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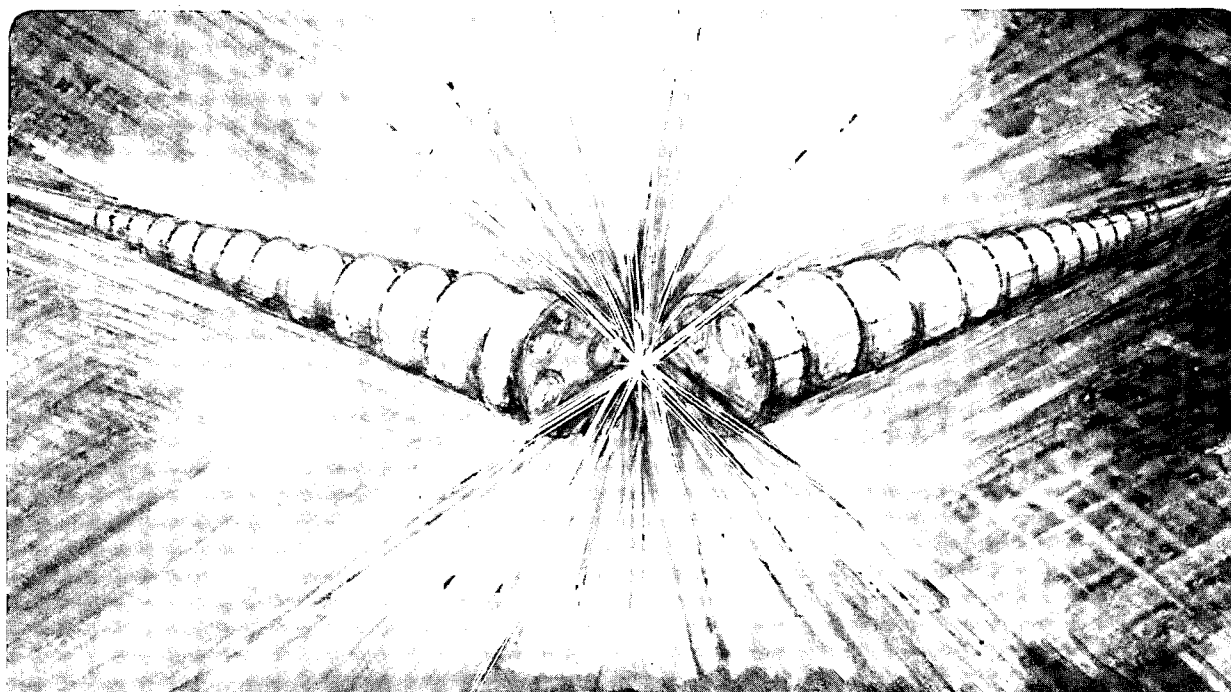
## Accelerator & Fusion Research Division

Presented at the Kent M. Terwilliger Memorial Symposium,  
Ann Arbor, MI, October 13, 1989

### Some Frontiers of Accelerator Physics

A.M. Sessler

October 1989



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## SOME FRONTIERS OF ACCELERATOR PHYSICS\*

Andrew M. Sessler  
Lawrence Berkeley Laboratory  
University of California  
Berkeley, CA 94720

October 13, 1989

## INTRODUCTORY REMARKS

When I think of MURA, I think of Ann Arbor; for I first came to Ann Arbor in the summer of 1955 as my initial experience with MURA. And then I think of yellow, and bright sunshine, and good times; for we had a small yellow house with lots of windows, and our first child was only 6 months old, and the world was all happiness, and work was such good fun, and we socialized a great deal. And I remember Kent.

There is a map of the midwest, used as a cover for MURA Reports, with little flags locating each participating university. Color that map yellow.

Later, I left the midwest, although I have returned many times for various professional needs. Increasingly, I have come for other reasons. Such as the death of a close friend's wife or the retirement of a former colleague.

I don't find retirements "yellow". Underneath, they are usually sad. Maybe "grey". I recall the retirement of Don Kerst which was especially colorless to me, for that very night I flew back to California so as to undergo brain surgery the next day. I remember thinking, optimistically, as I gave a fairly poor talk—certainly a distracted talk—at Don's retirement symposium, that perhaps the surgery would make me think better. My secretary probably hoped it would improve my handwriting. In the event, both hopes were dashed; I was lucky to come out of the hospital not much worse than when I went in.

And now this trip to the midwest. Ann Arbor is quite black; not yellow at all. Sure, there is good companionship, and it is wonderful to see old friends again, but underneath there is sadness at the loss of such a valued and good friend. I remember the warm companionship of Doris and Kent. I remember the manner in which Kent listened to new ideas, critically, and yet open to them, and how he often contributed with an insightful remark. I remember..... But, enough of that.

My subject today is Some Frontiers of Accelerator Physics and it is most appropriate to this Symposium for we at MURA laid the foundations upon which many of the modern developments are built. I say "we", and I really mean "we", for at MURA we were all involved in all activities. Sure, there were diverse authors and different papers, but we worked as a team and very little would have been accomplished by the same individuals working separately. For this Don Kerst deserves all of the credit.

We learned at MURA, for example, how to manipulate particles in longitudinal phase space. We developed—both theoretically and experimentally—stacking and phase displacement, large amplitude synchrotron motion, buckets and separatrices, capture and loss. In all of this Kent played a role, and all of this is the foundation upon

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which Free Electron Lasers, one of my topics, is based. Another of my topics is The Two-Beam Accelerator, and that, too, is based upon this solid foundation.

We learned at MURA, for example, how to manipulate particles in transverse phase space. We studied—both theoretically and experimentally—betatron motion, non-linear fixed points, stochastic motion, stable and unstable motion. In all of this Kent played a significant role, and much of this is the foundation upon which the Plasma-Based Adiabatic Compressor, one of my topics, is based.

## FREE ELECTRON LASERS

One of the frontiers of accelerator physics is the development of devices for the production of radiation. An individual has only to observe the number of synchrotron radiation facilities in existence, under construction, or under design, to appreciate the validity of this thought. (Recently I was in a town, in Japan, where four were under construction!) I shall not, however, discuss any of these devices. Rather I shall focus attention upon Free Electron Lasers (FEL), but you are to understand that the frontier is large and FELs are but one stretch of the radiation frontier, but a good stretch at that.

