



# Hybrid Linear and Nonlinear Programming Model for Hydropower Reservoir Optimization

Mustafa S. Dogan, A.M.ASCE<sup>1</sup>; Jay R. Lund, Dist.M.ASCE<sup>2</sup>; and Josue Medellin-Azuara, M.ASCE<sup>3</sup>

**Abstract:** Linear and nonlinear optimization models are common in hydropower reservoir modeling to aid system operators and planners. Different modeling techniques have their advantages and shortcomings. Linear optimization models are faster but less accurate, and nonlinear models are slower with better system representation. A hybrid linear and nonlinear hydropower energy reservoir optimization (HERO) model is introduced, where a hybrid optimization model sequentially solves the overall nonlinear hydropower optimization problem first with a faster-running linear programming (LP) approximation to improve an initial solution for a nonlinear programming (NLP) solution to significantly reduce NLP iterations and run time. The hybrid model is applied to six hydropower plants of California, with capacities of 13.5 to 714 MW. LP and NLP decisions are compared, and run time benchmarks of the LP, NLP, and hybrid LP-NLP models with different numbers of decision variables are presented. The hybrid model reduces the NLP run time by 79% to 88%, depending on model size, but still requires much more run time than the LP solution. For short-term operations with good inflow and energy price forecasts, where accuracy matters more and uncertainties are modest, the hybrid LP-NLP model has advantages. For long-term hydropower planning and management with many more decision variables and greater inflow uncertainty, the LP model, with its greater speed and sensitivity analysis, or stochastic models, representing some uncertainties, will often be preferred. DOI: 10.1061/(ASCE)WR.1943-5452.0001353. © 2021 American Society of Civil Engineers.

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## Introduction

Hydropower reservoirs store energy as higher-elevation water. Using potential energy difference (water head) between reservoir intake and tailwater levels, power is generated by vertical movement of water. Hydropower's lower operating cost than most other power sources provides an incentive to maximize hydropower generation in a power system with mixed generation sources (Hamlet et al. 2002; Madani et al. 2014). Hydropower also can provide operational flexibility by generating power on short notice (Chatterjee et al. 1998; Côté and Leconte 2016; Karimanzira et al. 2016) and additional ancillary services, such as peak and frequency regulation and spinning reserves (Li et al. 2013). Hydropower plants often are classified as (a) large-storage and low-head plants, (b) low-storage and high-head or run-of-river plants, or (c) pumped-storage plants (Pérez-Díaz and Wilhelmi 2010; Madani et al. 2014), with some plants overlapping these general types. Run-of-river plants run continuously and usually supply base power load, while plants with large storage capabilities and pumped-storage plants are more dispatchable and regulated for peak demands (Pérez-Díaz and Wilhelmi 2010).

Many hydropower reservoir operation algorithms use linear programming (LP) (van Do and Howard 1988; Madani and Lund 2009; Vicuña et al. 2011; Rheinheimer et al. 2016; Dogan et al. 2018), nonlinear programming (NLP) (Tejada-Guibert et al. 1990; Barros et al. 2003; Zhou et al. 2020), and dynamic programming (DP) (Grygier and Stedinger 1985; Mariño and Loaiciga 1985; Allen and Bridgeman 1986; Afshar et al. 1990; Zhao et al. 2012; Li et al. 2013) methods. LP has the advantages of fast evaluation and finding a globally optimal solution, but often requires problem simplification to fit an LP formulation, reducing the solution accuracy for the original nonlinear problem. NLP needs less simplification and nonlinear hydropower operations can be well represented. However, computing time increases exponentially with the number of decision variables, expanding with the number of plants, modeling horizon, and smaller time steps (Dogan 2019a). Barros et al. (2003) introduced an NLP model for Brazil's hydropower system, called SISOPT, where the successive LP and NLP models are initialized with an LP model derived from historical operations. SISOPT uses monthly time steps that can be sufficient for long-term planning, but too coarse for short-term decisions, where hourly or quarter-hourly time steps are desired. This paper presents an open-source hybrid linear programming–nonlinear programming (LP-NLP) hydropower optimization model, where an NLP model is initialized with linearized hydropower benefit curves for each time step and plant, requiring less system data. The hybrid hydropower energy reservoir optimization (HERO) model, developed with Pyomo, is independent from data, so any time step such as subhourly, hourly, or daily can be selected, depending on reservoir inflow and energy price data resolution. The current HERO model has a general network flow form, and additional linear constraints, such as ramping and water delivery constraints, can be added as needed. The following sections describe the model and quantify accuracy losses and run time benefits of LP, NLP, and the hybrid LP-NLP models for deterministic applications.

