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At present there is a great deal of interest in the features of the diffraction peak in elastic scattering, which can be derived on completely general grounds from the behavior of total cross sections, etc.

It is shown in the following that insofar as the width $\Gamma$ of the diffraction peak is concerned the bounds obtained previously[1)] (by means of a rather sophisticated handling of the Legendre expansion) are in fact the "crudest possible" bounds on $\Gamma$.

If $\Gamma(t)$ is the width of the diffraction peak when plotted against

$$t = -2k^2 (1 - \cos \theta) \quad ,$$

then it is known that[2)]:

(i) $\Gamma(t) < \text{constant}/\sigma_{total}$ provided $\sigma_{total}$ does not decrease as fast as $\text{constant}/k^2$ as $k$ approaches infinity.

(ii) If the scattering amplitude has the Regge behavior

$f(s, \theta) \equiv F(s, t) \approx g(t)s^{\alpha(t)}$ as $s$ approaches infinity,

then we have $\alpha(0) \leqslant 1$.

(iii) For $\sigma_{total} \geqslant$ constant and $\left(\frac{\sigma_{elastic}}{\sigma_{total}}\right)$ decreasing, both as $k$ approaches infinity, then $\Gamma(t)$ shrinks as

$$\Gamma(t) < \left(\frac{\sigma_{elastic}}{\sigma_{total}}\right) \cdot \frac{\text{constant}}{\sigma_{total}} .$$

We shall show that the above three bounds are already satisfied by the "crudest possible" upper bound of $\Gamma$ .

Consider the differential cross section

$$\frac{d\sigma}{d\Omega} = \left[\text{Im } f(\theta)\right]^2 + \left[\text{Re } f(\theta)\right]^2 \qquad (1)$$

as a function of $u = \cos\theta$ so that the elastic cross section is

$$\sigma_{elastic} = 2\pi \int_{-1}^{+1} \frac{d\sigma}{d\Omega}(u)\, du \qquad (2)$$

The optical theorem tells us that

$$\text{Im } f(\theta = 0) = \frac{k}{4\pi} \sigma_{total} \qquad (3)$$

so that

$$A = \frac{d\sigma}{d\Omega}(\theta = 0)$$

$$= \left(\frac{k}{4\pi} \sigma_{total}\right)^2 + [\text{Re } f(0)]^2 . \qquad (4)$$

We define the width $\Gamma(u)$ of the elastic peak as the width of $\frac{d\sigma}{d\Omega}$ versus $u$ at half the forward value, i.e. at the value $A/2$. (See fig. 1.)

Now it is clear that for fixed $A$ and $\sigma_{elastic}$, the greatest possible width is attained for $d\sigma/d\Omega$ approaching the rectangular shape shown in fig. 2, so that certainly we have

$$\Gamma(u) \leqslant \Gamma_M(u) , \tag{5}$$

and $\Gamma_M$ is essentially the crudest possible overestimate of $\Gamma(u)$ .

The results now follow from the trivial assertion that

$$\sigma_{elastic} \leqslant \sigma_{total} , \tag{6}$$

for

$$\sigma_{elastic} \cong 2\pi \times \Gamma_M(u) \times \frac{A}{2} , \tag{7}$$

and therefore

$$\Gamma(u) \leqslant \Gamma_M(u) \leqslant \frac{\sigma_{total}}{\pi A} . \tag{8}$$

Substituting from (4) we get

$$\Gamma(u) \leqslant \frac{16\pi}{k^2 \sigma_{total}} \times \frac{1}{1 + \left[\frac{\mathrm{Re}\ f(0)}{\mathrm{Im}\ f(0)}\right]^2} \tag{9}$$

This is sensible so long as we have $\Gamma_M(u) \leqslant 2$, i.e. as long as $\sigma_{total}$ does not decrease as fast as constant/$k^2$ as $k$ goes to infinity. For the width in $t$ we then have

$$\Gamma(t) \leqslant \frac{32\pi}{\sigma_{total}} \frac{1}{1 + \left[\frac{\mathrm{Re}\ f(0)}{\mathrm{Im}\ f(0)}\right]^2} , \tag{10}$$

which is result (i) .

If the scattering amplitude has a Regge type behavior then we have

$$F\,(s,t) \approx g\,(t) s^{\alpha(t)}$$

and (11)

$$\sigma_{total} \approx s^{\alpha(0)\,-\,1} \quad .$$

If we assume that the main variation with $t$ arises from the factor $s^{\alpha(t)}$, the width is proportional to $(\log s)^{-1}$ and (10) yields

$$\frac{1}{\log s} \lesssim \frac{\text{constant}}{s^{\alpha(0)\,-\,1}} \tag{12}$$

as $s$ approaches infinity. Therefore we must have $\alpha(0) \leqslant 1$, which is result (ii).

