UNIVERSITY OF CALIFORNIA

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Using Demographic and Geographic Traits to help ensure Fairness in Electoral Redistricting

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by

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ABSTRACT OF THE THESIS

Using Demographic and Geographic Traits to help ensure

Fairness in Electoral Redistricting

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Gerrymandering, the term used to describe the drawing of electoral districts to favor one political party or group of people over another, has become a pressing issue with a developing academic interest. A chief concern associated with gerrymandering is how to identify and quantify it in a way that is understandable and actionable. There have been many proposed methods to quantify the effects of gerrymandering, ranging from studies of compactness, analyses using simulated election and geographic data, and measures of partisan symmetry. These measures and methods help to establish and quantify the impacts of gerrymandering but often are not enough to help in prevention. We will define, implement and examine a new statistic that measures the geographic compactness and similarity of racial make-ups for each electoral district. The goal of our statistic is to create a measure that can detect geographic or demographic irregularities in any existing or

proposed districts that might suggest manipulation. This statistic could be used to help judge the fairness of proposals during the redistricting processes by helping to identify proposed districts that are drawn in an unintuitive way that might seek to favor one party over another.

The thesis of Cole Sanders is approved.

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1 Introduction

Gerrymandering was first used as a term to mock an electoral district drawn by Massachusetts Governor Elbridge Gerry in 1812. The electoral district stretched around the border of the state, connecting rural communities to larger towns in a serpent like fashion in order to create a district favorable to the Democratic-Republican Party. Many newspapers at the time commented that the district looked similar to a salamander, resulting in the term gerrymandering which is a combination of the Governor's last name and the term salamander (Caughey, Tausanovitch, & Warshaw, 2017). The term gerrymandering and its corresponding practice quickly gained traction around the nation. Gerrymandering came to describe the process of drawing electoral districts in a way to favor one party or class over another. However, even though the practice of gerrymandering has been in the public's perception since 1812, legal and academic institutions have struggled with ways to address and even measure the effects of gerrymandering.

2 Background

2.1 Legal Rulings and Legislation

It wasn't until 1962 that any decisive action was taken by the federal courts to remedy some of the effects of gerrymandering. In *Baker v. Carr* (1962) the Supreme Court ruled that electoral districts must be drawn in a way that makes the population in each district "as nearly equal as possible" and that any violations of this rule were a violation of the Equal Protection Clause of the Fourteenth Amendment" (Friedman & Holden, 2009, p. 596). While this did not completely address the issue of gerrymandering it did provide a new restriction on the redistricting process. This ruling was reaffirmed by the Supreme Court in *Wesberry v. Sanders* (1964) which gave the Federal Courts jurisdiction to "to impose their own district plan as part of their remedial powers" (Friedman & Holden, 2009, p. 596). These opinions by the court would create an important legal avenue through which gerrymandering claims could be tried. However, while these rulings did create a precedent, the Court expressed the view that partisan gerrymandering was "a political question beyond judicial remedy" (Caughey, Tausanovitch, & Warshaw, 2017, p. 455). Because of this, many early analyses of gerrymandering primarily tested the equal population assumption and examined the vast distribution of shapes electoral districts would take on to meet this assumption.

The Voting Rights Act of 1965 outlawed the drawing of electoral districts in ways which reduce the voting power of racial minorities. However, in order to challenge gerrymandering on the basis of racial discrimination, a discriminatory purpose needed to be established. In 1982, an amendment to the Voting Rights Act of 1965 removed the need to prove discriminatory purpose, making discrimination - even as a byproduct of otherwise fairly drawn maps - illegal and enforceable by the Federal Courts (Friedman & Holden, 2009, p. 597). This was affirmed by the

Supreme Court in *Thornburg v. Gingles* (1986), where the ruling "made the plaintiff's burden in vote dilution litigation substantially lighter" and determined that "if racial vote dilution was a by-product of such gerrymandering, the redistricting plan may be rejected" (Friedman & Holden, 2009, p.597). This ruling created a legal pathway to challenge racially motivated gerrymandering and resulted in a shift in the academic literature to focus on the distribution and spread of racial groups across electoral districts (Friedman & Holden, 2009, p.596). In the same year, the Supreme Court ruled in *Davis v. Bandemer* (1986) that "partisan gerrymandering was in principle judiciable" (Friedman & Holden, 2009, p.597). This ruling opened up the possibility of legal challenge to redistricting efforts that favored one party over another . However, this has never been enforced at the federal level due to the need to establish a discriminatory purpose. In the 30 years following the ruling in *Davis v. Bandemer* (1986), "not a single plaintiff [. . .] managed to persuade a court to strike down a plan on this basis" (Stephanopoulos & McGhee, 2015, p. 833). So far, the federal courts have only moved to correct claims of gerrymandering in cases that violate the equal population assumption and in those where racial minorities' voting power is being artificially diluted. This makes analyses of the demographic makeup of an electoral district one of the more practical and substantial ways to support claims of gerrymandering.

In *Vieth v. Jubelirer* (2004) the Supreme Court ruled that the United States Constitution offered no protection from electoral districts drawn with clear partisan bias. In its ruling, the Court expressed a concern that a measure of partisan bias would be "sufficiently hard" to develop and that partisan gerrymandering "may fall into the realm of non-justiciability" (Tam Cho & Liu, 2016, p. 351). Any measure of partisan bias, as a natural component, would have to define a state where no bias existed in an electoral district. This is problematic because there has been no legal or academic consensus into what constitutes a fair electoral district (Tam Cho & Liu, 2016, p. 352). However, Stephanopoulos and McGhee (2015, p. 833) noted that two years later in *League of United Latin American Citizens v Perry 548* (2006) the Supreme Court showed surprising enthusiasm for the concept of partisan symmetry, or the idea that “a district plan should treat the major parties symmetrically with respect to the conversion of votes to seats”. Justice John Paul Stevens claimed that partisan symmetry is “widely accepted by scholars as providing a measure of partisan fairness in electoral systems” (Stephanopoulos & McGhee, 2015, p. 833). Justices David Souter and Anthony Kennedy also expressed interest in the concept of partisan symmetry, although they believed further work was needed to develop a sufficient understanding (Stephanopoulos & McGhee, 2015, p. 833-834). This has led to renewed interest in establishing a measure of partisan bias on the assumption that district vote distributions should be symmetrical with respect to political parties. Although several Supreme Court Justices have supported the idea of partisan symmetry, others Justices have expressed concerns with its implications. Counter to Justice John Paul Steven’s claim, many scholars dispute the assumption of partisan symmetry (Cain, Tam Cho, Liu, & Zhang, 2018, p. 1525 & 1528). Because of this, measures of partisan bias still often have to defend and justify the theoretical basis for their existence. This is in contrast to measures of district compactness and demographic make-up where legal justifications already exist for these measures.

2.2 A Legislative Solution

A prominent legislative solution to address gerrymandering is to have an independent commission perform all electoral redistricting, taking the power out of elected officials’ control. This is ideal because it removes the obvious conflict of interest in having politicians select their own constituents. Other representative democracies, such as “Australia, Canada and the UK [...] have a long tradition of independent agencies overlooking the process of periodic reapportionment” (Bracco, 2013, p. 2). In the United States, 42 of the 50 States allow the state legislature to directly draw Congressional electoral district lines, and the 8 states that don’t do this allow varying degrees of oversight by the state legislature (Cain, 2012, p. 11). California has one of the strictest separations of the redistricting process from legislative control, where a commission of entirely independent and non-political citizens are selected to draw the electoral districts for the states (Fan, Li, Wolf, & Myint, 2015, p. 15). However, even though this has resulted in more compact and geographically centered districts, there have been “no changes in partisan bias” and “the outcomes of nonpartisan efforts to mitigate partisan gerrymandering in California remain unconvincing” (Fan, Li, Wolf, & Myint, 2015, p. 15). This seems to suggest the need for a more robust metric or form of analysis to help guide the independent counsels to avoid unintentional bias. While this legislative solution might provide a component to address gerrymandering, there is still a clear need to develop more comprehensive methods to understand and prevent gerrymandering.

2.3 Two Central Problems with Gerrymandering

When trying to understand and assess the impacts of gerrymandering we have to deal with two separate and often competing criteria. The two criteria “ usually called into play in the political debate are the safeguard of homogeneous communities, which would avoid irregularly shaped districts, and the creation of competitive districts, which could increase accountability and possibly also decrease polarization” (Bracco, 2013, p. 3). Traditionally electoral districts are expected to be drawn to group together communities of interest, in order to create a district in which the populous is geographically centered and reasonably homogeneous. However, since the Supreme Court has expressed interest in a measurement of partisan symmetry, efforts have expanded into finding a way to measure and correct for partisan bias in the redistricting process. Because these two problems are often at odds with each other, many of the prominent measures and methods proposed to help understand gerrymandering only address one of these problems.

2.4 Compactness

A very common and older set of metrics used to assess gerrymandering are the various measures of geographical compactness. Measures of compactness seek to normalize electoral district drawing and are used to enforce the more traditional problem of ensuring communities of interest are grouped together by enforcing strict rules on the spread, size, and shape an electoral district can have. Mathematically, compactness is a measure of how spread out boundary lines are from the center of a shape. The goal in the case of gerrymandering is to try to minimize the distance of any point along the boundary line of the district from the center point of that district. Compactness scores are ideal because they often have a legal basis for being used; “[m]any states have used compactness measures, albeit formed in different ways, as a major standard to limit the possibility of gerrymandering” (Fan, Li, Wolf, & Myint, 2015, p. 2). Further, the Supreme Court has acknowledged that “compactness is a ‘traditional’ districting principle that can be used to assess the constitutionality of districting plans” (Fan, Li, Wolf, & Myint, 2015, p. 2). Among the compactness measures available, three categories stand out as the most prominent in use: the Polsby-Popper Index, the Reock Index, and various Normalized Moment of Inertia measurements (Fan, Li, Wolf, & Myint, 2015, p. 2). All of these measures are motivated by the fact that a circle is the most compact two-dimensional object. The Polsby-Popper measure is the ratio of the area of a shape to the area of a circle that has the same perimeter as the shape. The Reock measure is the ratio of the area of a shape to the area of the minimum enclosing circle of the shape. The Normalized Moment of Inertia measure is the ratio between the Moment of Inertia of a shape and the Moment of Inertia of a circle with the same area (Fan, Li, Wolf, & Myint, 2015, p. 6). A useful extension of the Normalized Moment of Inertia measure is the Normalized Mass Moment of Inertia, which is a property that is well developed in physics and can be applied to redistricting by considering population to be mass.

