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Vicarious Learning Through Dialogic Scripted and Unscripted Videos: Orientations and Problem-Solving Behaviors

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Vicarious Learning Through Dialogic Scripted and Unscripted Videos: Orientations and Problem-Solving Behaviors

A dissertation submitted in partial satisfaction of the requirements
for the degree Doctor of Philosophy

in

Mathematics and Science Education

by

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2023

The dissertation of Michael Foster is approved, and it is acceptable in quality and form for publication on microfilm and electronically.

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University of California San Diego

San Diego State University

2023

DEDICATION

For Kyle,

It is impossible to put into words the influence you have had on my life. Without your passion and interest in math, I would never have pursued a math degree. While our friendship was hard at times, I would not trade a moment of it for the world.

For Sav,

You were one of the few people I told I was applying to the MSED program. You were the first person I told when I got accepted. When I am at my lowest, I think of your infectious attitude (and laugh), and I am reminded how important mindset is. I don't know where I would be right now had our paths not crossed at DePaul, but I know I wouldn't be here.

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ABSTRACT OF THE DISSERTATION

Vicarious Learning Through Dialogic Scripted and Unscripted Videos: Orientations and Problem-Solving Behaviors

by

Michael Foster

Doctor of Philosophy in Mathematics and Science Education

University of California San Diego, 2023
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Vicarious learning research is a burgeoning area of inquiry, which examines the learning of students who observe and are engaged with video- or audio-taped presentations of other people engaged in learning (Chi et al., 2008). In such studies, the students or *vicarious learners* (VLs) are positioned as indirect participants in a dialogue featured in the video- or audio-recording. Several projects have identified features of dialogic videos that benefit VLs. However, an important question remains: Does the nature of the dialogue—whether it is scripted or unscripted—matter?

For this study, I created two sets of dialogic videos, one featuring the unscripted dialogue of two precalculus students and the other featuring a scripted dialogue between a teacher (myself) and a precalculus student. These videos capture the inquiry process of the direct participants (i.e., the talent) engaged in a task sequence emphasizing covariational reasoning and culminated in the construction of the sine function. Four pairs of VLs were assigned to view either unscripted or scripted videos over five research sessions.

This study posed two research questions that explored differences across the treatments: one about how the VLs oriented toward the talent, and the second about the problem-solving behaviors of the VLs. To answer the first question, I analyzed the data qualitatively, using a priori and inductive codes. To answer the second question, I conducted thematic analysis, as described by Braun and Clarke (2004).

Results indicated that the VLs viewing unscripted dialogic videos consistently evidenced an emotionally-involved quasi-collaborative orientation toward the talent, (i.e., they acted *as if* they were in a collaborative group with the talent) with regular displays of emotion (e.g., surprise). In contrast, the VLs viewing the scripted videos evidenced a cognitively- and emotionally-distanced orientation toward the talent. Thematic analysis revealed a difference in the pairs' problem-solving behavior across three themes: (a) *patterns of* [video] *use*, (b) *idea justification*, and (c) *idea management*. For example, the VLs viewing unscripted videos frequently negotiated with ideas from the talent. In contrast, the VLs viewing scripted videos frequently repeated a solution from the talent without attending to the underlying mathematical meaning.

Chapter 1: Introduction

Online technologies have transformed our lives but have yet to transform the learning of mathematics (Borba et al., 2016). With information, literally, at our fingertips, educational outcomes can be forever shifted. However, as universities around the country learned during the COVID-19 pandemic, online technology is an educational tool that we have not fully learned how to leverage. If trends continue, and institutions of higher learning do not adapt, enrollment will likely suffer (Hartocollis & Levin, 2020).

In order to buck this trend, it is crucial for educational researchers to better understand the mechanisms behind students learning online or using online tools in classes. There is little question that learning by doing, enculturation into a practice, and a myriad of other hands-on, person-to-person, educational experiences positively influence students (e.g., Freeman et al., 2014; Theobald et al., 2020). Unfortunately, that is not always an option (e.g., for health or economic reasons) and a growing number of students have been choosing to pursue higher education through online formats (Blair et al., 2018). Thus, educators must consider how to best serve students using technology as a mediator of their educational experience.

One prevalent mode of online learning is through the use of instructional videos. Instructional videos can take a number of different forms, including asynchronous lectures (e.g., MIT OpenCourseWare; https://ocw.mit.edu/), virtual whiteboard recordings (e.g., Khan Academy; https://www.khanacademy.org/), animated students solving problems (e.g., Smile and Learn; https://www.smileandlearn.com/en/), or even real students solving problems (e.g., Project MathTalk; www.mathtalk.org). These instructional tools allow students to view content at their own pace and repeatedly (Lin & Michko, 2010; Vidergor & Ben-Amram, 2020), decentralize access to information (Parslow, 2012), and present alternatives to traditional instruction (e.g., flipped classrooms, Fyfield et al., 2019). The more options the better, but some forms of instructional videos may be more effective than others, particularly for conceptually deep subject material like mathematics.

In this chapter, I first explore some of the affordances and limitations of the dominant model of instructional videos, followed by a discussion of how an alternative video model, dialogic videos, has attempted to overcome these limitations. Second, I present a vignette illustrating the need for inquiry into how the nature of the dialogue of dialogic videos influences viewers. From there, I will explore the position of viewers of dialogic videos as vicarious learners and provide an overview of the empirical research on vicarious learning. Then I will provide an overview of the mathematics content of trigonometry, the content I will emphasize within my dialogic videos. Finally, I conclude with an introduction to the research questions that guided this dissertation.

Instructional Videos

Dominant Model

The most common and widely used instructional videos within mathematics education are those that follow the model set forth by Khan Academy (Bowers et al., 2012). This form of instructional video contains a voice, or “talking hand,” describing a procedure that is presented on a virtual whiteboard. As evidenced by Khan Academy’s vast library of virtual whiteboard videos, one affordance of this form of video is the ease of production. Under the goal of providing open access to content, the virtual whiteboard model of instructional videos has been extremely successful with millions of worldwide users (Kelly & Rutherford, 2017; Noer, 2012).

The main issue with this form of video—particularly for content covering mathematics—is its emphasis on procedures (Bowers et al., 2012; Klinger & Walter, 2022). With videos typically lasting less than 10 minutes, it is difficult for deep conceptual meaning to emerge for the presented mathematics (Danielson & Goldenberg, 2012). Within that time students can be reminded of or learn new procedures (e.g., how to find a derivative). However, establishing conceptual understanding takes more time (e.g., that a derivative is a function for instantaneous rate of change). Instead of challenging the status quo through a new medium of technology, the virtual whiteboard style of instructional videos serves as reinforcement for the conception of mathematics as a set of facts to be memorized and enacted given the proper context (Bowers et al., 2012).

Another issue for virtual whiteboard videos, and other forms of online instructional videos (e.g., asynchronous lectures), is the lack of student voices (Lobato et al., 2016). Virtual whiteboard videos emphasize the exposition of content by a knowledgeable teacher, as opposed to the inclusion of inexperienced student voices. Student voices are important because they can resonate with the thoughts and difficulties of the viewer (Chi et al., 2017). As online instructional videos, and distance learning more generally, grow in popularity, students may be left with fewer opportunities to engage with and experience fellow students' voices (McKendree et al., 1998). Instructional videos could fill that void, but not if they remain dominated by a model that focuses on exposition.

Dialogic Videos

An alternative to the dominant model of instructional videos is that of dialogic videos. Foremost, a dialogic video contains people engaged in a *dialogue*. Following Alrø and Skovsmose (2004), I take dialogue to be defined as a conversation that involves "a process of inquiry" (p. 235). This means that conversations captured within dialogic videos contain the video participants' attempts at creating new meaning or new ways of experiencing. This form of instructional video contains, and emphasizes, students' voices. There are typically two people within these videos: one student and a teacher (e.g., Muldner et al., 2014) or two students (e.g., Lobato et al., 2017). Importantly, the voices of the students within these videos are engaged in a process of meaning making. In turn, this meaning-making process may produce deeper conceptual understanding for the viewer than simply watching the exposition of content.

The most common form of dialogic video contains one student and one teacher and are modeled after tutor/tutee relationships (Craig et al., 2000). In the videos, the teacher is positioned as a tutor, and the student is positioned as a tutee learning the material. The creation of these videos varies. Some forms of these dialogic videos contain post-production and editing (e.g., Muller, Bewes, et al., 2008) and others contain minimal editing, capturing real students working with tutors in authentic tutoring sessions (e.g., Craig et al., 2000). Within the videos, the tutee works on a set of tasks with the help of the tutor, similar to a tutoring session. At times the tutor asks questions to guide the discussion, and at other times the tutor provides information. Importantly, these interactions are dialogic and contain the tutee's inquiry process. This style of video has seen use in physics, computer literacy, and language acquisition (Craig et al., 2009; Craig et al., 2000; Muller, Bewes, et al., 2008). In particular, dialogic videos using tutor/tutee dynamics have gained prominence within physics educational videos as a means of presenting common alternative conceptions from the tutee that the tutor can then correct (Muller, Bewes, et al., 2008).

Similarly, dialogic videos that contain two students engaged in a dialogue emphasize the importance of student voices and inquiry. There have been several recent efforts to create dialogic mathematics videos of this style (Kolikant & Broza, 2011; Lobato et al., 2017; Seethaler et al., 2020; Weinberg et al., 2022). The videos that have been most influential in my creation of videos for this study are those from Project MathTalk (Lobato et al., 2017). Within this form of video, students are given a set of tasks that they collaboratively work on and eventually solve. The videos of Project MathTalk showcase two synchronized videos: one video showing the two students engaged in a dialogue and the other showing their collaborative written work (Figure 1.1). Lobato et al. (2017) include an offscreen teacher that primarily serves to forward and guide the students. Emphasizing the students, the teacher does not add new information (other than the introduction of mathematical conventions) and does not appear on screen. Additionally, these videos contain post-production voiceovers and annotations. Again, these features do not add new information. Instead, the voiceover frames and recaps the content of each video. Similarly, annotations are used to highlight and emphasize specific features of the students' work (e.g., the red and blue arrows, dot/line, and words in Figure 1.1 are post-production annotations that appeared synchronously with the students' utterances).

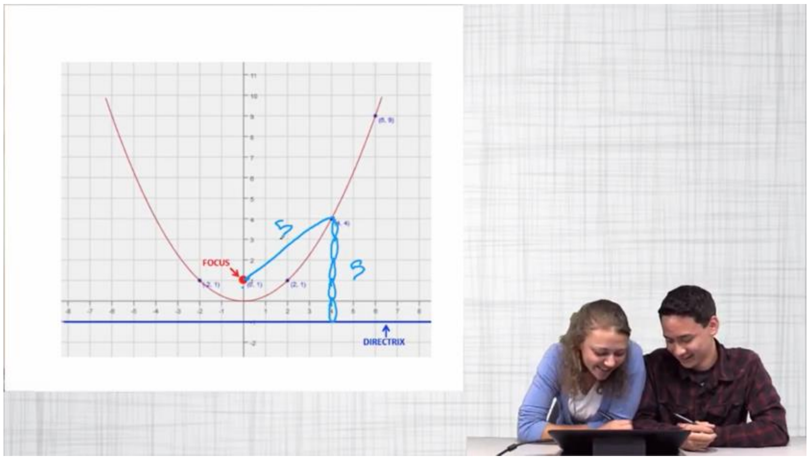


Figure 1.1
Image from Project MathTalk of Two Students in Dialogue

Features of Videos

Across all forms of instructional videos are a number of features that can benefit the viewer. In the following section, I will present an overview of several features identified within the literature that designers of videos, and dialogic videos in particular, can implement to best serve video viewers. Then I pose a question about a feature of dialogic videos in need of inquiry: Should the dialogue be scripted or unscripted?

Features

One feature is the length of the video. Khan (2012) suggests videos should be short (less than 10 minutes). Videos exceeding this length are less enjoyable, lose students' attention, and negatively impact learning (Slemmons et al., 2018). Further, viewers of the videos should be able to pause and replay segments or entire videos (Hampton, 2002). Finally, videos should contain some form of post-production that can draw attention to important features that viewers may otherwise not attend to (Trenholm et al., 2016).

Across the forms of dialogic videos presented above, the literature suggests a number of beneficial features. Foremost, dialogic videos should contain *alternative conceptions* and *productive struggle* (Baumeister et al., 2001; Chan et al., 1997; Chi et al., 2017; Tree & Mayer, 2008; Lobato & Walker, 2019; Muller et al., 2007; Muller, Bewes, et al., 2008). Following Smith et al. (1998), alternative conceptions are the ideas and beliefs that do not align with the accepted conception (i.e., the “correct” conception), but that can be a resource for progression towards an accepted conception. Productive struggle is the process of going from alternative to conventional conception. Alternative conceptions—and the subsequent negotiation process that resolves conflicting conceptions—can directly confront viewers’ beliefs (Muller et al., 2007; Muller, Bewes, et al., 2008) and can serve as a model for engaging in dialogue (McKendree et al., 1998). Similarly, productive struggle can demonstrate the social nature of learning and model for viewers that not knowing an answer is a part of being a learner (Chi et al., 2017; McKendree et al., 1998). In two forms of dialogic videos—student-tutor versus student-student—a difference emerges in the nature of the productive struggle and the eventual resolution. In the former, the tutor plays the role of correcting and directing the dialogue toward the correct conception. In the latter, the students engage in a lengthier process where they engage in argumentation and negotiation to arrive at an answer. Importantly, both dialogic formats can contain multiple alternative conceptions and eventual resolutions.

Another important feature of dialogic videos is that the speakers within the videos appear similar to the viewer (e.g., similar age, ability level, etc.) because viewers have been found to attend most to the words and actions of the individuals that most closely resemble themselves (Braaksmaet et al., 2002; Fowler & Mayes, 1999; Groenendijk et al., 2013; Mayes, 2015). For student-tutor and student-student dialogic videos, this suggests that the students contained within the video should be similar to the target audience of the video. For example, if the dialogic videos are covering calculus for undergraduates, then the student(s) within the videos should be undergraduates learning calculus. Unfortunately, the tutor, by definition, cannot match the ability level of the viewer. This suggests that student-student dialogic videos may position viewers to attend to more words and actions contained within the videos because the videos can contain more similar students than student-tutor videos.

Scripted Versus Unscripted Dialogue

One feature of dialogic videos that has not yet been explored empirically is whether or not the dialogue of the videos should be scripted or unscripted. Unscripted dialogue has been posited to be beneficial and more enjoyable to the viewers (Geertshuis et al., 2021), but to date, this has not been supported empirically. In the projects using student-tutor dialogic videos (e.g., Lobato et al., 2019), there is a bifurcation between studies implementing scripted and those implementing unscripted dialogue. Muller and colleagues made use of scripted dialogues, arguing that scripting is a way to ensure their physics dialogic videos contain common misconceptions (Muller et al., 2007; Muller, Bewes, et al., 2008). Chi and colleagues, on the other hand, captured authentic, unscripted, dialogue in their presentation of unedited tutoring sessions as a means of easily creating and disseminating tutoring experiences (Muldner et al., 2014) and because authentic confusion plays an important role in viewers' learning (Chi et al., 2017). Similar to Chi and colleagues, Lobato and colleagues, in their student-student dialogic videos, captured unscripted dialogic interactions and authentic alternative conceptions and confusion (Lobato et al., 2017). Similar to Muller and colleagues, Lobato and colleagues' videos involved a lengthy post-production process, which can be useful for drawing viewers' attention to specific features of the videos (Trenholm et al., 2016).

Importantly, both unscripted and scripted dialogic videos can be created to contain alternative conceptions, as these are an important feature (Baumeister et al., 2001; Chan et al., 1997; Chi et al., 2017; Lobato & Walker, 2019; Muller et al., 2007; Muller, Sharma, et al., 2008; Tree & Mayer, 2008). The role that alternative conceptions play for scripted and unscripted dialogic videos may differ, though. Particularly, the model of dialogic video created by Muller and colleagues makes use of scripted dialogue as a means of incorporating common alternative conceptions found within the literature (Muller et al., 2007; Muller, Sharma, et al., 2008). When a viewer hears a conception they hold presented and eventually refuted within a dialogic video, they may learn the correct conception. This was modeled after the conceptual change literature

(Muller, Sharma, et al., 2008). Muller and colleagues enacted this form of dialogue through a student-tutor model that saw the student presenting a scripted incorrect conception that the tutor then corrected.

A number of possible issues emerge from Muller and colleagues' model of scripted alternative conceptions being presented by a student and resolved by a tutor. One issue with this model of scripted dialogue lies in the norms it may establish. The tutor always presenting the correct conception can establish a norm for the viewers that the tutor is the only one who needs to be attended to and whose ideas need to be engaged with. If the options were between a student who frequently presents incorrect answers and a tutor who presents the correct ones, then it is inevitable that the viewer will favor the latter. This is different from a possible norm established in student-student dialogue (à la Project MathTalk), where either student may have the correct conception. If either student could present the correct conception, then both ideas must be attended to. Furthermore, if the pair of students are engaged in the co-construction of a solution, then the viewer's understanding requires engaging with both students' ideas.

Another possible issue arises when an alternative conception from the literature is presented by a student in a scripted dialogue and the viewers do not currently believe that conception. Knowing the tutor will eventually present the correct conception, the viewer can wait and ignore possibly confusing alternative ideas. If, instead, a conception emerges within the dialogue between two students that the viewer does not believe, then the viewer is positioned to not know whether or not it is accurate. To determine the accuracy, the viewer will have to engage with all conceptions and attend to the resolution between the two students.

Furthermore, I believe the scripted nature of dialogic videos pose several limitations. Scripted videos, whether they involve a student and a tutor or two students, have a possible issue in the nature of their scripted resolution. When a student-tutor or student-student pair enacts a scripted resolution, it is possible that the dialogue is not convincing for the viewers. If the viewer is not convinced, they are, at best, made aware that the initial concept was wrong and the final one is correct. Without being truly convinced of the answer presented within the script, the viewer may be left to take the conception provided axiomatically and without negotiation. While this may still occur for unscripted dialogues, I believe the presence of authentic, unscripted, student problem solving and resolution better positions viewers. This may be the case because viewers of dialogic videos attend more to the words of the students (Braaksma et al., 2002; Fowler & Mayes, 1999; Groenendijk et al., 2013; Mayes, 2015). Thus, if a resolution is presented by a student in that student's own words, as opposed to the words from a script, then the viewer is more likely to attend to it. Furthermore, Geertshuis et al. (2021) claim that the authenticity of unscripted dialogue is perceptible and appreciated by viewers. This implies that the inauthenticity of scripted dialogue may be equally perceptible. If a viewer perceives that the scripted student is convinced inauthentically, then a norm could be established for the viewer that it's okay to not be convinced and to just go along with the final resolution.

Finally, I believe that unscripted dialogue allows for the presence of more convincing resolution through authentic problem solving. Schoenfeld (2016) characterizes mathematical problem solving as containing cognitive processes and strategies used to identify, consider, and implement ideas to solve novel tasks. While it is possible to carefully script a dialogue so that it captures these features of problem solving, cognitive processes and strategies are necessarily contained in unscripted dialogue. If students are going to resolve their authentic confusion, then they will need to engage in problem solving. Students in scripted dialogue, on the other hand, can present the correct resolution without engaging in problem solving to produce that resolution. These possible limitations suggest that an empirical inquiry into the use of unscripted versus scripted dialogue is needed.

Summary

In sum, instructional videos offer a range of opportunities for learning. Videos following a virtual whiteboard format (à la Khan Academy) have successfully capitalized on a number of those opportunities (e.g., by offering open access to a vast library of educational resources). Ultimately, virtual whiteboards have been overly focused on procedures and exposition, resulting in an emphasis on a breadth of topics over depth (Bowers et al., 2012). Dialogic videos, on the other hand, put an emphasis on meaning-making, foregrounding the depth required for conceptually rich content like that of mathematics. While a number of important beneficial features of dialogic videos have been identified from the literature (e.g., the presence of misconceptions), one potentially influential feature, pertaining to the nature of the dialogue, remains untested—should the dialogue be scripted or unscripted?

Vignette

I present a vignette to illustrate a possibly unique affordance of unscripted dialogue: viewers of unscripted dialogic videos may be more open and willing to negotiate with alternative conceptions emerging from the dialogue of authentic and unscripted students. This willingness is evidenced by two viewers of unscripted dialogic videos, Desiree and Belinda. They show an ability to reproduce and even show affinity for an argument that they believe to be wrong. I will argue that if the dialogue of the videos had been scripted, the willingness to negotiate with what they believe to be a counterfactual would likely not have been shown.

Background

The vignette presented here is part of a data set collected by the Project MathTalk team during the summer of 2017 (for more details see Lobato et al., 2023). Within the study, two Grade 9 students, Desiree and Belinda, worked together through a series of tasks aimed at the construction of the formula for a general parabola starting from the geometric definition of a parabola. Mathematically, the goal was to derive the equation $y = \frac{(x-h)^2}{4p} + k$ (where the point (h,k) is the vertex, the point (h, $k+p$) is the focus, and the line $y = k-p$ is the directrix) using the geometric definition that a parabola is the set of points that are equidistant from a fixed point (called the focus) and a fixed line (called the directrix). Desiree and Belinda were guided through these tasks by a series of dialogic videos that contained a pair of students, called the *talent*, working towards the same goal. The talent, Sasha and Keoni, were high school students who engaged in unscripted dialogue as they attempted to make use of the geometric definition of the parabola. I use the term “talent,” in the sense of someone starring in a Hollywood film, not in the evaluative sense of “talented.”

This vignette takes place during Desiree and Belinda’s fifth session working with the videos. By this point, the participants had constructed a number of parabolas using the geometric definition. Furthermore, they had been able to derive, and confirm with the videos, equations for parabolas with a vertex at the origin when they were given the location of the focus (e.g., given a focus of (0,1) they knew the equation for the parabola was $y = \frac{x^2}{4}$, and if the focus was (0,2), they knew the equation was $y = \frac{x^2}{8}$). At the end of the fourth session, the participants were given a focus at (0,3) and, due to an error in their derivation (Figure 1.2), believed the equation of the parabola would be $y = \frac{x^2}{18}$. If they had completed their derivation without this error, their work would have produced the equation $y = \frac{x^2}{12}$.

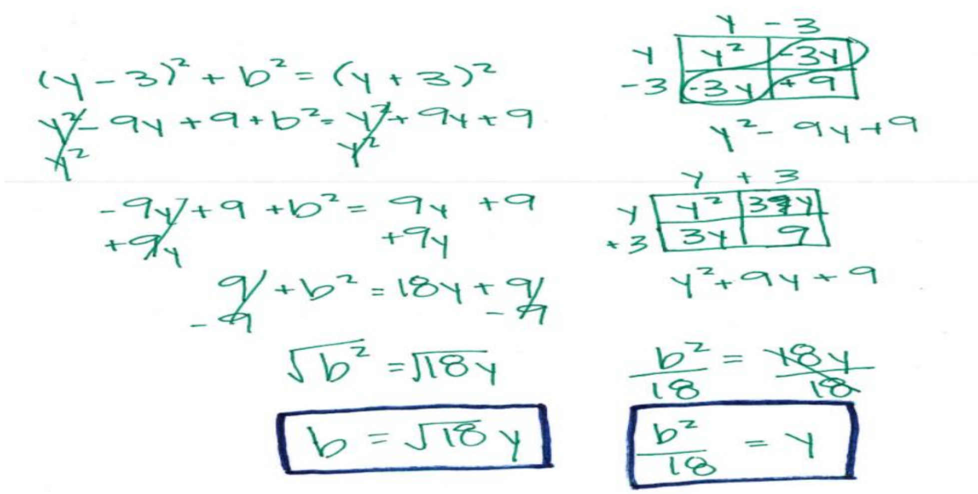


Figure 1.2
Error in Desiree and Belinda’s Calculation

I will begin by presenting an account of what occurs in the dialogic videos by describing the talent’s work. This will include transcripts of the dialogue from the dialogic videos. Then, I will describe the subsequent reaction to the talent's work by the viewers, Desiree and Belinda. Finally, I will discuss the way in which unscripted dialogue uniquely positions viewers to consider and negotiate with the dialogue of the dialogic videos.

The Talent

Research Session 5 began with Desiree and Belinda being told that the talent have not yet worked with the parabola that has a focus of (0,3), but that Sasha and Keoni are going to make a prediction based on the equations and shapes for the parabolas $y = \frac{x^2}{4}$ and $y = \frac{x^2}{8}$. Given this prompt, Desiree and Belinda started watching the video. Within the video, the talent created two predictions for a parabola with a focus at (0,3): $y = \frac{x^2}{16}$ and $y = \frac{x^2}{12}$. Sasha made the prediction that the equation will be $y = \frac{x^2}{16}$, and Keoni made the prediction of $y = \frac{x^2}{12}$.

Teacher:	What's the thinking behind each one of those [$y = \frac{x^2}{16}$ and $y = \frac{x^2}{12}$]?
Sasha:	I said 16 [on the denominator] because four times two and then eight times two.
Keoni:	But I thought four times three because this one was four [points to focus at (0,1)] this one was eight [points to focus at (0,2)] and then this one is twelve [points to focus at (0,3)].

The talent suggested two possible patterns. The first pattern was noticed by Sasha. She noticed a pattern where each denominator can be multiplied by two to find the next. She said that the first denominator, four, times two gives the second denominator of eight. She then predicted that the next denominator would be eight times two (or 16). The second pattern was noticed by Keoni. He noticed a relationship between the *y*-coordinate of the focus and the value in the denominator. Keoni noted when the focus was at (0,1) the denominator was ($4 \cdot 1$) and when the focus was at (0,2) the denominator was ($4 \cdot 2$). Thus, he predicted when the focus is (0,3) the denominator will be four times three (or 12). Neither of these equation matches the equation that Desiree and Belinda derived (Figure 1.2) Furthermore, both patterns noted by the talent are valid inferences from a two-point data set (i.e., given the numbers four then eight and being asked to produce the third number in the sequence). Consequently, the predictions by Sasha and Keoni are both reasonable. Before the talent can resolve their differences, the video ends and the viewers are left without a resolution. [In a later video, the talent check their predictions by deriving the new equation and determine that Keoni was correct.]

The Viewers

After watching this episode, Desiree and Belinda think that the talent are wrong. Through their previous work (Figure 1.2) they believe that the denominator should be 18. Significantly, even though Desiree and Belinda believe that their solution is correct and that both predictions from the talent are incorrect, they are able to recreate and negotiate with the talent's predications:

Belinda:	It's [the denominator] 18.
Researcher:	So you don't think either of their predictions are correct? What do you [Desiree] think?
Desiree:	It's gonna be 18 and they're not correct.
Researcher:	How did they get those two predictions?
Belinda:	Because they saw that…
Desiree:	it's multiplied by two.
Researcher:	What's multiplied by 2? Go ahead and write down what they're working with here.
Desiree:	*y* equals *x* squared… This [points to the 4 in the equation $y = \frac{x^2}{4}$] times 2 equals 8 [writes 4*2=8]. And this [while sweeping from (0,0) to (0,2)] is 1 more than this [sweeps from (0,0) to (0,1)].
Researcher:	Interesting. And so, for the next parabola, which is going to be this orange one, that has a focus of a distance of three from the vertex, what were the two predictions?
Desiree:	*y* equals *x* squared over 16 and *y* equals *x* squared over 12.
Researcher:	And how did they get both of those? Who had what? Sasha had which one?
Belinda:	Sasha had 16
Researcher:	What was her thinking?
Belinda:	She got that by multiplying by 2.
Desiree:	…by the 8.

Initially, Desiree and Belinda confidently acknowledged that they think both talent are incorrect. Then, when prompted, they were able to recreate Sasha's conjecture in two ways. First it was recreated by Desiree's coordination of four becoming eight when multiplied by two and then again it was recreated by Desiree and Belinda's coordinated response about Sasha's multiplying the eight by two. Taking Sasha's thinking further, the predication was also expanded upon with Desiree's sweeping gesture for the increase of one unit in the *y* coordinate of the focus. Desiree's sweeping gesture appeared to add a coordination between the change in the focus' location to the change in the denominator. Despite thinking Sasha is wrong, Desiree demonstrated an understanding of Sasha's idea in her explanation and evidenced a negotiation with Sasha's idea by expanding upon Sasha's prediction with a coordination of the change in the focus with the change in the denominator.

Desiree and Belinda's recreation of the talent's predictions continued. This time Belinda re-presented Keoni's argument and, in conclusion, Desiree and Belinda demonstrated an affinity for Keoni's argument–despite them reaching different solutions:

Researcher:	What was Keoni's thinking?
Belinda:	Keoni thought since it's 4 then add a 4 then add another 4, which would make 12.
Researcher:	If you didn't already know what it was, which one would you find more compelling?
Both:	[in unison, both Desiree and Belinda quickly point to Keoni's $y = \frac{x^2}{12}$ and giggle].

In this segment Belinda recreated Keoni's prediction. Belinda claimed that Keoni was adding four to the previous denominators. Belinda made a mathematically equivalent argument (multiplication versus repeated addition) for Keoni's pattern. At the same time, this version of Keoni's pattern lacked the coordination between the focus and the denominator that he used, but it still produced the same conjectured pattern of values in the denominator. Because Belinda did not merely reproduce Keoni's argument, it suggests that Belinda and Desiree have negotiated with his dialogue. At this point, both of the talent's arguments have been recreated by Desiree and Belinda, and these recreations suggest that negotiation is occurring with the dialogue of Sasha and Keoni.

Negotiation continued when Desiree and Belinda were asked which prediction they found more compelling. Desiree and Belinda quickly, in unison, and with a show of emotion (laughter) selected Keoni's pattern. Despite still thinking that both Keoni and Sasha were incorrect, Desiree and Belinda were able to weigh the differences between the predictions and find one more compelling than the other. This suggests a form of negotiation has occurred.

Discussion

Despite Desiree and Belinda Initially thinking that both of the predictions from the talent were incorrect, they were able to (a) recreate the logic behind each prediction, and (b) find one prediction more compelling than the other. Given that both of the predications are true for a pattern of two numbers (i.e., four plus four and four times two get you eight, thus the pattern could be linear or exponential) there is no way to accurately determine, at this stage, which prediction is correct. In fact, the derivation process Desiree and Belinda went through would suggest to them that both of the talent's predictions are incorrect, and equally so, because the talent did not predict a denominator of 18.

Despite believing both presented predictions are incorrect, the viewers of the unscripted dialogic videos negotiated with, and even extended the reasoning of, the talent's predictions. This is particularly interesting because the resolution of differences between Sasha and Keoni's ideas is a norm established throughout the four previous viewing sessions that Desiree and Belinda were a part of. While the video Desiree and Belinda viewed in this vignette did not contain a resolution, they previously experienced several instances where Sasha and Keoni had a difference of opinion that was eventually resolved. Despite this, Desiree and Belinda were convinced of their solution and claimed that both of the talent's predications were incorrect. Interestingly, Desiree and Belinda were able to, then, set their solution aside and think about which prediction they liked more. This was evidenced with their laughter and quick reaction towards Keoni's prediction when probed by the researcher. I believe that this setting aside and considering a (perceived) counterfactual would be unlikely to occur if the videos do not foster negotiation, and I believe scripted videos would not do so.

Foremost, if the patterns presented by Sasha and Keoni were presented by a tutee and a tutor, respectively, then the viewers would likely revisit their own work and doubt their own predictions. As I argued above, if a norm is established within the dynamics between a tutee and a tutor that the tutor will present the correct solution, then a viewer will primarily attend to the tutor's ideas. If, as was the case in this vignette, the viewers' solution was not presented by the tutor, then they would have reason to assume that their answer was incorrect. There would be no need to weigh counterfactuals or consider which solution they like more. Instead, efforts would be focused on coming to a solution that aligned with the tutors. This process would require little negotiation of the pattern presented by the tutee.

Furthermore, the negotiation that produced an extension of one of the talent's ideas would be unlikely to emerge. In this vignette, the prediction that will ultimately be rejected is the one that Desiree negotiated with and produced an extension of. If this prediction was presented by a tutee in a tutee-tutor video, then it may have been rejected and not negotiated with as the viewer waits for the tutor's prediction. Additionally, if a norm is established that the tutor presents the correct ideas, then the viewer may not think it is possible to add to or extend the ideas presented by the tutor. Instead of negotiating with the tutor's correct solutions, the viewer may focus on merely reproducing the tutors' work.

Geertshuis et al. (2021) claim that the authenticity of unscripted dialogue is perceptible and appreciated by viewers. I believe Desiree and Belinda's willingness to reject both predictions from the talent supports this claim. Because the talent were perceived to be authentic students working through the same set of tasks, Desiree and Belinda had no reason to think that the talent's ideas were any better than their own. This meant, to the viewers, that the talent could be wrong and that the talent's ideas were open for negotiation. Furthermore, because the talent frequently engages in their own negotiation process, it is possible that the viewers' negotiations are an enactment of behaviors they learned from using the videos. If Geertshuis and colleagues' claim is true and if it extends in the opposite direction (i.e., inauthenticity of scripted dialogue is equally perceptible), then viewers of scripted videos have little reason to negotiate with scripted dialogue. Instead of attempting to negotiate with ideas that appear to be set up only to be knocked down and replaced by the conclusion of the video, the viewer can instead wait until the end of the video where a culminating idea is presented.

Vicarious Learning

In this project I frame the observers of dialogic videos as *vicarious learners*. *Vicarious learning* is defined by Mayes (2015) as the indirect participation in, or observation of, a process of learning mediated by technology. For dialogic videos, a vicarious learner (VL) is then the person, or persons, indirectly participating in the dialogue presented in videos, animations, or audio files. In the following section, I provide an overview of the small but emerging empirical literature on vicarious learning. Specifically, I explore the findings on the orientation of VLs to the talent in the videos, the learning outcomes of VLs, and the behaviors of VLs while engaged with dialogic videos (see Chapter 2 for more details).

