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Optimal Power Allocation for Cooperative Multiple Access with Non-Ideal Full-Duplex Sources

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Abstract—This paper considers a cooperative multiple access (CMA) scheme where two full-duplex sources simultaneously send their own messages to a common destination. As part of their cooperation, each of the two sources also receives and relays part of the message from the other source. This paper develops algorithms for computing the optimal power allocation to achieve the maximal network throughput subject to residual self-interference (RSI) at the non-ideal full-duplex sources. As a generalization of a time-division duplex (TDD) scheme, we consider a three-phase CMA (3P-CMA) scheme which treats the TDD scheme as a special case. The throughput of the 3P-CMA scheme consistently exceeds that of the TDD scheme regardless of the RSI power gain. Also shown in this paper is a semi-closed form of the optimal power allocation when the three-phases are constrained to be identical, i.e., resulting in a one-phase CMA (1P-CMA) scheme. The optimal power allocation of the 1P-CMA scheme takes much less time to compute, and its network throughput is close to that of the 3P-CMA scheme when the RSI power gain is low.

Index Terms—Full-duplex, cooperative multiple access, partial decode-and-forward, achievable rate region, power allocation

I. INTRODUCTION

Full-duplex (FD) technology has the potential to double the throughput of point-to-point communications and has become a topic of wide interests, e.g., see [1] and the references therein. By simultaneously transmitting and receiving at the same frequency, FD technology can enhance the achievable rates and reliability of various fundamental networks such as relay channels [2].

When multiple sources help each other to transmit information to a common destination, it is called cooperative multiple access (CMA), which can be done via out-of-band conferencing [3]–[10] or in-band FD communications [11]–[18]. The latter is more spectrally efficient than the former. However, the FD communications in [11]–[18] are assumed to be ideal without any RSI.

In this paper, CMA with in-band FD communication is considered. But unlike [11]–[18], here we consider non-ideal FD sources where the power of RSI at each source is proportional to the power of the transmitted signal. With the existence of RSI, the power allocation problem becomes substantially different from that in [12]. By considering the RSI, this paper handles a problem which is more practical and more general. And the corresponding problem (also called optimal achievable rate problem) becomes much harder due to

more possible cases of the optimal solution. Furthermore, we adopt the space-time power scheduling concept from [19] to construct a three-phase FD decode-and-forward CMA scheme where the (transmit) power of each message component from each source is dependent on three phases (or three time windows). We further let the length of each phase be flexible. To be referred to as 3P-CMA, this three-phase scheme is a generalization of the time division duplex (TDD) scheme shown in [8] and outperforms the latter under any RSI power gain. Even if we force the three phases of the 3P-CMA scheme to be identical (i.e., making the power distributions the same over the three phases), the resulting scheme (to be called 1P-CMA) is still a generalization of the one-phase schemes in [5] and [12] due to the RSI consideration. It will be shown however that under a low level of RSI power gain, the 3P-CMA scheme performs similarly as the 1P-CMA scheme. A preliminary work of the 1P-CMA was reported in [20] while the current work is much more comprehensive.

The 3P-CMA scheme allows the power allocation for each of the two cooperative sources to be a function of three phases (or time windows). Since we consider two cooperative sources and one destination (comprising a network of three nodes), three phases are sufficient to provide all needed dimensions of freedom for optimal power allocation. If the first dimension corresponds to the first source multi-casting to the other two nodes, then the second dimension is needed for the second source to multi-cast to the other two nodes and also to act as a relay from the first source to the destination, and the third dimension is necessary for the first source to act as a relay from the second source to the destination. In other words, using three phases, the cooperation between two sources and one destination can be even done without any interference. For this reason, there is no need to consider four or more phases here. Also note that the 3P-CMA scheme not only exploits three phases but also an advanced coding scheme as detailed later.

The 3P-CMA scheme provides a flexible switch between TDD and FD, which is in contrast to [5] and [12]. We acknowledge that such a flexible switch is also shown in [29], which however addresses a single-source two-destination cooperative system (unlike our two-source one-destination cooperative system).

The overall structure of the power allocation algorithm that we can develop for 3P-CMA is rather simple and easy to implement. But its actual computational complexity is rather high, which is discussed in Section III. A significant reduction of the computational complexity for 3P-CMA does not seem a feasible task. For this reason, a large part of this paper

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(Section IV) is devoted to the development of a highly efficient algorithm for the 1P-CMA scheme. To achieve this, we need to exploit the rich structures inside the 1P-CMA power allocation problem as shown in this paper. It is because of this effort that we are able to demonstrate efficiently that the 1P-CMA scheme has a performance close to that of the 3P-CMA scheme under a low level of RSI power gain.

The rest of paper is organized as follows. Section II introduces the 3P-CMA scheme. In section III, we formulate and solve a power allocation problem which characterizes the achievable rate region of 3P-CMA. In section IV, we focus on the case of 1P-CMA. In section V, important numerical results are provided and discussed. Finally, section VI concludes this paper.

Fig. 1: Cooperative multiple access (CMA) with two non-ideal full-duplex (FD) source nodes and one destination node.

II. SYSTEM MODEL

Shown in Fig. 1 is a network of two non-ideal full-duplex (FD) source nodes and one destination node. The 3P-CMA scheme applied to this network has three phases which correspond to three different patterns of power allocations at the FD sources. For each phase (or each pattern of power allocations), the cooperation (i.e., coding and decoding) strategy is similar to that in [5].

Specifically, the 3P-CMA scheme is as follows. The transmissions from the sources to the destination consist of many blocks. For each block, there are three phases indexed by $j = 1, 2, 3$. We will let source $\bar{k}$ denote the counterpart of source k ($k = 1, 2$), i.e., if $k = 1$ then $\bar{k} = 2$, and if $k = 2$ then $\bar{k} = 1$. The encoding and decoding method here is a combination of the method in [5] and [8]. In block i , the message from source k is divided into two parts, one is $w_{k,0}(i)$ which is transmitted directly to the destination, the other is $w_{k,\bar{k}}(i)$ which is routed to the destination via source $\bar{k}$. For transmission from source k , the messages $w_{k,0}(i)$ and $w_{k,\bar{k}}(i)$ are further divided into three parts, i.e., $w_{k,0,j}(i)$ and $w_{k,\bar{k},j}(i)$ with $j = 1, 2, 3$ for the three phases. For relaying from source $\bar{k}$ to the destination, the message $w_{k,\bar{k}}(i)$ is re-divided into $w'_{k,\bar{k},j}(i)$ with $j = 1, 2, 3$, which can be different from $w_{k,\bar{k},j}(i)$.

Signaling: We let $x_{k,0,j}(i)$ be the coded signal of $w_{k,0,j}(i)$ and $x_{k,\bar{k},j}(i)$ be the coded signal of $w_{k,\bar{k},j}(i)$. We let $u_{k,j}(i)$ be a coded signal of both $w'_{k,\bar{k},j}(i)$ and $w'_{\bar{k},k,j}(i)$ at source k . Also note that for example $x_{k,0,j}(i)$ is not determined only by $w_{k,0,j}(i)$. Please see Appendix A.

Transmission at two sources: Without loss of generality, the transmission scheme at the i -th time slot is described as representation of the whole transmission scheme. As the

superposition block Markov coding scheme in [5] is adopted and $u_{k,j}(i-1)$ is encoded at the end of $(i-1)$ -th block, the transmitted signal $t_{k,j}(i)$ from source k is constructed as follows

$$t_{k,j}(i) = \sqrt{p_{k,0,j}}x_{k,0,j}(i) + \sqrt{p_{k,\bar{k},j}}x_{k,\bar{k},j}(i) + \sqrt{p_{u,k,j}}u_{k,j}(i-1), \quad (1)$$

where $p_{k,0,j}$, $p_{k,\bar{k},j}$ and $p_{u,k,j}$ are the powers associated with the terms of $x_{k,0,j}(i)$, $x_{k,\bar{k},j}(i)$ and $u_{k,j}(i-1)$, respectively. It follows that the total power from source k in phase j is $p_{k,j} = p_{k,0,j} + p_{k,\bar{k},j} + p_{u,k,j}$, which is invariant to the block index i .

Reception at three nodes: Then the signals received by the FD source nodes and the destination in phase j of block i are given as

$$r_{k,j}(i) = h_{\bar{k},k}t_{\bar{k},j}(i) + h_k\tilde{t}_{k,j}(i) + z_{k,j}(i), \quad k = 1, 2, \quad (2)$$

$$r_{0,j}(i) = h_{1,0}t_{1,j}(i) + h_{2,0}t_{2,j}(i) + z_{0,j}(i), \quad (3)$$

where $h_{\bar{k},k}$ is the channel gain from source $\bar{k}$ to source k , $h_k\tilde{t}_{k,j}(i)$ is the RSI at source k and modeled as circular symmetric complex Gaussian (CSCG) noise [21] and [22], i.e., $h_k\tilde{t}_{k,j}(i) \sim \mathcal{CN}(0, |h_k|^2 p_{k,j})$. Obviously, $|h_1|^2$ and $|h_2|^2$ are the power gains of RSI. Also, $z_{0,j}(i)$, $z_{1,j}(i)$, and $z_{2,j}(i)$ are the CSCG noises with mean zero and variances N_0 , N_1 , and N_2 , respectively. The length α_j of phase j is normalized to be such that $0 \leq \alpha_j \leq 1$ and $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

Decoding at the sources: According to (1) and (2), $r_{k,j}(i)$ includes the signal $x_{\bar{k},k,j}(i)$, which is encoded from message $w_{\bar{k},k,j}(i)$. Since the messages $w'_{1,2,j}(i-1)$ and $w'_{2,1,j}(i-1)$ at the $(i-1)$ -th time slot are known by source k , the signal $u_{\bar{k},j}(i-1)$ is known by source k . Thus, source k is able to decode the message $w_{\bar{k},k,j}(i)$ of node $\bar{k}$ from the signal $r_{k,j}(i)$ with the interference of signal $x_{k,0,j}(i)$ and RSI $h_k\tilde{t}_{k,j}(i)$. After decoding the messages $w_{\bar{k},k,j}(i)$ for $j = 1, 2, 3$, the message $w_{\bar{k},k}(i)$ is obtained by source k . Then, the messages $w_{\bar{k},k}(1)$ and $w_{\bar{k},k}(2)$ are re-divided into $w'_{\bar{k},k,j}(i)$ and $w'_{k,\bar{k},j}(i)$ with $j = 1, 2, 3$. Then $w'_{\bar{k},k,j}(i)$ and $w'_{k,\bar{k},j}(i)$ are encoded as $u_{k,j}(i)$, and sent out in the next block.

Decoding at the destination: Backward decoding [5] is adopted at the destination. The destination stores all the received signals and decodes the messages of the two source nodes from $(N-1)$ -th time slot to the first time slot. It is observed that the signal model of the proposed scheme is almost the same as that in [5] except the RSI at source nodes and the three-phase transmission.

Then, the following proposition follows from the similar argument as that used in [5]:

Proposition 2.1: Given the power constraint

$$\sum_{j=1}^3 \alpha_j p_{k,j} = \sum_{j=1}^3 \alpha_j (p_{k,0,j} + p_{k,\bar{k},j} + p_{u,k,j}) \leq \bar{p}_k, \quad (4)$$

the achievable rates of the two sources, $R_1 = R_{1,0} + R_{1,2}$ and $R_2 = R_{2,0} + R_{2,1}$, satisfy

$$R_{k,\bar{k}} \leq \sum_{j=1}^3 \alpha_j \log \left(1 + \frac{\gamma_{k,\bar{k}} p_{k,\bar{k},j}}{\gamma_{\bar{k},k} p_{k,j} + \gamma_{k,\bar{k}} p_{k,0,j} + 1} \right), \quad (5)$$

$$R_{k,0} \leq \sum_{j=1}^3 \alpha_j \log(1 + \gamma_{k,0} p_{k,0,j}), \quad (6)$$

$$R_{1,0} + R_{2,0} \leq \sum_{j=1}^3 \alpha_j \log(1 + \gamma_{1,0} p_{1,0,j} + \gamma_{2,0} p_{2,0,j}), \quad (7)$$

$$R_1 + R_2 \leq \sum_{j=1}^3 \alpha_j \log \left(1 + 2\sqrt{\gamma_{1,0}\gamma_{2,0}p_{u,1,j}p_{u,2,j}} + \sum_{k=1,2} \gamma_{k,0} p_{k,j} \right), \quad (8)$$

where source k transmits the messages $w_{k,0}(i)$ and $w_{k,\bar{k}}(i)$ at rates $R_{k,0}$ and $R_{k,\bar{k}}$, respectively, and $\gamma_{k,0} = \frac{|h_{k,0}|^2}{N_0}$, $\gamma_{k,\bar{k}} = \frac{|h_{k,\bar{k}}|^2}{N_k}$, and $\gamma_k = \frac{|h_k|^2}{N_k}$.

