

A marked correlation function for constraining modified gravity models

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Abstract. Future large scale structure surveys will provide increasingly tight constraints on our cosmological model. These surveys will report results on the distance scale and growth rate of perturbations through measurements of Baryon Acoustic Oscillations and Redshift-Space Distortions. It is interesting to ask: what further analyses should become routine, so as to test as-yet-unknown models of cosmic acceleration? Models which aim to explain the accelerated expansion rate of the Universe by modifications to General Relativity often invoke screening mechanisms which can imprint a non-standard density dependence on their predictions. This suggests density-dependent clustering as a ‘generic’ constraint. This paper argues that a density-marked correlation function provides a density-dependent statistic which is easy to compute and report and requires minimal additional infrastructure beyond what is routinely available to such survey analyses. We give one realization of this idea and study it using low order perturbation theory. We encourage groups developing modified gravity theories to see whether such statistics provide discriminatory power for their models.

Keywords: cosmological parameters from LSS – power spectrum – galaxy clustering

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1 Introduction

The observation that the expansion rate of the Universe is accelerating is one of the most puzzling aspects of our current cosmological model. Two classes of explanation have been investigated, one based on a modification of the contents of the Universe [1, 2] and one based on a modification of gravity (see e.g. Refs. [3–6] for recent reviews). At present there are no theoretically consistent, observationally allowed models which provide cosmic acceleration through modifications to gravity. Therefore, observational constraints on modifications to general relativity (GR) often focus on ‘generic’ features that some as-yet-to-be-determined models might be expected to have.

Large-scale structure surveys typically provide constraints on the distance scale and rate-of-growth of fluctuations through studies of baryon acoustic oscillations (BAO) and redshift-space distortions (RSD) [1, 2]. In combination with gravitational lensing surveys (of galaxies or the cosmic microwave background) a number of tests of GR on linear scales can be constructed [1, 2]. It is reasonable to assume that such analyses will be an integral part of analyses of future surveys as well, improving some tests of dark energy and modified gravity models. A question then arises what other analyses should ‘routinely’ be performed on future surveys, so that observational constraints are available for theorists and phenomenologists seeking to constrain next-generation models? Ideally these analyses should be simple to perform and report, while at the same time providing information beyond the standard analyses currently published.

In the absence of a specific theoretical framework this is a difficult question to answer. The tightest constraints will come from a model-by-model analysis, but more generic constraints can also be useful when investigating wide classes of models. One frequently encountered phenomenon in modified gravity models is a screening mechanism that forces model predictions to approach those of GR in regions of high density or strong gravitational potential. Conversely, signatures of modified gravity will show up in regions where gravity is weak. Screening is a property of many modified gravity models that offers potentially distinctive observational signatures.

Here we advocate the use of the density-marked correlation function [7] as an easy-to-compute statistic which may test future modified gravity models. Computation of the marked correlation function requires minimal modification to existing analysis frameworks,

and requires no further infrastructure beyond that which is routinely available for studying BAO and RSD. By weighting the pairs of galaxies by a ‘mark’ which depends on a local density estimate (e.g. increasing the weight of low density regions) it provides density-dependent information from the survey which may be of use in constraining future theories.

This paper introduces the inverse-density-marked correlation function and provides an exploration of its properties using low order Lagrangian perturbation theory. The latter is primarily for convenience – such statistics can also be calculated and explored using N-body simulations or mock catalogs which will give access to smaller scales where the signal may be larger. We shall return to this point in the conclusions. The outline of this paper is as follows. In section 2 we introduce the marked correlation function and quickly review Lagrangian perturbation theory and how to compute the marked correlation function within this framework. We present some results to build intuition on the marked correlation function in section 3. Finally we conclude in section 4.

2 Background

This section presents some background to set the stage and our notation. Our focus will be the prediction of the density-marked two-point function in redshift space, i.e. the marked correlation function. For technical reasons, we shall restrict ourselves here to the angle averaged (or monopole moment of the) correlation function though further information would almost certainly be contained in the higher moments.