There are several tradeoffs to each of these measures that make each useful in different contexts. The Reock measure, developed in 1961, was one of the first and most widespread compactness measures. The measure is simple to understand and easy to calculate, however its variation is tied to the area of the electoral district causing some comparison issues when comparing districts of different land sizes. The Polsby-Popper measure, while being one of the oldest and simplest measures has a very intuitive understanding. Its variation is not based on the area of land the district covers, and it is very sensitive to slight changes in boundaries, making it useful for identifying irregular and ragged boundaries. The various measures of Normalized Moment of Inertias are more computationally efficient and help correct for some of the difference in variation at different district sizes. Its extension, the Normalized Mass Moment of Inertia also gives an interesting look at the dispersion of the population in a shape (Fan, Li, Wolf, & Myint, 2015, p. 6). However, these two measures can be hard to interpret and translate to real world outcomes. Further, these measures fail to detect strange and ragged borders. Because of this, the Polsby-Popper index is more useful for ensuring that smooth and regular boundaries are being drawn for districts and the methods of Normalized Moment of Inertia are ideal for comparing area compactness.

2.5 Partisan Symmetry

The idea that electoral districts should be symmetrical with respect to party advantage is a relatively new idea encouraged by the Supreme Court. It has created a need to find a way to measure partisan bias in electoral district drawing. However a theoretical basis for these measures is still highly contested. In regards to calculating partisan bias, the "most commonly discussed measures derive from a symmetry concept: that is, in the same circumstances, partisan unfairness should not disadvantage one party more than the other. King-Gelman bias scores and the efficiency gap are the most prominent examples of partisan symmetry measures" (Cain, Tam Cho, Liu, & Zhang, 2018, p. 1528). We will consider the latest version of the King-Gelman bias scores implemented in their JudgeIT II election analysis program as well as the efficiency gap first proposed by Stephanopoulos and McGhee (2015) to understand how partisan bias is currently being measured.

2.6 King-Gelman Bias Scores

In their 1994 paper, King and Gelman were originally interested in multiple theoretical properties of a given election, including but not limited to partisan bias. They argued that by using a modified bootstrap technique, hypothetical elections under slightly different circumstances could be simulated in order to estimate several theoretical properties about the election. They "define hypothetical election results as the set of all possible election outcomes that could have occurred if all political conditions up to the start of the campaign were held constant and the campaign were run again" (Gelman & King, 1994, p. 517). By producing these simulated hypothetical elections, we now have a distribution of hypothetical elections to base our analysis on. Using King and Gelman's approach, we can "calculate a joint probability distribution for all quantities of interest [...] from this, all point estimates and standard errors can be calculated." (Gelman & King, 1994, p. 518). An important feature of the model is that these hypothetical elections are generated from a somewhat standard multivariate regression, and unlike "in many applications of regression modeling in the social sciences, the parameters β, σ^2, and λ are not themselves the goal of our analysis but rather intermediate quantities, used for estimating the distribution of hypothetical election quantities." (Gelman & King, 1994, p. 526). The quantities of interest, such as bias, are not being estimated in our regression but are being calculated by analyzing the hypothetical elections that we generate by varying the input values of our regression. In Gelman and King's model, bias is split into two parts, a statewide bias or nationwide bias and an electoral district bias (Gelman & King, 1994, p. 526). Splitting the bias serves to separate the large geographical biases of the state or nation from the district-specific biases. These electoral biases are estimated in each hypothetical election so a distribution of potential state and electoral biases is created, and an average and standard error can be calculated.

The bias scores generated using this method have two distinct advantages. First, they are statistically rigorous. Standard error and confidence bounds can be applied to our estimates of bias and further information can be extrapolated from the error of our generating regression. Second, this methodology can be used to calculate multiple different quantities of interest, including the set of possible seat distributions for each party. This would allow us to further quantify the likely damages a group or party received as the result of the bias in the electoral district maps (Gelman & King, 1994, p. 524). Despite this methodology's statistical strengths, it is weak in practical use. Any output generated by this method is dependent on the variables selected to be used in the regression. While Gelman and King (1994, p. 524) suggest possible models to be used, they admit there is no clear choice on what variables should be used. This makes the model flexible for analysis, but hard to implement as an enforcement mechanism unless a regression model design can be agreed upon. The methodology is also complicated and is often criticized for relying on ‘what could have happened’ instead of focusing on what did happen (Stephanopoulos & McGhee, 2015, p. 856). Also, because this method relies on a real election to generate hypothetical results, it is by nature a post-facto analysis and such a method could not be used to help in the redistricting process.

2.7 The Efficiency Gap

The efficiency gap is one of the more prominent measures of partisan bias in the general media due to its simple interpretation and formula. Its use in a few prominent and recent gerrymandering court cases has also ignited much interest for the measure. The efficiency gap “is simply the difference between the parties' respective wasted votes, divided by the total number of votes cast in the election. Wasted votes include both 'lost’ votes (those cast for a losing candidate) and ‘surplus’ votes (those cast for a winning candidate but in excess of what she needed to prevail). Each party’s wasted votes are totaled, one sum is subtracted from the other, and then, for the sake of comparability across systems, this difference is divided by the total number of votes cast” (Stephanopoulos & McGhee, 2015, p. 852). A key strength of the efficiency gap, apart from its simplicity, is that it relies only on actual observed outcomes. The creators of the efficiency gap, Stephanopoulos and McGhee (2015, p. 856) argued that “the efficiency gap is the superior metric because it more directly captures the essence of gerrymandering and does not require the estimation of hypothetical election results”. The efficiency gap's motivation, simplicity, and reliance on only observed outcomes are viewed by Stephanopoulos and McGhee as its most compelling features.

The features that are compelling to Stephanopoulos and McGee have also drawn criticism. Many have found fault with the motivation for the efficiency gap and partisan symmetry in general because it makes the implicit assumption that if electoral districts are drawn fairly, the amount of wasted votes should be roughly equal (or below the 7% threshold) (Stephanopoulos & McGhee, 2015, p. 856). However, party members are not spread randomly across the United States. For example, any "urban district likely uses Democratic votes inefficiently [… thus] a poor efficiency gap fulfills only a descriptive purpose, but not a diagnostic one" (Cain, Tam Cho, Liu, & Zhang, 2018, p. 1532). Since partisan members tend to group together, the assumption underlying the efficiency gap - and in partisan symmetry in general - introduces an overly simplified prospective. Further, compared to the more complex model proposed by Gelman and King (1994), the efficiency gap is criticized for failing to make any attempt to capture this natural partisan grouping. In contrast, Gelman and King's model includes a measure for calculating bias at a statewide level and uses it to help correct for the uneven distribution of partisan members. Because a distribution of efficiency gaps only exists at the statewide level (not even for all states), and the national level, the efficiency gap is only useful for analysis that are aggregating up in some fashion. Finally, the efficiency gap also has some strange mathematical properties noted by Dr. Wendy K. Tam Cho (2017, p. 35).; "Stephanopoulos and McGhee acknowledge that 'when one party receives more than 75 percent of the statewide vote - the efficiency gap can produce results that at first glance seem strange.' [...] Their solution is not to make any modifications to how the efficiency gap is calculated, but rather to say that these are rare occurrences that an analyst should flag ahead of time. Our point is that a general measure should exhibit mathematical properties that ensure it is measuring the quantity of interest, rather than needing to delete case where its behavior is erratic". While the efficiency gap does provide a simple and easy to understand measure of the possible effects of gerrymandering, it is criticized for its over simplification of the problem and its non-rigorous statistical nature.

2.8 Alternative Electoral District Maps Simulation

Another technique being pioneered in the field of gerrymandering is the simulation of alternative electoral district maps. The goal of these simulations is to provide a set of significantly different redistricting maps that meet or exceed the same requirements to which the original electoral district maps were drawn. These simulated maps provide a range of equally eligible maps to see whether partisan biases appear naturally in simulated drawings. Tam Cho and Liu (2016, p. 360) argue that, "For a partisan gerrymandering case, having a set of plans that are drawn without partisan considerations but exhibit comparable non-partisan metrics allows us to see how the alleged partisan considerations in the disputed plan substantively alter the outcomes that emerge from a less or nonpartisan process". The simulated maps are ideal for comparison because if a natural partisan bias is not found to be present in the majority of simulated district maps, the claim of other criteria inversely affecting partisan bias is already proven void. Further analysis of the simulated maps against the official map could show other non-stated criteria that the map was optimized against. At its core, "how competitive the districts could have been plainly goes a long way in helping us to understanding a single competitiveness number because it provides information on the range of values for other possible plans. In short, we need the context of a large number of possible plans to understand the characteristics of any single plan” (Tam Cho & Liu, 2016, p. 354). With a collection of hypothetical electoral maps, we can explore the realm of possible alternatives to understand the full range of options that were rejected in favor of the proposed or selected district mapping. In Gelman and King’s method, hypothetical elections were simulated based on real elections. Methods simulating useable electoral districts do not simulate in a bootstrap way, instead using advanced computer algorithms to identify other possible viable district drawings. These simulations are trying to construct a sampling distribution of viable electoral district maps.

The obvious benefit of this analysis is that we have a distribution of properties for the simulated electoral district maps to use in comparison to the official map, allowing a more statistically rigorous study to be conducted. Further, based on the simulated map’s expectations, a countless number of different metrics could be estimated for comparison against the official map, opening up a wider breadth of analysis that could be conducted. However, as Tam Cho and Liu (2016, p. 357) note, the amount of computer power required to even generate this smaller ideal set is very large, making this form of analysis limited in its applicability at present. Even if computational power was not an issue, the simulation of an ideal set of district maps is still based on fulfilling or maximizing some set of criteria. Since the set of maps to be simulated must be restricted in order to be computationally efficient, criteria must already be established and thresholds must be set. This makes the simulation of alternative electoral district maps an interesting and promising tool, but it still leaves the question open for what set of criteria to use. Having competing optimization criteria could also lead to further debate, because simulations trying to maximize on different measures have the potential to produce different sets of simulated maps, making the analysis highly conditional on the optimization criteria.