Proof: Please see Appendix A.

A key variation of this proposition from [5] is the term $\gamma_{\bar{k}} p_{\bar{k},j}$ in (5), which is due to the RSI power. The summations over j are due to the adoption of three phases. Note that the above scheme requires both sources 1 and 2 to know $h_{1,0}$ and $h_{2,0}$ (including their phases) so that $\sqrt{p_{u,1,j}}u_{1,j}(i-1)$ and $\sqrt{p_{u,2,j}}u_{2,j}(i-1)$ from sources 1 and 2 are completely constructive at the destination [23].

Remark 2.1: In (4)-(8), if all three phases in 3P-CMA are identical, i.e., the optimal power allocations of the three phases are the same, then the scheme become effectively a one-phase scheme. So the rate region of the three-phase scheme is no less than that of the one-phase scheme for all RSI power gain. Furthermore, the TDD scheme in [8] corresponds to a special form of power allocation in the above 3P-CMA scheme (i.e., $p_{k,0,1} = p_{k,0,2} = p_{u,k,1} = p_{u,k,2} = p_{k,\bar{k},3} = 0$ for $k = 1, 2$ and $p_{2,1,1} = p_{1,2,2} = 0$), and hence TDD cannot perform better than 3P-CMA (with optimized power distribution).

III. OPTIMAL POWER ALLOCATION FOR 3P-CMA

The achievable rate region of the scheme is the convex hull of the rate region $\mathcal{R}$, which is defined as $\mathcal{R} = \{(R_1, R_2) : (4) \sim (8)\}$. Since $R_1 = R_{1,0} + R_{1,2}$ and $R_2 = R_{2,0} + R_{2,1}$, we substitute $R_{k,\bar{k}} = R_k - R_{k,0}$ into constraint (5) and adopt Fourier-Motzkin Elimination to successively project out $R_{1,0}$ and $R_{2,0}$ from (5)-(7). Then, the rate pair (R_1, R_2) in $\mathcal{R}$ is subject to constraints (9)-(11) at the bottom of this page. Thus, $\mathcal{R}$ can be also written as $\mathcal{R} = \{(R_1, R_2) : (4), (9) \sim (11)\}$. A

point on the boundary of the rate region $\mathcal{R}$ can be calculated by maximizing R_1 for a fixed $R_2 = R_{2,\text{tar}}$ as follows

$$\begin{aligned} \text{P1 :} \quad & \max_{\{\alpha_j, p_{k,0,j}, p_{k,\bar{k},j}, p_{u,k,j}, \forall k, \forall j\}} R_1, \\ \text{s.t.} \quad & (4), (9) \sim (11). \end{aligned}$$

Due to RSI, the constraints (9)-(11) are all non-convex. Thus, Problem P1 is nonconvex even if the phase lengths are fixed. The approach based on Karush-Kuhn-Tucker (KKT) conditions as in [11] and [13] cannot be adopted here. Unlike [12], the complexity of the 3P-CMA problem does not seem to allow one to obtain the optimal solution analytically. Thus, we solve this problem by optimizing the lengths of the three phases and optimizing the power allocations alternately. For fixed power allocation, it is obvious that Problem P1 is a linear programming. For fixed phase lengths, Problem P1 is still non-convex, for which difference-of-convex-functions programming (DC programming) is adopted here. Namely, at each iteration of optimizing the power allocation, we use the following approximation:

$$\begin{aligned} & \log \left(1 + \frac{\gamma_{k,\bar{k}} p_{k,\bar{k},j}}{\gamma_{k,\bar{k}} p_{k,0,j} + \gamma_{\bar{k}} p_{\bar{k},j} + 1} \right) \\ & \approx \frac{1}{\gamma_{k,\bar{k}} p_{k,0,j,0} + \gamma_{\bar{k}} p_{\bar{k},j,0} + 1} [\gamma_{k,\bar{k}} (p_{k,0,j} - p_{k,0,j,0}) \\ & \quad + \gamma_{\bar{k}} (p_{\bar{k},j} - p_{\bar{k},j,0})] - \log(1 + \gamma_{k,\bar{k}} p_{k,0,j,0} + \gamma_{\bar{k}} p_{\bar{k},j,0}) \\ & \quad + \log(1 + \gamma_{k,\bar{k}} p_{k,\bar{k},j} + \gamma_{k,\bar{k}} p_{k,0,j} + \gamma_{\bar{k}} p_{\bar{k},j}), \end{aligned} \quad (12)$$

where $p_{k,0,j,0}$ and $p_{k,j,0}$ are the initial point of the variables $p_{k,0,j}$ and $p_{k,j}$, respectively. The approximation in (12) is based on the first-order Taylor series expansion of the second term in such an expression $\log \frac{A(x)}{B(x)} = \log A(x) - \log B(x)$. This type of approximation is a lower bound on $\log \frac{A(x)}{B(x)}$ which is part of the essence of the DC programming. An algorithm to solve Problem P1 is summarized below.

Remark 3.1: Algorithm 1 is a two-layer iterative algorithm. The number of iterations for the outer layer required is denoted by I_1 while the convergence rate is unknown. As for the inner layer, alternate optimization is adopted. At first, for fixed transmit power, linear programming is used to calculate α_j ($j = 1, 2, 3$). The corresponding complexity is $\mathcal{O}(N_1^3)$, where N_1 is the number of α_j (which is 3 in our case). Then, for fixed

$$R_1 \leq \sum_{j=1}^3 \alpha_j \log \left(1 + \frac{\gamma_{1,2} p_{1,2,j}}{\gamma_{2,2} p_{2,j} + \gamma_{1,2} p_{1,0,j} + 1} \right) + \sum_{j=1}^3 \alpha_j \log(1 + \gamma_{1,0} p_{1,0,j}), \quad (9)$$

$$R_2 \leq \sum_{j=1}^3 \alpha_j \log \left(1 + \frac{\gamma_{2,1} p_{2,1,j}}{\gamma_{1,1} p_{1,j} + \gamma_{2,1} p_{2,0,j} + 1} \right) + \sum_{j=1}^3 \alpha_j \log(1 + \gamma_{2,0} p_{2,0,j}), \quad (10)$$

$$\begin{aligned} R_1 + R_2 \leq \min & \left(\sum_{j=1}^3 \alpha_j \log \left(1 + 2\sqrt{\gamma_{1,0}\gamma_{2,0}p_{u,1,j}p_{u,2,j}} + \sum_{k=1,2} \gamma_{k,0} p_{k,j} \right) \right. \\ & \left. + \sum_{j=1}^3 \alpha_j \log(1 + \gamma_{1,0} p_{1,0,j} + \gamma_{2,0} p_{2,0,j}) + \sum_{j=1}^3 \alpha_j \sum_{k=1,2} \log \left(1 + \frac{\gamma_{k,\bar{k}} p_{k,\bar{k},j}}{\gamma_{k,\bar{k}} p_{k,j} + \gamma_{\bar{k}} p_{\bar{k},j} + 1} \right) \right). \end{aligned} \quad (11)$$

α_j , the optimal power is obtained by solving a convex problem from Problem P1 whose convergence rate is quadratic. The number of iterations required is denoted as I_2 . The complexity of each iteration of the inner layer is known to be $O(N^3)$ [28], where N is the number of unknown variables (which is 19 in our case). Thus, the total complexity of Algorithm 1 is $O(N^3 I_1 I_2 + N_1^3 I_1)$. The actual computing time will be illustrated later.

Algorithm 1 An algorithm for solving Problem P1

- 1: **Initialization:** Set $l = 0$, tolerance $\xi > 0$, and the feasible initial transmit powers $p_{k,0,j}^{(0)}$, $p_{k,\bar{k},j}^{(0)}$, $p_{u,k,j}^{(0)}$, and $p_{k,j}^{(0)}$.
 - 2: **Repeat**
 - Update $l = l + 1$, $p_{k,0,j,0} = p_{k,0,j}^{(l-1)}$ and $p_{k,j,0} = p_{k,j}^{(l-1)}$;
 - For fixed transmit powers, calculate the linear programming Problem P1 to obtain $\alpha_j^{(l)}$;
 - Linearize the concave parts of the constraints (9)-(11) in Problem P1 using (12) with initial powers $p_{k,0,j,0}$ and $p_{k,j,0}$;
 - For fixed $\alpha_j = \alpha_j^{(l)}$, solve the convex problem from Problem P1 using the CVX package [25] to obtain approximate solution $(p_{k,0,j}^{(l)}, p_{k,\bar{k},j}^{(l)}, p_{u,k,j}^{(l)})$;
 - 3: **Until** $R_1^{(l)} - R_1^{(l-1)} \leq \xi$
 - 4: **Output:** $(\alpha_j^{(l)}, p_{k,0,j}^{(l)}, p_{k,\bar{k},j}^{(l)}, p_{u,k,j}^{(l)})$ as the approximate solution.
-

IV. OPTIMAL POWER ALLOCATION FOR 1P-CMA

The previous algorithm for 3P-CMA is simple in structure but expensive in computational time. In this section, we will develop a computationally efficient algorithm for 1P-CMA. Although the derivation of this algorithm is lengthy, the benefit of the resulting algorithm is desirable. We will see later that the optimal power allocation for 3P-CMA is similar to that for 1P-CMA when the RSI power gain is low. Hence, an efficient algorithm for 1P-CMA as shown next is important.

When all three phases in 3P-CMA are identical, i.e., the optimal power allocations of the three phases are the same, they all reduce to 1P-CMA for which the corresponding power allocation problem is

$$\begin{aligned} \text{P2 : } & \max_{\{p_{k,0}, p_{k,\bar{k}}, p_{u,k}, \forall k \in \{1,2\}\}} R_1, \\ \text{s.t. } & (13) \sim (15) \\ & p_k = (p_{k,0} + p_{k,\bar{k}} + p_{u,k}) \leq \bar{p}_k, k = 1, 2 \end{aligned} \quad (16)$$

where we have used $p_{k,0,j} = p_{k,0}$, $p_{k,\bar{k},j} = p_{k,\bar{k}}$, and $p_{u,k,j} = p_{u,k}$ for all j .

Remark 4.1: If $\gamma_k = 0$, the above problem was solved in [11] numerically and [12] analytically. Here, the case of $\gamma_k > 0$ is considered. A connection between our problem and that in [12] can be seen if we replace $\gamma_{k,\bar{k}}$ in [12] by $\gamma_{k,\bar{k}}/(1 + \gamma_{\bar{k}} p_{\bar{k}})$. However, $p_{\bar{k}}$ for $k = 1, 2$ need to be found first before this replacement is possible. The optimal values of $p_{\bar{k}}$ here are not always the maximal available powers at the sources, which is unlike the case in [12]. In fact, finding the optimal values of $p_{\bar{k}}$ is a new challenge here.