2.1 Marked correlation function

The marked correlation function [8–13] is a generalization of the usual, configuration space, 2-point function. If each object is assigned a mark, m , the marked correlation function is defined as

$$\mathcal{M}(r) \equiv \frac{1}{n(r)\bar{m}^2} \sum_{ij} m_i m_j = \frac{1+W}{1+\xi} \quad (2.1)$$

where the sum is over all pairs of a given separation, r , $n(r)$ is the number of such pairs and \bar{m} is the mean mark for the entire sample. The second equality serves to define W and emphasizes that we expect $\mathcal{M} \rightarrow 1$ at large scales. In general the marked correlation function is very easy to evaluate if one is already able to evaluate the ‘normal’ correlation function. Its computation can be trivially integrated into a standard clustering pipeline with almost no overhead, and it can make use of the same masks, mock catalogs and codes as the standard analysis. Aside from a measure of density¹, it requires no additional survey products beyond those traditionally used for configuration-space clustering analyses.

The use of local density as a mark was suggested in Ref. [7]. In what follows we shall be interested in marks which are functions of a smoothed density field, ρ_R . There are numerous ways of estimating a density given a collection of objects (see e.g. [15–21] for some examples). The scale over which the density field can be estimated is tied to the mean separation of objects, which for current and future surveys aimed at BAO or RSD is likely to be close to linear (see Appendix A). If we assume² that the density which is used as a mark is in fact the

¹At least one estimate of the density is often computed as part of the analysis pipeline in order to do BAO reconstruction [14].

²Obviously there is no need to make this approximation in the data or if the prediction is generated from mock catalogs. It is made purely to facilitate a perturbative treatment of $\mathcal{M}(r)$.

linear density field, smoothed on some scale R , then it is straightforward to make an analytic prediction for the marked correlation function within Lagrangian perturbation theory.

The choice of mark is in principle arbitrary. The smaller the range of the mark the smaller the signal, but the more stable the result. Marks with a very large range can introduce noise. In general smoothly varying marks are preferred over very rapidly changing ones. If we had a particular signal we were looking for it would be possible to optimize the mark – for example we could use a value based on the screening maps introduced in Ref. [22] or on probability of being in a sheet or filament (e.g. Refs. [21, 23–26]; Refs. [21, 27] argue that some screening mechanisms are sensitive to the dimensionality of the surrounding structure). Highly complex marks could be simulated, but are unlikely to be analytically tractable. Marks which can be expressed in terms of the local Lagrangian density and its low order derivatives can be easily handled within Lagrangian perturbation theory (e.g. using the techniques described in Ref. [28]).

In the absence of a specific theoretical target, we choose simple functions of a smoothed density field. As an example we shall consider marks of the form

$$m = \left(\frac{\rho_\star + 1}{\rho_\star + \rho_R} \right)^p \quad (2.2)$$

where ρ_R is our smoothed density in units of the mean density, $\bar{\rho}$, and ρ_\star is a parameter we can adjust (an alternative would be $\exp[-\rho_R/\rho_\star]$). The contribution of low density regions can be enhanced by choosing $p > 0$. In general one would compute several of these marked correlation functions, obvious examples would be fixing p and varying ρ_\star over some range or fixing ρ_\star and varying p . Note that upweighting the low density regions is similar in spirit to computing the properties of voids, often suggested as a probe of modified gravity (e.g. Refs. [29–33]) but without the need to find voids and characterize their purity and completeness. It is another way to study clustering as a function of environment and a generalization of a void-galaxy cross correlation (e.g. Refs. [34–37] for recent studies in observations and simulations respectively). Like the void probability function, the marked correlation function involves an infinite tower of higher order correlation functions, though only a small fraction of the possible configurations. The marked correlation function is similar to statistics of the ‘clipped’ density field [38], which Ref. [39] advocated as a test of modified gravity, or to the log-transformed density field [40]. As such, the analytic development below may also be applicable to some limits of clipped or log-transformed correlation functions.

It will prove useful below to Taylor expand m in powers of the smoothed density contrast, δ_R ,

$$m \simeq 1 - \frac{p}{1 + \rho_\star} \delta_R + \frac{p(p+1)}{2(1 + \rho_\star)^2} \delta_R^2 + \dots \quad (2.3)$$

If $\rho_\star \gg 1$ the mark is slowly varying with δ and the expansion converges quickly. As we lower ρ_\star , or increase p , we require more terms to adequately describe the mark.

