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**Symmetry Breaking: The Standard Model
and Superstrings**

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Symmetry Breaking: The Standard Model and Superstrings* †

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The outstanding unresolved issue of the highly successful standard model is the origin of electroweak symmetry breaking and of the mechanism that determines its scale, namely the vacuum expectation value (vev) v that is fixed by experiment at the value

$$v = 4m_W^2/g^2 = (\sqrt{2}G_F)^{-1} \simeq \frac{1}{4}TeV. \tag{1}$$

In this talk I will discuss aspects of two approaches to this problem.

One approach is straightforward and down to earth: the search for experimental signatures, as discussed previously by Pierre Darriulat. This approach covers the energy scales accessible to future and present laboratory experiments: roughly $(10^{-9} - 10^3)GeV$. The lowest energy of about an eV is characteristic of neutrino oscillation experiments, which have an indirect connection in that neutrino masses—if they exist—arise, like all other masses, from gauge symmetry breaking. In terms of direct searches for the elementary Higgs particle of the minimal standard model, there has been a long-standing lower bound[1] on the Higgs mass of about 15 MeV from nuclear and atomic physics. If the top quark mass is as large as suggested by analyses[2] of $B - \bar{B}$ mixing data,[3] much of the Higgs mass range kinematically accessible in B or K decay is probably also ruled out.[4] In fact arguments[5] based on standard cosmology forbid such a light Higgs particle unless the top quark is nearly as heavy as the W—in which case one-loop effects[6] involving the top quark should induce $b \to s + H$ and/or $s \to d + H$ transitions at observable rates. LEP and SLC will soon probe Higgs masses up to those accessible in Z decay, and LEP II will probe beyond the Z mass through the process

$$e^+e^- \to Z + H. \tag{2}$$

Future facilities like the SSC, LHC and CLIC will be sensitive to a heavier Higgs via the decay $H \to 2W$ or $2Z$. There is an unfortunate window for Higgs masses in the range $2E_{LEP} - m_Z < m_H < 2m_W$ which will probably be inaccessible at any of the present and planned facilities. Aside from this gap, the SSC will be able to probe the Higgs mechanism up into the TeV region, as I will discuss in the first part of my talk. Happily (and not accidentally), this coincides with the maximum energy scale where theory dictates that some associated phenomenon must show up.

The second approach involves theoretical speculations, such as technicolor and supersymmetry, that attempt to explain the TeV scale. Ideally, one would like to derive this scale—together with all of observed physics!—from a Theory of Everything (TOE) that is perhaps manifest only at the Planck scale of about $10^{19}GeV$, and hence well beyond the reach of laboratory experiments. The second part of my talk will describe one attempt, based on superstring-inspired models, to descend from the Planck scale to the experimentally accessible scales of a TeV or less.

THE STANDARD MODEL AND THE TEV SCALE

In the minimal version of the standard model[7] the electroweak gauge symmetry is broken by the introduction of a complex scalar Higgs doublet:

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \frac{e^{i\vec{\theta}\cdot\vec{\tau}/v}}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho \end{pmatrix} \tag{3}$$

with a potential energy density:

$$V = \frac{\lambda}{4}(\rho^2 - v^2) \tag{4}$$

that depends only only on the modulus

$$|\varphi| \equiv \rho \equiv H + v, \tag{5}$$

as illustrated in Fig. 1. Here H is the physical Higgs field whose mass is determined by the potential of Fig. 1 as:
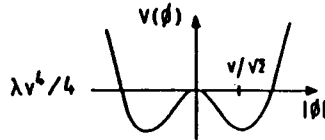
Figure 1
The tree potential of Equation 4.

$$m_H^2 = 2\lambda v^2. \tag{6}$$

The gauge symmetry breaking arises through the gauge invariant scalar kinetic energy density obtained by the substitution:

$$\partial_\mu \to D_\mu = \partial_\mu + i\vec{T} \cdot \vec{A}, \tag{7}$$

$$\partial_\mu \varphi \partial^\mu \overline{\varphi} \to D_\mu \varphi D^\mu \overline{\varphi} = \underbrace{\frac{g^2 v^2}{4}} W^+ W^- + m_Z^2 Z^2 + m_W W_\mu^+ \partial^\mu \overline{\varphi^+} + \cdots.$$

$$ \llcorner \equiv m_W^2 \tag{8}$$

The expansion (8) contains in particular vector boson mass terms; the identification of the W mass directly determines the vev v in terms of the Fermi constant G_F which is known from the neutron β-decay lifetime, Eq.(1).

The physical spectrum is manifest in the U-gauge, or unitary gauge, where the θ dependence of the last term in (3) is removed by a gauge transformation:

$$\varphi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho \end{pmatrix}, \quad \mathcal{L}_U = \mathcal{L}(H, W, Z, ...). \tag{9}$$

Higher order calculations are more easily performed in an R-gauge, or renormalizable gauge, where the unphysical degrees of freedom θ_i appear explicitly in the lagrangian:

$$\mathcal{L}_R = \mathcal{L}(H, \theta_i, W, Z, ...). \tag{10}$$

In the subsequent discussion I will make use of an "equivalence theorem" which states[8-10] that if one treats the θ_i in $\mathcal{L}_R$ as if they were physical fields, S-matrix elements obtained with the θ_i as external particles are the same, up to corrections of order $m_{W,Z}^2/E_{W,Z}^2$, as S-matrix elements with external longitudinally polarized W's and Z's (W_L, Z_L).