<sup>1</sup>Lecturer, Dept. of Civil Engineering, Aksaray Univ., Aksaray, Turkey 68100 (corresponding author). ORCID: <https://orcid.org/0000-0002-3378-9955>. Email: msahindogan@aksaray.edu.tr

<sup>2</sup>Distinguished Professor, Dept. of Civil and Environmental Engineering, Univ. of California, Davis, CA 95616. Email: jrlund@ucdavis.edu

<sup>3</sup>Associate Professor, Dept. of Civil and Environmental Engineering, Univ. of California, Merced, CA 95343. ORCID: <https://orcid.org/0000-0003-1379-2257>. Email: jmedellin-azuara@ucmerced.edu

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## Generalized Network Flow Representation

Network flow models use nodes and links to represent the physical system. Nodes represent point locations such as reservoirs or junctions, while links represent streams or canals [see Bazaraa et al. (2010) for more discussion]. Hydropower network flow optimization requires reservoir inflow and energy price inputs in addition to plant characteristics. Energy prices are assumed exogenous, largely unaffected by hydropower operations. Inflows and energy prices are assumed to be known with certainty in a deterministic model. Eq. (1) shows total revenue (dollars) from energy generated (Wh) as a function of density of water  $\rho$  (kg/m<sup>3</sup>), gravitational constant  $g$  (m/s<sup>2</sup>), plant efficiency  $\eta$  (constant), water head  $H$  (m), flow through turbines  $Q$  (m<sup>3</sup>/s), unit energy price  $p$  (\$/Wh), and time difference  $\Delta t$  (h). Storage ( $S$ ) and flow ( $Q$ ) are decision variables. Eq. (1) separates storage and flow variables to dynamically calculate head depending on storage in the NLP model. A single decision variable (flow) is used in the LP model

$$\text{Total Revenue} = \sum_r^R \sum_t^T \rho g \eta_r H(S)_{r,t} Q_{r,t} P_t \Delta t \quad (1)$$

where  $r$  = plants (or reservoirs);  $R$  = total number of plants in the network;  $t$  = time step; and  $T$  = total modeling horizon.

A third-degree polynomial relationship, suitable for rectangular and sloped reservoirs, is used to represent head as a function of storage for variable-head power plants. Polynomial parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $c$  are specific for each power plant and fit using observed storage and elevation data [Eq. (2)]

$$H(S) = \alpha S^3 + \beta S^2 + \gamma S + c \quad (2)$$

Because hydropower plants are often connected with streams or canals and water is allocated among them, multireservoir hydropower modeling fits a network flow framework. A typical hydropower network contains nodes and links, where nodes denote power plants and junctions, and links denote streams, canals, or pipelines. The general representation of a hydropower network flow representation contains an objective function to be maximized

$$\max_X z = \sum_i \sum_j f(X_{ij}) \quad (3)$$

subject to constraints

$$X_{ij} \leq u_{ij}, \quad \forall (i, j) \in \mathbf{A} \quad (4)$$

$$X_{ij} \geq l_{ij}, \quad \forall (i, j) \in \mathbf{A} \quad (5)$$

$$\sum_i X_{ji} - \sum_i a_{ij} X_{ij} = 0, \quad \forall j \in \mathbf{N} \quad (6)$$

where  $(i, j)$  indexes = origin and terminal nodes in time and space;  $X_{ij}$  = flow from node  $i$  to node  $j$  (decision variable);  $a_{ij}$  = amplitude and represents losses, such as evaporation and seepage;  $f(X)$  can be a linear or nonlinear objective function; Eqs. (4), (5), and (6) represent upper-bound, lower-bound, and mass balance constraints, respectively;  $\mathbf{A}$  = matrix of links (arcs); and  $\mathbf{N}$  = matrix of nodes.

