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On the Coulomb Sum Rule in the Relativistic Nuclear Many-body Problem

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Abstract

It is shown that in the relativistic many-body theory the Coulomb sum rule value is never exhausted in the space-like momentum transfer region. This implies that the Coulomb sum rule should be used with a particular caution to analyze deep inelastic electron scattering from nuclei.

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Currently, the Coulomb sum rule is of special interest relating to deep inelastic electron scattering from nuclei[1]. In the nonrelativistic quantum many-body theory, the Coulomb sum rule gives a simple and very useful relation between the static two-body correlation function and the inelastic electron scattering cross section integrated over the energy loss[2]. Quite recently a relativistic extension of the nonrelativistic Coulomb sum rule has been made by Walecka[3]. He also evaluated the relativistic two-body correlation function in the relativistic mean-field theory[4], which provides a concise and realistic description of nuclei[5], and observed that the anomalous magnetic moment of nucleon dominates the Coulomb sum rule value at high momentum transfer region ($q \gtrsim 2k_F$), indicating the importance of dynamical behavior of mesons in nuclei.

The Coulomb sum rule involves the integral of the Coulomb response function over the entire positive energy region. This is so even in the relativistic case[3]. Thus it is not apparent that the sum rule value is really exhausted, as commonly assumed, in the space-like momentum transfer region ($\omega < q$), which can be reached by the electron scattering.

In the following we shall carefully inspect the relativistic Coulomb sum rule by focusing on the distribution of the sum rule value over two different kinematical regions, i.e., space-like vs time-like ($\omega > q$) momentum transfer regions. After the analysis of the lowest order Coulomb response function in the relativistic mean-field theory, we shall see a remarkable upshot that the sum rule value of the Fermi sea nucleons is not exhausted in the space-like momentum transfer region, even in the extreme momentum transfer limit $q \to \infty$. The remaining part of the sum rule value is found in the time-like momentum region, mixed with the contribution from the Dirac sea nucleons, i.e., the vacuum polarization of baryon field.

To make the following argument transparent, we first assume that the baryon is a point-like Dirac particle with no anomalous magnetic moment. Then the relativistic Coulomb sum rule of Walecka[3)] is written as

$$S(q) \equiv \int_0^\infty d\omega\, W_{Coul.}(\underset{\sim}{q}^2,\omega)$$

$$= Z + \iint e^{-i\underset{\sim}{q}\cdot\underset{\sim}{x}}\, g(\underset{\sim}{x},\underset{\sim}{y})\, e^{i\underset{\sim}{q}\cdot\underset{\sim}{y}}\, d\underset{\sim}{x}d\underset{\sim}{y} \quad , \tag{1}$$

where $W_{Coul.}(\underset{\sim}{q}^2,\omega)$ is the Coulomb response function and $g(\underset{\sim}{x},\underset{\sim}{y})$ is a function defined by the trace of a product of proton charge operator $Q = (1 + \tau_3)/2$ and the baryon two-body correlation function over Dirac ($\alpha,\beta,\ldots = 1,\ldots,4$) and isospin ($k,l,\ldots = 1,2$) indices,

$$g(\underset{\sim}{x},\underset{\sim}{y}) = Q_{\alpha\beta,kl}\; g_{\alpha\beta,kl;\gamma\delta,ij}\,(\underset{\sim}{x},\underset{\sim}{y})\; Q^{+}_{\gamma\delta,ij} \quad , \tag{2}$$

$$g_{\alpha\beta,kl;\gamma\delta,ij}\,(\underset{\sim}{x},\underset{\sim}{y}) = \langle\psi^{+}_{\alpha k}(\underset{\sim}{x})\psi^{+}_{\gamma i}(\underset{\sim}{y})\psi_{\delta j}(\underset{\sim}{y})\;\psi_{\beta l}(\underset{\sim}{x})\rangle$$

$$- \langle\psi^{+}_{\alpha k}(\underset{\sim}{x})\psi_{\beta l}(\underset{\sim}{x})\rangle\langle\psi^{+}_{\gamma i}(\underset{\sim}{y})\psi_{\delta j}(\underset{\sim}{y})\rangle \tag{3}$$

where the symbol <...> indicates the ground state expectation value.

