TABLE II. Coefficients a and $b+r$ of $\log(\alpha c_i)$ vs $\log[2/(\rho_1/\rho_0+\rho_0/\rho_1)]$.

$\log(\alpha c_i) = a + b\log[2/(\rho_0/\rho_1+\rho_1/\rho_0)]$ when $\alpha = 10$			
	$0.1<\rho_1<\rho_0<10$		
	$M=100$	$M=30$	$M=10$
a; b; r	0.44; 0.06; 0.99	0.33; 0.06; 0.98	0.42; 0.04; 0.98

where γ_0 is the Lorentz factor and spans between 1 and 10.

The presence of a shear in cylindrical symmetry has not yet been considered and will presumably produce a cutoff in the instabilities where $\alpha > 2\pi/d$, with d (in jet radius units) the length characterizing the thickness of the shear.

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Effect of quasiconfined particles and $l = 2$ stellarator fields on the negative mass instability in a modified betatron

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A sufficient stability condition for the negative mass instability is derived. This condition is used to show that quasiconfined particles in a modified betatron create fields that can stabilize the negative mass instability. Additionally, it is shown that stellarator fields can inhibit this instability.

Recently there have been efforts to develop high current accelerators. Experiments with plasma betatrons have obtained currents below their design limits. Some attribute this to the negative mass instability.[1,2] Another proposed accelerator is the modified betatron.[3,4] The modified betatron is basically a betatron with a magnetic field along the beam axis. Calculations show that negative mass may also limit current in this device.[1,5–7] These calculations, though, do not consider quasiconfined particles in banana-like orbits, nor do they consider stellarator fields. In recent experiments with the University of California at Irvine (UCI) Modified Betatron[8,9] and at Maxwell Laboratories,[10] inductive charging[11,12] was used. Quasiconfined particles appear and are far more abundant than the accelerated beam particles. Additionally, $l=2$ stellarator fields have been proposed for beam focusing[6] and have been added to the (UCI) Modified Betatron.

A sufficient stability condition for negative mass is derived. This condition can be used to examine the effect of quasiconfined particles and stellarator fields. It is found that even the quasiconfined particle densities already obtained at UCI and Maxwell Laboratories will stabilize a beam to an energy of about 50 MeV. Also it is found that a stellarator field can stabilize negative mass.

To derive a sufficient stability condition, consider the motion of a beam particle in a toroidal configuration as seen in Fig. 1. If the beam develops a region of higher density (clump), then the negative mass instability will exist if the resulting particle motion is toward the clump. To see this consider Fig. 2. In Fig. 2 it can be seen that a clump in the beam will create electric fields that cause a beam particle to have a perturbed toroidal velocity away from the clump. However, a perturbed toroidal velocity away from the clump will not necessarily result in a perturbed position away from the clump. Specifically, $\delta v_\theta = R_0\,\delta\dot{\theta} - \dot{\theta}\delta x$ and a suitable choice of δx will cause a $\delta\dot{\theta}$ that will move the particle toward the clump (in θ). The criteria for this not to happen is clearly $\delta\dot{\theta}/\delta v_\theta > 0$, which is just

$$\frac{\delta\dot{\theta}}{\delta v_\theta} = \left(\dot{\theta}\frac{\delta x}{\delta v_\theta} + 1\right)\frac{1}{R_0} > 0, \tag{1}$$

where the previous expression for δv_θ has been used. Self-consistent Vlasov and fluid treatments find that when the above equation is evaluated for a modified betatron it describes a sufficient condition, but not a necessary condition, for stability.[5–7,2] Beam temperature is shown to relax this criteria. The above has the advantage that as long as the fields that a beam particle will experience are known, then

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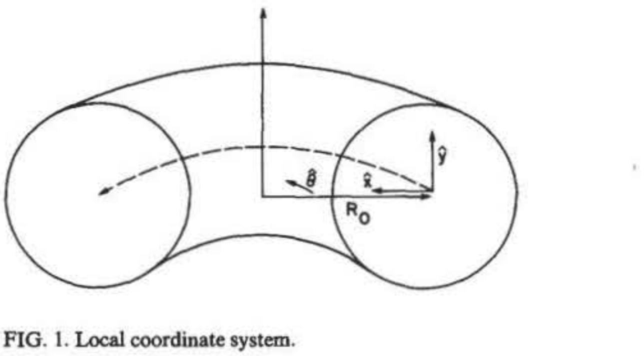

FIG. 1. Local coordinate system.

