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Authors

Swartz, M
Abrams, GS
Adolphsen, CE
[et al.](#)

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Search for Doubly Charged Higgs Scalars in Z Decay

M. Swartz,⁽⁴⁾ G. S. Abrams,⁽¹⁾ C. E. Adolphsen,⁽²⁾ D. Averill,⁽³⁾ J. Ballam,⁽⁴⁾ B. C. Barish,⁽⁵⁾ T. Barklow,⁽⁴⁾ B. A. Barnett,⁽⁶⁾ J. Bartelt,⁽⁴⁾ S. Bethke,⁽¹⁾ D. Blockus,⁽³⁾ G. Bonvicini,⁽⁷⁾ A. Boyarski,⁽⁴⁾ B. Brabson,⁽³⁾ A. Breakstone,⁽⁸⁾ F. Bulos,⁽⁴⁾ P. R. Burchat,⁽²⁾ D. L. Burke,⁽⁴⁾ R. J. Cence,⁽⁸⁾ J. Chapman,⁽⁷⁾ M. Chmeissani,⁽⁷⁾ D. Cords,⁽⁴⁾ D. P. Coupal,⁽⁴⁾ P. Dauncey,⁽⁶⁾ H. C. DeStaebler,⁽⁴⁾ D. E. Dorfan,⁽²⁾ J. M. Dorfan,⁽⁴⁾ D. C. Drewer,⁽⁶⁾ R. Elia,⁽⁴⁾ G. J. Feldman,⁽⁴⁾ D. Fernandes,⁽⁴⁾ R. C. Field,⁽⁴⁾ W. T. Ford,⁽⁹⁾ C. Fordham,⁽⁴⁾ R. Frey,⁽⁷⁾ D. Fujino,⁽⁴⁾ K. K. Gan,⁽⁴⁾ C. Gatto,⁽²⁾ E. Gero,⁽⁷⁾ G. Gidal,⁽¹⁾ T. Glanzman,⁽⁴⁾ G. Goldhaber,⁽¹⁾ J. J. Gomez Cadenas,⁽²⁾ G. Gratta,⁽²⁾ G. Grindhammer,⁽⁴⁾ P. Grosse-Wiesmann,⁽⁴⁾ G. Hanson,⁽⁴⁾ R. Harr,⁽¹⁾ B. Harral,⁽⁶⁾ F. A. Harris,⁽⁸⁾ C. M. Hawkes,⁽⁵⁾ K. Hayes,⁽⁴⁾ C. Hearty,⁽¹⁾ C. A. Heusch,⁽²⁾ M. D. Hildreth,⁽⁴⁾ T. Himel,⁽⁴⁾ D. A. Hinshaw,⁽⁹⁾ S. J. Hong,⁽⁷⁾ D. Hutchinson,⁽⁴⁾ J. Hylen,⁽⁶⁾ W. R. Innes,⁽⁴⁾ R. G. Jacobsen,⁽⁴⁾ J. A. Jaros,⁽⁴⁾ C. K. Jung,⁽⁴⁾ J. A. Kadyk,⁽¹⁾ J. Kent,⁽²⁾ M. King,⁽²⁾ S. R. Klein,⁽⁴⁾ D. S. Koetke,⁽⁴⁾ S. Komamiya,⁽⁴⁾ W. Koska,⁽⁷⁾ L. A. Kowalski,⁽⁴⁾ W. Kozanecki,⁽⁴⁾ J. F. Kral,⁽¹⁾ M. Kuhlen,⁽⁵⁾ L. Labarga,⁽²⁾ A. J. Lankford,⁽⁴⁾ R. R. Larsen,⁽⁴⁾ F. Le Diberder,⁽⁴⁾ M. E. Levi,⁽¹⁾ A. M. Litke,⁽²⁾ X. C. Lou,⁽³⁾ V. Lüth,⁽⁴⁾ J. A. McKenna,⁽⁵⁾ J. A. J. Matthews,⁽⁶⁾ T. Mattison,⁽⁴⁾ B. D. Milliken,⁽⁵⁾ K. C. Moffeit,⁽⁴⁾ C. T. Munger,⁽⁴⁾ W. N. Murray,⁽³⁾ J. Nash,⁽⁴⁾ H. Ogren,⁽³⁾ K. F. O'Shaughnessy,⁽⁴⁾ S. I. Parker,⁽⁸⁾ C. Peck,⁽⁵⁾ M. L. Perl,⁽⁴⁾ F. Perrier,⁽⁴⁾ M. Petradza,⁽⁷⁾ R. Pitthan,⁽⁴⁾ F. C. Porter,⁽⁵⁾ P. Rankin,⁽⁹⁾ K. Riles,⁽⁴⁾ F. R. Rouse,⁽⁴⁾ D. R. Rust,⁽³⁾ H. F. W. Sadrozinski,⁽²⁾ M. W. Schaad,⁽¹⁾ B. A. Schumm,⁽¹⁾ A. Seiden,⁽²⁾ J. G. Smith,⁽⁹⁾ A. Snyder,⁽³⁾ E. Soderstrom,⁽⁵⁾ D. P. Stoker,⁽⁶⁾ R. Stroynowski,⁽⁵⁾ R. Thun,⁽⁷⁾ G. H. Trilling,⁽¹⁾ R. Van Kooten,⁽⁴⁾ P. Voruganti,⁽⁴⁾ S. R. Wagner,⁽⁹⁾ S. Watson,⁽²⁾ P. Weber,⁽⁹⁾ A. Weigend,⁽⁴⁾ A. J. Weinstein,⁽⁵⁾ A. J. Weir,⁽⁵⁾ E. Wicklund,⁽⁵⁾ M. Woods,⁽⁴⁾ D. Y. Wu,⁽⁵⁾ M. Yurko,⁽³⁾ C. Zaccardelli,⁽²⁾ and C. von Zanthier⁽²⁾

