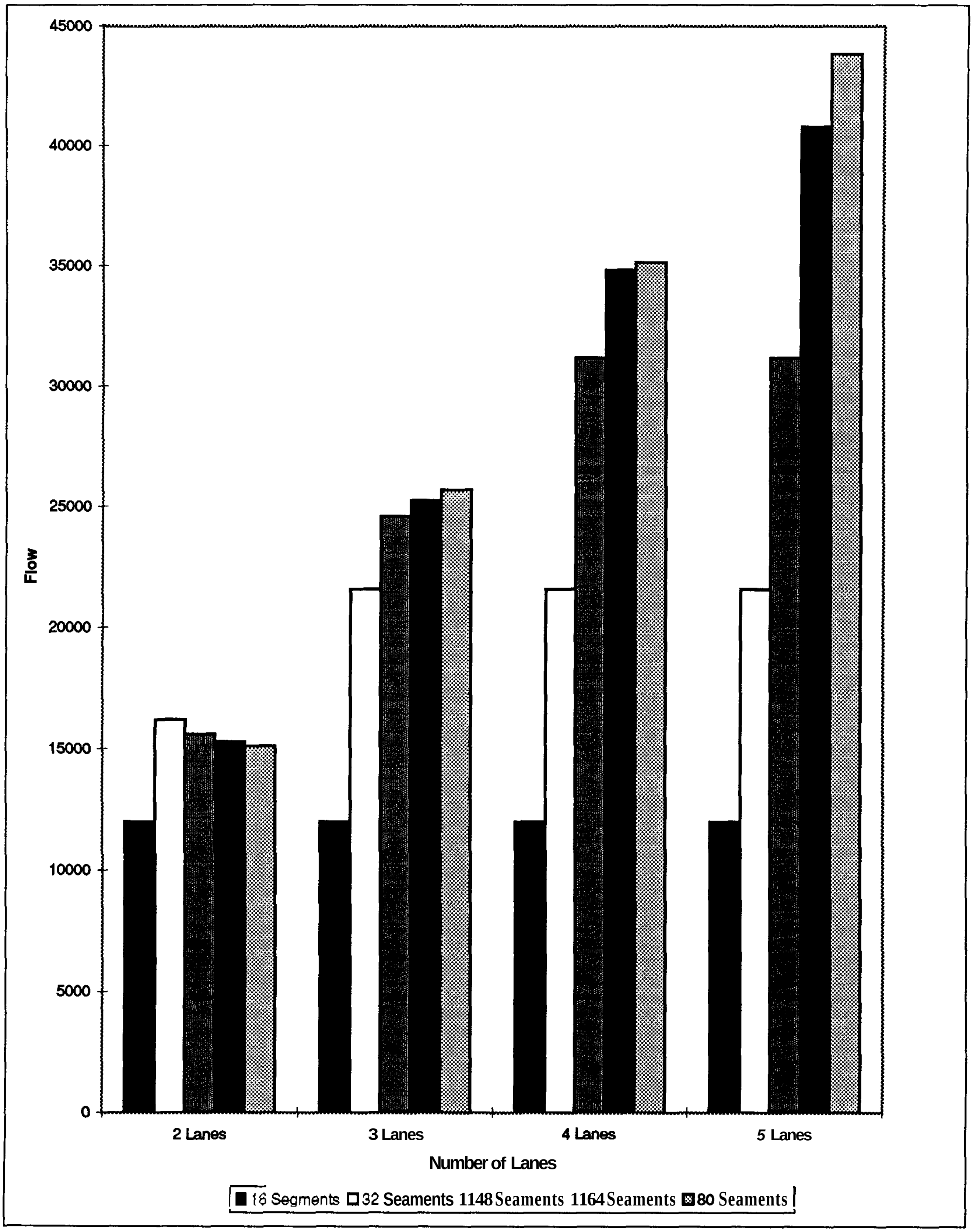


Figure 9: Total Flow versus Highway Length (Equalized OD)