The two-body correlation function summarizes all informtion about the dynamics of two baryon interaction inside the nuclei. It is, however, extremely difficult to carry out a reliable calculation of this function in the framework of the relativistic quantum field theory, and this is far beyond the scope of the present study. Instead, we simply evaluate it here in the relativistic mean-field theory. The prescription was given by Walecka in ref. 3); the baryon field is first expressed in terms of the Dirac spinors, which satisfy modified Dirac equations, $(i\not{k}+M^*)U(\underset{\sim}{k},\lambda) = 0$ and $(i\not{k}-M^*)V(-\underset{\sim}{k},\lambda) = 0$ with an effective mass M^*,

$$\psi_{\alpha 1}(x) = \frac{1}{\sqrt{\Omega}} \sum_{\underset{\sim}{k}\lambda\rho} [U(\underset{\sim}{k},\lambda)_\alpha \eta(\rho)_1 e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}} a_{\underset{\sim}{k}\lambda\rho} + V(-\underset{\sim}{k},\lambda)_\alpha \eta(\rho)_1 e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}} b^+_{\underset{\sim}{k}\lambda\rho}] \ . \quad (4)$$

Inserting this into Eq. (3) one may readily compute the ground state expectation value, noting that the ground state is composed of an equal number of protons and neutrons filled in the modified baryon spectra up to the Fermi level according to the Pauli principle. The trace in Eq. (2) is then carried out with the aid of the energy-projection relations, $\sum_\lambda U^+(\underset{\sim}{k},\lambda)_\mu U(\underset{\sim}{k},\lambda)_\nu = [(\underset{\sim}{\alpha}\cdot\underset{\sim}{k} + \beta M^* + E_{\underset{\sim}{k}})/(2E_{\underset{\sim}{k}})]_{\mu\nu}$ and $\sum_\lambda V^+(-\underset{\sim}{k},\lambda)_\mu V(-\underset{\sim}{k},\lambda)_\nu = [(\underset{\sim}{\alpha}\cdot\underset{\sim}{k} - \beta M^* + E_{\underset{\sim}{k}})/(2E_{\underset{\sim}{k}})]_{\mu\nu}$, with $E_{\underset{\sim}{k}} = \sqrt{\underset{\sim}{k}^2 + M^{*2}}$. We end up with the following result for the Fourier transform of the relativistic Pauli correlation function.

$$\int e^{-i\underset{\sim}{q}\cdot(\underset{\sim}{x}-\underset{\sim}{y})} g(\underset{\sim}{x},\underset{\sim}{y})d\underset{\sim}{x}d\underset{\sim}{y} = G^F(q) + G^D(q) \quad , \quad (5)$$

$$G^F(q) = -\sum_{\underset{\sim}{k}} \frac{(E_{\underset{\sim}{k}}+E_{\underset{\sim}{k}+\underset{\sim}{q}})^2 - q^2}{4E_{\underset{\sim}{k}}E_{\underset{\sim}{k}+\underset{\sim}{q}}} n_{\underset{\sim}{k}}\, n_{\underset{\sim}{k}+\underset{\sim}{q}} \quad (6)$$

$$G^D(q) = \sum_{\underset{\sim}{k}} \frac{(E_{\underset{\sim}{k}}-E_{\underset{\sim}{k}-\underset{\sim}{q}})^2 - \underset{\sim}{q}^2}{2E_{\underset{\sim}{k}}E_{\underset{\sim}{k}+\underset{\sim}{q}}} n_{\underset{\sim}{k}} - \sum_{\underset{\sim}{k}} \frac{(E_{\underset{\sim}{k}}+E_{\underset{\sim}{k}+\underset{\sim}{q}})^2 - \underset{\sim}{q}^2}{4E_{\underset{\sim}{k}}E_{\underset{\sim}{k}+\underset{\sim}{q}}} \quad (7)$$