$\delta x/\delta v_\theta$ can be found and this sufficient condition can be evaluated.

In recent modified betatron experiments that use inductive charging to form a beam, quasiconfined particles have been observed. To include the effect of these particles, consider the $\hat{x}$ equation of motion for a beam particle in a background of quasiconfined particles that have a constant density n':

$$\ddot{x} = -\frac{v_\theta^2}{R_0}\left(1+\frac{x}{R_0}\right) + \Omega_y v_\theta\left(1+\frac{sx}{R_0}\right) - \Omega_\theta \dot{y} + \frac{\omega_p^2}{2\gamma^2}(x-x_0) + \frac{\omega_p'^2}{2}x + \frac{\omega_p^2}{2}\left(\frac{a}{b}\right)^2 x_0, \qquad (2)$$

where $\omega_p'^2 = 4\pi n'e^2/(\gamma m)$, $\omega_p^2 = 4\pi ne^2/(\gamma m)$, n is the accelerated beam density (which has been taken to be a constant), v_θ is the particle toroidal velocity, x_0 is the center of the accelerated beam, the background beam position is taken to be stationary and at $x \approx 0$ (see Ref. 9), it has been assumed that the chamber wall is slotted so that there is no magnetic image field from the $\hat{\theta}$ beam motion, a is the beam radius, b is the minor radius of the torus, $\Omega_y = eB_y/(\gamma mc)$, $\Omega_\theta = eB_\theta/(\gamma mc)$, the betatron field is $B_y(1+sx/R_0)\,\hat{y} + B_y \times (sy/R_0)\hat{x}$, s is the betatron field index, the toroidal magnetic field is $B_\theta\hat{\theta}$, and $|x|/R_0 \ll 1$ has been assumed. Also, it has been assumed that the Budker parameter $n\pi a^2e^2/\gamma mc^2$ is small so that toroidal corrections are negligible. A prescription for including torodial effects is described in Ref. 13.

The above equation can be ensemble-averaged to find an equation for the beam position.[9,13] This leads to the following equation:

$$\ddot{x}_0 = -\frac{v_\theta^2}{R_0}\left(1+\frac{x_0}{R_0}\right) + \Omega_y v_\theta\left(1+\frac{sx_0}{R_0}\right) - \Omega_\theta \dot{y}_0 + \frac{\omega_p'^2}{2}x_0 + \frac{\omega_p^2}{2}\left(\frac{a}{b}\right)^2 x_0, \qquad (3)$$

where it has been assumed that all particles have the same v_θ. In Eq. (3) it can be seen that the solution for the beam position will consist of oscillations of the form $\cos(\omega t)$ (see Ref. 9, 13) about an equilibrium position. The equilibrium beam position (X_0) is the following:

$$X_0 = \frac{(v_\theta^2/R_0) - \Omega_y v_\theta}{[\omega_p^2(a/b)^2/2] + [\omega_p'^2/2] - (v_\theta^2/R_0^2) + (s\Omega_y v_\theta/R_0)} \qquad (4)$$

Similarly, Eq. (2) shows that the average particle position $\langle x\rangle$ is

$$\langle x\rangle = \frac{(v_\theta^2/R_0) - \Omega_y v_\theta + [(\omega_p^2/2\gamma^2) - \omega_p^2(a/b)^2/2]X_0}{(\omega_p^2/2\gamma^2) + (\omega_p'^2/2) - (v_\theta^2/R_0^2) + (s\Omega_y v_\theta/R_0)}. \qquad (5)$$