Fig. 1 illustrates a simple hydropower network for two time steps ( $t$  and  $t + 1$ ) and two plants ( $r$  and  $r + 1$ ) in series. Flow in the network originates from an artificial node called Source and ultimately leads to a node called Sink, for which Eq. (6) is skipped because there are only outgoing links from Source and incoming links to Sink. Adding more physical elements, such as reservoirs

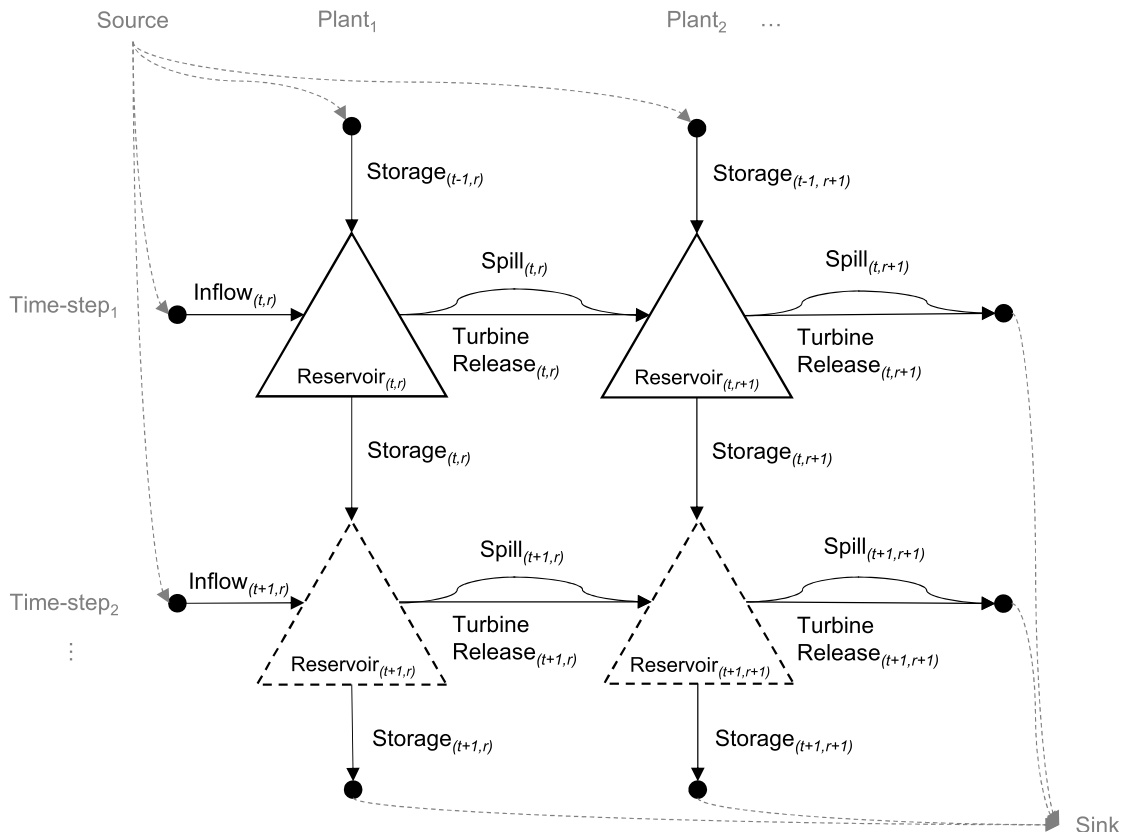


Fig. 1. Generic representation of the model with two time steps ( $t$  and  $t + 1$ ) and two plants ( $r$  and  $r + 1$ ) in series.

and canals, increases the model network horizontally, and adding more time steps to the network increases the network vertically, where the physical network is replicated and the only connection between time steps is reservoir storage [also discussed in Martin (1983)]. The expanded network flow based formulation is solved with NLP and LP methods and the hybrid method by sequentially running LP and NLP.

## NLP Model

Hydropower operations are substantially nonlinear, which is mostly from changing water head and storage. The NLP model dynamically represents water head based on the polynomial relationship [Eq. (2)] and for deterministic problems does not require further simplifying the objective function or constraints. To calculate head from storage, the NLP model separates flow  $X$  and storage  $Y$  decision variables. Converting Eq. (1) into a network flow framework, the NLP objective function becomes

$$\max_{X,Y} z = \sum_{m \in \mathbf{A}_F} \sum_{n \in \mathbf{A}_S} \rho g \eta_n (\alpha_n Y_n^3 + \beta_n Y_n^2 + \gamma_n Y_n + c_n) X_m p_m \Delta t - \sum_{m \in \mathbf{A}_F} p_m X_m \quad (7)$$