Figures 10 and 11 show total flow on highways that have two manual lanes, with **1** - 3 automated lanes. Figure 10 applies to the exponential-**16** origiddestination pattern, which can be compared to Figure **4.** Figure 11 applies to the exponential-32 pattern, which can be compared to Figure 5. Perhaps to no surprise, highways with the same number of lanes perform better when all of the lanes are automated, but the difference in performance declines as the lane-change coefficient increases. On the other hand, for a given number of automated lanes, addition of manual lanes provides an incremental capacity gain, ***so*** long as $c_{in} = c_{out}$ is larger on the automated lane than on the manual lane.

4.2 Characteristics of Solution

Lane-change patterns were examined for two **48** segment highways, one with an equalized origin-destination pattern and the other with an irregular pattern. In both cases, the highway had **3** automated lanes and $c_{in} = c_{out} =$ ***500*** meter-seconds. ***In both cases, lane changes from the rightmost lane to off-ramps and from on-ramps to the rightmost lane were not counted***

Figure 12 shows left and right lane changes, as a function of the distance from the destination for two origin-destination matrices. For both matrices, a regular pattern is observed. Right lane changes typically occur 2, 6, 10, ... segments fiom the destination (i.e., in the "neither" segment following an on-ramp segment), **and** left lane-changes typically occur **4, 8, 12,** ... segments from the destination (i.e., in off-ramp segments). On-ramp segments, and neither segments preceeding off-ramp segments, have few lane changes. These segment types are heavily taxed by traffic entering from on-ramps or leaving through off-ramps. The linear-program compensates for this traffic by minimizing lane-changes in other lanes.

Figure 12 also shows that right lane changes are concentrated close to the destination, expecially in the irregular pattern. We speculate that left lane changes are concentrated close to the destination (our destination based commodity formulation does