where $n_{\underset{\sim}{k}} = \theta(k - k_F)$ is the ground state distribution function. In ref. 3), only the first term $G^F(q)$ is retained in Eq. (5). Another term $G^D(q)$, which contains a divergent integral over $\underset{\sim}{k}$, arises due to the inclusion of the wave function $V(-\underset{\sim}{k},\lambda)$, which corresponds to the negative energy baryon state in the Dirac sea. This term was omitted in ref. 3), similar to the procedure to neglect the zero point motion energy of the baryon field in the mean-field theory[4]. As we shall see later, however, this term is very important to understand the distribution of sum rule value over the time-like momentum transfer region.

Now we shall compute the ω-integral of the Coulomb response function in Eq. (1) in the same approximation scheme and compare it to the above result. According to the linear response theory [2),6),7)], the Coulomb response function, $W_{Coul.} = W_{44}$, can be related to the fourth component of the imaginary part of the photon self-energy tensor in nuclei

$$W_{\mu\nu}(q,\omega) = -\frac{\Omega}{\pi}\,\mathrm{Im}\,\Pi_{\mu\nu}(q,\omega) \qquad (8)$$

where Ω is the volume of the system. This relation simply means that inelastic electron scattering is described, in one-photon exchange approximation, as the absorption of a virtual photon emitted by the electron in nuclei. In the mean-field approximation, the polarization tensor $\Pi_{\mu\nu}$ is reduced to the form

$$\Pi^{*}_{\mu\nu} = i\int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\,[\mathcal{J}_\mu G^{*}(k)\mathcal{J}_\nu G^{*}(k+q)] \qquad (9)$$

where $\mathcal{J}_\mu = Q\gamma_\mu$ is the electromagnetic vertex of the point-like Dirac baryon and the baryon propagator in the mean-field theory is given by

$$G^{*}(k) = [-i\not{k} + M^*]\left\{-\frac{1}{k_\mu^2+M^{*2}-i\epsilon} + 2\pi i\,\delta(k_\mu^2 + M^*)^2\,\theta(k_0)n_{\underset{\sim}{k}}\right\} \qquad (10)$$

Then, noting that $\Omega\int\frac{d^3k}{(2\pi)^3} = \sum_{\underset{\sim}{k}}$, a straightforward calculation leads to

$$\begin{aligned} W_{\mu\nu}(q,\omega) = &-\sum_{\underset{\sim}{k}} \frac{1}{4E_{\underset{\sim}{k}}E_{\underset{\sim}{k}+\underset{\sim}{q}}}\,\delta(\omega-E_{\underset{\sim}{k}+\underset{\sim}{q}} + E_{\underset{\sim}{k}})[n_{\underset{\sim}{k}}(1-n_{\underset{\sim}{k}+\underset{\sim}{q}}) + n_{\underset{\sim}{k}+\underset{\sim}{q}}(1-n_{\underset{\sim}{k}})]\,\mathcal{T}_{\mu\nu}(k_0= E_{\underset{\sim}{k}}) \\ &-\sum_{\underset{\sim}{k}} \frac{1}{4E_{\underset{\sim}{k}}E_{\underset{\sim}{k}+\underset{\sim}{q}}}\,\delta(\omega+E_{\underset{\sim}{k}} + E_{\underset{\sim}{k}+\underset{\sim}{q}})(1-n_{\underset{\sim}{k}})\,\mathcal{T}_{\mu\nu}(k_0= E_{\underset{\sim}{k}}) \\ &-\sum_{\underset{\sim}{k}} \frac{1}{4E_{\underset{\sim}{k}}E_{\underset{\sim}{k}+\underset{\sim}{q}}}\,\delta(\omega-E_{\underset{\sim}{k}} - E_{\underset{\sim}{k}+\underset{\sim}{q}})(1-n_{\underset{\sim}{k}+\underset{\sim}{q}})\,\mathcal{T}_{\mu\nu}(k_0= -E_{\underset{\sim}{k}}) \ , \end{aligned} \qquad (11)$$