subject to constraints

$$X_m \leq u_m, \quad \forall m \in \mathbf{A}_F \quad (8)$$

$$X_n \leq u_n, \quad \forall n \in \mathbf{A}_S \quad (9)$$

$$X_m \geq l_m, \quad \forall m \in \mathbf{A}_F \quad (10)$$

$$X_n \geq l_n, \quad \forall n \in \mathbf{A}_S \quad (11)$$

$$\left[ \sum_i X_{ji} - \sum_i Y_{ji} \right] - \left[ \sum_i a_{ij} X_{ij} - \sum_i a_{ij} Y_{ij} \right] = 0, \quad \forall j \in \mathbf{N}_F, \mathbf{N}_S \quad (12)$$

where  $m$  and  $n$  = flow and storage links of the physical network;  $i$  and  $j$  = origin and terminal nodes in a given link;  $\mathbf{A}_F$  and  $\mathbf{A}_S$  = sets of flow and storage links;  $u$  and  $l$  = upper- and lower-bound constraints, respectively;  $a$  = amplitude; and  $\eta$  = overall plant

efficiency. There are two parts in the objective function [Eq. (7)]: the first part represents nonlinear hydropower revenue; the second (linear) part penalizes spills. Released water can go through either turbines and revenue is generated or spillways with penalties to minimize spills. Eqs. (8)–(11) enforce upper-bound and lower-bound constraints on flow and storage links, while Eq. (12) enforces mass balance at every flow ( $\mathbf{N}_F$ ) or storage ( $\mathbf{N}_S$ ) node  $j$ . The Pyomo modeling platform connects to user-defined solvers. In this study, the NLP model is solved with IPOPT, an open-source large-scale nonlinear programming solver.

## LP Model

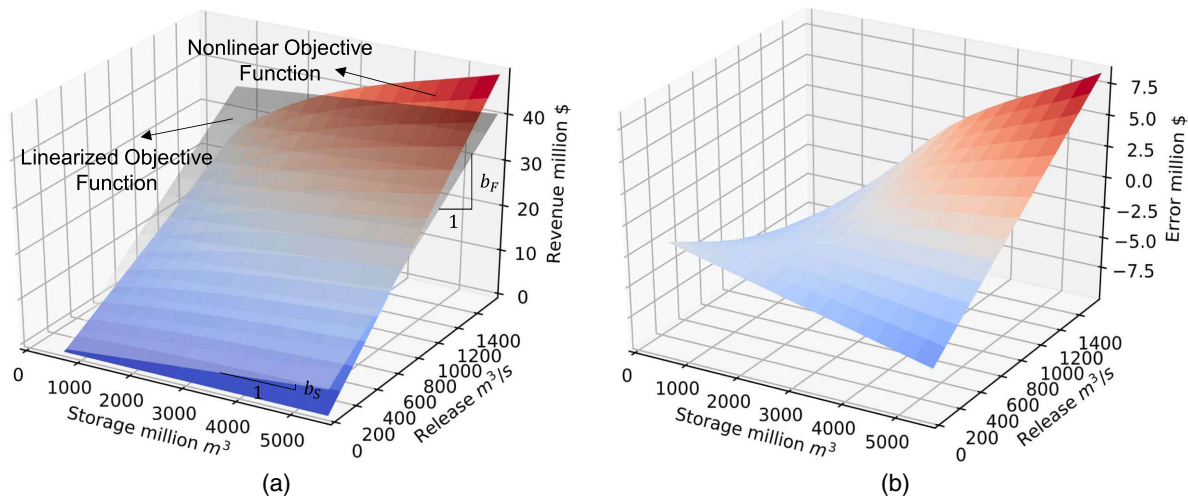
The LP model simplifies the nonlinear objective function by linearizing Eq. (1). Instead of dynamically calculating water head, power generation, and eventually revenue, the LP model uses prescribed unit benefit values  $b$ . These unit benefit values are calculated by fitting a linear surface [Eq. (13)] to nonlinear hydropower revenue curve [Eq. (14)] at each plant and time step, where coefficient of determination  $r^2$  is maximized (minimizing squared residuals) as shown in Eq. (15). In the equations,  $f_{LP}$  and  $f_{NLP}$  are linear and nonlinear two-dimensional hydropower revenue curves (objective functions) in Fig. 2(a)

$$f_{LP} = b_F X + b_S Y \quad (13)$$

$$f_{NLP} = \rho g \eta (\alpha Y^3 + \beta Y^2 + \gamma Y + c) X p \Delta t \quad (14)$$

$$r^2 = 1 - \frac{\sum_i (f_{NLP,i} - f_{LP,i})^2}{\sum_i (f_{NLP,i} - \frac{1}{N} \sum_i f_{NLP,i})^2} \quad (15)$$

where  $f_{NLP,i}$  is calculated using Eq. (1); and  $f_{LP,i}$  is calculated by optimizing unit benefits (slopes) of flow and storage,  $b$ , to maximize  $r^2$  for all plants and time steps. Keeping one corner at the origin, this process linearly approximates the nonlinear hydropower revenue curve with storage and turbine release decision variables. Linear benefits with optimized parameter  $b$  [used in Eq. (16)], nonlinear benefits, and errors between nonlinear and linear benefits are shown in Fig. 2. Errors are highest at corner points where storage and releases are maximum (plant capacity). Despite errors in the LP model, both models try to reach maximum storage and release, where benefit is the highest. The overall objective functions