We can combine the experimental value of G_F with the results (1) and (6) of the minimal theory to express the Higgs mass in the form:

$$m_H = \sqrt{2\lambda} v \simeq \frac{\lambda}{4\pi} TeV. \tag{11}$$

Since perturbation theory converges[11] only if $\lambda/4\pi < 1$, the conventional wisdom is that the Higgs mass is less than about a TeV. This conclusion leads to the well-known gauge hierarchy problem, which arises because scalar masses have quadratically divergent quantum corrections; e.g., at one loop

$$\Delta m_{scalar}^2 = a\Lambda^2/16\pi^2. \tag{12}$$

Technically, this is not a problem in the context of the renormalizable standard model; one simply sets the fully renormalized Higgs mass at its physical value, assumed to be less than a TeV. However a small Higgs mass is unnatural when the standard model is embedded in a more fundamental theory with much larger mass scales, like the GUT scale or the Planck scale. In this context we need a mechanism for damping the quadratic divergence in (12); this mechanism should be associated with a scale Λ such that

$$a\Lambda^2 < (4\pi TeV)^2.$$

The scale Λ must therefore be less than about 10 TeV unless the numerical factor a contains some suppression factor.

There are three standard mechanisms that are invoked for addressing the gauge hierarchy problem:

a) Compositeness. The loop corrections (12) could be sufficiently damped if the standard model is an effective theory in which the known fermions, for example, are composite objects with an inverse radius of confinement less than about a TeV. This possibility is disfavored by already existing experimental data, and I will not discuss it further. Possibilities for a compositeness scale much larger than the scale of electroweak symmetry breaking will be discussed at this meeting by Pati.

b) Technicolor.[12] This conjecture invokes new "technifermions" with gauge interactions that become strong at a scale

$$\Lambda \sim \Lambda_{TC} \sim 250 GeV \simeq v. \qquad (14)$$

c) Supersymmetry[13] (SUSY). In this case the effective cut-off is the superpartner mass splitting:

$$\Lambda \sim m_{fermion} - m_{boson} \equiv m_{SUSY} \qquad (15)$$

which should be less than a TeV. Experimental searches at e^+e^- colliders indicate a SUSY mass gap larger than 20 TeV and data from $p\bar{p}$ colliders suggest a mass gap larger than 90 TeV.

Why does SUSY help? Fermion masses are protected by chiral symmetry from large loop corrections. For every free massless fermion there is a global symmetry of the theory under which the left and right handed fermions undergo independent phase transformations:

$$q_L \to e^{i\alpha}q_L, \qquad q_R \to e^{i\beta}q_R. \qquad (16)$$

Because a massless fermion cannot be brought to rest, there is no communication between left and right spinning components. Gauge interactions preserve this chiral symmetry, so they cannot induce fermion masses in higher order. This means that to all orders the quantum-corrected fermion mass is proportional to the bare mass. For example, a fermion with bare mass m_F gets a one loop correction of the form

$$\Delta m_F = m_F A \ln(\Lambda/m_F), \qquad (17)$$

where $A \sim \alpha/4\pi$, so that even with a very large cut-off, the order of magnitude of a fermion mass is fixed by its tree-level value. The role of SUSY is to tie scalar masses to the already protected fermion masses via Eq.(15).

When gravity is taken into account, supersymmetry implies supergravity;[14] the massless spin-2 graviton G has a spin-$\frac{3}{2}$ superpartner, the gravitino $\tilde{G}$, whose nonvanishing mass is a measure of the scale of SUSY breaking. This would seem to imply a constraint

$$m_{\tilde{G}} \lesssim TeV, \qquad (18)$$

which is disfavored by cosmological arguments[15] unless the gravitino is lighter than about a keV. Before discussing how the bound (18) may be evaded, I will briefly consider the technicolor hypothesis and discuss electroweak physics at the TeV scale.

Technicolor is based on the empirical observation of quark condensation in QCD. That is, quark bilinears have a nonvanishing vev's:

$$< \bar{\psi}\psi > \sim \Lambda_{QCD}^3 \sim f_\pi^3 \neq 0, \qquad (19)$$

that break the global chiral symmetry $SU(N_F)_L \times SU(N_F)_R$, where N_F is the number of quark flavors, down to ordinary $SU(N_F)$, under which right and left handed fermions of the same flavor transform in the same way: $\alpha = \beta$ in Eq.(16). To each broken symmetry there corresponds a Goldstone boson; there are $N_F^2 - 1$ in this case and they are identified with the observed pseudoscalar mesons.

In addition to the spontaneous chiral symmetry breaking induced by the vev (19), chiral symmetry is explicitly broken by nonvanishing quark masses. In the real world, the up and down quark masses are small with respect to the QCD mass scale as determined by f_π or Λ_{QCD}. Therefore chiral symmetry is a good approximation for $N_F = 2$, with the $SU(3)$ triplet of pions as the Goldstone bosons. In this case the quark bilinear vev of Eq.(19) transforms as a doublet under the electroweak gauge group $SU(2)_L \times U(1)$. In the absence of any other source of electroweak symmetry breaking, the vector bosons W, Z would acquire masses of about 30 MeV by "eating" the pions, which would become their longitudinally polarized components.