Orientation of Vicarious Learners to the Videos

One question that emerges from the theoretical literature pertains to VLs' orientation toward the direct participants in the video or audio dialogues. The Scottish Vicarious Learning Project (SVLP) posited that VLs experience the orientation of *epistemic detachment* (McKendree et al., 1998). Epistemic detachment is a lack of emotional and cognitive connection to the dialogue. Because VLs are free from the pressure to participate directly in a dialogue, they can emotionally and cognitively remove themselves. With this metaphorical distance, an argument or idea from the dialogue can be neutrally evaluated. Emotionally, the viewer need not be concerned that their own idea needs to be presented in front of others. In theory, it does not matter to the VL whether an argument or idea they overhear is valid because they have no personal stake in it; thus, they can focus on evaluating it. To date, there have been no empirical findings reporting experiences of epistemic detachment.

In fact, the empirical findings presented by Project MathTalk suggest that epistemic detachment may not be omnipresent for vicarious learners. Specifically, Lobato and Walker (2019) found that the VLs in their study were not emotionally detached from the talent in the unscripted dialogic videos, nor did the VLs orient toward the talent as neutral observers. Instead, the VLs were reported as experiencing an orientation of *quasi-collaboration*, meaning that the VLs acted as if they were in a collaborative group with talent. While coding for experiences of quasi-collaboration, a number of emotions were also identified (e.g., warmth, trust, surprise, delight, and shared discomfort). This suggested that the VLs had a different orientation to the participants in dialogic videos than that of epistemic detachment.

The findings from Lobato and Walker (2019) are limited, but they warrant further exploration. Lobato and Walker (2019) reported on a single pair of Grade 9 students working together through their series of dialogic videos covering the math content of parabolas. Thus, further work can be used to expand the range of orientations towards the participants in videos, support or refute quasi-collaboration as an orientation, and provide supporting evidence for or against epistemic detachment. One way in which this can be done is by exploring different content in the videos and different age levels of VLs. It is an open question as to whether older or younger students would be less or more inclined to experience an emotionally-involved quasi-collaborative orientation toward the video participants. For instance, it is possible that undergraduate students viewing dialogic videos containing content specific to their skill level (e.g., college trigonometry, college calculus, etc.) would be more interested in answers than younger viewers and have a cognitively and emotional distance orientation toward the students in the videos.

Learning Outcomes

A number of research projects have sought to establish the effectiveness of vicarious learning through indirect participation in a dialogue via quantitative comparative studies of learning outcomes. This is motivated, in part, by the theoretical assumption of the Scottish Vicarious Learning Project (SVLP). The SVLP suggested that vicarious learning can be enhanced when the learning processes VLs indirectly participate in are dialogic (McKendree et al., 1998). Within these empirical studies, a commonality was the comparison of a treatment that used recordings (e.g., videos, audio tapes, etc.) that contained a single speaker (i.e., that were monologic) to recordings that contained multiple speakers (i.e., that were dialogic). The findings from these studies tend to be supportive of vicarious learning through dialogue, but there are conflicting results.

Findings

The majority of studies assessing the efficacy of vicarious learning support the claim that dialogues are better than monologues (e.g., Chi, Kang, et al., 2017; Chi, Roy, et al., 2008; Cox, et al., 1999; Craig et al., 2009; Craig et al., 2004; Driscoll et al., 2003; Gholson & Craig, 2006; Muldner et al., 2014; Muller et al., 2007; Muller, Sharma, et al., 2008). For example, in a set of studies Chi and colleagues position their dialogic videos as tutoring sessions with one speaker being the tutor and all other speakers being tutees. They then compared the learning gains from viewers of the dialogic videos to viewers of a single tutor presenting the same set of information (Chi et al., 2017; Chi et al., 2008; Craig et al., 2009; Craig et al., 2004; Muldner et al., 2014). These studies have found statistically significant differences, with medium to small effect sizes, in favor of the pre/post-test learning gains of the treatments who engaged with resources that contained multiple speakers, suggesting that two speakers are better than one.

A number of projects have found conflicting results, with either no statistical difference between viewers of monologic and dialogic videos or a difference in favor of the learning gains of VLs who engaged with monologic videos (e.g., Cooper et al., 2018; Monaghan & Stenning, 1998; Muller, Bewes, et al., 2008). For example, Cooper et al. (2018) found a statistically significant difference favoring the students who viewed monologic videos of their professor. This suggests that the presence of multiple speakers is not a sufficient condition to improve learning from instructional videos.

Interpretation

I interpret the empirical findings on learning outcomes as suggesting that VLs are best served by viewing dialogic videos, but the inconsistent findings suggest that further inquiry into the use of dialogic videos is needed. As such, I believe a lens placed on VLs' problem-solving behaviors as they engage with dialogic videos can provide support for the use of dialogic videos.

Following Schoenfeld (2016), I take problem solving to be the cognitive processes and strategies that emerge through one's engagement with a novel task. Recall, the production of conceptual understanding is a motivating goal in the creation of dialogic videos. This positions dialogic videos to contain novel tasks that will engage VLs in problem solving. While monologic videos may similarly produce conceptual understanding and engage viewers in problem solving, dialogic videos uniquely capture problem solving. If dialogue is defined as including a process of inquiry (Alrø & Skovsmose, 2004), then dialogic videos contain participants' problem solving. Thus, dialogic videos can not only engage VLs in problem solving but can also capture problem solving of the direct participants of the dialogue. With a lens placed on VLs' problem-solving behaviors as they engage with dialogic videos, it is possible to draw a connection between the new meaning making that is a part of dialogic videos (i.e., the inquiry process of the dialogue, Alrø & Skovsmose, 2004) and the ways in which the VLs engage in their own processes of meaning making (i.e., the VLs' problem-solving behaviors). Thus, through the identification of a connection between problem-solving behaviors of VLs and the dialogic videos they engage with, further evidence can be found supporting the use of dialogic videos.

Behaviors

A small body of work has applied an analytical lens on the effect that dialogic resources (e.g., voice recordings, audio files, animated dialogues, etc.) have on the VLs' behaviors as they engage with the dialogic resources and the ways in which VLs use these resources. The findings from this literature suggest that the individuals contained within the vicarious learning resources serve as a model for the VLs. The findings are limited, but the behaviors modeled range from content-specific practices like the construction of grammar trees to broader practices like posing deep questions (Bandura, 1965; Braaksma et al., 2004; Craig et al., 2000; Chi et al., 2017; Kuhn & Modrek, 2021; Mayes et al., 2002; Rummel & Spada 2005; Schunk et al., 1989; Twyford & Craig, 2017). This suggests that VLs not only learn the content as indirect participants but also behaviors that can benefit them as learners.

These findings are limited in scope and have largely drawn on data from written responses from the viewers of dialogic resources (e.g., Braaksma, et al., 2004; Craig et al., 2000; Kuhn & Modrek, 2021; Mayes et al., 2002). What is missing is an expanded lens on the behaviors that can be modeled, an expansion into where these behaviors are manifested, and an inquiry into how dialogic resources are used. For instance, a question emerges how viewing dialogic videos influences VLs' problem-solving behaviors as they make use of videos that contain problem solving. Indeed, Weinberg and Thomas (2018) identified a gap in the literature on how students use instructional videos. Thus, a lens into how VLs use dialogic videos is an important first step in answering a broader question. While behaviors can be evidenced in online forum discussions or other written work (e.g., posing deep questions; Craig et al., 2000), future work can identify a broader range of modeled behaviors by placing an analytical lens on VLs' activities while they engage with and make use of dialogic resources.

Trigonometry

Before presenting the research questions that guide this dissertation, this study needs to be grounded in a particular mathematical topic at the undergraduate level (my domain of interest). To advance the field's knowledge of vicarious learning, the mathematical topic needed to be: (a) conceptually rich, (b) mathematically important, and (c) traditionally routinized. Investigation of students' learning about the sine function fits these criteria. Conceptual richness ensures that the videos contain more than a set of procedures. Construction of the sine function is conceptually rich because it requires, among other skills, complex coordination between two changing quantities. The sine function is mathematically important, because it serves as an introduction to trigonometry, radians, and a way of reasoning that is fundamental to calculus (i.e., covariational reasoning, Moore, 2012). Traditionally routinized content within dialogic videos can be re-explored, showing its true complexity and serving to change students' perception of what mathematics is. Trigonometry is content that most students would have been exposed to in high school and, as I will review below (and in further detail in Chapter 3), it is overly routinized into a set of procedures.

Traditional instruction on the trigonometric functions emphasizes one of two approaches: triangle-first or circle-first (Demir & Heck, 2012; Moore, 2012). The triangle-first approach makes use of sine, cosine, and tangent to find missing side lengths of a triangle. The circle-first approach emphasizes the use of rational number multiples of π as the arguments of sine, cosine, and tangent to find coordinates on the unit circle. Instruction emphasizing the relationships between a triangle, or a circle, and the trigonometric functions can lead to a number of issues. When students are instructed with the triangle-first approach the sine function is introduced with the mnemonic of SOHCAHTOA which tells students that sine *is* opposite over hypothenuse. This perspective leads students to the conception that triangles are the argument of the trigonometric functions, meaning students feel like they cannot evaluate an expression like sin(20) without a triangle (Demir & Heck, 2012; Moore, 2012). If, instead, sine is introduced using the unit circle, then sine is frequently introduced as the *y* coordinate of special points whose respective outputs should be memorized (for example, memorizing $sin(\frac{\pi}{2}) = 1, sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$, $sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$, and $sin(\frac{\pi}{6}) = \frac{1}{2}$) (Çekmez, 2020; Demir & Heck, 2012; Moore, 2012). Defining sine as opposite over hypothenuse or as a list of values to memorize can leave students with a conception of sine as a number and not a function nor a measurable quantity (Moore, 2014).

Moore (2014) suggests that many of these issues with the triangle and circle approaches can be resolved with the implementation of *quantitative reasoning*. Following Thompson (1990), quantitative reasoning is the ability to reason about one's conception of measurable and relational features of objects. The sine function can be defined quantitatively as the function whose argument measures the number of radii in a length of arc, starting at (1,0), and whose output measures the vertical position of the endpoint of that arc. This quantitative definition of the sine function is rarely learned (Moore, 2014). Foremost, this definition relies on quantitative reasoning for angle–the argument of the sine function. Unfortunately, students struggle to reason quantitatively with angle (Alyami, 2020; Keiser, 2004; Moore, 2012; Yigit, 2014). Students struggle to see radian—a particular kind of angle measure—as little more than a result of a conversion formula with input in degrees (Moore, 2012). Seeing radian as a conversion from degrees then transitively embeds the meaning students have for degrees onto radians. Alyami (2020), in their review of the literature on angle, describes the common conceptions of degrees as qualitative. For example, students frequently view degrees as a label for the corner of a shape—a quality that the shape possesses. Instead, instruction on angle should emphasize the measurability, whether that measure is in degree or radians. With quantitative meaning for angle, then quantitative meaning for the sine function can be established.

Instead of starting with triangles or the unit circle, my videos use a graph-first approach which leverages a series of construction tasks that eventually culminate in the creation of the sine function. Construction of the graph of the sine function requires and fosters *covariational reasoning* (Moore, 2014). Covariational reasoning is a way of coordinating change between inputs and outputs of a function (Thompson & Carlson, 2017). For the sine function, the coordination required is between angle and vertical position. As described above, and in further detail in Chapter 3, vertical position is viewed quantitatively as a measurable height of a point traveling around a circle. Similarly, angle is viewed quantitatively as a measurable feature of the arc created by a point's path in terms of the number of radii that fit within the path (i.e., angle is measured as a radian, and radian is quantified as a ratio of a measurable arc length to the measurable radius). Covariational and quantitative reasoning are fundamental to students understanding of the trigonometric functions (Moore, 2014). As I explore in further detail in Chapter 3, establishing these ways of reasoning motivated and guided the creation of the sequence of tasks that are contained within my dialogic videos.

Research Questions and Significance

In the following section I introduce the research questions that guided this dissertation. Then, I conclude the chapter by discussing the significance of these questions.

1. How does the orientation of undergraduate vicarious learners toward the talent differ when the dialogue in the videos is scripted versus unscripted?
2. How do the problem-solving behaviors of vicarious learners engaged in learning to reason covariationally about the sine function differ when the dialogic videos that they are viewing and interacting with are scripted versus unscripted?

Research Questions

Across the findings presented above, and in greater detail in Chapter 2, is an inconsistency in the nature of the dialogue in dialogic videos. Some research teams used authentic and unscripted dialogue (e.g., Chi et al., 2017), while other teams scripted their dialogue (e.g., Muller et al., 2008). On the one hand, scripted dialogue has been posited to be a means for the inclusion of important misconceptions (Muller, Bewes, et al., 2008). On the other hand, unscripted dialogue has been posited to be beneficial and more enjoyable for VLs (Geertshuis et al., 2021). Ultimately, claims about scripted versus unscripted dialogue have not been tested empirically.

Because of this inconsistent implementation and lack of empirical data, my project adds to the field by applying a comparative lens to VLs' experiences with a series of scripted and a series of unscripted dialogic videos. The literature presents a number of design features for the creation of instructional videos that can best position VLs to learn (e.g., videos being less than 10 minutes, the presence and refutation of alternative conceptions, demographic similarities between talent and viewer, etc.), but there is no research comparing VLs use of scripted versus unscripted dialogic videos. Findings from this dissertation can, thus, add to the field's understanding of important features of dialogic videos influence on VLs.

Question 1

The first research question pertains to the orientation of VLs towards the talent in the videos. As presented above, the SVLP conjectured that VLs are uniquely positioned to be emotionally and cognitively distanced observers (McKendree et al., 1998). Empirically, Lobato and Walker (2019) found that this is not always the case. Instead, their VLs experienced an emotionally-involved quasi-collaborative orientation toward the talent in the unscripted dialogic videos. That is to say, the VLs behaved as if the talent were part of a collaborative group with them, and the VLs regularly expressed emotions such as warmth or shared confusion toward the talent.

As a part of this dissertation, I aim to extend the findings on orientations to a larger sample, a new age group, a new video type, and apply a comparative lens to the two modes of dialogue. By expanding the sample size, further exploration of both quasi-collaborative and epistemic detachment orientations can be conducted. Additionally, Lobato and Walker (2019) found experiences of quasi-collaboration for their high school VLs, but this project assesses the orientation of undergraduate students. It is possible that viewers at different age levels are more attuned to experiences of epistemic detachment or quasi-collaboration because of their age. Finally, it is possible that the nature of the dialogue influences these experiences. If, as Geertshuis et al. (2021) contend, the authenticity of unscripted dialogue is perceptible, then it is possible that this perception is more likely to produce emotional experiences and feelings of quasi-collaboration with the authentic students. On the opposite side, if inauthenticity is perceptible, then feelings of epistemic detachment may emerge from an indifference to the ideas presented by an inauthentic student and a focus, instead, being placed on final solutions.

Question 2

The second research question compares the problem-solving behaviors of VLs engaged with scripted versus unscripted dialogic videos. Mathematical problem solving includes the cognitive processes and strategies used to identify, consider, and implement ideas to engage with and solve novel tasks (Schoenfeld, 2016). The literature reviewed above suggests that indirect participation in dialogue can lead to improvements in learning outcomes, result in modeling of behavior of the talent by the VLs and can be optimized through a number of features. Missing across this literature is an understanding of the ways in which VLs engage with dialogic videos and how that influences their problem solving.

Weinberg and Thomas (2018) identified a gap in the literature on how students use instructional videos, with a focus in the literature primarily being placed on learning outcomes as opposed to uses of videos. By comparing the problem-solving behaviors across a small sample of VLs using either scripted or unscripted dialogic videos, this project offers the field a better understanding of both VLs' uses of the videos and how the nature of the dialogue influences those uses.

By studying the problem-solving behaviors of VLs this project is not only novel, but it also opens the door for expanded use of dialogic videos in mathematics education. For instance, if emergent problem-solving behaviors influenced by the dialogue of the videos include justifications, then dialogic videos could be impactful on justification as a mathematical practice. This could open the door to the production of dialogic videos covering introductory to proof content, where the central goal of most novel tasks is to construct a justification.

Chapter 2: Literature Review

The goal of this chapter is to locate my research questions within the present literature. I will begin with an overview of the current theoretical assumptions of vicarious learning and how those assumptions have been enacted in the literature. From there, I will provide a synthesis of the relevant literature on the mathematical constructs and content that I intend to highlight within my dialogic videos. This chapter concludes with a return to my research questions with added exposition in light of the literature presented.

Vicarious Learning

Following Mayes (2015), vicarious learning can be defined as the indirect participation in a process of learning mediated by technology. This section begins with a brief history of vicarious learning. I then present a number of theoretical assumptions on how vicarious learning can best be implemented.

History of Vicarious Learning

Vicarious learning was first introduced by Bandura (1965) in response to the dominant theory of the day, behaviorism. From a behaviorist's perspective, an individual learns through the creation and strengthening of stimulus-response chains (Thorndike, 1922). For Thorndike, behaviorism comes down to two principles: the law of exercise and the law of effect (Collins, 2002). The law of exercise says that the more a behavior is performed the stronger the stimulus-response connection (i.e., the adage that "practice makes perfect"). In education, this kind of thinking led to instructional approaches that emphasize rote memorization and, what we now would call, "drill and kill" styles of teaching (Erlwanger, 1973). Collins (2002) describes the law of effect as saying if the response from an individual's inputted stimulus is positive, then they are more likely to perform those actions (i.e., positive reinforcement to encourage a desired behavior). In education, positive feedback typically comes from the teacher or more expert other in the form of good grades or encouraging comments. For Bandura (1965), the central issue within behaviorism–and these two principles, in particular–is the lack of a lens on social influences.

Bandura (1965) posited vicarious learning as a means of capturing the influence of the social on the individual. Framed in the behaviorist language of stimulus-response, this initial definition of vicarious learning posits that individuals can learn, through observation, responses to stimuli they have not personally experienced. This definition is in opposition to both principles of behaviorism because an individual does not directly need to perform an action to strengthen a stimulus-response chain (i.e., counter to the law of exercise), and individuals do not need to receive a positive response to know what will happen if they perform a desired action (i.e., counter to the law of effect). Bandura (1965) notoriously illustrated this process with a study of children’s behavioral changes after watching an adult act aggressively towards a Bo-Bo doll. Using a treatment/control comparison method, a statistically significant difference was found between the groups. The treatment, who watched the aggression, mirrored the behavior and acted aggressively more frequently than those who did not. For Bandura, the children who demonstrated more aggressive behavior had vicariously learned that if they are given the stimulus of the Bo-Bo doll, then they were expected to respond aggressively towards it.

McKendree et al. (1998) revisit the idea of vicarious learning in light of the role language is posited to play by the leading learning theories of today (namely, Piagetian Constructivism and the many schools of thought stemming from the work of Vygotsky). McKendree and colleagues form the Scottish Vicarious Learning Project (SVLP), a group primarily interested in vicarious learning through dialogue mediated by technology. Thus, they began with a differentiation between monologue and dialogue in the process of knowledge acquisition/construction. A monologue, for the SVLP, is about transmission of information. Monologues are the traditional roles that professors have played by standing in the front of the classroom and bestowing knowledge upon others. This gives them authority, and everything they say is to be taken as a true proposition. From these assumed truths, implications are drawn. In terms of the formal logic, this can be expressed as $P \Rightarrow Q$ where a teacher gives you a list of Ps and tells you all of the resultant Qs, or they leave the “easy” Qs implicit. This typically leads to rote memorization of sets of rules and procedures. For a monologue-based-mathematics-class example, an instructor may tell their students that given an equation of the form $ax^2 + bx + c = 0$, then use the quadratic equation to find x. The issues emanating from monologic instruction are well documented (Wells & Arauz, 2006), and inserting a student voice into the classroom through dialogue is important (e.g., Freeman et al., 2014).

For McKendree et al (1998), dialogue is a social process between multiple people constructing and deriving implications. Through dialogue, assumptions can be negotiated, and the many possible implications can be explicitly shared. Logically, the Ps, of $P \Rightarrow Q$, are discussed and all Qs are collectively derived and made explicit. Dialogue has been demonstrated to be a fruitful alternative to monologue-based lectures (Freeman et al., 2014), and the SVLP posits that students observing dialogue (vicarious learners) can reap similar learning benefits as the direct participants.

Theoretical Assumptions

A vicarious learner (VL) is an individual that is indirectly participating in learning (Mayes, 2015). The SVLP posits that vicarious learning can be beneficial to VLs in two ways. First, VLs are able to listen in on and observe explicit connections made within a dialogue. Explicit connections stem from a dialogue technique that is unique to formal education. Dialogue is frequently used to make derivations explicit (i.e., the connection between the P and a Q explicit). McKendree et al. (1998) say, “educational discourse is full of explicit derivations because there is a continual need to check that assumptions are indeed shared and that interpretations are aligned” (p. 115). The use of explicit connections allows fellow students and, the SVLP posits, VLs to see the connections they have missed and to see a model of the process of deriving. Illustrating this discourse technique, Luria (1976) provided the infamous exchange of a teacher attempting to engage a student in this form of deduction,

Teacher: All bears in Omsk are white. One day Ivan went to Omsk and saw a bear. What color was it?
Student: I don't know, I wasn't there. You will have to ask Ivan!

Students encultured into the practice of formal schooling may know that the teacher is asking them to connect the implications of the two sentences. If all bears in Omsk are white, then the bear Ivan saw in Omsk was white. While this specific discourse practice is less frequent outside of schools (as was the case for the culture of the student in the excerpt above), the practice of drawing conclusions from a set of assumptions is a relevant skill for everyday life. Observing dialogue allows for students to see others' connections, ones they may have missed, and the process of connection making, serving as a model that can be learned from.

The second way the SVLP posits VLs benefit from their position of indirect participants of a dialogue is through the experience of *epistemic detachment* (McKendree, et al., 1998). The experience of epistemic detachment is one of cognitive and emotional distance, which allows arguments in an observed dialogue to be analyzed from a more neutral position. As McKendree et al. (1998) state:

> When spectating, there is a 'lower processing load' both emotional and cognitive. The student is not as emotionally caught up in trying to defend a position or struggle with a new idea publicly. There is less of a cognitive load when they concentrate on the content and process of what is being said. (p. 117)

This means that VLs are both literally and metaphorically distanced. McKendree et al. (1998) view this metaphorical distance as beneficial for a VL's ability to listen and digest another person's claims. If a VL is listening, not pressured to think of a contribution, and is unbiased towards the dialogue, then they will have a lowered interactional load. The SVLP conjecture that a lowered interactional load results in a lowered cognitive load. With a lowered cognitive load, the VL may be able to digest, interpret, and, thus, experience what is being said in the dialogue. Accepting the claim of epistemic detachment, one could argue that the lowered cognitive load on indirect participants allows for more learning than the direct participants of a dialogue who are burdened by their emotional attachment to an idea and the need to think of something to say next.

In sum, the SVLP suggests VLs may benefit from indirect engagement in dialogue, and all the benefits that dialogue offers, because of the lowered emotional and cognitive demand, as well as periphery benefits of the dialogue serving as a model of behavior. To date, many of these assumptions remain untested.

Empirical Findings on Vicarious Learning

In the following section I will explore the empirical literature on vicarious learning. The literature covers a number of different forms of vicarious learning resources (e.g., transcribed dialogues, videos containing dialogues, etc.). As my project will use videos, I will pay special attention in this review to the research conducted on the use of dialogic videos (i.e., videos that contain an inquiry process). The literature will be broken into two categories: (a) learning gains as a result of indirect participation and (b) modeling behaviors from the dialogue VLs indirectly participate in. In the first section, I will explore the field's framing of vicarious learning experiences as akin to peer tutoring, leveraging claims based on the positive effect of peer-tutoring (Bowman-Perrott et al., 2013; Cohen et al., 1982). In the second section, I will present findings that suggest that the dialogue and participants of the dialogic videos serve as a model of behaviors for the VLs.

Learning as a Vicarious Learner

The SVLP suggests that listening in on a dialogue is better than listening in on a monologue. One common way that this distinction has been tested by the field is through the lens of peer tutoring, with peer tutoring being a form of dialogue and traditional instruction being a monologue. Because peer tutoring has been shown to be one of the most effective forms of instruction (Bowman-Perrott et al., 2013; Cohen et al., 1982), it is argued that vicarious learning resources (e.g., videos, transcribed dialogue, etc.) that capture tutor/tutee interactions will result in learning gains (Chi et al., 2008).

A substantial portion of the vicarious learning literature has focused on comparing the learning gains on pre/post-test measures from vicarious learning treatments who engaged with dialogic material to treatments who used monologic material (e.g., Chi et al., 2017; Chi et al., 2008; Cox, et al., 1999; Craig et al., 2009; Craig et al., 2004; Driscoll et al., 2003; Gholson & Craig, 2006; Muldner et al., 2014; Muller et al., 2007; Muller, Sharma, et al., 2008). For example, Craig et al. (2009) created four treatments in their unscripted instructional video study: monologue alone, monologue pairs, dialogue alone, and dialogue pairs. Monologue alone and monologue pairs were participants who watched monologic videos alone or in pairs, respectively. Likewise, dialogue alone and dialogue pairs watched dialogic videos along or in pairs, respectively. The monologic videos of this study contained an expert (30+ years teaching) in physics and consisted of the expert working through examples and explaining their process. The dialogic videos used the same expert, serving as a peer tutor, and a semi-experienced tutee (second-year physics student, while the material in the video was first-year topics). Craig et al. (2009) found a statistically significant difference in the long-term retention in favor of the treatments who used the dialogic videos. This suggests that viewing videos that contain a tutee engaging with a tutor is more beneficial for long term learning gains than watching a lecture from a single expert.

In another study, Muller et al. (2007) compared scripted monologic to scripted dialogic videos covering the physics topic of quantum tunneling. The two video types were constructed to be as similar as possible, with the same graphics presented and the same content covered in both videos. The primary difference was the presence of scripted questions and scripted alternative conceptions posed by a tutee to a more knowledgeable tutor in the dialogic videos, while the monologic treatment contained only the peer tutor presenting the same material as a lecture. Using a pre/post-test model, the difference in the treatments' improvement on pre/post-test mean scores was found to be statistically significant, in favor of the viewers of the dialogue.

Chi and colleagues have found similar results and even added a comparative lens to the learning of the tutees contained within their unscripted dialogic videos (Chi et al., 2017; Chi et al., 2008; Muldner et al., 2014). In one instance, Muldner et al. (2014) created unedited monologic videos and unedited dialogic videos that contained an expert lecturer alone or the lecturer engaged in peer-tutoring with a tutee, respectively. In addition to comparing the post-test scores across treatments, comparisons were made with the post-test scores of the tutees contained in the dialogic videos. A statistically significant difference was found between the treatments' post-test scores, in favor of dialogic viewers with a small effect size (d = 0.16). Interestingly, average post-test scores were nearly identical among the tutees and the dialogue treatment (mean of 81.7% and 81.2%, respectively). This finding suggested that dialogic videos are better than monologic videos and that dialogic videos may be just as impactful as tutor-to-tutee instruction (a form of instruction that has been widely heralded as the most impactful, Bowman-Perrott et al., 2013; Cohen et al., 1982). Together, the findings from these comparative studies suggest that videos containing dialogue about content lead to more changes in vicarious learners' understandings than videos that contain a monologue of content. But replication is an important, and often under exemplified, part of research (Head et al., 2015; Makel & Plucker, 2014).

While the majority of the research presented thus far suggests that instructional videos should contain a dialogue, the results are mixed. A number of projects have found no statistically significant difference between dialogue and monologue vicarious learning resources, or they have found statistically significant results favoring monologue (Cooper et al., 2018; Monaghan & Stenning, 1998; Muller, Bewes, et al., 2008). For instance, Cooper et al. (2018) conducted a study that compared students' performances on first-year undergraduate physics quizzes. Prior to taking the quizzes, participants either watched monologic videos containing their professor or dialogic videos containing their professor and a second-year physics student, who served as the tutee. Participants of this study were randomly assigned to one of two groups. The first group watched monologic videos every week before taking the weekly quizzes for weeks one to four of the course, and then they watched dialogic videos every week before taking the quizzes for weeks five to eight. The second group started with the dialogic videos for the first half of the course and switched to the monologic videos for the latter half. Cooper et al. reported a statistically significant difference in quiz scores in favor of the monologic video viewers.

In sum, the results are mixed. Several projects suggest that VLs' learning outcomes benefit from vicarious learning materials, particularly videos, that are designed to mirror peer tutoring through the use of multiple voices engaged in a dialogue. Importantly, this is not always the case (e.g., Cooper et al., 2018), imply that injecting multiple voices is not a sufficient condition. The mixed results on vicarious learning suggest that further inquiry is needed into how VLs interact with and engage with dialogic videos. It is possible that more important than the presence of multiple voices is the invitation to negotiate within the videos. It is notable that Cooper et al.'s (2018) videos contained the VLs' professor. It is possible that this created a viewing environment for the monologic videos that overcame the learning benefits of dialogic videos reported by other projects.

The findings reported above, then, suggest that dialogic videos more frequently invite negotiation, but further work needs to be done to investigate how negotiation and engagement can better be fostered. In the next section I explore another possible benefit of vicarious learning resources that contain multiple speakers: dialogic learning resources may serve as a model of behavior for VLs.

Modeled Behavior as a Vicarious Learner

In addition to the learning outcomes found through indirect participation, observation has been posited to produce modeling of the observed behavior (Bandura, 1965; McKendree et al., 1998). Videos that contain multiple participants engaged in a dialogue can model constructive dialogic behavior, the process of deductive reasoning, or even more general content-specific practices. If an instructional video takes the form of a lecture (i.e., what I have been calling a monologue), then there is little behavior to be modeled–only information to be processed and stored by the viewer. Empirically, the literature suggests that VLs do model their behaviors after the people–particularly, the tutees–contained in vicarious learning resources, with modeled behavior ranging from content-specific techniques to writing processes (Bandura, 1965; Braaksma et al., 2004; Craig et al., 2000; Chi et al., 2017; Mayes et al., 2002; Rummel & Spada 2005; Schunk et al., 1989; Twyford & Craig, 2017).

Going back to the origins of vicarious learning, Bandura (1965) found the aggressive behavior of adults served as a model for the children observing the behavior. Since then, findings for the vicarious learning of behaviors have ranged from more thorough planning in writing (Braaksma, et al., 2004) to asking more follow-up questions (Chi et al., 2017). In Twyford and Craig (2017), a control/treatment model was used to test how observing learning goal orientation (LGO) influenced VLs. The control of this experiment viewed dialogic videos and the treatment observed dialogic videos that contained added LGO excerpts. The LGO excerpts contained in the treatment's videos emphasized the tutee's desire to learn (e.g., "I want to learn because learning is fun"), as opposed to a desire to avoid failure (e.g., "I want to learn because I don't want a bad grade"). Comparing pre/post-test surveys of orientation, the LGO treatment was found to have a statistically significant increase in their learning goal orientations and a moderate change in goal avoidance orientation. The control, on the other hand, was found to have no change in LGO and a statistically significant increase in goal avoidance orientation after engaging with the challenging material covered in the dialogic videos. These findings suggested to the researchers that the tutees in the dialogic videos were modeling goal orientation for the VLs.

Mayes et al. (2002), as a part of the SVLP, found similar results with a collection of vicarious learning resources that emphasized explicit derivation over exposition. In this study the treatment that received the dialogue with derivation resources had online discussions that contain more derivation. Mayes et al. (2002) claim, “the [vicarious] learner is given a model of what it means to *become* [emphasis added] an expert in the subject” (p. 215). This suggests that the vicarious learning resources were serving as a model for learning (i.e., a model for transitioning from novice to expert) that the VL could follow. While learning still occurs in monologue/single-voiced vicarious learning resources, the process of becoming an expert will not be present and, thus, cannot be modeled. Only when the VL sees the changes that occur through the dialogue of an observed learner can these behaviors be modeled and learned by the VL.

The opportunity to model the behavior of interlocutors (i.e., the people engaged in dialogue) within a dialogue and the trend in the research on monologic versus dialogic videos suggest that videos should contain multiple voices. Monologic videos could still serve as a model for some behaviors for VLs. For example, the language used within exposition videos can include explicit deductions, careful description of thoughts and procedures, and even learning goal orientations. Ultimately, monologic videos fall short of those that contain a dialogue because single voices, by definition, cannot model dialogue (as an inquiry) practice. Furthermore, under the present model of monologic videos, where an expert presents information, authentic confusion and resolution or learning could not be present. Thus, if dialogic processes, resolution of confusion, and learning processes are desirable behaviors to be modeled, then the videos should be dialogic.

In the next section, I explore how dialogic practices, authentic confusion, and learning uniquely modeled within dialogic video–as well as a number of other design features–play an influential role on VLs’ learning and their experience of the dialogue as containing internally persuasive voices.

Design Features

In order to best support the cognitive growth of VLs, a number of important features emerge from the literature. Based on the literature, I infer that vicarious learning resources, particularly dialogic videos, should: (a) be constructed so that the students contained in the videos are similar to the VLs (including similar skill level, age, etc., Braaksma et al., 2002; Fowler & Mayes, 1999; Groenendijk et al., 2013; Mayes, 2015), (b) contain productive struggle and alternative conceptions (Baumeister et al., 2001; Chan et al., 1997; Chi et al., 2017; Fox, Tree, & Mayer, 2008; Lobato & Walker, 2019; Muller et al., 2007; Muller, Bewes, et al., 2008), and (c) contain deep-level questions (e.g., questions that contain a comparison or causal antecedent, Chi et al., 2017; Craig, et al., 2000; Craig et al., 2006; Driscoll et al., 2003; Gholson & Craig, 2006; Gholson et al., 2009). I elaborate on each feature below.