Free Electron Lasers have attracted a great deal of attention because they operate across a wide range of the electromagnetic spectrum, because they are easily tunable, and because they are capable of high power operation. The range of operating wavelengths has already been demonstrated to a considerable degree, as has the tunability. Considerable effort (many hundreds of millions of dollars) has been put into the development of high power lasers, with resultant significant theoretical and technological advance, but only small experimental advance.

The range of wavelengths in which FELs may provide tunable, coherent radiation is shown in Fig. 1. In Fig. 2 we show the range of wavelengths, and the associated peak power, of operating FELs. One can note that there is considerable basis for the ranges described in Fig. 1.

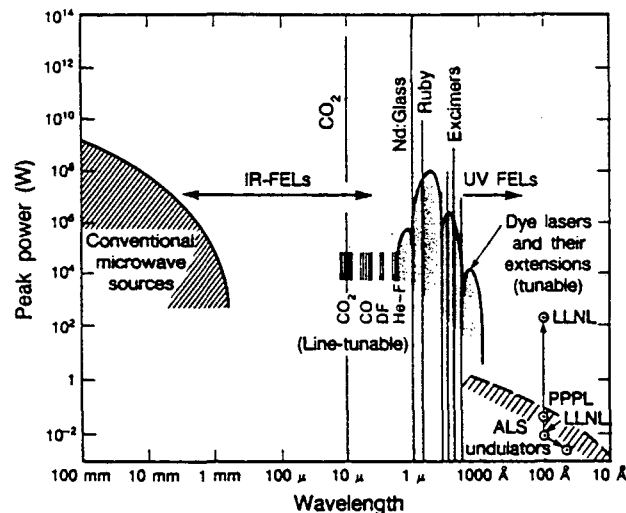


Fig. 1. FELs may provide tunable, coherent radiation in the IR and UV spectral regions (Figure courtesy of K-J. Kim)

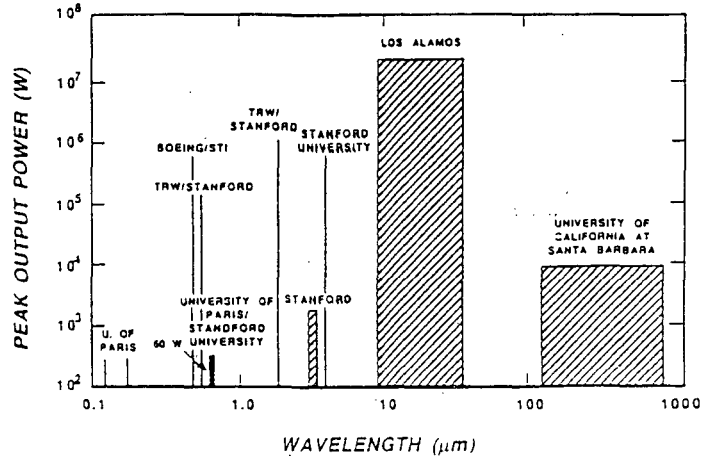


Fig. 2. FEL oscillators span a broad wavelength range. (Figure courtesy of K-J. Kim)

As a result of the attractive features of FELs, quite a number of applications for them have been envisioned. We list in Table I some of the applications considered. It is, however, fair to say that most of these applications are still in the future. A more complete discussion of these applications may be found in the literature.<sup>1</sup>

Having touched upon the reason for considering FELs, I would like now to turn our attention to the principle of operation of an FEL. Of course there are many hundreds of papers in the literature on FELS, and we can only discuss the subject very superficially here, but I believe we can quite cover, and clearly understand, the essential elements of FEL operation.

An FEL is described, to very good accuracy, by a set of equations first derived by Kroll, Morton & Rosenbluth (KMR).<sup>2</sup> These equations are:

$$\begin{aligned}
 \frac{d\gamma_j}{dz} &= -\frac{\omega a_s a_w}{2c} \frac{f_B}{\gamma_j} \sin\psi_j, \\
 \frac{d\psi_j}{dz} &= k_w + \frac{d\phi_s}{dz} - \frac{\omega}{2c\gamma_j^2} \left[ 1 + \frac{a_w^2}{2} \right], \\
 \frac{da_s}{dz} &= f_B \frac{\omega_{pe}^2 a_w}{k_s c^2} \left\langle \frac{\sin\psi}{\gamma} \right\rangle, \\
 \frac{d\phi_s}{dz} &= f_B \frac{\omega_{pe}^2 a_w}{a_s k_s c^2} \left\langle \frac{\cos\psi}{\gamma} \right\rangle.
 \end{aligned} \tag{1}$$

The equations are in a form given by Szoke, Neil and Prosnitz.<sup>3</sup> The subscript  $j$  refers to the  $j$ -th particle. The notation employed is given in Table II.

The KMR Equations are very easy to understand. The first two equations (really  $2N$  equations) simply describe the longitudinal motion of  $N$ -particles in a wiggler of static field  $B_w$  and wavelengths  $\lambda_w$ , subjected also to a wave of frequency  $\omega$ , described by amplitude  $a_s$  and phase  $\phi_s$ . In fact, the  $2N$  equations are

**Table I. FEL Applications**

1. Condensed Matter Studies
  - Surface science (IR)
  - Semiconductors (IR)
  - Superconductors (IR)
  - Magnetic properties (IR)
2. Non Linear Plasma Studies
  - Heating ( $\mu$ -wave)
  - Current drive in tokamaks ( $\mu$ -wave)
3. Non-Linear QED Studies
  - ( $1\mu\text{m}$  or  $10\mu\text{m}$ )
4. Non-Linear Optics and Microwaves
5. Inertial Fusion ( $\sim 1/2\mu\text{m}$ )
6. Chemistry
  - Molecular excitations (IR)
  - Dynamic reactions ( $1\text{-}10\mu\text{m}$ )  
[Pump-Probe]
  - Crossed photon-molecular beams  
( $1000\text{-}2000\text{ \AA}$ )
7. Biology
  - Microscopy
  - DNA studies
  - Cell response
8. Medicine
  - Surgery ( $1\text{-}10\mu\text{m}$ )
  - Photo-reactions (IR)
  - Angiography
9. Accelerators
  - IFEL
  - TBA ( $1\text{-}10\text{ mm}$ )
10. Photo-reactions
  - Fixing polymers
  - Making drugs
11. Isotope separation (ENEA)
12. Military uses (LLNL, LANL, Boeing/ST)

nothing but the equations, highly developed by MURA, to describe rf motion in an accelerator. The remaining two equations describe the growth of the coherent wave. It is only the last two equations which differentiate an FEL from rf acceleration; i.e. the coherent wave is allowed to change—self-consistently—whereas it is prescribed in rf acceleration.