Technicolor mimics QCD by replacing gluons by technigluons, quarks by techniquarks and pions by technipions. The difference is in the scale at which the technicolor interaction becomes strong:

$$\Lambda_{QCD} \sim f_\pi \simeq 100 MeV \to \Lambda_{TC} \sim v \simeq 250 GeV, \qquad (20)$$

so that

$$m_W \simeq 30 MeV \to m_W \simeq 80 GeV. \qquad (21)$$

and the technipions become the longitudinally polarized components of W, Z.

Let us return now to the "Mexican Hat" potential of Fig. 1, and consider what happens when the Higgs mass m_H, and therefore the coupling constant λ, becomes arbitrarily large.[11] Quite generally, if φ is an n-component complex spinor:

$$\varphi = \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_n \end{pmatrix},$$

the lagrangian

$$\mathcal{L} = \partial_\mu \varphi \partial^\mu \overline{\varphi} - \frac{\lambda}{4}(\rho^2 - v^2)^2 \tag{22}$$

is invariant under the global orthogonal group $SO(2n)$. Consider first the case $n = 1$, with

$$\varphi = \frac{\rho}{\sqrt{2}} e^{i\theta/v}. \tag{23}$$

If we let the coupling constant $\lambda \to \infty$ the field variable ρ is fixed at its ground–state value

$$\rho = v, \quad \varphi \to \frac{v}{\sqrt{2}} e^{i\theta/v} \tag{24}$$

because quantum excitations about that value cost an infinite amount of energy. Then the remaining nontrivial part of the lagrangian, i.e. the kinetic energy term

$$\mathcal{L} \to \frac{1}{2} \partial_\mu \theta \partial^\mu \theta \tag{25}$$

describes a free massless particle. In the general case $n > 1$,

$$\varphi = \frac{1}{\sqrt{2}} e^{i\theta/v} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \rho \end{pmatrix} \to \frac{1}{\sqrt{2}} e^{i\theta/v} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ v \end{pmatrix} \tag{26}$$

where θ is an $n \times n$ traceless hermitian matrix with $2n - 1$ independent elements that correspond to the massless Goldstone boson fields θ_i that arise when $SO(2n)$ is broken spontaneously to $SO(2n - 1)$. Then since θ does not commute with its derivatives, the kinetic energy term in (22) is non trivial and contains derivative interactions:

$$\mathcal{L} \to \frac{1}{2} \partial_\mu \theta^i \partial^\mu \theta^j (\delta_{ij} + \frac{\theta_i \theta_i}{v^2 - \theta^2}). \tag{27}$$

This yields scattering amplitudes that increase with the center of mass energy E as E^2/v^2, and partial wave tree unitarity is violated[9] for $E \gtrsim \sqrt{\pi} v$.

For $n = 2$, that is, for the spontaneous symmetry breaking pattern

$$SO(4) \cong SU(2) \times SU(2) \to SO(3) \cong SU(2), \tag{28}$$

the lagrangian (27) describes the effective low energy ($E \lesssim GeV \sim m_\rho$, etc.) theory for QCD with the identification

$$\pi_{1,2,3} \leftrightarrow \theta_{1,2,3}$$

or

$$\pi^\pm, \pi^0 \leftrightarrow \theta^\pm, \theta^0.$$

In the case that $m_H \gg v$, the same lagrangian describes the effective low energy ($m_W \ll E \lesssim TeV$) theory for the longitudinally polarized vector bosons of the electroweak sector:[16,9]

$$W_L^\pm, Z_L^0 \leftrightarrow \theta^\pm, \theta^0.$$

Strong W_L, Z_L scattering would occur[9] at colliders via the Bremsstrahlung and rescattering mechanism of Fig. 2. The production rates for $pp \to Z_L Z_L$ and $W_L^+ W_L^-$ plus anything
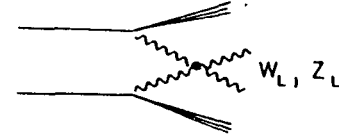
Figure 2

Vector boson fusion process for diboson production via strong W_L, W_L rescattering in fermion collisions.

are shown in Fig. 3 for both the tree level[8,9] and one–loop corrected[17] amplitudes. The amplitudes are those derived from the lagrangian (27), except that unitarity constraints, which damp the high energy behavior, have been imposed in both cases. Because the theory defined by (27) is not renormalizable, the one–loop corrections depend logarithmically on a cut–off Λ, taken to be $3 \, TeV$ in Fig. 3, and terms of order E/Λ (that cannot be reliably

Figure 3

$W_L^+ W_L^-$ and $Z_L Z_L$ pair production rates[17] in pp collisions at $\sqrt{s} = 20$ and $40 TeV$ with a rapidity cut $|y| < 1.5$ and a cut–off $\Lambda = 3 TeV$.

