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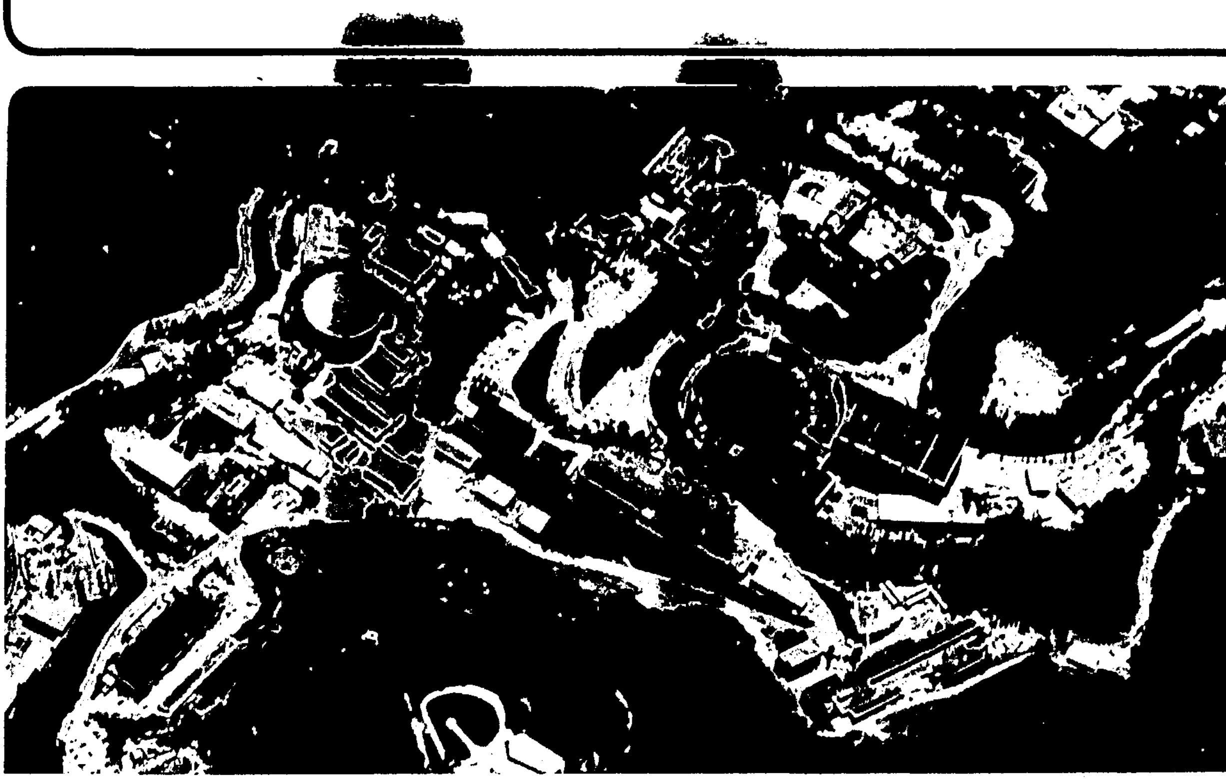

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"STOCHASTIC REGULARIZATION OF SCALAR ELECTRODYNAMICS"*

Zvi Bern

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720, U.S.A.

ABSTRACT

A regularization scheme, first proposed by Breit, Gupta, and Zaks and based upon the Langevin equation of Parisi and Wu, is used to regularize scalar electrodynamics. This scheme is shown to preserve the masslessness of the photon and the tensor structure of the photon vacuum polarization at the one loop level. The scalar wavefunction renormalization, Z_2, is shown to be equal to the one photon vertex renormalization, Z_1, to all orders of the stochastically regularized theory.

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1. Introduction

Several years ago Parisi and Wu [1] introduced stochastic quantization. While equivalent to the more standard methods of quantization, their procedure offers some interesting insights into quantum field theory. For example, they showed that gauge theories could be constructed without the need for gauge fixing so that the Gribov ambiguity could be circumvented. They also realized that the ideas inherent in the Langevin equation are strongly connected to Monte Carlo computer simulations.

Breit, Gupta, and Zaks [2] have proposed the possibility of making use of the ideas inherent in stochastic quantization to regularize field theories. However, the conclusion reached by these authors was that the applicability of this stochastic regularization to perturbation calculations is problematic. The claim was, that although the symmetries of the theory are preserved, the naive conservation laws are not preserved, so that stochastic regularization may not be a satisfactory scheme. However, the relevance of this fact to regularization and renormalization is not clear. For example, the method of higher covariant derivatives (Pauli-Villars) ruins the conservation of the naive Noether currents, but is certainly a good regularization scheme for gauge theories [3]. Another objection [4] that has been raised to the stochastic regularization scheme is that the identity,

$$\left\langle \phi(x_1) \frac{\delta S[\phi]}{\delta \phi(x_2)} \right\rangle = \delta^4(x_1 - x_2) \,, \tag{1.1}$$

where $S[\phi]$ is the action, is modified by the loop corrections in quadratically divergent theories, such as $\lambda\phi^4$ theory. Gauge theories are a different matter because, at least in the "gluon channel", they are only *superficially* quadratically divergent, if the regularization scheme preserves the gauge invariance. If in the stochastic regularization scheme such quadratic divergences don't cancel, the scheme would fail anyway. If these quadratic divergences do cancel, identity (1.1) in the "gluon channel" is, in fact, preserved. This paper will show that at the one loop level the quadratic divergences in the photon propagator of scalar electrodynamics do indeed cancel in the stochastic regularization scheme.

As a simple example of a gauge theory, this paper discusses the stochastic regularization of scalar electrodynamics. The infinite part of the photon self energy is calculated to one loop order using the stochastic regularizer and the infinite part of the photon vacuum polarization tensor is shown automatically to come out transverse, as it should. The photon does not acquire a mass at the one loop level, because at zero external momentum the photon vacuum polarization is shown to vanish. By a diagrammatic calculation it is shown that the Ward identity that equates the scalar wavefunction renormalization, Z_2 to the one photon vertex renormalization, Z_1, holds to all orders of the stochastically regularized theory.

This paper is divided into five main sections. Section 2 contains a brief overview of the ideas inherent in stochastic quantization that are needed in order to understand the regularization scheme. Section 3 gives an example of how the stochastic regularizer works for the case of a scalar theory, while Section 4 contains the explicit one loop calculation for scalar electrodynamics. Section 5 contains the proof of the Ward identity to all orders of perturbation theory. In Section 6 the conclusions and comments are given.

2. Overview of Stochastic Quantization

Stochastic quantization is based upon some well known ideas in nonequilibrium statistical mechanics [5]. For simplicity, at first, the stochastic quantization of a single scalar field, ϕ, with action, $S[\phi]$, will be considered. The usual starting point of stochastic quantization [1] is the Langevin equation,

$$\frac{\partial\phi(x,t^5)}{\partial t^5} = -\frac{\delta S[\phi]}{\delta\phi(x,t^5)} + \eta(x,t^5)\,, \tag{2.1}$$

in which t^5 is a fictitious fifth-time variable, not to be confused with physical time and x represents the four physical space-time dimensions. Here, η is a five dimensional random field with Gaussian probability distribution,

$$\langle F[\phi(\eta)]\rangle_\eta \equiv \frac{\int D\eta\; F[\phi(\eta)]\exp(-\frac{1}{4}\int \eta^2(x,t^5)d^4x dt^5)}{\int D\eta\;\exp(-\frac{1}{4}\int \eta^2(x,t^5)d^4x dt^5)}\,. \tag{2.2}$$

By evaluating the generating functional, $\langle \exp(\int J\eta\, d^4x dt^5)\rangle_\eta$, all the n-point η correlation functions can easily be calculated. After a simple calculation the two point correlation is found to be

$$\langle \eta(x,t^5)\eta(x',t^{5\prime})\rangle_\eta = 2\,\delta^4(x-x')\,\delta(t^5-t^{5\prime})\,, \tag{2.3}$$

while all other *connected* η correlations vanish.