Proposition 4.1: Let $(p_k^*, p_{k,0}^*, p_{k,\bar{k}}^*, p_{u,k}^*)$ denote the optimal values of $(p_k, p_{k,0}, p_{k,\bar{k}}, p_{u,k})$ as the solution to Problem P2. It follows that:

- 1) If $\gamma_{1,0} \leq \gamma_{1,2}/(1 + \gamma_2 p_2^*)$ and $\gamma_{2,0} \leq \gamma_{2,1}/(1 + \gamma_1 p_1^*)$, then $p_{1,0}^* = p_{2,0}^* = 0$, i.e., neither of sources 1 and 2 should send direct message $w_{1,0}(i)$ or $w_{2,0}(i)$ to the destination.
- 2) If $\gamma_{1,0} \leq \gamma_{1,2}/(1 + \gamma_2 p_1^*)$ and $\gamma_{2,0} > \gamma_{2,1}/(1 + \gamma_1 p_1^*)$, then $p_{1,0}^* = p_{2,1}^* = 0$, i.e., source 1 must not send any direct message $w_{1,0}(i)$ to the destination, and source 2 must not use source 1 as relay.
- 3) If $\gamma_{1,0} > \gamma_{1,2}/(1 + \gamma_2 p_2^*)$ and $\gamma_{2,0} \leq \gamma_{2,1}/(1 + \gamma_1 p_1^*)$, then $p_{2,0}^* = p_{1,2}^* = 0$, i.e., source 2 must not send direct message $w_{2,0}(i)$ to the destination, and source 1 must not use source 2 as relay.
- 4) If $\gamma_{1,0} > \gamma_{1,2}/(1 + \gamma_2 p_2^*)$ and $\gamma_{2,0} > \gamma_{2,1}/(1 + \gamma_1 p_1^*)$, then the convex hull of the region $\mathcal{R}$ obtained by solving Problem P2 does not change subject to one of the followings: $p_{1,2}^* = p_{2,1}^* = 0$ (the conventional multiple access), or $p_{1,0}^* = p_{2,1}^* = 0$ (only source 2 acts as relay), or $p_{2,0}^* = p_{1,2}^* = 0$ (only source 1 acts as relay).

Proof: Please see Appendix B.

Remark 4.2: Proposition 4.1 extends Proposition 1 in [11] in that the latter is for the case where $\gamma_k = 0$, and furthermore the latter only refers to the maximization of the right hand side (RHS) of the sum rate constraint (15).

Proposition 4.2: The rate region $\mathcal{R}$ obtained by solving Problem P2 has the same convex hull with the rate region $\mathcal{R}_1$ obtained by finding the maximal of the optimal R_1 of the four problems in Table I for which the following additional constraint is needed

$$R_1 \leq \log \left(1 + 2\sqrt{\gamma_{1,0}\gamma_{2,0}}\sqrt{p_{u,1}p_{u,2}} + \sum_{k=1,2} \gamma_{k,0}p_k \right) - R_{2,\text{tar}}. \quad (17)$$

$$R_1 \leq \log \left(1 + \frac{\gamma_{1,2}p_{1,2}}{\gamma_2 p_2 + \gamma_{1,2}p_{1,0} + 1} \right) + \log (1 + \gamma_{1,0}p_{1,0}) \quad (13)$$

$$R_{2,\text{tar}} \leq \log \left(1 + \frac{\gamma_{2,1}p_{2,1}}{\gamma_1 p_1 + \gamma_{2,1}p_{2,0} + 1} \right) + \log (1 + \gamma_{2,0}p_{2,0}) \quad (14)$$

$$\begin{aligned} R_1 + R_{2,\text{tar}} \leq \min & \left(\log \left(1 + 2\sqrt{\gamma_{1,0}\gamma_{2,0}}\sqrt{p_{u,1}p_{u,2}} + \sum_{k=1,2} \gamma_{k,0}p_k \right), \right. \\ & \left. \log (1 + \gamma_{1,0}p_{1,0} + \gamma_{2,0}p_{2,0}) + \sum_{k=1,2} \log \left(1 + \frac{\gamma_{k,\bar{k}}p_{k,\bar{k}}}{\gamma_{\bar{k}}p_{\bar{k}} + \gamma_{k,\bar{k}}p_{k,0} + 1} \right) \right) \end{aligned} \quad (15)$$

TABLE I: Four subproblems of Problem P3

Problem P3-a	Problem P3-b
$\begin{aligned} & \max_{\{R_1, p_{k,\bar{k}}, p_{u,k}\}} R_1, \\ \text{s.t. (17),} \\ & R_1 \leq \log \left(1 + \frac{\gamma_{1,2} p_{1,2}}{\gamma_{2,0} p_{2,0} + 1} \right), \\ & R_{2,\text{tar}} \leq \log \left(1 + \frac{\gamma_{2,1} p_{2,1}}{\gamma_{1,0} p_{1,0} + 1} \right), \\ & p_k = p_{k,\bar{k}} + p_{u,k} \leq \bar{p}_k, \quad k = 1, 2, \\ & p_{k,\bar{k}} \geq 0, \quad p_{u,k} \geq 0; \end{aligned} \quad (18)$	$\begin{aligned} & \max_{\{R_1, p_{1,0}, p_{2,1}, p_{u,k}\}} R_1, \\ \text{s.t. (17), (19),} \\ & R_1 \leq \log (1 + \gamma_{1,0} p_{1,0}), \\ & p_1 = p_{1,0} + p_{u,1} \leq \bar{p}_1, \\ & p_2 = p_{2,1} + p_{u,2} \leq \bar{p}_2, \\ & p_{k,0} \geq 0, \quad p_{k,\bar{k}} \geq 0, \quad p_{u,k} \geq 0; \end{aligned} \quad (22)$
Problem P3-c	Problem P3-d
$\begin{aligned} & \max_{\{R_1, p_{2,0}, p_{1,2}, p_{u,k}\}} R_1, \\ \text{s.t. (17), (18),} \\ & R_{2,\text{tar}} \leq \log (1 + \gamma_{2,0} p_{2,0}), \\ & p_1 = p_{1,2} + p_{u,1} \leq \bar{p}_1, \\ & p_2 = p_{2,0} + p_{u,2} \leq \bar{p}_2, \\ & p_{2,0} \geq 0, \quad p_{1,2} \geq 0, \quad p_{u,k} \geq 0; \end{aligned}$	$\begin{aligned} & \max_{\{R_1\}} R_1, \\ \text{s.t.} \quad & R_1 \leq \log (1 + \gamma_{1,0} \bar{p}_1), \\ & R_{2,\text{tar}} \leq \log (1 + \gamma_{2,0} \bar{p}_2), \\ & R_1 + R_{2,\text{tar}} \leq \log (1 + \gamma_{1,0} \bar{p}_1 + \gamma_{2,0} \bar{p}_2). \end{aligned}$

Proof: According to Proposition 4.1, the convex hull of $\mathcal{R}$ does not change if one of these four conditions: $p_{1,0}^* = p_{2,0}^* = 0$ or $p_{1,0}^* = p_{2,1}^* = 0$ or $p_{2,0}^* = p_{1,2}^* = 0$ or $p_{1,2}^* = p_{2,1}^* = 0$ is added according to the corresponding channel situations. These four conditions directly correspond to the four subproblems P3-a to P3-d. Thus, the convex hull of the rate region $\mathcal{R}_1$ (which is obtained by finding the maximal of the optimal R_1 of P3-a to P3-d) contains the convex hull of the rate region $\mathcal{R}$. Besides, the optimal R_1 of Problem P2 is no less than the maximal of the optimal R_1 of P3-a to P3-d. Thus, the convex hull of the rate region $\mathcal{R}$ also contains the convex hull of the rate region $\mathcal{R}_1$. ■

Remark 4.3: Compared with [12], our problem here for 1P-CMA differs in two key aspects. First, Problem P2 here is decomposed into four subproblems P3-a to P3-d. But in [12] there are eight subproblems. Second, here the optimal R_1 of each of the four subproblems must be obtained first before the solution to Problem P2 can be found by finding the best of the four. This is because RSI is power dependent. More specifically, $\gamma_{k,\bar{k}}/(1 + \gamma_{\bar{k}} p_{\bar{k}})$ depends on $p_{\bar{k}}$. In [12], RSI is assumed to be zero, and hence their original problem can be reduced to a subproblem solely based on the knowledge of the channel gains.

Remark 4.4: Each of the subproblems P3-a to P3-d has a physical meaning. P3-a is for the case where the two sources help each other's transmissions without transmitting the direct message $w_{k,0}(i)$. P3-b and P3-c are each corresponding to a case where one (and only one) of the two sources transmits direct message $w_{k,0}(i)$ to the destination and also works as a relay at the same time. P3-d corresponds to the case of a conventional multiple access channel. Since the rate region $\mathcal{R}$ is bigger than the rate region corresponding to Problems P3-b and P3-c, it is meaningful to study the case that two sources cooperate with each other.

Remark 4.5: P3-a and P3-b are quasi-convex, and P3-c and P3-d are convex. Although all of these subproblems can be solved numerically using existing software packages,

we will show a semi-closed form solution for each of these subproblems.

A. Solution to Subproblem P3-a

In this subsection, the solution to P3-a is derived. The derivation is divided into three steps:

- 1) Two lemmas are given to prove that there are six possible optimal solutions;
- 2) Five of the possible solutions are figured out according to the constraints that hold with equalities at the optimal point and one of these are proved to be suboptimal;
- 3) The remaining five possible solutions are proved to be optimal in different regions of R_2 .

Step 1

Lemma 4.1: The solution to P3-a is obtained when the transmit power of one of the sources equals the maximal transmit budget, i.e.,

$$p_{1,2}^* + p_{u,1}^* = \bar{p}_1 \text{ or } p_{2,1}^* + p_{u,2}^* = \bar{p}_2 \quad (23)$$

where the asterisk denotes the optimal power allocation.

Proof: If $p_{1,2} + p_{u,1} < \bar{p}_1$ and $p_{2,1} + p_{u,2} < \bar{p}_2$, then $p_{k,\bar{k}}$ and $p_{u,k}$ for $k = 1, 2$ can be increased by a common factor, which increases the RHSs of (17)-(19) and hence R_1 . ■

For fixed p_1 and p_2 , the equivalent cooperative channel gains $\gamma'_{k,\bar{k}} = \gamma_{k,\bar{k}}/(1 + \gamma_{\bar{k}} p_{\bar{k}})$ are fixed. Thus, P3-a has the same form as one in [12]. According to [12], we have

Lemma 4.2: All (17)-(19) hold with equality for the optimal solution of Problem P3-a if and only if

$$2^{R_{2,\text{tar}}} \left(1 + \frac{\gamma_{1,2} p_1}{1 + \gamma_{2,0} p_2} \right) - (1 + \gamma_{1,0} p_1 + \gamma_{2,0} p_2) \geq 0 \quad (24)$$

with $p_1 = p_1^*$ and $p_2 = p_2^*$.