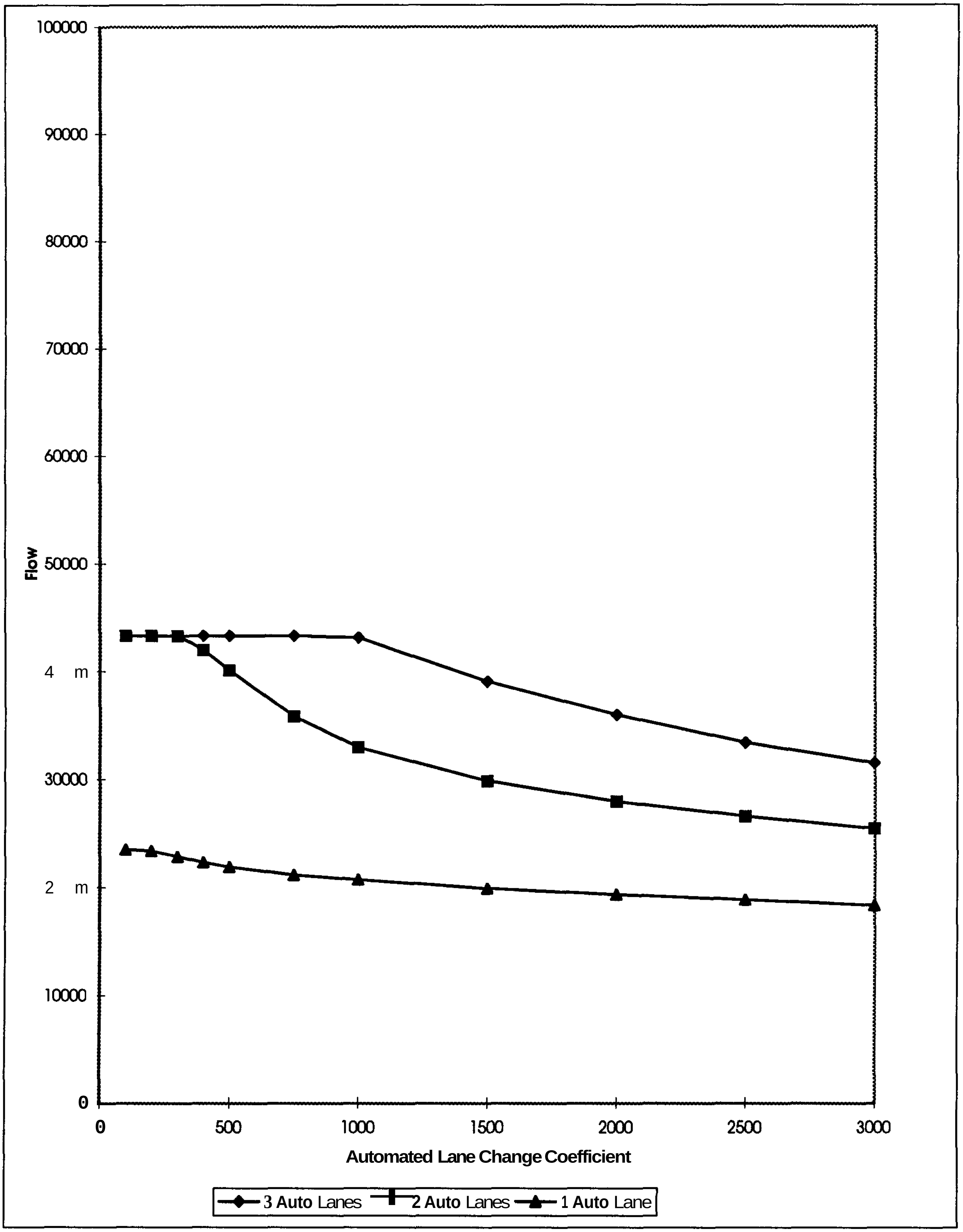


Figure 10: Total Flow for Highway with 2 Manual Lanes (Exponential-16 OD, 48 Segments)