**Fig. 2.** (a) Linear and nonlinear objective functions; and (b) errors showing residuals between nonlinear and linear curves for one plant and time step.

[Eqs. (7) and (16)] are a superposition of Fig. 2(a) for all plants and time steps.

After calculating  $b_{ij}$  by maximizing Eq. (15), the LP model's objective function and constraints can be written as

$$\max_X z = \sum_i \sum_j b_{ij} X_{ij} \quad (16)$$

subject to constraints

$$X_{ij} \leq u_{ij}, \quad \forall (i, j) \in \mathbf{A} \quad (17)$$

$$X_{ij} \geq l_{ij}, \quad \forall (i, j) \in \mathbf{A} \quad (18)$$

$$\sum_i X_{ji} - \sum_i a_{ij} X_{ij} = 0, \quad \forall j \in \mathbf{N} \quad (19)$$

where  $i$  and  $j$  = origin and terminal nodes in space and time;  $\mathbf{N}$  and  $\mathbf{A}$  = sets of nodes and links;  $X$  = flow (decision variable);  $b$  = unit benefit;  $u$  = upper bound;  $l$  = lower bound; and  $a$  = amplitude. The objective function [Eq. (16)] is a sum of benefits, and Eqs. (17)–(19) enforce upper-bound, lower-bound, and mass balance constraints, respectively. Parameter  $b$  is positive for turbine links, where revenue is generated and slightly negative for spill links to minimize revenue losses. The LP model is solved with GLPK, an open-source large-scale linear programming solver.

### HERO Hybrid LP-NLP Model

The HERO model uses the faster LP model to reduce iterations needed in the NLP model, with its better system representation but slower computing time. Both LP and NLP models use the same network, defined by the number of plants and time steps in the modeling horizon. Despite the LP model's accuracy losses and resulting errors discussed subsequently, LP efficiently points toward an optimal feasible area. In a sequential optimization, the same network problem is solved with a linear approximation [Eq. (15)], and then LP decision outputs are used to initialize an NLP optimization problem, sometimes called a warm start for the NLP. Any solution can help initialize the nonlinear model. Using LP outputs to warm

start the NLP model greatly reduces NLP run time. The current model represents six hydropower plants of California: Shasta, Folsom, New Melones, and Pine Flat have large storage capacities, while Keswick and Nimbus, located downstream of Shasta and Folsom, respectively, have small storage capacities. Dogan (2019a) provides details of modeled hydropower plants, such as storage and turbine release capacities and head-storage relationships. The model has a rather flexible temporal resolution. Given fine-resolution (hourly) energy price and reservoir inflow data, users can choose different time steps, such as hourly, daily, monthly, or a day-hour scheme.

### Comparing LP and NLP Decisions

Both LP and NLP models use the same network structure, similar to Fig. 1, but with slightly different representation of operations and equations. The NLP model separates flow and storage decisions and dynamically calculates head and hydropower revenue. The LP model has a single decision variable (flow) and uses unit benefit values  $b$  to calculate total benefit. If all nonlinearities of hydropower optimization are represented, the NLP model is assumed to perfectly represent operations. Accuracy losses occur with linearization of the objective function in the LP model. The linear objective function underestimates hydropower revenue when operations are at plant capacities (storage and release) and overestimates it when storage is high but release is low and storage is low but release is high (Fig. 2).

The LP and NLP models are compared for dry and wet periods. The dry period, with 3,541 decision variables, is from June 1 to September 1, 2018, with a daily time step and an average 23 m<sup>3</sup>/s reservoir inflow. The wet period, with 3,466 decision variables, is from January 1 to April 1, 2017, with a daily time step and an average 310 m<sup>3</sup>/s reservoir inflow. Initial and ending storage values are set to half of the storage capacity for modeled reservoirs. Differences in decision variable outputs are due to residuals (errors) shown in Fig. 2(b). Despite the same network and properties, different objective functions drive operations, resulting in output differences.