calculated) have been neglected. Higher orders in the loop expansion entail higher powers of $(\Lambda/4\pi v) \sim (\Lambda/\pi TeV)$, so the loop expansion converges for $\Lambda \lesssim 3 TeV$. The one–loop corrected rates should therefore be approximately correct over the center–of–mass energy region
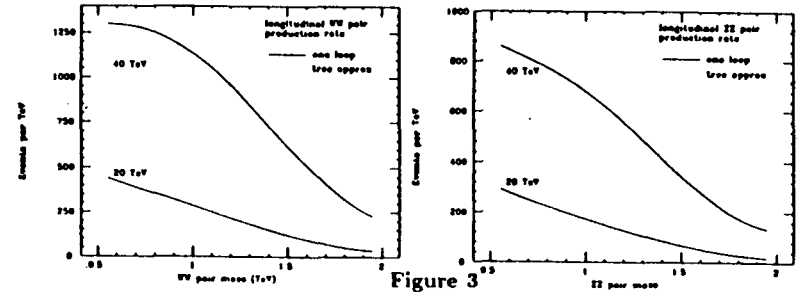
$$m_W^2 << E^2 << \Lambda^2 \lesssim 3 TeV,$$

where the cut–off Λ represents the scale at which the effective theory defined by (27) breaks down. For example, Λ could be the actual physical Higgs mass m_H or the mass scale of a richer resonance spectrum. Below this scale, the yields of Fig. 3 would produce an enhancement of the processes

$$pp \rightarrow \begin{pmatrix} ZZ \\ WW \\ WZ \end{pmatrix} + X \tag{29}$$

relative to the "background" arising from $q\bar{q}$ annihilation into W, Z pairs, which scales as E^{-2}. An enhancement for invariant pair masses $E > (.5 - 1)TeV$ should be observable[9,18] at the SSC operating at maximum design energy and luminosity. Note that the final states in (29) include $W^{\pm}W^{\pm}$ pairs, which occur at a lower rate, but have no quark annihilation background.

The lagrangian (27) [including gauge couplings via the substitution (7)] was obtained as a limit of a renormalizable theory, and the rates shown in Fig. 3 can be obtained[19] as well by performing the calculations in that theory, and then taking the limit $m_H \rightarrow \infty$. One obtains identical results in this way making the identification $m_H = \Lambda$.

However, the results are much more general than the standard model. They are valid[20] in any model in which the symmetry breaking sector—i.e. the sector of the eaten Goldstone bosons—has a global $SU(2) \times SU(2)$ that assures the tree level relation:

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_w} = 1. \tag{30}$$

Radiative corrections to (30) arise from gauge and Yukawa couplings that explicitly break the chiral symmetry. Therefore, if there is no Higgs particle—or other state connected with the symmetry breaking mechanism— lighter than a TeV, the rates shown in Fig. 3 are quite generally valid up to corrections of order m_W/E, $(E/\Lambda)^2$ and $\rho - 1$—and up to resonance effects. Depending on how closely a strongly interacting symmetry breaking sector mimics

the pion sector, one could expect, for example (as in technicolor), a $J = 1$ resonance with mass

$$m_{J=1} \sim \frac{v}{f_\pi} m_\rho \sim 2 TeV. \tag{31}$$

What's wrong with a strongly interacting minimal model symmetry breaking sector with $m_H >> TeV$? Aside from theoretical considerations that cast doubt on the consistency of a self–interacting scalar field theory, we cannot evade the gauge hierarchy problem in this way because we run into the same problem with quantum corrections. At one loop the effective low energy lagrangian (27) is modified according to[21]

$$\mathcal{L}_{Classical} \rightarrow Z\mathcal{L}_{Cl} + O(\ln(\Lambda/v)) + \text{finite}. \tag{32}$$

The factor

$$Z = 1 + \frac{(1 - N)}{16\pi^2} \frac{\Lambda^2}{v^2} \tag{33}$$

can be absorbed into a renormalization of the $2n - 1$ fields θ_i I the vev v:

$$\varphi_{Ren} = Z^{\frac{1}{2}}\varphi, \quad v_{Ren} = Z^{\frac{1}{2}}v. \tag{34}$$

For $n = 4$ the renormalized vev is related to the bare one by

$$v_{Ren} = \left(1 - \frac{\Lambda^2}{v^2}\frac{2}{32\pi^2}\right)v = \left(1 - \left[\frac{\Lambda}{4\pi v}\right]^2\right)v, \tag{35}$$

which cannot "naturally" be kept small if the correction—i.e. the cut–off —is arbitrarily large. In the minimal model, the cut–off is the Higgs mass itself, and the corrections will be $\lesssim O(1)$ for $v \simeq \frac{1}{4}TeV$ only if

$$m_H \lesssim \pi TeV. \tag{36}$$

The bottom line is that some mechanism is needed to damp quadratically divergent radiative corrections and provide an understanding of the observed scale of $250 \, GeV$.

To gain further insight into possible mechanisms for suppressing scalar masses, consider why the pion is so light. A good empirical formula for the pion mass is:

$$m_\pi^2 \simeq \frac{m_{u,d}}{f_\pi} m_\rho^2. \tag{37}$$

The squared pion mass is suppressed by the ratio of the explicit chiral symmetry breaking scale (the light quark mass $m_{u,d}$) relative to the scale (f_π) of spontaneous symmetry

9

10

breaking, Eq.(19), that forces the existence of a would–be massless Goldstone boson. This dimensionless ratio is scaled by the physical cut–off, $\Lambda \sim m_\rho$, of the effective low energy pion theory.