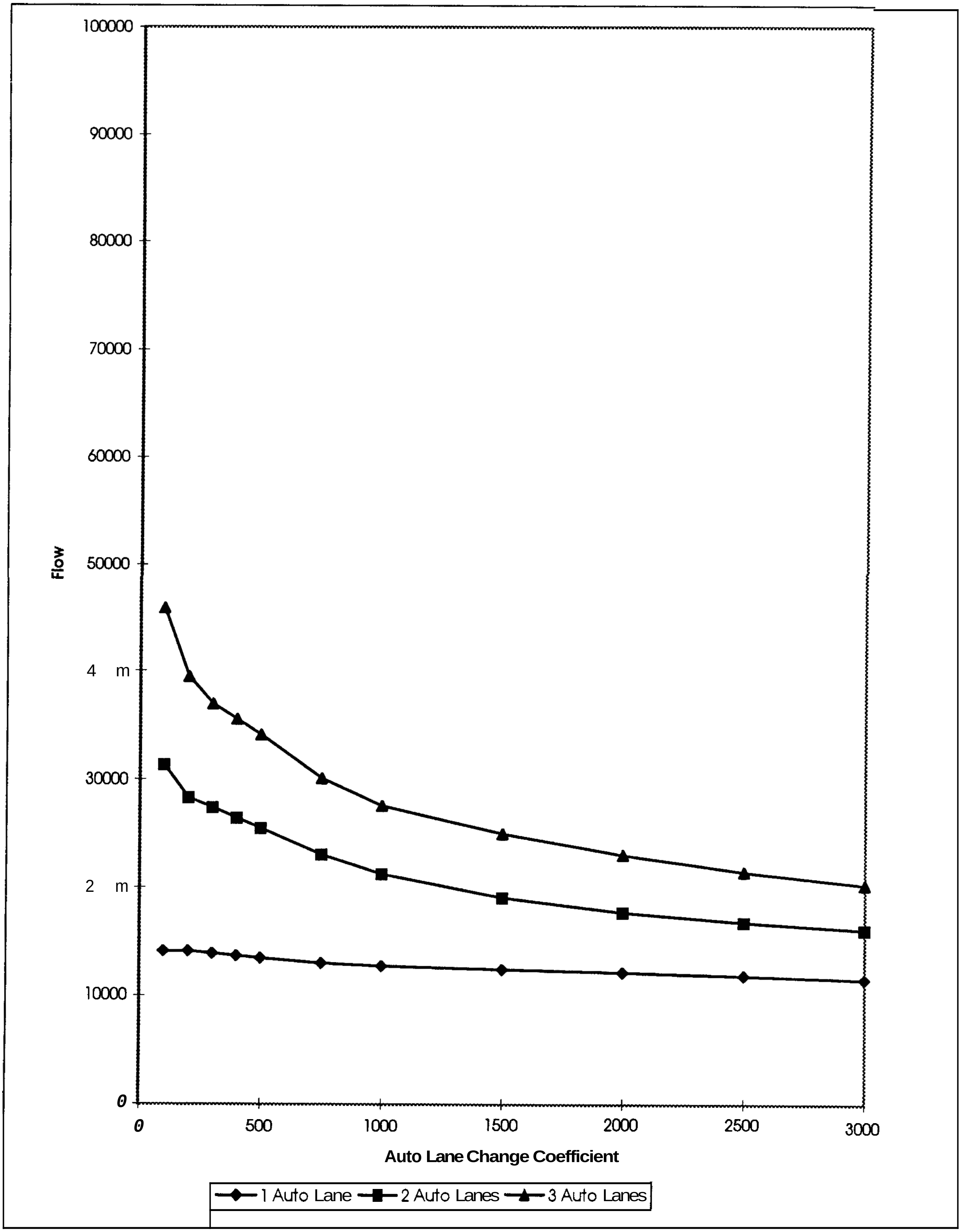