2.2 Lagrangian perturbation theory

The Lagrangian approach to cosmological structure formation was developed in [41–49] and traces the trajectory of an individual fluid element through space and time. It has been well developed in the literature, and extended to include effective field theory corrections and a generalized bias model [28, 50, 51]. In this exploratory foray we will use the first order solution, linear in the density field (also known as the Zeldovich approximation [41]) and a

simple local Lagrangian bias scheme. It is plausible, though by no means proven, that a low order expansion may work better for a statistic that emphasizes low density regions than for a statistic focused on high density peaks. In any event, the extension of the theory to higher orders, and to more complex biasing schemes, is straightforward and can be attempted if it proves useful.

We shall closely follow the notation in Refs. [46, 52], in particular we define

$$\langle \delta(\mathbf{q}_1)\delta(\mathbf{q}_2) \rangle = \langle \delta_1\delta_2 \rangle_c = \xi(\mathbf{q} = \mathbf{q}_1 - \mathbf{q}_2) \quad , \quad (2.4)$$

$$\Delta_i = \Psi_i(\mathbf{q}_1) - \Psi_i(\mathbf{q}_2) \quad , \quad A_{ij} = \langle \Delta_i\Delta_j \rangle_c \quad \text{and} \quad U_i = \langle \delta_1\Delta_i \rangle_c \quad (2.5)$$

with Ψ_i the Lagrangian displacement (i.e. the final position \mathbf{x} is related to the initial position \mathbf{q} of a fluid element or dark matter particle by $\mathbf{x} = \mathbf{q} + \Psi(\mathbf{q})$). The two-point function of the density is the usual correlation function, ξ , which is a function only of the (Lagrangian) separation \mathbf{q} . The other two 2-point functions are the auto-correlation of the relative Lagrangian displacement, Δ_i , which we denote A_{ij} and the cross-correlation of the displacement and density, U_i . These are also only functions of the separation, \mathbf{q} .

We shall assume that our tracers have local Lagrangian bias [46]

$$1 + \delta(\mathbf{x}, t) = \int d^3q F[\delta_L(\mathbf{q})] \delta_D[\mathbf{x} - \mathbf{q} - \Psi(\mathbf{q}, t)] \quad (2.6)$$

specified by F . We shall describe F via its low-order bias expansion, with $b_1 = \langle F' \rangle$ and $b_2 = \langle F'' \rangle$ the Lagrangian bias parameters [46, 47].

As discussed above we assume that our mark can be expressed as a function of the smoothed, linear theory density field (an extension to derivatives of this smooth field, e.g. shear, is straightforward in principle). We shall denote the smoothed overdensity as δ_R and write

$$U_i^R = \langle \delta_{R,1}\Delta_i \rangle_c \quad , \quad \xi_{R,1} = \langle \delta_1\delta_{R,2} \rangle_c \quad \text{and} \quad \xi_R = \langle \delta_{R,1}\delta_{R,2} \rangle_c \quad . \quad (2.7)$$

The zero-lag correlators can be written as $\sigma_{R,1}^2$ and σ_R^2 . Writing the mark as $G[\delta_R]$ and performing the same bias expansion as for F with coefficients B_1 and B_2 we have the mean mark $\bar{m} = 1 + b_1B_1\sigma_{R,1}^2 + \dots$.

The marked correlation function can now be computed using standard methods [46–48, 53]. Let us define Ξ such that

$$\mathcal{M}(r) = \frac{1 + W}{1 + \xi} = \frac{\Xi(b_i, B_i)}{\Xi(b_i, B_i \equiv 0)} \quad (2.8)$$