To see how a similar mechanism could supplement the role of supersymmetry in suppressing elementary scalar masses, consider the action for N free, massless real scalars minimally coupled to gravity:

$$S_G = \frac{1}{2} \int d^4x \sqrt{g} \left(g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - m_P^2 R \right), \qquad (38)$$

where $g_{\mu\nu}$ is the space–time metric and R is the space–time curvature. Graviton exchange, Fig. 4a, gives a contribution to scalar self energies. In unbroken supergravity these contributions are exactly cancelled by gravitino exchange, Fig. 4b. When SUSY is broken and the gravitino acquires a mass, the cancellation is no longer exact, and one expects a contribution to the scalar masses of order
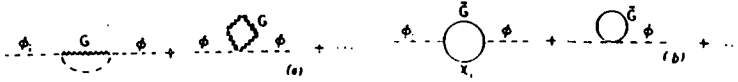

Figure 4

Contributions to scalar (φ) self energy from (a) graviton (G) and (b) gravitino ($\tilde{G}$)loops. In Fig. 6b χ_i is the fermionic superpartner of φ_i.

$$m_{scalar}^2 \sim \frac{m_{\tilde{G}}^2}{16\pi^2} \frac{\Lambda^2}{m_P^2}. \qquad (39)$$

If there is no other scale in the theory, the cut–off is presumably equal to the Planck mass m_P, suggesting (instead of (18)) the bound

$$m_{\tilde{G}} \lesssim 10 TeV \qquad (40)$$

to assure scalar masses below a TeV.

However the action (38) (as well as its supersymmetric extension) is invariant under global $SO(N)$ transformations among the scalar fields. This means that the fully quantum corrected effective lagrangian can depend on the scalar fields only through invariant field operators:

$$\mathcal{L}_{eff} = \mathcal{L}_{eff}(\sum_i |\varphi_i|^2, \sum_i \partial_\mu \varphi_i \partial^\mu \overline{\varphi}^i, ...). \qquad (41)$$

If the lagrangian (41) is such that the energy density is minimized for a scalar field configuration with

$$< |\varphi^2| > \neq 0, \qquad (42).$$

the global $SO(N)$ symmetry will be spontaneously broken to $SO(N-1)$ and there must be $N-1$ massless Goldstone bosons. Explicitly, one can use an $SO(N)$ transformation to cast the vev's of the scalar fields in the form

$$< \varphi >= \begin{pmatrix} \varphi_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \qquad (43)$$

Then only φ_1 acquires a mass of the order of (39), and the other scalars remain strictly massless to all orders[22] in the effective theory defined by the supersymmetric extension of the action (38).

In the real world scalars have other interactions, in particular gauge interactions, that explicitly break the global $SO(N)$ symmetry and might induce scalar masses of order $\alpha \Lambda^2$. Suppose however that the gravitino acquires a mass through the vev of a gauge singlet scalar and that SUSY is unbroken in the gauge sector. One loop contributions to the scalar masses from the gauge–gaugino sector cancel because of SUSY. At two loops there can be contributions involving both the gravity sector, which knows that SUSY is broken, and the gauge sector, which breaks $SO(N)$. Their combined effects might be expected to generate scalar masses of order

$$m_{scalar}^2 \sim \frac{\alpha}{(4\pi)^3} \frac{\Lambda^2}{m_P^2}, \qquad (44)$$

implying a bound

$$m_{\tilde{G}} \lesssim 10^5 TeV, \qquad (45)$$

which at least is weak enough to avoid problems with cosmology.[23]

One anticipated source of scalar masses at the two–loop level is from gaugino masses generated at one loop, which are a priori of the form:

$$m_{\tilde{G}} = am_{\tilde{G}} \frac{\Lambda^2}{m_P^2} + \frac{m_{\tilde{G}}^3}{m_P^2}(b\ln(\Lambda^2/m_{\tilde{G}}^2) + c) \qquad (46)$$

with a, b, c of order $1/16\pi^2$. However it was noted[24] some time ago that gravitino loop contributions to the coefficients a and b of the divergent terms cancel. If this were the only

two–loop contribution to the scalar masses, it would suggest

$$m^2_{scalar} \sim \frac{\alpha}{4\pi} m^2_G \sim \frac{\alpha}{(4\pi)^3} \frac{m^6_G}{m^4_P},$$ (47)

giving a bound

$$m_G \lesssim 10^{14} GeV.$$ (48)

In fact, it was subsequently shown[25] that the gravitino loop contributions to m_3 cancel completely. On the other hand, the real world is still more complicated. Scalars also have Yukawa couplings—needed in the standard model to generate fermion masses—but the above discussion illustrates how the cancellations provided by supersymmetry may combine forces with partial global symmetries of gravitational couplings so as to strongly suppress scalar masses. In the remainder of this talk I will discuss a class of models suggested by superstring theory[26,27] that possess such features.

SUPERSTRING–INSPIRED MODELS

The most popular candidate at present for the TOE is a string theory, according to which elementary "particles" are not particles at all, but rather the lowest vibrational modes of tiny strings that have an extension of the order of the Planck length, about $10^{-33}cm$. When supersymmetry is included, this "superstring" theory provides the only known possibility for a consistent quantum theory of gravity. It suggests that space-time is actually ten dimensional, but with six dimensions curled up with a radius comparable to the Planck length.