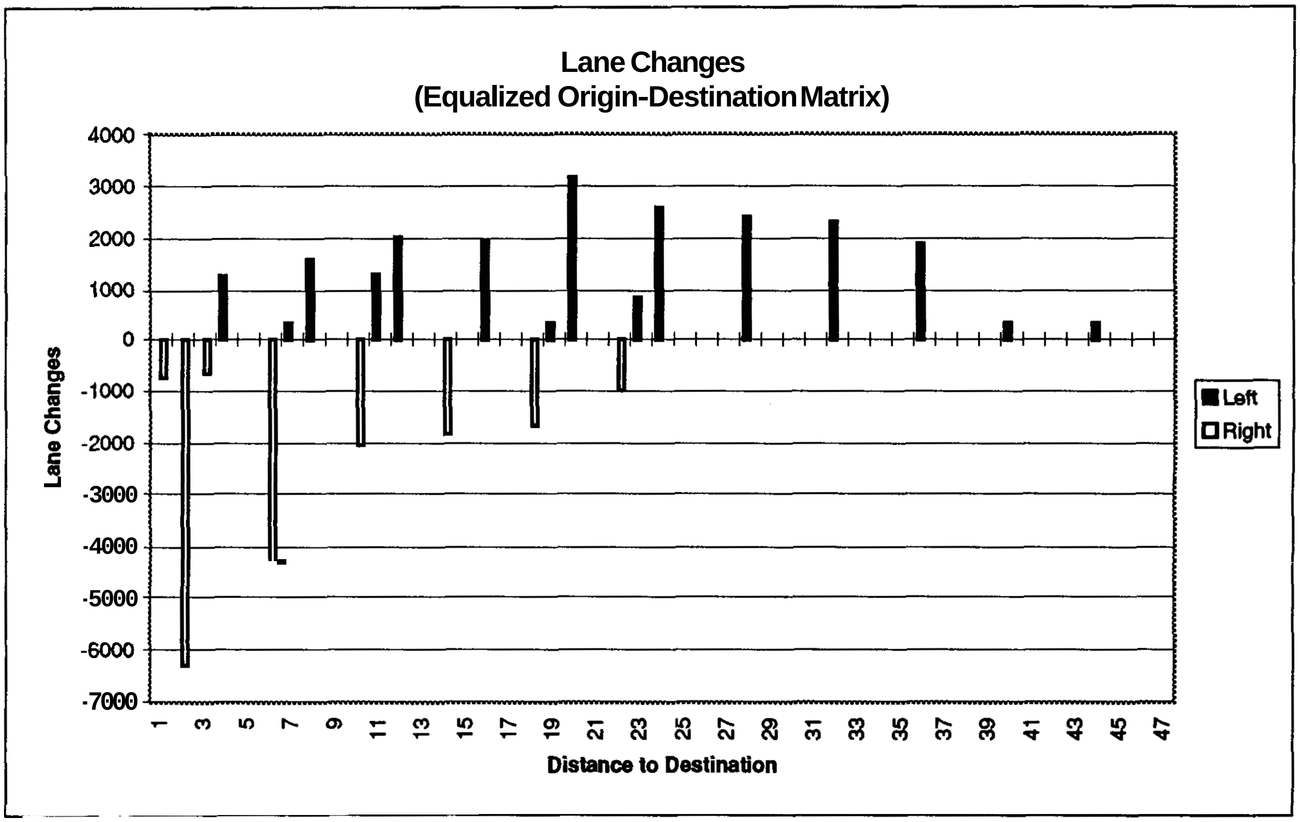