3 Extending Compactness

3.1 Motivation

While there are many avenues being explored in the study of gerrymandering, there appears to be a need for a more sophisticated measure that summarizes the more traditional problem associated with partisan redistricting. We want to develop a measure that is able to detect and quantify irregularities in any existing or proposed districts that might suggest manipulation. This is in contrast to measures of partisan bias which seek to capture the effect of gerrymandering, instead of the root cause. Further, we avoid the principle of partisan symmetry in general because "partisans are not randomly dispersed across geography. Rather, they cluster in nonrandom ways, causing redistricting to produce natural partisan bias" (Cain, Tam Cho, Liu, & Zhang, 2018, p. 1530). This limits the effectiveness of measures of partisan bias to be used as a tool to help fight or even correctly assess gerrymandering. Our proposed measure is one that could be used in the drawing process, before votes have even been cast. It also has the potential to be used as a criteria in simulating district maps. If a simple measure could reasonably detect the extent that a given district mapping is showing signs of possible partisan manipulation, it could be optimized to give, at least, a starting set of ideal electoral district mappings. We believe that compactness gets us part of the way there and has a long history, both in practical implementations and in research. However, it seems clear that this alone is not enough to help assess when an electoral district is being gerrymandered.

3.2 Race and Majority-Minority Districts

Racial makeup of a district has long been used to assess the legality of an electoral district under the Voting Rights Act of 1965 and later amendments. In efforts to enforce the law, courts have ordered the creation of majority-minority districts in electoral district maps to help ensure minority representation in Congress (Sauter 2013; Cameron, Epstein and O'Halloran 1996). A majority-minority district is an electoral district where the majority of residents in that district identify as a minority race in the United States. The merits of these districts have long been contested (Hayes & Mckee, 2011; Sauter 2013; Cameron, Epstein & O'Halloran 1996; Barreto, Segura & Woods, 2004; Lubin 1999; Grofman, Griffin & Glazer, 1992). Debates center on what outcomes actually advance the interest of minority groups in the United States, with minority turnout for elections, the racial makeup of Congress, and the passage of minority-supported legislation all being suggested as ideal outcomes (Hayes & Mckee, 2011; Cameron, Epstein & O'Halloran 1996; Barreto, Segura & Woods, 2004; Lubin 1999; Grofman, Griffin & Glazer, 1992). Early research by Grofman, Griffin and Glazer (1992) focused specifically on black communities and found evidence that electoral districts that create large black majorities in districts increase the likelihood that black Congressmen and Congresswomen are elected to Congress. However, this often came at the cost of decreasing numbers of Democratic House Representatives (Grofman, Griffin & Glazer, 1992).

Cameron, Epstein and O'Halloran (1996) also focused on black communities and used the roll call behavior of House Representatives on pieces of legislation supported by the majority of black Americans to measure the effectiveness of majority-minority districts in the United States. Their findings showed that outside of southern states, black representation is “best served by distributing black voters equally among all districts” and that passing legislation supported by minority populations is often at odds with electing minority candidates to Congress (Cameron, Epstein & O'Halloran, 1996). They also suggested that outside the south, majority-minority district were not necessary to electing minority candidates to Congress (Cameron, Epstein &

O'Halloran, 1996). Lublin (1999) latter criticized Cameron, Epstein and O'Halloran's analysis for neglecting the role of Latinos and other minority populations in electing minority, and specifically black, candidates to Congress and refutes their claim that such efforts always result in net increases for Republicans in Congress. Lubin (1999) asserts that majority-minority districts, when all minority groups are considered, are essential in efforts to electing more minority candidates into Congress. There is also evidence to support that majority-minority districts help increase voter participation in minority populations that can result in large advantages for up-ballot candidates in the state, such as Governors and Senators who in most states are elected through a statewide popular vote (Barreto, Segura & Woods, 2004).

Whether majority-minority districts are beneficial to minority populations is still debated. Their influence on election results are well-supported. Because of this, measures of racial makeup of a district are essential to understanding if districts are being gerrymandered to favor one party over another. However, since there is no academic consensus on whether majority-minority districts are beneficial overall or definitively favor one party over another, we will default to current court rulings on the matter which tends to favor majority-minority districts (Sauter, 2013; Cameron, Epstein & O'Halloran, 1996; Barreto, Segura & Woods, 2004). Under the Voting Rights Act of 1965 and later amendments, the creation of majority-minority districts became a popular solution to ensuring minority representation in public offices (Cameron, Epstein & O'Halloran, 1996; Barreto, Segura & Woods, 2004). In multiple court cases involving redistricting, including a Supreme Court case, a minimum threshold of 65% of the total population needs to identify as a minority race for the district to be considered a majority-minority district, although other standards have also been used (Cameron, Epstein & O'Halloran, 1996; Sauter, 2013). However, there seems to be a preference by the Court to have districts with large majorities of white or non-white populations. Thus, in our statistic we will follow this preference and give preference to districts with increasingly large white or non-white populations.

3.3 Definition

Our statistic will be made up of two component scores, measuring the compactness of electoral district boundaries and the percentage of white versus non-white residents in each district. These measures will range from 0 to 1, where scores closer to 1 are perceived as more desirable. Our final statistic will be the product of the two component measures, so it will also range from 0 to 1. The statistic acts as an extension of the compactness scores, with an added racial make-up component, and provides a simple way to compare two prominent characteristics of an electoral district. Districts that are low in compactness and have slim majorities of white and non-white residents seem more likely to be created for partisan gain, and the effects compound each other. Our immediate interest will be drawn to those electoral districts that score close to 0, suggesting one of three possibilities: low compactness scores and slim majorities, extremely low compactness scores, or extremely slim majorities. By taking the product of the two scores, we ensure that a reductions in both scores are compounded, that large reductions in a single score is enough to pull down the overall score, and that tradeoffs for small reductions in one score that leads to large gains in another are rewarded. This creates a flexible and sensitive measure that can be used to compare sets of electoral district map proposals. We will refer to the statistic as the Electoral District Similarity Score, because it measures two factors of geographic and demographic similarity in an electoral district.

3.4 Compactness Score

For the Compactness Score we will use the Polsby-Popper Index, which is defined by the ratio of the area of a shape to the area of a circle that has the same perimeter as the shape (Fan, Li, Wolf, & Myint, 2015, p. 6). For our purposes, we will define the Polsby-Popper Index as

$$4\pi A \div P^2$$

where A and P are the area and perimeter of the electoral district, respectively. The Polsby-Popper method is chosen because it is a commonly used metric in the study of gerrymandering and because it punishes for jagged and unsmooth boundaries, helping to detect more subtle manipulation (Fan, Li, Wolf, & Myint, 2015, p. 6). The measure returns a score of 1 for a circle, which is the most compact two dimensional shape (Fan, Li, Wolf, & Myint, 2015, p. 6). However, circles are not a natural or ideal form for an electoral district, so for clearer reference a square receives a score of 0.785. The Polsby-Popper Index naturally scales the effects of compactness, offering a natural comparison point in which to judge two dimensional shapes that is not dependent on Area.

3.5 White and Non-White Similarity Score

For our measure of racial similarity, we identify and group the residents in each district as either white or non-white. While this is a simple grouping, it accurately captures the majority/minority dichotomy in most districts and is consistent with current implementations and ruling of the Voting Rights Act (Sauter, 2013; Cameron, Epstein & O'Halloran 1996; Barreto, Segura & Woods, 2004). Using a binary grouping also provides an easier way to quantify the effects of an electorate with a white majority. We define White and Non-White Similarity Score as:

$$2 * |W - .5|$$

where W is the percentage of white residents in the electoral district. Since we are concerned with measuring how homogeneous a district is, we are only concerned with how large the white or non-white group is past a simple majority. We take the absolute value because this allows us to treat both groups – white and non-white - equally, simply measuring how much larger the majority grouping is past a simple majority. We multiply by 2 to scale our output from 0 to 1. Thus, this score rewards districts where there is a large majority of either white or non-white residents by resulting in scores closer to 1.

4 Analysis of the 115th United States Congress Electoral District Mapping

To proceed with developing our understanding of the statistic and its properties, we will analyze the electoral district mapping used to elect the 115th United States Congress. We specify which Congressional session we are looking at because although electoral district maps are only redrawn every ten years, court rulings and changes in state laws may result in adjustments being made prior to every Congressional election. Our electoral district mapping data is collected from the United States Census Bureau in the form of shapefiles from the Tigerline program and our demographic data for each electoral district comes from the American Community Survey, using the most recent 5-year survey which was conducted in 2017.

4.1 Compactness Score

Compactness Scores for the 115th United States Congress electoral district map take on a roughly normal - though slightly skewed - distribution. With a mean of 0.24 and a standard deviation of 0.124, Compactness Scores for most districts are relatively low and there spread can be seen in the figure below.

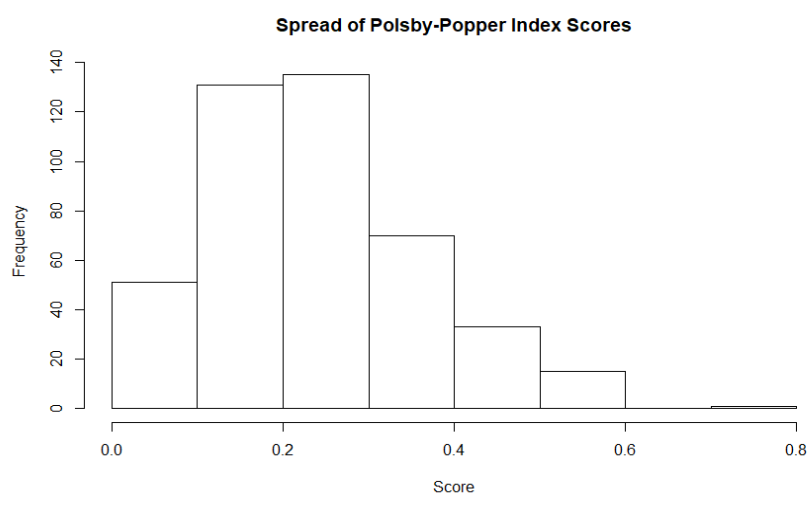

Figure 4.1: Distribution of Polsby-Popper Index Scores (115th Congress)

A map of the spread of compactness scores across the Continental United States is shown below.