⁽¹⁾Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720

⁽²⁾University of California, Santa Cruz, California 95064

⁽³⁾Indiana University, Bloomington, Indiana 47405

⁽⁴⁾Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

⁽⁵⁾California Institute of Technology, Pasadena, California 91125

⁽⁶⁾Johns Hopkins University, Baltimore, Maryland 21218

⁽⁷⁾University of Michigan, Ann Arbor, Michigan 48109

⁽⁸⁾University of Hawaii, Honolulu, Hawaii 96822

⁽⁹⁾University of Colorado, Boulder, Colorado 80309

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We describe a search for the decay of the Z boson into pairs of doubly charged Higgs bosons with the Mark II detector operating at the SLAC Linear Collider. Each Higgs boson is assumed to decay into a same-sign lepton pair. No event candidates are found in a sample of 528 Z decays. At the 95% confidence level, this result excludes the region of leptonic coupling $g_{\ell\ell} > 3 \times 10^{-7}$ and Higgs-boson mass $6.5 < M_H < 36.5 \text{ GeV}/c^2$ for isotriplet (left-handed) Higgs bosons. Isosinglet (right-handed) Higgs bosons are excluded in the same $g_{\ell\ell}$ interval and in the mass interval $7.3 < M_H < 34.3 \text{ GeV}/c^2$.

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This Letter presents the result of a search for the decay of the Z boson into a pair of doubly charged Higgs bosons. Doubly charged Higgs bosons are a feature of many theories that extend the standard model to include left-handed Majorana mass terms for the neutrino sector. If the Majorana mass terms are generated by the vacuum expectation value of a neutral Higgs field, the theory must also include singly and doubly charged Higgs states. The members of the Higgs triplet couple only to lepton pairs and to each other via gauge bosons.

The simplest theory of this type, the Gelmini-Roncadelli model,¹ contains a single doubly charged state that couples only to left-handed charged-lepton pairs. The third component of (left-handed) weak iso-

spin for this state has unit value ($I_3^L = 1$). A slightly more complicated example is the left-right-symmetric model² which contains a doubly charged Higgs state that couples to left-handed charged-lepton pairs (with $I_3^L = 1$) and a doubly charged state that couples to right-handed charged-lepton pairs (with $I_3^L = 0$).

Most of the existing limits on doubly charged Higgs bosons³⁻⁵ are functions of the coupling strength of the Higgs boson to lepton pairs ($g_{\ell\ell}$) and of the Higgs-boson mass (M_H). The only limit that extends to small values of $g_{\ell\ell}$ is rather weak,⁵ $M_H > 14 \text{ GeV}/c^2$ with 90% confidence. We therefore seek to improve the small-coupling limit by searching for the process $e^+e^- \rightarrow Z \rightarrow H^{++}H^{--} \rightarrow \ell^+\ell^+l^-l^-$, where ℓ and l may or may

not be the same species of charged lepton.⁶

The production cross section for doubly charged Higgs scalars at the Z pole is moderately large. The partial width for the decay $Z \rightarrow H^{++}H^{--}$ (Γ_{HH}) is given by the tree-level expression^{7,8}

$$\Gamma_{HH} = \frac{G_F M_Z^3}{6\pi\sqrt{2}} (I_3^L - 2\sin^2\theta_W)^2 \beta^3, \quad (1)$$

where G_F is the Fermi coupling constant, M_Z is the mass of the Z boson,⁹ $\sin^2\theta_W$ is the electroweak mixing parameter,⁹ and β is the Higgs-boson velocity in the e^+e^- center-of-mass frame. In the limit $\beta \rightarrow 1$, the rate of left-handed Higgs-boson ($I_3^L = 1$) production is approximately 14% larger than that for a charged-lepton species and the rate of right-handed Higgs-boson ($I_3^L = 0$) production is about 14% smaller than the charged-lepton rate.