The connection to the standard formulation of quantum field theory is arrived at by evaluating the equal fifth-time expectation values. That is, it is possible to prove that

$$\lim_{t^5\to\infty}\langle\phi(x_1,t^5)\phi(x_2,t^5)\ldots\phi(x_n,t^5)\rangle_\eta = \frac{\int D\phi\;\phi(x_1)\phi(x_2)\ldots\phi(x_n)e^{-S[\phi]}}{\int D\phi\; e^{-S[\phi]}}\,, \tag{2.4}$$

where $S[\phi]$ is the four dimensional action. Note that on the right hand side of the equation the field, ϕ, is a function of the four physical space-time dimensions, while on the left hand side of the equation, ϕ is a function of the five dimensional extended space. By starting the Langevin system at $t^5_0 = -\infty$, the system is equilibrated for any finite fifth-time, so there is no need to take the limit of infinite fifth time to make the correspondence to the standard formulation of field theory.

There are quite a few proofs in the literature of the equivalence of stochastic quantization to the standard procedures of quantization. One way to make the connection is by defining the Fokker-Planck probability [6], which describes the probability density of finding the field ϕ at a given value under the Langevin dynamics. By deriving an evolution equation for the Fokker-Planck probability, it is possible to show that for essentially arbitrary initial conditions, at equilibrium, the Fokker-Planck probability reduces to the probability density of the ordinary formulation. There are also proofs based on the various perturbative expansions of stochastic quantization [7]. Another rather elegant proof makes use of a hidden supersymmetry [8].

The Langevin equation can be used to perturbatively solve quantum field theories. In general, the lagrangian will consist of a kinetic term plus an interaction potential. Thus, the Langevin equation is

$$\frac{\partial\phi(x,t^5)}{\partial t^5}+(-\partial^2+m^2)\phi(x,t^5)=-V'(\phi(x,t^5))+\eta(x,t^5)\,, \tag{2.5}$$

where $V'(\phi)$ is the derivative of the potential with respect to the field ϕ. One way to handle this equation is with the method of Green functions.

$$\frac{\partial G(x-x',t^5-t^{5\prime})}{\partial t^5}+(-\partial^2+m^2)G(x-x',t^5-t^{5\prime})=\delta^4(x-x')\,\delta(t^5-t^{5\prime})\,. \tag{2.6}$$

The causal Green function in coordinate space is

$$G(x-x',t^5-t^{5\prime})=\theta(t^5-t^{5\prime})\int\frac{d^4p}{(2\pi)^4}\,e^{-ip(x-x')}e^{-(p^2+m^2)(t^5-t^{5\prime})}\,. \tag{2.7}$$

The Green function can be used to rewrite the differential equation as an integral equation,

$$\phi(x,t^5)=\int d^4x'\int_{-\infty}^{\infty}dt^{5\prime}\;G(x-x',t^5-t^{5\prime})[\eta(x',t^{5\prime})-V'(\phi(x',t^{5\prime}))]\,, \tag{2.8}$$

that contains the initial condition that the field vanishes at $t_0^5=-\infty$, as well as the causality requirement. To simplify matters, a compact notation is introduced.

$$G_{x1}\equiv G(x-x_1,t^5-t_1^5)\,,\qquad \eta_1\equiv\eta(x_1,t_1^5)\,,\qquad \int_1\equiv\int d^4x_1\int_{-\infty}^{\infty}dt_1^5\,. \tag{2.9}$$

By iteration the integral equation (2.8) can be solved as a perturbative series.

$$\phi(x,t^5)=\int_1 G_{x1}\eta_1-\int_1 G_{x1}V'\Big(\int_2 G_{12}\eta_2-\int_2 G_{12}V'\Big(\int_3 G_{23}\eta_3-\cdots\Big)\Big)\,. \tag{2.10}$$

An explicit example of how the the Langevin equation can be used to generate a perturbation series is the massive scalar ϕ^4 theory. To the first order in the coupling constant the field is given from equation (2.10) to be

$$\phi(x,t^5)=\int_1 G_{x1}\eta_1-\frac{\lambda}{3!}\int_1 G_{x1}\Big[\int_2 G_{12}\eta_2\Big]^3+\cdots \tag{2.11}$$

The tree diagrams corresponding to the perturbation series are given in Figure 1. Each line corresponds to a Green function, while the crosses at the ends of the diagrams represent the noise term, η. The vertex factors are the same as for ordinary Feynman diagrams, up to a possible combinatoric factor.

The loop diagrams come about by piecing together the tree diagrams (Fig. 2). For example, the two point correlation function is

$$\begin{aligned}\langle\phi(x,t^5)\phi(x',t^{5\prime})\rangle_\eta &= \left\langle\int_1\int_2 G_{x1}G_{x'2}\eta_1\eta_2\right\rangle_\eta \\ &\quad - \frac{\lambda}{3!}\left\langle\int_1\int_2 [G_{x1}G_{x'2}+G_{x'1}G_{x2}]\eta_1\left[\int_3 G_{23}\eta_3\right]^3\right\rangle_\eta + O(\lambda^2)\,.\end{aligned} \tag{2.12}$$

From equation (2.2), the n-point η correlation functions are sums of products of delta functions. The delta functions can be thought of as glue that holds the tree diagrams together to form the n-point ϕ correlations. As will be discussed in the next section, stochastic regularization consists of smearing the delta function glue in fifth-time.

The zeroth order contribution is given by

$$\begin{aligned}\langle\phi(x,t^5)\phi(x,t^{5\prime})\rangle_\eta^{(0)} &\equiv D(x-x',t^5-t^{5\prime}) \\ &= 2\int_1 G_{x1}G_{x'1} \end{aligned} \tag{2.13}$$

$$= \int\frac{d^4p}{(2\pi)^4}\, e^{-ip(x-x')}\frac{e^{-(p^2+m^2)|t^5-t^{5\prime}|}}{p^2+m^2}\,. \tag{2.14}$$

Therefore, in momentum space, the zeroth order free propagator is given by

$$D_{12}(p) = \frac{e^{-(p^2+m^2)|t_1^5-t_2^5|}}{p^2+m^2}\,, \tag{2.15}$$

where the subscript on $D_{12}(p)$ refers only to the fifth-time coordinate. After replacing the η correlations with the appropriate delta functions and combining terms that differ only by dummy indices, the first order contribution is given by

$$\langle\phi(x,t^5)\phi(x',t^{5\prime})\rangle_\eta^{(1)} = -2\lambda\int_1\int_2\int_3 [G_{x1}G_{x'2}G_{21}G_{23}G_{23}+G_{x'1}G_{x2}G_{21}G_{23}G_{23}]\,. \tag{2.16}$$

By explicit evaluation, it is easy to check that for $t^5 = t^{5\prime}$, the same result is obtained as by using ordinary Feynman diagrams.

3. Stochastic Regularization

Since there is an extra dimension present in the Langevin approach, the infinities can be smeared without destroying any symmetries that are present in the corresponding four dimensional theory. The preservation of the symmetries that are present in the infinite theory is crucial to finding a satisfactory regularization scheme. A time smeared system is known as a non-Markovian system

[5]. In general, such a system can be expected to be less divergent than its Markovian counterpart. From the perturbative point of view, stochastic regularization can be thought of as preventing the loops of the correlation functions from completely closing on themselves in the fifth-time.