Fig. 3 compares LP and NLP model outputs (daily flow and storage in million cubic meters) for the dry and wet periods.

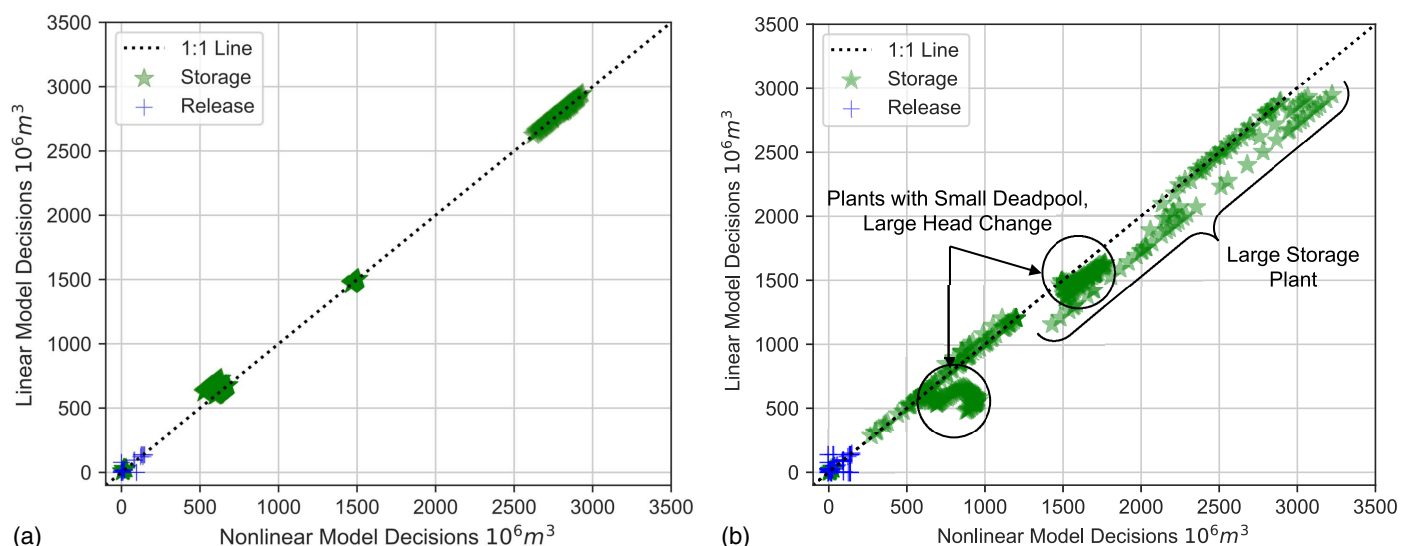


Fig. 3. LP and NLP model decision variables (daily release and storage) comparison for (a) dry; and (b) wet seasons.

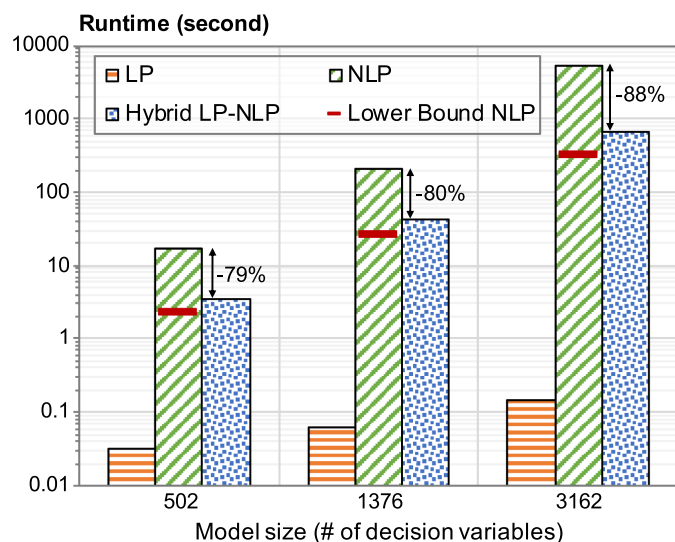


The lesser operational flexibility in the dry period leads to more agreement between LP and NLP model decisions. Wet period decisions span a wider range of decision values, and the LP model underestimates some high-storage decisions. Most large differences between the LP and NLP models are from only three of six reservoirs, under specific conditions. The LP decisions in the wet season are less reliable in two cases: (a) the largest differences are from reservoirs with low dead pool and large head variations (New Melones and Pine Flat), where nonlinearities are higher; and (b) reservoir with large storage capacity (Shasta), where the LP model mostly underestimates storage [for high-storage decisions, the LP objective function curve is below NLP, shown in Fig. 2(a)].