A doubly charged Higgs boson can decay into a same-sign lepton pair or into a W boson and a singly charged Higgs boson. We assume that the former decay mode is dominant. The Higgs-boson lifetime (τ_H) is then given by the expression^{5,7}

$$\tau_H^{-1} = \sum_{\ell} \frac{g_{\ell}^2}{8\pi} M_H \left(1 - \frac{2m_{\ell}^2}{M_H^2} \right) \left(1 - \frac{4m_{\ell}^2}{M_H^2} \right)^{1/2}, \quad (2)$$

where m_{ℓ} is the lepton mass. The Higgs bosons are therefore short lived ($\lesssim 10^{-12}$ sec) unless the coupling constants g_{ℓ} are very small.

The search that is described here makes use of a data sample that was collected with the Mark II detector at the SLAC Linear Collider (SLC). The sample corresponds to a total integrated luminosity of 19.7 nb^{-1} taken at nine different center-of-mass energies near the Z pole. The sample includes 455 events that are identified as hadronic final states.¹⁰ Incorporating Eq. (1) into a complete calculation of the Higgs-boson pair production rate (including the effects of initial-state radiation), we predict that the Mark II data would include approximately 22 left-handed (or 16 right-handed) Higgs-boson pairs if the Higgs-boson mass is $15 \text{ GeV}/c^2$ (near the current limit on M_H).

A detailed description of the Mark II detector can be found in Ref. 11. Most of the information used in this analysis is provided by the 72-layer drift chamber. The chamber is immersed in a 4.75-kG magnetic field. Isolated charged-particle tracks are reconstructed with high efficiency ($\sim 99\%$) in the region of polar angle $|\cos\theta| < 0.80$. In the forward regions, $0.80 < |\cos\theta| < 0.92$, the reconstruction efficiency decreases from 99% to approximately 80%. The detector is also instrumented with an electromagnetic calorimeter that consists of three segments. The barrel section detects electrons and photons in the region $|\cos\theta| < 0.72$. The two end-cap calorimeters extend the region of angular coverage to $|\cos\theta| < 0.96$. The detector is triggered by two or more

charged tracks in the region of polar angle $|\cos\theta| < 0.76$, or by the localized deposition of at least 3.3 GeV of energy in the barrel calorimeter or 2.2 GeV in one of the end-cap calorimeters.

Of the six possible four-lepton final states, the most difficult to detect is the one consisting of four τ leptons. The strategy of this analysis is to define a set of topological selection criteria that can identify the four- τ final state with high efficiency. It is clear that such criteria select four-lepton final states that contain two or more stable leptons with comparable or larger efficiency.

Since 90% of all four- τ events decay into six or fewer charged particles, we consider only those event candidates that contain six or fewer charged tracks that project into a cylindrical volume of 2 cm radius and 6 cm length that is centered on the interaction point of the SLC. In order to suppress two-photon events and badly accepted hadronic final states, we require that the scalar sum of the track momenta be at least $10 \text{ GeV}/c$.

Isolated τ leptons appear as isolated single tracks or as low-mass clusters of tracks. The four-vectors of the charged tracks are thus subjected to a mass-based clustering algorithm. Initially, each track is defined to be a cluster. The pair of clusters with the smallest invariant mass is merged if its mass is less than $2.0 \text{ GeV}/c^2$. The procedure is repeated until all pairs of clusters have invariant masses larger than $2.0 \text{ GeV}/c^2$.

We expect most $H^{++}H^{--}$ events to appear as four-cluster events. There is a reasonable probability, however, that a cluster occurs in one of the forward regions, $|\cos\theta| > 0.80$, and is not detected (approximately 30% of all events fall into this category). We therefore require that each event candidate contain either three or four clusters of energy larger than 1.0 GeV. The net charge of each cluster must be unity. The event must not contain any clusters with charges larger than unity. The net event charge must be zero for four-cluster candidates or unity for three-cluster candidates.