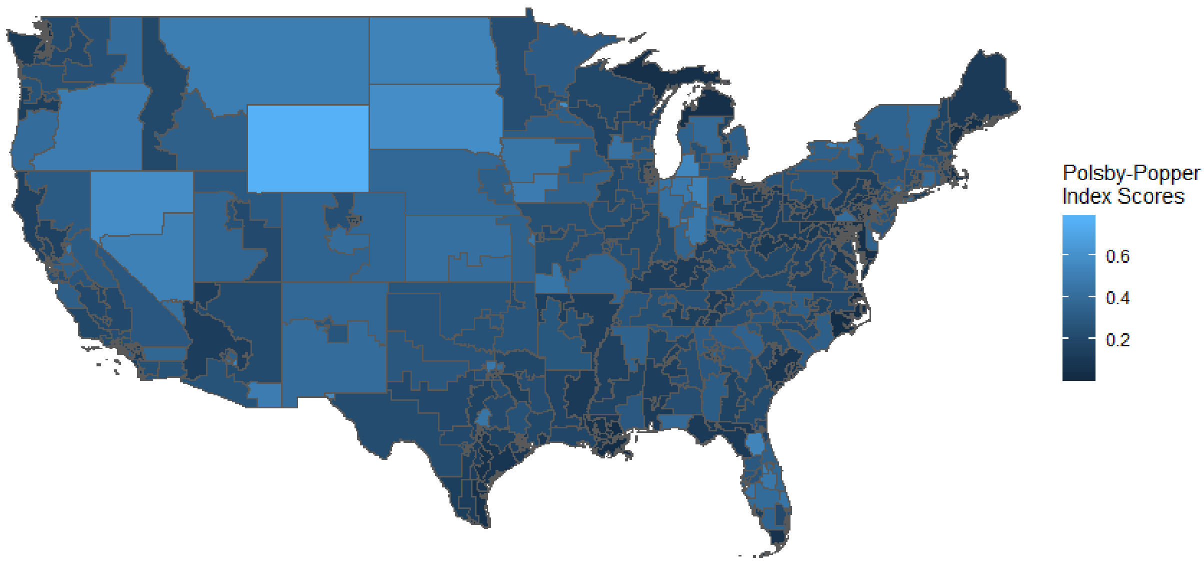

Figure 4.2: Map of Polsby-Popper Index Scores (115th Congress)

At first glance, it seems that some of the most compact scores are in states in the northern Midwest, which each only have one electoral district. Wyoming in particular appears to be the most compact. Wyoming only has a single congressional district, making it impossible to gerrymander in the state. Looking at the top performing districts nationwide, only two of these single-district states make it into the top ten. Districts in high population states like New York, Texas, and Florida are also in the top ten.

	Congressional District	Compactness Score
1	Congressional District (at Large) (115th Congress), Wyoming	0.772
2	Congressional District 1 (115th Congress), Nevada	0.584
3	Congressional District 2 (115th Congress), Nevada	0.582
4	Congressional District (at Large) (115th Congress), South Dakota	0.581
5	Congressional District 16 (115th Congress), Texas	0.569
6	Congressional District 4 (115th Congress), Minnesota	0.567
7	Delegate District (at Large) (115th Congress), District of Columbia	0.551
8	Congressional District 15 (115th Congress), New York	0.550
9	Congressional District 6 (115th Congress), Michigan	0.545
10	Congressional District 3 (115th Congress), Florida	0.533

Table 4.1: Top 10 Polsby-Popper Index Scores (115th Congress)

4.2 White and Non-White Similarity Score

White and Non-White Similarity Scores for the 115th United States Congress electoral district map do not have a clearly identifiable distribution, but they appear to be closest to a uniform distribution. With a mean of 0.522 and a standard deviation of 0.241, White and Non-White Similarity Scores for most districts have a relatively wide and frequent spread across the possible scores.

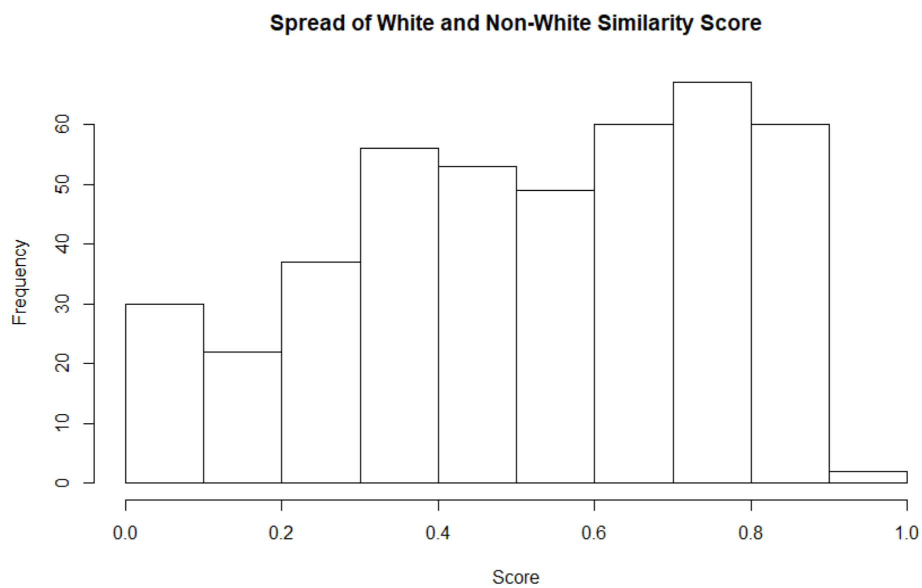

Figure 4.3: Distribution of White or Non-White Similarity Scores (115th Congress)

A map of the spread of White and Non-White Similarity Scores is shown in Figure 4.4. It is worth noting that states with relatively low minority populations are naturally likely to perform better for this measure. For instance, the states in the Northern Midwest score very high. This is a natural consequence and is consistent with our intentions because in states with very low minority populations there is less opportunity to gerrymander on a racial basis.

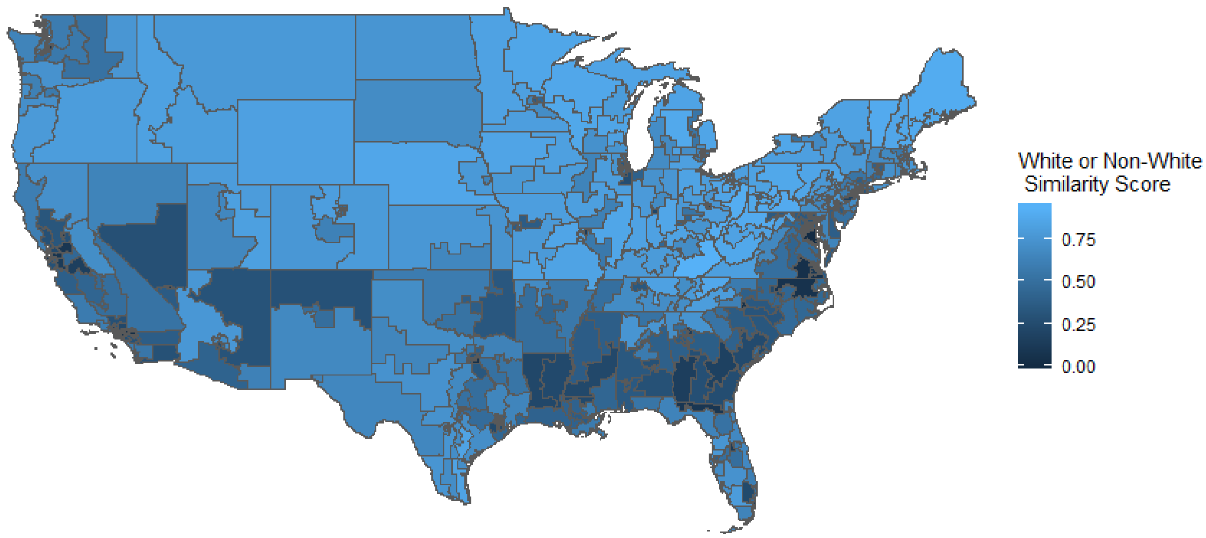

Figure 4.4: Map of White or Non-White Similarity Scores (115th Congress)

Some states have electoral districts with radically different scores, which could ostensibly be drawn in another way to produce more consistent scores within the state. We also have states where every electoral district receives a very low score, suggesting that a large non-white population has been split up among all of the electoral districts. In both cases, the low-scoring districts will serve to bring down the overall score of the state.

4.3 Electoral District Similarity Scores

Our Electoral District Similarity Score is the product of the past two scores, resulting in a distribution with mean 0.13 and standard deviation 0.098. The highest score received in the 115th United States Congress electoral district map is 0.638 and the median score is 0.11. Less than 10% of electoral districts for the 115th United States Congress receive a score of less than 0.02. Similarly, less than 10% of electoral districts receive a score above 0.28.

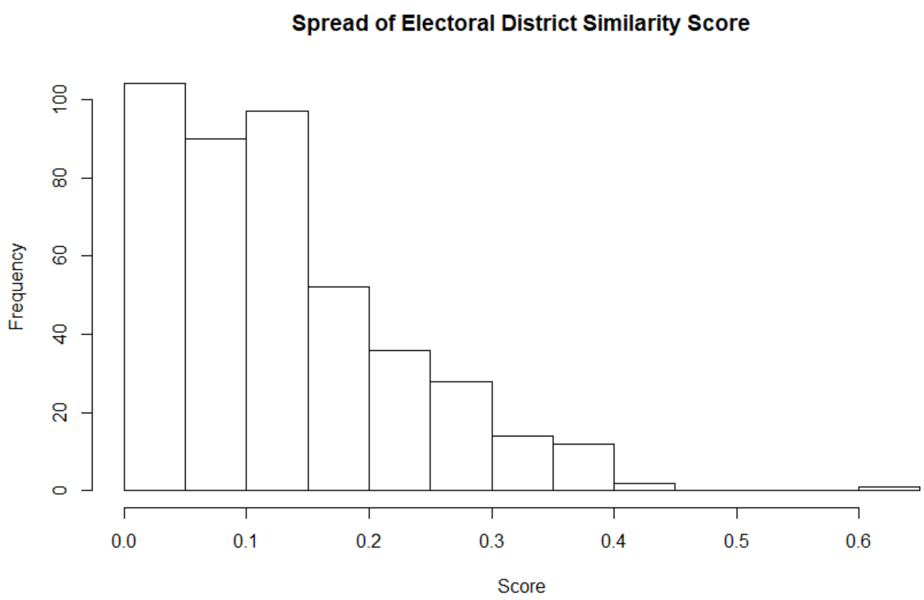

Figure 4.5: Distribution of Electoral District Similarity Scores (115th Congress)

Figure 4.6 is a mapping of Electoral District Similarity Scores across the 115th United States Congress electoral district map, first with all states, then excluding Wyoming in Figure 4.7, which is an outlier in the data set.