The first design feature is particular to vicarious learning resources that take the form of dialogic videos. Students contained within the videos should be similar to the target audience (i.e., the VLs and the tutees in the video should be similar in age, ability, etc., Braaksma, et al., 2002; Chi, 2013; Fowler & Mayes, 1999; Groenendijk, et al., 2013; Mayes, 2015). Chi (2013) found that VLs attended more to the utterances of the tutee than to the tutor within their dialogic videos and found the utterances of the tutee were easier to comprehend for the VLs. Similarly, Braakmsa et al. (2002) found that VLs who were categorized as weak learners benefited more from observing weak models, and VLs who were considered better learners benefited more from observing good models. One possible explanation for these findings is the fact that the tutee's ideas are similar to that of the observer, allowing for the dialogue to resonate with the VLs' ideas. Since the tutor is positioned as an expert demonstrating their proficiency, their ideas are likely to be experienced as they were a teacher's. The tutee, on the other hand, is positioned as a learner–just like the VL is. If the tutee is understood to be a learner, then it is not expected for everything the tutee says to be correct. The VL must then process what is being said and negotiate with their ideas. The more similar observer and observed (VL and tutee, respectively) are, the easier it will be for the VL to recognize the position the tutee is in, seeing them as learners, and resonating with their struggle and inquiry processes.

The next design feature pertains to the content of the voices within vicarious learning resources: the dialogue should contain multiple conceptions (Baumeister et al., 2001; Chan et al., 1997; Fox, Tree, & Mayer, 2008; Lobato & Walker, 2019; Muller, Bewes, et al., 2008; Schunk et al., 1989). Illustrating the power of videos designed to contain multiple conceptions, Muller, Bewes, et al. (2008) conducted a monologic versus dialogic study with 364 physics students who were placed in one of four instructional video treatments: exposition, extended exposition, refutation, or dialogue. The exposition was a monologic-style video that contained an expert presenting the material. The extended exposition followed the same format as the exposition, with additional information added to clarify specific details. The refutation videos were, again, monologic videos, but also contained explicit mention of common misconceptions and refutations of them. Finally, the dialogue treatment viewed scripted videos containing an actor asking questions and presenting common alternative conceptions as their own to an expert peer-tutor. Comparing pre/post-test gains showed that both treatments that contained alternative conceptions, the refutation and dialogue videos, had a marginal difference, but a statistically significant difference when compared to the exposition and extended exposition treatments (with a reported effect size of 0.79 and 0.83 for refutation and dialogue videos compared to exposition, respectively). This finding suggested that the content of the voices within the videos was playing an important role in VLs' learning, and VLs can best be served by speech that contains the refutation of alternative conceptions (i.e., videos that directly address multiple conceptions).

One reason that VLs may benefit from resources that contain multiple conceptions is from the increased cognitive demand produced by overhearing incorrect answers and the subsequent corrections. Stein et al. (2000) define cognitive demand as “the kinds of thinking needed to solve tasks” (p. 5) and suggest that an increase in cognitive demand is a desirable goal of educators. Conducting a follow up to their previous study, Muller, Sharma, et al. (2008) reported that participants in treatments who engaged with videos with alternative conceptions self-reported the videos as more cognitively demanding and, again, found that the alternative conceptions treatment performed better than the treatments without alternative conceptions (with a moderate effect size of $d = 0.36$). This finding suggested to Muller and colleagues that the cognitive demand experienced by the VLs played a role in their improved performance. If it is true that vicarious learning resources should be cognitively demanding, then the vicarious learning resources that contain multiple conceptions will be successful.

Another way to increase the cognitive demand on students and simultaneously improve their learning outcomes is through deep-level questions (Pashler et al., 2007). Deep-level questions include asking interpretation questions, comparison questions, and causal antecedent questions. A number of projects have found that VLs who observed deep-level questions posed within a dialogue led to improved learning outcomes (Driscoll, et al, 2003; Craig, et al., 2000; Craig, et al., 2006; Gholson & Craig, 2006; Gholson et al., 2009). For example, Craig et al. (2006) manipulated the kind of questions VLs heard across two dialogic video viewing treatments and found a relationship between the type of question and the amount of learning. In one treatment, VLs heard simple, factual, and closed questions posed by a tutee. For the other treatment, the same tutee posed complex, causal, predictive, and evaluative questions. The latter treatment was classified as experiencing deep-level questions, and the VLs of this treatment were found to have learned a statistically significant amount more than the former treatment. Once again, the implication was that because deep-level questions are more demanding and the VLs who were exposed to those cognitively demanding questions performed better, then more cognitive demand on the VL is a desirable feature of vicarious learning resources.

importantly, the claim that increasing cognitive demand is beneficial to VLs runs counter to the claim of the SVLP that VLs have less cognitive demand because of epistemic detachment. Either epistemic detachment does not lower the cognitive load on VLs, or VLs are not always experiencing epistemic detachment. In the following section, I will explore the latter possibility.

Project MathTalk

My work follows the model of dialogic video creation set forth by Lobato and colleagues as a part of Project MathTalk (www.mathtalk.org). Lobato et al. (2019) state that their videos are "unscripted videos in which secondary school students explore sources of confusion and resolve their own dilemmas by arguing for and against particular ways of reasoning" (p. 2). This design is a novel take on the dialogic video model. Instead of one voice coming from an expert tutor and another coming from a learning tutee, the voices emphasized in Project MathTalk are of two learners, called the talent, working together as they problem solve.

As a part of this project, Lobato and Walker (2019) call into question the claim that VLs are always experiencing epistemic detachment. In their case study, Lobato and Walker (2019) found that the VLs oriented toward the talent as if they were all in the same collaborative group, which they called a quasi-collaborative orientation. Lobato and Walker were investigating how two VLs engaged in a series of tasks that accompanied dialogic videos of the talent. The VLs engaged in five different categories of quasi-collaborative behavior: (a) identifying the talent's mathematical personalities (e.g., noting that one talent liked to find short-cuts while the other was more methodical), (b) predicting the talent's mathematical actions (e.g., thinking one talent would act in one way and the other talent in another), (c) coordinating activity with the talent (e.g., the VLs comparing their methodologies with that of the talent), (d) acting as if the talent could engage in their work (e.g., the VLs talking to the videos as if the talent could hear them), and (e) speaking about being in a community of learners with the talent (e.g., sharing confusion with the talent helped them feel less isolated).

Applying the methodology set forth by Evans et al. (2006), Lobato and Walker (2019) performed a second analytical pass to code and count emotions evidenced through tone of voice, intonation, body movements, facial expressions, gestures, utterances (e.g., laughter), and emphasis on particular words/phrases. The presence of these emotions during the five categories of interactions resulted in the claim that VLs were experiencing a feeling of emotionally-involved quasi-collaboration. While these emotions were not omnipresent (a number of instances of what could be considered emotionally neutral were present), the VLs generally experienced positive emotions towards and aligned themselves with the talent. This suggested that VLs are not always experiencing epistemic detachment, and epistemic detachment is not the only way for VLs to learn from observing dialogue.

The model of dialogic videos created by Lobato and colleagues captures many of the key features highlighted thus far in this review. Foremost, the videos contained multiple voices engaged in a dialogue. Uniquely, both of the voices within the videos come from learners positioned analogously to a tutee. If VLs attend more to the voices emanating from those who are most similar to them (e.g., Chi, 2013), then replacing the contrasting voice of the expert–frequently positioned as a tutor–with a learner's voice should increase the amount of material attended to within a video. Additionally, the videos highlight students who are struggling with the content and present alternative conceptions that are eventually resolved. In addition to the cognitive benefits of this feature (e.g., Muller et al., 2008), Lobato and Walker (2019) found that the presence of struggle within the videos was appreciated by the VLs.

The final feature of the Project MathTalk videos is that the dialogue within the videos was authentic and unscripted. The struggle and resolution captured (and appreciated by the VLs) is a real struggle that the talent experienced with their mathematical tasks. In the final section I will explore the different uses of unscripted and scripted dialogue within the literature on vicarious learning.

Scripted Versus Unscripted Dialogues

Whether VLs are better positioned to learn from and engage with dialogic videos that are scripted or unscripted is an open question. Muller et al. (2007) posited that scripting the dialogue allows for a breadth of possible alternative conceptions to be presented and refuted.

On the other hand, Muldner et al. (2014) position the creation of unscripted videos as easier for development. The work of Muldner and colleagues captured authentic tutee-tutor sessions and tasked their participants to view and engage with the unedited videos. If the videos are easier to develop, then a large library can be created to serve a large number of students. While it is the case that multiple conceptions can easily be scripted into instructional videos, and videos that capture authentic tutor sessions can be easy and quick to produce, I disagree that these are reasons to create scripted or unscripted videos.

As I have argued above, instructional videos should contain alternative conceptions, but I believe the assumption that only scripted videos can contain a breadth of alternative conceptions is not the case. If the videos and, importantly, the tasks contained within the videos are carefully constructed, then unscripted videos can capture a number of alternative conceptions. As described above, Lobato and Walker (2019) illustrated that unscripted videos can still capture alternative conceptions. Similarly, Chi et al. (2017) found that unscripted dialogue allowed for an impactful authentic struggle to emerge within their videos. Muller and colleagues suggest alternative conceptions are a benefit of scripting dialogue, but it is evident that this can still be produced with unscripted dialogue as well. In turn, unscripted videos that seek to capture a number of alternative conceptions would not benefit from ease of production. Unlike the model of vicarious learning video created by Muldner and colleagues, where authentic tutor sessions were simply recorded and used at a later time by VLs, creating videos that contain authentic alternative conceptions may take time and deliberate construction. Lobato et al. (2017) describe their postproduction process as involving an editing, annotation, and narration process, all of which involve significantly more time than presenting raw footage of a tutoring session (for further details on the production of unscripted videos à la MathTalk, see Chapter 3).

In addition to the fact that authentic dialogue can capture alternative conceptions, it is possible that alternative conceptions presented by authentic students would be more impactful. Geertshuis et al. (2021) state,

> [Unscripted dialogue] ensured that the language that was used was that of the students. It was their voice and lexicon born out of their current knowledge and understanding, not something imagined and fabricated by a tutor. The feedback from students confirms that this will help our videos resonate with observers. (p. 116)

Unscripted dialogue resonates with viewers because it sounds like something they would hear in a class, personally think, or actually say. If VLs better attend to the utterances of learners that are characteristically similar to them (e.g., similar skill level, age, etc., Braaksma, et al., 2002; Chi, 2013; Fowler & Mayes, 1999; Groenendijk, et al., 2013; Mayes, 2015), then this effect may hold for the actual words and utterances that sound similar to those of a VL.

For this project, I offer a qualitative lens that attempts to measure the benefits and differences in scripted/unscripted videos. This will include a comparison of what, if any, emotional orientation emerges and a comparison of problem-solving behaviors of the VLs as they engage with scripted versus unscripted videos. In the next section I will explore the mathematical context in which this study is situated, trigonometry.

Mathematics

In the following section I will explore two constructs fundamental to students thinking with respect to mathematics: quantitative and covariational reasoning. After exploring these constructs, I will present an overview of the literature on trigonometry and provide a connection to the role quantitative and covariational reasoning plays in trigonometry.

Quantitative Reasoning

Quantitative reasoning is a well-studied and polysemous construct within mathematics educational research (Mayes et al., 2012). For the purposes of this project, I will take quantitative reasoning to mean a form of logic that focuses on the relationships between objects that can be quantified (Smith & Thompson, 2007). Quantifiable objects can range from people's ages to the distance between celestial bodies. Importantly, they can be compared using relations like “older” and “further.” For example, if a student is quantitatively reasoning with distance, then they could compare two distances traveled without attending to any specific numbers. Broadly, if an object can be measured in some way, then students should be able to reason about the object–with or without measuring it (Thompson, 1990). Instruction that attempts to instill quantitative reasoning will thus de-emphasize the process of operations and, instead, shine a light on the meaning of the operations and comparisons between quantifiable objects. The use of quantitative operations will allow students to see the resultant as a value, a number and a unit, and not just a number (Stevens & Moore, 2017).

A number of researchers have built off this conception of quantitative reasoning (e.g., Chazan, 2000; Hackenberg, 2010; Lobato & Siebert, 2002; Steffe & Olive, 2009) and applied it to mathematical content ranging from fractions (Ellis, 2011) to trigonometric functions (Moore, 2012). One major affordance of quantitative reasoning presented in this work is the student’s ability to generalize and reason algebraically (Chazan, 2000; Ellis, 2011; Lobato & Siebert, 2002). Students with a strong foundation in quantitative reasoning have a support structure for flexible, and more general, reasoning (Smith & Thompson, 2008). The focus, for these students, is pushed away from symbolic expressions and, instead, meaningful and intuitive connections to algebraic knowledge are made. For example, Lobato and Siebert (2002) found students’ quantitative reasoning was influential on their conceptions of slope and linear functions. Slope needs to have deeper meaning than find a “rise,” find a “run,” and perform a computation. Instead, slope can be seen, quantitatively, as a measurement of an appropriate attribute associated with the rate of change of a function (e.g., speed, density, sweetness, steepness, etc.). While quantitative reasoning can greatly strengthen algebraic reasoning, it is also important for conceptualization of the content of trigonometry (Moore, 2014) and beyond (e.g., Rasmussen, 2001).

I view quantitative reasoning as fundamental to students' conceptualizations of angle. As I will explore later, students often see angle as a number that labels a part of a shape (Cekmez, 2020; Demir & Heck, 2013; Moore, 2012). It is important in trigonometry that students come to see angle as a value representing a ratio of lengths. With respect to radians, the ratio is the length around a circle (i.e., arc length) to the length of the radius. Quantitatively, this can mean radians represent the number of radii that fit in a given length. Similarly, for degrees, the quantitative meaning is how many units of $1/360^{th}$ of the circumference fit within a given length. With the ability to compare and reason with angle as a ratio, students' quantitative reasoning can be leveraged in the construction of the sine and cosine functions (Moore, 2014). Fundamental to the use of quantitative meaning for angle in this constructive process will be students' abilities to use covariational reasoning, another form of reasoning that is deeply entangled with quantitative reasoning (Castillo-Garsow, 2012; Ellis, 2011; Moore, 2014; Moore et al. 2009).

Covariational Reasoning

Carlson et al. (2002) defined covariational reasoning as the coordination between changes in one variable with another. This kind of reasoning is evident in our everyday life when, for example, we reason that it was colder on December 15^{th}, 1987, than on July 15^{th}, 1987 in Chicago. In this example, we know that as the time of year varies, so does the temperature. This allows us to conclude something about the temperature through the coordination of time of year, and the typical seasonal weather of Chicago. In mathematics, covariational reasoning is fundamental to the reasoning required for calculus, specifically for reasoning with instantaneous rates of change – in other words, reasoning with the derivative (Thompson & Carlson, 2017).

Carlson et al. (2002) created a coding scheme used to assess undergraduate students' covariational reasoning. The scheme resulted in a hierarchy of five mental actions. Mental Action 1 (MA1) is defined as the coordination of the value in one variable with the change of value in another variable. MA2 is evidenced through the coordination of the direction of change in one variable with the direction of change in the other. For example, a student acting with MA1 would be able to claim that $\sin(0) = 0$, but would not be able to approximate $\sin(0.1)$. A student with MA2 might reason that the argument of $\sin(0.1)$ has increased and claim that the output must have increased as well. At Mental Action 3 (MA3) a student can coordinate amounts of changes in two variables (e.g., finding the slope of a linear function). MA4 extends the coordination of amounts of change to the coordination of average rates of change (e.g., finding the slope of a function over defined intervals). Finally, MA5 is defined as the coordination of instantaneous rates of change for a given function with continuous changes in the output (e.g., finding the derivative of a function).

Castillo-Garsow (2012), extending the idea of covariational reasoning, was interested in students' developing mental images of continuous functions. Connecting covariational and quantitative reasoning, Castillo-Garsow (2012) says, "in reasoning covariationally about a continuous function, students must reason about a quantity, about continuity, and about change" (p. 55). Covariational reasoning with functions, particularly the graphs of functions, requires the coordination of changes in input with changes in output and, simultaneously, the coordination of reasoning quantitatively with reasoning about continuity.

Castillo-Garsow (2012) posits two kinds of covariational reasoning: chunky-continuous and smooth-continuous reasoning. A chunky-continuous view of a function is a process of creating discrete chunks that describe what has already happened to a function over a continuous segment. This is analogous to viewing all functions as piecewise defined, where each segment of the piecewise function represents a chunk. Smooth-continuous, on the other hand, is a view of change as it occurs and typically involves imagery of change occurring constantly over time. For example, Castillo-Garsow illustrates smooth-continuous reasoning with a student who, when discussing the change in time over one year, does not jump from zero to one year, but instead reasoned about time progressing month to month, day to day, hour to hour, etc. For trigonometry, Demir and Heck (2012) found students unable to reason with arguments between the standard radian values (i.e., they could evaluate $\sin(0)$ and $\sin\left(\frac{\pi}{2}\right)$, but could not approximate $\sin(0.5)$). Applying Costillo-Garsow's chunky vs. smooth lens suggests that the students held a chunky-continuous view of the sine function, with chunks defined by multiples of π. Smooth-continuous reasoning for the sine function would involve the coordination of change in height with distance around the circumference (i.e., an appeal to the – rarely learned – definition of the sine function, Moore, 2014). While not a sufficient condition for smooth-continuous reasoning, a student who has smooth-continuous reasoning for the sine function would be able to approximate $\sin(0.5)$ through the coordination of increasing input and increasing output on the interval $(0, \frac{\pi}{2})$.

Thompson and Carlson (2017) combine the contributions of Carlson et al. and Castillo-Garsow into one developmental hierarchy: the Major Levels of Covariational Reasoning. At the lowest level, students will have no level of coordination between variables and will only attend to change in one variable. From there, students begin to reason that change happens in a sequence of change in one variable followed by change in the other and repeating in lockstep. Thompson and Carlson call this the precoordination of values level. The next level, gross coordination of values, is analogous to Carlson et al.'s Mental Action 2. At this level, students will be able to reason about the coordinated effect that changing one variable has on the other. For example, a student reasoning at this level could coordinate that increasing *x* results in a decrease in *y*. Importantly, this coordination is not yet simultaneous (e.g., first *x* increases and then *y* decreases). The fourth level marks the coordination of values with the goal of creating a set of ordered pairs, change in the variables is beginning to be seen as simultaneous. Finally, the last two levels are chunky, and then, smooth continuous covariation. Building off of Costillo-Garsow, chunky covariation includes reasoning about simultaneous changes in two variables and describing this change with the creation of segments of discrete length, i.e., chunks.

Smooth continuous covariation is then an extension of chunky, but the discrete segments are now seen as continuous and smooth. As these levels are seen as hierarchical, the goal is for students to develop smooth continuous covariational reasoning.

Thompson and Carlson (2017) conclude, “quantitative and covariational reasoning affords students powerful ways to conceptualize and represent dynamical phenomena” (p. 461). If students are able to reason with quantitative and covariational reasoning, then they will be positioned to visualize, represent, and think deeply about measurable objects, especially complex objects like the derivative of a function. In the next section, I will explore the mathematical content I intend to instill quantitative and covariational reasoning into trigonometry.

Trigonometry

Research on students’ conceptions of trigonometry is broken into two content areas: angle and functions. Angles play a foundational role within trigonometry. An angle, measured in degrees or radians, is the argument of the trigonometric functions (sine, cosine, tangent, cosecant, secant, and cotangent). If students are going to reason covariationally about a trigonometric function, then they need to coordinate changes in the angle with changes in the respective output of the function. For example, the cosine function measures the horizontal displacement of a point traveling around a circle, and the amount of displacement depends on the position of the point defined by the arc length traveled around the circle’s circumference. To establish this kind of reasoning students would benefit from a deeper understanding of the argument. In this way, it is important that students have quantitative meaning for angle.

Moore (2012) conducted a textbook analysis and found seven different definitions for angle. These definitions included uses of degree (e.g., defining 1 degree as $1/360^{th}$ of the circumference of a circle) and uses of radian (e.g., defining radians as the unit such that 2π radians is equal to 360°). Moore found that these definitions were frequently introduced in separate contexts, as if degrees are for one type of angle and radians were for another.

Degrees are introduced first in geometry classes to label part of a shape (e.g., 90° labels the corners of a square). Then radians are introduced in trigonometry classes with the unit circle and an accompanying degrees-to-radians conversion formula for trigonometric functions. Under instruction that positions radians as a unit conversion from degrees, the conceptual meaning students have for degrees is transferred to students' conceptions of radians (Moore, 2009).

Moore (2012) found that students resulting conception for angle in terms of degrees was dominated by seeing degree as a label or by seeing the size of a degree as dependent on the size of the shape (i.e., students believe that if side length AB and side length BC are longer than side length CD and side length DE, then ∠ABC is bigger than ∠CDE). Another conception of angle emerges with difficulty in identifying 0°,180°, and 360° (Keiser, 2004; Yigit, 2014). In their study of pre-service math teachers, Yilgit (2014) described their participants' conceptualizations of angle as falling into one of three categories: 0-line, 1-line, and 2-line. 0-line angles correspond to angles measuring 0 or 360 degrees and can be visually represented by a point or a circle. 1-line angles have a measure of 180 degrees and are visualized by a line segment. Few pre-service teachers were able to identify either of these two forms of angle. The most common conceptualization of angle corresponded to 2-line angles, where two rays meet at a point and the smaller region created represents an angle. These findings fit with the interpretation of angle as a label. Yilgrit (2014) suggests that angle as openness could allow students to extend their conception of angle to representations beyond 2-line images, and perhaps to a view that 2-line representations create two open regions that can be measured using angles.

In their own textbook analysis, Alyami (2020) called the conceptualization of angle as a label for a shape and the determination of magnitude of an angle by the length of its rays *qualitative meanings* for angle. A qualitative meaning for angle is a conception of angle as a quality as opposed to a quantifiable. Much like a triangle has a quality of being three-sided, two rays meeting at a point have a quality that can be described with an numerical angle. Instead, instruction on angle should emphasize the measurability of angles and foster quantitative reasoning (Alyami, 2020; Moore, 2009).

Quantitative reasoning involves seeing an angle as a ratio of the distance around a circle to the total distance around the circle. Moore (2012) suggests that angles represent openness through a definition of angle as "the fractional amount of any circle's circumference subtended by the angle's rays" (p. 78). This definition means, if there are two points on the circumference of a circle, then between them is a measurable distance around the circumference of the circle. This segment of the circle's circumference is called an arc, and the distance is the arc length between the two points. Angle is then the ratio of the arc length to the circle's circumference. Importantly, this definition invokes a measurement: a measurement of the length around the arc of a circle proportioned to its total circumference. Figure 2.1 illustrates the progression in the construction of an angle $\angle \mathrm{cba}$. First, construct a circle centered at b with radius, r, equal to the length of the line segment $\overline{\mathrm{ba}}$, i.e., $\mathrm{r} = \left\|\overline{\mathrm{ba}}\right\|$ (Figure 2.1b). Then extend $\overline{\mathrm{bc}}$ to point c' such that the line segment $\overline{\mathrm{bc}'}$ is equal to the radius of the circle, i.e., $\left\|\overline{\mathrm{ba}}\right\| = \left\|\overline{\mathrm{bc}'}\right\|$ (Figure 2.1c). Let the resultant arc, from c' to a, be represented by $\mathrm{c}' \frown \mathrm{a}$ (Figure 2.1d). If the circumference of the circle is $2\pi \cdot \left\|\overline{\mathrm{ba}}\right\|$, then the measure of the angle is the length of the arc of a circle proportioned to its total circumference (i.e., $\angle \mathrm{cba} = \frac{\left\|\mathrm{c}' \frown \mathrm{a}\right\|}{2\pi \cdot \left\|\overline{\mathrm{ba}}\right\|}$). Using this definition of angle affords quantitative reasoning with openness. If an angle is more open, then the angle is larger. Notice, this reasoning did not require exact measurement, merely comparison of measurables. With a view of angle as measurable, it is possible for students to avoid some of the conceptions presented above.

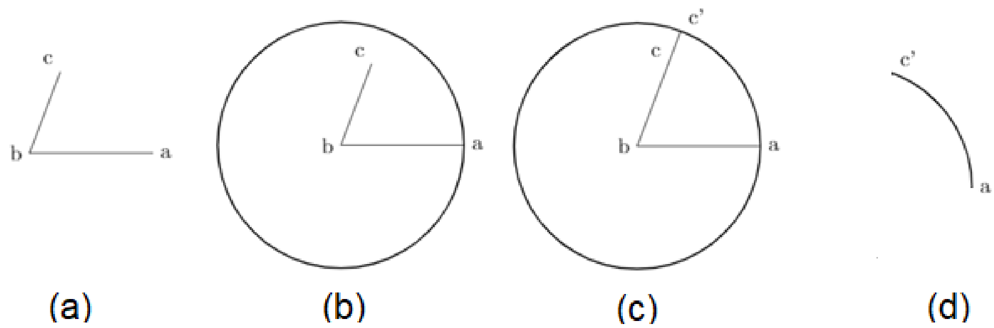


Figure 2.1
Constructing a Radian

Moore (2012) continued his textbook analysis with a collection of definitions for the sine function from trigonometry textbooks. The four definitions he found can be broken into two approaches: the triangle-first approach and the circle-first approach. The triangle-first approach is trademarked by the introduction of the trigonometric functions as a means of finding missing side lengths of triangles through the use of the acronym SOHCAHTOA (Sine = Opposite side length over Hypotenuse side length, Cosine = Adjacent side length over Hypotenuse side length, Tangent = Opposite side length over Adjacent side length). The triangle-first approach obscures what the argument and the meaning of the trigonometric functions historically are (Van Sickle, 2011). Empirically, Demir and Heck (2012) found their students felt that they needed a triangle to evaluate trigonometric functions. If they were given $\sin(2)$, then they would either try to draw a triangle or claim not enough information is given because there is no triangle. This suggested that the overemphasis on the relationship between trig functions and triangles leads to misconceptions of the arguments (i.e., the argument of the trigonometric functions is a triangle, when it is an angle).

On the other hand, the circle-first approach begins with the unit circle. Using this approach, the sine function is defined as the second coordinate of a point on the unit circle (Moore, 2012). This approach is branded with an overemphasis on the output of the trigonometric functions at argument values that are rational number multiples of the unit circle's circumference–that is, memorization of sine and cosine for the inputs $\pi, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$, 0, and 2π (Moore, 2012). Under this approach, students may struggle to approximate, or even conceptualize, the value of the trig functions at values between multiples of π (Çekmez, 2020; Demir & Heck, 2012). For example, Demir and Heck (2012) found that students could evaluate trigonometric functions like $\sin\left(\frac{\pi}{3}\right)$, but they could not approximate $\sin(1)$. Similarly, Çekmez (2020) found students struggled to evaluate trigonometric functions for all real number arguments. When given radians as multiples of π they converted to degrees. When given rational-valued radians the students assumed it meant degrees (e.g., thinking that $\sin(2.4) = \sin(2.4°)$).

In both cases, the triangle- and circle-first approaches, trig functions are introduced as evaluative tools that are rarely connected back to their graphs (Demir & Heck, 2012). Instead, an emphasis on the construction of the trig functions with quantitative meaning of their arguments (i.e., reasoning with angle as a measurement) can foster the covariational reasoning required for a deeper understanding of the trigonometric functions. Covariational reasoning is fundamental to the use of functions, where *y* varies with *x*, and calculus, where changes in the variance between *x* and *y* is studied (Carlson et al., 2002). For the trigonometric functions–in particular, the sine and cosine functions–what must be coordinated is the change in the distance around the unit circle (i.e., the quantitative meaning for angle) and the vertical or horizontal displacement for sine and cosine, respectively. Construction of the trigonometric functions will require students to see the distance created along the circumference of the unit circle as a point travels counterclockwise. This distance must then be coordinated with the change in the height of the point (i.e., the change in the *y* value of the point's changing coordinates) for the sine function and the change in the horizontal position (i.e., the change in the *x* value of the point's changing coordinates) for the cosine function.

Further details about the specific tasks I will be using to foster covariational and quantitative reasoning are explored in Chapter 3, the methods section. In the following section, I re-present my research questions. This will include further exposition in light of the literature reviewed here.

Research Questions

Two research questions will frame this project. Research Question 1 (RQ1) is about the VLs' orientation. The target of this question is the emotional relationship the VLs feel towards the talent and the way the VLs position themselves with respect to the talent. The second research question is the VLs' problem-solving behaviors. The target of this question is the ways in which the VLs use the videos in their work on novel tasks. As will be explored in further details in Chapter 3, this study contained two treatments: one viewing scripted dialogue and the other viewing unscripted dialogue. There were four participants in each treatment, broken into two pairs. Each pair viewed their respective type of dialogic videos over the course of five one-hour and fifteen-minute-long viewing sessions. The following subsection explores the two research questions and expands upon my hypotheses entering this study.

RQ1: Orientations

RQ: How does the orientation of undergraduate vicarious learners toward the talent differ when the dialogue of the talent is scripted versus unscripted?

Elaboration

This question is aimed at VLs' orientations toward the talent across two dimensions: (a) collaborative versus voyeuristic, and (b) emotional versus unemotional. On the one hand, the SVLP positioned vicarious learners as neutral or distanced from the direct participants through epistemic detachment (McKendree, et al., 1998). On the other hand, Project MathTalk demonstrated that VLs can be emotionally involved and orient towards the talent in the videos as if they were all in a collaborative group (Lobato & Walker, 2019).

I view the distinction between when VLs are experiencing epistemic detachment or an emotionally-involved quasi-collaborative orientation as analogous to experiencing sympathy or empathy, respectively. If Person A is sympathetic to the suffering of Person B, then Person A has an understanding of the suffering of Person B. If, on the other hand, Person A is empathetic to the suffering of Person B, then Person A has gone through–or is going through–similar suffering as Person B and is feeling similar emotions. The key difference between these experiences is the emotional experience of Person A. Sympathy is understanding without feeling, and empathy is understanding with feeling. This distinction holds for the orientations of epistemic detachment versus emotionally-involved quasi-collaboration. When a VL is epistemically detached they gain an understanding of arguments and cognitive processes (e.g., how to plot a parabola) by viewing from a literal and metaphorical distance (i.e., without feeling). Through quasi-collaboration, arguments and cognitive processes are experienced and understood from a literal distance, but a metaphorical closeness (i.e., with feeling).

I intend this question to build off of the findings of Lobato and Walker (2019), but for a different demographic of participants (undergraduates here), and to extend those findings through the comparison of orientation of VLs in scripted versus unscripted treatments. This study is exploratory in nature. In addition to extending the analysis to a new population, it also is a needed step toward a larger *N*. With a sample size of four pairs of VLs, this study will explore the nature of VLs orientation for a population four times that of the extant literature. Findings from this study can then serve as a foundation for survey-type questions for future quantitative analysis of vicarious learner's orientations.

Hypotheses

I hypothesize that VLs engaged with unscripted dialogic videos are more likely to take on a quasi-collaborative and emotional orientation toward the talent than the VLs viewing the scripted videos. Lobato and Walker (2019) posit that the VLs' mathematical ability could be predictive of the quasi-collaborative and emotional orientations. In that study, the VLs were earning in the B to D range in high school algebra and frequently assessed themselves as prone to struggling in math classes. That characteristic will remain constant for this work with a population of samples from college algebra courses. Students participating in college algebra courses, overwhelmingly, performed poorly on placement exams and can be purposefully sampled such that they will be likely to experience challenges with the content of the videos. If VLs are going to orientate towards the talent in a positive way, then they must have experiences that can foster such feelings. In other words, the VLs will need to be able to empathize with the talent. Specifically, experience with struggle in mathematics, especially if both talent and VLs struggle on the same thing, can promote a sense of working together to overcome those struggles. While Lobato and Walker (2019) found an emotional connection in their secondary school students towards the struggle of the talent, even postsecondary school students have been found to empathize with the struggle contained within the statement of a hypothetical student task (Rasmussen et al., 2020). Furthermore, Mayes (2015) asserts that VLs attend more to the voices of the tutee because the shared ability, language, and appearance produce an empathetic connection for the VL toward the tutee.

Importantly, the struggle of the talent needs to be perceived as authentic if the VLs are going to empathize with it. As Geertshuis et al. (2021) suggested, scripted dialogue will be, at times, perceptibly inauthentic, and I believe that the VLs will be aware that the misconceptions are not organically presented. Similar to the “uncanny valley” principle, where the perceived human-ness of a robot drastically decreases as robots get more similar to humans, I believe the VLs in the scripted treatment will experience less empathy towards the talent’s struggle because the misconceptions are nearly “human” like but are not “human.” Thus, the VLs in the scripted treatment are more likely to take on an orientation of epistemic detachment than the VLs in the unscripted treatment. The alternative to this hypothesis would be a prevalence of epistemic detachment across both treatments. Foremost, this could occur because of the change in demographics of this study (i.e., the participants being undergraduate students). If undergraduates' motivation for viewing and engaging with the videos is dominated by a desire for answers, then episodes that contain authentic confusion could be perceived as a waste of time. The difference between authentic and inauthentic dialogue would largely be irrelevant to VLs of this mindset and epistemic detachment would be their default modus operandi. Additionally, there exists the possibility that VLs will have a negative emotional experience with the talent. While this has not been found in the extant literature, there remains the possibility that features of the talent (e.g., not outgoing, not expert enough, etc.), features of the videos (e.g., confusion in the scripted treatment is distracting, dialogue in the scripted treatment is inauthentic, etc.), or features of the participants (e.g., mathematical skill level, undergraduate VLs and talent, etc.) could lead to different findings than positive emotions from Lobato and Walker (2019) or the conjectured epistemic detachment of McKendree et al. (1998).