There are no event candidates in the Mark II data sample that satisfy the selection criteria. The number of background events that are expected to satisfy the selection criteria is estimated from several Monte Carlo simulations. We estimate the contributions from two-photon processes,¹² radiative lepton-pair production,¹³ and low-multiplicity hadronic Z decays¹⁴ to be 0.01, 0.11, and 0.52 event, respectively.

In order to interpret this result, we have performed a Monte Carlo simulation of doubly-charged-Higgs-boson production and decay for the Mark II detector. The effect of initial-state bremsstrahlung is simulated using the structure-function approach of Nicosini and Trentadue.¹⁵ Each Higgs boson is allowed to decay isotropically into a same-sign lepton pair or into $\ell\gamma$ according to an approximate distribution.¹⁶ The Monte Carlo program simulates the displacement of the decay vertices due to finite Higgs-boson lifetimes. The τ simulation includes

final-state spin effects for the dominant single-prong decay modes ($\mathcal{L}\nu\bar{\nu}$, $\rho\nu$, and $\pi\nu$ final states). In order to simulate the effect of machine-related backgrounds upon the event reconstruction, the raw data from the Monte Carlo simulation are mixed with real raw data from random triggers of the apparatus. The mixed data are then subjected to trigger emulation and to the complete Mark II event-reconstruction program.

Using the Monte Carlo simulation, we find that the efficiency of the detector and selection criteria (ϵ_H) for τ final states is insensitive to the τ chirality.¹⁷ The overall efficiency is therefore independent of I_3^L . We also find that the acceptance for muon final states is smaller than that for electron final states by a few percent. This difference is due entirely to the presence of the neutral-energy trigger which detects electrons but not muons. To simplify the analysis, we choose to ignore this difference and to use the (more conservative) muon acceptance for both species of stable lepton.

The efficiency function ϵ_H therefore depends upon the τ -pair branching ratio of the Higgs boson (B_τ), the Higgs-boson mass, and the Higgs-boson lifetime. The efficiency increases steeply with increasing M_H at low masses and becomes independent of mass in the region $M_H > 30$ GeV/ c^2 . The function ϵ_H does not vary with the Higgs-boson lifetime in the region $\tau_H < 30$ ps and decreases slowly with increasing τ_H . In the high-mass, short-lifetime region, ϵ_H increases from 63% at $B_\tau = 1.0$ to 81% at $B_\tau = 0.0$.

We extract limits on M_H and the couplings $g_{\mathcal{L}\mathcal{L}}$ from the ratio of the observed number of events to the number of detected hadronic Z decays, R_H . Hadronic Z decays are defined by selection criteria that are given in Ref. 10. The measured ratio is $R_H = 0/455$. Using binomial statistics,¹⁸ the 90%- and 95%-confidence limits on R_H are $R_H^{90} = 5.07 \times 10^{-3}$ and $R_H^{95} = 6.61 \times 10^{-3}$, respectively.

The actual limits are determined by finding the values of M_H and τ_H for which the expected value of R_H is equal to the experimentally derived limit. The expected value of R_H is given by the expression

$$R_H = \frac{\epsilon_H(M_H, B_\tau, \tau_H) N_H(M_H, I_3^L)}{\epsilon_{qq} N_{qq} + \epsilon'_H(M_H, B_\tau, \tau_H) N_H(M_H, I_3^L)}, \quad (3)$$

where N_H is the expected number of produced Higgs-boson pairs of mass M_H and isospin I_3^L (correctly summed over all of the Mark II energy-luminosity points), $\epsilon_{qq} = 0.953$ is the efficiency to detect a hadronic event,¹⁰ N_{qq} is the expected number of produced hadronic events (summed over all energy-luminosity points), and ϵ'_H is the probability that a produced doubly-charged-Higgs-boson event fails the selection criteria and satisfies the hadronic event-selection criteria. Although this technique is insensitive to the absolute luminosity scale of the experiment, it is reassuring to note that the product $\epsilon_{qq} N_{qq}$ is calculated to be 451 events, in

excellent agreement with the observed number of events (455).

We account for systematic uncertainties on the τ branching ratios (0.4%) and on the tracking efficiency (1.5%) and for the statistical uncertainty of the Monte Carlo calculation (1.4%) by reducing ϵ_H by the linear sum of the effects. Since the simulation of the trigger is known to underestimate its efficiency, no additional correction is made for trigger efficiency. The systematic uncertainty on the hadronic event-detection efficiency is included by increasing ϵ_{qq} by 1σ (0.6%).

The ratio N_H/N_{qq} is sensitive to our choice of $\sin^2\theta_W$.⁹ We account for this effect by taking N_H/N_{qq} to be equal to its minimum value in the interval $\sin^2\theta_W = 0.233 \pm 0.007$. Note that the minima of the $I_3^L = 0$ and 1 cases occur at opposite ends of the interval. The result of this procedure is to reduce the expected value of R_H by 6.9% in the $I_3^L = 0$ case and by 4.2% in the $I_3^L = 1$ case.