with $\mathcal{T}_{\mu\nu} = q_\mu q_\nu - q^2\delta_{\mu\nu} - (q_\mu + 2k_\mu)(q_\nu + 2k_\nu)$ where $q_\mu = (\underset{\sim}{q}, i\omega)$ and

and $k_\mu = (\underset{\sim}{k}, ik_0)$. The current conservation, $q_\mu W_{\mu\nu} = 0$, can be readily recovered from this result. The above result was first obtained by Chin in the context of the study of collective mode[6]. Using Eq. (11) one can separate the Coulomb sum rule value into two parts,

$$\int_0^\infty d\omega\, W_{\mathrm{Coul.}}(q,\omega) = \int_0^\infty d\omega\, W_{44}(q,\omega) = S^{SL}(q) + S^{TL}(q) \tag{12}$$

where $S^{SL}(q)$ and $S^{TL}(q)$ correspond to the <u>space-like</u> energy integral and the <u>time-like</u> energy integral of $W_{44}(q,\omega)$ respectively

$$S^{SL}(q) \equiv \int_0^q d\omega\, W_{44}(q,\omega) = \sum_{\underset{\sim}{k}} \frac{(E_{\underset{\sim}{k}}+E_{\underset{\sim}{k}+\underset{\sim}{q}})^2 - \underset{\sim}{q}^2}{4E_{\underset{\sim}{k}}E_{\underset{\sim}{k}+\underset{\sim}{q}}}\, n_{\underset{\sim}{k}}(1 - n_{\underset{\sim}{k}+\underset{\sim}{q}}) \tag{13}$$

$$S^{TL}(q) \equiv \int_q^\infty d\omega\, W_{44}(q,\omega) = -\sum_{\underset{\sim}{k}} \frac{(E_{\underset{\sim}{k}}-E_{\underset{\sim}{k}+\underset{\sim}{q}})^2 - \underset{\sim}{q}^2}{4E_{\underset{\sim}{k}}E_{\underset{\sim}{k}+\underset{\sim}{q}}}\, (1 - n_{\underset{\sim}{k}+\underset{\sim}{q}}) \tag{14}$$

If one compares this result with the previous one for the two-body correlation function, Eqs. (5)-(7), then the following relations can be found

$$S^{SL}(q) = Z + G^F(q) + \Delta G(q) \quad , \tag{15}$$

$$S^{TL}(q) = \sum_{\underset{\sim}{k}} 1 + G^D(q) - \Delta G(q) \quad , \tag{16}$$

where

$$\Delta G(q) = \sum_{\underset{\sim}{k}} \frac{(E_{\underset{\sim}{k}}-E_{\underset{\sim}{k}+\underset{\sim}{q}})^2 - \underset{\sim}{q}^2}{4E_{\underset{\sim}{k}}E_{\underset{\sim}{k}+\underset{\sim}{q}}}\, n_{\underset{\sim}{k}} \quad . \tag{17}$$

The striking observation here is that the sum rule value, $Z + G^F(q)$, which results from the neglect of antinucleon wave function, does not coincide with the <u>space-like</u> energy sum of the Coulomb response function. The difference $\Delta G(q)$ is always negative and has the limiting behaviors, $\Delta G(0) = 0$ and