Thus one sees that the KMR Equations are a 1D description, very similar to that employed to describe rf motion in accelerators, but now extended to very much shorter wavelength radiation. Much of the intuitive development, acquired through years of accelerator work, can be—and has been—applied to FEL operation.

Let me describe just one such use of understanding acquired from accelerators; namely the use of "tapering" in an FEL. The concept of tapering was

**Table II. Notation**

|                   |   |                                                                       |
|-------------------|---|-----------------------------------------------------------------------|
| $B_w$             | = | peak magnetic field of a linear wiggler                               |
| $\lambda_w$       | = | wiggler period                                                        |
| $k_w$             | = | $2\pi/\lambda_w$                                                      |
| $a_w$             | = | $\frac{e B_w}{\sqrt{2}mc^2k_w}$                                       |
| $\omega$          | = | angular frequency of the signal field                                 |
| $\lambda_s$       | = | wavelength of the signal                                              |
| $k_s$             | = | $2\pi/\lambda_s$ = peak electric field of a linearly polarized signal |
| $a_s$             | = | $\frac{eE_s}{\sqrt{2}mc^2k_s}$                                        |
| $\phi_s$          | = | electric field phase with respect to a vacuum plane wave              |
| $\gamma_j$        | = | Lorentz factor                                                        |
| $c$               | = | velocity of light                                                     |
| $v_j$             | = | $(k_s+k_w)z - w_st + \phi_s$                                          |
| $\omega_{pe}^2$   | = | $\frac{4\pi n_e e^2}{m}$                                              |
| $n_e$             | = | density of electrons                                                  |
| $m$               | = | mass of electrons                                                     |
| $\langle \rangle$ | = | average over particles $j$                                            |
| $f_B$             | = | $J_0(\xi) - J_1(\xi)$                                                 |
| where             |   |                                                                       |
| $\xi$             | = | $\frac{a_w^2}{2(1+a_w^2)}$                                            |

developed by Kroll, Morton & Rosenbluth in Ref. 2; one should note that Morton was at MURA during the late 50's when rf manipulation of particles was so extensively studied and so highly developed.

The condition for FEL synchronism, often called the resonance condition (i.e., that  $\frac{dy}{dz} = 0$  for a central particle) is:

$$\lambda_s = \frac{\lambda_w}{2\gamma^2} (1 + a_w^2) . \quad (2)$$

As a particle loses energy synchronism cannot be maintained if  $\lambda_w$  and  $a_w$  are constants; i.e., if the wiggler is untapered.



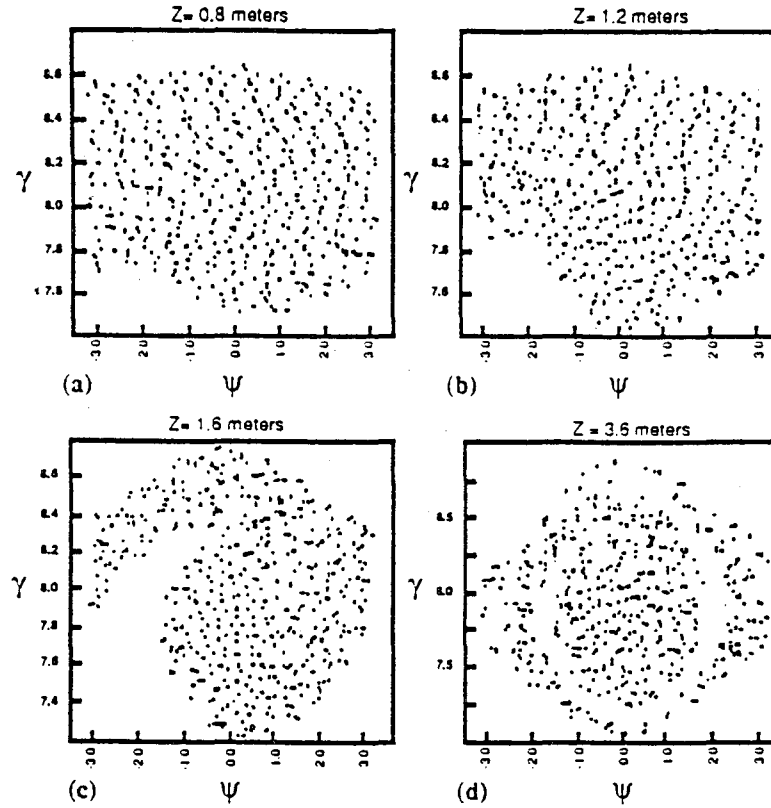
The effect of taper has been studied in hundreds of numerical simulations. Here, I show the result of tapering studies using particular parameters; namely those of the Electron Laser Facility (ELF) at Livermore. The parameters are given in Table III. The results of a 1-D numerical simulation for an un-tapered wiggler is given in Fig. 3 and Fig. 4. For a tapered wiggler the results are shown in Fig. 5. [Figs. 3, 4 and 5 are due to Efreim Sternbach.]

**Table III. ELF Parameters**

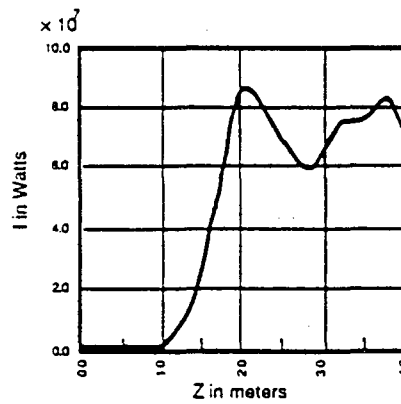
|                         |                                |
|-------------------------|--------------------------------|
| Beam Energy             | 4.14 Mev                       |
| Beam Energy in $\gamma$ | 8.1                            |
| Beam Current            | 800 amperes                    |
| Wiggler wavelength      | 9.8 cm                         |
| Radiation frequency     | 34.6 GHz                       |
| Waveguide width         | 9.8 cm                         |
| Waveguide height        | 2.9 cm                         |
| Input radiation power   | 60KWatts/TE <sup>01</sup> mode |
| Wiggler Peak Field      | 4.67 KGauss                    |
| Energy Spread           | $\pm 1$ MeV                    |

Although the effect of tapering has been studied in many hundreds of simulations, it has only been studied in a few experiments. In Fig. 6 we show the dramatic results achieved on ELF.<sup>4</sup> The extracted power and efficiency was as high as 1.86 GW and 45% in the best case.