Remark 4.6: According to lemma 4.1, there are three possibilities for the solution to P3-a (i.e., only the left equation holds, only the right equation holds, or both equations hold). According to lemma 4.2, each of the three possibilities

TABLE II: Power allocation of Problem P3 for different value of $R_{2,\text{tar}}$ with lower bound $R_{2,l}$ and upper bound $R_{2,u}$

Problem	$R_{2,l}$	$R_{2,u}$	B	$p_{1,0}^*$	$p_{1,2}^*$	$p_{u,1}^*$	p_1^*	$p_{2,0}^*$	$p_{2,1}^*$	$p_{u,2}^*$	p_2^*
(P3-a)	$\max(C_2, C_3)$	C_1	≥ 0	0	$\Phi_{1,2}$	$\bar{p}_1 - p_{1,2}^*$	$\bar{p}_1$	0	$\Phi_{2,1}$	$p_2^* - p_{2,1}^*$	(36)
		C_0	< 0	0	$\bar{p}_1$	0	$\bar{p}_1$	0	$\Phi_{2,1}$	0	$\Phi_{2,1}$
		C_0		0	$\Phi_{1,2}$	$p_1^* - p_{1,2}^*$	(50)	0	$\Phi_{2,1}$	$\bar{p}_2 - p_{2,1}^*$	$\bar{p}_2$
		C_2		0	p_1^*	0	(54)	0	$\bar{p}_2$	0	$\bar{p}_2$
		C_3		0	$\bar{p}_1 - p_{u,1}^*$	(58)	$\bar{p}_1$	0	$\Phi_{2,1}$	$\bar{p}_2 - p_{2,1}^*$	$\bar{p}_2$
(P3-b)	$\max(C_4, C_5)$			$\Phi_{1,0}$	0	$p_1^* - p_{1,0}^*$	(50)	0	$\Phi_{2,1}$	$\bar{p}_2 - p_{2,1}^*$	$\bar{p}_2$
		C_5		p_1^*	0	0	(65)	0	$\bar{p}_2$	0	$\bar{p}_2$
		C_4		$\bar{p}_1 - p_{u,1}^*$	0	(67)	$\bar{p}_1$	0	$\Phi_{2,1}$	$\bar{p}_2 - p_{2,1}^*$	$\bar{p}_2$
(P3-c)	$\max(C_2, C_6)$	C_6	≥ 0	0	$\Phi_{1,2}$	$p_1^* - p_{1,0}^*$	$\bar{p}_1$	$\Phi_{2,0}$	0	$\bar{p}_2 - p_{2,1}^*$	(36)
			< 0	0	$\bar{p}_1$	0	$\bar{p}_1$	$\Phi_{2,0}$	0	0	$\Phi_{2,0}$
				0	$\bar{p}_1 - p_{u,1}^*$	(58)	$\bar{p}_1$	$\Phi_{2,0}$	0	$\bar{p}_2 - p_{2,0}^*$	$\bar{p}_2$
(P3-d)				$\bar{p}_1$	0	0	$\bar{p}_1$	$\bar{p}_2$	0	0	$\bar{p}_2$

Note: $C_0, C_1, C_2, C_3, C_4, C_5$, and C_6 are given by (40), (43), (51), (52), (62), (63), and (71). B is given by (39) for Problem P3-a, and (72) for Problem P3-c. For $k = 1, 2$, $\Phi_{k,\bar{k}} = (2^{R_k} - 1)(\gamma_{\bar{k}} p_{\bar{k}}^* + 1)/\gamma_{k,\bar{k}}$ and $\Phi_{k,0} = (2^{R_k} - 1)/\gamma_{k,0}$ with R_1 being maximized and $R_2 = R_{2,\text{tar}}$. Additional guidance on how to use this table is shown in Remarks 4.9 and 4.10.

comprises two cases (i.e., the inequality (24) holds or does not hold). The total number of possibilities is six.

Step 2 We discuss each of the six possibilities below.

(P3-a-1): The transmit power of source 1 is the maximal transmit budget, i.e., $p_1^* = \bar{p}_1$, and (24) holds with $p_1 = p_1^* = \bar{p}_1$ and $p_2 = p_2^* < \bar{p}_2$.

According to lemma 4.2, (17)-(19) hold with equality for the optimal solution of Problem P3-a. It follows from (18) and (19) that $p_{1,2}^* = \Phi_{1,2}$ and $p_{2,1}^* = \Phi_{2,1}$, where

$$\Phi_{k,\bar{k}} = (2^{R_k} - 1)(\gamma_{\bar{k}} p_{\bar{k}}^* + 1)/\gamma_{k,\bar{k}}. \quad (25)$$

To find p_2^* in $\Phi_{2,1}$, substituting $p_{1,2}^* = \Phi_{1,2}$, $p_{2,1}^* = \Phi_{2,1}$, and $p_1 = \bar{p}_1$ into (17) yields

$$2^{R_1+R_{2,\text{tar}}} - 1 = \gamma_{1,0}\bar{p}_1 + \gamma_{2,0}p_2 + 2\sqrt{\gamma_{1,0}\gamma_{2,0}} \cdot \sqrt{(\bar{p}_1 - \Phi_{1,2})(p_2 - \Phi_{2,1})}, \quad (26)$$

which shows a connection between R_1 and p_2 . Applying differentiation to the above equation yields

$$D \frac{dR_1}{dp_2} = \gamma_{2,0} + \frac{\sqrt{\gamma_{1,0}\gamma_{2,0}}}{\sqrt{(\bar{p}_1 - \Phi_{1,2})(p_2 - \Phi_{2,1})}} \cdot \left(\bar{p}_1 - \Phi_{1,2} - (p_2 - \Phi_{2,1}) \frac{(2^{R_1} - 1)\gamma_2}{\gamma_{1,2}} \right), \quad (27)$$

where

$$D = \ln 2 \cdot 2^{R_1+R_{2,\text{tar}}} + \ln 2 \cdot 2^{R_1} \frac{\sqrt{p_2 - \Phi_{2,1}}(\gamma_2 p_2 + 1)}{2\sqrt{\bar{p}_1 - \Phi_{1,2}}\gamma_{1,2}}. \quad (28)$$

The optimal p_2 is obtained when (27) equals to zero, i.e.,

$$\sqrt{\gamma_{2,0}(\bar{p}_1 - \Phi_{1,2})(p_2 - \Phi_{2,1})} = \sqrt{\gamma_{1,0}} \left((p_2 - \Phi_{2,1}) \frac{(2^{R_1} - 1)\gamma_2}{\gamma_{1,2}} - (\bar{p}_1 - \Phi_{1,2}) \right). \quad (29)$$

According to (29), p_2^* is the solution of a quadratic equation

$$a_1 p_2^{*2} + b_1 p_2^* + c_1 = 0, \quad (30)$$

where a_1, b_1 , and c_1 are given as

$$a_1 = \gamma_{2,0}\alpha_1 + 4\gamma_{1,0}\alpha_1^2, \quad (31)$$

$$b_1 = -(4\gamma_{1,0}\alpha_1 + \gamma_{2,0})(\bar{p}_1 + \beta_1), \quad (32)$$

$$c_1 = \gamma_{2,0}\Phi_{2,1}\varepsilon_1 + \gamma_{1,0}(\bar{p}_1 + \beta_1)^2, \quad (33)$$

with

$$\alpha_1 = \frac{\gamma_2(2^{R_1} - 1)}{\gamma_{1,2}}, \quad \beta_1 = \frac{(2^{R_1} - 1)(\gamma_2\Phi_{2,1} - 1)}{\gamma_{1,2}}, \quad (34)$$

$$\varepsilon_1 = \bar{p}_1 - \frac{(2^{R_1} - 1)}{\gamma_{1,2}}. \quad (35)$$

The solution of (30) is

$$p_2^* = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1c_1}}{2a_1}. \quad (36)$$

By substituting (36) into (17), we know that R_1 satisfies $f_1(2^{R_1}) = 0$ with

$$f_1(2^{R_1}) = \left[2a_1 \left((2^{R_1+R_{2,\text{tar}}} - 1)N_0 + \gamma_{1,0}\bar{p}_1 + 2\gamma_{1,0}\beta_1 \right) + b_1(\gamma_{2,0} + 4\gamma_{1,0}\alpha_1) \right]^2 - (\gamma_{2,0} + 4\gamma_{1,0}\alpha_1)^2 \cdot (b_1^2 - 4a_1c_1). \quad (37)$$

With the optimal power allocations $p_{1,2}^* = \Phi_{1,2}$, $p_{2,1}^* = \Phi_{2,1}$, and (36), it follows that

$$p_{u,k}^* = p_k^* - p_{k,\bar{k}}, \quad k = 1, 2. \quad (38)$$

Note that $p_2^*, p_{1,2}^*, p_{2,1}^*, p_{u,1}^*$, and $p_{u,2}^*$ are all functions of 2^{R_1} .

The inequality (24) with $p_1 = \bar{p}_1$ and $p_2 = p_2^*$ cannot be verified before calculating p_2^* . Whether p_2^* given by (36) is the optimal p_2 is still unknown without verifying the inequality (24) with $p_1 = \bar{p}_1$ and $p_2 = p_2^*$. To fix this, the following lemma is given

Lemma 4.3: (24) holds with $p_1 = \bar{p}_1$ and $p_2 = p_2^*$ if and only if (24) holds with $p_1 = \bar{p}_1$ and $p_2 = \Phi_{2,1}$ under the condition of $p_2^* \leq \bar{p}_2$.

Proof: Please see Appendix C.

From (24) with $p_1 = \bar{p}_1$ and $p_2 = \Phi_{2,1}$, it follows that

$$B \doteq 2^{R_{2,\text{tar}}} \left(1 + \frac{\gamma_{1,2}\bar{p}_1}{1 + \gamma_{2,0}\Phi_{2,1}} \right) - (1 + \gamma_{1,0}\bar{p}_1 + \gamma_{2,0}\Phi_{2,1}) \geq 0. \quad (39)$$

Lemma 4.4: p_2^* is increasing with $R_{2,\text{tar}}$ when $B \geq 0$.

Proof: Please see Appendix D.

Since p_2^* is increasing with $R_{2,\text{tar}}$, to ensure that p_2^* given by (36) satisfies power constraint (21), i.e., $p_2^* \leq \bar{p}_2$, $R_{2,\text{tar}}$ must satisfy $R_{2,\text{tar}} \leq C_1$ where C_1 is the threshold point of $R_{2,\text{tar}}$ corresponding to

$$p_2^* = \bar{p}_2, \quad (40)$$

in which p_2^* is given by (36).

(P3-a-2): The transmit power of source 1 is the maximal transmit budget, i.e., $p_1^* = \bar{p}_1$, and (24) does not hold with $p_1 = \bar{p}_1$ and $p_2 = p_2^*$.

According to lemma 4.3, $B < 0$. Then,

$$\log \left(1 + \frac{\gamma_{1,2}\bar{p}_1}{1 + \gamma_{2,0}\Phi_{2,1}} \right) < \log (1 + \gamma_{1,0}\bar{p}_1 + \gamma_{2,0}\Phi_{2,1}) - R_{2,\text{tar}}. \quad (41)$$

Since R_1 is only bounded by (17) and (18), to maximize R_1 , one of (17) and (18) holds with equality for the optimal solution of Problem P3-a. According to (41), (18) holds with equality for the optimal solution of Problem P3-a. Consequently, $p_{1,2}^* = \bar{p}_1$ and $p_{u,1}^* = 0$. To reduce the interference power p_2 in (18), p_2^* is the minimal p_2 , i.e., $p_{2,1}^* = p_2^* = \Phi_{2,1}$. The rate of source 1 is given as

$$R_1 = \log \left(1 + \frac{\gamma_{1,2}\bar{p}_1}{\gamma_{1,0}\bar{p}_1 + 1} \right). \quad (42)$$

It should be noted that p_2^* satisfies $p_2^* \leq \bar{p}_2$. Thus, $R_{2,\text{tar}}$ satisfies

$$R_{2,\text{tar}} \leq C_0 = \log \left(1 + \frac{\gamma_{2,1}\bar{p}_2}{1 + \gamma_{1,0}\bar{p}_1} \right). \quad (43)$$

In the case of $R_{2,\text{tar}} > C_0$, if $p_1^* = \bar{p}_1$ and $p_2^* < \bar{p}_2$, the transmit power can not support the transmission of source 2. Thus, $p_2 = \bar{p}_2$ which will be discussed in (P3-a-3) and (P3-a-4).

(P3-a-3): The transmit power of source 2 is the maximal transmit budget, i.e., $p_2^* = \bar{p}_2$, and (24) holds with $p_1 = p_1^*$ and $p_2 = \bar{p}_2$.