$\Delta G(\infty) = -Z/2$. Since $G^F(q) = 0$ for $q \geq 2P_F$, this implies that only a half of the total charge of the system is visible by the electron scattering with extremely large momentum transfer. The missing part of the sum rule value appears in the time-like region, mixed with the antinucleon (or Dirac sea nucleon) sum rule value, as shown in Eq. (16). If one takes the non-relativistic limit by setting $M^*/q \to \infty$ and $M^*/p_F \to \infty$, then $\Delta G(q)$ vanishes and thereby one sees that the sum rule value of the Fermi sea nucleon is completely exhausted in the space-like momentum transfer region. It is the relativistic kinematics that makes $\Delta G(q)$ finite and shifts a part of sum rule value out of sight. [Because of the close resemblance between $\Delta G(q)$ and the density dependent term in $G^D(q)$ (see Eq. (7)) except for a factor 2, one may interpret that $\Delta G(q)$ arises due to the Pauli correlation between the Fermi sea nucleon and the Dirac sea nucleon. Also note that the divergent term $\sum_{\tilde{k}} 1$ in Eq. (16), corresponding to the total charge of the baryons in the infinite Dirac sea, has to be also added in the right-hand side of Eq. (1) for completeness.]

In Fig. 1 the numerical result of $S^{SL}(q)$ normalized by the total charge Z is compared to the total normalized sum rule value, $1 + G^F(q)/Z$, for the relevant momentum regions. The space-like component of the sum rule value strongly depends on the effective mass of the baryon, while the total sum rule value remains very close to the nonrelativistic result. With a small effective mass $M^* = 0.56$ M we find 15% reduction in the space-like component of the sum rule value at $q = 2k_F$, and with $M^* = 0.70M$ this number is reduced to 11%.

So far we have neglected the presence of the anomalous magnetic moment of nucleon and the nucleon electromagnetic form factor. The inclusion of these effects can be made by simply replacing the electromagnetic vertex operator in Eq. (9) by $\tilde{\mathcal{J}}_\mu = F_1(q)\gamma_\mu + F_2(q)\sigma_{\mu\nu}q_\nu$. If we assume the intrinsic nucleon properties do not change in nuclei, although this might not be the case[8,9], then we can take $F_1(q) = (1 + \tau_3)/2$ and $F_2(q) = [\lambda_p(1+\tau_3)/2 + \lambda_n(1-\tau_3)/2]/(2M)$, where $\lambda_p = 1.7928$ and $\lambda_n = -1.9131$ are the static anomalous moments of proton and neutron respectively and the overall free nucleon form factor $f_{sn}(q^2)$ is removed. Corresponding sum rule value is obtained by substituting the effective charge operator defined by $\tilde{Q} = \tilde{\mathcal{F}}_4\gamma_4$ into Eq. (2) and by replacing Z by $[1 + \underset{\sim}{q}^2\lambda_p^2/(2M)^2]Z + [\underset{\sim}{q}^2\lambda_n^2/(2M)^2]N$. The result is shown in Fig. 2. Again we find a considerable difference between the total sum rule value C(q) of the Fermi sea nucleon [this is exactly what was evaluated by Walecka[3]] and its space-like visible part. Note that the difference becomes minimum at $M^* \sim 0.35M$ around $q = 2p_F$, converting the order of the curves for different effective masses from that of Fig. 1.

In summary, we have studied the distribution of the Coulomb sum rule value over the space-like and time-like momentum transfer regions by analyzing the lowest order Coulomb response function. In the present calculation the nucleus is virtually viewed as a relativistic Fermi gas of baryon with an effective mass M^*, and, therefore, only the Pauli correlation between baryons is taken into account. Our key observation was that the Coulomb sum rule value of the Fermi sea nucleon is incompletely exhausted in the space-like momentum transfer region and thus, in principle, it cannot be entirely seen by the inclusive electron scattering. Of course the actual experimental situation would be far more complicated because of the exchange current[10] and the production of real mesons[11] and baryon resonances[12], which are not included in the present study, and certainly requires more elaborate study including these effects. Although we used a specific model description of the nuclear many-body system, one may consider that our finding is one of the general aspects of the relativistic many-body problem.

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Figure Captions

Fig. 1. Space-like component of the Coulomb sum rule value [see Eq. (13)]. The total sum rule value of the Fermi sea nucleon (dashed line) does not depend much on the effective baryon mass M* and appears very close to the nonrelativistic result.

Fig. 2. Space-like component of the Coulomb sum rule value with nucleon anomalous magnetic moments. C(q) is the total sum rule value of the Fermi sea nucleon obtained by Walecka[3)] [We set M* = M here to evaluate C(q)].