There are at least two choices for fifth-time smearing the Langevin system. Either the Langevin equation or the probability distribution of the random noise, η, can be smeared. By studying the first order correction in the $\lambda\phi^4$ theory, it is possible to show that the non-Markovian Langevin equation,

$$\frac{\partial\phi(x,t^5)}{\partial t^5} = -\int dt^{5\prime}\,\alpha_\Lambda(t^5 - t^{5\prime})\frac{\delta S[\phi]}{\delta\phi(x,t^{5\prime})} + \eta(x,t^5)\,, \tag{3.1}$$

where α_Λ is a smearing function, can at best only remove two degrees of divergence in the perturbation theory. Quadratically divergent integrals become logarithmically divergent, and there does not exist a regularization function that does better.

The other possibility is to smear the η probability functional [2]. In this scheme, the Langevin equation is left alone, while equation (2.2) is replaced by

$$\langle F[\phi(\eta)]\rangle_\eta \equiv \frac{\int \mathcal{D}\eta\; F[\phi(\eta)]\exp(-\frac{1}{4}\int \eta(x,t^5)\alpha_\Lambda^{-1}(t^5 - t^{5\prime})\eta(x,t^{5\prime})d^4x\,dt^5 dt^{5\prime})}{\int \mathcal{D}\eta\;\exp(-\frac{1}{4}\int \eta(x,t^5)\alpha_\Lambda^{-1}(t^5 - t^{5\prime})\eta(x,t^{5\prime})d^4x\,dt^5 dt^{5\prime})}\,. \tag{3.2}$$

This changes the η correlation to

$$\langle \eta(x,t^5)\eta(x',t^{5\prime})\rangle_\eta = 2\,\delta^4(x-x')\,\alpha_\Lambda(t^5 - t^{5\prime})\,. \tag{3.3}$$

The smearing functions α_Λ and α_Λ^{-1} are functional inverses of each other, in the sense that

$$\int dt^{5\prime\prime}\,\alpha_\Lambda(t^5 - t^{5\prime\prime})\,\alpha_\Lambda^{-1}(t^{5\prime\prime} - t^{5\prime}) = \delta(t^5 - t^{5\prime})\,. \tag{3.4}$$

The hope is that, because

$$\lim_{\Lambda\to\infty} \alpha_\Lambda(t^5 - t^{5\prime}) = \delta(t^5 - t^{5\prime})\,, \tag{3.5}$$

as Λ becomes infinite, the original theory is recovered.

Since the Langevin equation is unaffected by the stochastic regularization, the physical field is the same as in the unregularized case, so that

$$\langle \phi(x,t^5)\phi(x',t^{5\prime})\rangle_\eta^{(0)} = \int_1\int_2 G_{x1}G_{x'2}\langle \eta_1\eta_2\rangle_\eta\,. \tag{3.6}$$

In this case, however, the two point η correlation is given by equation (3.3). Working in physical momentum space the zeroth order propagator is

$$D_{12}^\Lambda(p) \equiv 2\int dt_3^5 \int dt_4^5\; G_{13}(p)G_{24}(p)\,\alpha_\Lambda(t_3^5 - t_4^5) \tag{3.7}$$

$$= 2\int_{-\infty}^{t_1^5} dt_3^5 \int_{-\infty}^{t_2^5} dt_4^5\, e^{-(t_1^5 - t_3^5)(p^2+m^2)}e^{-(t_2^5 - t_4^5)(p^2+m^2)}\alpha_\Lambda(t_3^5 - t_4^5) \tag{3.8}$$

$$= 2\int \frac{dE}{2\pi}\, e^{-iE(t_1^5 - t_2^5)}\frac{\tilde{\alpha}_\Lambda(E)}{(p^2+m^2)^2 + E^2}\,, \tag{3.9}$$

where the Fourier transform of the smearing function, $\tilde{\alpha}_\Lambda(E)$, has been introduced. Since there is an extra power of p^2 in the denominator over the ordinary Feynman propagator, a reduction of two degrees of divergence can be obtained, if $\tilde{\alpha}_\Lambda(E)$ cuts off for large values of E. Since all loops in the perturbative expansion of an arbitrary theory contain at least one factor of $\tilde{\alpha}_\Lambda(E)$, the logarithmically divergent loops can be expected to be rendered finite.

It is a little more difficult to regularize a theory whose diagrams are quadratically divergent. For example, the first order correction to the scalar propagator in ϕ^4 theory, is

$$\begin{aligned}\langle\phi(x_1,t_1^5)\phi(x_2,t_2^5)\rangle_\eta^{(1)} &= -\frac{\lambda}{3!}\left\langle\int_3\int_4 [G_{14}G_{23}+G_{13}G_{24}]\eta_3\left[\int_5 G_{45}\eta_5\right]^3\right\rangle_\eta \\ &= -\frac{\lambda}{2}\int\frac{d^4k}{(2\pi)^4}\,e^{-ik(x_1-x_2)}\int dt_3^5\,[D_{13}^\Lambda(k)G_{23}(k)+D_{23}^\Lambda(k)G_{13}(k)] \\ &\quad\times\int\frac{d^4p}{(2\pi)^4}\,D_{33}^\Lambda(p)\,.\end{aligned} \tag{3.10}$$

It is possible to find a necessary condition on the set of functions that can be used as regularizers by studying the loop of the first order correction [2]. In this case, the loop is decoupled from the rest of the diagram, so the loop can be studied by itself. The loop is given by

$$L=\int\frac{d^4p}{(2\pi)^4}\,D_{33}^\Lambda(p) \tag{3.11}$$

$$=2\int_0^\infty dt_4^5\int_0^\infty dt_5^5\int\frac{d^4p}{(2\pi)^4}\,e^{-(t_4^5+t_5^5)(p^2+m^2)}\alpha_\Lambda(t_4^5-t_5^5) \tag{3.12}$$

$$\sim\frac{2\alpha_\Lambda(0)}{(4\pi)^2}\int_0^\epsilon dT^5\,\frac{1}{T^5}\,. \tag{3.13}$$

In order for the integral to be finite, a necessary condition on the regularization function is that [2]

$$\alpha_\Lambda(0)=0\,. \tag{3.14}$$

Using the Fourier transform of the smearing function, $\tilde{\alpha}_\Lambda(E)$, condition (3.14) can be rewritten as

$$\int\frac{dE}{2\pi}\,\tilde{\alpha}_\Lambda(E)=0\,. \tag{3.15}$$

Therefore, to remove quadratic divergences, the support of $\tilde{\alpha}_\Lambda(E)$ is not positive. The generating functional in Euclidean space, in general, won't be well defined as can be seen by looking at the generating functional written in terms of the Fourier transformed fields.

$$Z[J]=\frac{\int\mathcal{D}\eta\,\exp(-\int\frac{d^4p}{(2\pi)^4}\frac{dE}{2\pi}\,[\frac{1}{4}|\eta(p,E)|^2/\tilde{\alpha}_\Lambda(E)-J^*(p,E)\phi(p,E)])}{\int\mathcal{D}\eta\,\exp(-\int\frac{d^4p}{(2\pi)^4}\frac{dE}{2\pi}\,[\frac{1}{4}|\eta(p,E)|^2/\tilde{\alpha}_\Lambda(E)])}\,. \tag{3.16}$$

This action is unbounded from below, which seems to rule out the nonperturbative usefulness of the stochastic regularizer for quadratically divergent theories [9]. For logarithmically divergent theories, such as supersymmetric theories, the nonperturbative usefulness of the stochastic regularizer is not ruled out.