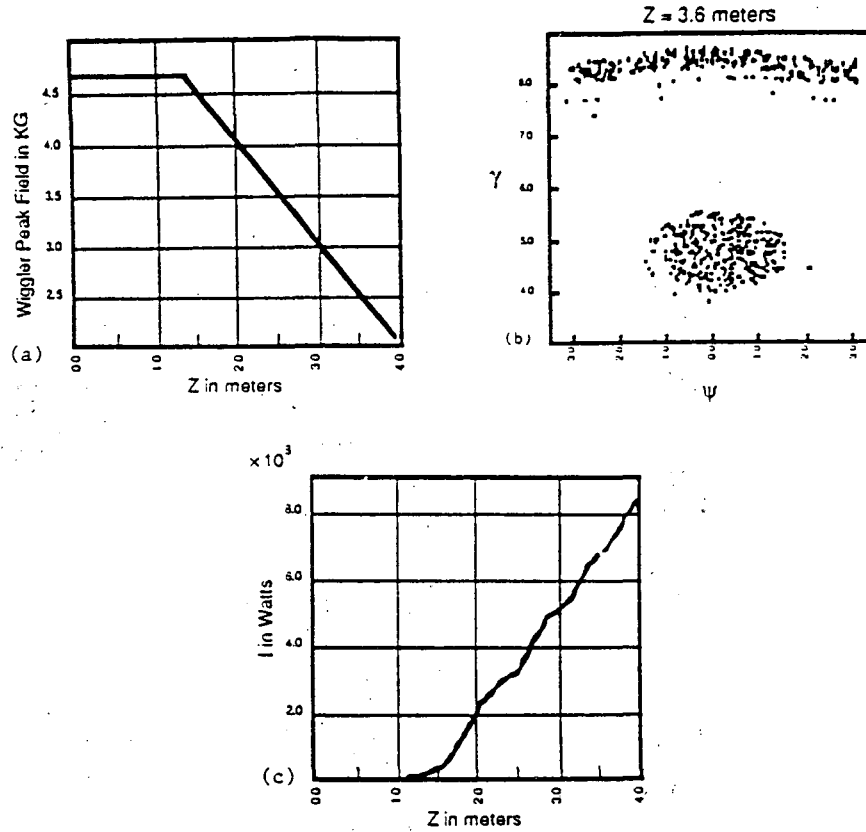
There are many other aspects of FELs and the interested reader will want to study the literature starting, perhaps, with some review articles or even the text on the subject.<sup>5</sup> Hopefully, the comments here have been adequate to show the motivation for developing FELs; i.e. why they constitute one of the "frontiers of accelerator physics". Perhaps, just to demonstrate the accuracy of that statement, the reader will want to examine Table IV in which are listed present worldwide FEL projects.



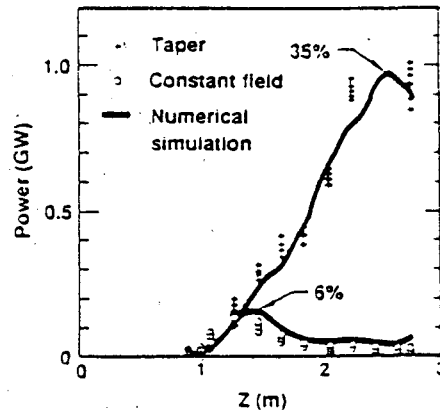
**Fig. 3.** Numerical simulations of ELF (Table III). Particle energy vs. phase is shown for four different distances down the wiggler.



**Fig. 4.** Numerical simulation for ELF (Table III) showing microwave power as a function of distance down the wiggler.



**Fig. 5.** Numerical simulations for a tapered ELF. The tapering is shown in (a); the resultant trapping of particles in (b) and the output microwave power in (c).



**Fig. 6.** Power vs. wiggler length as measured experimentally at ELF, with comparison with numerical simulations. The numerical simulation was performed with a code (FRED) that has space charge and 3-D effects included. Tapering enhanced the extraction efficiency from 6% to more than 35%.

**Table IV. World-Wide FEL Projects**

|             |                                                                                                                                                                                                                                                                                                            |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| China       | Shanghai (pulsed)<br>Beijing (rf)<br>Chengdu (4f)                                                                                                                                                                                                                                                          |
| Japan       | JAERI 4+2 projects (rf)<br>Osaka (induction linac)                                                                                                                                                                                                                                                         |
| Germany     | Darmstadt (superconducting)<br>Dortmund DELTA (storage ring)                                                                                                                                                                                                                                               |
| Israel      | Technion<br>Weizmann (DC)                                                                                                                                                                                                                                                                                  |
| France      | ACO (sr)<br>Super ACO (sr)<br>CLIO (RF)<br>1 project (induction linac)                                                                                                                                                                                                                                     |
| Netherlands | FOM Institute, Nieuwegein FELIX (rf)                                                                                                                                                                                                                                                                       |
| Italy       | ENEA (Microtron)<br>INFN (sc)<br>Milan (ARES (sc), ELFA (sc))<br>Padua (DC)                                                                                                                                                                                                                                |
| England     | Oxford (DC)<br>Liverpool                                                                                                                                                                                                                                                                                   |
| USSR        | Novosibirsk                                                                                                                                                                                                                                                                                                |
| USA         | UCSB 2 projects (DC)<br>Vanderbilt (rf)<br>LLNL Palladin (induction)<br>LLNL MTX (induction)<br>BNL (TOK) (sr)<br>BNL (rf)<br>Stanford Mark III (rf)<br>Stanford (sc)<br>LASL (rf)<br>Boeing/Spectra Physics (rf)<br>Hughes<br>Florida (DC)<br>NBS (microtron)<br>MIT<br>Columbia (induction)<br>Duke (rf) |

Projects Terminated: UK and Bell Labs

## TWO BEAM ACCELERATOR

A very large frontier of accelerator physics is the development of a TeV-class linear electron-positron collider. This development involves, amongst other things, damping rings, beam emittance preservation, accurate beam monitors, and final focus physics. I shan't be able to discuss the wide range of devices and approaches that accelerator physicists have deployed along this frontier, but rather I limit myself to just one device. You must understand that the Two-Beam Accelerator (TBA) isn't one of the major devices, it may never be used, but it is a personal favorite. You are to take it as representative of the frontier of linear colliders.