To evaluate the $\delta x/\delta v_\theta$ term in the sufficient condition for stability [Eq. (1)] the average position of the particles will be used. In the above, let $\langle x\rangle \to \langle x\rangle + \delta x$ and $v_\theta \to v_\theta + \delta v_\theta$. This yields the following expression for $\delta x/\delta v_\theta$:

$$\frac{\delta x}{\delta v_\theta} \cong \frac{\gamma^2\Omega_y}{[\omega_p^2(a/b)^2/2] + (\omega_p'^2/2) - (1-s)\Omega_y^2}, \qquad (6)$$

where $\Omega_y = v_\theta/R_0 = \dot{\theta}$ has been assumed. Using the above in Eq. (1), the following sufficient stability condition is obtained:

$$\frac{\gamma^2\Omega_y^2}{[\omega_p^2(a/b)^2/2] + (\omega_p'^2/2) - (1-s)\Omega_y^2} + 1 > 0. \qquad (7)$$

The result for a conventional betatron can be recovered from the above if $\omega_p^2, \omega_p'^2 \to 0$, in which case the above becomes $\gamma^2 < (1-s)$, which is never satisfied since $0 < s < 1$ is needed for particle orbit stability.[1] The beam self-fields and the quasiconfined particles are stabilizing. In the UCI Modified Betatron, the stabilizing term from the quasiconfined particles is quantitatively larger than the accelerated beam self-field term. Specifically, the above stability condition is satisfied for a particle energy of

$$\gamma < (4\pi n'e^2/m)(R_0/c)^2, \qquad (8)$$

where $\Omega_y R_0 = v_\theta \cong c$ and $s = \frac{1}{2}$ have been assumed. In recent experiments in the UCI Modified Betatron, the quasiconfined particles have a line density of about 10^{12} cm^{-1} (the accelerated beam line density was about 10^{10} cm^{-1}). Using this the density of the quasiconfined particles (n') is about 2×10^{10} cm^{-3}; since $R_0 = 40$ cm, Eq. (8) is then satisfied for $\gamma \lesssim 100$, which corresponds to a particle energy of 50 MeV. In the experiment at Maxwell Laboratories $n' \approx 10^{10}$ cm^{-3} and $R_0 = 50$ cm, so Eq. (8) is satisfied for $\gamma \lesssim 120$ or an energy of 60 MeV. It should also be noted that the injection method used in the UCI Modified Betatron is still under

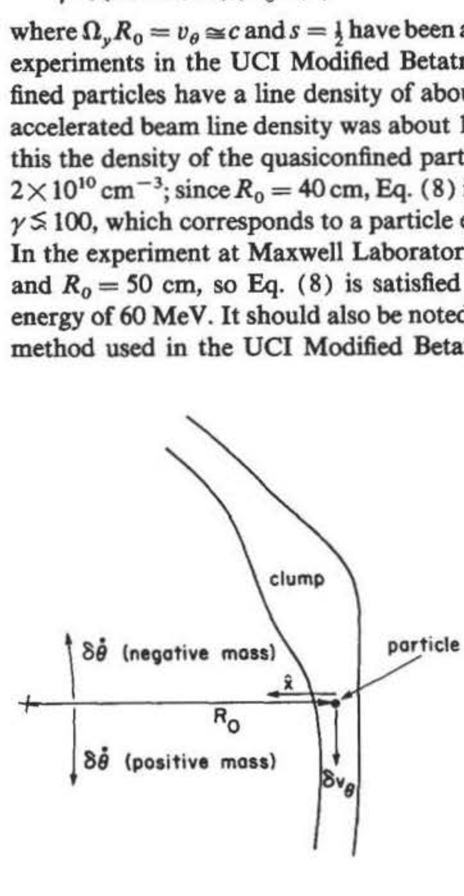

FIG. 2. Negative mass instability. A particle will have a perturbed velocity away from the clump. Because of the geometry, this could cause a perturbed $\delta\theta$ toward the clump, which would create an instability.

development and these particle densities should ultimately be increased. Also, without the quasiconfined particles the criterion in Eq. (7) is satisfied only to an energy of about 2 MeV in the UCI Betatron.