Up to third order in the bias expansion (and second order in clustering)

$$\begin{aligned} \Xi = & \int d^3q \frac{1}{(2\pi)^{3/2}|A|^{1/2}} e^{-(1/2)(q_i - r_i)(A^{-1})_{ij}(q_j - r_j)} \\ & \times \left\{ 1 + b_1^2 \xi_L - 2b_1 U_i g_i - [b_2 + b_1^2] U_i U_j G_{ij} - 2b_1 b_2 \xi_L U_i g_i + \dots \right. \\ & + B_1^2 \xi_R - 2B_1 U_i^R g_i - [B_2 + B_1^2] U_i^R U_j^R G_{ij} - 2B_1 B_2 \xi_R U_i^R g_i + \dots \\ & + 2b_1 B_1 (\xi_{R,1} - 2U_i U_j^R G_{ij}) - 2B_1^2 b_1 g_i U_i \xi_R - 2b_1^2 B_1 g_i U_i^R \xi \\ & \left. - 2(b_2 + b_1^2) B_1 g_i U_i \xi_{R,1} - 2(B_2 + B_1^2) b_1 g_i U_i^R \xi_{R,1} + \dots \right\} \quad (2.9) \end{aligned}$$

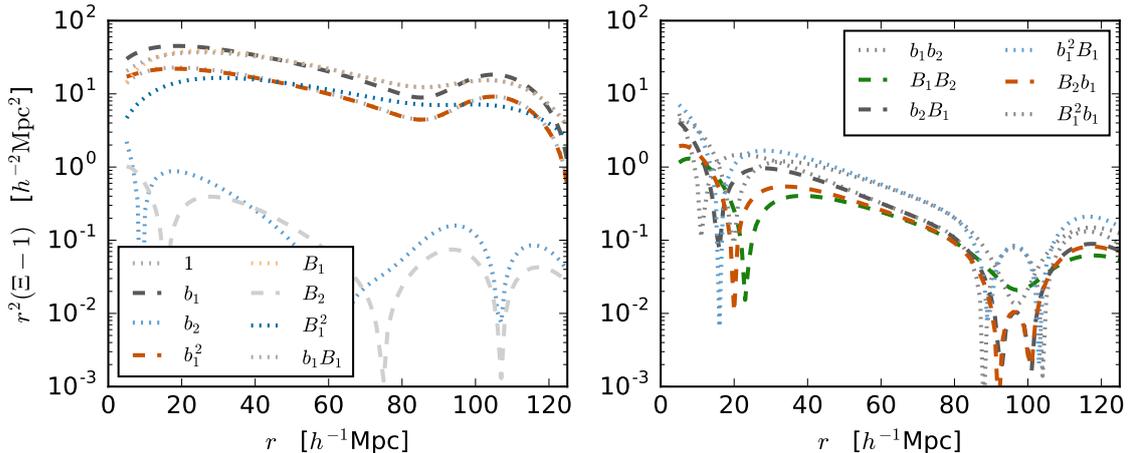


Figure 1. The contributions to Ξ , in real space, as a function of r . We have assumed a Gaussian smoothing with a width of $10 h^{-1}\text{Mpc}$ to define ρ_R and a ΛCDM cosmology at $z = 0.55$ to define the linear theory power spectrum. (Left) The first 8 contributions, up second order in the bias expansion. The 1 and b_1^2 terms are nearly indistinguishable. (Right) The 3rd order terms, which are generally small on large scales.

where $g_i = (A^{-1})_{ij}(q_j - r_j)$ and $G_{ij} = (A^{-1})_{ij} - g_i g_j$ (not to be confused with our mark function, $G[\delta_L]$).

Fig. 1 shows the contributions to Ξ for a ΛCDM cosmology at $z = 0.55$. We have defined the density, ρ_R , appearing in our mark using a Gaussian of width $10 h^{-1}\text{Mpc}$. Fig. 1 demonstrates that the sum is dominated on large scales by terms involving 1, b_1 and B_1 , with the b_2 and B_2 terms being significantly smaller. Keeping only the contribution from the 1 term in the $\{\dots\}$ gives the Zeldovich approximation for the matter correlation function. On large scales this looks like a smoothed version of the linear theory correlation function [54]. Including non-zero b_1 describes biased tracers, with the standard, large-scale, Eulerian bias $b = 1 + b_1$. The terms involving B_i are the terms which characterize the density dependence of the mark (i.e. the 1st and 2nd derivatives of $G[\delta_L]$ with respect to δ_L) so $B_1 < 0$ describes a mark which emphasizes lower densities. The b_1^2 and B_1^2 terms also look very much like a smoothed version of the linear theory correlation function on large scales (arising as they do from an almost-convolution of ξ with the Gaussian in Eq. 2.9). The b_1 and B_1 terms have a similar shape to the “1”, but an amplitude about twice as large (for $b_1 = B_1 = 1$) [47, 48, 53]. These facts will help to explain the behavior we will see below in the marked correlation function.