Figure 11: Total Flow for Highway with 2 Manual Lanes (Exponential-32, 48 Segments OD)



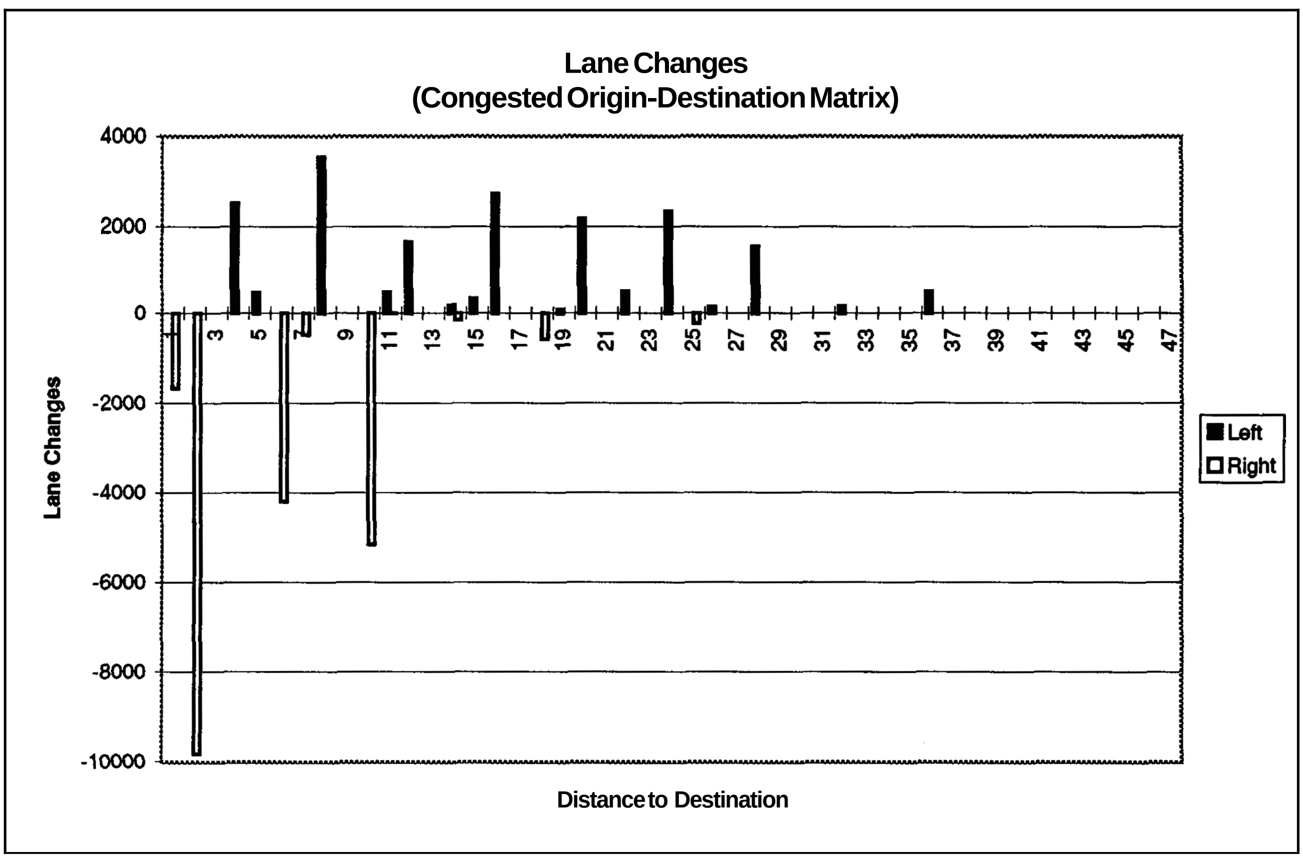


Figure 12: Lane Changes for the (a) Equalized and (b) Irregular OD patterns

not readily allow **us** to count this quantity). Combined, these results suggest that the formulation might be simplified by limiting left lane changes to off-ramp segments in the vicinity of the vehicle's origin, and limiting right lane changes to neither segments in the vicinity of the destination.

The final graph, Figure **13,** shows cumulative lane changes (right and left) for the two origin-destination patterns. ***As*** should be expected, lane changes are much more concentrated in the irregular pattern close to the middle of the highway. Segment 20 absorbs over half of the **left** lane changes, and segment **26** absorbs about **30%** of the right lane changes. These "weaving" segments are system bottlenecks, limiting total network flow. Nevertheless, with some exceptions, lane changes still occur in regular **4** segment intervals, suggesting that the formulation can be simplified even when the origin-destination pattern is irregular.

5. CONCLUSIONS

This paper has developed and tested a linear programming based model that assigns traffic to lanes on an automated highway with the objective of maximizing total network flow. The formulation assumes a fixed origin-destination pattern (expressed on a proportional basis) and a workload based capacity formulation. Although computational efficiency was not the focus of the research, **CPU** times were reasonable for research purposes, allowing highways with up to 80 segments and ***5*** lanes to be optimized.

It can be concluded **from** the ***set*** of experiments that conditions may exist whereby increasing the number of AHS lanes beyond **2** or **3** only provides incremental capacity gains. This would be true if the lane-change coefficient exceeds 1000 meter-seconds, especially when origin-destination patterns are irregular. In all cases, increasing the