According to lemma 4.2, (17)-(19) hold with equality for the optimal solution of Problem P3-a. Substituting $p_{1,2}^* = \Phi_{1,2}$, $p_{2,1}^* = \Phi_{2,1}$, and $p_2 = \bar{p}_2$ into (17) yields

$$2^{R_1+R_{2,\text{tar}}} - 1 = \gamma_{1,0}p_1 + \gamma_{2,0}\bar{p}_2 + 2\sqrt{\gamma_{1,0}\gamma_{2,0}} \cdot \sqrt{(p_1 - \Phi_{1,2})(\bar{p}_2 - \Phi_{2,1})}. \quad (44)$$

which shows a connection between R_1 and p_1 . By adopting the similar processing which is used to obtain (37) from (26), the optimal R_1 is formulated as a solution of the equation

$f_2(2^{R_1}) = 0$ from (44) where

$$f_2(2^{R_1}) = \left[4a_2\gamma_{2,0} \left(\alpha\beta(2^{R_{2,\text{tar}}} - 1) - \varepsilon - \frac{1}{2}\bar{p}_1 \right) + 2a_2(2^{R_1+R_{2,\text{tar}}} - 1) + (\gamma_{1,0} + 4\gamma_{2,0}\alpha)b_2 \right]^2 - (\gamma_{1,0} + 4\gamma_{2,0}\alpha)^2(b_2^2 - 4a_2c_2), \quad (45)$$

where

$$\alpha = \frac{\gamma_{1,1}(2^{R_1} - 1)}{\gamma_{2,1}}, \quad \beta = \frac{\gamma_{2,1}\bar{p}_2 + 1}{\gamma_{1,2}}, \quad (46)$$

$$\varepsilon = \frac{(2^{R_{2,\text{tar}}} - 1)}{\gamma_{2,1}} - \bar{p}_2, \quad a_2 = 4\gamma_{2,0}\alpha^2 + \gamma_{1,0}\alpha, \quad (47)$$

$$b_2 = (\gamma_{1,0} - 4\gamma_{2,0}\alpha) \left(\varepsilon - \alpha\beta(2^{R_1} - 1) \right), \quad (48)$$

$$c_2 = \gamma_{2,0} \left(\varepsilon - \alpha\beta(2^{R_1} - 1) \right)^2 - \gamma_{1,0}\varepsilon\beta(2^{R_1} - 1). \quad (49)$$

Furthermore, the corresponding power allocation scheme is given by $p_{1,2}^* = \Phi_{1,2}$, $p_{2,1}^* = \Phi_{2,1}$, and (38), and

$$p_1^* = \frac{-b_2 \pm \sqrt{b_2^2 - 4a_2c_2}}{2a_2}. \quad (50)$$

According to Proposition 4.1, $\gamma_{1,2}p_1^*/(1 + \gamma_{2,0}\bar{p}_2) \geq \gamma_{1,0}$. Thus, the LHS of (24) is increasing with p_1 . Following a similar proof as for lemma 4.3, the following lemma can be obtained:

Lemma 4.5: (24) holds with $p_1 = p_1^*$ and $p_2 = \bar{p}_2$ if and only if (24) holds with $p_1 = \bar{p}_1$ and $p_2 = \bar{p}_2$, i.e.,

$$R_{2,\text{tar}} \geq C_2 = \log \left(\frac{(1 + \gamma_{1,0}\bar{p}_1 + \gamma_{2,0}\bar{p}_2)(1 + \gamma_{1,0}\bar{p}_1)}{1 + \gamma_{1,0}\bar{p}_1 + \gamma_{2,1}\bar{p}_2} \right). \quad (51)$$

Following a similar proof as for lemma 4.4, we have

Lemma 4.6: p_1^* in (P2-a-3) is decreasing with $R_{2,\text{tar}}$ when $R_{2,\text{tar}} \geq C_2$.

We know that p_1^* given by (50) needs to satisfy power constraint (21), i.e., $p_1^* \leq \bar{p}_1$. Thus, $R_{2,\text{tar}} \geq C_3$ where C_3 is the threshold point of $R_{2,\text{tar}}$ that satisfies

$$p_1^* = \bar{p}_1, \quad (52)$$

where p_1^* is given by (50).

(P3-a-4): The transmit power of source 2 is the maximal transmit budget, i.e., $p_2^* = \bar{p}_2$, and (24) does not hold with $p_1 = p_1^*$ and $p_2 = \bar{p}_2$.

According to lemma 4.5, $R_{2,\text{tar}} < C_2$. Then,

$$\log \left(1 + \frac{\gamma_{1,2}\bar{p}_1}{1 + \gamma_{2,0}\bar{p}_2} \right) < \log (1 + \gamma_{1,0}\bar{p}_1 + \gamma_{2,0}\bar{p}_2) - R_{2,\text{tar}}. \quad (53)$$

In similar with (P3-a-2), (18) holds with equality for the optimal solution of Problem P3-a. Consequently, $p_{1,2}^* = p_1^*$ and $p_{u,1}^* = 0$. Hence, p_1^* is the maximal p_1 , i.e.,

$$p_{1,2}^* = p_1^* = \frac{1}{\gamma_{1,1}} \left(\frac{\gamma_{2,1}\bar{p}_2}{2^{R_{2,\text{tar}}} - 1} - 1 \right). \quad (54)$$

R_1 is given as

$$R_1 = \log \left(1 + \frac{\gamma_{1,2}p_{1,2}^*}{\gamma_{2,0}\bar{p}_2 + 1} \right), \quad (55)$$

the corresponding power allocation scheme is given as $p_{2,1}^* = p_2^* = \bar{p}_2$.

It should be noted that p_1^* needs to satisfy $p_1^* \leq \bar{p}_1$, i.e.,

$$R_{2,\text{tar}} \geq C_0 = \log \left(1 + \frac{\gamma_{2,1}\bar{p}_2}{1 + \gamma_1\bar{p}_1} \right). \quad (56)$$

(P3-a-5): The transmit powers of the two sources are the maximal transmit budgets, i.e., $p_1^* = \bar{p}_1$ and $p_2^* = \bar{p}_2$, and (24) holds with $p_1 = \bar{p}_1$ and $p_2 = \bar{p}_2$.

According to lemma 4.2, (17)-(19) hold with equality for the optimal solution of Problem P3-a, when (24) holds with $p_1 = \bar{p}_1$ and $p_2 = \bar{p}_2$, i.e., $R_{2,\text{tar}} \geq C_2$. Similar to [12], the optimal R_1 is given as

$$R_1 = \log \left(1 + \frac{\gamma_{1,2}p_{1,2}^*}{1 + \gamma_2\bar{p}_2} \right). \quad (57)$$

The corresponding powers are given as $p_{2,1}^* = \Phi_{2,1}$, $p_{u,2}^* = p_2^* - p_{2,1}^*$,

$$p_{u,1}^* = \frac{2A_1B_1 + p_{u,2}^* - \sqrt{(2A_1B_1 + p_{u,2}^*)^2 - 4A_1^2B_1^2}}{2A_1^2}, \quad (58)$$

$p_{1,2}^* = p_1^* - p_{u,1}^*$, where

$$A_1 = \frac{\gamma_{1,2}}{2\sqrt{\gamma_{1,0}\gamma_{2,0}}(1 + \gamma_2\bar{p}_2)}, \quad (59)$$

$$B_1 = \frac{2^{R_{2,\text{tar}}} \left(1 + \frac{\gamma_{1,2}\bar{p}_1}{1 + \gamma_2\bar{p}_2} \right) - (1 + \gamma_{1,0}\bar{p}_1 + \gamma_{2,0}\bar{p}_2)}{2\sqrt{\gamma_{1,0}\gamma_{2,0}}}. \quad (60)$$

(P3-a-6): The transmit powers of the two sources are the maximal transmit budgets, i.e., $p_1^* = \bar{p}_1$ and $p_2^* = \bar{p}_2$, and (24) dose not hold with $p_1 = \bar{p}_1$ and $p_2 = \bar{p}_2$.

This case can be viewed as a special case of (P3-a-2) or (P3-a-4). So the corresponding optimal R_1 is smaller than the optimal R_1 obtained in (P3-a-2) or (P3-a-4). Thus, the case is not the optimal solution of Problem P3-a.

Step 3: One of (P3-a-1)~(P3-a-5) is proved next to be the optimal solution of Problem P3-a when $R_{2,\text{tar}}$ is in a given region.

To do that, the following lemma is first obtained:

Lemma 4.7: $C_1 \leq C_0 \leq C_3$.

Proof: If $C_1 > C_0$ is true, then

$$\begin{aligned} \log \left(1 + \frac{\gamma_{2,1}\bar{p}_2}{1 + \gamma_1p_1^{(1)}} \right) &\geq C_1 = \log \left(1 + \frac{\gamma_{2,1}p_{2,1}^{(1)}}{1 + \gamma_1p_1^{(1)}} \right) \\ &> C_0 = \log \left(1 + \frac{\gamma_{2,1}\bar{p}_2}{1 + \gamma_1\bar{p}_1} \right), \end{aligned} \quad (61)$$

where $p_{2,1}^{(1)}$ and $p_1^{(1)}$ are the optimal $p_{2,1}$ and p_1 given by (P3-a-1) when $R_{2,\text{tar}} = C_1$. Thus, $p_1^{(1)} < \bar{p}_1$ which contradicts the fact that $p_1^{(1)} = \bar{p}_1$. Thus, $C_1 \leq C_0$ is true. Similarly, $C_0 \leq C_3$ is true. ■

According to lemma 4.7 and the discussion in step 2), the optimal solution of Problem P3-a given by Table II can be verified case by case, i.e., we have:

Proposition 4.3: The optimal solution of Problem P3-a is given as in Table II.

Proof: See Appendix E.

Remark 4.7: The optimal R_1 in (P3-a-1) can be solved by a bisection search. For fixed optimal R_1 , (37) is a quadratic equation of $R_{2,\text{tar}}$ which can be solved in a closed form expression. Thus, by using the fact that the optimal R_1 is decreasing with $R_{2,\text{tar}}$, the optimal $R_1 = R_1'$ can be obtained for fixed $R_{2,\text{tar}} = R_2'$ by a bi-section search, as shown in Algorithm 2. Similarly, since the optimal p_2^* is increasing with $R_{2,\text{tar}}$ in (P3-a-1) and the optimal p_1^* is decreasing with $R_{2,\text{tar}}$ in (P3-a-3), C_1 and C_3 can also be obtained by a bi-section search. The method for obtaining C_1 and C_2 is given in Algorithm 3. Note that two layers of iteration is required to obtain C_1 .

Remark 4.8: The solutions in (P3-a-1)~(P3-a-5) have clear physical meanings:

- 1) (P3-a-1) and (P3-a-3) show a tradeoff of transmit power at one source when the transmit power of the other source is fixed. Since the power of RSI at each source is increasing with the transmit power of each source, too much transmit power may hinder the reception at the source. On the other hand, too little power may also decrease the transmission rate.
- 2) In (P3-a-5), since each source transmits signals using full power budgets, the equivalent cooperative channel $\gamma'_{k,\bar{k}}$ is fixed. Thus, the solution of (P3-a-5) is similar to the solution of the cooperation scheme with ideal FD sources [12].
- 3) In (P3-a-2) and (P3-a-4) where the two sources do not help with the transmission of each other, the transmit power of one source may not be its budget. This is because the two sources still need to decode the messages $w_{1,2}(i)$ and $w_{2,1}(i)$ of their counterparts in these two situations.

Algorithm 2 An algorithm for obtaining optimal R_1 in Case (P3-a-1)

- 1: **Initialization:** Set tolerance $\xi_2 > 0$, and the rates $R_{2,\text{tar}} = R_2'$, $R_1 = R_{1,0}$, $R_u = R_{u,0}$, and $R_l = R_{l,0}$.
- 2: **Repeat**
 - For fixed $R_1 = (R_l + R_u)/2$, view $f_1(2^{R_1}) = 0$ in (37) as a quadratic equation of $R_{2,\text{tar}}$, solve the equation $f_1(2^{R_1}) = 0$, and then choose bigger one $R_{2,1}$ of the positive solutions whose corresponding power allocation is feasible for Problem P3-a;
 - If $R_{2,1} > R_2'$, update $R_l = R_1$; otherwise, update $R_u = R_1$;
- 3: **Until** $R_u - R_l \leq \xi_2$.
- 4: **Output:** R_u as the approximate solution.