Effective four dimensional field theories[28] suggested by superstrings typically have gauge groups much larger than that of the standard model. Part of the symmetry can be broken by the so–called Hosotani or Wilson–loop mechanism,[29] in which lines L of gauge flux are trapped around holes in the compact six dimensional manifold:

$$< \int_L dL^m A_m > \neq 0,$$ (49)

where $m = 4, ..., 9$ is a Lorentz index in the compact manifold. In the four dimensional theory the vev (49) has the effect of an adjoint Higgs. The gauge bosons corresponding to the broken symmetries acquire masses on the order of the compactification scale Λ_{GUT}. The remaining unbroken gauge group is assumed to be broken further to the standard model by conventional Higgs mechanisms; the corresponding gauge bosons may have masses in the TeV region and

thus be observable at proposed supercolliders. There may also be additional generations of quark and leptons, depending on the topology of the compact manifold. "Massless" modes in four dimensions—i.e. those that end up with masses much smaller than the Planck scale—correspond to zero eigenfunctions of the Laplacian on the compact manifold:

$$\sum_{m=4}^{9} \frac{d^2}{dx^2_m} \varphi = 0.$$ (50)

The number of solutions depends on the number of holes in the manifold. In the following I will denote by y_i the complex scalars (squarks, sleptons, Higgs,...) of the "observed" sector.

In addition, superstring-inspired models contain a "hidden sector" of particles that couple to ordinary matter only with gravitational strength. These include the dilaton ϕ of ten dimensional supergravity and various scalars associated with the topology of the compact manifold. One of these is the "breathing mode" or "compacton" σ whose vev determines the overall size of the compact manifold:

$$r \sim 8\pi G_N < e^{2\sigma} >_{vac} \equiv m_P^{-1} < e^{2\sigma} >_{vac}.$$ (51)

The scalars

$$s \equiv 2ReS = 2\phi^{-\frac{3}{2}} e^{3\sigma}, \quad t \equiv 2ReT = 2\phi^{\frac{1}{2}} e^{\sigma} + |y|^2,$$ (52)

form[30] two gauge singlet chiral supermultiplets together with their fermionic partners and two pseudoscalars that are the four dimensional relics of an antisymmetric tensor field A_{MN}, $M, N = 0, ...9$, of the ten dimensional theory:

$$\partial_\mu ImS \propto \epsilon^\nu_{\mu\nu\rho\sigma} A_{\rho\sigma},$$

$$ImT \propto \epsilon^{lm} A_{lm},$$ (53)

where $\epsilon^{lm} = 0, \pm 1$ is a normalized antisymmetric tensor.

In heterotic string[27] inspired models there is also a hidden matter sector assumed to consist only of the gauge bosons and their gaugino superpartners. The hidden gauge group is E_8 or some subgroup thereof. If it is at least as large as $SU(3)$ it is asymptotically free and therefore becomes strongly coupled, like QCD, at some scale Λ_c. One expects[31] that, as in QCD, the strongly coupled fermions (the hidden gauginos) condense at this scale:

$$<\bar{\tilde{g}} \tilde{g}>_{vac}^{hidden} \propto h \neq 0.$$ (54)

The vev (54) spontaneously breaks supersymmetry. Another conjectured[32] source of SUSY breaking is a nonvanishing vev of the antisymmetric three-form $H_{lmn} = \partial_l A_{mn}$:

$$< H >_{vac} \propto c \neq 0 \qquad (55)$$

via a mechanism similar to the gauge symmetry breaking via Wilson loops, Eq.(49), except that the H-flux is trapped over a three dimensional surface S in the compact 6-manifold, and satisfies a quantization condition[33] of the form:

$$< \int_S dS^{lmn} H_{lmn} > = 2\pi n. \qquad (56)$$

Either nonvanishing vev (54) or (55) by itself would induce a positive cosmological constant, but together they can conspire to cancel the vacuum energy in the presence of SUSY breaking. Dine et al.[32] used these ideas to obtain an effective supergravity theory for the observed sector in four dimensions with a scalar potential which is schematically of the form (in reduced Planck mass units: $m_P = 1$):

$$V = \frac{1}{s}(t - |y|^2)^{-3}\left\{(t - |y|^2)[g^2 |y|^4 + G_Y^2 |y|^4 + |G_Y y^3 + c + f(s)h|^2\right\} \qquad (57)$$

where g and G_Y generically represent the gauge and Yukawa coupling constants, respectively. Note that the potential (57) is the sum of three positive semi-definite terms. The first two terms, proportional to $|y|^4$, force $y_i = 0$ at the ground state. Then the third term,

$$V(y = 0) = \frac{1}{st^3}|c + f(s)h|^2 \equiv U, \qquad (58)$$

determines a relation among the parameters c and h and the vev of s:

$$< c + f(s)h > = < U^{\frac{1}{2}} > = 0. \qquad (59)$$

The tree level vacuum has the following features:

a) The cosmological constant vanishes:

$$< V > = < U > = 0. \qquad (60)$$

b) The gauge nonsinglet scalar masses vanish:

$$m_y^2 = (\frac{\partial^2 V}{\partial y^2})_{vac} = 0 \qquad (61)$$

because the only quadratic dependence on the y_i is in the multiplicative factor $(t - |y|^2)$ whose coefficient vanishes at the ground state. This feature is related[34,35] to the fact that the potential (57) is partially invariant under a nonlinearly realized "Heisenberg" symmetry under which the scalars transform as:

$$\delta y_i = \alpha_i, \qquad \delta t = y_i^* \alpha_i + \alpha_i^* y_i, \qquad (62)$$

with the y_i as Goldstone bosons. Since this symmetry is explicitly broken by gauge and Yukawa couplings, one would expect scalar masses to be generated by quantum corrections.

c) The gauginos of the observable sector are massless; their masses can be shown to be given by

$$m_{\tilde{g}} = < |U^{\frac{1}{2}}| > = 0. \qquad (63)$$

As a result the gauge interactions will not induce scalar masses at one loop.