4. Stochastic Regularisation of Perturbative Scalar Electrodynamics

The manifestly covariant gauge fixed four dimensional action of euclidean scalar electrodynamics is

$$S[A_\sigma, \phi^\dagger, \phi] = \int d^4x \left[-\frac{1}{2} A_\mu (T_{\mu\nu} + \frac{1}{\lambda} L_{\mu\nu}) \partial^2 A_\nu + |(\partial_\mu - ieA_\mu)\phi|^2 + m^2 |\phi|^2 \right] . \tag{4.1}$$

Using the standard Feynman diagrammatical techniques, the quantum corrections to the vacuum polarization in scalar electrodynamics can easily be calculated. In doing the calculation, care must be taken, because the diagrams are infinite [10]. For example, the first order correction to the vacuum polarization in euclidean space is given by (Fig. 3):

$$\Pi_{\mu\nu}(k) = -2e^2 \int \frac{d^4p}{(2\pi)^4} \frac{\delta_{\mu\nu}}{p^2 + m^2} + e^2 \int \frac{d^4p}{(2\pi)^4} \frac{(2p+k)_\mu (2p+k)_\nu}{[(k+p)^2 + m^2](p^2 + m^2)} . \tag{4.2}$$

Using a naive momentum cutoff, Λ, on the integrals, to leading order in the cutoff, one obtains

$$\Pi_{\mu\nu}(k) \sim -\frac{e^2 \Lambda^2}{16\pi^2} \delta_{\mu\nu} . \tag{4.3}$$

Thus, this naive regularizer explicitly breaks gauge invariance by giving the photon a mass.

An example of a well known gauge invariant regularization scheme is dimensional regularization [3]. In this scheme the dimension of space-time is "analytically continued" to $4-\epsilon$ dimensions, where the integral is finite. In this case, the photon mass correction contributions of the two diagrams just cancel to give a gauge invariant vacuum polarization.

$$\Pi_{\mu\nu}(k) = \frac{1}{3} \frac{e^2}{(4\pi)^2} (k_\mu k_\nu - k^2 \delta_{\mu\nu}) \ln \frac{\Lambda^2}{m^2} + \text{regular terms} , \tag{4.4}$$

where the usual connection, $\frac{2}{\epsilon} \leftrightarrow \ln \Lambda^2$, has been made and where Λ is a cutoff parameter with units of momentum.

As first discussed by Parisi and Wu [1], it is possible to formulate gauge theories without the need for gauge fixing, by using stochastic quantization. The gauge invariance manifests itself by a nonequilibrating random walk in the gauge parameter space. Since the physically interesting quantities are gauge invariant, the wandering in the gauge parameter space is essentially irrelevant. In fact, as Parisi and Wu pointed out, it is possible to rewrite the Langevin equations in terms of gauge invariant fields. Another simple way to avoid the nonequilibration of the abelian gauge field is by introducing a gauge fixing term, since the property that gauge fixing is unnecessary is unimportant for this study of regularization.

The Langevin equations of the gauge fixed scalar electrodynamics are

$$\frac{\partial\phi}{\partial t^5} = (\partial^2 - m^2)\phi - ieA_\mu\partial_\mu\phi - ie\partial_\mu(A_\mu\phi) - e^2A_\mu A_\mu\phi + \eta \tag{4.5}$$

$$\frac{\partial\phi^\dagger}{\partial t^5} = (\partial^2 - m^2)\phi^\dagger + ieA_\mu\partial_\mu\phi^\dagger + ie\partial_\mu(A_\mu\phi^\dagger) - e^2A_\mu A_\mu\phi^\dagger + \eta^\dagger \tag{4.6}$$

$$\frac{\partial A_\mu}{\partial t^5} = (T_{\mu\nu}\partial^2 + \frac{1}{\lambda}L_{\mu\nu}\partial^2)A_\nu - ie\phi^\dagger(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)\phi - 2e^2A_\mu\phi^\dagger\phi + \eta_\mu\,, \tag{4.7}$$

with unsmeared expectation values defined by

$$\langle F[A_\sigma,\phi^\dagger,\phi]\rangle_\eta \equiv \frac{\int D\eta_\mu D\eta^\dagger D\eta\; F[A_\sigma,\phi^\dagger,\phi]\exp(-\frac{1}{4}\int[\eta_\nu^2 + 2\eta^\dagger\eta]\,d^4x\,dt^5)}{\int D\eta_\mu D\eta^\dagger D\eta\exp(-\frac{1}{4}\int[\eta_\nu^2 + 2\eta^\dagger\eta]\,d^4x\,dt^5)}\,. \tag{4.8}$$

The causal Green function for the photon Langevin equation is

$$G_{\mu\nu}(x,t^5) = \theta(t^5)\int\frac{d^4k}{(2\pi)^4}\,e^{-ikx}\Big[T_{\mu\nu}(k)e^{-k^2t^5} + L_{\mu\nu}(k)e^{-k^2t^5/\lambda}\Big]\,, \tag{4.9}$$

while in the unregularized theory the zeroth order propagator is

$$\begin{aligned}D_{\mu\nu}(x,t^5) &\equiv \langle A_\mu(x,t^5)A_\nu(0,0)\rangle_\eta^{(0)}\\ &= \int\frac{d^4k}{(2\pi)^4}\,e^{-ikx}\Big[\frac{T_{\mu\nu}(k)}{k^2}\,e^{-k^2t^5} + \frac{\lambda L_{\mu\nu}(k)}{k^2}\,e^{-k^2t^5/\lambda}\Big]\,,\end{aligned} \tag{4.10}$$

where $T_{\mu\nu}(k)$ and $L_{\mu\nu}(k)$ are respectively the transverse and longitudinal projection operators. The two point functions for the scalars are given in equations (2.7) and (2.14). As with ordinary Feynman diagrammatic calculations the simplest gauge to use is Feynman gauge, where $\lambda = 1$. Henceforth, the Feynman gauge will be used exclusively.

An example of a function that satisfies the condition of equation (3.14), and renders the loops finite, is [2]

$$\alpha_\Lambda^{(d)}(t^5 - t^{5\prime}) = \frac{\Lambda^4|t^5 - t^{5\prime}|}{2}\,e^{-\Lambda^2|t^5 - t^{5\prime}|}\,, \tag{4.11}$$

The superscript refers to the fact that the Fourier transform of the above regularization function has a double pole structure. For calculational purposes it is easier to use a function whose Fourier transform has a single pole structure. Namely,

$$\alpha_\Lambda^{(s)}(t^5 - t^{5\prime}) = \frac{\Lambda^2}{2}\,e^{-\Lambda^2|t^5 - t^{5\prime}|}\,, \tag{4.12}$$

which does not satisfy the requirements of a quadratic divergence regularization function. The two functions are related by

$$\alpha_\Lambda^{(d)}(t^5 - t^{5\prime}) = -\Lambda^4\frac{\partial}{\partial\Lambda^2}\Big[\frac{\alpha_\Lambda^{(s)}(t^5 - t^{5\prime})}{\Lambda^2}\Big]\,. \tag{4.13}$$

Therefore, $\alpha^{(s)}$ can be used until a divergent integral is to be evaluated, where equation (4.13) will be used to replace $\alpha_\Lambda^{(s)}$ with $\alpha_\Lambda^{(d)}$, within the calculation.

Since this section is only concerned with the perturbative one loop expansion of the photon propagator, the photon random noise field need not be fifth-time smeared, since only scalars appear within the loops. Using equations (3.8) and (4.12), the zeroth order regularized scalar two point function is

$$D_{12}^{(s)}(p) = e^{-|t_1^5 - t_2^5|(p^2+m^2)}\left[\frac{\Lambda^2}{(p^2+m^2)(p^2+m^2+\Lambda^2)} - \frac{\Lambda^2}{(p^2+m^2)^2-\Lambda^4}\right] + e^{-\Lambda^2|t_1^5-t_2^5|}\frac{\Lambda^2}{(p^2+m^2)^2-\Lambda^4} \, . \tag{4.14}$$

Note that the apparent singularity at $p^2 + m^2 = \pm\Lambda^2$ is fictitious.