B. Solution of Problem P3-b

Following a similar proof as for Problem P3-a, the solution of Problem P3-b is given as follows:

(P3-b-1): The transmit power of source 2 is the maximal transmit budget, i.e., $p_2^* = \bar{p}_2$, and the two sources transmit the common messages, i.e., $p_{u,k}^* > 0$. R_1 is a solution of a

Algorithm 3 An algorithm for obtaining C_1 (or C_2)

- 1: **Initialization:** Set tolerance $\xi_3 > 0$, $C_u = C_{u,0}$, and $C_l = C_{l,0}$.
- 2: **Repeat**
 - For fixed $R_{2,\text{tar}} = (C_l + C_u)/2$, calculate the optimal R_1 by using Algorithm 2 (or by solving (45)), and then obtain p_2 (or p_1) according to (36) (or (50));
 - If $p_2 > \bar{p}_2$ (or $p_1 < \bar{p}_1$), update $C_u = R_{2,\text{tar}}$; otherwise, update $C_l = R_{2,\text{tar}}$;
- 3: **Until** $C_u - C_l \leq \xi_3$.
- 4: **Output:** C_u as the approximate solution.

quadratic equation i.e., $f_2(2^{R_1}) = 0$, with $\beta = \frac{1}{\gamma_{2,0}}$ and the other parameters are given in (46)-(49). The corresponding power allocation scheme is given in (50), $p_{2,1}^* = \Phi_{2,1}$, $p_{1,0}^* = \Phi_{1,0}$, $p_{u,1}^* = p_1^* - p_{1,0}^*$, and $p_{u,2}^* = p_2^* - p_{2,1}^*$. The above case is optimal under the condition of $R_{2,\text{tar}} \geq \max(C_4, C_5)$ where C_4 is the solution of $R_{2,\text{tar}}$ subject to

$$p_1^* = \bar{p}_1, \quad (62)$$

and p_1^* is given by (50), and C_5 is given by

$$C_5 = \log \left(\frac{1 + \gamma_{1,0}\bar{p}_1 + \gamma_{2,0}\bar{p}_2}{1 + \gamma_{1,0}\bar{p}_1} \right). \quad (63)$$

(P3-b-2): The transmit power of source 2 is the maximal transmit budget, i.e., $p_2^* = \bar{p}_2$, and the two sources do not transmit the common messages, i.e., $p_{u,k}^* = 0$. R_1 is given by

$$R_1 = \log \left(1 + \gamma_{1,0}p_{1,0}^* \right). \quad (64)$$

The corresponding power allocation scheme is given by $p_{2,1}^* = p_2^* = \bar{p}_2$ and

$$p_{1,0}^* = p_1^* = \frac{1}{\gamma_1} \left(\frac{\gamma_{2,1}\bar{p}_2}{2^{R_{2,\text{tar}}} - 1} - 1 \right). \quad (65)$$

The above case is optimal when $R_{2,\text{tar}} \leq C_5$.

(P3-b-3): The transmit powers of the two sources are the maximal transmit budgets, i.e., $p_1^* = \bar{p}_1$ and $p_2^* = \bar{p}_2$, and the sources also transmit the common messages, i.e., $p_{u,k}^* > 0$. R_1 is given by

$$R_1 = \log \left(1 + \gamma_{1,0}p_{1,0}^* \right). \quad (66)$$

The corresponding power allocation scheme is given by $p_{2,1}^* = \Phi_{2,1}$, $p_{u,2}^* = p_2^* - p_{2,1}^*$ and

$$p_{u,1}^* = \frac{2A_2B_2 + p_{u,2}^* - \sqrt{(2A_2B_2 + p_{u,2}^*)^2 - 4A_2^2B_2^2}}{2A_2^2}, \quad (67)$$

$p_{1,0}^* = \bar{p}_1 - p_{u,1}^*$, where $A_2 = \gamma_{1,0}/2\sqrt{\gamma_{1,0}\gamma_{2,0}}$,

$$B_2 = \frac{2^{R_{2,\text{tar}}} (1 + \gamma_{2,0}\bar{p}_2) - (1 + \gamma_{1,0}\bar{p}_1 + \gamma_{2,0}\bar{p}_2)}{2\sqrt{\gamma_{1,0}\gamma_{2,0}}}. \quad (68)$$

The above case is optimal under the condition $C_5 \leq R_1 \leq C_4$.

C. Solution of Problem P3-c

The solution of Problem P3-c can be obtained similarly as that of Problem P3-a, which is given as follows:

(P3-c-1): The optimal transmit power of source 1 is the maximal transmit budget, i.e., $p_1^* = \bar{p}_1$, and the two sources transmit the common messages, i.e., $p_{u,k}^* > 0$. The optimal R_1 is one solution of the sextic equation, i.e., $f_1(2^{R_1}) = 0$, where

$$c_1 = \gamma_{2,0}P_{2,0}^*\varepsilon_1 + \gamma_{1,0}(\bar{p}_1 + \beta_1)^2, \quad (69)$$

$$\beta_1 = \frac{(2^{R_1} - 1) (\gamma_{2,1}P_{2,0}^* - 1)}{\gamma_{1,2}}, \quad (70)$$

and the other parameters are given in (31)-(35). The corresponding power allocation scheme is given by (36), $p_{1,2}^* = \Phi_{1,2}$, $p_{2,0}^* = \Phi_{2,0}$, $p_{u,1}^* = p_1^* - p_{1,2}^*$, and $p_{u,2}^* = p_2^* - p_{2,0}^*$. The above case is optimal under the condition of $R_{2,\text{tar}} \leq C_6$ and $B \geq 0$, where C_6 is the solution of $R_{2,\text{tar}}$ subject to

$$p_2^* = \bar{p}_2, \quad (71)$$

and p_2^* is given by (36) with c_1 and β_1 given by (69) and (70), and B is given by

$$B = 2^{R_{2,\text{tar}}} \left(1 + \frac{\gamma_{1,2}\bar{p}_1}{1 + \gamma_{2,0}P_{2,0}^*} \right) - (1 + \gamma_{1,0}\bar{p}_1 + \gamma_{2,0}P_{2,0}^*). \quad (72)$$

(P3-c-2): The optimal transmit power of source 1 is the maximal transmit budget, i.e., $p_1^* = \bar{p}_1$, and the two sources do not transmit the common messages, i.e., $p_{u,k}^* = 0$. R_1 is given by (42), the corresponding power allocation scheme is given by (42), $p_1^* = p_{1,2}^* = \bar{p}_1$, and $p_2^* = p_{2,0}^* = \Phi_{2,0}$. This case is optimal under the condition $B < 0$.

(P3-c-3): The optimal transmit powers of the two sources are the maximal transmit budgets, i.e., $p_1^* = \bar{p}_1$ and $p_2^* = \bar{p}_2$, and the two sources transmit the common messages, i.e., $p_{u,k}^* > 0$. R_1 is given by (57). The corresponding power allocation scheme is given by (58), $p_{2,0}^* = \Phi_{2,0}$, $p_{u,2}^* = \bar{p}_2 - p_{2,0}^*$, and $p_{1,2}^* = \bar{p}_1 - p_{u,1}^*$. This case is optimal under the condition $R_1 \geq \max(C_2, C_6)$.

Remark 4.9: According to Proposition 4.2, we need to calculate the optimal/maximum R_1 of Problems P3-a to P3-d which as a function of $R_{2,\text{tar}}$ determines the achievable rate region of 1P-CMA. Note that the solutions of Problems P3-a, P3-b and P3-c listed in Table II correspond to Cases (P3-a-1)-(P3-a-5), (P3-b-1)-(P3-b-3) and (P3-c-1)-(P3-c-3) shown in Section IV in that order. To obtain the solution in Cases (P3-a-1), (P3-a-3), (P3-b-1), and (P3-c-1), the optimal R_1 needs to be obtained first by solving equations (such as $f_1(2^{R_1}) = 0$ as shown in (37)) for the corresponding cases. Besides, Algorithm 2 is used to solve similar equations for Cases (P3-a-1) and (P3-c-1). After that, the optimal power allocation can be derived according to Table II. The power allocation of the other cases can be obtained according to Table II directly.

Remark 4.10: For Problem P3-a, computing C_1 is relatively expensive due to two-layer iterations. But if one only needs to know the maximum R_1 for a given $R_{2,\text{tar}}$, there is no need to compute C_1 as shown next. First, we calculate C_0 , C_2 , C_3 and B . If $R_{2,\text{tar}}$ falls in the regions corresponding to Cases (P3-a-2)-(P3-a-4) shown in Table II, then the optimal powers can

be obtained directly in closed forms, or otherwise the optimal powers can only be obtained after the optimal value of R_1 is found. For Case (P3-a-1), the optimal R_1 needs to be searched. For Case (P3-a-5), the optimal R_1 has a closed form. If the optimal R_1 and power allocation given by Case (P3-a-1) is feasible for Problem P3-a, then the optimal R_1 of Problem P3-a is given by Case (P3-a-1), or otherwise the optimal R_1 is given by Case (P3-a-5).

On the other hand, if C_1 has been calculated, then there is a closed form relationship between the maximum R_1 and $R_{2,\text{tar}}$, which determines the edge of the rate region. For example, for Case (P3-a-1), the equation $f_1(2^{R_1}) = 0$ in (37) shows the closed form relationship. Similar situations hold for Cases (P3-a-2)-(P3-a-5).

For Problem P3-c, computing C_6 is also relatively expensive due to two-layer iterations. The role of C_6 for Problem P3-c is similar to that of C_1 for Problem P3-a. For Problem P3-b, C_4 must be computed by using Algorithm 3, which is however not as expensive as computing C_1 or C_6 . With C_4 available, the close form solution of the optimal R_1 versus $R_{2,\text{tar}}$ is available.

Remark 4.11: A bisection search of R_1 is used to obtain the optimal solution of Problem P3-a subject to $0 \leq R_{2,\text{tar}} \leq C_1$, and another bisection search of R_1 is used to obtain the optimal solution of Problem P3-c subject to $0 \leq R_{2,\text{tar}} \leq C_6$. The rest of the computations are in closed forms. The convergence rate of bisection search is linear [26], and for each iteration in Problems P3-a and P3-c a single-variable quadratic equation is solved. Denote the number of required iterations by I_3 . Then the complexity of the 1P-CMA is $O(I_3)$. Without knowing the relationship among I_1 , I_2 and I_3 , it is not possible to precisely compare the complexities of 1P-CMA and 3P-CMA. But simulation shows that the 1P-CMA algorithm is much less costly than the 3P-CMA algorithm and the TDD scheme in [8]. A comparison of computational times for 1P-CMA, 3P-CMA and TDD will be shown later.

V. NUMERICAL RESULTS

In this section, we compare the optimized rate regions obtained from our proposed algorithms with those from the conventional MAC and TDD schemes [8]. To compare our RSI-dependent schemes with the zero-RSI scheme in [12], we define a naive (RSI-dependent) 1P-CMA scheme to be the scheme in [12] with the cooperative channel power gain chosen to be $|h_{\bar{k},k}|^2/(1 + |h_{\bar{k}}|^2 \bar{p}_{\bar{k}})$ where $\bar{p}_1$ and $\bar{p}_2$ are the full powers at the sources. We set the maximal transmit power of the two sources at $\bar{p}_1 = \bar{p}_2 = 20\text{dB}$ and the powers of all CSCG noises at $N_0 = N_1 = N_2 = 0\text{dB}$. Due to the reciprocity of the channel between the sources, we set $h_{1,2} = h_{2,1}$. The convex hull is drawn according to the algorithm in [24]. When computing the achievable rates of the 3P-CMA scheme, the initial points of power allocation and lengths of phases are given in four different ways:

- 1) Constant point: the power allocation is given as: $p_{1,2,1} = \bar{p}_1$, $p_{2,1,2} = \bar{p}_2$, $p_{u,1,3} = \bar{p}_1$, $p_{u,2,3} = \bar{p}_2$, and the other powers are zeros. The lengths of phases are given as $[\alpha_1, \alpha_2, \alpha_3] = \frac{1}{3}[1, 1, 1]$.
- 2) Random point: For fixed $R_1 = 0$, we compute the optimal R_2 using a random initialization of power allocation and

(a) The achievable rate region for different $|h_{1,2}|^2$

(b) The achievable rate region for different $|h_1|^2$

Fig. 2: The achievable rate region of the 1P-CMA and 3P-CMA schemes

lengths of phases. Then for each R_2 , we compute the optimal R_1 using the optimal power allocation and lengths of phases from the previous step as the initialization.