If one examines the history of particle accelerators, and this is shown in the famous Livingston graph of Fig. 7, one notes firstly a very large increase in available energy (more than an order for every decade in years) and, secondly, that the general increase is a result of constantly developing new techniques and methods. Linear colliders, first suggested by Richter, are the newest practical idea for extending the range of electron-positron circular colliders.<sup>6</sup>

Linear colliders require very powerful power sources and a great deal of research is being done on the development of suitable sources. Alternatively, an accelerator could have a single drive beam rather than thousands of drive klystrons.<sup>7,8</sup> A schematic of a Two-Beam Accelerator (TBA) is shown in Fig. 8. There are a number of versions of a TBA; one employs a free electron laser and is shown in Fig. 9, another employs relativistic klystrons and is shown in Fig. 10.

We have already noted that an FEL can be a remarkably powerful source of radiation appropriate to a linear collider (between 1 cm and 2.5 cm). Recently a relativistic klystron has been shown to produce hundreds of megawatts of peak power and therefore, also, to be a viable alternative for the TBA.<sup>9</sup>

Besides the various technological developments which are required for a TBA, there are questions of principle involved. First, one is concerned about maintaining longitudinal bunching during the process of de-acceleration (i.e., radiation generation) and re-acceleration (presumably by an induction unit). As can be seen, from the work done years ago at MURA, in fact longitudinal bunching can be maintained if the drift regions and re-acceleration units are not too long.

A second question has to do with the sensitivity of the rf accelerating wave to imperfections in the re-acceleration units and to variations in the drive beam energy and current. Considerable effort has been devoted to this subject. With the understanding and development at present allowing (for reasonable tolerances) about 100 de-acceleration and re-acceleration periods.<sup>10</sup> This is not as long as one might have wished, but it is long enough to make the TBA still interesting.

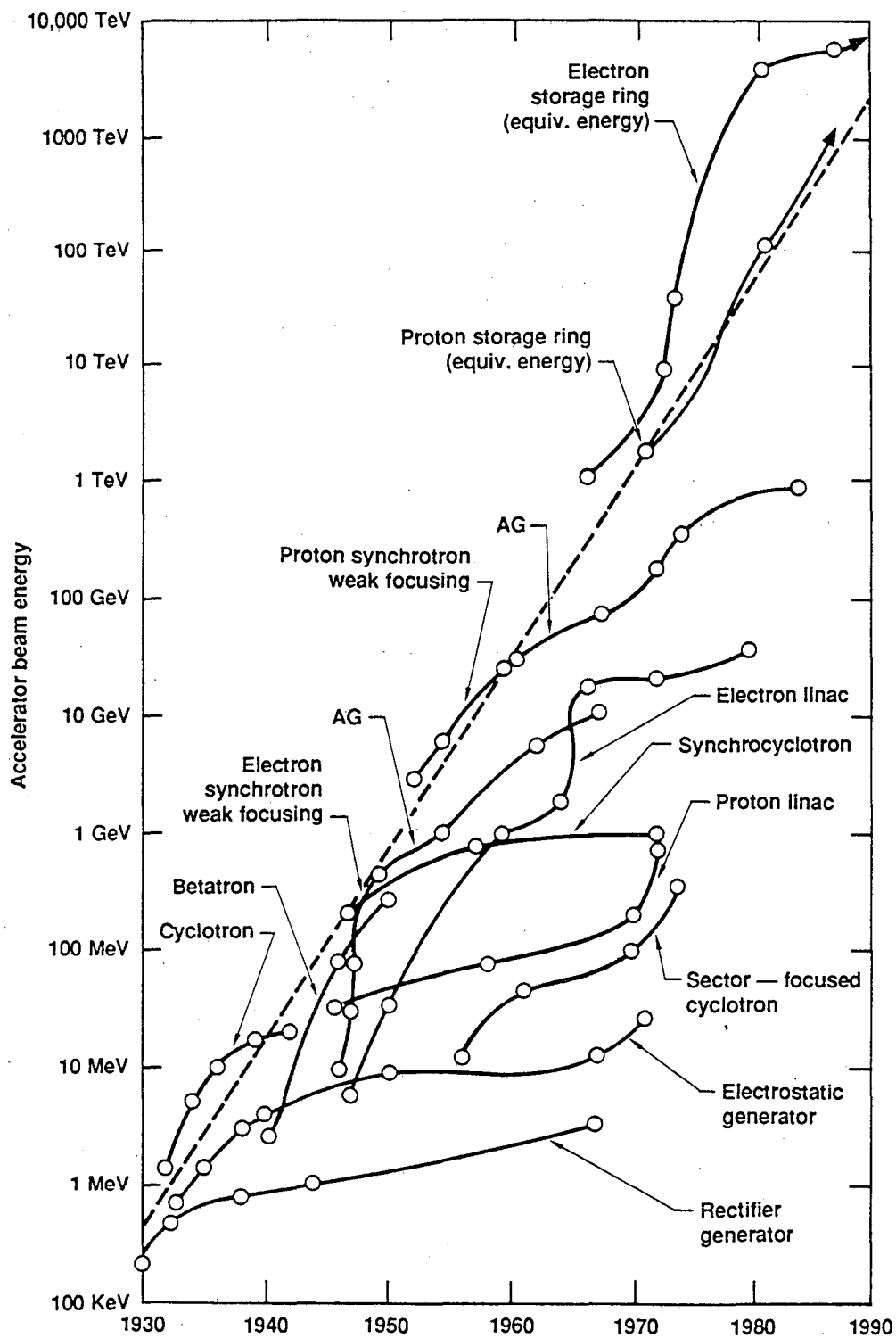


Fig. 7. The energy of particle accelerators shown as a function of year.