RQ2: Problem-Solving Behaviors

> RQ: How do the problem-solving behaviors of vicarious learners engaged in learning to reason covariationally about the sine function differ when the dialogic videos that they are viewing and interacting with are scripted versus unscripted?

Elaboration

In this question I am interested in tracking the VLs' problem-solving behaviors as they engage with the videos and comparing those behaviors for students engaging with scripted versus unscripted videos. As a baseline, I expect all participants to enter the study with a similar level of understanding of trigonometry and similar covariational reasoning. A similar starting baseline is important for engaging students in problem solving. Following Schoenfield (2016), problem solving describes the processes and strategies used while working to solve a novel task. Thus, it is important for students to have similar starting conceptions in order for tasks to be perceived as novel and for problem solving to emerge.

I hypothesize that all participants of this study (the talent and VLs) will have a triangle-based (also known as a ratio-based) understanding of the trigonometric functions. A triangle-based approach is trademarked by its overemphasis on the use of the acronym SOHCAHTOA

(Demir & Heck, 2012). Under traditional instruction, students are presented with a triangle, a side length, and an angle and use SOHCAHTOA to solve for a missing value. This leads to conceptions of the trigonometric functions that are inseparable from triangles. My lessons, on the other hand, will provide a circle-based approach, where the argument for trigonometric functions are distances along the arc of a unit circle (Moore, 2012). National documents, such as the *Common Core State Mathematics Standards* (CCSSM; National Governors Association Center for Best Practices, 2010), reinforce the triangle-based approach. Trigonometry taught at the high school level in California uses the triangle-first approach in geometry classes. In fact, every mention of applying the trigonometric functions within the CCSSM for geometry, students' first exposure to these functions, includes the use of triangles. It is not until the unit circle is introduced that students learn to "[e]xtend the domain of the trigonometric functions using the unit circle" (National Governors Association Center for Best Practices, 2010, p. 71). Overwhelmingly, these attempts to extend the domain fall short (Demir & Heck, 2012; Moore, 2012). Given my participants are undergraduates, they will likely have experienced the triangle-based form of instruction in their high school mathematics courses. This ought to create a similar baseline conception of trigonometric functions.

Hypotheses

One reason that a difference can emerge is if the scripted videos are experienced as authoritative. Without an authentic student voice, I believe that VLs are likely to experience the scripted videos as if they were lectures containing information to be memorized. The talent of the scripted treatment would play the role of a teacher posing rhetorical questions to the class that the teacher then answers before any kind of classroom discussion can take place. VLs in the scripted treatment would then use the videos as a resource for answers and ignore the ways of reasoning presented in the videos. Not paying attention to the ways of reasoning could then directly affect the VLs covariational reasoning and their quantitative reasoning, which are both important parts of students' understanding of trigonometry (Demir & Heck, 2012; Moore, 2012).

On the other hand, I hypothesize that the VLs in the unscripted treatment will experience their respective videos as open for negotiation and engagement. The authentic-student-dialogue feature of the unscripted videos will, at minimum, foreground the talents' ways of reasoning. This exposure to ways of reasoning that are similar to the VLs can influence the VLs' problem solving. As shifts happen for the talent, shifts will happen for the VLs. When the talent posit an idea, the VLs can come to recognize it is a suggestion and not something to be accepted a priori. Additionally, if VLs perceive that the talent in the unscripted videos are in the process of their own negotiation of meaning, then the VLs have few words to take as authoritative. The talent are not presenting information to be memorized and words to be accepted, instead they are negotiating meaning for themselves. Thus, an indirect participant may do the same.

Alternatively, it is possible that no difference between the treatments will emerge from their problem-solving. No difference could emerge if, for example, the tasks alone mediate learning. If, as Simon et al. (2010) suggest, the tasks are the primary vehicle for learning, then the videos and ways of reasoning evidenced in the videos do little more than reiterate what the VLs have already done/said. Another possibility is for the VLs viewing the unscripted videos to disengage with the videos and evidence little problem solving. This is possible if the authentic confusion contained in the unscripted videos negatively impacts the students. This would then result in the VLs using the videos as if they were expository. It is possible that the undergraduate participants are experienced enough with the construction of their knowledge that seeing peers engage in that process causes interference and is overly distracting.

Chapter 3: Methods

The previous two chapters have argued that the phenomenon of vicarious learning has been understudied with respect to key design features and mechanisms of learning. Namely, comparisons of scripted and unscripted dialogue have been made, but not tested, and learning–as an outcome–has been shown, but not fully understood. It has also been argued that experiences may differ between vicarious learners (VLs) using scripted versus unscripted instructional videos. In this chapter, the methods used to explore differences between treatments, scripted versus unscripted, will be presented.

The methods for this project are broken into five main sections: research questions, video lesson preparation, filming, data collection, and analysis. In the first section, an explicit connection will be made between the research design and the research questions. The Video Lesson Preparation section focuses on the development of Realistic Mathematics Education (RME) inspired lessons for the filming of both treatments. The Filming section describes how I produced the unscripted dialogic and scripted dialogic videos. The Data Collection section provides details about the vicarious learners, the kinds of data that were collected, and the data-collection processes. Finally, the Analysis depicts how that data will be used in service of my two research questions.

Research Questions

The purpose of this dissertation is to further the fields understanding of vicarious learning by answering the following research questions (RQs):

1. How does the orientation of undergraduate vicarious learners toward the talent differ when the dialogue of the talent is scripted versus unscripted?
2. How do the problem-solving behaviors of vicarious learners engaged in learning to reason covariationally about the sine function differ when the dialogic videos that they are viewing and interacting with are scripted versus unscripted?

To investigate these questions, participants were placed in one of two treatments, unscripted or scripted. The unscripted treatment viewed dialogic videos that captured authentic learning of two students working to construct the sine function. The scripted treatment viewed instructional videos that contain one student and one teacher acting out a learning scenario that is modeled after the unscripted videos (for further details about the differences in the videos see Filming section below). For both treatments, videos were viewed in pairs. While viewing, the pairs engaged in their own set of tasks aimed at the construction of the sine function. There were five total viewing sessions for each pair.

To answer the orientation question, data was collected from viewing sessions and from interviews at the end of each viewing session. During the sessions, particular attention for analysis was paid to explicit references to the talent and their work. Additionally, the analysis looked into comments made while the videos played and comments after the videos concluded. Finally, data was collected from the interviews consisting of the VLs' responses to questions about how the VLs felt about the session and the videos. Analysis of this question attempted to build off of the findings of Lobato and Walker (2019) and began with the application of their quasi-collaborative framework.

Answering research question 2 relied on the use of thematic analysis. As characterized by Braun and Clarke (2006), thematic analysis was used to identify and describe patterns of meanings (i.e., themes) within my participants' problem-solving behaviors. Collecting data from their viewing session allowed for an analysis of the ways in which each pair of VLs interacted with the videos and each other. Following Schoenfield (2016), I take problem solving to occur while students engage with novel problems that engage cognitive processes and strategies to identify, consider, and implement ideas. Analysis of this question begins to assess how VLs use dialogic videos in their mathematical problem solving.

Video Lesson Preparation

In the following section I introduce the guiding principles that provided the structure that led to the creation of a sequence of tasks that were used for the lessons in the dialogic videos.

Then I illustrate a number of tasks that were used and conclude with pilot data that led to the refinement of my task sequence.

Lesson Structure

I created five lessons surrounding the construction of the sine function and the development of quantitative meaning for angle. The initial function of these lessons was for filming; thus, I will refer to them as the lessons-for-filming. Later, the lessons-for-filming will be adapted for the viewers, the VLs, with the creation of lessons-for-viewing. The lessons were constructed using RME principles and activity types (*situated*, *reference*, *general*, and *formal*; Gravemeijer, 2020) with the goal of roughly one activity type per lesson. In RME, situated activities are set in the context of a realistic activity. Importantly, realistic here means realizable (i.e., realistic is anything that a student can imagine). Reference activities are those that call back, implicitly or explicitly, to the situational one. General activities are those activities that allow for interpretations and actions independent of the original context. Lastly, formal activities are the activities involving the use of formal terminology and symbols. This results in a new realistic context made possible through the previous three phases.

The progression through activity types leads to new mathematical realities (i.e., once engaged in the formal activity phase, some feature(s) of the mathematics are newly realistic for the student). More generally, the formal activity phase serves as a springboard for future phases of activity types, beginning in a situation that is newly realizable. For the purposes of my lessons, the goal of these lessons culminated in formal symbolization of the sine function. Part of this goal requires the formalization of the terminology for the argument of the sine function, thus a subgoal included the formal symbolization of angle in radians. Given the constraints of the number of lessons, it was unlikely that a new mathematical reality would emerge, but these activity phases provided the framework for my unit.

Emergent Modeling

While the activity phases framed the creation of the lessons-for-filming, the overarching structure of the lessons was produced through the use of *emergent modeling*. Gravemeijer (2020) describes emergent modeling as the progression through five steps in lesson construction. In the following section I provide an overview of the five steps of emergent modeling, and in the subsequent subsection explore the details of each step for my unit.

The first step was to identify the instructional goals. Instructional goals include the content goals (i.e., what you want students to learn) and effective strategies for attaining those goals (i.e., how you expect them to learn). The next three steps are where the lessons, and the specific tasks therein, began to take form. The second step of emergent modeling is to identify a central model. This model served as the situational activity and needed to offer an extension toward meeting the goals of Step 1. This is one of the most important steps in the process as one is attempting to pick a realistic model that will be placed at the foundation of the students' reasoning. The third step is to identify instructional starting points. The instructional starting points is informed by the literature reviewed and an understanding of what your students currently know. The fourth step is to design a series of sub-models and identify the *overarching model*. The overarching model is the broad body of the lesson, and the sub-models are the connective tissue. At this step the progression between activity types and between individual tasks takes form. Following this step, the first iteration of the lesson is complete.

The fifth, and final, step of emergent modeling is to reflect on the sequence. Reflection allows for changes to be made to the lesson goals, structure, or individual tasks for subsequent implementations of the lesson. This process was used to create my *lessons-for-viewing* (i.e., the analogous set of lessons for the VLs) and for the creation of the videos. Reflection allowed me to consider what kinds of tasks should be omitted, sequences should be changed, and to consider what tasks the VLs should be engaged in while viewing the videos.

Instructional goals

The sequence of tasks for the lessons-for-filming were framed under the instructional goal of establishing student's quantitative reasoning (à la Thompson, 1993) with angle and covariational reasoning for the sine function. As presented in Chapter 2, the literature on trigonometry suggests a hole in students' conceptions for angles (Moore, 2012, 2014). Common student conceptions of angle include angle as a label for the corners of a shape (Alyami, 2020; Demir & Heck, 2012; Moore, 2014), as located between two lines but not one line or zero lines (i.e., students struggle to locate 180 and 360 degrees, Keiser, 2004; Yigit, 2014), and as dominated by degrees (i.e., radians are merely conceived of through a conversion formula, Moore, 2012, 2013). Alyami (2020) juxtaposes these ways of reasoning with angle as qualitative instead of quantitative. I emphasized the latter within the lessons.

Introductory instruction on trigonometry typically begins either with triangle relationships or with the unit circle (Demir & Heck, 2012; Moore, 2012). Students who are introduced to the trigonometric functions as a means of finding missing side lengths of triangles, often, struggle to evaluate trigonometric functions when they are not given a triangle (i.e., they implicitly view the triangle as the mathematical argument of the trigonometric functions, Çekmez, E. 2020; Van Sickle, 2011). Students who are introduced to trigonometry via the unit circle typically have conceptions of the trigonometric functions that are dominated by the rote memorization of values on the unit circle (e.g., $\sin\left(\frac{\pi}{6}\right), \cos\left(\frac{3\pi}{2}\right)$, etc.). The unit circle approach can lead to a number of difficulties when students are asked to approximate the output of a trigonometric function when the argument is not a rational number multiple of π (Çekmez, E. 2020; Demir & Heck, 2012). For example, students may interpret sin(1) as sin(1°). Demir and Heck (2012) suggest an alternative approach, one that starts with a graph. Introducing the trigonometric functions in conjunction with their graph can afford a view of the trigonometric functions as distinct from triangles and distinct from discrete sets of values around the unit circle.

The findings from the literature suggest that an instructional unit on trigonometry ought to emphasize angle as a measurable quantity and emphasize deeper understanding for the inputs and outputs of the trigonometric functions (Alyami, 2020; Çekmez, E. 2020; Demir & Heck, 2012; Moore, 2012). Thus, the literature suggested two instructional goals: (a) establish quantitative meaning for angle and (b) to instill strong covariation reasoning for the inputs and outputs of the sine function.

Quantitative Reasoning. The quantitative meaning for angle, instructional goal (a), that I attempted to establish was a view of angle as a measurable portion of a circle and that can be measured in units of radians. Angles were not explicitly used in the first three lessons, but a seed for the conception of angle as measurable was planted. Over the course of the students' work with the Circling the *N*-gon task sequence (Figure 3.1) the repeated emphasis on distance embedded a kind of quantitative measurability for angle. In other words, as the shape of the *n*-gon approaches a circle the students needed to reason quantitatively about the distance around the shape. This likely comes from an appeal to their conception of the perimeter. This means that distance traveled was viewed as a measurable quantity. Then, when the shape became a circle, they could extend their ways of reasoning to an understanding that this distance is still measurable. Formally, this distance is an arc length. To make use of the students' quantitative meaning for arc length, radians were then introduced.

Create a series of graphs that relate the distance traveled by the point A to the height of the point as it travels counterclockwise around each of the following *n*-gons.

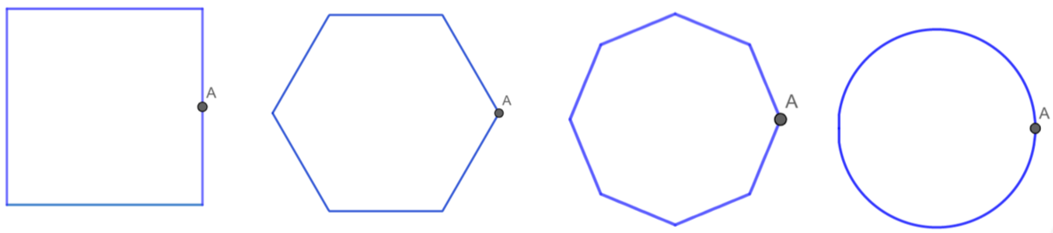


Figure 3.1
N-gon Task Sequence

To embed students' conceptions for angle as radian with measurability, radians were defined in terms of the number of radii that fit in an arc length (i.e., arc length per radius). Quantitative reasoning with radians can be demonstrated with a fluid understanding of the relationship between proportionally equivalent segments of arc length for circles of varying radius (Moore, 2013). In other words, an emphasis was placed on the students' ability to see equivalence in radians for arc lengths of proportionally higher magnitude.

Covariational Reasoning. Instructional goal (b) was aimed at the progression through Thompson and Carlson's (2017) hierarchy toward the transition from chunky to smooth covariational reasoning (Table 3.1). In the first set of tasks students were successful if they were able to coordinate synchronous changes between input and output. This was, likely, not the starting point for my college algebra students' covariational reasoning (Thompson & Carlson, 2017). Through the use of dynamic simulations, synchronous changes were emphasized (Moore, 2012). As the lesson progressed, students were able to make use of simultaneous changes between variables to create a graph that will appear, from the expert's vantage point, as a chunky view of the sine function. While the tasks did not emphasize this, the graphs create piecewise approximations of the sine function. This fact was left implicit to the students. Thinking in terms of chunks of fixed size and reasoning about the rate of change over that whole segment is a hallmark of chunky covariational reasoning (Castillo-Garsow, 2012). Through the use of regular polygons in the Circle an *N*-gon task sequence (Figure 3.1), the distance traveled is fixed to the side length and can foster this way of reasoning. Thus, chunky covariational reasoning was evidenced when students are able to coordinate differences in the slopes over intervals of the same length. For example, if the students are comparing line segments with different slopes, then chunky covariational reasoning would be evidenced if the students are able to notice that the point has traveled the same distance but the change in height was greater in one segment than in the other.

Table 3.1
Thompson and Carlson (2017) Levels of Covariational Reasoning

Level	Description
No coordination	There is no evidenced image of variables varying together. Change occurs in one variable with no coordination of the other.
Precoordination of values	Asynchronously, an image of the variables changes is evidenced. The coordination is such that change occurs in one variable, and then change occurs in the other.
Gross coordination of values	An image of the variables changing together without a view of coordinate pairs of values is present. Simultaneous change is understood to be occurring, but with a loose link between the specific changes in the quantities values.
Coordination of values	Coordination is occurring between values of one variable with values of another variable emerge with the goal of creating a set of ordered pairs.
Chunky continuous covariation	Changes in variables are seen to be occurring simultaneously over intervals of fixed size.
Smooth continuous covariation	Changes in one variable are occurring simultaneously with change in another variable and this change is envisioned both smooth and continuous.

As tasks progress, the chunks will become progressively smaller and a chunky to smooth transition can begin to take place. While students may not fully be able to coordinate the continuous change in the rate of change for this function (i.e., what Thompson and Carlson call smooth covariational reasoning), they will have begun their progression towards a smooth covariational reasoning for the sine function. This can be evidenced if students are able to discuss the way in which change is occurring in their graph over a variety of intervals. For example, a student is evidencing smooth covariational reasoning if they can reason and compare the rate of change in height for differing changes in distance traveled. In other words, while the goal is for students to attain a smooth covariational level of reasoning, it was sufficient if they were able to start to reason with chunks of different sizes, one step beyond what Thompson and Carlson called chunky covariational reasoning.

Importantly, instructional goals (a) and (b) are related. The inputs, or arguments, of the trigonometric functions are angles. Thus, establishing quantitative reasoning with the argument of the sine function was be leveraged to quantitatively reason about angles. The relationship between these two goals inspired the central models of the lessons.

Central Model and Instructional Starting Point

The central model of the lessons was the Ferris wheel and the creation of a sequence of graphs representing the relationship between distance traveled by a point around a polygon of *n* sides to the height of the point (i.e., the Circling an *N*-gon task sequence, Figure 3.1). Use of the Ferris wheel context for trigonometry has gained some prominence (e.g., Johnson et al., 2017; Stevens & Moore, 2016). This context provides a realizable situation in which a student can think about the journey a rider takes around the wheel and the resultant covarying quantities. For the sine function, one can think about the distance traveled by the rider and compare that to the height of the rider. Circling an *N*-gon, on the other hand, will serve as the central model for the graph-first approach to trigonometry. Through a sequence of tasks, starting with a square, the Circling an *N*-gon context will walk students through a piecewise construction of the graph of the sine function. Following Thompson and Carlson (2017), this process will allow for the creation of chunky covariational reasoning that can eventually transition towards smooth covariational reasoning.

The plan was for instruction to begin with the presentation of a static image of the Ferris wheel as an introduction to the context. Eventually, this image will be put into motion with the goal of extending students covariational reasoning. At the initial, static, stage, students presented with the task of coordinating the distance around the Ferris wheel and the vertical position are likely to think of change happen asynchronously: first distance changes and then height changes (Thompson & Carlson, 2017). This way of reasoning can then be extended through the use of dynamic simulations (e.g., in the Which One? Task, where students must select which red line corresponds with the change in distance, Figure 3.2). This can lead to a shift in covariational reasoning to a coordinated change in variables. In the initial framing of this task (see Table 3.2 below), the goal was merely to ground students in the context and to initiate their thinking in terms of quantities. The task statement is broadly about identifying measurable features, as opposed to starting to think about the sine function with the coordination of distance and height. Through extensions that reference this situational activity, the mental image of the Ferris wheel will be an important tool for student's thinking throughout the lessons.

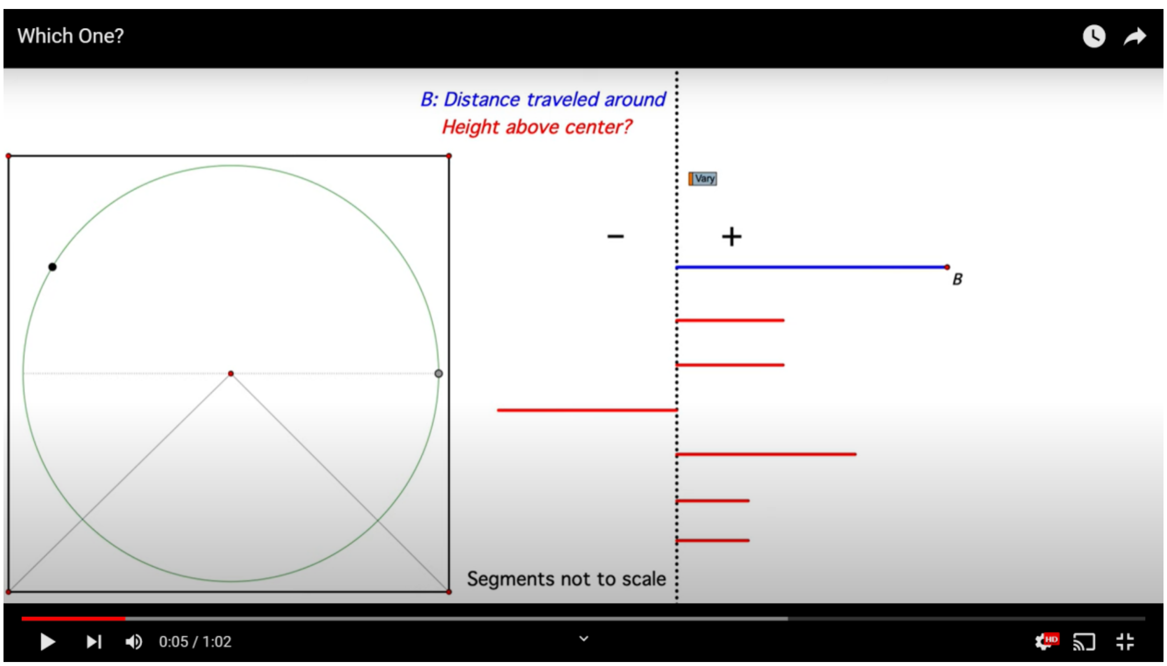


Figure 3.2
Which One? Task

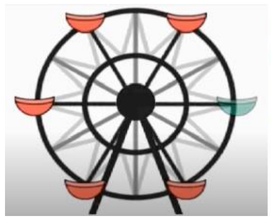
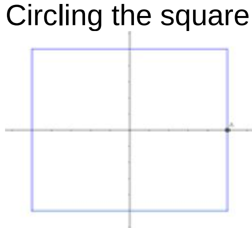
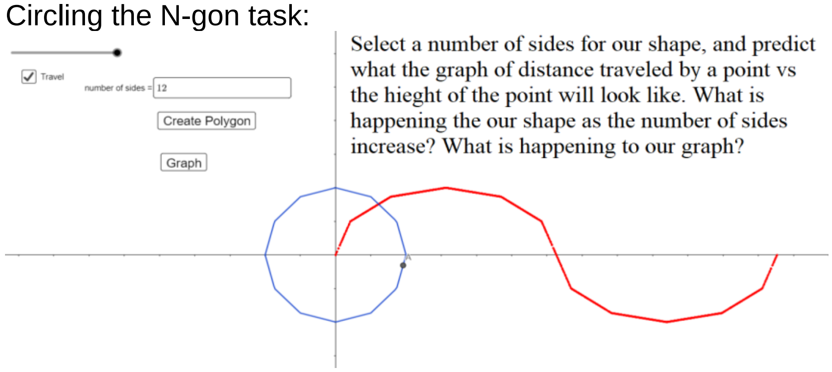
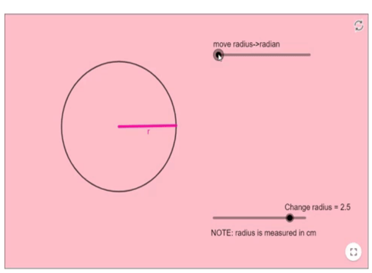
Progression of Activities

The progression of activities for the five lessons is summarized in Table 3.2. Following the Ferris wheel task, the plan was for students to be given a square and to be tasked with creating a graph that represents the relationship between distance traveled by a point around the shape and the height of the point. This situation is still embedded in the Ferris wheel context, with the square being positioned as the inventor of the Ferris wheel's first attempt at creating the ride. Imagining the point is a rider, the students will need to think about how the distance traveled counterclockwise around the square is related to the height of the rider. Covariational reasoning will play a fundamental role in students' success in this task. The goal was for the emergence of covariational reasoning with coordination of values (distance around a shape and vertical position/height are changing simultaneously) and the beginning of the transition from chunky to smooth covariational reasoning.

In addition to generalizing this task for squares of different sizes (a comparison that will allow the students to generalize the shape of the graph), students then generalize the shape of the graphs that compare distance traveled by a rider to the height of the rider as the number of the sides begins to increase. This task makes use of a GeoGebra applet that allows the students to make predictions for the shape of their graph based on the number of sides, and then test their predictions. This continues the progression from chunky to smooth covariational reasoning and will culminate in the extension of the *N*-gon into a circle, and the resulting graph into the sine function. This resulting graph will informally be referred to as the circle graph. Taking the generalization process one step further, the students will also be tasked with determining the general shape of the smooth curve for circles of different sizes. After the sine curve is constructed, but prior to telling the students what exactly they have created, the next task will involve defining radians.

The primary motivation for the introduction of the terminology of radians is to establish important points on the smooth graph the students created. Through the exploration of an applet that visually depicts the relationship between a radian and the radius of a circle (see Radian Task in Table 3.2), students will be tasked with defining what a radian is. The definition that this applet emphasizes is radian as a number of radii. Quantitatively defining radian as the number of radii means that determining how many radians an arc length represents is equivalent to determining the number of radii that fit in the arc length.

Table 3.2
Selection of Key Tasks and Associated Conceptual Goals

Conceptual goal	**Task**
Grounding in a realizable context and initiate thinking in terms of the measurability of distance and comparison between distances (quantitative reasoning with distance)	Ferris wheel Task:  Imagine you are a rider in the blue car of this Ferris Wheel traveling counterclockwise. What are some of the measurable features of this context?
Covariational reasoning with coordination of values and the beginning of the transition from chunky to smooth covariational reasoning	Circling the square task:  Image the point A is a rider on a square Ferris wheel. Create a graph that shows the relationship between distance traveled around the square and the height of the point above the center of the square.
Covariation at the chunky level continues to be smoothed out	Circling the N-gon task: Select a number of sides for our shape, and predict what the graph of distance traveled by a point vs the hieght of the point will look like. What is happening the our shape as the number of sides increase? What is happening to our graph? 
Quantitative meaning for radian as the number of radii in an arc length	Defining a radian task:  Use the top slider to start the animation. What do you think the point of the animation is? What does the animation attempt to show in terms of the relationship between a radian and a radius?

The goal is for the students to make a generalization for radians that is analogous to their generalization of a smooth graph. Namely, the goal for this sequence of tasks is for the students to recognize that radians are radius invariant, or that, much like the shape of the smooth graphs, the radius does not have an effect on the number of radians in a circle. This definition for radian is then be applied to the smooth graph to find and label a number of key values (e.g., the end point of the graph at 2π radians). The goal of this sequencing of tasks is to leverage students' quantitative reasoning used in the construction of the smooth graph. They will have been repeatedly positioned to view the *x*-axis of their constructed graph as a measurable quantity (distance around a shape). Through the introduction of radian, they can now apply the same way of reasoning to the proportion of number of radii in a segment of arc length. Finally, the formal terminology of sine is introduced (i.e., they will be told that the function they have been creating is called sine and sine of *x* is function that takes angle as an input and outputs the height) and the Which One? task (Figure 3.2) is revisited.

Pilot of Tasks

The final step in emergent modeling is to reflect on the sequence. As part of that reflection, I piloted some of the tasks with a student matching the criterion sampling I will be using for this project, an undergraduate student at a Southwestern United States university currently enrolled in college algebra (Patton, 1990). The 1-hour pilot session was conducted over Zoom and all work was done via a virtual whiteboard.

The focus of this pilot was to see how the student was able to reason covariationally within the Circling the Square task. Upon presentation of the task to create a graph that shows the relationship between the distance traveled by a point around a square and the height of the point, a number of quantifiable features became clear to the student based on their experience with squares.

Student: So, I am looking at the point, and the point is five over. Which means it's equidistant, five and five and five and five and five and five. Which means that every length is ten… It travels 40 points, so the height is 8.

Here, the student assigned a value to the distance from origin to the starting location of the point. With this distance defined as 5, she was able to reason correctly that the distance around the square would be 40. However, it is unclear where the height of 8 came from or what it meant to the student. Regardless, finding a specific value was not the assigned task.

Directing the student back to the creation of a graph that modeled the situation, it became clear that the creation of the graph required more than just the coordination of distance and height. Because of the presentation of the square within a coordinate plane, the student was forced to coordinate the use of multiple axes. This difficulty was evidenced by the student's recreation of the square on their graph. When prompted to reason about the height and distance traveled on the first segment of the square, the student was able to find a few points.

Instructor:	If we are at the starting location, how far has the point traveled and what is its height?
Student:	Well, it's 0.
Instructor:	Okay, what is 0?
Student:	It hasn't moved so the distance is 0 and the height is 0.
Instructor:	What if the point moved to the first corner? What would happen to the distance?
Student:	It would have gone 5.
Instructor:	And what about the height?
Student:	5 up.
Instructor:	What if the point hasn't reached the corner yet, can we say anything about the distance traveled and the height of the point?
Student:	Sure, I could send the point like 3 units and the height would be 3 too.

Using these three points, (0,0), (3,3) and (5,5), the student drew a line (Figure 3.3). In an attempt to suggest that this line cannot represent what is happening everywhere on the square, the student was asked about the next segment of the square.

Instructor:	What happens to the height of the square after the point has gone 5 units?
Student:	At 5 it becomes y = 5. The line becomes y = 5.
Instructor:	For how long?
Student:	From -5 to 5.

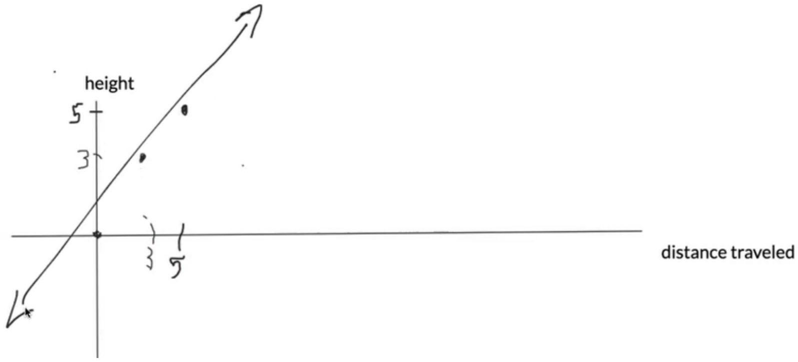


Figure 3.3
Student's Graph for the First Segment of the Square

The student then drew the line $y = 5$ and placed a point on this line at what would appear to be (-5, 5) (Figure 3.4). At this point the student appears to be recreating the square. The domain of the constructed graph is not consistent and the graph is not piecewise defined.

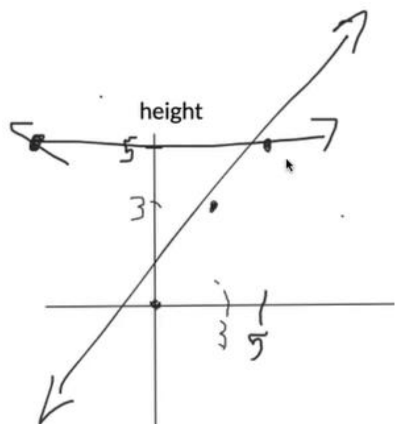


Figure 3.4
Student's Continued Work

Unfortunately, the covariational reasoning was not evidenced by the student over the entire perimeter of the square. At this stage, it is important for the student to reason with a synchronized coordination of variables. In other words, a successful student will need to be able to see change happening in one variable as it is happening in the other and to see this change happening around the shape. Upon reflection of the lesson's sequence, a new task was added to aid in covariational reasoning. Based on the pilot student's success in reasoning about the first segment of the square an Elevator Task (Figure 3.5) was created to slow the progression of difficulty. In this task, the point is presented as a person in an elevator. The goal of the task is to create a graph that represents the relationship between the distance traveled by the rider and the height of the rider. To encourage quantitative reasoning, the elevator is given a height of 10 meters. Initially, the point will only be going up. Once an accurate graph is created then the context will be extended to going down.

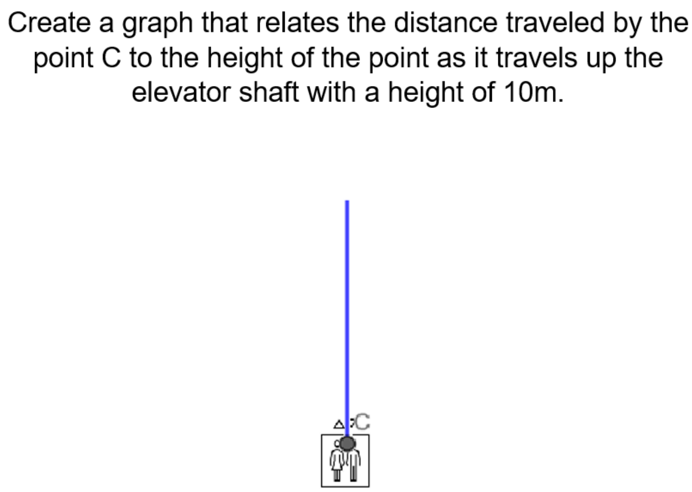


Figure 3.5
Elevator Task

There are two goals of this task that became clear upon the pilot experience. The first was to ease the students into covariational reasoning. There is more coordination required than just that between the quantities of distance and height. There is a need to coordinate the quantities presented in the context of the first graph containing the task to the related, but different, quantities required for the creation of a second graph. In other words, the student must coordinate a graph showing horizontal position and vertical position (Figure 3.5) to a constructed graph showing distance traveled and vertical position. The second goal of this task is to get students to think about the constraints on the domain on their graph. In the context of an elevator, there is a very realizable restraint on how high one can go. The extension task, modeling a return trip back down the elevator, will allow for discussion of piecewise functions and the creation of a graph through multiple functions defined over piecewise domains.