- 3) Using the result of the TDD scheme as the initial point.
- 4) Using the result of 1P-CMA as the initial point.

It should be noted that each result of 3P-CMA is obtained by choosing the best of the solutions based on these four initializations.

Shown in Fig. 2(a) is a comparison of the achievable rate regions of different schemes under different values of $h_{1,2}$ but subject to $|h_1|^2 = |h_2|^2 = 0.01$ and $|h_{1,0}|^2 = |h_{2,0}|^2 = 0.01$. For $|h_{1,2}|^2 = 0.01$ (which is relatively small), the achievable rate regions of 1P-CMA and TDD scheme are almost the same as the conventional MAC. In other words, the solution of Problem P2 is now given by Problem P3-d which corresponds to the conventional MAC. Also note the rate region of the conventional MAC is invariant to $h_{1,2}$. For some larger values of $|h_{1,2}|^2$, none of the 1P-CMA and the TDD scheme has a consistent advantage over the other. But if $|h_{1,2}|^2 = 0.05$, for any fixed R_1 , R_2 of the 1P-CMA is almost 1 bit larger than that of the conventional MAC, and also surpasses that

Fig. 3: The achievable rate of the 1P-CMA, naive 1P-CMA, and 3P-CMA and TDD schemes when $R_{2,\text{tar}} = 0$.

of TDD scheme. The above suggests that for a sufficiently large $h_{1,2}$, the 1P-CMA has a larger rate region than the TDD scheme. Moreover, in all cases, the 3P-CMA scheme has a larger rate region than the 1P-CMA and TDD scheme. In Fig. 2(a), we only plot the rate region for the naive 1P-CMA with $|h_{1,2}|^2 = 0.0225$, since naive 1P-CMA and 1P-CMA have the same rate region when $|h_{1,2}|^2 = 0.01$ or $|h_{1,2}|^2 = 0.05$. We see that 1P-CMA outperforms naive 1P-CMA when R_1 is small or large.

Shown in Fig. 2(b) is a comparison of achievable rate regions under different values of the RSI power gain $|h_1|^2 = |h_2|^2$ but subject to $|h_{1,2}|^2 = 0.09$ and $|h_{1,0}|^2 = |h_{2,0}|^2 = 0.01$. We see that the 1P-CMA can have a larger rate region than the TDD scheme if the RSI power gain is not too high. Also note that the rate region of the TDD scheme is invariant to the RSI power gain. The rate region of the 3P-CMA subject to $|h_1|^2 = |h_2|^2 = 0.01$ or $|h_1|^2 = |h_2|^2 = 0.08$ is not shown on this figure. But these two regions are almost the same as that of 1P-CMA with $|h_1|^2 = |h_2|^2 = 0.01$ or that of the TDD scheme with $|h_1|^2 = |h_2|^2 = 0.08$, respectively. Here, for naive 1P-CMA, we only used $|h_1|^2 = 0.04$ and $|h_1|^2 = 0.08$, since the naive 1P-CMA and 1P-CMA schemes have the same rate region when $|h_1|^2 = 0.01$. We see again that 1P-CMA outperforms naive 1P-CMA when R_1 is small or large.

Shown in Fig. 3 is a comparison of achievable rates under different values of the RSI power gain $|h_1|^2 = |h_2|^2$ but subject to $|h_{1,2}|^2 = 0.03$, $|h_{1,0}|^2 = |h_{2,0}|^2 = 0.01$, and $R_1 = 0$. We choose $R_1 = 0$, because the 1P-CMA outperforms naive 1P-CMA when R_1 is small or large and the case $R_1 = 0$ is representative. We see that the 1P-CMA can have a larger rate than the TDD scheme if $|h_1|^2$ is not too large. Also note that the rate region of the TDD scheme is invariant to the RSI power gain. Besides, 1P-CMA has the same performance with naive 1P-CMA when $|h_1|^2$ is small and outperform naive 1P-CMA when $|h_1|^2$ is large. The 3P-CMA scheme outperforms TDD scheme and 1P-CMA scheme for all $|h_1|^2$ as discussed in Remark 2.1.

Shown in Fig. 4(a) is a ratio of the achievable rate of the

TDD (or HD) scheme over that of 3P-CMA under different values of the RSI power gain $|h_1|^2 = |h_2|^2$ with 1000 samples of channel realization, and shown in Fig. 4(b) are the ratios of the achievable rates of the 1P-CMA over those of 3P-CMA. The achievable rate is defined as the maximum of the smaller one of R_1 and R_2 , i.e., $\max_{\text{power-allocation}} \min(R_1, R_2)$. The $x\%$ base line of $R_{\text{HD}}/R_{3\text{-phases}}$ means that $x\%$ of the realizations are above this line. The channel power gains $|h_{1,0}|^2$, $|h_{2,0}|^2$, and $|h_{1,2}|^2$ obey the exponential distributions with the expected values 0.01, 0.01, and 0.09, respectively. It is obvious that the 1P-CMA has the same performance as the 3P-CMA when the power gain of RSI is low. The 3P-CMA tends to have the same performance with the TDD scheme when the power gain of RSI is large.

Shown in Fig. 4(c) is a ratio of the achievable rate of the 3P-CMA with $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{3}$ over that of 3P-CMA with optimal values of α_1 , α_2 and α_3 . The other settings are the same with Fig. 4(a). We see that 3P-CMA with constant α_i has the same performance as 3P-CMA with optimal α_i when the power gain of RSI is low.

Shown in Fig. 4(d) is a cumulative distribution function of the ratio of the computing time of the 1P-CMA over that of the 3P-CMA, and the ratio of the computing time of the 1P-CMA over that of the TDD scheme, which were based on the same computations for Fig. 4(a) and Fig. 4(b). We see that the 1P-CMA takes about 0.2% time of the 3P-CMA and about 0.3% time of the TDD scheme on average.

VI. CONCLUSION

This paper has studied a full-duplex (FD) decode-forward (DF) source cooperative multiple access (CMA) scheme where two FD sources simultaneously send their own signals to the destination and help each other by a partial DF protocol enhanced with backward decoding at the destination. To characterize the achievable rate region of the scheme, we first considered a three-phase based CMA called 3P-CMA and presented a simple algorithm for 3P-CMA optimal power allocation. While this algorithm is simple, its computational complexity is high. We then considered a single-phase based CMA called 1P-CMA. While similar to some prior CMA schemes, 1P-CMA imposed a new challenge due to the residual self-interference (RSI) of non-ideal full-duplex cooperative sources. A large part of this paper is devoted to deriving a closed-form solution of the 1P-CMA optimal power allocation. Although tedious in details, the 1P-CMA algorithm is not only optimal for the one-phase case but also highly efficient in computational time. Simulation results have been provided to show the performance differences in achievable rate regions between 3P-CMA and 1P-CMA, and between 3P-CMA and a prior state-of-art scheme called time-division-duplex (TDD) or half-duplex (HD) scheme. The performance advantage of 3P-CMA (with optimal power allocation) over TDD is significant especially for low and medium RSI power gains. At the same time, for low and medium RSI power gains, 1P-CMA and 3P-CMA have comparable performances. This makes 1P-CMA a good choice in practice due to its efficient power allocation algorithm derived in this paper. Furthermore, the

Fig. 4: The comparison of 1P-CMA, 3P-CMA, and the TDD scheme over achievable rate and computing time

computational time for 1P-CMA is shown via simulation to be a small fraction of that for 3P-CMA and that for TDD.

APPENDIX A PROOF OF PROPOSITION 2.1

Due to the limited space, we present a brief proof for Proposition 2.1 here. In block i , the signals $x_{k,0,j}(i)$, $x_{k,\bar{k},j}(i)$ and $u_{k,j}(i)$ in (1) are given as

$$x_{k,0,j}(i) = \tilde{x}_{k,0,j}(w_{k,0,j}(i), w'_{1,2,j}(i-1), w'_{2,1,j}(i-1)), \quad (73)$$

$$x_{k,\bar{k},j}(i) = \tilde{x}_{k,\bar{k},j}(w_{k,\bar{k},j}(i), w'_{1,2,j}(i-1), w'_{2,1,j}(i-1)), \quad (74)$$

$$u_{k,j}(i) = \tilde{u}_{k,j}(w'_{1,2,j}(i-1), w'_{2,1,j}(i-1)), \quad (75)$$

where $\tilde{x}_{k,0,j}$, $\tilde{x}_{k,\bar{k},j}$ and $\tilde{u}_{k,j}$ are the codebooks generated from the independent and identically distributed Gaussian distribution $\mathcal{CN}(0, 1)$. Note that the signals depend on the messages $(w_{1,0,j}(i), w_{2,0,j}(i), w_{1,2,j}(i), w_{2,1,j}(i))$ of the current block and the messages $(w'_{1,2,j}(i-1), w'_{2,1,j}(i-1))$ of the previous

block. Also note that $w_{k,\bar{k},j}(i)$ for all j constitute the same message $w_{k,\bar{k}}(i)$ that $w'_{k,\bar{k},j}(i)$ for all j constitute.

First, we consider the decode at the source at block i . According to (1) and (2), $r_{k,j}(i)$ includes the signal $x_{\bar{k},k,j}(i)$, which is encoded from message $w_{\bar{k},k,j}(i)$. With the messages $w'_{1,2,j}(i-1)$ and $w'_{2,1,j}(i-1)$ and the signal $u_{\bar{k},j}(i-1)$ known by source k according to the description in Section II, source k decodes the message $w_{\bar{k},k,j}(i)$ of node $\bar{k}$ from the signal $r_{k,j}(i)$ with the interferences due to the signal $x_{\bar{k},0}(i)$ and RSI $h_{\bar{k}}\tilde{r}_k(i)$. Thus, to ensure the perfect reconstruction of the message $w_{\bar{k},k,j}(i)$, the rate $R_{\bar{k},k,j}$ of phase j is bounded by

$$R_{\bar{k},k,j} \leq \log \left(1 + \frac{\gamma_{k,\bar{k}} P_{k,\bar{k},j}}{\gamma_{\bar{k}} P_{\bar{k},j} + \gamma_{k,\bar{k}} P_{k,0,j} + 1} \right). \quad (76)$$

By using Fourier-Motzkin Elimination, the constraints (76) for $j = 1, 2, 3$ can be proven to be equivalent to (5).

Then, following the Appendix in [5], the perfect reconstruction of $w_{k,0,j}(i)$ and $w'_{k,\bar{k},j}(i)$ for each j and i can

be done at the destination where the backward decoding is adopted. Note that there is no new message transmitted in the last block. Since signal $r_{0,j}(i)$ includes the messages $(w_{1,0,j}(i), w_{2,0,j}(i), w_{1,2,j}(i), w_{2,1,j}(i), w'_{1,2,j}(i-1), w'_{2,1,j}(i-1))$ and the messages $w_{1,2,j}(i)$ and $w_{2,1,j}(i)$ were obtained in the previous step in the backward order, the destination can decode $(w_{1,0,j}(i), w_{2,0,j}(i), w'_{1,2,j}(i-1), w'_{2,1,j}(i-1))$ from the signal $r_{0,j}(i)$. After $w'_{1,2,j}(i-1)$ and $w'_{2,1,j}(i-1)$ ($j = 1, 2, 3$) are obtained, the messages $w_{1,2}(i-1)$ and $w_{2,1}(i-1)$ are known, and so are the messages $w_{1,2,j}(i-1)$ and $w_{2,1,j}(i-1)$. The process repeats in the backward order. Therefore, the decoding at each phase at the destination is almost same as that described in the Appendix in [5] except the re-division of the messages $w_{1,2}(i)$ and $w_{2,1}(i)$ over the three phases. So the sufficient conditions for the perfect reconstruction of $w_{k,0,j}(i)$ and $w'_{k,\bar{k},j}(i)$ are given by (6)-(8).