The seven Langevin diagrams of the one loop correction to the photon propagator in scalar electrodynamics are given in Figure 4. Since only physical expectation values are of interest, the external fifth-times are taken to be equal. Introducing the simplifying notation

$$a \equiv p^2 + m^2 \, , \qquad b \equiv (p+k)^2 + m^2 \, , \tag{4.15}$$

the diagrams with no external momenta in the loop (Fig. 4a) are given by

$$P_{\rho\sigma}^{(d)\,1}(k) = -4e^2\delta_{\mu\nu}\int dt_1^5 \, D_{01}^{\rho\sigma}(k)\, G_{01}^{\sigma\nu}(k) \int \frac{d^4p}{(2\pi)^4}\left(-\Lambda^4\frac{\partial}{\partial \Lambda^2}\right)\left\{\frac{D_{11}^{(s)}(p)}{\Lambda^2}\right\} \tag{4.16}$$

$$= -\frac{2e^2\delta_{\rho\sigma}}{k^4}\int \frac{d^4p}{(2\pi)^4}\,\frac{\Lambda^4}{a(a+\Lambda^2)^2} \, . \tag{4.17}$$

The other diagrams are significantly more complicated because of the intertwining of the external legs with the loop. In order to simplify the expressions, the vertex factors will be written as

$$V_{\mu\nu} \equiv e^2(2p+k)_\mu(2p+k)_\nu \, . \tag{4.18}$$

The diagram in Figure 4b is given by

$$P^{(d)^2}_{\rho\sigma}(k) = \int dt_1^5 \int dt_2^5\, G^{\rho\sigma}_{01}(k) G^{\sigma\nu}_{02}(k) \int \frac{d^4p}{(2\pi)^4}\, V_{\mu\nu}\, D^{(d)}_{12}(p+k) D^{(d)}_{12}(p) \tag{4.19}$$

$$= \frac{\delta_{\rho\mu}\delta_{\sigma\nu}}{k^2}\Big(-{\Lambda_1}^4 \frac{\partial}{\partial {\Lambda_1}^2}\Big)\Big(-{\Lambda_2}^4 \frac{\partial}{\partial {\Lambda_2}^2}\Big)\int \frac{d^4p}{(2\pi)^4}\, V_{\mu\nu} \frac{1}{b^2 - {\Lambda_1}^4} \frac{1}{a^2 - {\Lambda_2}^4}\Big[\frac{{\Lambda_1}^2{\Lambda_2}^2}{ab(a+b+k^2)}$$

$$- \frac{{\Lambda_2}^2}{a(a+k^2+{\Lambda_1}^2)} - \frac{{\Lambda_1}^2}{b(b+k^2+{\Lambda_2}^2)} + \frac{1}{(k^2+{\Lambda_1}^2+{\Lambda_2}^2)}\Big]\Big|_{\Lambda_1=\Lambda_2=\Lambda} \tag{4.20}$$

$$= \frac{\delta_{\rho\mu}\delta_{\sigma\nu}}{k^4}\Big(-{\Lambda_1}^4 \frac{\partial}{\partial {\Lambda_1}^2}\Big)\Big(-{\Lambda_2}^4 \frac{\partial}{\partial {\Lambda_2}^2}\Big)\int \frac{d^4p}{(2\pi)^4}\, V_{\mu\nu} k^2$$
$$\times\ [({\Lambda_1}^2 + a + b + k^2){\Lambda_2}^4 + ({\Lambda_2}^2 + a + b + k^2){\Lambda_1}^4$$
$$+ (3k^2 + 2b + 2a){\Lambda_1}^2{\Lambda_2}^2 + k^2(2k^2 + 3b + 3a)({\Lambda_1}^2 + {\Lambda_2}^2)$$
$$+ (a+b)^2({\Lambda_1}^2 + {\Lambda_2}^2) + k^4(k^2 + 2b + 2a)$$
$$+ k^2(a^2 + 3ab + b^2) + ab^2 + a^2b]$$
$$/\ [ab(a+b+k^2)({\Lambda_1}^2 + b)({\Lambda_2}^2 + a)({\Lambda_1}^2 + a + k^2)$$
$$\times\ ({\Lambda_2}^2 + b + k^2)({\Lambda_1}^2 + {\Lambda_2}^2 + k^2)]\big|_{\Lambda_1=\Lambda_2=\Lambda}\,, \tag{4.21}$$

where the two regularization parameters are distinguished, in order to be able to differentiate individually each of the two regularization functions contained within the diagram. Later Λ_1 will be set equal to Λ_2. The diagrams in Figure 4c contribute a value of

$$P^{(d)^3}_{\rho\sigma}(k) = 2\int dt_1^5 \int dt_2^5\, D^{\rho\sigma}_{01}(k) G^{\sigma\nu}_{02}(k) \int \frac{d^4p}{(2\pi)^4}\, V_{\mu\nu}\, D^{(d)}_{12}(p+k) G_{21}(p) \tag{4.22}$$

$$= \frac{\delta_{\rho\mu}\delta_{\sigma\nu}}{k^4}\int \frac{d^4p}{(2\pi)^4}\, V_{\mu\nu}\Big(-\Lambda^4\frac{\partial}{\partial\Lambda^2}\Big)\Big\{\frac{1}{b^2-\Lambda^4}\Big[\frac{1}{k^2+a+\Lambda^2} - \frac{\Lambda^2}{b(k^2+a+b)}\Big]\Big\} \tag{4.23}$$

$$= \frac{\delta_{\rho\mu}\delta_{\sigma\nu}}{k^4}\int \frac{d^4p}{(2\pi)^4}\, V_{\mu\nu}\Big(-\Lambda^4\frac{\partial}{\partial\Lambda^2}\Big)\frac{(k^2+a+b+\Lambda^2)}{b(b+\Lambda^2)(a+b+k^2)(a+k^2+\Lambda^2)}\,. \tag{4.24}$$

Similarly the last two diagrams can be evaluated. The values are identical to the diagrams just calculated, as can be shown either by symmetry or by shifting the variables of integration. Therefore, the diagrams in Figure 4d contribute a value of

$$P^{(d)^4}_{\rho\sigma}(k) = P^{(d)^3}_{\rho\sigma}(k)\,. \tag{4.25}$$

In order to make the theory finite the results obtained by using $\alpha^{(s)}_\Lambda$ are taken and differentiated in order to obtain the results by using $\alpha^{(d)}_\Lambda$. For calculational purposes it is better to use the form of the vacuum polarization that contains no apparent singularities. After truncating the external photon lines the vacuum polarization of the photon is

$$\Pi^{(d)}_{\mu\nu}(k) \equiv \Pi^{(d)^1}_{\mu\nu}(k) + \Pi^{(d)^2}_{\mu\nu}(k) + \Pi^{(d)^3}_{\mu\nu}(k) + \Pi^{(d)^4}_{\mu\nu}(k)\,, \tag{4.26}$$

where

$$\Pi^{(d)^1}_{\mu\nu}(k) = -2e^2\delta_{\mu\nu}\int \frac{d^4p}{(2\pi)^4}\,\frac{\Lambda^4}{a(a+\Lambda^2)^2}\,, \tag{4.27}$$