Reflection through the pilot experience has been helpful for the refinement of the lessons and continued in the refinement of the tasks-for-viewing.

Video Creation: Filming and Post-Production

Following the completion of the lessons-for-filming, I filmed the unscripted and then the scripted videos. The unscripted dialogic videos were filmed first. Then the unscripted videos were used to inform the dialogue of a scripted version of the videos. The unscripted talent's emergent conceptions were used to inform *misconceptions* in the scripted videos. The moments that are translated into scripted misconceptions are those that were most productive for the unscripted talent's progression on the tasks. This model follows Muller, Bewes, et al.'s (2008) use of misconceptions in their scripted physics videos. Informed by the conceptual change literature, this format of video contains a tutee, what I call the talent, and a tutor, what I call a teacher, working on a set of problems. Importantly, the set of problems are identical to those contained in the unscripted videos. During the scripted videos, the talent work on a task and present a scripted misconception. The teacher then asks guiding questions and draws attention to the specific mistakes being made. On filming days, the talent and the teacher would begin each task by reviewing the unscripted talent's work on that task. This included going over the misconceptions the talent presented in the unscripted videos, their progression through the task, and the final solution presented in the unscripted videos. During filming, variation in the precise language was permitted by the talent in the scripted videos (e.g., saying, "I'm not sure…" versus "I think so…"), but deviations from the ideas presented by the talent resulted in reshoots or the teacher guiding the talent back to the language of the script.

The central difference between the video types is authenticity. Within the unscripted videos, authentic moments of insight and confusion emerge. The scripted videos then recreate these moments through the talent who may not truly hold the belief they are presenting. In addition to not holding the professed belief, the talent of the scripted treatment may not believe the resolution (i.e., because of the more expert language of the script, difficulty of the concept, or any number of other reasons the talent may not fully be convinced by the scripted explanation).

Another difference between the videos was the necessity to establish norms for the interactions between the talent and the off-screen teacher in the unscripted videos. Three norms were negotiated to moderate their work and ensure the videos contained useful dialogue: co-construction (i.e., the talent worked together on their solutions), non-judgment (i.e., the talent were open and responsive to each other's suggestions and ideas), and rephrasing (i.e., the talent repeated their understanding of what their partner had suggested). My goal was to negotiate these norms at the beginning of the first filming sessions and then to reinforce them during each subsequent session by asking both of the talent to share, react, and reflect on their own and their partner's work. Not all of these norm-establishing moments are included in the final produced unscripted videos, but the moments that are particularly salient and productive for the talent's work on tasks are included. The role of the talent in the scripted videos is not one of interlocutor, thus these norms were written into the dialogue of the scripted videos. Instead of a process of co-construction, the talent worked independently while the on-screen teacher guides and asks leading questions. On the other hand, without interlocutions between the talent in the unscripted videos, no progress could be made on the tasks.

Filming and Post-Production

Filming took place in the Project MathTalk filming studio. The studio is equipped with cameras that can be used to capture the talent and their work. The camera was positioned on the talent (and teacher for the scripted videos) and recording software captured their work on a Wacom Cintiq device (i.e., a pen display that allows users to write and work on web pages or Microsoft Word). Filming of the unscripted dialogic videos took place over five 1-hour sessions. During the filming, the unscripted talent sat together at the filming table and were guided by an off-screen teacher/director. The teacher's role was to guide the talent by providing them with new tasks, encouragement, reminders of the collaborative norms, and to ask guiding questions.

Similarly, filming of the scripted dialogic videos took place over five sessions. During the filming, the scripted talent sat and worked with a teacher at the filming table. The teacher played the same role as the teacher in the unscripted sessions but asked scripted guiding questions. Following filming the videos were edited. The edited videos contain the talent (and teacher for the scripted videos) as they work on tasks, the work they produced on a Wacom Cintiq device, and screen recordings of the production of their work on applets/calculators (Figure 3.6). Each lesson created approximately 30 minutes of footage that was broken into four to six shorter video episodes, none exceeding ten minutes in length. The use of short videos is a convention set forth by Khan Academy (Ibrahim et al., 2012; Khan, 2012) and mirrored in the dialogic videos created by other researchers (e.g., Chi et al., 2006; Muller, Sharma, et al., 2007). In addition to the video recordings of the talent and the screen recordings of their work, post-production of the videos added voiceovers and highlighting.

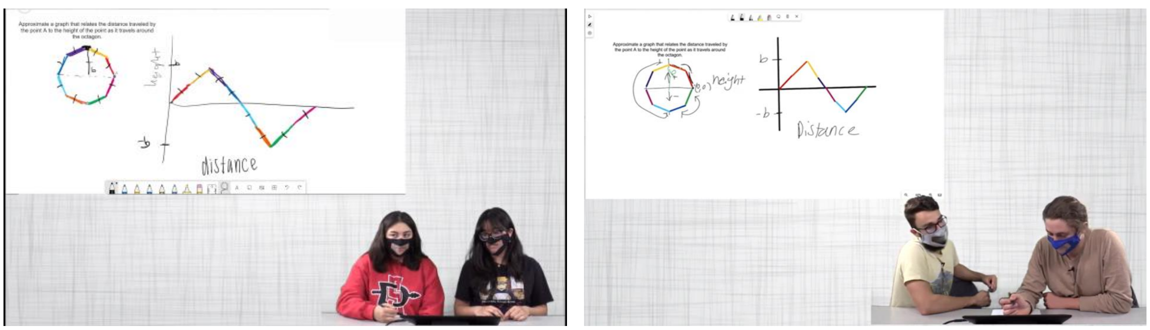


Figure 3.6
Image from Unscripted and Scripted Dialogic Videos, with Talent and Work Visible

The post-production process followed the model of Project MathTalk. The short voiceovers introduce and summarize the tasks at the beginning and the end of each video. Significantly, the voiceovers were recorded in such a way that the same recording was used for both unscripted and scripted videos (e.g., specific names were not used in the voiceovers, with the talent simply referred to as the students). Highlights were used to draw attention to the work of the talent (e.g., if the talent were talking about a point on a graph, that point was highlighted as if it were being pointed at). Similar to the voiceovers, highlighting was only used symmetrically (i.e., if highlighting was present in the unscripted videos, then that same highlighting was present in the scripted video). The goal of the post-production editing was to preserve the work—and voice—of the talent and, for the scripted videos, the teacher. Thus, postproduction editing did not add any new information, merely repeated and emphasized what had already been done. Upon completion, the videos were hosted on YouTube, where they were accessed for the vicarious learners viewing sessions.

Participants

Three students were recruited for the filming of the dialogic videos. These students were referred to as the “talent”. Two of the talent, Zoe and Gisele, were featured in the unscripted videos, and the third talent, Adrian, accompanied the teacher—whose role was filled by the researcher—in the scripted videos. Purposeful sampling was used to find both talent and later the VLs, specifically the strategy of criterion sampling (Patton, 1990). The criteria of importance for this study were: undergraduate students enrolled in college pre-calculus (recruited through a local Hispanic Serving Institution), moderate mathematical ability (assessed by their instructor), and good communication skills (based on a small screening interview). Selecting students from college pre-calculus ensured that the talent would be at a mathematical level slightly higher than the VLs (see data collection for details about VLs). Moderate mathematical ability was important to ensure productive struggle can be captured. Finally, good communication skills was important to ensure the dialogue of the dialogic videos is productive.

Data Collection

Following the filming and post-production of the video lessons, four pairs of VLs were recruited. Recruitment of VLs followed a similar process of criterion sampling used to find the talent contained within the video types (Patton, 1990). The one difference was VLs were recruited from college algebra courses at the same university as the talent. The goal of this criterion was to produce a model of behavior in the talent that the VLs could make use of. The use of pairs was purposeful as pairs have been shown to benefit the most from vicarious learning treatments, as compared to individuals viewing and learning from videos (Chi et al., 2008). Two pairs were assigned to each treatment, and the two pairs watched their respective videos over five 60-90 minute lesson-for-viewing sessions. For convenience, the pairs randomly selected to view the videos containing unscripted dialogue are referred to as Unscripted Pair 1 and 2. Similarly, the pairs selected to view the scripted dialogic videos are referred to as Scripted Pair 1 and 2. Table 3.3 presents a key of the talent within the videos and the pairs of vicarious learners (pseudonyms were used). Participants were financially compensated for their participation.

Table 3.3
Key of Talent's and Vicarious Learners' Names

Nature of Dialogue in Video	Name of the Talent within the Video	Treatment Name	Vicarious Leaners
Unscripted	Zoe and Gisele	Unscripted Pair 1	Sarah and Osiris
		Unscripted Pair 2	Becca and Nichole
Scripted	Adrian and the teacher	Scripted Pair 1	Camila and Alex
		Scripted Pair 2	Lucas and Jane

The research sessions were recorded with two cameras, one captured the students and the other their work. Following Dreyfus et al. (2001) the pairs were analyzed as one unit. As a unit, inferences will not be made about what an individual believes and, instead, claims will be made about the pair as a whole (e.g., the pair was evidencing categories of quasi-collaboration, etc.). The research sessions for both scripted and unscripted treatments will follow the same procedures.

Research Sessions

At the beginning of each research session, VLs were given a *paired task* (see Appendix A for all paired tasks). These paired tasks align with the tasks presented in the videos. When the task was particularly complex, then the tasks were the same. When the task was less complex, then the paired task was slightly different (e.g., the numbers were changed, but the problem statement was constant). After they were given the task, the VLs were prompted to read the questions together. During the viewing sessions, they were encouraged to either watch the video together and then work on the task, or work on the task and then watch. The videos were presented on a laptop placed between the VLs. When viewing the videos, the VLs had control of their viewing. Both VLs were encouraged to pause the video at any time to discuss anything they are noticing or any insights they have gleaned. If they opted to work on the task prior to viewing the video, then they were prompted to view the videos at the end to see how the work in the video compared to their own. If they were stuck, then they were encouraged to rewatch the videos, but this did not occur across any of the pairs.

While the VLs worked on their tasks and watched the videos the researcher sat across the room from them. When they are stuck or ready to explain what they had done the researcher sat across the table from the pair and asked them to walk through their solutions, their thinking, and their use of the videos (e.g., if they paused the videos at any moment they will be asked what prompted this, if they completed the task then viewed the videos they will be asked how their solutions compared, etc.). In an effort to establish a dialogic norm, when one VL finished responding the researcher repeated the questions—verbatim—and directed them explicitly to the other VL. If both took turns responding, then this was not necessary. The goal was to have the pairs respond together and for each VL to participate (see Appendix B for the Research Session Protocol).

Research Instruments

In service of answering Research Question 1, short (less than five minutes) experience-based clinical interviews were conducted at the end of every session with each pair. The purpose of these interviews was to help assess the VL’s orientation toward the talent. In these interviews, VLs were asked to reflect on their experience during the session. Specifically, the VLs were asked about moments that contain confusion—scripted misconceptions or authentic confusion for the respective video types. At the end of each research session, the VLs were asked, “How did you feel when you saw the students were confused?” and “Do you have any reflections from today?” The VLs responses and written work from these clinical interviews were collected in the same manner as the data from the video viewing.

Analysis

Preliminary analysis of the data from the vicarious learners in both treatments, for all the research questions, began with the creation of *descriptive accounts*. A descriptive account is a document used to describe what happens during differentiable episodes without inferences (Miles & Huberman, 2002). A descriptive account was created for all the clinical interviews and all of the research sessions. Then an initial data reduction pass was made to eliminate episodes that would not be used to answer any of the research questions, such as when the VLs quietly watched the videos. Further data reduction is described in the subsections below pertaining to each research question.

The data that will be used to populate my descriptive accounts is dependent upon each of my research questions. For the first research question, I drew from data collected during post-session interviews, any episode where the VLs make explicit mention of the videos or the talent within the videos, and moments of reflection implicitly targeted at the videos or talent (e.g., verbalizations in response to questions posed by the talent within the videos). For the second research question, two episodes were identified containing rich and characteristic problem-solving behaviors for Unscripted Pair 1 and Scripted Pair 1. After the descriptive accounts were created, segments were coded and transcribed. In the following sections, I will describe the process of coding, further justify data reduction, and explore further details about the analysis of each of my research questions.

Analysis for Research Question 1

RQ1: How does the orientation of undergraduate vicarious learners toward the talent differ when the dialogue of the talent is scripted versus unscripted?

The creation of the descriptive accounts for RQ1 began with data reduction. Initially, all 20 research sessions were separated into two categories, videos paused and videos playing. During the segments when the videos were paused, data reduction excluded portions of the research sessions in which the VLs worked on math tasks without mentioning the talent or when the VLs shared their reasoning with the researcher without referring to the talent. Next, during the segments when the videos were playing, data reduction excluded the moments when the VLs viewed the videos without talking, Together, this left moments when the VLs explicitly talked about the talent or their work, reflective moments about the videos (i.e., response to clinical interviewer questions and reflective statements at the conclusion of a video), and moments when the VLs were talking while the video played.

Using these descriptive accounts, experiences of *quasi-collaboration* will be coded for. Following Lobato and Walker (2018), quasi-collaboration is an orientation of VLs toward the talent. There are fire different categories of quasi-collaborative behavior I initially coded for: (a) identifying the talent's mathematical personalities (e.g., noting that one talent liked to find short-cuts while the other was more methodical), (b) predicting the talent's mathematical actions (e.g., thinking one talent would act in one way and the other talent in another), (c) coordinating activity with the talent (e.g., the VLs comparing their methodologies with that of the talent), (d) acting as if the talent could engage in their work (e.g., the VLs talking to the videos as if the talent could hear them), and (e) speaking about being in a community of learners with the talent (e.g., sharing confusion with the talent helped them feel less isolated).

Following Miles et al., (2018), an additional inductive pass was made over the data resulting in three emergent categories of orientation: (a) responding directly to the talent, (b) seeing the talent as a source of answers, and (c) evaluating the talent's ideas. Responding directly to the talent was evidenced by answering questions the talent posed, reacting verbally to the talent's ideas, and expanding on the talent's ideas without explicit interpretation (e.g., cutting the videos off and completing the talents thoughts). Engaging with the talent in this way emerged as a category of quasi-collaborative orientation. While answering the talent's questions and reacting to their ideas, the VLs built off of the talent's thinking and evidencing a form of collaboration. Furthermore, Mueller et al. (2012) identified expanding and building on others' arguments and ideas as a primary form of co-construction, which is a key form of collaboration within mathematics.

The final two emergent categories of orientations were characteristically different than the identified categories of quasi-collaboration. As such, seeing the talent as a source of answers and evaluating the talent's ideas were identified as categories of a distanced orientation. Seeing the talent as a source of answers emerged as a way in which the VLs turned to the talent as a source of authority. This category included the VLs' reproducing the talent's work with explanation or verbalized interpretation, the VLs' explicit appeals to the talent as a reason for a solution, and the VLs' use of the talent's work as a validation tool. The final category of orientation, evaluating the talent's ideas, emerged through the VLs' explicit assessment of whether they believed the talent were presenting a valid idea or not. Importantly, these moments lacked elaboration and consisted strictly of an evaluation. Contrasting with the categories of quasi-collaboration, these categories of a distanced orientation evidenced passive observation or spectating the dialogue, as opposed to collaboratively engaging with the talent. For further elaboration of each of the categories of orientation, see Chapter 4.

Following the coding for these categories of orientation, a second phase of analysis was conducted to identify the VLs' shows of emotions. Following Evans et al.'s (2006) textual analysis, each moment coded for a category of orientation was analyzed for emotions (e.g., frustration, appreciation, happiness, etc.). This phase began with the identification of four signs of emotions: (a) utterances (e.g., "I feel happy", laughter, or giggling); (b) tone of voice (e.g., adding inflection to individual words); (c) intonation (e.g., changes in cadence); and (d) body language (e.g., leaning in toward the video, gesturing toward the videos, or facial expressions).

Using these identified signs of emotions, an inference was made about the emotion being expressed by the VLs.

In conclusion, treatments were then compared to support or refute the hypothesis that undergraduate students in the unscripted versus scripted video viewing treatments will orient toward their respective treatments differently. This comparison included a comparison in frequencies of categories of orientations, frequencies of shows of emotions, and a comparison in the nature of the VLs' categories of orientations.

Analysis for Research Question 2

> RQ2: How do the problem-solving behaviors of vicarious learners engaged in learning to reason covariationally about the sine function differ when the dialogic videos that they are viewing and interacting with are scripted versus unscripted?

Analysis for RQ2 began with the identification of Scripted Pair 1 and Unscripted Pair 1's work on Square Task 1 and the Octagon Task. These pairs were selected for several reasons. Foremost, Scripted Pair 1 and Unscripted Pair 1 had a similar group composition (i.e., a male and a female) and a similar dynamic (i.e., both pairs engaged with one another in a similarly collaborative way). Additionally, both pairs were at a similar mathematical ability level and struggled on the same tasks and with the same concepts presented within the videos. In contrast, Scripted Pair 2 rarely evidenced any form of engagement with the videos nor amongst themselves. This pair seemed focused on working independently on the tasks and getting through the videos. As such, Scripted Pair 1 provided a richer verbal trace, allowing for more meaningful assessment of their problem-solving behaviors and conclusions to be drawn from their problem-solving behaviors. Unscripted Pair 2, on the other hand, had a similarly rich verbal trace, to Unscripted Pair 1 and Scripted Pair 1, but one of the VLs from the pair revealed during data collection that they had previously taken courses up through integral calculus. This positioned Unscripted Pair 2 at a more advanced mathematical level than the other pairs and the talent. Given the goal of this research question was to explore the VLs problem-solving behaviors on novel tasks aimed at establishing and pushing the VLs' covariational reasoning, Unscripted Pair 2's experience in concepts such as the derivative made them unsuitable for analysis.

After Scripted Pair 1 and Unscripted Pair 1 were identified for analysis, thematic analysis was used to identify patterns of meanings with their problem-solving behaviors. As systematized by Braun and Clarke (2006), thematic analysis was used to identify and describe themes within the data. This methodology was applied for three reasons: (a) thematic analysis allows for the use of a priori and emergent codes, (b) the process of theme refinement ensures analysis stays close to the data, and (c) thematic analysis has demonstrated flexibility in its use in mathematics education to answer a variety of research questions (e.g., Apkarian et al., 2023; Ellis et al., 2015; Garcia-Garcia & Delores-Flores, 2021; Rupnow & Sassman, 2022).

Thematic analysis is a six-phase process (Braun & Clarke, 2006). During Phase 1, analysis began by familiarizing myself with the data. As described above, this phase constituted the construction of descriptive accounts. During Phase 2, initial codes were generated. This phase included low inferential identification of the VLs' problem-solving behaviors and the VLs' uses of the videos (e.g., pausing and playing the videos). During Phase 3, general themes and subthemes were created. As a part of Phase 3, patterns and clusters within the initial codes were identified, such as a pattern for pausing the video to discuss moments of insights. During Phase 4, themes identified in Phase 3 were clarified and refined. During Phase 5, final themes are defined and named. Finally, during Phase 6, this research report was produced. Further details about each phase and its entailments are presented in Chapter 5.

Chapter 4: Orientation Toward the Talent

In this chapter, I answer Research Question 1, which is restated below:

> **Research Question 1:** How does the orientation of undergraduate vicarious learners toward the talent differ when the dialogue of the talent is scripted versus unscripted?

This study aims to identify how undergraduate vicarious learners (VLs) oriented toward the students (i.e., the talent) in dialogic instructional videos and to determine how those orientations differed when the talent were enacting a scripted versus unscripted dialogue. To date, the field has conceptualized two contrasting ways VLs orient toward the talent of dialogic videos: (a) epistemic detachment and (b) emotionally-involved quasi-collaboration.

McKendree et al. (1998), as a part of the Scottish Vicarious Learning Project (SVLP), hypothesized that vicarious learning positions the indirect participants in a dialogue (i.e., the VLs) to experience *epistemic detachment*. The SVLP argued that epistemic detachment is the distanced orientation of spectators, produced through experiences of lowered cognitive demand and a neutral emotional stance. With VLs literally distanced from the direct participants of a dialogue, VLs are not cognitively engaged in thinking about how they can contribute to a dialogue and are positioned to passively observe and think about what is being said. At the same time, emotional detachment can arise through avoidance of the social pressures germane to classroom dynamics, where students may have fears of their ideas sparking laughter or having an emotional stake in a friend’s idea. McKendree and colleagues conclude, when unencumbered by undue cognitive demands and emotions, VLs are freed up to attend to the validity of an argument and to learn.

Lobato and Walker (2019) defined quasi-collaboration, on the other hand, as the way that the Grade 9 VLs of their study were orienting toward the talent as if the VLs were in a collaborative group with them. Experiences of quasi-collaboration indicate that VLs are actively and cognitively engaged with the talent, as opposed to passively observing and absorbing information presented within a dialogue in a cognitively effortless way. Furthermore, in their study of high school VLs' orientations toward the talent of their unscripted videos, Lobato and Walker (2019) reported that quasi-collaboration was an emotionally-involved orientation, with their VLs evidencing an emotional connection to the talent. Given these contrasting orientations, the goal of this chapter is to advance the field's understanding of how undergraduate VLs orient toward the talent within dialogic videos and how the nature of the dialogue (scripted versus unscripted) influences VLs' orientations.

Review of the Methods

As discussed in Chapter 3, during each research session, pairs of VLs viewed dialogic videos containing either an unscripted dialogue between a pair of students or a scripted dialogue between a teacher (the author) and a student. In the following section, I present an overview of the methods (see Chapter 3 for further details). This section is separated into three subsections: (a) participants, (b) data, and (c) data analysis.

Participants

Eight students enrolled in college algebra courses were recruited for participation in this study. The students were separated into pairs and the pairs were randomly assigned to view either the scripted or unscripted dialogic videos. For convenience, the two pairs of VLs viewing the scripted videos are referred to as Scripted Pair 1 and Scripted Pair 2, and the pairs viewing the unscripted videos are referred to as Unscripted Pair 1 and Unscripted Pair 2 (Table 4.1).

Table 4.1
Key of Talent's and Vicarious Learners' Names

Nature of Dialogue in Video	Name of the Talent within the Video	Treatment Name	Vicarious Leaners
Unscripted	Zoe and Gisele	Unscripted Pair 1	Sarah and Osiris
		Unscripted Pair 2	Becca and Nichole
Scripted	Adrian and the teacher	Scripted Pair 1	Camila and Alex
		Scripted Pair 2	Lucas and Jane

Each pair participated in five research sessions lasting roughly an hour and fifteen minutes each.

Data

Analysis of this research question made use of data collected over all 20 research sessions, while the VLs engaged with the videos and while the VLs responded to reflection questions. As described in Chapter 3, all 20 video-recorded research sessions were watched and separated into two categories: video paused or video playing. Next, during the paused segments of data, all instances were recorded when the VLs were talking about the talent, when the VLs were making comparisons with the talent, and the VLs' responses to reflection questions (i.e., "How did you feel when you saw the students were confused?" and "Do you have any reflections from today?") at the end of each session. Finally, during segments in which the videos were playing, all instances were recorded when the VLs were talking. These moments captured instances where the VLs seemed to be reacting or talking to the videos. Because these moments did not contain explicit mention of the talent or the talent's work, an inference was made based on the temporal link that the reactions were in response to the talent. For each entry, the session number and timestamp were recorded, as well as an inference-free descriptive account of the surrounding moment.

Data Analysis

To analyze VLs' orientations toward the talent, a coding scheme was created using a mixed qualitative approach combining both a priori and inductive codes. Foremost, Lobato and Walker's (2019) coding scheme for categories of quasi-collaborative orientation was applied (see Table 4.2 for a description of these codes). During this coding process, three additional categories were induced. One of the induced categories aligned with the quasi-collaborative orientation, but the other two induced behaviors constituted another form of orientation, called a distanced orientation (see Table 4.2). Distanced orientations stand in contrast to quasi-collaborative orientations and are evidenced by a passive observation of the dialogue and a lack of collaborative engagement with the talent. While coding for these categories, each instance represents an engagement by the VLs that evidenced their orientation toward the talent. The overall trend of these engagements then speaks to how the VLs were orienting toward the talent. After coding for orientations, shows of emotions were identified during each engagement that evidenced an orientation.

Table 4.2
Orientations: Description of Categories

Orientation	Code	Description
Quasi-collaborative	Coordinating activity with the talent	Tracking and interpreting the work of the talent and then comparing it to their (the VLs') own work
	Predicting the talent's mathematical actions	Predictions relating to what the talent are going to do mathematically
	Acting as if the talent could engage with their work	Statements directed toward the talent that acted as if the talent could engage with those statements
	Understanding the talent's mathematical personalities	Statements about patterns in the talent's behaviors that characterized the talent's general mathematical propensities and practices
	Being in a community of learners	VLs' formation of a sense of community with the talent
	Responding to the talent's ideas, suggestions, and questions	Engagement with ideas and questions from the talent as if they were directed at them
Distanced	Seeing the talent as a source of answers	Appeal to the talent as a source of their solutions
	Evaluating the talent	Assessment of ideas presented by the talent that lacked elaboration or interpretation

Orientations

Lobato and Walker (2019) reported five categories of quasi-collaboration: (a) coordinating activity with the talent, (b) predicting the talent's mathematical actions, (c) acting as if the talent could engage with the VLs' work, (d) understanding the talent's mathematical personality, and (e) being in a community of learners. Of these five categories, all forms of engagements that evidenced an orientation emerged except for *predicting the talent's mathematical actions*.

Lobato and Walker (2019) defined *coordinating activity with the talent* as the "comparisons that the VLs made between their work and that of the talent" (p. 187). The coordination of activity allows for the VLs to see similarities and differences between their work and that of the talent. Comparisons can be made to solutions, methods, labels, ideas, and justifications. In this study, I expand *coordinating activity with the talent* to include another important way that the VLs were coordinating their activity, through interpretations. Going beyond just comparisons with the work of the talent, Barron (2000) argues that an important piece of coordinating activity is interpretation. As one engages in the collaborative process of coordinating activity, the ability to see similarities and differences relies on a process of making interpretations of another's work. Additionally, interpretation is a way to ensure "the ideas generated were considered common resources" (Barron, 2000, p. 417). In other words, interpretation is a way to check one's understanding and make sure a collaborative group is on the same page. To capture the role that interpretation played across the VLs' experiences with the talent, the a priori category of *coordination of activity with the talent* was expanded to include instances of explicit interpretation.

The central meaning of the VLs *acting as if the talent could engage with their work* was a kind of breaking of the fourth wall but in the reverse direction. Lobato and Walker (2019) characterize this category as containing instances in which their VLs appeared to speak directly to the talent. This was evidenced in the creation of a temporal ordering where the VLs attempted to help the talent as if the talent could act in response. This category draws on the difference between cooperation and collaboration. In the former, work is done in parallel across a group and then shared. In the latter, work is characterized by a back-and-forth, co-

constructive, effort. By design, instructional videos position the viewers to take information, but when VLs *acted as if the talent could engage with their work*, they were attempting to give information back and, thus, collaborate with the talent.

Understanding the talent's mathematical personalities entails the VLs noticing patterns and making statements about the talent that characterized the talent's general mathematical propensities and practices. I take mathematical personality to broadly be defined as one's strengths, weaknesses, and ways of doing mathematics (e.g., "Mike does a lot of mental arithmetic," "Mike struggles with remainders," etc.). Lotan (2003) argues that an important aspect of group and collaborative work is the opportunity to recognize peers' strengths and range of problem-solving strategies. As the VLs engaged with the videos, instances of their *understanding of the talent's mathematical personalities* illustrated that a sense of collaboration was being felt.

The final a priori category of quasi-collaboration, *being in a community of learners*, is presented by Lobato and Walker (2019) as capturing moments when VLs expressed a "sense of shared endeavor" and were "learning *with* (not just *from*) the talent, figuring things out together, and sharing a similar process of revising and editing one's work" (p. 192). Being in a community of learners means trust and warmth are felt, which is a requirement for willful collaboration (Kreijns et al., 2003).

In addition to these a priori categories constituting quasi-collaboration identified by Lobato and Walker (2019), three additional categories were induced from the data: (a) responding directly to the talent, (b) seeing the talent as a source of answers, and (c) evaluating of the talent's ideas. *Responding directly to the talent* included answering questions the talent posed, reacting to the talent's ideas, and expanding on the talent's ideas. By answering the talent's questions and reacting to the talent's ideas, the VLs acted as active participants in the dialogue instead of passive observers. These moments of reacting to the talent at times contained an evaluation, but they also showed an engagement with the idea. In contrast to the code of *acting as if the talent could engage with their work*, *responding directly to the talent* was a way in which the VLs acted as if the talent wanted the VLs to engage directly with their work (i.e., as if the talent were breaking the fourth wall). This was evident when the VLs were responding directly to the talent by expanding and building on the talent's thinking. Mueller et al. (2012) identified expanding on others' arguments as the primary way in which students engage in a co-constructive process, a key form of collaboration in mathematics. In this way, responding directly to the talent is a category of quasi-collaborative orientation.

Importantly, this category differentiates itself from the category of *coordinating activity with the talent* because utterances that indicated the VLs were responding directly to the talent did not contain explicit comparisons and they did not center on an interpretation. A critical reader may suggest that interpretation is required for one to expand upon or even react to an idea. Although an inference can be made that the VLs are simultaneously interpreting and *responding directly to the talent*, the exchanges coded into this category do not explicitly contain an interpretation and instead are characteristic of responding to, as opposed to reproducing, what the talent have said or done.

When the VLs were *seeing the talent as a source of answers,* there was a distanced orientation evidenced by utterances and engagement with the talent's ideas that appealed to them as a source of authority. This included the VLs' reproduction of work from the talent without explanation or verbalized interpretation, explicit appeals to the talent as a source of authority, and the validation of their work with the talent's ideas. When the VLs of this study were *seeing the talent as a source of answers*, they were passively, instead of collaboratively, engaging with the talent's ideas and operating from the position of a passive observer or spectator.

The final category of this study, the *evaluating the talent's ideas*, was another emergent category of distanced orientation. Primarily, the VLs evaluated the talent's ideas by explicitly assessing whether they believed the idea or not. Evaluations evidenced a lack of collaboration because they lacked elaboration. If an elaboration was present, such as a reason for the VL's (dis)belief in the talent's idea, the utterance was considered an interpretation and was coded under the category of *coordinating activity with the talent*. *Evaluating the talent's ideas* positioned the VLs as passive observers who were assessing the talent and their dialogue without responding to or engaging with the meaning within the talent's dialogue.

Shows of Emotions

In addition to coding for these different forms of orientation toward the talent, a second phase of analysis was conducted on the VLs' shows of emotions. In response to the SVLP's claim of emotional distance within epistemic detachment, Lobato and Walker's inquiry into quasi-collaborative orientations included an identification of the VLs' shows of emotions. Following Lobato and Walker (2019), this final analytical pass was made by viewing each identified moment of orientation toward the talent for an indication that the VLs were exhibiting a show of emotions. This method followed Evans et al.'s (2006) textual analysis to infer emotions (e.g., tenderness, frustration, or delight) by attending to: (a) utterances (e.g., "I feel happy", laughter, or giggling); (b) tone of voice (e.g., adding inflection to individual words); (c) intonation (e.g., changes in cadence); and (d) body language (e.g., leaning in toward the video, gesturing toward the videos, or facial expressions).

Results

The remainder of this chapter is organized into two sections. In the first section, I present a brief quantitative overview that illustrates differences in the way the VLs in the two treatments (viewing either unscripted or scripted videos) oriented toward the talent in the videos. In the second section, I begin by providing a detailed characterization and illustrative example(s) of each code. Then, similarities and differences between and within the treatments are explored for each code. Finally, shows of emotions are presented for each code.

Quantitative Overview

In this section, I provide evidence showing that the orientation of the VLs toward the talent differed between the VLs who viewed unscripted versus scripted dialogic videos. This evidence draws on a quantitative overview of the frequency of each category constituting the orientations of quasi-collaboration and distanced. The main purpose of this section is to illustrate the magnitude with which a difference was found in the coded data. I conclude this sclaim that the VLs in the two treatments were oriented differently toward their respective talent.

Following the coding process described above and in further detail in Chapter 3, all coded data were inputted into an Excel spreadsheet. These entrees were broken down by pairs and session numbers. The data included the timestamp, the allotted code, and the transcription of the surrounding utterance. A basic frequency count was then used to calculate the number of occurrences of each code in each research session for each pair.

Looking at the occurrences of each code showed, foremost, that the VLs viewing unscripted videos had more engagements that evidenced an orientation toward their talent than the VLs viewing the scripted videos. Table 4.3 shows the total amount of occurrences of each of the seven coded categories for all four pairs of VLs. In total, there were 402 instances coded for evidencing an orientation toward the talent. The two pairs of VLs in the unscripted dialogue treatment accounted for 290; the VLs in the scripted dialogue treatment accounted for 112. The difference in these frequencies suggests that the nature of the dialogue within the videos was influencing the VLs' orientations.