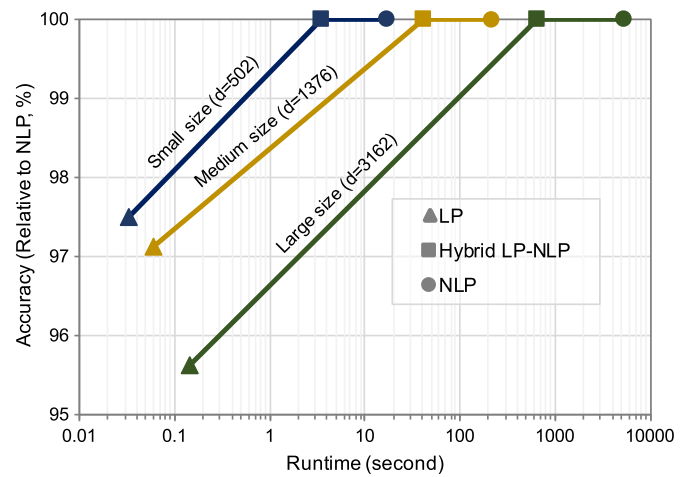
## Run Time Benchmarks

Model sizes can vary by the number of hydropower plants in the network, modeling period, and time step. More hydropower plants, longer time periods, and finer time steps increase the number of decision variables and model sizes. Each model is run for three model sizes with an increasing number of decision variables, 502, 1,376, and 3,162, respectively. All model sizes have six hydropower plants with different modeling periods. The hybrid LP-NLP model, which initializes the NLP model with the LP model's solution, reduces run time from the NLP model by 79%, 80%, and 88% in small, medium, and large models, respectively (Fig. 4). Initializing the NLP model with the NLP solution can represent a lower-bound NLP run time needed to verify that an initial solution (in this case from the NLP itself) is the optimal solution. Multiple LP iterations or other warm start techniques could not improve on this lower bound. Fig. 4 shows there is still some room (between the lower-bound NLP and the hybrid LP-NLP) to further reduce NLP iterations and run time if a better initial solution (than LP) is provided, such as piecewise LP or successive LP, especially for the large model size, but most computational gains are already realized.

Fig. 5 compares accuracy and solver run times of LP, NLP, and hybrid LP-NLP models. Accuracy losses in the LP model result from differences in the objective function and relative to the



**Fig. 4.** Solver runtimes of LP, NLP, and hybrid LP-NLP models and lower-bound NLP with different model sizes. Small, medium, and large models have 502, 1,376, and 3,162 decision variables. Solver runtime does not include time for model creation and postprocessing.



**Fig. 5.** Result accuracies and solver runtimes of LP, NLP, and hybrid LP-NLP models with three model sizes. Solver runtime does not include time for model creation and postprocessing.

NLP model in Eq. (20), where  $\hat{f}$  is the average objective function (revenue) value. Accuracy losses accumulate with larger LP sizes. LP run time increases exponentially, and grows much faster for larger NLP and hybrid LP-NLP models. The hybrid model, where the NLP model is initialized with LP outputs, significantly reduce run time of the NLP model without affecting accuracy

$$\text{Accuracy}(\%) = 100 \times \left( 1 - \frac{|\hat{f}_{LP} - \hat{f}_{NLP}|}{\hat{f}_{NLP}} \right) \quad (20)$$

The initialization with the LP model reduces the number of NLP iterations, while run time still increases exponentially and can pose a problem for long-term operations. The NLP model may not converge to an optimal feasible solution with many decision variables. In addition, price and inflow uncertainties increase as the modeling horizon increases. So, the LP model's ability to run for long periods in a short run time can become more important than the NLP model's better system representation of storage-dependent head for long-term planning decisions. The LP model also returns dual values for constraints that are useful for sensitivity analysis. These dual values show economic benefits of expanding existing capacity or adding new infrastructure to the network (Nover et al. 2019). The proposed hybrid LP-NLP model is more suitable for short-term operations, while the LP model is adequate for long-term planning with its better run time and other advantages.