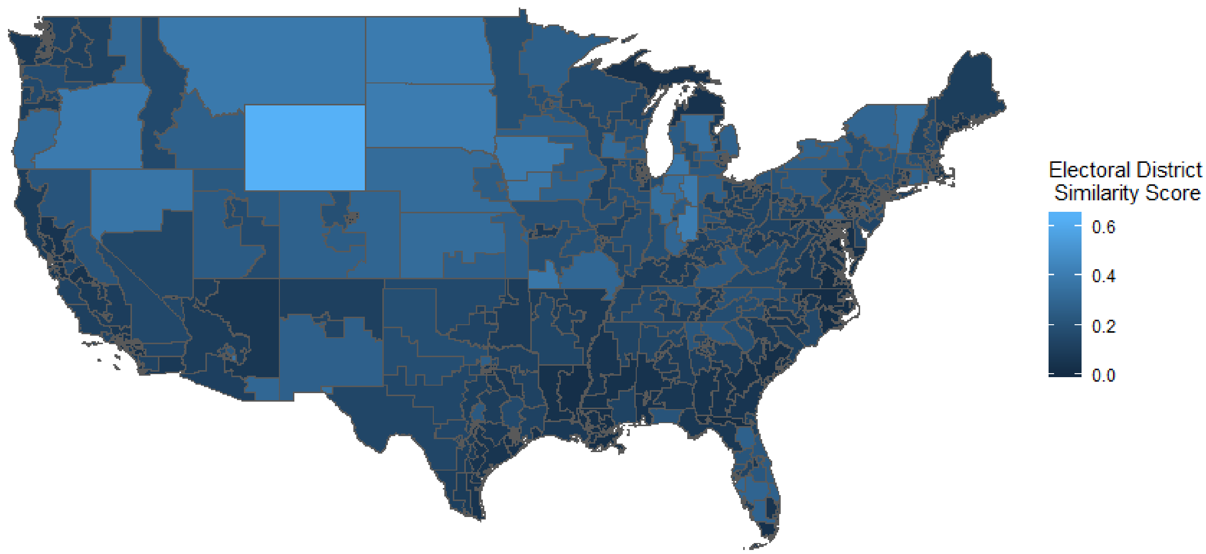

Figure 4.6: Map of Electoral District Similarity Scores (115th Congress)

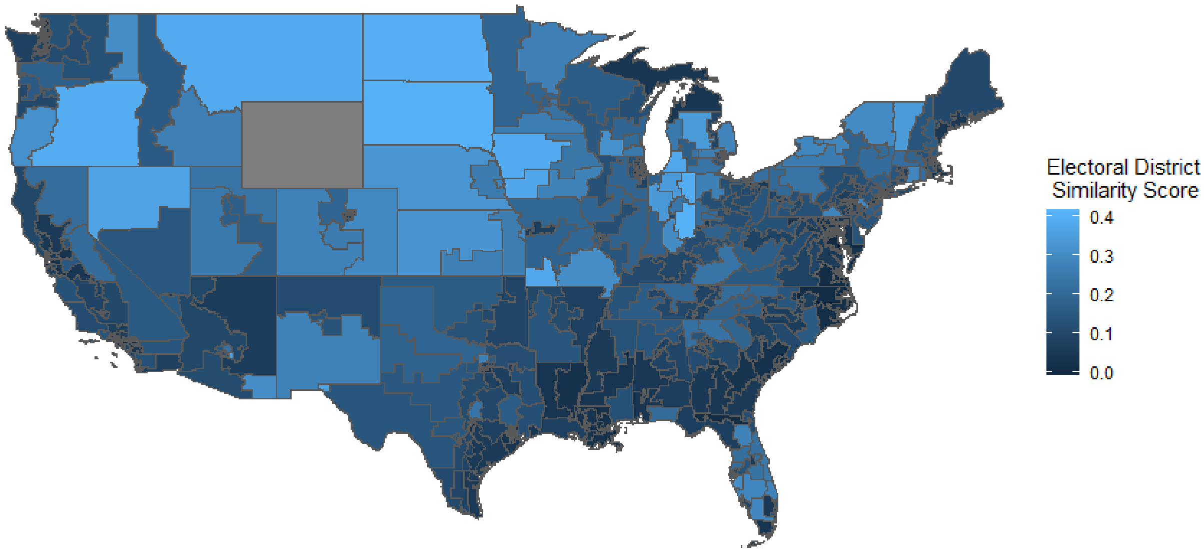

Figure 4.7: Map of Electoral District Similarity Scores without Wyoming (115th Congress)

Our interest is immediately drawn to those electoral districts that score the lowest, suggesting low similarity in the underlying residents and districts drawn in non-intuitive ways. A list of the electoral districts with the lowest Electoral District Similarity Scores is shown in Table 4.2.

	Congressional District	Electoral District Similarity Score	Compactness Score	White and Non-White Similarity Score
1	Congressional District 5 (115th Congress), Maryland	0.0000	0.1858	0.000
2	Congressional District 7 (115th Congress), Massachusetts	0.0006	0.0758	0.008
3	Congressional District 41 (115th Congress), California	0.0027	0.1933	0.014
4	Congressional District 7 (115th Congress), New York	0.0027	0.1042	0.026
5	Congressional District 2 (115th Congress), Maryland	0.0029	0.0312	0.094
6	Congressional District 12 (115th Congress), California	0.0032	0.3163	0.01
7	Congressional District (at Large) (115th Congress), Alaska	0.0033	0.0109	0.306
8	Congressional District 18 (115th Congress), Texas	0.0041	0.0822	0.05
9	Congressional District 1 (115th Congress), Pennsylvania	0.0052	0.0820	0.064
10	Congressional District 9 (115th Congress), Washington	0.0059	0.1541	0.038

Table 4.2: Top 10 Electoral District Similarity Scores (115th Congress)

4.4 Correlation of Component Scores

A possible concern about the Electoral District Similarity Score is how the component scores relate to each other. More specifically, we do not know whether there a relationship between the two component scores that could lead to a confounding variable that we are not capturing. However, we find that the correlation between the Compactness Scores and the White and Non-White Similarity Scores for the 115th Congress is 0.1477. A correlation of this magnitude is relatively weak, and help support the case for the two component scores by showing that they both bring different and relevant data.

4.5 Electoral District Similarity Score and The Efficiency Gap

With our Electoral District Similarity Scores calculated for the 115th Congress, we can compare our statistic to the Efficiency Gap, one of the more prominent measures used to assess gerrymandering. We calculate the Efficiency Gap based on the House Representatives election held in each district for the 115th Congress (2016 Election). For this analysis, we ignore third party candidates and only compare votes casted for the two major parties. We also remove districts where incumbents ran unchallenged in the General Election. This leaves us with 393 electoral districts to calculate wasted votes for, and from these we can calculate the Efficiency Gap of each state. These values are mapped below where negative values represent states where more Democratic votes are wasted, and positive values where more Republican votes are wasted.

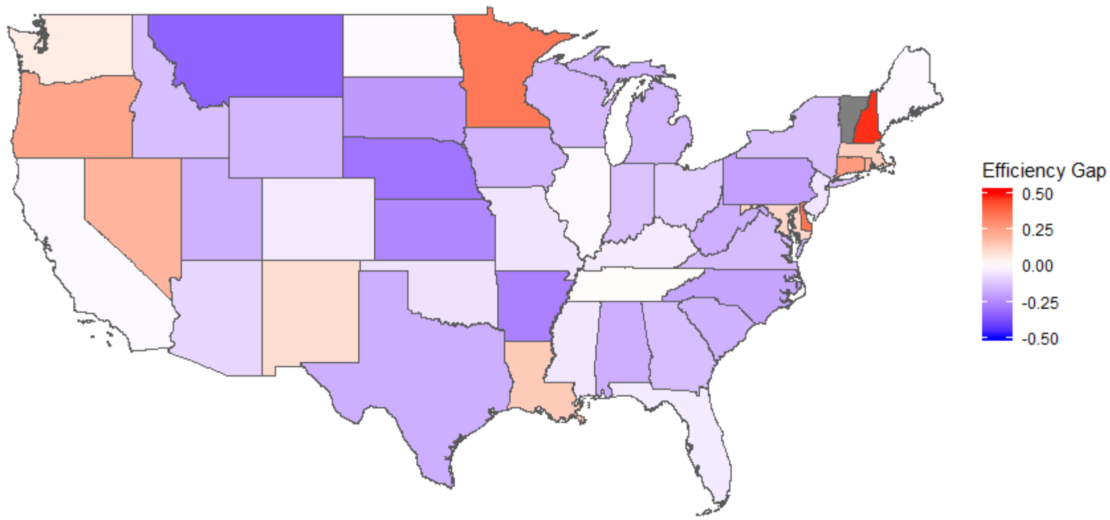

Figure 4.8: Map of Efficiency Gaps at State Level (115th Congress)

Efficiency Gaps are generally calculated at the State level, but we will diverge from this to make a better comparison to our statistic and calculate the Efficiency Gaps for each congressional district for the 2016 election. Below are those scores; congressional districts that are grey had no incumbent challengers, so no Efficiency Gap could be calculated.