In the short-lifetime region, the intervals of M_H that are excluded with 90% confidence and with 95% confidence are listed in Table I for several values of B_τ and I_3^L . The upper limits on M_H are due to the β^3 suppression of the cross section. The lower limits are due to the loss of efficiency as M_H becomes small. The efficiency function for stable leptons falls sharply at the cluster mass of 2.0 GeV/ c^2 . The 90%- and 95%-confidence limits occur quite close to this point. For $B_\tau = 0.5$, the number of events with four stable leptons is sufficient to exclude values of M_H down to the τ -pair threshold. The short-lifetime constraint requires that there be at least one coupling constant in the region $g_{\mathcal{L}\mathcal{L}} \gtrsim 7.4 \times 10^{-7} / \sqrt{M_H}$ (M_H in GeV/ c^2). This implies that the dominant coupling(s) be larger than $\sim 5 \times 10^{-7}$ in the small-mass region and $\sim 1 \times 10^{-7}$ in the high-mass region.

The excluded regions overlap and significantly extend the existing small-coupling limits on M_H (which are independent of I_3^L).⁵ The 90%-confidence limit for $B_\tau = 0$ is extended from approximately 21.5 to 39.7 GeV/ c^2

TABLE I. The intervals of M_H that are excluded at 90% confidence and at 95% confidence for left-handed ($I_3^L = 1$) and right-handed ($I_3^L = 0$) doubly charged Higgs bosons. The excluded intervals are tabulated as a function of the τ branching ratio B_τ . They are valid in the region of coupling constant $g_{\mathcal{L}\mathcal{L}} \gtrsim 5 \times 10^{-7}$.

I_3^L	B_τ	90% limit (GeV/ c^2)	95% limit (GeV/ c^2)
0	1.0	$6.5 < M_H < 36.4$	$7.3 < M_H < 34.3$
0	0.5	$3.6 < M_H < 37.7$	$3.6 < M_H < 36.0$
0	0.0	$2.0 < M_H < 38.5$	$2.0 < M_H < 36.7$
1	1.0	$5.9 < M_H < 38.2$	$6.5 < M_H < 36.6$
1	0.5	$3.6 < M_H < 39.2$	$3.6 < M_H < 37.9$
1	0.0	$2.0 < M_H < 39.7$	$2.0 < M_H < 38.4$

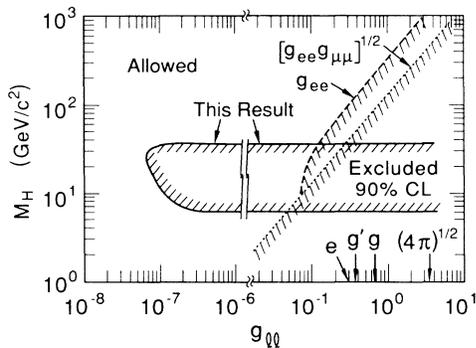


FIG. 1. The 90%-confidence contours of M_H vs the leptonic coupling strength g_{ll} that are obtained from several processes. The excluded regions are indicated by the shaded side of each contour. The result of this search is shown as the solid contour (the limit is independent of lepton flavor). The limit (Ref. 5) that is obtained from the limit on muonium to antimuonium conversion is shown as a dotted line ($\sqrt{g_{ee}g_{\mu\mu}}$ is plotted along the horizontal axis). The limit (Ref. 5) that is obtained from the Bhabha-scattering data of several experiments at the SLAC and DESY storage rings PEP and PETRA is shown as a dashed curve (g_{ee} is plotted along the horizontal axis). For reference, the sizes of the coupling constants g , g' , and e are indicated. The strong-coupling limit occurs at the value $\sqrt{4\pi}$.

($38.5 \text{ GeV}/c^2$) for left-handed (right-handed) Higgs bosons. For $B_\tau=1$, the 90% limit is extended from 14 to $38.2 \text{ GeV}/c^2$ ($36.4 \text{ GeV}/c^2$) for left-handed (right-handed) Higgs bosons.

In order to illustrate the dependence of the limits upon the mass and coupling constants, the least restrictive 90% limit ($B_\tau=1$, $I_3^L=0$) is plotted in g_{ll} - M_H space in Fig. 1 (the solid curve). Note that it extends to $g_{ll}=7.2 \times 10^{-8}$. The limit is compared with two rather specific limits from Ref. 5.

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