d) There are no "A-terms", that is, terms cubic in y that are proportional to the "superpotential" $W(y)$, represented schematically in (57) by the term $G_Y y^3$. Specifically, we have

$$V = |s^{-\frac{1}{2}}t^{-\frac{3}{2}}G_Y y^3 + U^{\frac{1}{2}}|^2 + O(y^4) = 2G_Y y^3 < s^{-\frac{1}{2}}t^{-\frac{3}{2}}U^{\frac{1}{2}} > + ... \qquad (64)$$

The first term on the right in the expansion (64) is the A-term. Its vanishing, as for gaugino masses, is directly related to the vanishing of the cosmological constant, Eq.(60).

d) The vev of the field t is undetermined at tree level, and so, therefore are the various scales of theory. These are the compactification scale (or GUT scale, where all gauge couplings—both hidden and observed— are unified):

$$\Lambda_{GUT} \sim r^{-1} \sim e^{-2\sigma}m_P = < (st)^{-1} >, \qquad (65)$$

the scale of hidden gaugino condensation:

$$\Lambda_c \sim e^{-\frac{1}{2b_0 r^3}}\Lambda_{GUT} \sim < (st)^{-1}exp(\frac{-3S}{2b_0}) >, \qquad (66)$$

and the gravitino mass:

$$m_{\tilde{G}} = < (st^3)^{-1}\tilde{f}(s,h) > . \qquad (67)$$

Binétruy, Dawson, Hinchliffe and I have studied[36] one-loop corrections to the effective theory described above. These are of course infinite, because the theory is nonrenormalizable.

However, if the underlying theory is finite, the apparent divergences of the effective theory should be damped at the physical scales Λ_c and Λ_{GUT} above which the effective low energy theory is not valid. We can hope that, as for the (gauged[37] or ungauged[21]) nonlinear sigma-model of Eq.(27), loop corrections evaluated using the effective low energy theory correctly reproduce the low energy limit of loop corrections to the underlying (finite or renormalizable) theory, when the cut-off Λ is replaced by the appropriate physical scale. This scale is known only as to order of magnitude—the precise value of the cut-off depends on the details of the way in which contributions from the physics above that scale cancel the apparent divergences. We have therefore introduced "uncertainty factors" η in our analysis, e.g. $\Lambda \rightarrow \eta_1 \Lambda_c$ or $\eta_2 \Lambda_{GUT}$, to cover our ignorance.

Quite generally, we found that the ground state equations for the one–loop corrected effective potential admit three possible solutions:

1) The potential is unbounded from below, which means an infinite, negative cosmological constant, and the gravitino mass is infinite. This is obviously not a physically acceptable solution.

2) There is a unique global minimum with vanishing cosmological constant and vanishing gravitino mass. This is equally unacceptable because if SUSY is unbroken at tree level, it cannot be broken perturbatively to any order. The point is that, aside from the constraint (58), the nonperturbatively induced vev's (54) and (55) are undetermined at the classical level. If the classical degeneracy is lifted at one–loop in a way that forces them to vanish, higher order corrections cannot change this situation.

3) The potential is positive semi-definite with a degenerate ground state. The gravitino mass (67) can be nonvanishing but its value, as well as that of the other scales, (65) and (66), remains undetermined. However most of the tree level degeneracy is lifted, so that ratios of scales are determined at one loop. Moreover, because of the quantization condition (56), the vacuum degeneracy—which is conceivably related to the vanishing of the cosmological constant—is discrete for a given compact manifold. This means that there is not an associated massless Goldstone boson, as would be the case for a continuous vacuum degeneracy, if it persisted to all orders in perturbation theory.

The content of the (hidden plus observed) matter spectrum determines whether or not 1) is the case. If it is not, the choice between 2) and 3) depends on the details of the cut-off mechanism, i.e., on the "uncertainty parameters", which for consistency should be of the

order of unity. We searched numerically for solutions of type 3), which indeed were found for $\eta \sim 1$ and values of the gauge coupling constant at the GUT scale:

$$\alpha_{GUT} = <(4\pi s)^{-1} > \simeq (0.06 - 1). \tag{68}$$

The potential for a sample solution is shown in Fig. 5 in the $(c, t^{-1} \propto m_{\tilde{G}}^{1/3})$ plane. For a given topology of the compact manifold, allowed vaccua correspond to intersections of the $< V = 0 >$ valley in the potential with lines of constant c:
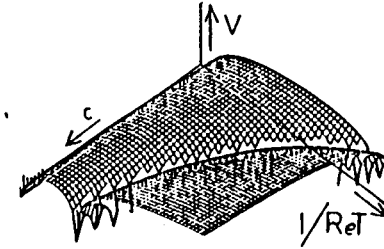

Figure 5

The one–loop effective potential in the (c, t^{-1}) plane for fixed values of the other dynamical variables in the case where a minimum exists for finite $m_{\tilde{G}}$.

$$c = n\Delta c, \tag{69}$$

where the quantization condition (56) determines the increment Δc:

$$\Delta c \simeq 10^3 I. \tag{70}$$

The factor 10^3 in (70) comes from various factors of π, and I is the inverse ratio of the surface integral over dS in (56) to the square root of the six-dimensional volume integral over the compact manifold M_6:

$$I = \left[\int_{M_6} d^6 x \sqrt{g^6} \right]^{\frac{1}{3}} \Big/ \int_S dS^{lmn} \epsilon_{lmn} \gtrsim 1. \tag{71}$$