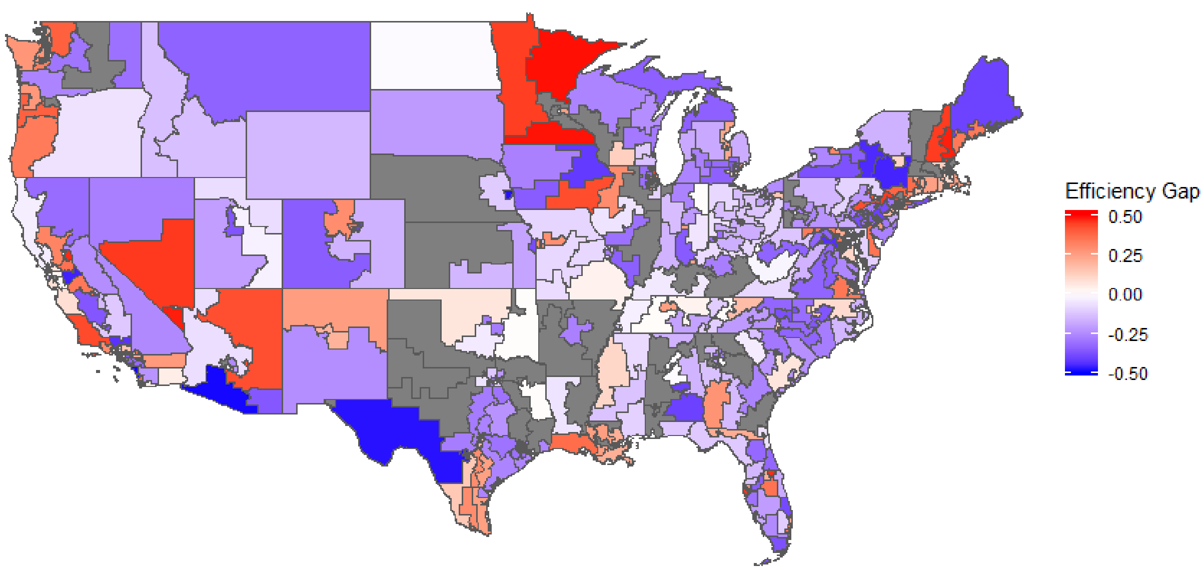

Figure 4.9: Map of Efficiency Gaps at District Level (115th Congress)

We now will compare the relationship between the two measures, but first we'll take the absolute value of the Efficiency Gap. We do not care which district a party favors, just how strongly it favors one over the other. Figure 4.10 is a scatterplot comparing the two measures. There seems to be a weak relationship between the two. The two scores have a weak correlation of .06 and appear to almost be randomly distributed.

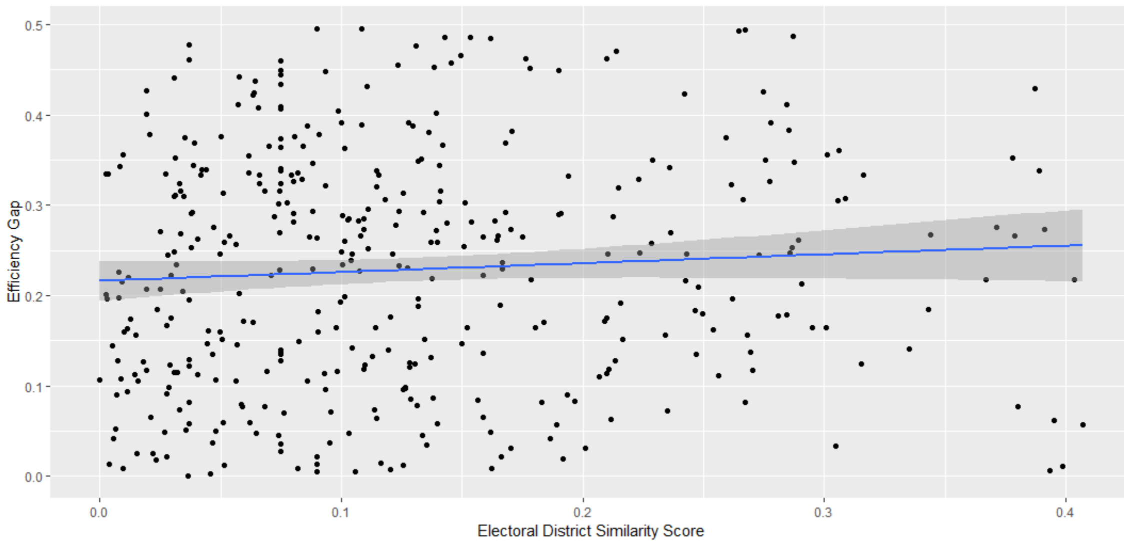

Figure 4.10: Plot of Electoral District Similarity Score vs Efficiency Gaps (115th Congress) (One Outlier Removed)

The weak correlation is an interesting finding, although not completely unsurprising as each statistic seeks to capture something different about the electoral district. The Efficiency Gap cares only about raw votes for the two major parties and does not consider other trends, such as natural partisan bias, in its calculation. The Electoral District Similarity Score on the other hand does not care about votes at all and instead looks at an electoral district's characteristics. Also the Efficiency Gap does not account for natural bias present in a state. Many of the high performing states under the Electoral District Similarity Score are from more rural states, where Republicans traditionally enjoy a large advantage in elections. This natural bias present makes it hard to reconcile these findings, other than noting the interesting trend that on average efficiency gaps are not significantly changed in districts with high Electoral District Similarity Score.

5 Case Studies

5.1 Case Study No. 1: 2011 Texas Redistricting

The Electoral District Similarity Score can be used to help assess previous Court interventions in the redistricting process. We hope to see improvement in the Electoral District Similarity Score that would justify a court stepping in to modify or order a redrawing of an existing electoral map. A prominent example that we will examine is the 2011 Texas Congressional District map that was ordered to be redrawn after the map failed to get approval from the Federal Court in Washington D.C., as required by the Voting Rights Act and later amendments (Sauter, 2013). We will compare the old congressional map and new congressional map to see if the Electoral District Similarity Scores show an improvement after the court ordered redrawing.

We will use data collected from the Texas State Government website, where files for the proposed district mappings are saved. The maps were created by combining 2010 Census Blocks and each proposed map contains a table relating all of the Census Blocks to the districts which contain them. We will use the 2010 Census Block findings to calculate the percentage of White Identifying residents in each district. We then create our data set by combining the shapefile of the proposed and enacted plans with the percentage of White Identifying residents in each district for each plan. With these two datasets, an Electoral District Similarity Score can be calculated for each map.

The new congressional map received an average Electoral District Similarity Score of 0.104 compared to the old congressional map, which averaged a score of 0.062 across the 36 electoral districts. For the new congressional map, many of the districts' scores become substantially higher than they previously were. In the old congressional map, only two districts were above a score of 0.1. In the new congressional map, 18 congressional districts receive a score above 0.1, and 11 districts are above the national average of 0.13. Running Welch's T-test on the two plans electoral district's scores shows that the increase is statistically significant at the 0.05 level, with a p-value of 0.0163. The range and mapping of scores for each district can be found in Figure 5.1, as well as a histogram comparing the spread of district score in Figure 5.2.

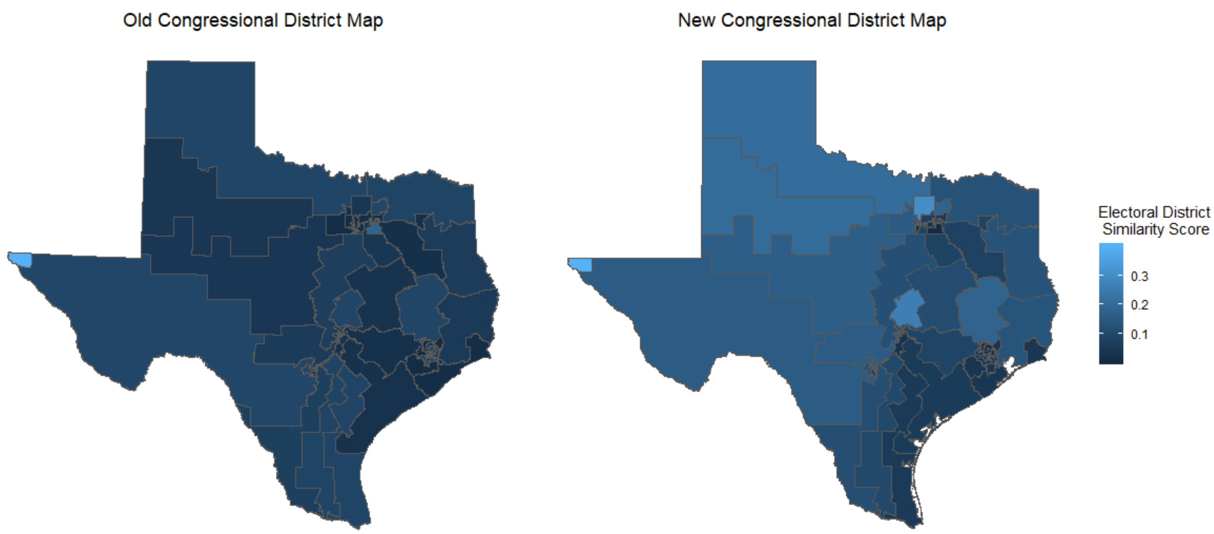

Figure 5.1: Maps of Electoral District Similarity Score in 2011 Texas Redistricting

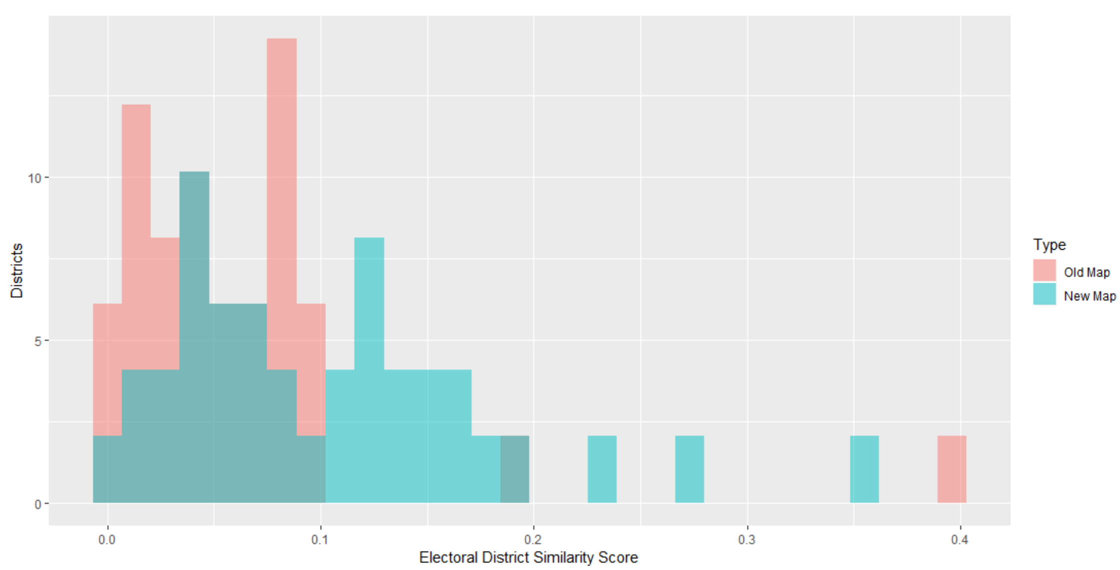

Figure 5.2: Distributions of Electoral District Similarity Score in 2011 Texas Redistricting

When the Federal Court in the Washington D.C did not grant a summary judgment in the approval of Texas' original congressional map, it cited Dallas and Fort Worth as an area of concern (Sauter, 2013). In Figure 5.3, we can see the modifications made to the old congressional map and the improved scores received as a result. Many of the congressional districts are only slightly modified in the new map compared to the old. This causes many districts' overall shapes to appear similar and many districts' improvement in our Electoral District Similarity Score comes from more efficiently grouping together majority and minority groups rather than increasing the compactness of the districts. This trend seems to be true for the whole state.