In redshift space we define the angle-averaged \mathcal{M} by first averaging the numerator and denominator over angle, and then performing the division. This is different from averaging the ratio, but serves to make the denominator the monopole of the redshift space correlation function. With this definition, to go into redshift space we simply multiply the line-of-sight components of U and A by $1 + f$ before doing the d^3q integral, then perform the division [47, 53]. As the Zeldovich approximation does a relatively good job of describing the monopole of the redshift-space correlation function but a poor job on the higher moments [53] we shall restrict attention to this angle-averaged statistic here. Then the relative sizes of the terms

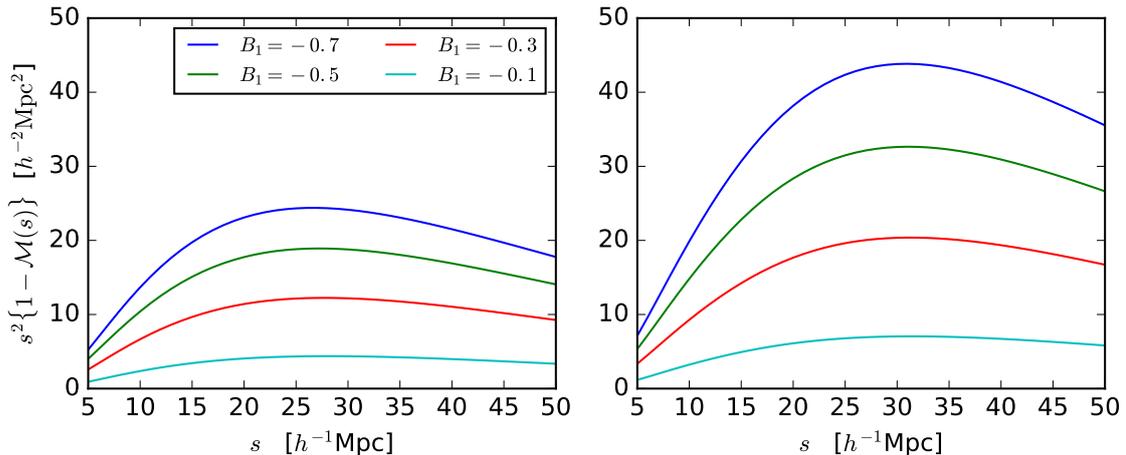


Figure 2. The monopole of the marked correlation function for unbiased ($b_1 = b_2 = 0$; left) and biased ($b_1 = 1, b_2 = 0$; right) tracers assuming $B_2 = B_1^2$. We plot $s^2(1 - \mathcal{M})$ to allow the use of a linear y -axis scale and because on large scales we expect $1 - \mathcal{M} \propto \xi_0$ (see text).

do not change much between real- and redshift-space and Fig. 1 remains a good guide to the structure of the theory.

3 Results

With the formalism in hand we can now explore the behavior of \mathcal{M} as we change the bias of the tracer and the mark. To begin we consider the marked correlation function for an unbiased tracer of the density field ($b_1 = b_2 = 0$) with the same definition of density as above. Motivated by Eq. (2.3) we shall take $B_1 < 0$ and $B_2 = B_1^2$. Fig. 2 (left) shows the angle-averaged marked correlation function at $z \simeq 0.55$ for three values of B_1 . Given the structure in Fig. 1 and recalling that $B_1 < 0$ we see $\mathcal{M} < 1$. To enhance the dynamic range we have plotted $s^2[1 - \mathcal{M}]$, which we see is qualitatively similar to $r^2\xi$ as might be expected by the structure of the terms (see below).