Once the value of c is fixed (possibly by cosmology?), all the scales of the effective low energy theory are fixed; we find for the numerical solutions:

$$m_{\tilde{G}} \sim \frac{1}{3}\Lambda_c \sim \frac{1}{10}\Lambda_{GUT} \sim (0.07 - 1.8)m_P/\sqrt{c}, \tag{72}$$

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where part of the uncertainty is connected with the size of the gauge group in the hidden sector (the larger the group, the smaller the gravitino mass). Using (70) and (71) this gives

$$m_{\hat{G}} \lesssim (2-60) \times 10^{-3} m_P/\sqrt{n} \sim (0.4-12) \times 10^{16} GeV/\sqrt{n}. \tag{73}$$

Assuming a solution of type 3), we can expand the effective one–loop corrected lagrangian around its ground state configuration to study soft supersymmetry breaking terms in the observable sector. To do this we have to consider two different energy scales of the effective four dimensional theory. The tree potential (57) is valid below the scale Λ_c of gaugino condensation. Loop corrections for this effective theory in fact generate no observable soft SUSY breaking for reasons very similar to those operative at tree level. Scalar masses vanish at one loop[38] due to a combination of effects:[35] SUSY prevents the gauge and Yukawa sectors from generating one-loop scalar masses; the partial Heisenberg symmetry, Eq.(62), forbids mass terms to arise from the coupling of scalars to the gauge singlet sector; and finally matter–hidden sector interference terms vanish due to the tree level condition (59). The vanishing of gaugino masses and A–terms at one loop, as at tree level, is directly related to the vanishing of the cosmological constant at that order.[36]

Between the scale Λ_c and the compactification scale Λ_{GUT}, the effective theory is a four dimensional supergravity theory with freely propagating hidden gauginos: $h = 0$ in Eq.(57). Including one–loop corrections from this region does not change[36] the general features of the possible solutions 1) – 3) above. [In fact these additional corrections appear to be necessary to avoid[36] the unacceptable case[39] 1).] Their contributions to soft SUSY breaking in the observable sector have not yet been fully evaluated. Here I will offer some educated guesses as to the result, and assign my own personal confidence levels to the various possibilities. In this spirit, I predict (95% c.l.) that one–loop corrections in the effective four dimensional field theory generate no scalar masses or A–terms. The situation regarding gaugino masses is more uncertain; the possibilities are:

1) (10% c.l.) Gaugino masses get quadratically divergent corrections and are therefore of order

$$m_{\tilde{g}} \sim \frac{m_{\tilde{G}} \Lambda_{GUT}^2}{(4\pi m_P)^2} \sim (10^9 - 3 \times 10^{13})\frac{TeV}{n^{3/2}}. \tag{74}$$

This would in turn generate a two–loop scalar Higgs mass of order

$$(m_H)_{2-loop} \left(\frac{80-2500}{\sqrt{n\tilde{I}}}\right)^3 TeV. \tag{75}$$

2) (40% c.l.) Gaugino masses are at most logarithmically divergent at one loop and therefore of order[25]

$$m_{\tilde{g}} \sim \frac{m_{\tilde{G}}^3}{(4\pi m_P)^2} \sim (10^7 - 3 \times 10^{11})\frac{Tev}{n^{3/2}}, \tag{76}$$

generating a Higgs mass of order

$$(m_H)_{2-loop} \sim \left(\frac{17-550}{\sqrt{n\tilde{I}}}\right)^3 TeV. \tag{77}$$

The numerical estimates in Eqs. (74)–(77) are based on the results (69)–(73) of our analysis[36] using a specific approximation to the one-loop potential. To the extent that this numerical analysis can be trusted (which is much less reliable than the more general qualitative results), we need a factor ranging between 15 and 2500 (modulo various other uncertainties) in the parameter $\sqrt{c} \propto \sqrt{n\tilde{I}}$, in order to get a Higgs mass less than a TeV. Of course n and therefore c could be arbitrarily large, and therefore $m_{\hat{G}}$ arbitrarily small. However Eq.(72) suggests that $m_{\hat{G}}$ cannot be much below the GUT scale, which we expect to be closer to the Planck scale than to the TeV scale. It is interesting that the minimum value of c suggested by (70) and (71) assures as least a mild hierarchy among the Planck scale and the GUT or gravitino mass scale, which could be magnified, via, e.g. Eq.(77), into a sufficiently large hierarchy between those scales and the scale of electroweak symmetry breaking.

However there is a third possibility, namely

3) (50% c.l.) No gaugino masses are generated by one-loop corrections in the effective four dimensional field theory. Then scalar masses may occur only at a very high order in perturbation theory, with many factors of $(4\pi)^{-2} \sim 10^{-2}$ accounting for the observed gauge hierarchy. Of course there may be other sources[40] of two-loop contributions to scalar masses, but as we don't fully understand the origin of cancellations that occur at the one–loop level, there is no way of guessing the contributions of higher order effects. It may turn out that the dominant contribution to scalar masses comes from higher Kaluza-Klein or string modes, entailing higher powers of $m_{\hat{G}}/m_P$ as suppression factors.

The main lesson to be drawn from this discussion is that there are a variety of mechanisms in effective supergravity theories inspired by superstrings that might produce an electroweak breaking scale that is very much smaller that the natural scale of the theory, namely the Planck mass.

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