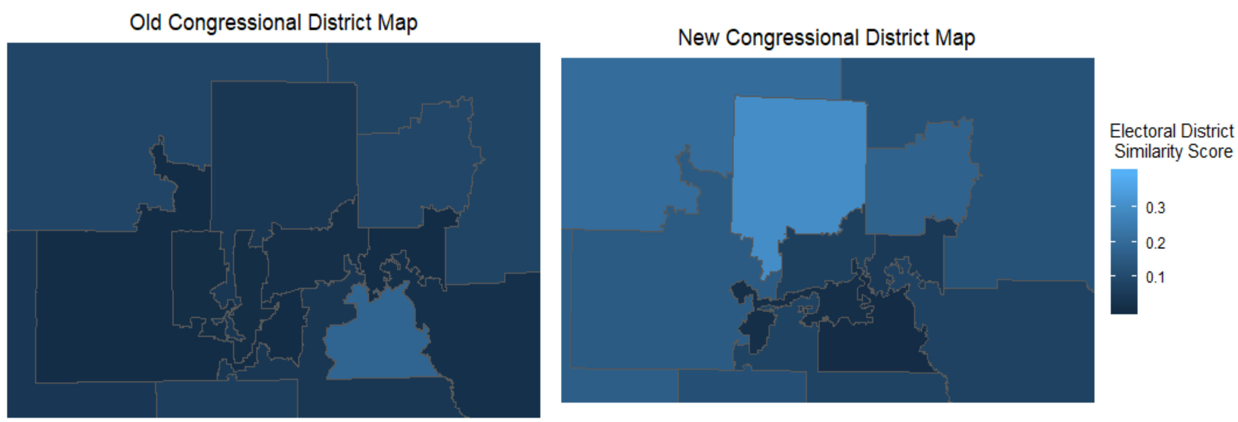

Figure 5.3: Maps Comparing Old and New Plan for Dallas and Fort Worth Area

As a component of our Electoral District Similarity Score, we calculate a White and Non-White Similarity Score to capture how large the majority population is when residents are split into White and Non-White groupings. Adding this racial component to our score is critical in showing how the new congressional mapping actually improved the fairness of the electoral districts over the old congressional maps. If we look only at compactness using the Polsby-Popper Index, the average for the new congressional mapping is only slightly improved, 0.1956 compared to 0.1909. Under a Welch's T-test this difference is not statistically significant, with a p-value of 0.85. Many of the new congressional map's districts are less compact than the old congressional map, but the distribution for White and Non-White Similarity Score is vastly improved. The average White and Non-White Similarity Score increases from 0.330 to 0.515 under the new plan, a statistically significant difference at the 0.01 level with a p-value of 0.00075. The distributions for both are shown in Figure 5.4.

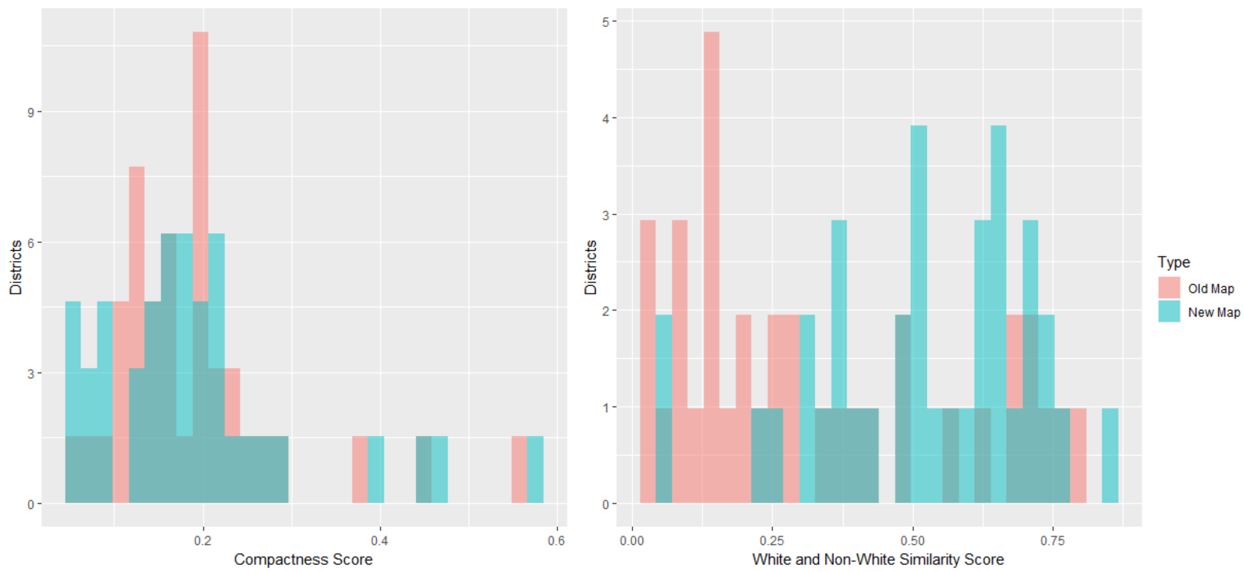

Figure 5.4: Distributions of Compactness and White and Non-White Similarity Scores in 2011 Texas Redistricting

The difference between the two congressional maps highlights the limits of compactness scores alone and the need to incorporate the underlying racial make-up of a district in some fashion. With Compactness Scores alone, not much change will be seen in the redrawn congressional maps, however with our White and Non-White Similarity Score we can see that the redrawing resulted in more efficient groupings of the Majority and Minority populations in Texas.

5.2 Case Study No. 2: 2018 Pennsylvania Redistricting

In *League of Women Voters et al. v. Commonwealth of Pennsylvania et al.* (2018), the Pennsylvania Supreme Court struck down the current congressional district mapping implemented by the state legislature in 2011 on the basis of partisan gerrymandering (Grofman & Cervas, 2018). Traditionally Pennsylvania is considered as “one of the most competitive states in the nation” but since 2011 Republicans have secured decisive advantages in recent elections (Royden, Li, & Rudensky, 2018). The ruling noted that the current district mapping was decidedly not “responsive”, requiring historically large margins for Democrats to gain any additional congressional seats (Royden, Li, & Rudensky, 2018). In *League of Women Voters et al. v. Commonwealth of Pennsylvania et al.* (2018), the Pennsylvania Supreme Court ordered the state to draw new congressional districts. We will again compare the old congressional maps to the new ones to see if the Electoral District Similarity Score shows any improvement in the new mapping.

In our analysis of the 115th Congressional Districts, Pennsylvania was still using congressional districts that it had originally drawn in 2011. We will use this data to compare with the new congressional district mapping which we retrieved from the Pennsylvania State Redistricting website. From the site we retrieved shapefiles representing district lines and estimates for the percent of residents identifying as white in each district. The new congressional districts first went into effect in the 2018 election for the 116th Congress. Our analysis shows that the court ordered redrawing of congressional districts led to a significant improvement in Electoral District Similarity Scores, with the average score rising from 0.12 under the old map to 0.221 under the new map. Running a Welch’s T-test on the two maps’ scores shows that the increase is statistically significant at the 0.01 level, with a p-value of 0.001608. With an average score of 0.221 under the new map, this puts Pennsylvania well above the national average of 0.13. The distribution of scores for the district is mapped in Figure 5.5 and given in Figure 5.6.

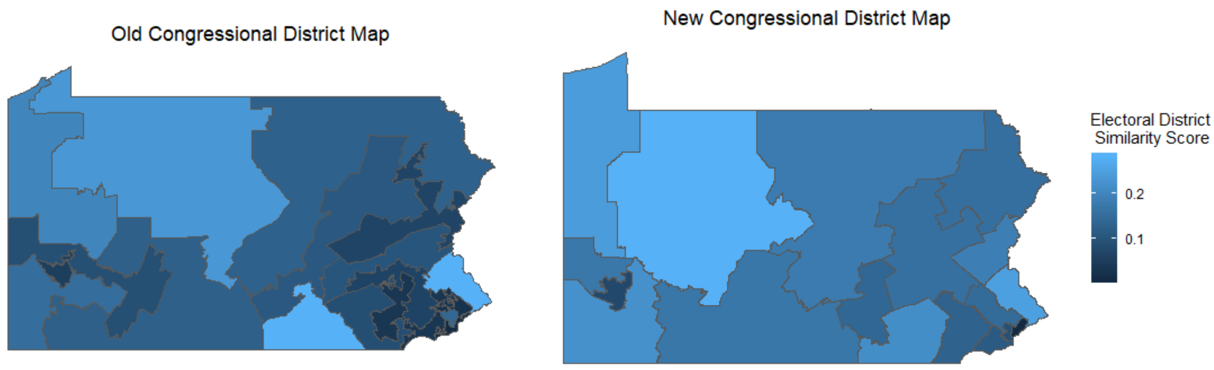

Figure 5.5: Maps of Electoral District Similarity Score in 2018 Pennsylvania Redistricting