$$\Pi^{(d)^2}_{\rho\sigma}(k) = k^2\left(-\Lambda_1{}^4\frac{\partial}{\partial \Lambda_1{}^2}\right)\left(-\Lambda_2{}^4\frac{\partial}{\partial \Lambda_2{}^2}\right)\int \frac{d^4p}{(2\pi)^4} V_{\mu\nu}$$
$$\times \left[(\Lambda_1{}^2 + a + b + k^2)\Lambda_2{}^4 + (\Lambda_2{}^2 + a + b + k^2)\Lambda_1{}^4 \right.$$
$$+ (3k^2 + 2b + 2a)\Lambda_1{}^2\Lambda_2{}^2 + k^2(2k^2 + 3b + 3a)(\Lambda_1{}^2 + \Lambda_2{}^2)$$
$$+ (a+b)^2(\Lambda_1{}^2 + \Lambda_2{}^2) + k^4(k^2 + 2b + 2a)$$
$$\left. + k^2(a^2 + 3ab + b^2) + ab^2 + a^2b\right]$$
$$/\left[ab(a + b + k^2)(\Lambda_1{}^2 + b)(\Lambda_2{}^2 + a)(\Lambda_1{}^2 + a + k^2)\right.$$
$$\left.\times\ (\Lambda_2{}^2 + b + k^2)(\Lambda_1{}^2 + \Lambda_2{}^2 + k^2)\right]\Big|_{\Lambda_1=\Lambda_2=\Lambda}, \tag{4.28}$$

$$\Pi^{(d)^3}_{\mu\nu}(k) = e^2 \int \frac{d^4p}{(2\pi)^4} \frac{\Lambda^4(2p+k)_\mu(2p+k)_\nu}{b(b+\Lambda^2)(a+b+k^2)(a+k^2+\Lambda^2)}$$
$$\times \left[\frac{a+b+k^2+\Lambda^2}{b+\Lambda^2} + \frac{a+b+k^2+\Lambda^2}{a+k^2+\Lambda^2} - 1\right], \tag{4.29}$$

$$\Pi^{(d)^4}_{\mu\nu}(k) = e^2 \int \frac{d^4p}{(2\pi)^4} \frac{\Lambda^4(2p+k)_\mu(2p+k)_\nu}{a(a+\Lambda^2)(a+b+k^2)(b+k^2+\Lambda^2)}$$
$$\times \left[\frac{a+b+k^2+\Lambda^2}{a+\Lambda^2} + \frac{a+b+k^2+\Lambda^2}{b+k^2+\Lambda^2} - 1\right]. \tag{4.30}$$

Although these integrals may seem quite formidable, only a few of the terms will contribute to the infinite part of the vacuum polarization.

A fundamental consequence of the gauge invariance of scalar electrodynamics is that the photon does not acquire a mass by the higher order corrections to the vacuum polarization. Setting the external momentum to zero, the exact mass correction to the photon can be found. Explicitly,

$$\Pi^{(d)}_{\mu\mu}(0) = \Pi^{(d)^1}_{\mu\mu}(0) + 2\Pi^{(d)^3}_{\mu\mu}(0) \tag{4.31}$$

where,

$$\Pi^{(d)^1}_{\mu\mu}(0) = -8e^2 \int \frac{d^4p}{(2\pi)^4} \frac{\Lambda^4}{(p^2+m^2)(p^2+m^2+\Lambda^2)^2} \tag{4.32}$$

and

$$2\Pi^{(d)^3}_{\mu\mu}(0) = 4\Lambda^4e^2 \int \frac{d^4p}{(2\pi)^4} \frac{p^2}{p^2+m^2}\left[\frac{1}{(p^2+m^2)(p^2+m^2+\Lambda^2)^2} + \frac{2}{(p^2+m^2+\Lambda^2)^3}\right] \tag{4.33}$$
$$= 4\Lambda^4e^2 \int_0^1 dz \int_0^\infty \frac{dp^2}{(4\pi)^2} \frac{6zp^4}{(p^2+m^2+z\Lambda^2)^4} \tag{4.34}$$
$$= 8\Lambda^4e^2 \int_0^1 dz \int_0^\infty \frac{dp^2}{(4\pi)^2} \frac{2zp^2}{(p^2+m^2+z\Lambda^2)^3} \tag{4.35}$$
$$= 8e^2 \int \frac{d^4p}{(2\pi)^4} \frac{\Lambda^4}{(p^2+m^2)(p^2+m^2+\Lambda^2)^2}. \tag{4.36}$$

Thus, the desired result is

$$\Pi^{(d)}_{\mu\mu}(0) = 0\,, \tag{4.37}$$

and the mass correction vanishes.

A direct evaluation of the finite parts of the vacuum polarization with the stochastic regularizer is fairly involved and will not be discussed here. We have computed only the infinite part of the vacuum polarization for nonzero external momentum. The contribution to the vacuum polarization of the simplest diagrams is from equation (4.27).

$$\Pi^{(d)}_{\mu\nu}{}^{1}(k) = -2e^2\delta_{\mu\nu}\int \frac{d^4p}{(2\pi)^4}\frac{\Lambda^4}{(p^2+m^2)(p^2+m^2+\Lambda^2)^2} \tag{4.38}$$

$$= -\frac{e^2}{8\pi^2}\delta_{\mu\nu}\left[\Lambda^2 - m^2\ln\left(\frac{m^2+\Lambda^2}{m^2}\right)\right]. \tag{4.39}$$

The next contribution is given by equation (4.28). By power counting, the integral in equation (4.28) is finite even before differentiating with respect to Λ^2. Note that the only possible singularity as $\Lambda \to \infty$ is logarithmic. In fact, since an ultraviolet divergence in Λ^2 can only occur when there is an infrared divergence in m^2, the terms with no such divergence in m^2 can immediately be eliminated as being finite. As a further simplification, k^2 can be set to zero within the integral, without affecting the leading order in Λ^2. Also m^2 can be be neglected except where it is needed to prevent an infrared divergence within the integral. After performing all these simplifications, equation (4.28) is reduced to

$$\Pi^{(d)}_{\mu\nu}{}^{2}(k) = {\Lambda_1}^4{\Lambda_2}^4 e^2k^2\delta_{\mu\nu}\frac{\partial}{\partial{\Lambda_1}^2}\frac{\partial}{\partial{\Lambda_2}^2}\int\frac{d^4p}{(2\pi)^4}\frac{1}{2p^2(p^2+m^2)}\frac{{\Lambda_1}^2{\Lambda_2}^2}{(p^2+{\Lambda_1}^2)^2(p^2+{\Lambda_2}^2)^2}\bigg|_{\Lambda_1=\Lambda_2=\Lambda}$$
$$+\text{ regular terms}\,. \tag{4.40}$$

This integral can be evaluated with the usual Feynman parameterization to arrive at the result,

$$\Pi^{(d)}_{\mu\nu}{}^{2}(k) = \frac{1}{2}\delta_{\mu\nu}\frac{e^2k^2}{(4\pi)^2}\ln\frac{\Lambda^2}{m^2} + \text{regular terms}\,, \tag{4.41}$$

where all terms that are finite as $\Lambda \to \infty$ have not been calculated. In the remaining contributions from equations (4.29) and (4.30), k^2 can be neglected compared to Λ^2. As usual, this type of integral is done by first Feynman parameterization and then evaluating the momentum integrals. After neglecting all the terms that are finite as $\Lambda \to \infty$, the result is

$$\Pi^{(d)}_{\mu\nu}{}^{3}(k) = \Pi^{(d)}_{\mu\mu}{}^{4}(k)$$
$$= \frac{e^2}{(4\pi)^2}\left[\delta_{\mu\nu}\left(\Lambda^2 - m^2\ln\frac{\Lambda^2}{m^2} - \frac{5}{12}k^2\ln\frac{\Lambda^2}{m^2}\right) + \frac{1}{6}k_\mu k_\nu\ln\frac{\Lambda^2}{m^2}\right] + \text{regular terms}\,. \tag{4.42}$$

By adding everything together, the momentum independent pieces cancel and the infinite part of the one loop vacuum polarization is found to be

$$\Pi^{(d)}_{\mu\nu} = \frac{1}{3}\frac{e^2}{(4\pi)^2}(k_\mu k_\nu - k^2\delta_{\mu\nu})\ln\frac{\Lambda^2}{m^2} + \text{regular terms}\,. \tag{4.43}$$

This is precisely the correct value, as was obtained by using dimensional regularization.