APPENDIX B PROOF OF PROPOSITION 4.1

Problem P2 of maximizing R_1 for any given $R_2 = R_{2,\text{tar}}$ includes a special case that maximizes the RHS of (13), (14), and (15) which correspond to cases 1)-3). For any given p_k , we can lump it with $\gamma_{k,\bar{k}}$ to form

$$\gamma'_{k,\bar{k}} = \frac{\gamma_{k,\bar{k}}}{1 + \gamma_{k,\bar{k}} p_{\bar{k}}}, \quad (77)$$

which makes the Problem P2 virtually the same as that in the Proposition 1 of [11] in terms of maximizing (15). Hence, the statements of cases 1)-3) in Proposition 4.1 hold for maximizing the right hand side of (15).

(13) is rewritten as

$$R_1 \leq \log \left(1 + \frac{\gamma_{1,0} p_{1,0}}{1 + \gamma'_{1,2} p_{1,0}} \right) + \log \left(1 + \gamma'_{1,2} (p_{1,0} + p_{1,2}) \right). \quad (78)$$

For fixed p_k and $p_{u,k}$, the second term on the RHS of (78) is fixed, and (78) only depends on $p_{1,0}$. The RHS of (13) reaches its maximum as follows in two cases: 1) if $\gamma_{1,0} < \gamma'_{1,2}$, then $p_{1,0}^* = 0$; 2) if $\gamma_{1,0} \geq \gamma'_{1,2}$, then $p_{1,0}^* = p_1 - p_{u,1}$, i.e., $p_{1,2}^* = 0$. Namely, the statements of cases 1)-3) in Proposition 4.1 hold for maximizing the RHS of (13). Similarly, the statements of cases 1)-3) in Proposition 4.1 hold for maximizing the RHS of (14). According to the analysis above, the RHS of constraints (13), (14), and (15) can be maximized at the same time in cases 1)-3). Thus, cases 1)-3) are proved.

In the following, case 4) is proved. Since R_1 is only bounded by (13) and (15), to maximize R_1 , one of (13) and (15) holds with equality for the optimal solution of Problem P2. Besides, (14) may also hold with equality for the optimal solution of Problem P2. If (15) holds with equality and (13) and (14) hold with inequality, then the RHS of (15) reaches its maximum for the optimal solution of Problem P2. According to the Proposition 1 of [11], one of the three conditions in case 4) holds for maximizing the RHS of (15). Thus, case 4) is true.

If (14) hold with equality for the optimal solution of Problem P2 when $\gamma_{1,0} > \gamma'_{1,2}$ and $\gamma_{2,0} > \gamma'_{2,1}$, then R_1 is

bounded by

$$\begin{aligned} R_1 &= R_1 + R_{2,\text{tar}} - R_{2,\text{tar}} \\ &\leq \log \left(1 + \frac{\gamma'_{1,2} p_{1,2}}{\gamma'_{1,2} p_{1,0} + 1} \right) + \log \left(1 + \frac{\gamma_{1,0} p_{1,0}}{\gamma_{2,0} p_{2,0} + 1} \right) \\ &= \log \left(\frac{\gamma_{1,0} p_{1,0} + \gamma_{2,0} p_{2,0} + 1}{\gamma'_{1,2} p_{1,0} + 1} \right) \\ &\quad + \log \left(\frac{\gamma'_{1,2} (p_{1,0} + p_{1,2}) + 1}{\gamma_{2,0} p_{2,0} + 1} \right), \end{aligned} \quad (79)$$

where the inequality is obtained by substituting (14) and (15) into the first equality. According to [12], if $p_{2,0}^* \geq (\gamma_{1,0}/\gamma'_{1,2} - 1)/\gamma_{2,0}$, then $p_{1,0}^* = p_{2,1}^* = 0$ or $p_{1,2}^* = p_{2,1}^* = 0$; if $p_{2,0}^* < (\gamma_{1,0}/\gamma'_{1,2} - 1)/\gamma_{2,0}$, the constraint (13) in Problem P2 can be rewritten as

$$R_1 \leq \log \left(1 + \frac{\gamma_{1,0} p_{1,0}}{\gamma_{2,0} p_{2,0} + 1} \right), \quad (80)$$

and the convex hull obtained from Problem P2 with constraint (80) is contained by the convex hull of the rate region obtained from Problem P2 with (13) holding with equality.

Similar to the proof when (14) holds with equality, if (13) holds with equality for the optimal solution of Problem P2, then $p_{2,0}^* = p_{1,2}^* = 0$ or $p_{1,2}^* = p_{2,1}^* = 0$. In conclusion, case 4) holds under every possible condition.

APPENDIX C PROOF OF LEMMA 4.3

Since the RHS of (24) is decreasing with p_2 for fixed $p_1 = \bar{p}_1$ and $p_2^* \geq p_{2,1}^*$, (24) holds with $p_1 = \bar{p}_1$ and $p_2 = p_{2,1}^*$ if (24) holds with $p_1 = \bar{p}_1$ and $p_2 = p_2^*$. In the following, (24) is proved (by contradiction) to hold with $p_1 = \bar{p}_1$ and $p_2 = p_2^*$ when (24) holds with $p_1 = \bar{p}_1$ and $p_2 = p_{2,1}^*$. If (24) does not hold when $p_1 = \bar{p}_1$ and $p_2 = p_2^*$, then

$$\log \left(1 + \frac{\gamma_{1,2} \bar{p}_1}{1 + \gamma_2 p_2^*} \right) < \log (1 + \gamma_{1,0} \bar{p}_1 + \gamma_{2,0} p_2^*) - R_{2,\text{tar}}. \quad (81)$$

Since R_1 is only bounded by (17) and (18), at least one of (17) and (18) holds with equality for the optimal solution of Problem P3-a. Moreover, according to (81), (18) holds with equality for the optimal solution of Problem P3-a. Thus, to obtain the optimal R_1 , $p_{1,2}^* = \bar{p}_1$. Since (24) holds with $p_1 = \bar{p}_1$ and $p_2 = p_{2,1}^*$ and (24) does not hold with $p_1 = \bar{p}_1$ and $p_2 = p_2^*$, then $p_2^* > p_{2,1}^*$. According to (18), R_1 can be increased by decreasing p_2 . Thus, R_1 with $p_1 = \bar{p}_1$ and $p_2 = p_{2,1}^*$ is bigger than the optimal R_1 . This contradicts the property of the optimal p_2 (i.e., p_2^*). And (24) must hold with $p_1 = \bar{p}_1$ and $p_2 = p_2^*$.

APPENDIX D PROOF OF LEMMA 4.4

This lemma is proved by contradiction. According to $p_{2,1}^* = \Phi_{2,1}$, $p_{2,1}^* = \Phi_{2,1}$ is increasing with $R_{2,\text{tar}}$ in this case. If $p_2^* =$

non-increasing with $R_{2,\text{tar}}$, then $p_{u,2}^*$ is decreasing with $R_{2,\text{tar}}$ by using (20). Substituting $p_{u,k}^* = \bar{p}_k - \Phi_{k,\bar{k}}$ into (29) yields

$$\sqrt{\gamma_{1,0}} \frac{p_{u,1}^*}{p_{u,2}^*} + \sqrt{\frac{p_{u,1}^*}{p_{u,2}^*}} - \frac{\sqrt{\gamma_{1,0}\gamma_2}}{\gamma_{1,2}} (2^{R_1} - 1) = 0, \quad (82)$$

which is a quadratic equation of $\sqrt{p_{u,1}^*/p_{u,2}^*}$. Besides, the optimal R_1 is non-increasing with $R_{2,\text{tar}}$, otherwise, a value of R_1 , which is bigger than the optimal R_1 , can be achievable. Hence, the quadratic function on the LHS of (82) is shifted up in cartesian coordinates as $R_{2,\text{tar}}$ increases. Therefore, the positive solution of (82) is decreasing with $R_{2,\text{tar}}$. Consequently, $p_{u,1}^*$ is decreasing with $R_{2,\text{tar}}$. Thus, $p_{u,2}^*$ is increasing with $R_{2,\text{tar}}$. By using (18), the optimal R_1 is increasing with $R_{2,\text{tar}}$, which contracts with the facts that optimal R_1 is non-increasing with $R_{2,\text{tar}}$. Thus, p_2^* must increase with $R_{2,\text{tar}}$.

APPENDIX E PROOF OF PROPOSITION 4.2

Since the solutions given by (P3-a-1)~(P3-a-5) are all the possible optimal solutions according to the discussion in step 2), we only need to prove that one solution is better than the other solutions.

If $R_{2,\text{tar}} \leq C_1$ and $B \geq 0$, then the solution given in (P3-a-1) can be proved to be the optimal solution of Problem P3-a. First, according to lemma 4.3, (24) holds with $p_1 = \bar{p}_1$ and $p_2 = p_2^*$ when $B \geq 0$. According to lemma 4.2, the optimal R_1 obtained by (P3-a-1), where all (17)-(19) hold with equality, is better than that of (P3-a-2). Second, by using lemma 4.7, $R_{2,\text{tar}} \leq C_1 \leq C_0 \leq C_3$. According to the definition of C_3 , the optimal p_1 given by (P3-a-3)-(P3-a-4) surpasses the power budget $\bar{p}_1$. Finally, since $p_1^* < \bar{p}_1$ according to the definition of C_1 , $p_1 = \bar{p}_1$ is suboptimal which means that R_1 given by (P3-a-1) is larger than R_1 given by (P3-a-5).

If $R_{2,\text{tar}} \leq C_0$ and $B < 0$, then the solution given in (P3-a-2) can be proved to be the optimal solution of Problem P3-a. First, according to the discussion in (P3-a-1), (P3-a-1) does not have a rational solution. Second, since $R_{2,\text{tar}} \leq C_0 \leq C_3$, the optimal p_1 given by (P3-a-3)-(P3-a-4) surpasses the power budget $\bar{p}_1$. Finally, since the RHS of (24) is decreasing with p_2 , if $B < 0$ then it follows that (24) does not hold with $p_1 = \bar{p}_1$ and $p_2 = \bar{p}_2$. Thus, according to lemma 4.2, (P3-a-5) is not the optimal value of Problem P3-a.

A similar proof as (P3-a-1) and (P3-a-2) can be adopted for (P3-a-3) and (P3-a-4).

If $\max(C_1, C_2) \leq R_{2,\text{tar}} \leq C_3$, then the solution given in (P3-a-5) can be proved to be the optimal solution of Problem P3-a. Since $R_{2,\text{tar}} \geq C_1$, p_2 given by (P3-a-1) surpasses the power budget $\bar{p}_2$. Since $R_{2,\text{tar}} \leq C_3$, p_1 given by (P3-a-3) surpasses the power budget $\bar{p}_1$. Besides, since $p_{u,1}^* = p_{u,2}^* = 0$ in (P3-a-2) and (P3-a-4), by using inequality (17),

$$R_1^{(\text{II})} + R_{2,\text{tar}} \leq \log(1 + \gamma_{1,0}\bar{p}_1 + \gamma_{2,0}\bar{p}_2) \leq R_1^{(\text{V})} + R_{2,\text{tar}}, \quad (83)$$

where $R_1^{(\text{II})}$ is the rate obtained by (P3-a-2) or (P3-a-4), and $R_1^{(\text{V})}$ is the rate obtained by (P3-a-5). Thus, the solution given in (P3-a-5) is the solution of Problem P3-a.

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