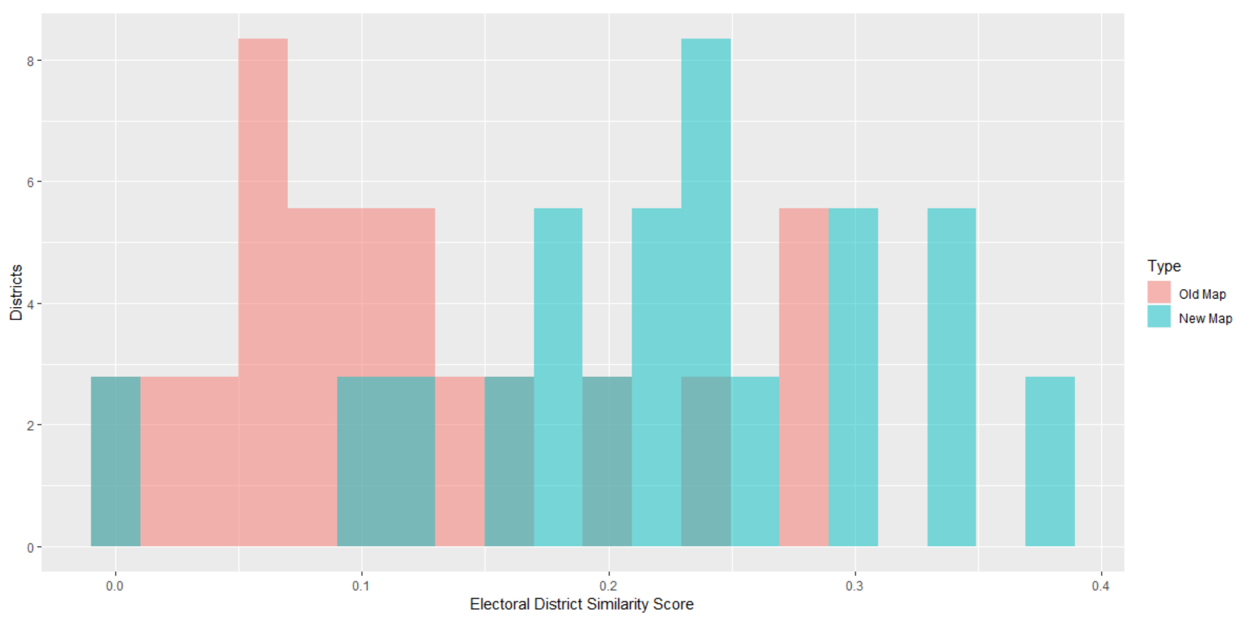

Figure 5.6: Distributions of Electoral District Similarity Score in 2018 Pennsylvania Redistricting

An interesting difference compared to the improvements made in the 2011 Texas Redistricting case is that while in Texas the increase in Electoral District Similarity Scores was largely attributed to increases in the White or Non-white Similarity Scores, in Pennsylvania it is the Compactness Scores that cause the majority of the increase. In Pennsylvania the average

Compactness Score almost doubled under the new congressional district map, increasing from 0.175 to 0.336. A Welch's T-test shows that this difference is very significant, with a p-value of less than 0.00003. However the average White Non-white Similarity Score for the state only slightly increased from 0.6722 to 0.6757, an insignificant change with a p-value of 0.9653. The distribution for both scores in Pennsylvania is given below.

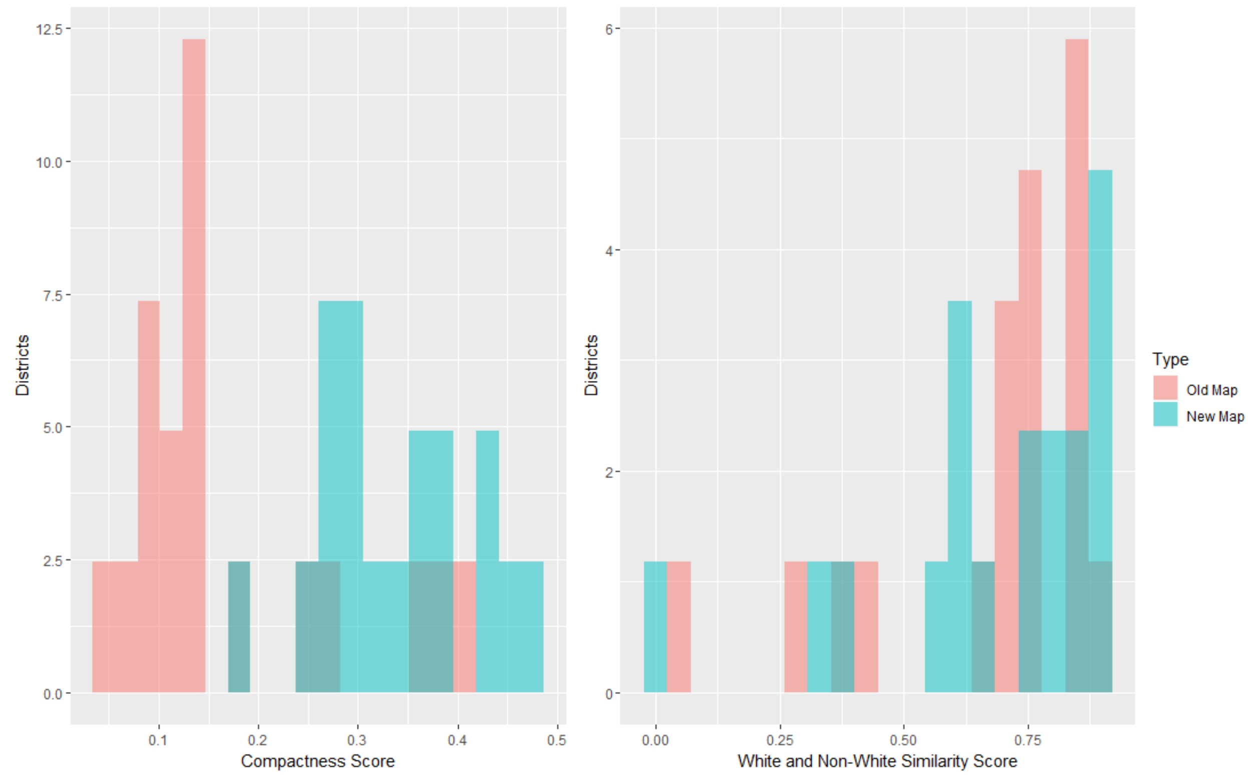

Figure 5.7: Distributions of Compactness and White and Non-White Similarity Scores
In 2018 Pennsylvania Redistricting

In the new congressional district mapping for Pennsylvania, the improvement in Electoral District Similarity Scores can largely be attributed to the increased compactness of the districts. These two case studies highlight the duality of our statistic and the need for race and compactness to be considered together.

6 Discussion

6.1 Comparing Plans

In the previous two case studies we used a Welch's T-test to see if there was a statistically significant difference between different congressional district plans proposed for the same state. Welch's T-test drops assumptions of equal variance and is more robust when comparing skewed distributions. This makes it ideal to be used in inference on the location of our Electoral District Similarity Scores. The practical applications of the Electoral District Similarity Score are mainly in comparing competing plans for electoral district mapping, as shown in the case studies. We can calculate Electoral District Similarity Scores for each district in a proposed plan and then perform Welch's t-test on the competing plans to see if any perform significantly better than the others. Welch's t-test can also be generalized to more than two samples, allowing for more robust and quicker comparison of multiple plans. Using inference in this way can allow a logical argument to be built around whether or not one redistricting plan is significantly different than another. We can interpret statistical significance to mean that one plan is significantly less likely to be drawing minority or majority districts in a compact and intuitive way. A limitation of this inference is in states with low populations and only a few electoral districts. While Electoral District Similarity Scores can still be useful in comparison with such low sample sizes, proving a statistical significance might be impossible. Even in such cases, the scores can still be used to help quantify the differences between two competing plans.

6.2 Further Research

Further research on Electoral District Similarity Scores would be greatly aided by advanced simulation methods that generate electoral districts, such as those used by Tam Cho and Liu (2016). Using Electoral District Similarity Scores as an optimizing criterion in a computer algorithm to generate electoral districts would provide an interesting analysis on the types of redistricting plans that are ideal for each state. Generating Electoral districts randomly or based on other criteria could also allow us to build a sampling distribution of Electoral District Similarity Scores for states with few electoral districts, allowing statistical inference to be used to judge their proposed redistricting plans. Such methods would be very enlightening but would require intensive computing power.

While a racial make-up component in our score seems necessary to properly identify and highlight those electoral districts with unintuitive mappings, further research could be done on how to represent it. In our Electoral District Similarity Scores we used a sort of absolute loss function, equally weighting each percentage increase above the simple majority in an electoral district. An argument could be made that increasing the majority white or non-white group from 51% to 52% of residents should not be weighted the same as an increase from 58% to 59%. If we believe that an increase closer to a simple majority should be weighted less, then a quadratic loss function could be used instead. This sort of change would likely make the racial component much more dominant in pulling down scores than the compactness score. Another avenue to explore is a hinge loss function, where after a certain percentage majority is established, anything over that value is assigned an automatic score of 1 for the White Non-white Similarity Score. In some way this might be more consistent with current court rulings on the matter (Cameron, Epstein & O'Halloran 1996; Sauter, 2013). This could also be problematic by hiding the effects that may be present in a district passed a 65% majority. In our score we chose a simple criteria to represent how similar in white and non-white groupings a district is. However, there is likely merit in choosing to model this in other ways that warrants further research.

6.3 Conclusion

Electoral District Similarity Scores are intended to fulfill a specific role in helping prevent partisan gerrymandering and in highlighting those districts which are the least similar in geographic and racial groupings. We identify these districts through unintuitive shapes, slim majorities, or a combination of both where similarity in geographic and demographic characteristics seem to be sacrificed for some unknown criteria. We demonstrated how Electoral District Similarity Scores could be used to identify areas of concern in an existing congressional district map. We looked at two prominent partisan redistricting cases and the Electoral District Similarity Scores showed that in both cases the court ordered redistricting improved the congressional mapping. These cases provided concrete examples of the need to incorporate both Compactness Scores and White Non-white Similarity Scores into the Electoral District Similarity Scores that were consistent with court findings. Using these cases we also established a practical use case and developed what inference would mean in comparing two competing redistricting plans. Through our analysis and cases studies we have shown how Electoral District Similarity Scores could be a viable extension of compactness scores to help aid in the redistricting discussion.

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