Finally, if we are given information on the ratio $\left(\sigma_{elastic}/\sigma_{total}\right)$ and if the conditions of (iii) are satisfied, then (8) is strengthened to

$$\Gamma\,(u) \leqslant \Gamma_M\,(u) = \left(\frac{\sigma_{elastic}}{\sigma_{total}}\right) \frac{\sigma_{total}}{\pi A} \tag{13}$$

and result (iii) follows easily via (9) or (10) .

We have thus shown that the quoted results (i) to (iii) relating the width of the diffraction peak to the behavior of $\sigma_{total}$ and $\left(\sigma_{elastic}/\sigma_{total}\right)$ are exceedingly coarse bounds in the sense that $\Gamma_M$ is a very crude upper bound on $\Gamma$ . It may well be, however, that it is not possible to get <u>essentially</u> better bounds with the given input information[3].

REFERENCE

* This work done under the auspices of the U. S. Atomic Energy Commission.

1) A. Martin, Phys. Rev. 129, 1432 (1963).

2) Strictly speaking, in Martin's paper these results depend on the assumption that $d\sigma/d\Omega\ (\theta) \backsimeq [\text{Im } f(\theta)]^2$, which is certainly reasonable for small $\theta$ and high energies.

3) By essentially better bounds we mean bounds in which a more restrictive $k$ dependence can be deduced. For instance, the fact that for the above definition of the width, Martin's result gives the constant in (i) to be about $30\pi$ compared with $32\pi$ in Eq. (10), is not considered a significant difference.

# FIGURE LEGENDS

Fig. 1. Typical differential cross section.

Fig. 2. Limiting form of the differential cross section for maximum width at fixed area and fixed value at $\theta = 0$ .