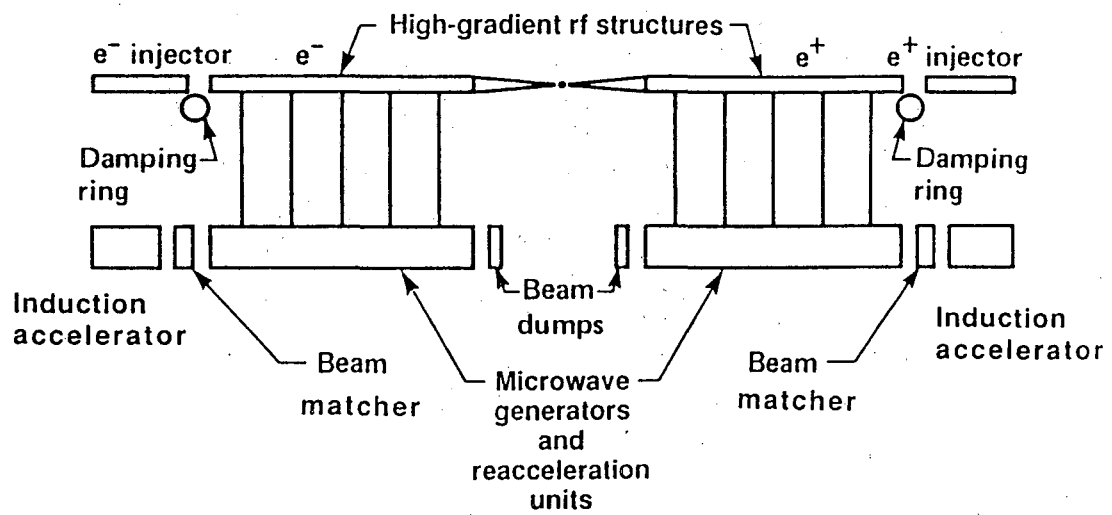
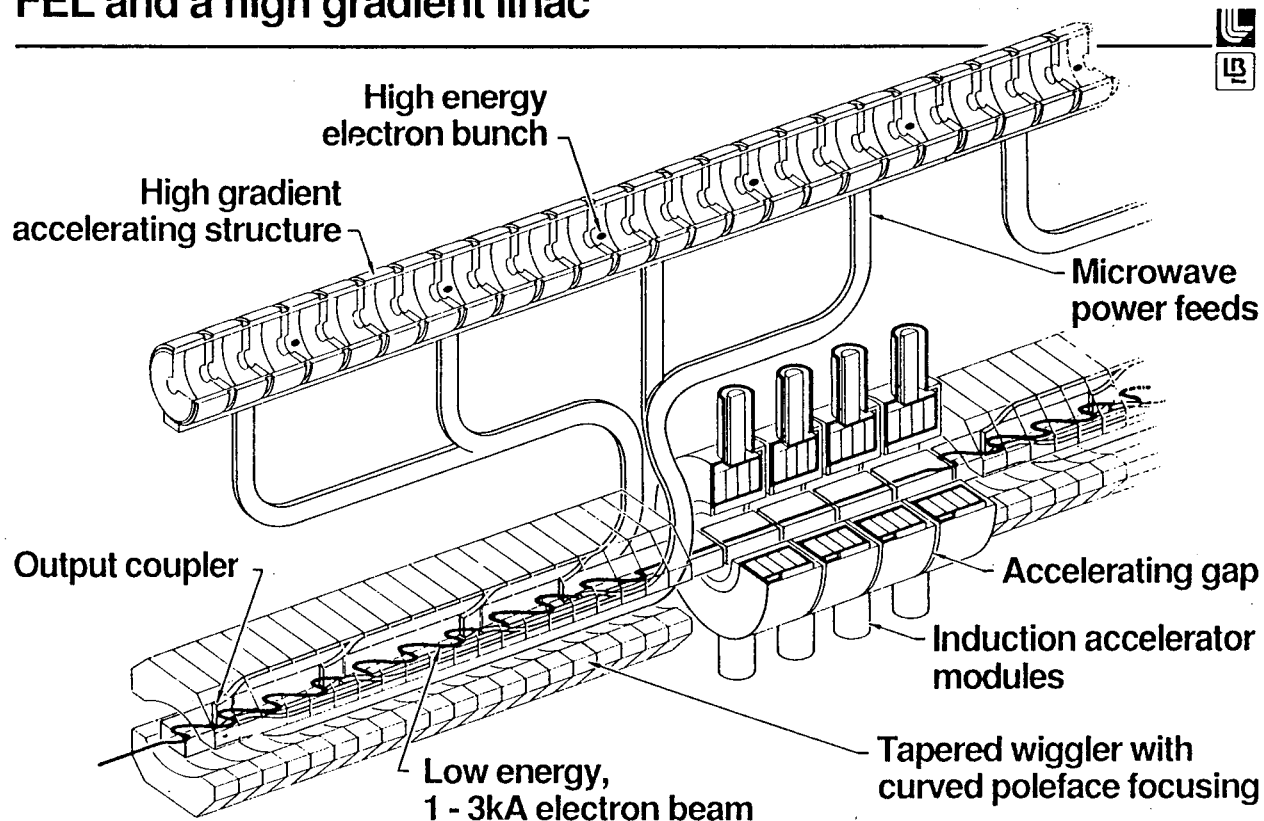


Fig. 8. Schematic of a two-beam accelerator (TBA). The low-energy drive beam provides power to the high-energy ( $\approx \text{TeV}$ ) beam).