Table 4.3
Summary of Frequency of Each Category

Code	Unscripted Pair 1	Unscripted Pair 2	Scripted Pair 1	Scripted Pair 2	Total
Coordinating Activity with Talent	103	82	26	16	227
Acting as if the Talent can Engage	2	1	0	0	3
Understanding the Talent's Math Personalities	4	11	3	0	18
Being in a Community of Learners with Talent	6	3	0	0	9
Responding Directly to the Talent	37	25	5	1	68
Seeing the Talent as a Source of Answers	5	2	32	8	47
Evaluating the Talent	6	3	10	11	30
Total	163	127	76	36	402

Another difference that emerged across the treatments was in the nature of the VLs' orientations. As presented in Table 4.2, the seven codes were clustered into two orientations: quasi-collaborative and distanced. Figure 4.1 illustrates the distribution of these orientations across the viewers of unscripted versus scripted videos. The VLs who viewed the unscripted videos more frequently evidenced a quasi-collaborative orientation toward their talent than the VLs in the other treatment. A distanced orientation, on the other hand, was more frequent for the viewers of the scripted videos. This trend continued when comparing orientations as a percentage. In total, the VLs who viewed unscripted videos were coded for categories of a quasi-collaborative orientation for 94.5% of their total engagements that evidenced an orientation. The VLs who viewed the scripted videos, on the other hand, evidenced categories of a distanced orientation for 54.5% of their total engagements that evidenced an orientation.

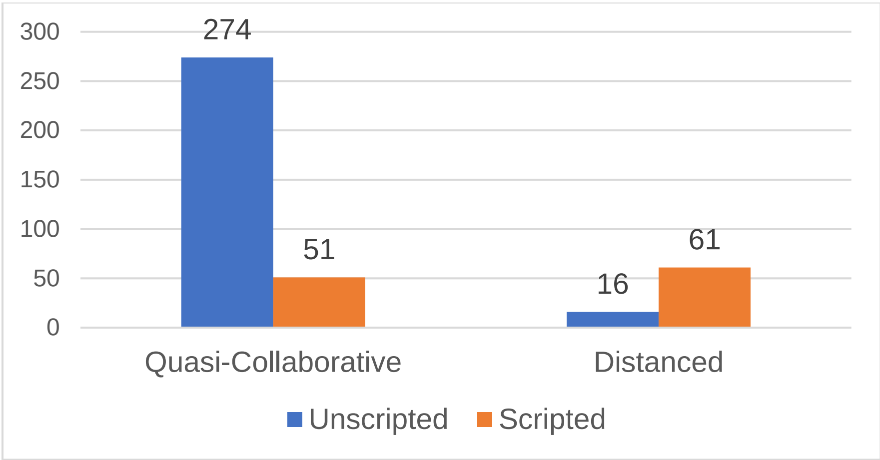


Figure 4.1
Frequency of Orientations for VLs Viewing Unscripted vs. Scripted Videos

Finally, a quantitative difference emerged in the VLs' shows of emotions while they oriented toward their respective talent. Across the 402 total engagements that evidenced an orientation, 117 were coded for a display of emotions. Breaking this distribution down by pair showed a similarity in displays of emotions within treatments and a large difference between them. Figure 4.2 illustrates the number of engagements that evidenced an orientation with a show of emotions versus the total engagements that evidenced an orientation for each pair. In total, Unscripted Pair 1 had 52 engagements that evidenced an orientation with a show of emotions, Unscripted Pair 2 had 59, Scripted Pair 1 had six, and Scripted Pair 2 had zero. As a percentage of their total engagements that evidenced an orientation, 31.9%, 46.5%, 7.9%, and 0.0% of Unscripted Pair 1, Unscripted Pair 2, Scripted Pair 1, and Scripted Pair 2's engagements that evidenced an orientation were with a show of emotions, respectively. The differences presented within this section suggest the nature of the dialogue within the instructional videos influenced not only the kinds of orientations the VLs had but also the degree of emotional involvement with those orientations.

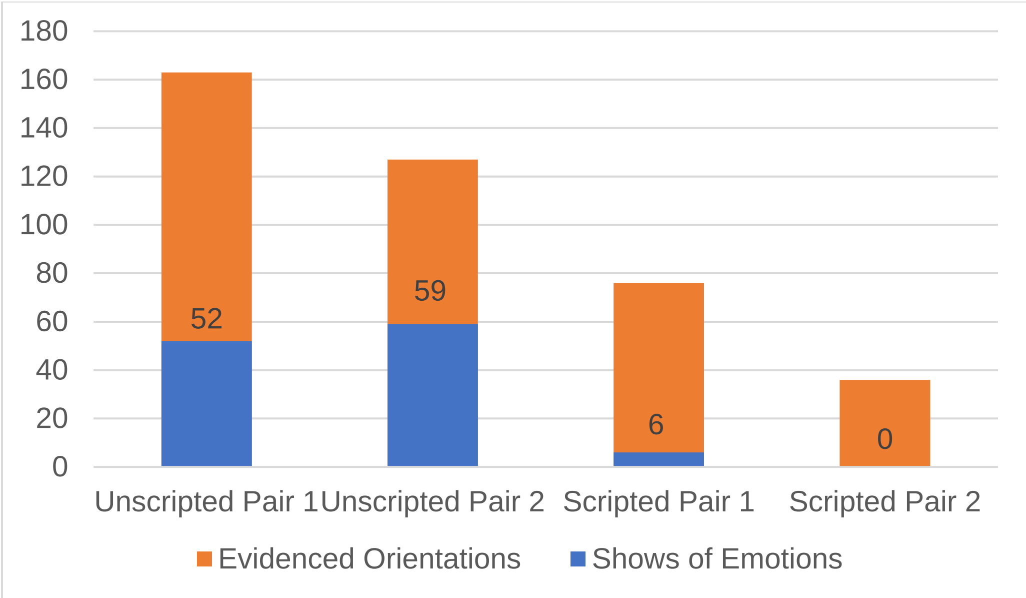


Figure 4.2
Shows of Emotions Compared to all Orientations for Each Pair

Categories of Vicarious Learners' Orientations

The initial phase of qualitative analysis consisted of coding the data using the quasi-collaboration categories presented by Lobato and Walker (2019). What emerged, was a difference in the way in which the undergraduate students of this study were engaging with and experiencing the ideas and solutions presented by the talent in the treatments' respective videos. In the following section, I describe and provide an example of each code, a priori and inductive. Finally, I will illustrate, when relevant, the difference between and among treatments' orientations by comparing the frequencies of each category, the emotions shown, and the nature of occurrences of each category.

Coordinating Activity with the Talent

The most prevalent way in which VLs evidenced an orientation toward their respective talent came in the category of *coordinating activity with the talent*. Lobato and Walker (2019) describe the category of coordinating activity as evident when VLs make comparisons of their methods, labels, ideas, and justifications to the talent's methods, labels, ideas, and justifications.

Consider an instance of comparison as coordination of activity from Unscripted Pair 1 during Research Session 5. In this episode, the VLs created an approximate graph for the Circle Task (Figure 4.3), but they struggled to describe their graph in terms of the height versus the distance traveled for a point as it journeyed around a circle.

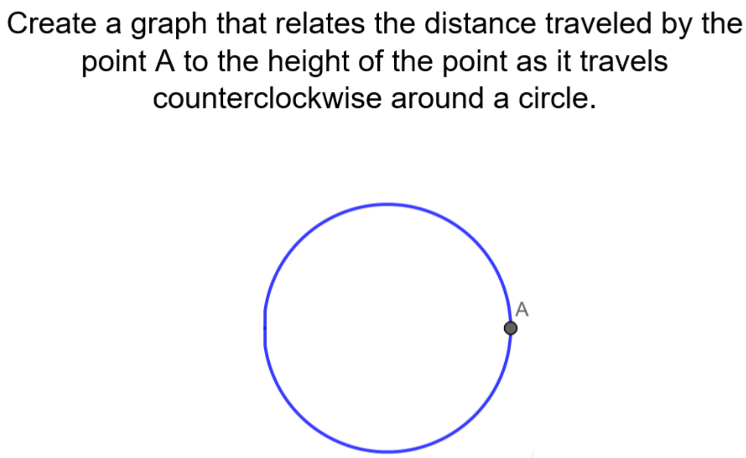


Figure 4.3
The Circle Task

Prior to watching the video, the VLs created a graph (Figure 4.4a). Included in this graph were points, corresponding to what mathematically represents inflection and extrema points, that the VLs appeared to coordinate with points on their circle (Figure 4.4b). Significantly, the VLs included these points but were not able to describe what the points meant or their significance. While watching the video of the talent describing their circle graph and plotting similar points, which the talent called "turning points," one of the VLs, Sarah, noticed a similarity and exclaimed, "That's what we did!" Later, at the conclusion of the video, Sarah expanded on what similarity she saw:

Sarah: **We did the same thing**! With the turning points.
Researcher: Divided it?
Osiris: Yeah. Into different quadrants. First [sweeps from the 12 o'clock to 3 o'clock region of their circle] second [sweeps from the 9 o'clock to 12 o'clock region of their circle] third [sweeps from the 6'oclock to 9 o'clock region of their circle] fourth [sweeps from the 3 o'clock to 6 o'clock region of their circle].
Researcher: How do you think Zoe and Gisele are doing right now with this task?

Sarah: I mean, **they did the same exact thing as we did**. And **they explained it the same exact way**. Especially when they split it up into quadrants. And there is not exactly like a point, but you can see where it reached its maximum height and when it's going down again. We kind of showed that right here [points to her graph]. This would be the first quadrant [points to the first quarter of their graph]. This would be the second [points to the second quarter of their graph]. This would be the third [points to the third quarter of their graph]. This would be the fourth [points to the final quarter of their graph]

(a) (b)

Figure 4.4
Unscripted Pair 1's Approximate Circle Task Graph and Circle, Broken into Quadrants

In this episode, Sarah presented two clear articulations of *coordination of activity with the talent* through what she saw as identical to what she and Osiris were doing. Specifically, Sarah coordinated the points she and her partner had been using on their graph with the points that the talent used. She also concluded that the points carried the same meaning. Prior to this moment, the VLs were unable to articulate what these points meant. Through a comparison with the talent's work, the VLs began to explicitly explain two meanings they had for these points: (a) the points on their graph have corresponding points on the circle indicating different quadrants and (b) these points are significant for the change in height.

Further examples of the VLs' comparisons of ideas with the ideas of the talent are presented in Table 4.4. To illustrate the range of the VLs' *coordination of activity with the talent* through comparison, these examples vary across the pairs as well as the research sessions. Additionally, the examples include instances of the VLs making comparisons to the talent's labels, ideas, what was said, solutions, and procedures.

Table 4.4
Examples of the VLs Comparing as Coordinating Activity with the Talent

Treatment	Vicarious Learner's Statement	Location of Utterance
Unscripted Pair 1	"I like how they did it. It's the same thing, I just like how they labeled it".	Session 2 1:07:07
	"Hmm they are doing half of x [instead of x for the side length]… Yeah it's the same".	Session 2 1:19:38
	"I said that a while back"	Session 4 1:00:31
Unscripted Pair 2	"Now they're getting into what I'm doing. Does it slow down? Does it speed up?"	Session 1 45:26
	"Beautiful. They got what we got."	Session 2 34:43
	"I think their scale might have been better than ours … so that is one thing I really liked about it [their graph]."	Session 2 1:13:24
	"Our's was basically similar"	Session 3 33:10
Scripted Pair 1	"Then again, it's also hypothetical. He's [the talent is] not saying that's the right way to do it. He's just saying 'if' the line started like that… so both answers [the VLs' answer and the talent's answer] could still be very much right."	Session 2 24:01
	"Hey look [points at screen]. They're going with your idea."	Session 2 52:54
Scripted Pair 2	"It's the same thing. It's just that the different starting point. Because the starting point is 5 it's not 0."	Session 2 6:45
	"[Another difference is] that he [the talent] wrote $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ [as he says this, he writes what the talent wrote under where he had written, ½ π and 3/2 π]."	Session 5 8:45

Another important part of the VLs' *coordination of activity with the talent* was their interpretations. Barron (2000) contends that interpretation plays a vital role in coordinating activity. As individuals engage in a collaborative process, their ability to coordinate activity through the identification of similarities and differences depends on their ability to interpret the work of others. Moreover, interpretation is an important way of identifying common knowledge and understanding. As such, *coordinating activity with the talent* was expanded to include the VLs' interpretations. Interpretation of the talent's ideas and inscriptions became an implicit task for the VLs, with interpretations emerging consistently across pairs and research sessions, typically in response to the conclusion of a video.

As an illustrative example, consider an episode during Unscripted Pair 2's first engagement with the videos during Research Session 1. After being informed that they could work and view the video in any order, they elected to start by working on Elevator Task 1 (Figure 4.5).

Create a graph that relates the distance traveled by the point C to the height of the point as it travels up the elevator shaft with a height of 10m.

Figure 4.5
Elevator Task 1

After the VLs re-read the task, they agreed that the given height corresponded to the top of the elevator shaft, but they quickly went silent and agreed to turn to the video. The VLs viewed the video for a little over a minute until one of the talent, Zoe, suggested the graph would be a "direct relationship." Hearing this, Becca asked Nichole to pause the video and offered an interpretation:

> Becca: I feel like they're already reaching this problem. She's [Zoe is] saying it'll be a direct relationship. And I guess that's the question I had. Are we operating it like a math question or like a real elevator? For a math question it would just be a constant straight line [gestures a linear $y = x$ line]. You're going up.

In this segment, Becca presented her understanding of the talent's idea about the graph. Becca interpreted the talent as suggesting the graph will be a "direct relationship." She then extended her interpretation and suggested what shape this would produce for a graph, namely a "constant straight line." Beyond the reproduction of an idea presented in the video, Becca's interpretation allowed her to make progress on the task, as demonstrated by her verbal and gestural indication of what she thought the solution "for a math question" would be.

Table 4.5 includes several additional examples across pairs and research sessions, illustrating the many ways in which interpretation as a way of *coordinating activity with the talent* occurred. In addition to being a central part of making comparisons with the talent, interpretation was a means by which pairs coordinated their understanding and made sure they had a common understanding with their partner. As such, the included examples show the VLs interpreting inscriptions, starting assumptions, and conclusions.

Table 4.5
Examples of the VLs Interpreting as a form of Coordinating Activity with the Talent

Treatment	Vicarious Learner's Statement	Location of Utterance
Unscripted Pair 1	"I think that **they [the talent] think that** you start at 10 meters and you stay at 10 meters walking. When in fact the green guy [gestures to stick figure in the video] would start at zero because he hasn't moved yet."	Session 1 49:23
	"**They're [the talent are] making** the *y* value *b*. so if this was *b* [relabels the *y* value as *b* on their graph], then this would stay constant at a *b*. but you can't say this specific point [points to the middle of the second segment on the hexagon] is *b*. you'd have to say this entire side [spans segment 2] would be *b*."	Session 3 22:57
Unscripted Pair 2	"**They [the talent] do make a good point**. Height and distance are the same in this problem … height and distance mean the same thing".	Session 1 34:32
	"That's when **they [the talent] were talking about how** like it's constantly moving. So, it's constantly passing from segment to segment to segment…"	Session 4 14:49
Scripted Pair 1	"I see **his [the talent's] point**. Like technically it is round [the shape]. So, you're gonna have to keep that [does sinusoidal gesture with hand] because there are no technical "lines" [does air quotes]."	Session 4 28:58
	"Specifically in the end when **he [the talent] was like 'all circles are the same'**, it doesn't matter which sides you give us because either way there is always 6.28 radian."	Session 5 37:01
Scripted Pair 2	"It was weird that **he [the talent] thought that** the 5 to -5 was… it says that the whole side was 10, but it started in the middle instead of the bottom. But then he started it [points to her graph's origin] at 0, and it didn't start at 0 it started at 5 [points to point *A* on the square]."	Session 2 6:45
	"**He [the talent] just kind of assumed that** since you are starting here [points to point *A*] you'd be starting at 0 and moving up but this is halfway through a 10-meter line so halfway through 10 is 5. Since we put the height here [points to the start of their graph] you would put that as 5 but he put it as 0 and there's no like 0 height besides like the end corners I guess."	Session 2 10:35

Frequency. *Coordination of activity with the talent* was the most common way in which the VLs' evidenced an orientation. As reported in Table 4.3, 227 occurrences across both treatments were coded as coordination, out of a total of 402 instances of evidence for the expression of an orientation toward the talent. Figure 4.6 displays the frequency of these occurrences across all four pairs and a percentage indicating the ratio of this category to each pair's total engagements that evidenced an orientation. Looking at the frequency counts for Unscripted Pair 1 and Unscripted Pair 2, the number of times these VLs *coordinated their activity with the talent* was similar (103 versus 82, respectively), and the difference can be accounted for by looking at the *coordination of activity with the talent* as a percentage of all engagements that evidenced an orientation (63.19% and 64.57% for Unscripted Pair 1 and Pair 2, respectively). This suggests that the VLs who viewed unscripted videos evidenced an orientation at a similar frequency when they were *coordinating their activity with the talent*. Scripted Pair 1 and Scripted Pair 2 had a more pronounced difference in frequency and percentages, with frequency in favor of Scripted Pair 1 (27 versus 16, respectively) and percentage in favor of Scripted Pair 2 (35.53% and 44.44%).

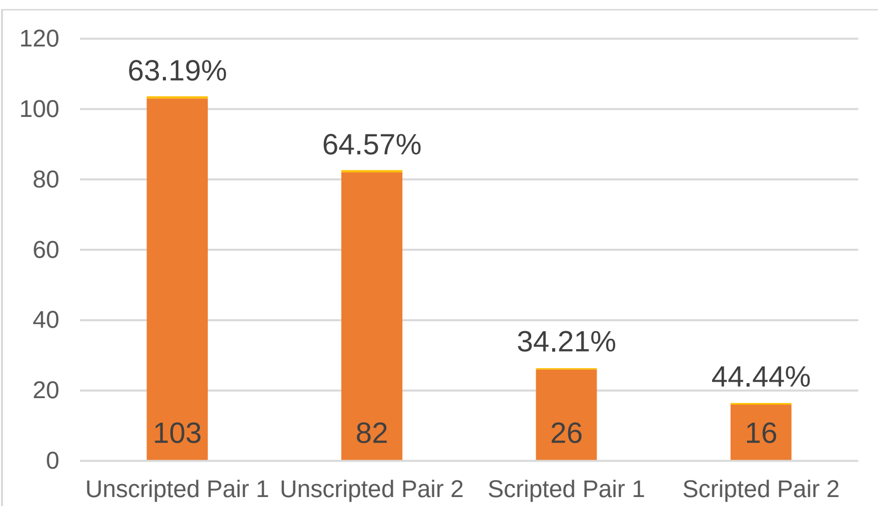


Figure 4.6
Frequency of Coordination of Work with the Talent's Work

Looking across treatments shows a large difference with frequency counts and a smaller, but stark, difference when comparing respective percentages. In total, the VLs viewing unscripted videos *coordinated activity with the talent* 185 times, or for 63.79% of their total engagements that evidenced an orientation. On the other hand, the VLs viewing the scripted videos coordinated their activity with the talent just 43 times, or for 38.39% of their total engagements which evidenced an orientation. These differences suggest that *coordinating activity with the talent* was engendered more when the talent's dialogue and struggle were unscripted.

Emotions. A stark difference emerged between the treatments in VLs' shows of emotion while they *coordinated their activity with the talent*. Of the 227 engagements that evidenced an orientation coded as *coordination of activity with the talent*, 59 were coded as containing a show of emotions—58 of which came from the pairs viewing unscripted videos. Both Unscripted Pair 1 and Unscripted Pair 2 had 29 coordinations of activity that included shows of emotions, whereas Scripted Pair 2 had none and Scripted Pair 1 had one. These shows of emotion while *coordinating their activity with the talent* included instances of surprise, annoyance, and pleasure.

For example, while Unscripted Pair 1 watched the talent find specific values on their circle graph using radians, the VLs showed emotions with their laughter and body language. During Research Session 5, the VLs attempted to find the exact value on their *x*-axis of the one-quarter point on their graph (i.e., at the point $(\frac{\pi}{2}, 0)$). Knowing the endpoint on their graph was at $x = 2\pi$ (or 6.28), the VLs attempted to calculate the "exact value" of $\frac{2\pi}{4}$ (see Figure 4.7). While Osiris carried out this computation the video played and showed the talent identifying this point as simply $\frac{\pi}{2}$. Seeing this, Sarah started to laugh and said, "Oh! Look what she is doing. You don't have to do all that math…" In response, Osiris rolled his eyes and sighed in an exasperated manner. This suggested while coordinating their inscriptions for their graph with the talent's inscriptions the VLs saw the talent present an easier method and reacted with both surprise (as indicated by Sarah's laughter) and annoyance (as indicated by Osiris's body language).

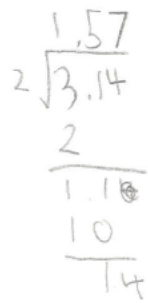


Figure 4.7
Osiris's Work to Find the "Exact Value" of $\frac{2\pi}{4}$

Scripted Pair 1's lone show of emotion while *coordinating activity with the talent* emerged during Research Session 2 when they identified a similarity between their work and that of the talent. After working on Square Task 3 (Figure 4.8), the VLs had a disagreement and created two separate graphs (Figure 4.9). Unable to negotiate with one another, they turned to the video. While the video played Camila noticed a similarity and exclaimed, "Hey look [points at screen]! They're going with your idea [looks at Alex, who grins widely]." Camila's enthusiasm and exclamation indicated an excitement for Alex, and Alex's facial expression showed the pleasure he felt to be vindicated. This example shows a clear comparison of work between the VLs and the talent, thus it was a coordination of activity. Interestingly, it is strongly related to an emergent category of distanced orientation that dominated Scripted Pair 1's orientations, *seeing the talent as a source of answers* (explored in further detail below in the subsection on Seeing the Talent as a Source for Answers). As opposed to a feeling of shared mathematical endeavor with the talent, the emotion of pleasure at being vindicated was in response to the talent determining which of the VLs were correct. While feelings of pleasure from being vindicated while viewing the videos were prevalent for Scripted Pair 1 (see subsection on Seeing the

Talent as a Source for Answers), the VLs' explicit comparison of work with the talent was not. Thus, it is unsurprising that this example was one of the few instances in which Scripted Pair 1 *coordinated activity with the talent* and one of the only instances with a show of emotions.

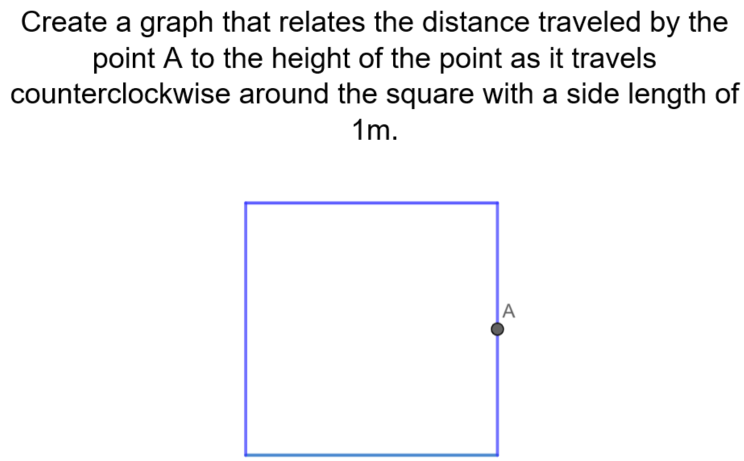


Figure 4.8
Square Task 3

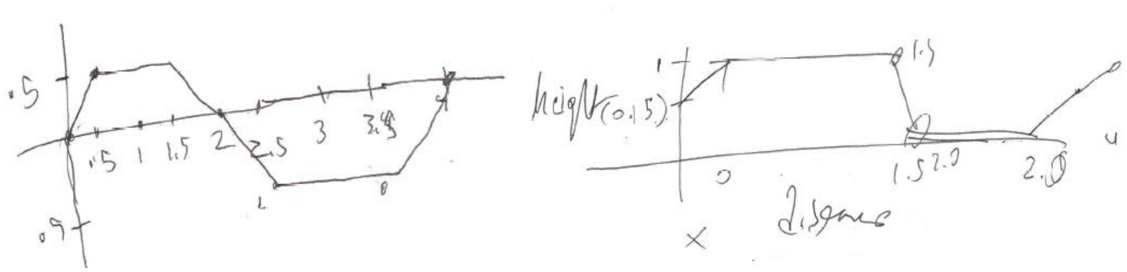


Figure 4.9
Alex's and Camila's Graphs for Square Task 3

Acting as if the Talent Could Engage with Their Work

Acting as if the talent could engage with their work was evidenced when the VLs attempted to interact directly with the talent. This category of quasi-collaborative orientation is characterized by actions and utterances directed at the talent that indicated the VLs were trying to give the talent information.

For example, during the fourth research session for Unscripted Pair 1, Osiris attempted to help the talent when he thought they were making a mistake. After working on the Octagon

Task (Figure 4.10), Unscripted Pair 1 struggled to think about how the change in the height over the first segment was different from the change in height over the second segment.

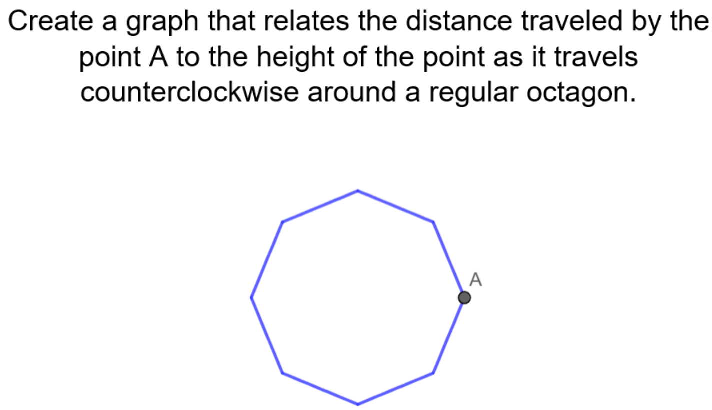


Figure 4.10
The Octagon Task

Indeed, this struggle led them to create three graphs with varying levels of detail (Figure 4.11). After creating these graphs, the VLs turned to the video. In the video, the talent exhibited a similar difficulty, creating two graphs (Figure 4.12) but struggling to articulate the difference in changes in height over subsequent segments of the shape. To clarify their confusion, the talent created an equation that represented the height gained over the first two segments, $b = b_{red} + b_{yellow}$. Prior to the talent explaining their equation, Osiris paused the video and attempted to fix an issue that he saw with the talent's equation (Figure 4.13), saying "I feel like it would be *b* red [writes b_{red}], parenthesis, and then the angle. If you multiply the angle, it will be more accurate." He then went on to explain that he was helping the talent:

Osiris: I don't know if they [the talent] mean *b* red *b* yellow as actually the length of the line or they mean the actual height from this point here [points to position vertically below the first corner of the octagon] to this point here [sweeps up to the first corner]. And so on, from right here to right here [sweeps pen horizontally over and then vertically, tracing the base and height of a triangle, up to the top of the octagon]. But, from how they're explaining it, I think they mean the length. **So I corrected them**. Well, not corrected them but like you know **give my idea**.

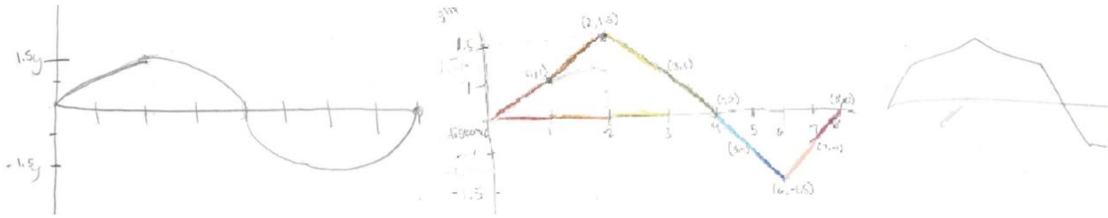


Figure 4.11
Three Graphs Created by Unscripted Pair 1

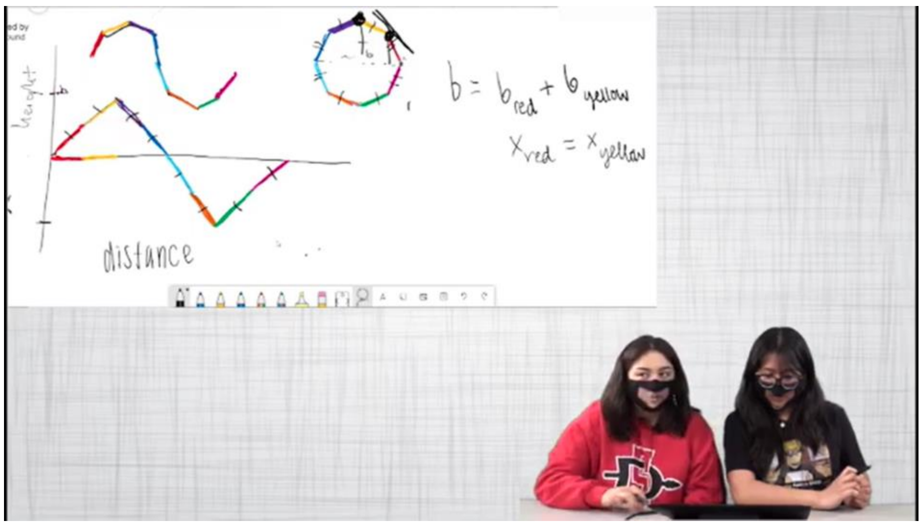


Figure 4.12
Screenshot of the Talent's Work and Equation for Height

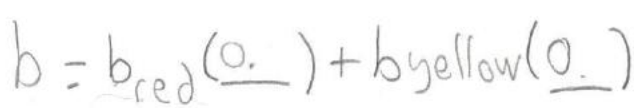


Figure 4.13
Osiris's Alternative Equation for Height

This episode illustrates a way in which the VLs *acted as if the talent could engage with their work*. Osiris felt like he could "correct" the talent and "give [his] idea" to them when he noticed they made a mistake in their equation. Osiris even went so far as to write the equation he thought the talent would need (Figure 4.13). His hesitation regarding the claim that he "corrected" them indicates that Osiris knew a back-and-forth exchange of ideas with the talent was not possible, but his utterance suggests he acted *as if* it was.

Frequency. Acting as if the talent could engage with their work occurred just three times across the pairs (twice for Unscripted Pair 1 and once for Unscripted Pair 2). Given the infrequency of this category of quasi-collaboration, it is difficult to make comparisons across the pairs or to assess the difference in the nature of these codes. While this category was rare, it is of note that *acting as if the talent could engage with their work* did emerge for the undergraduate students of this study.

In their study, Lobato and Walker (2019) reported that their Grade 9 VLs knew that the talent could not actually engage with their work but still acted as if they could. The few instances of this category in this study suggest that this form of orientation is more than a playful form of engagement for secondary students. Additionally, given it only emerged for the VLs who viewed the unscripted videos, *acting as if the talent can engage with their work* may be a form of quasi-collaboration uniquely fostered by unscripted dialogic videos.

Emotions. Of the three occurrences of this category, Unscripted Pair 2's lone occurrence was coded for a show of emotions. During their first research session, Unscripted Pair 2 came to a point in the videos that they disagreed with. In the video, the talent were expressing confusion about the validity of using the same units (meters) for both the *x*- and *y*-axis. While the video played, the VLs explicitly disagreed with and argued against the talent's idea. After the video concluded, Nichole reflected, "We were like arguing with *them* [laughs and gestures towards the laptop while turning to look at her partner]!" This display of emotions continued as both women laughed together and recounted where they thought the talent were making a mistake. In this exchange, Nicole acknowledged that they were *acting as if the talent could engage with their work* when they were disagreeing with the talent. Furthermore, Nichole's inflection at the end of her utterance suggested she was surprised that they had engaged in this way. Their shared laughter echoed how strange this was, but also indicated enjoyment of this seemingly absurd form of engagement.

Understanding the Talent's Mathematical Personalities

Lobato and Walker (2019) say, "This category refers to statements that the VLs made about the mathematical personalities of the talent" (p. 183). Broadly conceived, mathematical personality includes one's mathematical strengths, weaknesses, and practices. For example, in the Lobato and Walker study, the VLs noted the mathematical personality of one of the talent as being "methodical and repeating calculations or derivations" while the other talent "looked for shortcuts" (p. 184). Thus, VLs' engagements that evidenced an orientation through *understanding the talent's mathematical personalities* included statements the VLs made about the talent's propensities and ways of doing mathematics.

For example, during Research Session 5, Unscripted Pair 2 started to identify patterns they were seeing in the talent's struggles. While watching a video in which the talent attempted to leverage their definition of a radian to generalize their graph for the Circle Task (Figure 4.3), a question arose about the units. In the video, the talent were quickly able to make use of radians for their *x*-axis (i.e., for distance around a circle), but they were hesitant to use radians for their *y*-axis (i.e., for the height of the point as a distance measured in radians). In the video, Gisele says, "Yeah… we'll put it [the units for the *y*-axis] in radians… I'm just a bit confused about circles!" Hearing this, Nichole relented, "Sames [laughing]..." Later, after a lengthy discussion in which the VLs decided radians can be used as the units for both *x*- and *y*-axis, Nichole reflected on the talent's struggle:

> Nichole: I see a common pattern with them [the talent], where they have a hard time finding their *x*- and *y*-axis. Like in the elevator one [elevator tasks, e.g., Figure 4.5], they said the distance was like… they said it was very different from the height.

In this exchange, Nichole identified a pattern in the talent's mathematical personality in the difficulty the talent were experiencing with units of their graph, and the talent's struggle resonated with her. Specifically, she connected this struggle to a video she viewed in Research Session 1, during which the talent struggled to identify the units for the *x*- and *y*-axis in Elevator

Task 1 (Figure 4.5). In the Standards for the Preparation of Secondary Mathematics Teachers (National Council of Teachers of Mathematics, 2020) struggle is identified as a mathematical practice central to rigorous mathematics. Indeed, identifying appropriate units was a weakness of the talent, but the presence of the talent's struggle with units produced significant development in their quantitative meaning of their graphs. Thus, Nichole's identification of the talent's weakness with units indicated an understanding of the talent's mathematical personalities. Furthermore, when she commented, "Sames," in response to the talent's confusion Nichole implied she related to their struggle. By identifying with their struggle, Nichole was engaged in more than just a comparison of work. She was picking out a part of the talent's weakness and determining it was something she also saw as a weakness in herself.