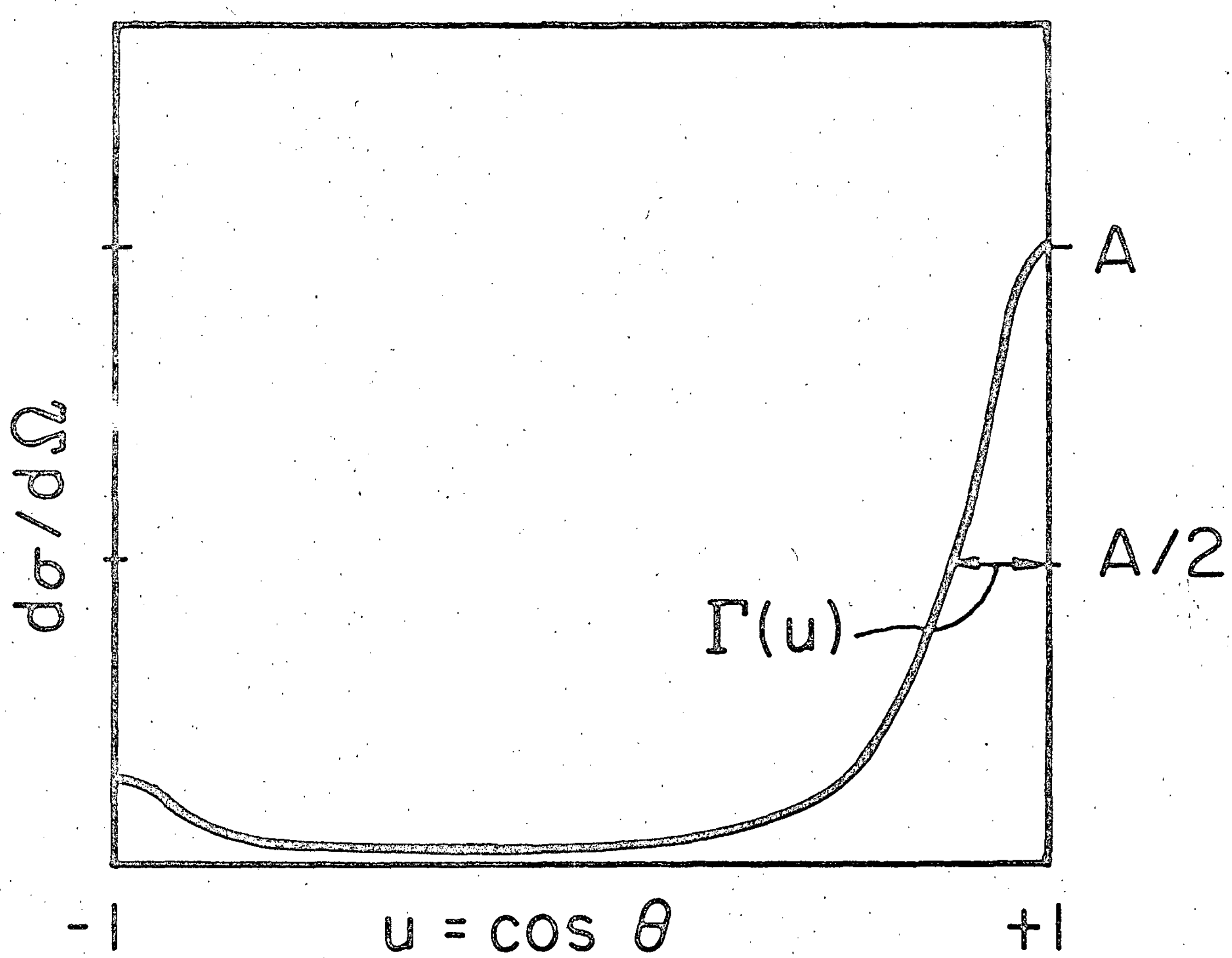


Fig. 1.

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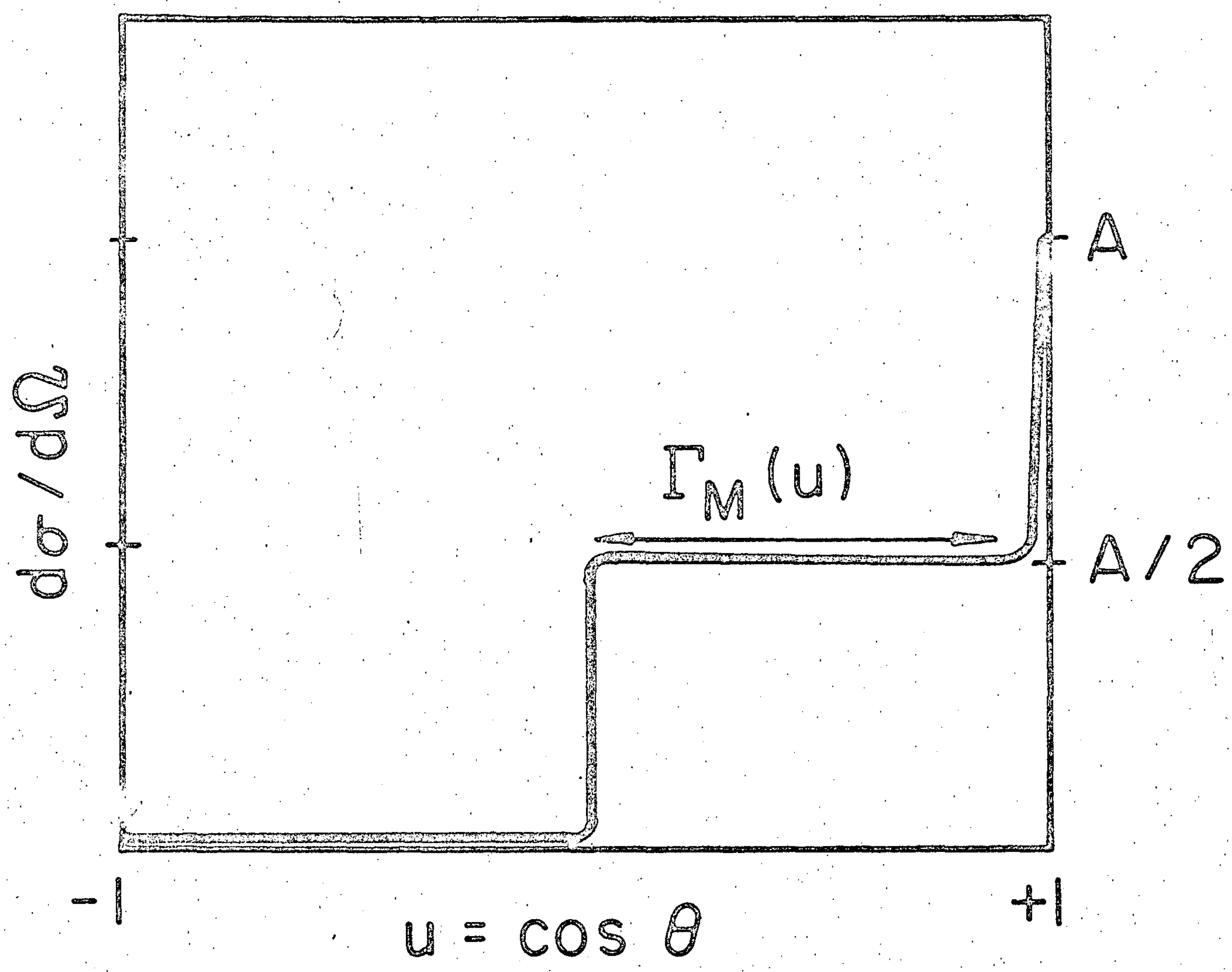


Fig. 2.

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