## The Two Beam Accelerator (TBA) consists of a high power microwave FEL and a high gradient linac

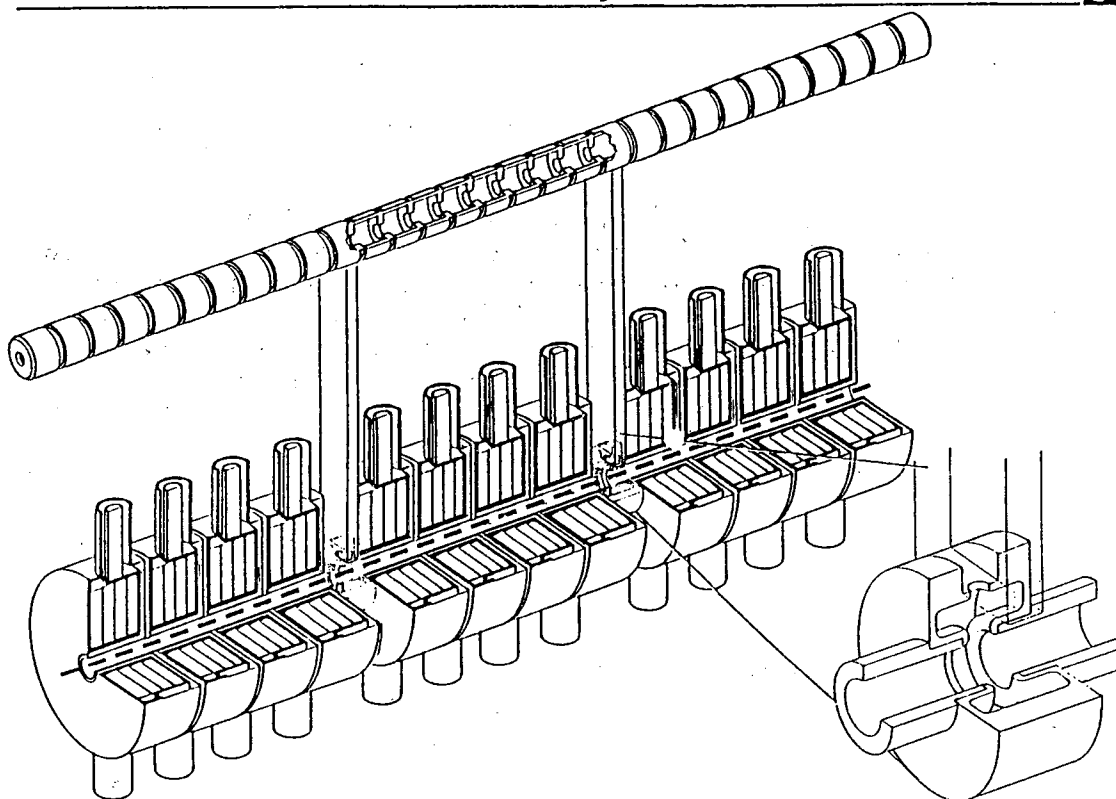


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**Fig. 9.** An artist's drawing of the free-electron laser (FEL) version of the two-beam accelerator (TBA). In this version, microwaves are generated from the drive beam by FEL action.





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**Fig. 10.** An artist's drawing of the relativistic klystron (RK) version of the two-beam accelerator (TBA). In this version, microwaves are generated from the drive beam by RK action in the (small) transfer cavities.

## PLASMA-BASED ADIABATIC COMPRESSOR

Many think that the future of particle accelerators is in plasma physics. Others say, "My God, accelerators are difficult enough to design, build and operate; plasmas will finish the field off". Observing the large progress made towards harnessing fusion energy—in the 1950s it was 30 years away and in the late 1980s it is only 25 years away—we cannot help but be impressed with the field of plasma physics.

Nevertheless a frontier of accelerator physics is that facing on new ideas, many of which involve plasmas. I won't cover beat-wave accelerators, plasma wake-field accelerators, plasma lenses, and plasma compensators; to say nothing of inverse Cherenkov accelerators or laser cooling. I would like, however, to discourse on one device; namely, the Plasma-Based Adiabatic Compressor. Please take it as representative of the frontier of new ideas.

The Plasma-Based Adiabatic Compressor is simply a device in which the focusing forces are made to increase along the axis, so that particles moving through the device are subjected to an ever-increasing focusing force.<sup>11</sup> Simple as this is, and anyone knowing about adiabatic damping—a concept well-explored by

those at MURA—will appreciate that this device will focus particles, it has some quite remarkable features as we shall soon see.

The Compressor consists of an underdense plasma (plasma density less than the beam density) through which a beam of electrons is sent. The plasma electrons are ejected radially and the remaining (essentially rigid) ions provide focusing for the beam electrons. In all of the analysis we don't invoke the plasma beyond the concept just described; the plasma is essential, however, as the focusing force it provides corresponds to an effective magnetic field in the mega-gauss range, which cannot be provided in any other manner. The reader should note that concept of ion focusing is well established both theoretically and experimentally.

Single particle dynamics in the Compressor can be described by

$$\frac{d^2y}{ds^2} + K(s)y = 0, \quad (3)$$

where we only consider, for simplicity, one transverse coordinate  $y$ . The solution can be written in the Twiss form

$$y(s) = \beta^{1/2}(s)\cos[\psi(s) + \phi] \quad (4)$$

where the parametric functions satisfy

$$\begin{aligned} \frac{d\beta}{ds} &= -2\alpha(s), \\ \psi(s) &= \int^s \frac{ds'}{\beta(s')}. \end{aligned} \quad (5)$$

Taking the increase of focusing strength such that

$$\frac{d\beta}{ds} = \text{constant} \quad (6)$$

we conclude that

$$\begin{aligned} \beta(s) &= \beta_0 - 2\alpha_0 s, \\ \alpha(s) &= \alpha_0, \end{aligned} \quad (7)$$

and the focusing force  $K(s)$  becomes

$$K(s) = \frac{1 + \alpha_0^2}{\beta^2} = \frac{1 + \alpha_0^2}{(\beta_0 - 2\alpha_0 s)^2}. \quad (8)$$

Thus one sees that the parameter  $\alpha_0$  characterizes the degree of adiabaticity of the device. The expression for  $K(s)$  describes how the plasma density must be tailored with distance  $s$ .

One of the important characteristics of the Compressor is its insensitivity to the energy of a particle. This is important, in very high energy applications, where a particle undergoing focusing radiates a considerable amount of energy. As a result of this radiation the particle energy is reduced—but because of the nature of focusing in the adiabatic Compressor, the particle is still focussed.

Radiation by a particle going through the Compressor is not as important as in a normal focusing system, but it is not a negligible effect either. In a "normal" system, i.e. a focusing system consisting of discrete elements, there is a severe limit on the resultant beam size at the focus. In fact, the beam cannot be made arbitrarily small in size, no matter how powerful the focusing system is, as was first observed by Oide.<sup>12</sup>

The argument, in qualitative terms, goes the following way. Consider a very powerful lens put quite near the focal spot. Quantum fluctuations in the amount of radiation by an electron going through the powerful lens will (because of the chromatic effect of any lens) cause a smear in the position of the focal point and hence some non-zero transverse size even at the focus. The effect is a result of the uncertainty in the radiation (any determined effect can be corrected for) and hence the final beam size involves Plank's constant. One might be tempted to circumvent the limit by creating the focus with a weak lens far from the focal point. True, in this case the radiation will be less (and hence the quantum fluctuations also reduced), but now one only needs a small error in focusing strength (due to chromatic effects) to create smear, because of the long "lever arm" of the far-away lens. One can show that the limit on beam size is essentially independent of lens strength (and hence position).

The Adiabatic Compressor does not suffer from the Oide limit, but radiation effects do need to be considered. Their consideration leads to some interesting consequences. A particle going through a compressor will be focussed into an ever-smaller transverse size while undergoing transverse oscillations of ever-increasing rapidity. As a result it will radiate ever more. Clearly, for the Focuser to be interesting the amount of radiated energy must be less than the particle energy.