As discussed by Ishikawa [4], a modification in the identity

$$\left\langle \phi(x_1)\frac{\delta S[\phi]}{\delta\phi(x_2)}\right\rangle = \delta^4(x_1 - x_2) \tag{4.44}$$

can occur in stochastically regularized quadratically divergent scalar field theories. The leading behavior of quadratically divergent loops is proportional to Λ^2, while the external legs of the Langevin diagrams may possess a Λ^{-2} dependence. The combination of these two factors can yield an extra finite nonzero contribution, in the limit that the cutoff becomes infinite.

Although it is not clear what the relevance of this fact is to regularization and renormalization, it is straight forward to show that no problem occurs at the one loop level in the gluon channel of stochastically regularized scalar electrodynamics. In order for there to be a possibility of modifying

$$\left\langle A_\mu(x_1)\frac{\delta S[A_\sigma,\phi^\dagger,\phi]}{\delta A_\nu(x_2)}\right\rangle = \delta_{\mu\nu}\,\delta^4(x_1 - x_2)\,, \tag{4.45}$$

where $S[A_\sigma,\phi^\dagger,\phi]$ is the action of scalar electrodynamics (4.1), the photon random noise field should also be fifth-time smeared. Keeping only the leading behavior of the quadratically divergent loops, explicit calculation shows that there is no modification of identity (4.45). The coefficients of the various factors of Λ^2 that occur in the one loop evaluation of the left hand side of equation (4.45) can be obtained by comparison to the results for the various contributions to the photon vacuum polarization. Just as the quadratic divergences proportional to Λ^2 have cancelled in the vacuum polarization, the quadratic divergences cancel in the explicit evaluation of the left hand side of (4.45), and no modification of the identity occurs in the stochastic regularization of scalar electrodynamics. Of course, in the charged scalar channel, the identity analogous to (4.44) would again be quadratically divergent. In the case of pure Yang-Mills or QCD with fermions, one would expect the whole phenomenon to disappear, because all quadratic divergences are spurious.

5. Diagrammatic Proof of the Ward Identity to All Orders

By working with the standard Feynman diagrams, it is possible to prove the Ward identity [10], (in Feynman gauge)

$$\lim_{q\to 0} q^2 V_\sigma(p, p-q) = -e\frac{\partial S(p)}{\partial p_\sigma}\,, \tag{5.1}$$

where $V_\sigma(p, p-q)$ is the complete three point function (Fig. 5) and $S(p)$ is the complete scalar propagator (Fig. 6). A regularization scheme that preserves this identity implies that $Z_1 = Z_2$. The proof using the Langevin formulation is analogous to the proof using ordinary Feynman diagrams. The main difference is that there are two types of two point functions to consider. In the Langevin perturbative expansion, the external photon line can either be $D^{12}_{\mu\nu}(q)$ or $G^{12}_{\mu\nu}((q)$. In this section all orders of perturbation theory are being considered, so the probability density of the photon noise field must also be fifth-time smeared. In Feynman gauge, the stochastically regularized two point functions, using $\alpha^{(d)}_{\Lambda}$, are

$$G^{12}_{\mu\nu}(q) = \delta_{\mu\nu} e^{-q^2(t_1^5 - t_2^5)} \theta(t_1^5 - t_2^5) \,, \tag{5.2}$$

and

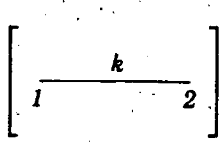
$$D^{12}_{\mu\nu}(q) = -\delta_{\mu\nu} \Lambda^4 \frac{\partial}{\partial \Lambda^2} \left[e^{-|t_1^5 - t_2^5| q^2} \left(\frac{1}{q^2(q^2+\Lambda^2)} - \frac{1}{q^4 - \Lambda^4} \right) + e^{-\Lambda^2 |t_1^5 - t_2^5|} \frac{1}{q^4 - \Lambda^4} \right] . \tag{5.3}$$

Since $G^{12}_{\mu\nu}(q)$ does not have a pole at $q^2 = 0$, vertex diagrams whose external photon line is $G^{12}_{\mu\nu}(q)$, do not contribute to the Ward identity (5.1). Vertex diagrams whose external photon line is $D^{12}_{\mu\nu}(q)$ do contribute, but in a simple way, because

$$\lim_{q \to 0} q^2 D^{12}_{\mu\nu}(q) = \delta_{\mu\nu} \,. \tag{5.4}$$

The proof of the Ward identity will proceed by showing that inserting a photon at zero momentum, q, at a given point in a typical Langevin diagram and then multiplying by q^2, is the same as differentiating that part of the Langevin diagram with respect to the momentum flowing through that point. Summing over all the possible ways to insert the photon into a diagram is therefore equal to summing over all the possible ways of differentiating with respect to the momentum flowing through the scalar lines. The sum over a closed scalar loop vanishes, because the loop momentum is integrated over. Thus, only the derivatives with respect to the momentum flowing through the scalar line that begins and ends externally are left. This is just the Ward identity (5.1).

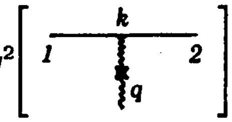
All that remains to be done is to explicitly check that inserting a photon at zero momentum at a given point in a Langevin diagram is indeed equivalent to differentiating that part of the diagram with respect to the momentum flowing through the scalar. There are two types of scalar two point functions that appear within a typical Langevin diagram and one type of vertex factor that explicitly contain momentum dependence. Since



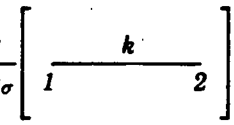
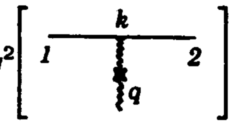
$$\equiv G_{12}(k) = e^{-(k^2+m^2)(t_1^5 - t_2^5)} \theta(t_1^5 - t_2^5) \,, \tag{5.5}$$

$$-e \frac{\partial}{\partial p_\sigma} G_{12}(k) = 2ek_\sigma (t_1^5 - t_2^5) G_{12}(k) \,, \tag{5.6}$$

where $k = p - \sum p_i$, p is the scalar line momentum, and the p_i are the momenta of internal photons that are attached to the scalar. On the other hand,

$$\lim_{q\to 0} q^2$$

$$= 2ek_\sigma \int dt_3^s \, G_{13}(k) G_{32}(k) = 2ek_\sigma (t_1^s - t_2^s) G_{12}(k) \,. \tag{5.7}$$