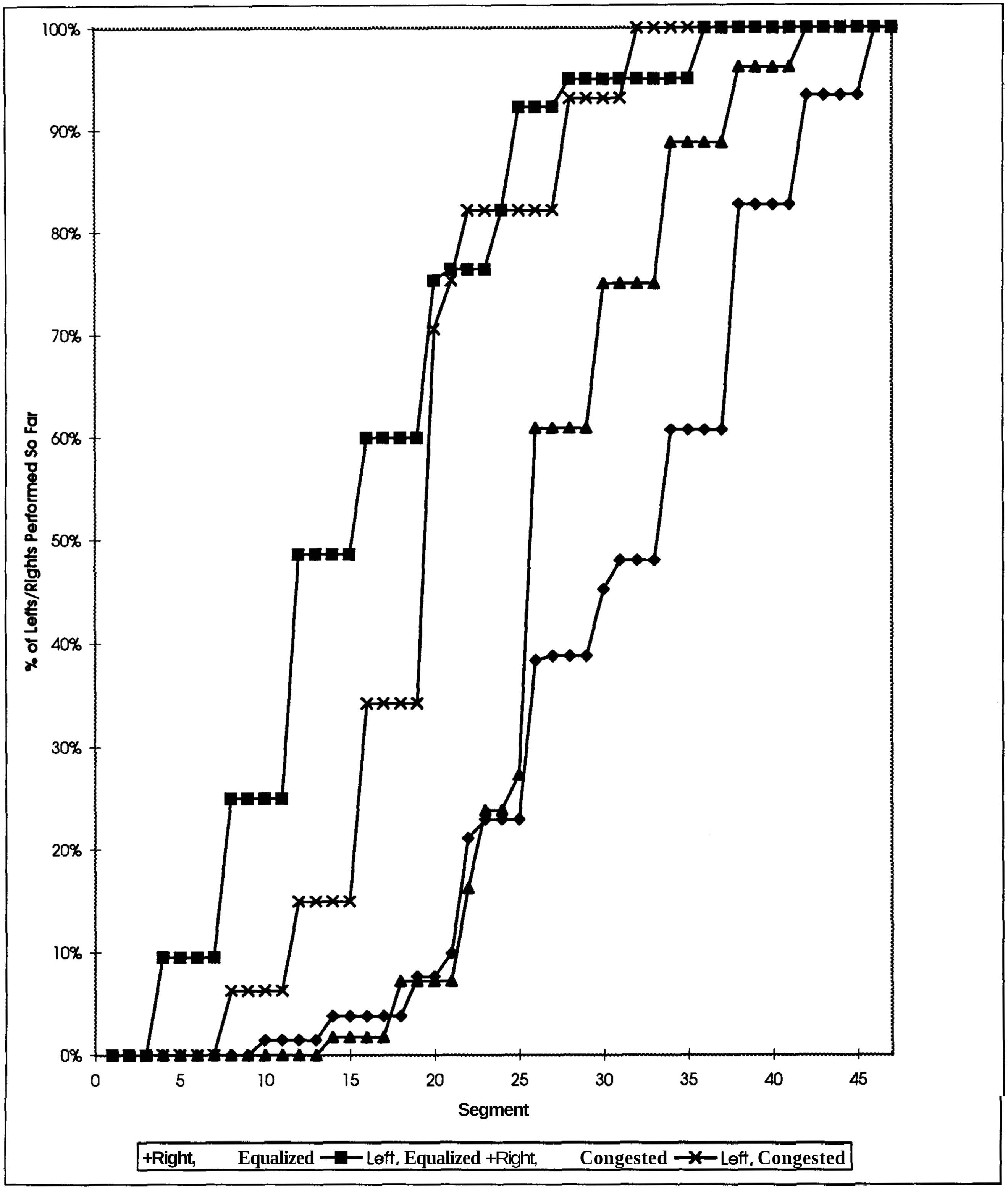


Figure 13: Cumulative Percentage of Lane Changes on a 3 Lane, 48 Segment Highway

number **of** lanes provides decreasing marginal returns, due to the added overhead for lane changes.

Future research will concentrate on dynamic formulations. This will entail discretizing time, and optimizing flows on a period by period basis. To solve such problems within reasonable computation time, it appears that the formulation will have to be simplified. This may include restrictions on the direction of lane changes within each segment, and probibiting lane changes when vehicles are distant from either the origin or the destination. It may also prove practical to employ path-based or decomposition-based methods to exploit the fact that the number of reasonable paths for any origin-destination pair **is** likely to be small.

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