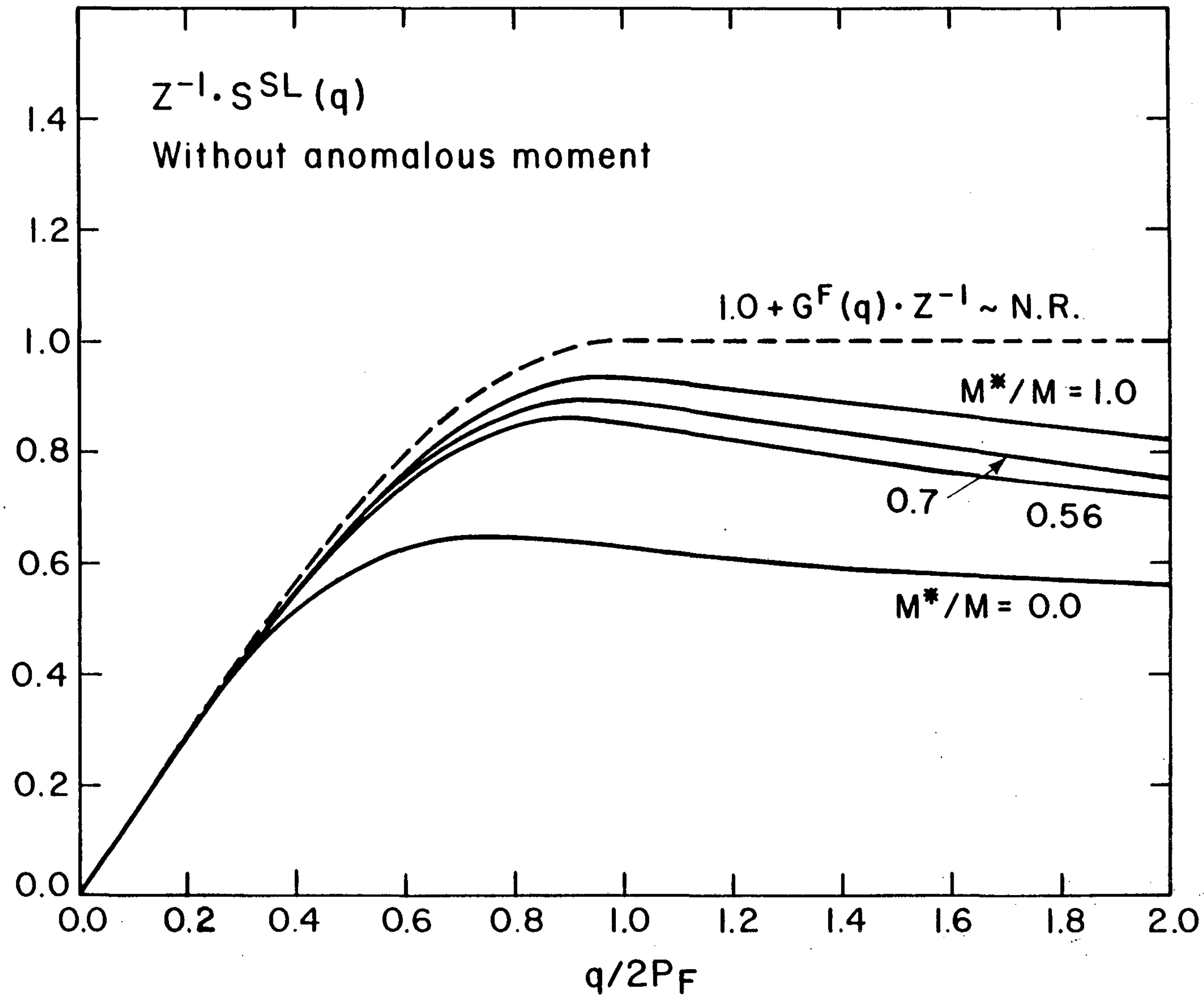

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Fig. 1

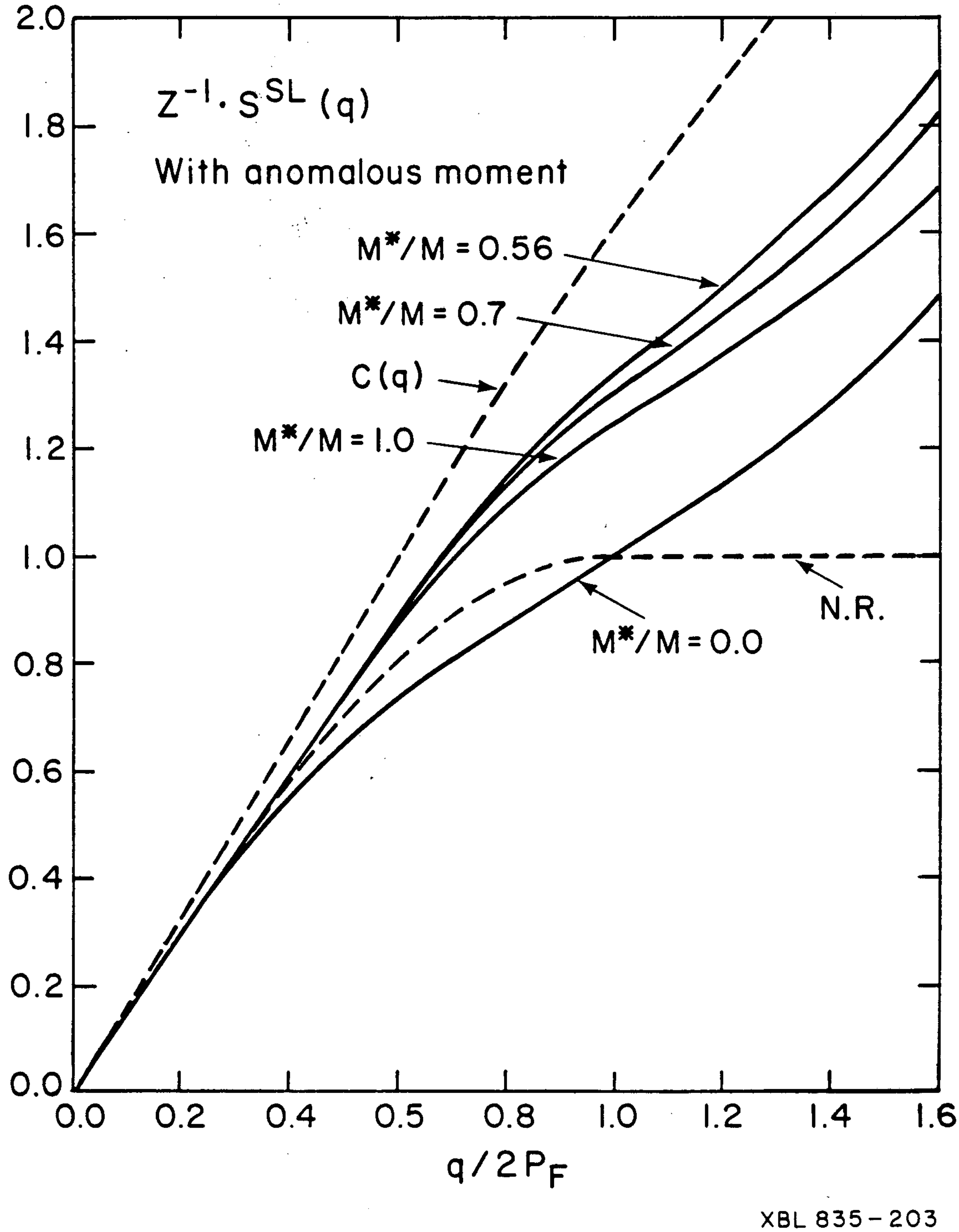

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Fig. 2

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