To gain some intuition let us consider just the lowest order term, linear in B_1 (still with $b_1 = b_2 = 0$). This term is $-2B_1 U_i^R g_i$. Recalling that $U_i^R = \langle \delta_R \Delta_i \rangle$ we see that this term describes the density dependence of the (infall) velocity in GR (and Lagrangian PT). On large scales the $-U_i g_i$ contribution is very similar to ξ_L [48, 53], so this term makes $\mathcal{M} - 1$ look like $(1 + 2B_1)\xi_L$. If the density dependence of infall velocity is modified from its GR form we would expect to see a departure in the measured dependence of \mathcal{M} on B_1 (through changes in ρ_* or p) on the scales where this modification manifests. Knowing the B_1 and r -dependence of this modification would provide valuable information as to the form of the departure from GR. If the linear correlation function, ξ_L , is not affected by the departure from GR, then we expect the U effect to dominate on large scales. Only on smaller scales the other terms, e.g. $G_{ij} U_i U_j$ describing more complex density-velocity-velocity correlations, become comparable in size to the U_i terms (Fig. 1).

As a second example we consider biased tracers. For definiteness we have taken $b_1 = 1$ (corresponding to a large-scale, Eulerian bias of 2) and $b_2 = 0$ (though the results are quite

insensitive to b_2). Fig. 2 shows that the structure of \mathcal{M} is quite similar to the unbiased case, but the amplitude is increased. A future survey such as DESI³ or Euclid⁴, covering many Gpc^3 of volume, would be able to measure \mathcal{M} with per cent level precision on scales of many Mpc.

Our analytic calculation explains the major trends we see here. For large s , $\mathcal{M} \simeq 1 + W - \xi$. On scales above the smoothing scale, R , the contributions $\xi \simeq \xi_{R,1} \simeq \xi_R$ and $U^R \simeq U$. Further, the contributions from $-U_i g_i$ are almost the same as those from ξ , as mentioned above. Thus on large scales $\Xi \simeq 1 + (1 + b_1 + B_1)^2 \xi_0$ and

$$1 - \mathcal{M}(s) \simeq \xi - W \simeq -B_1(2 + 2b_1 + B_1)\xi_0 + \dots \quad (3.1)$$

This gives a simple prediction for the scaling of the amplitude with the mark (e.g. choice of ρ_*) and the bias of the tracer, valid on large scales and to low order in perturbation theory.

If the theory of gravity is modified, in a density dependent way, we would expect the ingredients going into this calculation to also be modified. We mentioned above the example of $U_i = \langle \delta \Delta_i \rangle$, describing the correlation between velocities and densities. This could be modified from its form in GR if there are additional, density-dependent forces at work. Precise prediction of the functional form of any deviations would require a calculation within a specific model, and the scale at which these departures occur is likely to be model dependent. With a suitably flexible bias model, low order perturbation theory (such as presented above) suffices for GR predictions on large scales. To probe for departures occurring on smaller scales corrections to this model must be computed. Higher order terms in the bias expansion and higher order terms in the dynamics can be systematically computed if desired. This would allow us to push to quasi-linear scales, but not to fully non-linear scales. A model for non-linear clustering would need to be derived from numerical simulations. This is straightforward, given a suitable model for the halo occupancy of the galaxies. Mock catalogs can also include the effects of masking or missing data, complex marks and other survey non-idealities.

4 Conclusions

The growth of large-scale structure as observed in modern cosmological surveys offers one means of testing general relativity on the largest scales. Constraints on the distance scale, growth rate and deflection of light (from BAO, RSD and lens modeling respectively) have become standard and such analyses are likely to be performed on all future surveys. In the absence of a single, compelling model of modified gravity it is difficult to know how to augment these ‘standard’ analyses so as to best constrain modifications.

Many models which modify gravity invoke a screening mechanism that forces the predictions to become equal to those of GR in regions of high density or strong gravitational potential. This suggests that generic tests for the density dependence of the growth of structure could be added to our list of ‘standard’ analyses and might provide useful in constraining models of modified gravity in the future. In this paper we have pointed out that an inverse-density-marked correlation function, $\mathcal{M}(r)$, is very easy to compute and report, requiring little computational overhead, code development or additional survey products.