Identifying patterns in the talent's struggle was a common theme for Unscripted Pair 2. In addition to identifying the talent's difficulty with the units of their axes, the talent's struggles throughout the videos were attended to by Unscripted Pair 2 (see Table 4.6). The commonality across these examples is Unscripted Pair 2's identification of weaknesses in the talent's work and the belief that the talent's struggle is due to a lack of planning. Once again, with mathematical personality broadly defined to capture characteristics of the talent's work while engaged in mathematical problem solving, weaknesses become an important part of their mathematical personality—given the fact that Unscripted Pair 2 identified the weaknesses of the talent across all of their research sessions, it appears that Unscripted Pair 2 came to see struggling as a part of the talent's mathematical personality.

Table 4.6
Understanding the Talent's Mathematical Personality of Strugglers

Speaker	Vicarious Learner's Statement	Time of Utterance
Nichole	"I think **they [the talent] just needed to plan it out first**. Instead of just directly drawing the graph. **Because that makes a lot of errors**, if you just go in with graphing it instead of planning each point."	Session 2 39:25
Becca	"[While laughing] Do they know that the height is different from the distance?"	Session 3 27:40
Becca	"I've been caught up before [laughs] … I think having that experience **gave me a little more foresight that they [the talent] might not have**."	Session 4 32:31
Becca	"I don't think they were making mistakes per se. I think **they are having a very hard time making sense** of what they are looking at. They know that they got this all down on paper but they're not really so sure what they're looking at anymore."	Session 5 18:30

Frequency. Explicitly *understanding the talent's mathematical personalities* was an infrequent form of quasi-collaborative orientation for the undergraduate students of this study. As previously indicated in Table 4.3, this category accounted for a total of 18 of the 402 engagements that evidenced an orientation toward the talent across the treatments. Figure 4.14 illustrates the frequencies across all four pairs and the percentage that this form of quasi-collaborative orientation made up of the VLs' total engagements that evidenced an orientation toward the talent. Unscripted Pair 2 stood out from the rest of the pairs. Over their five research sessions, Unscripted Pair 2 had 11 engagements that evidenced an orientation identified as *understanding the talent's mathematical personalities*. This accounted for the largest percentage of their total engagement, at 8.66%. Unscripted Pair 1 and Scripted Pair 1 had a percentage of their total engagements that evidenced an orientation in this category of quasi-collaborative orientations of 2.48% and 3.90%, respectively. Scripted Pair 2 had none.

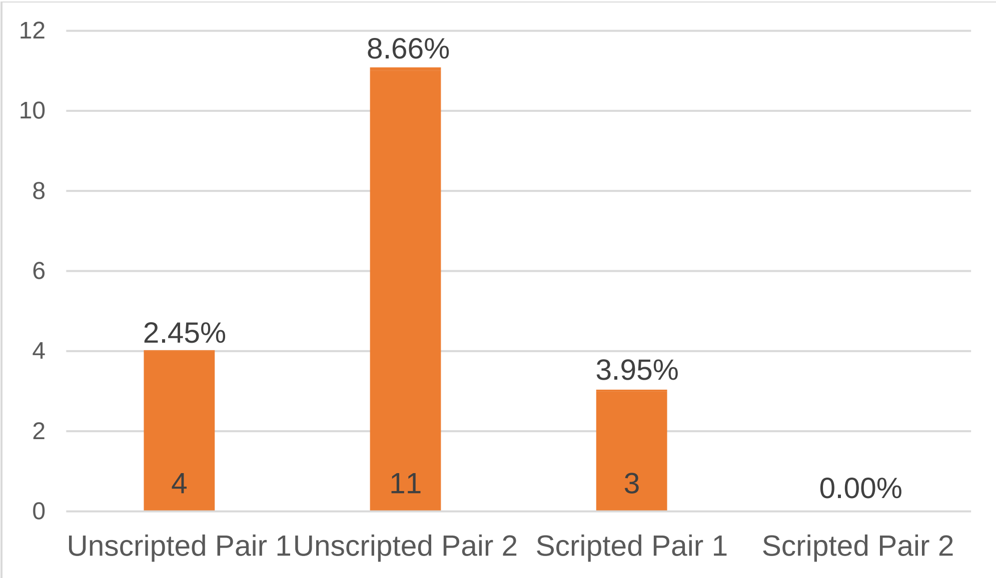


Figure 4.14
Frequency that the VLs Identified the Talents' Mathematical Personalities

Emotions. One key difference that emerged between the treatments, was in the emotions shown as the VLs *identified the talent's mathematical personalities*. Of the 18 instances, nine occurred with a show of emotions. Unscripted Pair 1 had one show of emotion, Unscripted Pair 2 had eight, and Scripted Pair 1 had none. The emotions shown by the unscripted treatment included feeling uncomfortable and appreciation for seeing the talent's struggle. This was evidenced by laughter, body language, and tone of voice (e.g., Nichole's inflection when she relented "sames," in response to the talent's struggle) while they simultaneously identified with the mathematical personality of the talent (see Table 4.6 for examples of instances of laughter).

Being in a Community of Learners

Being in a community of learners captures the ways in which the VLs expressed a sense of belonging in a group with the talent and having a sense of a shared undertaking. At the end of Research Session 4, during the daily exit reflection with Unscripted Pair 1, an illustrative exchange occurred. This session had been particularly difficult for the VLs. They had spent a large portion of their time working on the tasks, arguing with each other, and disagreeing with ideas expressed in the videos. In fact, the talent also struggled with the content of this session. In short, this session was difficult and full of struggle for the VLs. While the VLs reflected on the videos from the day, the VLs articulated that they appreciated seeing struggle from fellow students in the videos:

Researcher:	Do you guys have any reflections or comments before we wrap up for the day?
Osiris:	You should do a Khan Academy website. I feel like this is a cool idea. Making actual students do it. Because Khan Academy is like a guy doing it on a black screen. When there’s actual students on the screen doing it and getting it wrong or right, I feel like it’s giving me a layer of deeper understanding.
Researcher:	Do you like seeing that [student confusion]?
Osiris:	Yeah. I feel like it’s way better than Khan Academy. I feel like in Khan Academy it’s easy to get lost and get no understanding of it. It’s just a guy saying it. I mean he knows it he knows how to do it; you know what I mean? So, for someone else who doesn’t know how to do it too, if you’re learning it, **it makes you feel like you’re more involved**.

In this exchange, Osiris seemed to express a sense of community he was feeling with the talent. Initially, he articulated a difference between the videos of this study and a traditional form of instructional video. The central difference that Osiris attended to was the presence of an “actual student” working on tasks, who is “getting it wrong or right.” Osiris then connected the presence of the talent to a better understanding he felt he now had and feeling “more involved” with the talent as a fellow learner. In sum, this exchange illustrated Osiris’s experience of learning *with* the talent (as opposed to learning *from* Khan Academy), sharing a process of struggle with the talent (e.g., the amount of struggle Unscripted Pair 1 evidenced in this session and Osiris’s recognition of the talent’s struggle), and an overall feeling of being in a community of learners with the talent.

Frequency. *Being in a community of learners with the talent* was a form of quasi-collaborative orientation that only emerged for the pairs viewing the unscripted videos. In total, there were nine instances in which the VLs evidenced *being in a community of learners* with the talent. Unscripted Pair 1 evidenced this category six times and Unscripted Pair 2 three times. As explored in further detail below, this category contrasts with the category of *seeing the talent as a source of answers* (see the subsection of Seeing the Talent as a Source of Answers for more details). The differences in the frequency of these two categories suggests that the VLs viewing scripted videos oriented as learning *from* the talent, while the VLs viewing unscripted videos oriented as learning *with* the talent.

Emotions. Of the nine instances of being in a community of learners, five were with a show of emotions (three from Unscripted Pair 1 and two from Unscripted Pair 2). Shows of emotions included appreciation, happiness, and a sense of relief evidenced by explicit verbal appreciation of the talent, body language, and laughter. After watching the talent make a breakthrough, Unscripted Pair 1 displayed a show of emotions while expressing an appreciation for learning with the talent. After the video ended, Sarah reflected, “They [points to screen] were talking about how this [the *y*-axis] would also be meters, and I am like *oh my god* [laughs, leans back in her chair, and throws hands in the air]…” Sarah’s inflection of her utterance “oh my god” indicated the talent’s breakthrough helped her, and her laughter and gestures suggested that she was happy and relieved to be through her struggle.

Similarly, Unscripted Pair 2 came to explicitly appreciate struggles that they themselves did not have. Reflecting on seeing the talent make mistakes, Becca said, “I’m pretty impatient and all that… [but] I did like to see them get to create the graph after [their mistake] and see them ultimately get it. And get to see them check their work. That was helpful. It made me think about it a little differently and have some new ideas.” Initially, Becca expressed the frustration she felt while seeing the talent make mistakes. Her tone here was low and suggested she saw this as a fault. Then, her tone picked up as she expressed an appreciation for the talent’s breakthrough and indicated the talent’s progression led to her own growth in understanding.

Responding Directly to the Talent

The final category of a quasi-collaborative orientation, *responding directly to the talent*, emerged from this study as a way in which the VLs engaged with the talent’s ideas during and after the videos as if the talent's ideas were directed at the VLs. This form of quasi-collaborative orientation was a way in which the VLs actively participated in the dialogue by reacting and engaging with the talent's ideas and by answering questions the talent posed.

For example, in Research Session 1 the VLs in Scripted Pair 1 elected to watch the video prior to working on Elevator Task 1 (Figure 4.5). In the video, the talent, Adrian, presents a scripted moment of confusion about the units for the *x*- and *y*-axis. During this exchange, Adrian expresses confusion and says, "I'm just not sure what that distance would be, as far as units." Hearing this, the VLs paused the video and responded:

> Camila: What do you think the units would be? Because he [points to laptop] was questioning.
> Alex: Meters. Because we are using meters once, so why would we change it up?

In this exchange, the VLs rephrased the talent's statement into a question and then responded directly to that question.

In a second example, during Research Session 3 with Unscripted Pair 1, Osiris responded directly to the talent and expanded on their thinking when he completed their sentence. During the video, the talent worked to describe their graph (Figure 4.15) for the Hexagon Task (Figure 4.16), and they were coming to an explanation that coordinated a change in height with a change in distance for the point moving counterclockwise around the hexagon. In the video, while explaining her thinking for height, Zoe said, "…for the dark blue, light blue, green part of the graph it would be…" Hearing this segment in the video and Zoe's hesitation, Osiris (a VL) responded, "It would be negative." Continuing with the video, Zoe (one of the talent) says, "Dark blue would reach negative b…" Through Osiris's (the VL's) completion of Zoe's (the talent's) sentence, he was indicating that he was an active participant. Further, he demonstrated an expansion of Zoe's idea by completing her thought.

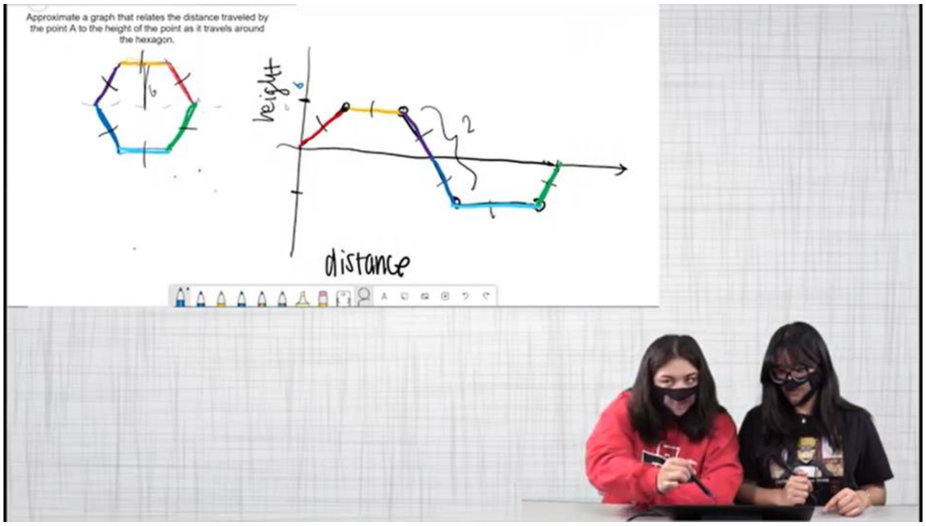


Figure 4.15
Zoe and Gisele Explaining Their Graph for the Hexagon Task

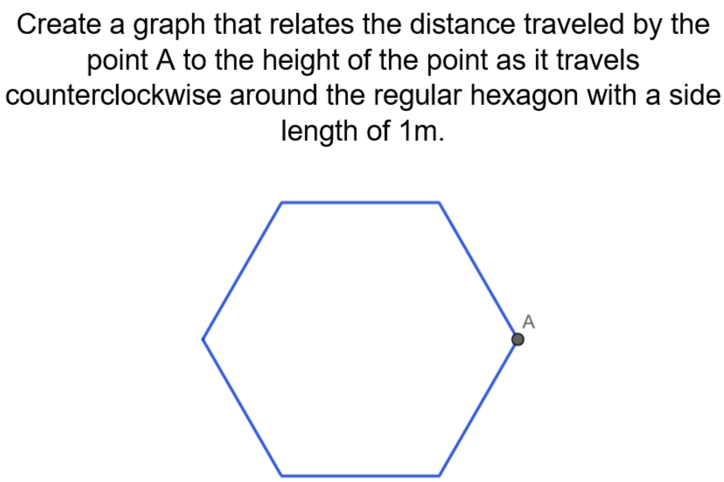


Figure 4.16
The Hexagon Task

Frequency. Across all pairs and research sessions, there were 68 instances coded as *responding directly to the talent*. As illustrated in Figure 4.17, this category of quasi-collaborative orientation was much more frequent for the VLs viewing unscripted videos. Unscripted Pair 1 had more instances of *responding directly to the talent* than Unscripted Pair 2 (37 and 25, respectively). Unscripted pair 1 had a similar ratio of this category of quasi-collaborative orientation to the total engagements that evidenced an orientation to Unscripted Pair 2 (22.98% and 19.69%, respectively). On the other hand, *responding directly to the talent* was rare for both pairs viewing scripted videos. Scripted Pair 1 evidenced this category of quasi-collaboration five times (for 6.58% of their total engagements that evidenced an orientation), and Scripted Pair 1 one time (for 2.78% of their total engagements). The similarity in the frequencies and percentages of this form of quasi-collaborative orientation were preserved within treatments. Across treatments, there was a stark difference. Viewers of unscripted dialogic videos had far more of *responding directly to the talent* than viewers of the scripted videos.

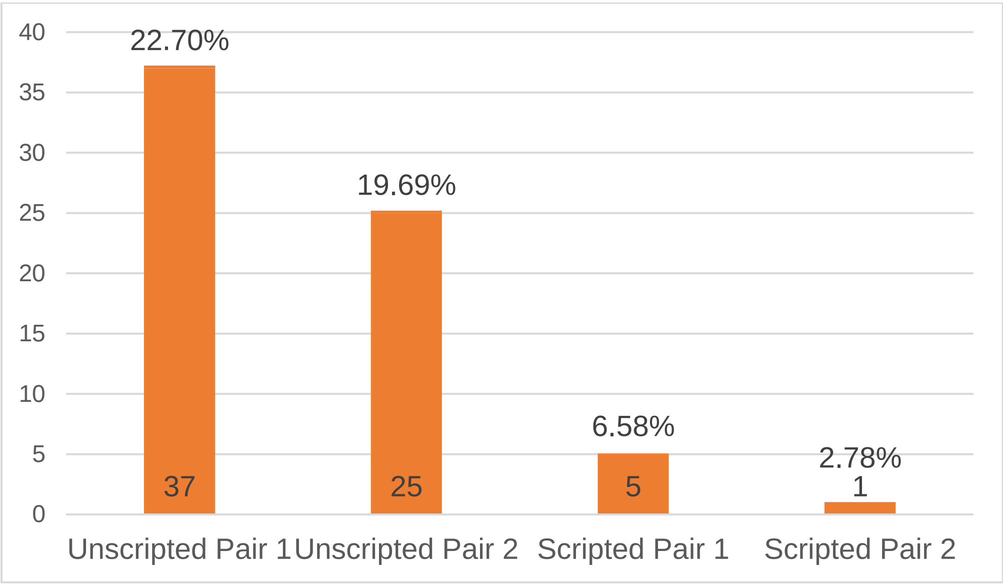


Figure 4.17
Frequency of Responding Directly to the Talent

Emotions. Of the instances of *responding directly to the talent*, 51.47% (35 of the total 68) were with a show of emotions. As a percentage, this category evidenced the most shows of emotions. Shows of emotions while the VLs *responded directly to the talent* were most prevalent for the viewers of the unscripted videos. Unscripted Pair 1 and Unscripted Pair 2 had 16 and 18 shows of emotions, respectively, while Scripted Pair 1 had one show of emotions. As a percentage of total engagements that evidenced an orientation, Unscripted Pair 2 was particularly emotionally involved in this category of quasi-collaboration (with 72% versus Unscripted Pair 1's 43.24%).

There were many ways in which emotions emerged while VLs evidenced this category of quasi-collaborative orientation toward the talent. For example, during Research Session 5, the VLs in Unscripted Pair 1 watched a video with the talent completing the Circle Task (Figure 4.3). During the video, Gisele (one of the talent) wondered aloud, “What would the units be…?” Hearing this, Sarah (a VL) responded to this question and emphatically said, “Radians!” Here, Sarah displayed eagerness and excitement to be engaged with the talent’s thinking. In another example, during Unscripted Pair 2’s Research Session 3, the VLs viewed a video in which the talent explained their graph for the Octogen Task. During this video, Gisele expressed confusion about the changes in height as changing over different segments of their graph. Eventually, Zoe convinced Gisele, and Gisele enthusiastically stated, “I get it! I get it! I get it!” Hearing this both VLs looked at one another, laughed, and Becca stated, “I was starting to get frustrated!” The VLs evidenced an excitement for Gisele’s progress and an explicit frustration with her struggle. Their shared glance suggested a mutual understanding of what they just watched, their laughter indicated Gisele’s eagerness was endearing to the VLs, and Becca’s statement was an explicit response to the talent’s struggle as something that tested her patience.

Differences in the Nature of Responses to the Talent. Making comparisons across treatments is difficult, given the infrequency with which the VLs viewing scripted videos evidenced *responding directly to the talent*. Across the pairs of VLs who viewed the unscripted videos, a difference emerged in the nature of their responses. For Unscripted Pair 1, they answered questions and finished the ideas of the talent. Unscripted Pair 2, on the other hand, frequently corrected the talent.

Consider two contrasting examples that occurred while Unscripted Pair 1 and Unscripted Pair 2 watched the same video, during their respective Research Session 1. During this video, the talent worked on Elevator Task 1 (Figure 4.5) but struggled to determine how they should handle the units on their axes. While watching this video, Unscripted Pair 2 responded directly to the talent by correcting them. In the video, Zoe wondered if the units on their x-axis, labeled distance, had to do with speed saying, “using the equation distance equals rate times time… but we don’t have a time. Technically you can measure distance in meters per second…” Hearing this, Becca (a VL) responded saying, “You wouldn’t be like *I live forty miles per hour away*.” Beyond just evaluating the talent and claiming they were wrong, Becca responded with why she thought it was wrong and corrected Zoe (the talent).

This can be compared to the way in which Unscripted Pair 1 responded directly to the talent by extending their thinking. Moments later in the video, the talent started to reason that both axes can be measured in meters. Zoe said, “We drew our graph to be proportional. So, it would make sense if the height is in meters, then the distance would be proportional in meters.” Hearing this, Unscripted Pair 1 responded directly. Sarah paused the video and exclaimed, “Oh my gosh! Hold on. That makes so much sense. They’re traveling in meters. Say they did 10 meters. The distance is also 10 meters.”

Both instances illustrate a willingness to engage directly with the talent—an important part of collaboration. However, the way they engaged suggests a difference between the pairs of VLs. Unscripted Pair 2 engaged by arguing against the talent, and Unscripted Pair 1 engaged by build on an idea they agreed with. Both pairs were met with the same moment of struggle in the video and extended the reasoning presented by the talent. In response to this struggle, Unscripted Pair 2 responded by extending the talent’s thinking to another example—traveling home—which allowed the VLs to argue against it. On the other hand, Unscripted Pair 1 responded to the talent’s breakthrough and excitedly extended the talent’s idea to its logical conclusion (i.e., not only are distance and height proportional, but they are also identically 10 meters). Despite the pairs responding to different moments within the same video, these examples illustrate a trend in the pairs. Specifically, Unscripted Pair 2 attended, and responded, directly to the talent’s ideas that they disagreed with and corrected the talent. Unscripted Pair 1, on the other hand, did not focus on correcting the talent and instead responded directly to the ideas that seemed to resonate with them by answering the talent's questions and adding to the talent's ideas.

Seeing the Talent as a Source of Answers

Seeing the talent as a source of answers was the first emergent form of engagement that evidenced an orientation categorized as distanced. This category of distanced orientation emerged as a way in which the talent were used as a resource for the completion of the tasks and a source of solutions. *Seeing the talent as a source of answers* constituted passive observation and lacked collaboration with the talent, as opposed to actively watching and engaging with the ideas and solutions presented by the talent.

A clear example in which the VLs from Unscripted Pair 2 were *seeing the talent as a source of answers* occurred during Research Session 5. After watching the talent begin to make use of the units of radians to find specific distances on their graph for the Circle Task (Figure 4.3), the VLs began integrating radians into their own graph. As a part of this work, the VLs coordinated a specific point, $(\frac{\pi}{2}r, r)$, which they had labeled on their graph (Figure 4.18), with a point on their circle.

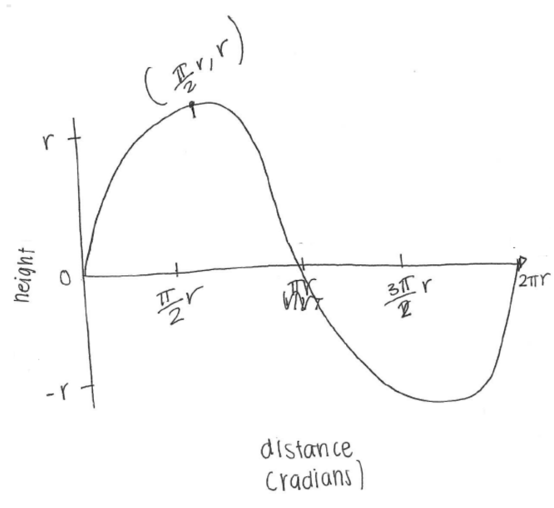


Figure 4.18
Unscripted Pair 2's Graph for the Circle Task

Nichole identified the twelve o'clock position on their circle as corresponding with the distance of $\frac{\pi}{2}$. When probed by the researcher how she knew, Nichole appealed to the authority of the talent and stated, "Because I remembered from **their [the talent's] video** [points to the laptop while laughing aloud]." Explicitly, Nichole cited the talent as the source of her answer. Her emotive response (i.e., open mouth laughter) suggested that she knew this was not a mathematically rich justification, but her justification was still based on the talent's work.

As a second example, consider a moment when Scripted Pair 1 appeared to be *seeing the talent as a source for answers*. During Scripted Pair 1's Research Session 2, the VLs had established a pattern for working on the given task, debating their solutions, failing to convince one another, "agreeing to disagree," and finally viewing the video. Explicitly, the VLs turned to the video containing the paired task to settle their disagreements. As Alex stated, "[let's work on the task first] because we argue, then we see who's right [points to computer]." During Research Session 2, this sentiment was echoed on several other occasions for this pair (see Table 4.7), with several explicit appeals to the talent's ideas and work. This pattern of action, culminating in an appeal to the "gentleman in the video" as the arbiter of their debates, persisted throughout Scripted Pair 2's research sessions.

Table 4.7
Scripted Pair 1's Repeated Appeal to the Talent as a Source for Answers

Speaker	Vicarious Learner's Statement	Time of Utterance
Camila	"The height. because he [the talent] said it was…"	14:11
Camila	"The smart man [the talent] in the computer told me so I had to follow."	34:50
Camila	"When I saw the video, I saw that the graph was at 3.5. what? Where'd it go? Can I… [rewinds the video]. It starts at 3.5 right there [points to screen]."	38:15
Alex	"The gentleman [the talent] in the video said this [points to his line spanning the center of the square] is the line we should use."	47:21
Alex	"The gentleman [the talent] in the video said mine was wrong."	47:45

As a final example, consider an exchange for Scripted Pair 2 as they worked on Square Task 2 (Figure 4.19) during Research Session 2. Previously, the VLs had created a graph for Square Task 1 (Figure 4.20), where they decided to call the starting height on the point's journey 5 meters. They then watched two videos. In the first video, the talent worked on Square Task 1 and produced a slightly different graph than the VLs, based on a different starting assumption for the height of point *A*. In this first video, the talent called the starting height 0 meters. In the second video, the talent worked on Square Task 2 under the same assumption that the VLs had been working under.

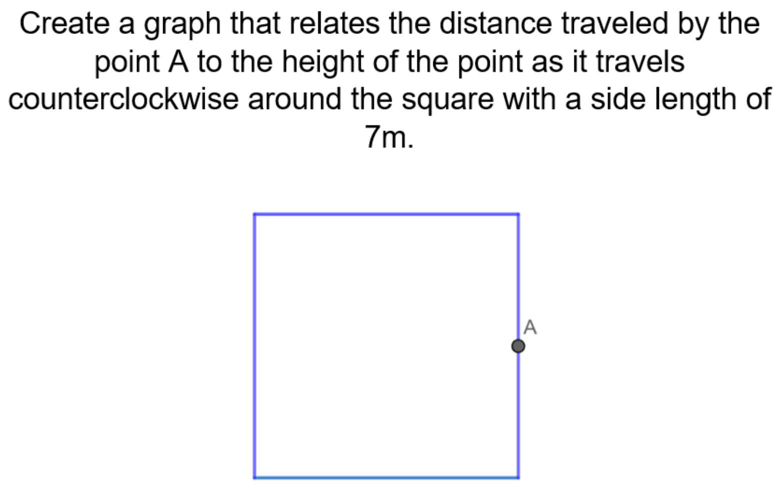


Figure 4.19
Square Task 2

In this second video, the talent labeled the starting point as having a height of half the side length (e.g., if the square had a side length of 7 meters, the height of the starting point would be defined as 3.5 meters). Although both videos contained a valid approach, the VLs attended only to the method that aligned with their work:

Researcher: Any thoughts or reflections on that [the video]?
VLs: Nope [both shake head]
Researcher: How did the work done in that [video] relate to what you have done here [points to VLs work, Figure 4.20]?
Jane: **It shows we were right.**
Researcher: Okay. Could you say a little bit more about that?
Jane: That we were right putting the starting position at five [on Square Task 1]. Since that is halfway of the whole length of the side. And

that's where the dot starts [points to point *A* on their square] its journey around the square. And yeah.

Research: What do you think Lucas?

Lucas: I agree … **As the person showed in the video the starting point is at 3.5 not 0. So that solidified our other graph.**

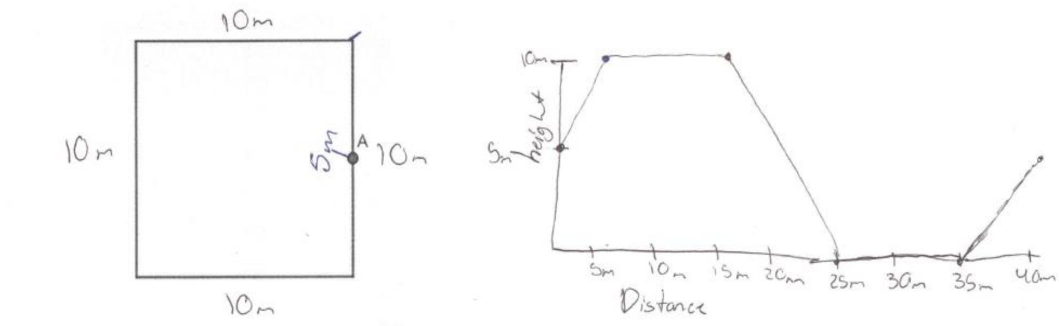


Figure 4.20
Scripted Pair 2's Work from Square Task 1

This exchange illustrated how the VLs were *seeing the talent as a source of answers* through validation of their work. According to Jane, seeing the talent produce a graph similar to their own "shows we were right." After watching two videos containing two methods, differing only in their starting assumption, the most salient detail for the VLs was the information that confirmed their work. This is further evidenced 30 seconds later when the researcher asked the VLs if they could create a graph like the one in the first video of this episode. Jane quickly shook her head and said, "I wouldn't [be able to create the graph], because I would be worried about the negative. I'd be confused." Even though she had seen a video that successfully dealt with the negatives and worked under the starting assumption that the height of point *A* is 0 meters, Jane said she would not be able to. This suggests that the VLs were not engaging collaboratively with the talent's ideas and were instead passively observing the talent and using their solutions as a validation tool.

Frequency. In total, the VLs evidenced *seeing the talent as a source for answers* 47 times. For the VLs viewing the unscripted videos, this category of orientation was uncommon. Unscripted Pair 1 had five occurrences and Unscripted Pair 2 had two (making up 3.07% and 1.58% of their total engagements that evidenced an orientation, respectively). On the other hand, the VLs viewing the scripted videos evidenced *seeing the talent as a source of answers* more frequently. Scripted Pair 1 evidenced this category of distanced orientation 32 times (or for 42.10% of their total engagements that evidenced an orientation), and Scripted Pair 2 evidenced this form of distanced orientation eight times (or for 22.22% of their total engagements that evidenced an orientation). In fact, for Scripted Pair 1, this was the most common way in which they evidenced an orientation toward the talent and they, uniquely, *saw the talent as a source of answers* in all five of their research sessions.

Emotions. Across the 47 instances where the VLs were *seeing the talent as a source of answers*, just four were with a show of emotions. For example, during Research Session 3, Unscripted Pair 1 could not come to an agreement on how to handle height for the Octagon Task (Figure 4.10). Osiris recalled how he thought the talent were treating the height, but Sarah disagreed. While rewatching the video, Osiris leaned in, looked at Sarah, and pointed to the screen. Seeing this, Sarah smiled, paused the video, and said, "*You're* right…". This episode evidenced an eagerness by Osiris when he heard the video confirm his answer and he leaned in. Additionally, when Sarah smiled as if to say she had been caught in a lie and placed an inflection on "*you're*" it demonstrated a disappointment in her realization that she was wrong.

In a second example, at the end of Research Session 3, Scripted Pair 1 celebrated seeing their work confirmed by the talent. While working on the Octagon Task, the VLs struggled to resolve a disagreement in their graphs and turned to the videos. While watching the talent create a graph, Camila stated, "*Oh*! I'm so smart!" and pumped her fists in the air. Camila's utterance and gesture indicated her emotion of pleasure that her answer was vindicated by the talent's work. She excitedly claimed she was right, and her gesture was a sign of celebration of the confirmation of her work with the talent's work. While shows of emotions when the VLs *saw the talent as a resource for answers* were rare (occurring just 8.51% of the time), the emotions were overt and typically in response to being vindicated.

Differences in the Nature of Seeing the Talent as a Source of Answers. The nature of the instances when the pairs were *seeing the talent as a source of answers* also suggested a difference in this form of distanced orientation. Consider two contrasting examples from Unscripted Pair 1 and Scripted Pair 1. During Research Session 3, Unscripted Pair 1 came to a point where they could no longer progress on the Hexagon Task (Figure 4.16). Turning to the video, Sarah said, "I want to see what they did… I'm just a little confused as well". Here, Sarah explicitly expressed an interest in seeing the talent's work, and her expression of confusion indicated she thought the talent could help her understanding.

This instance stands in contrast to a similar moment during Research Session 2 for Scripted Pair 1. While working on Square Task 3, Camila and Alex could not come to an agreement and Camila said, "I don't know why I am getting confused… let's just watch the video." Both instances showed the VLs reaching a point where they needed help to progress on their task and they explicitly turned to the videos. The difference was in the nature of their turn to the videos. In the former example, Sarah suggested she wanted to "see what" the talent did, which indicated she might learn something from their work and that the talent will be helpful in coming to an answer (i.e., she acted as if she needed support). In the latter example, Camila shut down the conversation, citing her confusion, and wanted to turn to the video as a means of settling her confusion (i.e., she no longer wanted to work on the task, and was ready for the answer to be revealed). Indeed, as shown previously in Table 4.7, Scripted Pair 1 frequently turned to the talent to settle their debates.

Evaluating the Talent

The final category of a distanced orientation, the *evaluation of the talent's ideas*, was induced from the data. While evaluating, VLs were making interpretation and elaboration-free statements about their belief or disbelief in the talent's ideas.

Instances of the *evaluation of the talent's ideas* were present across all pairs and most research sessions. Included in these occurrences were evaluations of specific ideas from the talent, surface-level details (e.g., calling an unknown height *b* versus *h*), and general endorsements of the talent (e.g., "that [what the talent said] made sense"). Table 4.8 presents several evaluations that the VLs made as statements without interpretation and that the VLs did not further elaborate upon.