## Limitations

Method limitations here include perfect hydrologic foresight, a single revenue-maximizing objective, and the assumption of exogenous energy prices. For short-term operations, 1–3 days with hourly time steps, operators often have good hydrologic foresight. However, for long-term operations or highly variable and uncertain short-term conditions, perfect hydrologic foresight may suggest overly optimistic operations, flattening effects of extreme events such as droughts and floods. Most modeled reservoirs are multipurpose, but their short-term objective is often to maximize hydropower revenue within water supply release schedules and flood operation constraints. Finally, a price-taking (or exogenous energy price) approach is employed, yet energy prices often fluctuate with

```

warmstart = True

# call hydropower model class
hp_model = HYDROPOWER(start,end,step,overall_average=False,warmstart=warmstart,
                      flow_path=flow_path,price_path=price_path,plants=plants)

# ***** Linear Model *****

# preprocess to create input network matrix
hp_model.preprocess_LP(plot_benefit_curves=False)
# create pyomo hydropower lp model
hp_model.create_pyomo_LP(datadir='model/data_lp.csv',display_model=False)
# solve lp model
hp_model.solve_pyomo_LP(solver='glpk',stream_solver=True,display_model_out=False,
                       display_raw_results=False)
# postprocess and save results as csv time-series
hp_model.postprocess(save_path='outputs/linear_model')

# ***** Nonlinear Model *****

# preprocess to create input network matrix
hp_model.preprocess_NLP(warmstart_path='outputs/nonlinear_model')
# create pyomo hydropower model
hp_model.create_pyomo_NLP(datadir='model/data_nlp.csv',display_model=False)
# solve the model
hp_model.solve_pyomo_NLP(solver='ipopt',stream_solver=True,display_model_out=False,
                        display_raw_results=False,max_iter=3000,mu_init=1e-9)
# postprocess and save results as csv time-series
hp_model.postprocess(save_path='outputs/nonlinear_model')

```

**Fig. 6.** Example code for creating linear and nonlinear models and warm start.

energy market conditions. Given the relatively small share of energy generation the selection of hydropower facilities represent, this assumption is expected to hold for a wide range of generating conditions and periods of analysis. To improve the model, more network elements and policy and operating constraints, such as minimum flow or storage requirements, can be added and explored from this basic model.

## Conclusion

A hybrid sequential linear and nonlinear hydropower reservoir optimization model was developed. Taking advantage of faster LP run times and a fuller system representation by an NLP model, the hybrid model runs the LP and NLP models sequentially, using LP outputs to initialize the NLP model to reduce its iterations and run time. This warm start initialization significantly reduces run time for the NLP model without affecting accuracy of results. The hybrid LP-NLP run time might be further reduced up to a lower-bound NLP time needed for an NLP to verify an optimal solution if a more accurate solution than LP, such as piecewise LP or successive LP, is provided. The LP and NLP models also can run separately. Despite this reduction with a warm start, the hybrid LP-NLP model still requires much more run time than the LP model, especially for large model sizes. So, for short-term operations, where good system representation is important, the hybrid LP-NLP model can be preferred. For long-term operation planning and management decisions with many more decision variables and higher hydrologic and energy price uncertainties, the LP model or other solutions may be preferred.

LP and NLP decisions are similar in the dry season, with less operational flexibility. Because water availability increases in the wet season, differences between the LP and NLP models increase. The LP model is less reliable when water availability is high, head changes nonlinearly with storage, and storage capacity is large. Such hybrid modeling also can be applied to other NLP problems. LP's advantage of fast run-time calculation makes it a good candidate as an initial solution to NLP problems where run time is important.

## Appendix. Example Model Code

The HERO model (Dogan 2019b) was developed with Pyomo, a high-level optimization modeling language in Python. In the Python script file main.py, the warmstart = True option connects LP and NLP models created from HYDROPOWER() Python class (Fig. 6). When creating a hydropower model (hp\_model in Fig. 6), the file paths of reservoir inflow and energy prices and power plants must be defined. Class functions are defined in hydropower.py. LP and NLP also can run separately, without linking two models by turning warm start option off (warmstart = False). Once a model run is finished, the postprocess() Python function organizes outputs and creates time series of reservoir storage, release, hydropower generation, and revenue. Model runs are grouped in two categories: (a) short-term operating decisions with an hourly time step over a few days or a week period; and (b) long-term operation planning decisions with a daily or monthly time step over years.

## Data Availability Statement

Some or all data, models, or code generated or used during the study are available in a repository online in accordance with funder data retention policies (Dogan 2019b).

## Acknowledgments

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