To illustrate the form that $\mathcal{M}(r)$ takes in the standard theory, we present the lowest order Lagrangian perturbation theory calculation. In keeping with our perturbative approach

³<http://desi.lbl.gov>

⁴<http://sci.esa.int/euclid>

we have emphasized large scales, such as can be easily probed with upcoming surveys aimed at BAO and RSD. Of course it may be that modifications to gravity are more easily seen on smaller scales with density fields estimated from denser samples of galaxies (or other objects). While this likely invalidates the perturbative calculation presented earlier, it is relatively straightforward to generate predictions on such scales from simulations given a suitably refined model for bias (e.g. populating halos in N-body simulations with galaxies using an HOD model [55] or a semi-analytic model [56]). Indeed, even if such statistics prove unconstraining for the modified gravity models of the future, the density information they encode can be very valuable for validating and refining the bias model used to model BAO and RSD statistics, e.g. breaking degeneracies [7] or testing for assembly bias. Should the population of galaxies in a survey depend on properties of a dark matter halo beyond its mass (e.g. formation time or concentration) which correlates with large-scale environment we expect to see a departure from the simplest theoretical models which neglect such effects. The nature of this departure can be understood from the theory above and parameterized in a very flexible way. Since the GR predictions are so well known, as long as modifications to gravity are not fully degenerate with these effects, we should still be able to disentangle them.

There are several obvious lines of development. First, this statistic could be computed on existing modified gravity simulations to get a sense for the size of the effect as a function of scale and the ideal redshift and number density of tracer for this test. This would help to calibrate expectations, but cannot be taken as definitive since existing models are either ruled out or do not explain acceleration with modified gravity and we do not know how the predictions would differ in a model which could explain our Universe. Second, a study should be undertaken of the best density estimate and to what extent noise in this estimate adversely affects the results. Third, if further investigation warrants, this model can be extended to higher order in perturbation theory or to include a dependence on the derivatives of the density or the dimensionality of the structure. Within the Lagrangian framework it is relatively straightforward to include marks which can be expressed in terms of initial density and its derivatives. Finally, it is worth investigating whether marked statistics for auto- and cross-correlations of imaging and spectroscopic surveys could yield other, valuable constraints on modifications to GR. It is straightforward [47, 48, 52] to modify the formulae in this paper to account for cross-correlations of biased and marked tracers, thus opening the possibility to predict a range of other statistics.

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A The smoothing scale

In the text we have assumed that the smoothed density field, estimated from the galaxies, can be reasonably well approximated by the linear density field. In this appendix we provide

a back-of-the-envelope argument that this is likely to be true for surveys which are designed to study BAO or RSD.

Most surveys focused on large-scale structure adjust their sampling so that $\bar{n}P$ is of order a few, where P is the power spectrum evaluated at a convenient scale, often $0.2 h \text{ Mpc}^{-1}$. If we assume that the smoothing scale that defines the density in our mark, ρ , scales with the mean interobject separation, $\bar{n}^{-1/3}$, this implies ρ is close to linear.

As an example, imagine $P(k) = P_*(k_*/k)$ for some fiducial k_* . This is approximately the slope of the CDM power spectrum on quasi-linear scales today. Further imagine we smooth the field with a filter of size $R = r\bar{n}^{-1/3}$ with $r \sim 1$. We have

$$\sigma^2(R) = \int \frac{k^2 dk}{2\pi^2} P(k) W^2(k; R) = \frac{k_*^3 P_*}{2\pi^2} \int d\kappa \kappa W^2(\kappa; k_* R) \quad (\text{A.1})$$

with $\kappa = k/k_*$. For a Gaussian of width R

$$\sigma^2(R) = \frac{k_*^3 P_*}{2\pi^2} \frac{1}{2(k_* R)^2} \quad (\text{A.2})$$

To simplify this, let us choose k_* so that $k_*^3 P_*/2\pi^2 = 1$. This means k_* is where the dimensionless power is unity. Now if we say $\bar{n}P_* = \nu$ then

$$k_* R = \left(\frac{2\pi^2}{P_*} \right)^{1/3} r \bar{n}^{-1/3} \simeq \frac{2.7 r}{\nu^{1/3}} \quad (\text{A.3})$$

and hence

$$\sigma(R) = \frac{\nu^{1/3}}{2^{5/6} \pi^{2/3} r} = 0.26 \frac{\nu^{1/3}}{r} \quad (\text{A.4})$$

This suggests that for modest ν and $r \geq 1$ a kernel density estimate should return an approximately linear density field. If the galaxy field is highly biased, then the matter field which it represents will be even closer to linear.

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