Let us quantify this thought. The energy loss per unit length, divided by the energy of a particle, is

$$\frac{d\mu}{\mu} = (2/3) r_0 \alpha^3 \frac{1}{\rho^2}, \quad (9)$$

where  $r_0$  is the classical electron radius and  $\rho$  is the bending radius of the trajectory the particle is forced to go on. In the Compressor, the radius of curvature  $\rho$  is related to the displacement  $y$  and the  $\beta$ -function by

$$\frac{1}{\rho} = y/\beta^2. \quad (10)$$

Roughly, the particle loses energy for a distance  $\beta$ , and hence letting  $\delta$  be the fractional energy loss of the particle going through the Focuser, we have

$$\delta = \beta \frac{d\mu}{\mu} = (2/3) r_0 \frac{\gamma^2 \epsilon_N}{\beta^2}, \quad (11)$$

where we have introduced the normalized emittance  $\epsilon_N = \gamma^2/\beta$ .

Notice that  $\delta$  describes a purely classical effect. Requiring  $\delta < 1$  implies

$$\beta > ((2/3) r_0 \gamma^2 \epsilon_N)^{1/2}, \quad (12)$$

and hence (from the expression for  $\epsilon_N$ )

$$\sigma \equiv \langle y^2 \rangle^{1/2} > ((2/3) r_0 \epsilon_N)^{1/4}. \quad (13)$$

(A more careful analysis would replace the factor of  $2/3$  by  $(1 + \alpha_0^2)^2/6\alpha_0$ . The optimum choice—in this range—of  $\alpha_0 = 1/\sqrt{3}$  means that this factor is 0.513.) It can be seen that this limit does not involve Plank's constant; in principle—but not in practice—it can be exceeded.

It is not correct, however, in many cases, to compute the energy loss in a purely classical manner; quantum effects can dramatically change the above conclusions. As a particle traverses the Compressor it oscillates more and more, and consequently emits photons of ever higher energy. It is clear that if  $\omega$  is the frequency of a photon, then  $\hbar\omega$  cannot be larger than the energy of the emitting particle. Now classically the radiation has a pattern which is well-known; namely, the power radiated increases as  $\omega^{1/3}$  until an exponential cut-off occurs at the critical frequency  $\omega_c = \gamma^3 c/\rho$ . Our above estimates are correct if  $\gamma mc^2 \gg \hbar\omega_c$ , but not otherwise.

We may estimate quantum effects by equating  $\hbar\omega_c$  and  $\gamma mc^2$  (at this point quantum effects come in quickly and significantly as one rides up the exponential):

$$\gamma mc^2 = \hbar\omega_c = \hbar\gamma^3 c/\rho. \quad (14)$$

Employing the expressions for  $\rho$  and  $\epsilon_N$  we have

$$\beta^3 = \gamma^3 \lambda_c^2 \epsilon_N, \quad (15)$$

where we have introduced the Compton wavelength  $\lambda_c = \hbar/mc$ .

Inserting this value for  $\beta$  into the expression for the fractional energy loss,  $\delta$ , we obtain

$$\delta = (2/3)r_0 \frac{\alpha^2 \epsilon_N}{[\gamma^3 \chi^2 \epsilon_N]^{2/3}} = \frac{2}{3} \alpha \left( \frac{\epsilon_N}{\chi_c} \right)^{1/3} \quad (16)$$

where we have introduced the fine structure constant  $\alpha$ . Equating  $\delta$  to unity we find a critical emittance:

$$\epsilon_c \equiv \epsilon_N = \left( \frac{3}{2} \right)^3 \frac{\chi_c}{\alpha^3} = 3.34 \times 10^{-6} \text{ m}. \quad (17)$$

(A more careful analysis replaces the numerical factor  $(3/2)^3$  by  $\frac{15^3}{2^3 22} \frac{\alpha_0^3}{(1+\alpha_0^2)^2}$ .

The optimum choice—in this range—of  $\alpha_0 = \sqrt{3}$  yields  $\epsilon_c = 6.17 \times 10^{-6} \text{ m}$  for the critical emittance.)

Above the critical emittance a beam going through the Compressor will be subject to the limit on beam size,  $\sigma$ , derived above. For a sufficiently fine beam, however, having an emittance less than the critical emittance, the beam can be focused down to a value much less than the limit applicable in the classical regime. Explicit expressions are given in Ref. 11.

It is interesting to note that the critical emittance,  $\epsilon_c$ , only involves fundamental constants. Furthermore it is interesting to note that  $\epsilon_c$  is quite accessible; i.e. not exceeding small at all.

Finally, Table V consists of three examples of the adiabatic compressor. The first is a possible experiment, the second is a device built for the Stanford Linear Collider and the third is an example, for a future linear collider, where the beam size is made much smaller than the Oide limit. For all the examples the required plasma density is given. Note, first of all, that these densities are non-trivial to attain in practice and, secondly, that the plasma will cause background effects which probably will be significant and which have not yet been considered.

**Table V.** Three Examples of the Adiabatic Compressor

|                                | SLAC End Station     | SLC                  | TLC                  |
|--------------------------------|----------------------|----------------------|----------------------|
| <u>Initial Beam Properties</u> |                      |                      |                      |
| $E_0$ [GeV]                    | 15                   | 50                   | 500                  |
| $\epsilon_n$ [m]               | $1 \times 10^{-4}$   | $3 \times 10^{-5}$   | $1 \times 10^{-8}$   |
| $\sigma_0$ [ $\mu$ m]          | 20                   | 3                    | $5 \times 10^{-3}$   |
| $\beta_0$ [cm]                 | 12                   | 3                    | 0.25                 |
| <u>Focuser Properties</u>      |                      |                      |                      |
| $\alpha_0$                     | $5.0 \times 10^{-2}$ | $1/\sqrt{3}$         | $\sqrt{3}$           |
| L [cm]                         | 119                  | 2.6                  | 0.07                 |
| $n_0$ [cm <sup>-3</sup> ]      | $1.2 \times 10^{14}$ | $8.4 \times 10^{15}$ | $1.8 \times 10^{19}$ |
| $n^*$ [cm <sup>-3</sup> ]      | $1.2 \times 10^{18}$ | $8.4 \times 10^{19}$ | $1.8 \times 10^{23}$ |
| <u>Final Beam Properties</u>   |                      |                      |                      |
| $\delta$                       | Negligible           | 3%                   | 1%                   |
| $\sigma^*$ [ $\mu$ m]          | 2                    | 0.3                  | $0.5 \times 10^{-3}$ |

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LAWRENCE BERKELEY LABORATORY  
TECHNICAL INFORMATION DEPARTMENT  
1 CYCLOTRON ROAD  
BERKELEY, CALIFORNIA 94720