It has been speculated that $l = 2$ stellarator fields can increase the stability of the modified betatron.[6] The effect of stellarator fields can be analyzed using the same procedure that was used with the quasiconfined particles. The $\hat{x}$ equation of motion for a modified betatron with a stellarator field is the following, assuming $|x| \ll R_0$ (see Ref. 14):

$$\ddot{x} = -\frac{v_\theta^2}{R_0}\left(1 + \frac{x}{R_0}\right) + \Omega_y v_\theta\left(1 + \frac{sx}{R_0}\right) - \Omega_\theta \dot{y} + \frac{\omega_p^2}{2\gamma^2}(x - x_0) + \Omega_s v_\theta\left(\cos(j\theta)\frac{y}{R_0} \mp \sin(j\theta)\frac{x}{R_0}\right) + (\omega_p^2/2)(a/b)^2 x_0, \quad (9)$$

where the stellarator field in the $\hat{y}$ direction has been assumed to be $-B_s(x/R_0)\sin(j\theta) + B_s(y/R_0)\cos(j\theta)$, $\Omega_s = eB_s/(\gamma mc)$, and j is twice the number of rotations of the stellarator windings around the torus. To find the equilibrium x position from the above, the beam oscillations must be found since the terms such as $\cos(j\theta)\,y$ will have a nonzero average contribution that depends on the oscillatory behavior of y. To determine this, a Hill equation must be solved and the following is obtained for the equilibrium beam position[14]:

$$X_0 = \frac{(v_\theta^2/R_0) - \Omega_y v_\theta}{\Omega^2 - \{\Omega_s^2/[j^2 + j(B_\theta/B_y) + (\Omega/\Omega_y)^2]\}}, \quad (10)$$

where $\Omega^2 = [\omega_p^2(a/b)^2/2] - (v_\theta^2/R_0^2) + (s\Omega_y v_\theta/R_0)$ and it has been assumed that the appropriate stability conditions are satisfied.[14] From the above, the ratio $\delta x/\delta v_\theta$ can be obtained and Eq. (1) can be used to find the following stability condition:

$$\frac{\gamma^2\Omega_y^2}{\Omega^2 - \{\Omega_s^2/[j^2 + j(B_\theta/B_y) + (\Omega/\Omega_y)^2]\}} + 1 > 0. \quad (11)$$

If $\Omega_s = 0$, then the condition $\omega_p^2(a/b)^2 > 2(1-s)\Omega_y^2\gamma^2$ is sufficient for stability as it is in the modified betatron. However, when $\gamma^2\Omega_y^2 > \omega_p^2(a/b)^2$, then the stellarator fields can still satisfy the stability criterion. Specifically, if the following is satisfied then the above sufficient stability condition is satisfied:

$$\gamma^2 < B_s^2/jB_\theta B_y, \quad (12)$$

where $j > 0$, $j < B_\theta/B_y$, and $|\Omega/\Omega_y| \ll 1$ have been assumed. In a typical experiment with the UCI Modified Betatron with $l = 2$ stellarator fields, $j = 12$, $B_s/B_y \approx 100$, and $B_y/B_\theta \approx 1/25$; Eq. (11) shows that for $\gamma \lesssim 3$ the sufficient stability condition is satisfied. The experiments at UCI do not show the negative mass instability up to $\gamma \approx 9$.

This discussion applies to the conventional negative mass instability that does not include coupling to the transverse modes of the beam. It is consistent with the UCI experimental results for the modified betatron with and without stellarator windings. The beams are observed to be stable below the transition energy for currents of a few hundred amperes. Recently it has been predicted that the modified betatron should be unstable below the transition energy.[15,16] The unstable mode is a hybrid transverse mode that can have a higher growth rate than the usual negative mass instability at high currents. At the present current level (up to 500 A), this instability has not been observed.

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