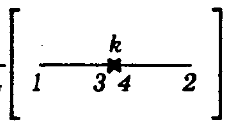
Therefore, diagrammatically,

$$-e\frac{\partial}{\partial p_\sigma}$$

$$= \lim_{q\to 0} q^2$$

$$. \tag{5.8}$$

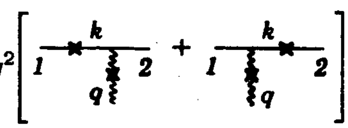
Similarly,

$$D_{12}(k) = \int dt_3^s dt_4^s \, \alpha_{34}^A \, G_{13}(k) G_{24}(k) \,, \tag{5.9}$$

yields

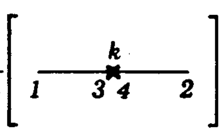
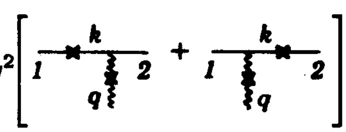
$$-e\frac{\partial}{\partial p_\sigma}$$

$$= 2ek_\sigma \int dt_3^s dt_4^s \, \alpha_{34}^A \, [(t_1^s - t_3^s) + (t_2^s - t_4^s)] \, G_{13}(k) G_{24}(k) \,. \tag{5.10}$$

Attaching the photon in the two possible ways results in

$$\lim_{q\to 0} q^2$$

$$= 2ek_\sigma \int dt_3^s \int dt_4^s \int dt_5^s \, \alpha_{34}^A \, G_{13}(k) G_{25}(k) G_{54}(k) + 2ek_\sigma \int dt_3^s \int dt_4^s \int dt_5^s \, \alpha_{34}^A \, G_{15}(k) G_{53}(k) G_{24}(k) \tag{5.11}$$

$$= 2ek_\sigma \int dt_3^s \int dt_4^s \, \alpha_{34}^A G_{13}(k) G_{23}(k) \times [(t_2^s - t_4^s) + (t_1^s - t_3^s)] \,. \tag{5.12}$$

Thus, diagrammatically,

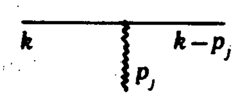
$$-e\frac{\partial}{\partial p_\sigma}$$

$$= \lim_{q\to 0} q^2$$

$$. \tag{5.13}$$

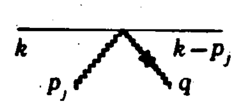
The reader may have noted that the fact that adding an external truncated photon to $D_{12}(k)$ is equivalent to differentiating with respect to the momentum flowing through the scalar is already contained in the fact that attaching an external truncated photon to $G_{12}(k)$ is equivalent to differentiating $G_{12}(k)$. However, the point was to explicitly show that the regularizer does not affect the results.

Differentiating the one photon vertex factor yields the two photon vertex factor.

$$-e\frac{\partial}{\partial p_\sigma}\left[e\Big(2p - 2\sum p_i - p_j\Big)_\mu\right] = -2e^2\delta_{\mu\sigma}\ , \tag{5.14}$$

or diagrammatically,

$-e\dfrac{\partial}{\partial p_\sigma}\Big[$ 

$\Big] = \lim\limits_{q\to 0} q^2 \Big[$ 

$\Big]$, (5.15)

where only the vertex factor is to be differentiated on the left hand side. Thus, it follows that the Ward identity (5.1) holds to all orders of perturbation theory.

6. Conclusions and Comments

This paper showed that the the stochastic regularizer does, in fact, yield the correct gauge invariant infinite part of the one loop photon vacuum polarization. The Ward identity that equates the scalar wavefunction renormalization to the one photon vertex renormalization was shown to hold to all orders of perturbation theory. Of course, it is possible that above the one loop level, stochastic regularization breaks down, but the Ward identity would still hold. These results seem to indicate that the stochastic regularizer may be useful as a regularizer that preserves the symmetries and *relevant* identities that are present in the corresponding infinite theory. As noted previously, for logarithmically divergent theories there may be nonperturbative applications, but this requires a more detailed examination. Although this paper dealt only with scalar electrodynamics, it should be possible to extend the results presented in this paper to fermions [11] as well as to nonabelian gauge theories [12].

7. Acknowledgements

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FIGURE CAPTIONS

Fig. 1. Perturbative Langevin expansion of ϕ in $\frac{\lambda\phi^4}{4!}$ theory.

Fig. 2. Expansion of two point function in $\frac{\lambda\phi^4}{4!}$ theory.

Fig. 3. One Loop correction to the photon propagator in SED using ordinary Feynman diagrams.

Fig. 4. One Loop correction to the photon propagator in SED using Langevin diagrams.

Fig. 5. Complete three point function in scalar electrodynamics.

Fig. 6. Complete scalar propagator in scalar electrodynamics.

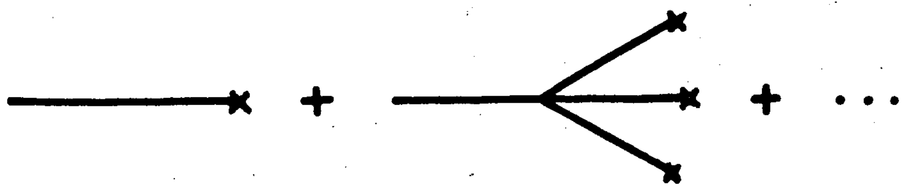

Figure 1

Figure 2

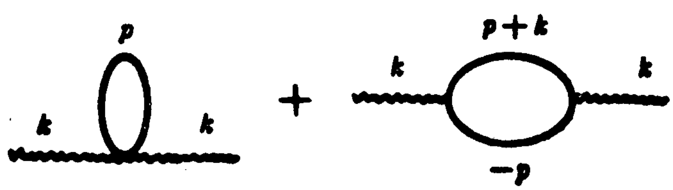

Figure 3

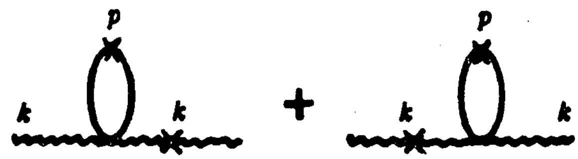

Figure 4a

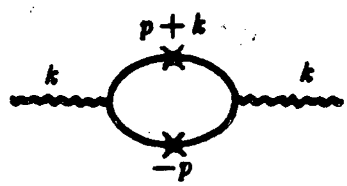

Figure 4b

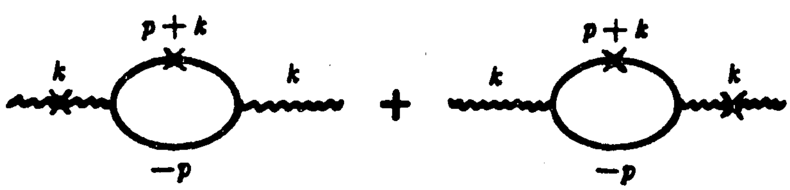

Figure 4c

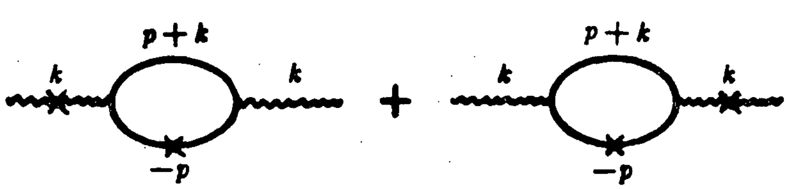

Figure 4d

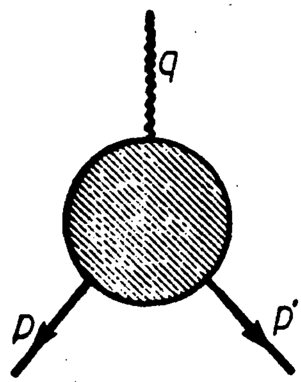

Figure 5

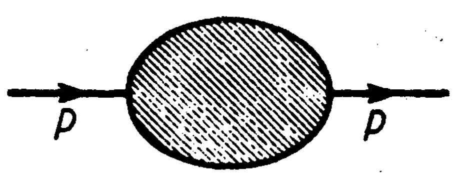

Figure 6