Table 4.8
VLs' Orientation through the Evaluation of the Talents' Ideas

Group	**Vicarious Learner's Statement**	**Location of Utterance**
Unscripted Pair 1	"Except their coordinate for the bottom is wrong. They put zero 62 when it should be 62 zero."	Session 2 9:52
	"I don't believe that."	Session 5 26:25
Scripted Pair 1	"Yeah, that's accurate."	Session 2 11:28
	"I disagree… with the second graph. The one that's like a really weird, rounded version."	Session 2 47:20
	"I disagree with the fact that they are going over the maximum minimum."	Session 3 1:06:28
Unscripted Pair 2	"I agree."	Session 4 3:46
	"They were incorrect with their labeling of their graph once they threw in what units they were doing. Since we know we are working in radians."	Session 5 24:08
Scripted Pair 2	"That makes sense."	Session 1 54:51
	"Once he was correct that he [the talent] did it the way I envisioned the graph looking… so I agree with the end [of the video]"	Session 3 33:55
	"I agree with that graph [the talent's], because ours is just like it"	Session 5 7:48

Frequency. Figure 4.21 illustrates the distribution of the 30 total occurrences of the VLs *evaluating the talent*'s ideas. Looking at the percentage of this category of distanced orientation for each pair to their total engagements that evidenced an orientation shows a clear trend. For Unscripted Pair 1 and Unscripted Pair 2, *evaluating the talent* was an infrequent occurrence

(3.68% and 2.36%, respectively), and *evaluating the talent* was slightly more common for Scripted Pair 1 (13,16%). For Scripted Pair 2, this category of orientation was one of the most common ways they evidenced an orientation toward the talent (30.56% of their total engagements that evidenced an orientation), and it was present in all five of their research sessions.

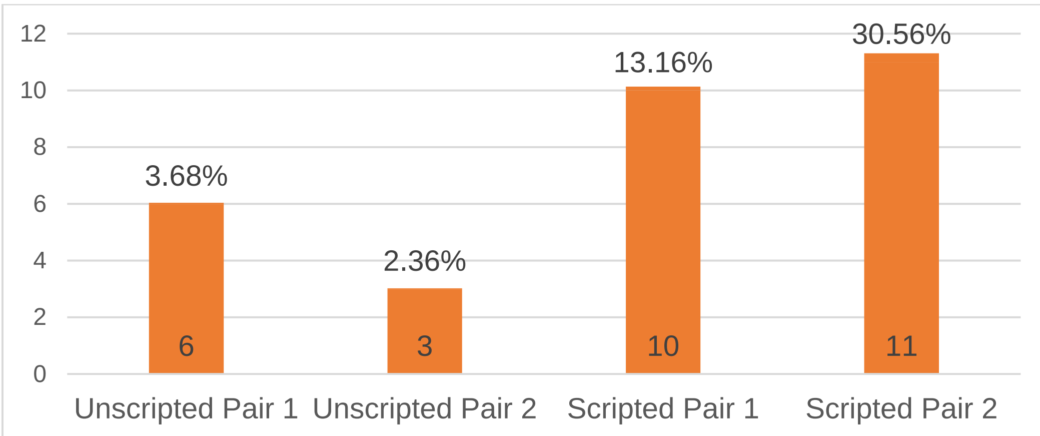


Figure 4.21
Frequencies of Evaluating the Talent's Ideas

Emotions. Shows of emotions while making *evaluations of the talent's ideas* occurred in just two cases, once for each of Unscripted Pair 1 and Unscripted Pair 2. Unscripted Pair 1 evidenced nervousness and Unscripted Pair 1 evidenced frustration. For Unscripted Pair 1, their show of emotion occurred in Research Session 1 while evaluating a specific detail on the talent's graph (i.e., the VLs noticed the talent flipped the coordinates of a point). Sarah said while laughing, "[I agree with everything] except their coordinate for the bottom is wrong. They put (0, 62) when it should be (62, 0)." Her laughter indicated that she was nervous about disagreeing with the talent. For Unscripted Pair 2, their lone show of emotions while *evaluating the talent's ideas* occurred at the start of Research Session 5. While watching the video, Becca listened to the talent claim that "[the graph for the square task is different from the graph for the circle task because] the circle graph has no stagnant segments." Hearing this, Becca laughed and stated, "I *agree*." Becca's laughter and intonation of "agree" indicated that she felt the talent were making a simple point and that she was frustrated with the pace of the video, a form of emotion she had previously expressed (see Being in a Community of Learners).

Differences in the Nature of Evaluating the Talent's Ideas. One difference in the nature of the VLs' evaluations of the talent emerged in the unique way that Unscripted Pair 1 showed hesitation to correct the talent and to claim the talent were wrong. Of Unscripted Pair 1's six *evaluations of the talent's ideas*, three contained hesitations. In Research Session 1, Sarah prefaced her evaluation of a mistake she noticed by saying, "I'm pretty sure they're gonna figure it out, but…" Later, in the same session, Osiris noticed a mistake but hesitated in his evaluation and said, "I think that this graph [that the talent created]… I think that it's, I can't say wrong, but I think it's incorrect to use this kind of graph to determine this question." Finally, in Research Session 2, Osiris began an evaluation in a way similar to the other pairs and said, "They [the talent] completely botched it." In response, Sarah walked his statement back and claimed, "I think they're gonna get it." In all three of these examples, Unscripted Pair 1 claimed that talent had a mistake, but their prefaces and walking back of strong statements, like the talent being "wrong," indicated they were hesitating in their evaluations. Their hesitation stands in stark contrast to the more characteristic examples presented in Table 4.8, where the VLs (Unscripted Pair 2 and both pairs who viewed the scripted videos) would simply deem the talent correct/incorrect or agree/disagree with the talent and move on.

Discussion

Reflecting on the results allows me to return and respond to the research question that guided this chapter:

> **Research Question 1:** How does the orientation of undergraduate vicarious learners toward the talent differ when the dialogue of the videos is scripted versus unscripted?

The central difference between the orientations of the undergraduate VLs toward their respective talent is that Scripted Pair 1 and Scripted Pair 2 evidenced more instances of epistemic detachment, and Unscripted Pair 1 and Unscripted Pair 2 evidenced a more emotionally-involved quasi-collaborative stance. I support this conclusion by first making a case that the VLs viewing the scripted videos were orienting toward the talent through epistemic detachment. Then I present an argument that the VLs viewing the unscripted videos were orienting through an emotionally-involved quasi-collaboration. In conclusion, I return to my hypothesis and explore the possible reasons for the differences in orientations of the VLs toward the talent across treatments.

Epistemically-Detached Orientation

I argue that the two forms of a distanced orientation, presented in the Results section of this chapter, align with what the SVLP called *epistemic detachment*. Recall, McKendree et al. (1998) conjectured that VLs may experience epistemic detachment because of a lowered cognitive demand and because they are not emotionally invested in arguments or the dynamics of a classroom (e.g., they aren't worried about what they will say when called on). McKendree and colleagues claimed that when VLs are unencumbered by emotions they are free to attend to the validity of an argument. Under the code of *evaluating the talent's ideas*, this is what emerged. The VLs, primarily those viewing the scripted videos, passively observed the dialogue and determined what ideas from the talent they believed in and what ideas they did not believe in. This suggests that the VLs viewing the scripted videos were less cognitively engaged because they did not expand upon or interact with the ideas beyond evaluating them (i.e., they merely attended to the validity). Furthermore, these evaluations appeared with little to no shows of emotions (just two of the 30 evaluations were with a show of emotions, and none for the VLs viewing the scripted videos). Together, the VLs' lack of cognitive engagement and the lack of emotions suggests that *evaluating the talent's ideas* was a form of epistemic detachment.

Similarly, when the VLs *saw the talent as a source of answers*, the VLs were not positioning themselves as a member of a collaborative group. Instead, the VLs were minimally engaging with the talent's ideas and were focusing on the answers within the videos. In other words, the VLs were not attending to the substance of the talent's arguments and were, instead, focused on the reproduction of solutions for their own work or the validation of their work. Thus, the VLs (particularly, the VLs viewing the scripted videos), when *seeing the talent as a source of answers*, had a lower cognitive engagement with the talent. Furthermore, as with *evaluating the talent's ideas*, *seeing the talent as a source of answers* had few shows of emotions (four of the 47 instances of *seeing the talent as a source of answers* were with a show of emotions). As such, *seeing the talent as a source of answers* was a form of epistemic detachment.

Above, I have argued that the category of a distanced orientation without emotional displays aligns with what the SVLP called epistemic detachment. Recall, Scripted Pair 1 and Scripted Pair 2 evidenced more, as a frequency and a percentage, categories of a distanced orientation. Thus, they appeared to be epistemically detached. Furthermore, Scripted Pair 1 and Scripted Pair 2 also engaged in less quasi-collaborative orientations and fewer shows of emotions across all engagements that evidenced an orientation toward the talent than the VLs viewing the unscripted videos. Given that quasi-collaboration stands in contrast to experiences of epistemic detachment and the argument that distanced orientations are forms of epistemic detachment, the results of this chapter suggest that the VLs who viewed the scripted videos had an epistemically-detached orientation toward the talent.

Emotionally-Involved Quasi-Collaborative Orientation

I argue that the VLs viewing the unscripted videos had an emotionally-involved quasi-collaborative orientation toward the talent. First, the frequency with which Unscripted Pair 1 and Unscripted Pair 2 were coded for engagements that evidenced an orientation suggests that they were more engaged with the talent (see Table 4.3). After all, a defining feature of many of these codes was the inclusion of explicit mention of the talent or the talent's ideas.

Second, the VLs viewing unscripted videos more frequently engaged in forms of quasi-collaborative orientation, as a raw count and as a percentage of all their engagements that evidenced an orientation. Thus, Unscripted Pair 1 and Unscripted Pair 2 were not only more engaged with their respective talent, but they were more frequently engaged with their talent like collaborative partners (i.e., quasi-collaboratively). Consider two categories of quasi-collaborative orientation that dominated Unscripted Pair 1 and Unscripted Pair 2's engagements with the videos: (a) *coordinating activity with the talent* and (b) *responding directly to the talent*. Following and expanding upon Lobato and Walker (2019), *coordinating activity with the talent* occurred when the VLs made explicit comparisons with and interpretations of the talent's work. As the most frequent category of quasi-collaborative orientation toward the talent, Unscripted Pair 1 and Unscripted Pair 2 made numerous connections between their work and the talent's work and often used these connections to extend their own thinking. Through their frequent interpretations, the VLs viewing the unscripted videos evidenced an interest in engaging with and making sure they had a shared understanding of the talent's ideas. Taken together, the VLs' comparisons and interpretations suggest that they were cognitively engaged with the talent and the talent's ideas.

Responding directly to the talent evidenced a similar sense of collaboration for the VLs viewing the unscripted video toward the talent. This emergent form of quasi-collaborative orientation toward the talent demonstrated a way in which the VLs were cognitively engaged through actively participating in the dialogue by reacting and making explicit use of the talent's ideas. Much in the same way that two students working together may co-construct a solution by piggybacking off one another's ideas, *responding directly to the talent* evidenced collaboration through co-construction.

There were a few instances in which the VLs viewing unscripted videos provided evidence of the orientation categories that marked the VLs viewing scripted videos (i.e., forms of a distanced orientation). However, there were often differences in the ways in which a particular category was instantiated for VLs viewing unscripted versus scripted videos. For example, there was a difference between Unscripted Pair 1 and Scripted Pair 1's evidence for the code of *seeing the talent as a source of answers*. Unscripted Pair 1 would turn to the talent for help, and Scripted Pair 1 would turn to the talent for answers. This difference suggests that Unscripted

Pair 1 was more cognitively engaged with the dialogue and the talent, as opposed to being passive observers. Another difference emerged while *evaluating the talent*; Unscripted Pair 1 was the only pair that showed reluctance in their evaluations. This suggests Unscripted Pair 1 was empathizing with their talent's struggles and did not want to judge them, implying they were more engaged with the talent.

Finally, Unscripted Pair 1 and Unscripted Pair 2's emotional involvement with the talent was evidenced in their shows of emotions across all of their forms of quasi-collaborative and distanced orientations. Furthermore, shows of emotions were evidenced more frequently during the engagements that evidenced an orientation for Unscripted Pair 1 and Unscripted 2 (31.9% and 46.5%, respectively) than Scripted Pair 1 and Scripted Pair 2 (7.9%, and 0.0%, respectively). This suggests, relative to the VLs viewing the scripted videos, Unscripted Pair 1 and Unscripted Pair 2 were emotionally involved. Furthermore, the consistent shows of emotions from the VLs viewing the unscripted videos indicates that the categories of orientations were not something they felt they had to do, but instead, because those categories of orientations were something they were emotionally invested in doing. Thus, I have argued that the VLs were quasi-collaborating with the talent and that their engagements that evidenced an orientation were emotionally involved. I conclude, then, that the VLs viewing the unscripted videos were taking an emotionally-involved quasi-collaborative orientation toward the talent.

Revisiting my Hypothesis

To conclude this chapter, I return to my hypothesis from Chapter 2. After designing this study but before collecting data, I had hypothesized that the VLs viewing the unscripted videos would tend to orient toward the talent in the videos with an emotionally-involved quasi-collaborative stance, while the VLs viewing the scripted videos would experience more epistemic detachment. This was the case.

A central reason for my hypothesized difference between the treatments was based on the VLs' experiences of empathy with the talent. I conjectured that the presence of struggle by the talent would be relatable to the VLs, and thus would produce empathetic experiences. This appeared to be the case. Further, I conjectured that the VLs who viewed the scripted videos would be less empathetic because the inauthenticity of the dialogue would be perceptible (i.e., the VLs would sense the talent's struggle was not authentic). Given the differences observed in the treatments, I believe this was the case.

It is beyond the scope of this chapter, and the research methods of this study, to assess the VLs' perceptions of the authenticity of the dialogue, but I believe an emotional (i.e., empathetic) connection to the talent in the unscripted videos was evidenced by Unscripted Pair 1 and Unscripted Pair 2's shows of emotions. Additionally, as argued above, Unscripted Pair 1 showed empathy in their reluctance to *evaluate the talent*. For example, when Osiris attended to a mistake the talent were making, he said, "I think that this graph [that the talent created]… I think that it's, I can't say wrong, but I think it's incorrect to use this kind of graph to determine this question." In this evaluation, Osiris was reluctant to say the talent were "wrong" and merely concluded that it was not right for "this question." I believe this reluctancy emerged as the pair empathized with the talent and could image how it feels to be harshly corrected in math courses. This only emerged for the VLs viewing the unscripted dialogue.

In fact, on one occasion, Alex, a member of Scripted Pair 1, explicitly called into question the authenticity of the talent. Specifically, during their fourth research session, Alex asked, "Does he [points to laptop] know what he is talking about? The guy that helped you. Or does he just pretend not to know." Alex's question was not answered, and no other questions were asked about the authenticity of the talent's struggle. This moment suggests that much like the *uncanny valley principle* (i.e., the idea that the robot-ness of a near-human android becomes rapidly more perceptible when the android becomes nearly human), VLs have the ability to perceive inauthenticity.

While work needs to be done to assess how VLs discern inauthenticity in scripted videos, the results of this chapter suggest there was a difference between scripted versus unscripted treatments. The VLs viewing unscripted videos and the VLs viewing scripted videos differed in the shows of emotions, frequency of engagements that evidenced an orientation, and ways in which they oriented (emotionally-involved quasi-collaborative orientation for the VLs viewing the unscripted videos and epistemically-detached orientation for the VLs viewing the scripted videos), and these differences may have been a product of an empathetic connection (or lack thereof) to the treatment's respective talent.

Chapter 5: Problem-Solving Behaviors

In this chapter, I answer Research Question 2, which is restated below:

> **Research Question**: How do the problem-solving behaviors of vicarious learners engaged in learning to reason covariationally about the sine function differ when the dialogic videos that they are viewing and interacting with are scripted versus unscripted?

The goal of this study was to identify what problem-solving behaviors emerged from the vicarious learners (VLs) as they engaged with the mathematics within the videos and to determine if their behaviors differed when the videos they engaged with were scripted versus unscripted. Broadly conceived, problem solving includes the cognitive processes and strategies used to identify, consider, and implement ideas to solve novel problems (Schoenfeld, 2016). Recall, during the research sessions, the VLs were given novel problems in the form of paired tasks (i.e., tasks that were either identical or had differing numbers from the tasks within the videos). The VLs then had autonomy in their use of their videos, and they could choose to watch the videos and work on their tasks as they saw fit. As such, viewing and engaging with the dialogic videos were integral to the VLs' problem-solving behaviors.

Review of Methods

Over the course of five research sessions, pairs of VLs viewed dialogic instructional videos that contained either two talent engaged in unscripted dialogue (i.e., the unscripted videos) or one teacher (the researcher) and one talent carrying out a scripted dialogue (i.e., the scripted videos). In the following section, I present an overview of the methods (see Chapter 3 for further details). This section is separated into three subsections: (a) participants, (b) instruments, and (c) thematic analysis.

Participants

In total, eight participants were recruited for this study. Each participant was assigned a partner, creating four pairs. Each pair was then assigned a video type, scripted or unscripted. For convenience, the VLs viewing the unscripted videos are referred to as Unscripted Pair 1 and

2, and the VLs viewing the scripted videos are referred to as Scripted Pair 1 and 2. Table 5.1 provides a key of each pair, the treatment they were a part of, and the talent contained within the videos.

Table 5.1
Key of Talent's and Vicarious Learners' Names

Nature of Dialogue in Video	Name of the Talent within the Video	Treatment Name	Vicarious Leaners
Unscripted	Zoe and Gisele	Unscripted Pair 1	Sarah and Osiris
		Unscripted Pair 2	Becca and Nichole
Scripted	Adrian and the teacher	Scripted Pair 1	Camila and Alex
		Scripted Pair 2	Lucas and Jane

The analysis of this chapter focuses on Scripted Pair 1 and Unscripted Pair 1. Scripted Pair 1 and Unscripted Pair 1 were identified because of their similar dynamic (i.e., both pairs got along well), their similar mathematical ability (i.e., the pairs appeared to perform similarly on the tasks), and their willingness to engage with the videos and one another. This was not the case for the other pairs. Foremost, during data collection, Unscripted Pair 2 stated that they had previously taken mathematics courses up through integral calculus. As such, this pair was at a more advanced mathematical level than the content of the videos. Given that the goal of this chapter was to explore the VLs' problem solving on novel tasks, Unscripted Pair 2's expertise made them unsuitable for analysis. I then selected Scripted Pair 1, rather than Scripted Pair 2, for the comparative analysis, because Scripted Pair 1 provided a richer verbal trace, which made it easier to draw conclusions from their problem-solving behaviors.

Instruments

The mathematics contained within both sets of dialogic videos centers on a sequence of tasks where the students are asked to consider a point moving around a shape. Within this context, the problem the students are solving is to construct a graph that relates the distance traveled by the point to the height of the point. The tasks provide a sequence of *n*-sided polygons whose graph approaches the sine function as *n* increases. Fundamental to the task progression is the integration of rich covariational reasoning that incrementally increases in complexity for each subsequent task (i.e., provides a way for students to transition from chunky to smooth covariational reasoning). This sequence culminates in the development of covariational reasoning for the resultant sine function (for more details about the videos and task sequence, see Chapter 3: Methods).

To illustrate the VLs' problem-solving behaviors, Unscripted Pair 1 and Scripted Pair 1's work on Square Task 1 (Figure 5.1) and the Octagon Task (Figure 5.2) were analyzed. These tasks were identified because they captured the treatments' work on the same task, the pairs have a similar starting and ending position for both tasks, and the episodes are characteristic of both pairs' behaviors across all their respective research sessions.

Create a graph that relates the distance traveled by the point A to the height of the point as it travels counterclockwise around the square with a side length of 10m.

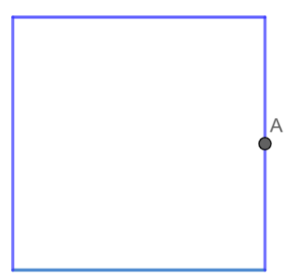


Figure 5.1
Square Task 1

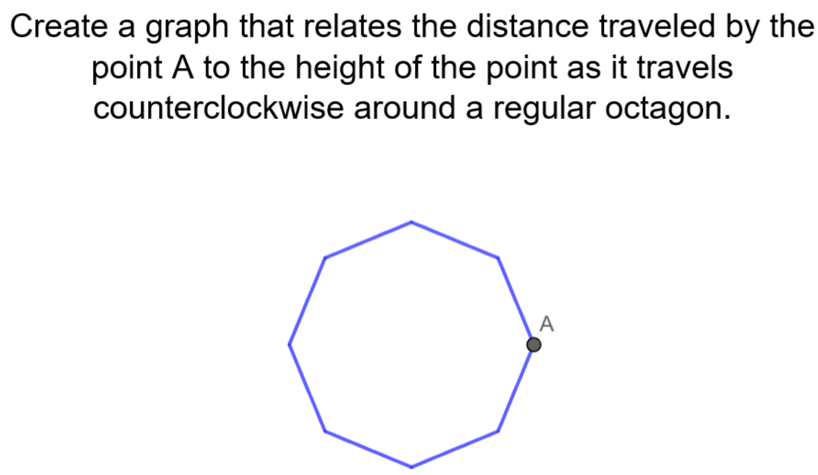


Figure 5.2
The Octagon Task

Additionally, both Square Task 1 and the Octagon Task represent significant moments in the task sequence where a layer of added complexity in covariational reasoning is required to solve the task and a pivotal step towards the construction of the sine function is made. Square Task 1 marks the first time that students must coordinate three different changes in height with changes in distance (i.e., if $f(x)$ is the final graph for the square task, then $f(x)$ has three unique instantaneous rates of change: 1, 0, and -1). This way of reasoning is crucial as the number of sides increase and the shape approaches a circle because the number of unique instantaneous rates of change also increases. The Octagon Task was the penultimate task and required students to reason with a rate of change in height that was changing, non-zero everywhere, and constantly differed from the change in distance (i.e., if $f(x)$ is the final graph for the Octagon Task, then $f(x)$ has five unique instantaneous rates of change, and none of them was 1 or -1). This, again, is a pivotal step in students' covariational reasoning that was leveraged in their thinking about how the change in height is changing for a point moving around a circle. In sum, this segment of data allowed for a rich comparison of the VLs on tasks that required problem solving and pushed the students' (both the talent and the VLs) covariational reasoning.

Thematic Analysis

Thematic analysis (TA), as systematized by Braun and Clarke (2006), was used to identify and describe patterns of meanings or themes within this data. This methodology was applied for three reasons: (a) TA allows for the use of both a priori and emergent codes, (b) the process of defining and refining themes ensures analysis stays close to the data, and (c) the flexibility of TA has supported its use in mathematics education to answer a variety of research questions (e.g., Apkarian et al., 2023; Ellis et al., 2015; Garcia-Garcia & Delores-Flores, 2021; Rupnow & Sassman, 2022).

Braun and Clarke (2006) delineated six phases within thematic analysis. Phase 1 begins with the familiarization of oneself with the data. Phase 2 calls for the generation of initial codes. Phase 3 entails the creation of general themes and subthemes. Phase 4 involves reviewing and clarifying themes created in Phase 3. Phase 5 requires defining and naming themes. Phase 6 involves the production of the research report.

For this study, Phase 1 began with the review of Scripted Pair 1 and Unscripted Pair 1's work on Square Task 1 and the Octagon Task. This included the VLs' time spent watching the videos on the talent's work on these tasks and the VLs' reflections on their own work. This phase culminated in the creation of descriptive accounts of the videotaped data, which consisted of a narrative of what transpired with minimal inference (Miles & Huberman, 1994). Phase 2 began the identification of problem-solving behaviors, using a low level of inference. For example, the VLs' interactions with the videos were identified, which resulted in codes of their uses of the videos (e.g., pausing, rewinding, and altering the playback speed). During Phase 3, themes and subthemes began to emerge through the grouping of codes. These groupings allowed for patterns to be identified in the VLs' behaviors and engagement with the mathematical content of the videos. Phase 4 allowed the groupings identified in Phase 3 to be concentrated, with the interrelationships between groupings to be clarified. As a result, more refined themes and subthemes emerged.

For this study, Phase 4 also involved a literature review. As described above, thematic analysis allows for the use of both a priori and induced codes. At this stage, the literature review was used to identify possible frameworks that had already been established and could be used as themes grounded in the literature. For example, the VLs' use of the videos was a theme identified as a form of problem-solving behavior. Through conducting a literature review, Yoon et al.'s (2021) framework for patterns of video use was determined to align with the VLs' manipulations of the videos and VLs' engagements while the videos played. As a result, previous codes were recoded to better align with Yoon and colleagues' framework.

Next, Phase 5 entailed the defining and naming phase. Table 5.2 presents an overview of the results of this process. In total, three themes were identified: (a) *patterns of use*, (b) *idea justification*, and (c) *idea management*. Each theme and subtheme are defined and characterized in further detail in the Results section below.

Finalizing these three themes and their subthemes concluded Phase 5 of TA. Phase 6, or the production of the research report phase, was the writing of this chapter. This chapter is organized around each of the themes identified. Within each section, the themes are described, a characteristic example from the VLs' work is presented, and a connection to the VLs' problem-solving behaviors is illuminated. I then conclude this chapter with a discussion of the results.

Table 5.2
Themes and Subthemes

Theme	Description	Subtheme	Description of Subtheme
Patterns of Use	The VLs' problem-solving behaviors while they played or manipulated the videos	Browsing	Problem-solving behaviors relating to the VLs' pausing or playing the videos
		Social Interaction	Comments toward one another about the videos while the videos played
		Information Seeking	VLs' work while the videos play that evidenced a coordination of information from the videos and their work (e.g., note-taking)
		Environment Configuration	Manipulations to the videos (e.g., adjusting video playback speed) that influenced VLs' viewing experiences
Idea Justification	The ways in which the VLs substantiated their claims to their partners while working on novel tasks and engaging with dialogic videos	Appeal to an Authority	The reference to an external source as support for a claim
		Substantive Justification	The use of an example, fact, or deductive reasoning as a justification of a claim
Idea Management	The ways in which the VLs made use of ideas, solutions, and meanings expressed within the videos	Repetition	Reproduction of an idea, solution, or other work presented within the video by the VLs, without engaging with the associated meaning of the idea, solution, or other work
		Negotiation	The use of an idea, solution, or other work presented within the videos by the talent, while engaging and extending the associated meanings of the idea, solution, and other work

Results

The following section explores the problem-solving behaviors of the VLs from Unscripted Pair 1 and Scripted Pair 1 as they engaged with two important tasks and the dialogic videos associated with these tasks. This section contains three subsections, one for each theme.

Within each theme is a descriptor of the theme and an exploration of the subthemes that constitute it.

Theme 1: Patterns of Use

One way in which the VLs' problem-solving behaviors emerged was in the way they used the videos. The theme of *patterns of use* centers on what the VLs did while they played or manipulated the videos. In total, four subthemes were identified: (a) *browsing*, (b) *social interaction*, (c) *information seeking*, and (d) *environment configuration*.

The four subthemes of *patterns of use* are adaptations of Yoon et al.'s (2021) categories of patterns of use. In their study, Yoon and colleagues tracked the ways in which their undergraduate students used an asynchronous instructional mathematics video while watching at home by themselves. The lesson was not part of broader coursework and was offered as an optional lesson on quantifiers. Using their video hosting software, Yoon and colleagues were able to capture 11 different forms of interactions with the video, what Yoon et al. (2021) called "log behavior" (p. 1). Included in these forms of interactions were playing and pausing the video, adding a comment to the video, and seeking specific segments of the video (i.e., dragging the play head to specific timestamps). The 11 interactions were clustered into four categories: (a) *browsing*, (b) *social interaction*, (c) *information seeking*, and (d) *environment configuration*.

The category of *browsing* included uses such as pausing, playing, and bookmarking segments of the video. The use of the video for *social interaction* captured the ways in which viewers interacted with other viewers. This consisted of placing comments on specific segments of the video and replying to other students' comments. *Information seeking* included what the authors called "filtering" and "annotation" which was "how students actively sought information to enhance their understanding of the video's topic" (Yoon et al., 2021, p. 4). Annotation, for example, was the authors' shorthand for clicking on annotations from the teacher within the video. Finally, *environment configuration* was defined as the way in which the viewers optimize their environment through manipulations of the video for effective learning. This included re-viewing segments of the video, adjusting the volume, and “seeking,” or skipping ahead.

While Yoon et al. (2021) identified specific patterns of use of their instructional video, the nature of my study allowed for behaviors to include what happened while the videos were playing. Specifically, the video used by Yoon and colleagues had features such as commenting, annotating, and sliding, which are not applicable to this study. As such, analogous behaviors and patterns of use were identified. For example, the commenting feature functioned as a means of communicating online with other viewers in the Yoon et al. study. For this study, the viewers watched in pairs and could communicate directly. As such, the category of *social interaction* was expanded to include talking while the videos were playing.

Similarly, I extended the code of *information seeking*. Yoon and colleagues characterized information seeking as including the ways in which information from the video was used for some purpose or goal. For this study, the VLs often sought information from the dialogic videos for the purpose of solving a paired task. Consequently, I characterize *information seeking* in the context of my study as the VLs’ coordination of information from the videos and their work (e.g., note-taking). For example, if the talent made a claim that the total distance around a shape is 40 meters and then the VLs wrote down 40 meters, this moment was categorized as information seeking. Finally, the fourth category, *environment configuration*, was extended to capture an emergent use of the videos, watching the video with a playback speed of times two. This pattern of use appeared to function in an analogous way of skipping ahead, a use that Yoon and colleagues reported under this category. The first category, *browsing*, was kept the same, with playing and pausing identified and analyzed.

The following section is organized around the four subthemes of *browsing*, *social interaction*, *information seeking*, and *environment configuration*. Within each subtheme, the specific *patterns of use* that were coded under each subtheme are detailed. Additionally, differences between the *patterns of use* for Unscripted Pair 1 and Scripted Pair 1 are characterized.

Theme 1a: Browsing

The *pattern of use* of *browsing* was captured in two forms: playing and pausing the videos. In the following two subsections, the behavior of browsing through playing the video is explored and described for both pairs of VLs, followed by evidence of the novel way in which Unscripted Pair 1 browsed through pausing the videos.

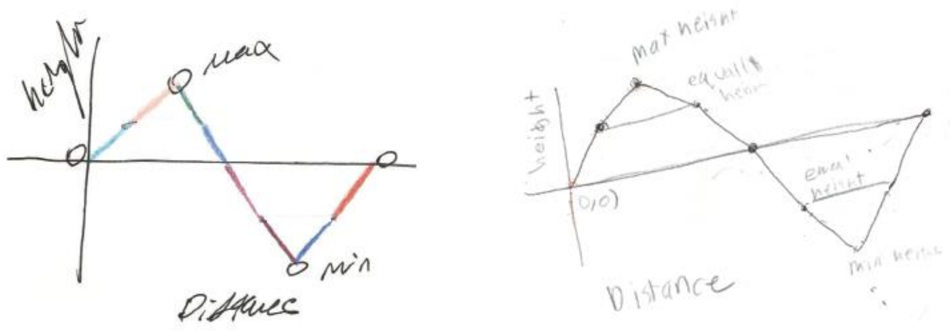
Browsing Through Playing. This section focuses on the apparent purpose that playing the videos served for Unscripted Pair 1 versus Scripted Pair 1. Unscripted Pair 1 seemed to play the videos for two purposes: (a) to gather ideas about how to start working on a task and (b) to help them progress when stuck on a novel task. During Research Session 2, Unscripted Pair 1 turned to the videos to gather ideas about how to start Square Task 1. As Sarah finished rereading the task statement for Square Task 1, she stated, "… the point as it traveled… we can watch this first [points to screen], and then we can kinda… [trails off and plays the video]." The long pauses between her utterances and her incomplete thought as she played the video indicated she was unsure how to begin the task. Less than a minute into the video, Sarah paused the video and was able to start working on the task (for more details, see the Browsing Through Pausing subsection below). During this work, Sarah confirmed her initial confusion with the task when she reflected, "I don't know why it [the task] didn't click [points at screen]." Here, Sarah implied that the meaning of the task statement hadn't "clicked" but now did, and her gesture toward the screen indicated that the video had sparked in her some comprehension of the task.

Unscripted Pair 1 also appeared to play the videos to settle confusion when they got stuck during their problem solving. For example, during Research Session 3, Sarah and Osiris were able to make initial progress on the Octagon Task (Figure 5.2) but quickly turned to the videos. While they worked on the Octagon Task, Sarah created a sketch (Figure 5.3) that she described as an “up and down kind of deal.” After creating this sketch, the pair attempted to draw a more precise graph, but they struggled to account for the height. After a lengthy back-and-forth, Sarah suggested they turn to the videos, “I want to see what they did… I’m a little confused.” Explicitly, Sarah’s pattern of use for the video (i.e., browsing through playing) was a means of settling her confusion and progressing on the task.

Figure 5.3
Sarah’s Sketch for the Octagon Task

Similarly, Scripted Pair 1 evidenced playing the video to help them progress when stuck on a novel task, but, uniquely, this use faded and evolved into playing the videos to settle their debates. Initially, Scripted Pair 1 appeared to play the video to help them progress on Square Task 1 during Research Session 2. After being reminded by the researcher that they could watch the video first or work on the task, Camila said, “Let’s try it [solving the problem] out first.” After they spent about two minutes discussing how to start the graph and rereading the task statement three separate times, the VLs silently played the video and watched the video from beginning to end without saying a word. Given their explicit goal of trying the task first, the lack of progress on the task, and the repeated reading of the task statement suggests that Scripted Pair 1 played the video because they were confused by the task and saw the videos as a resource to help them progress on the novel task.

As Scripted Pair 1 progressed through the research sessions, they played the videos for a different reason, to settle their debates. While working on the Octagon Task during Research Session 3, Camila and Alex separately created two similar graphs (Figure 5.4).



(a) (b)

Figure 5.4
(a) Camila's Graph and (b) Alex's Graph for the Octagon Task

After the VLs watched an episode where the talent creates a similar, imprecise, graph and then adjusts it to account for changing changes in height, Camila decided to create a new graph (Figure 5.5). This resulted in a lengthy exchange between the VLs about the accuracy of the different graphs (for an overview of this exchange, see Theme 3b, Scripted Pair 1).

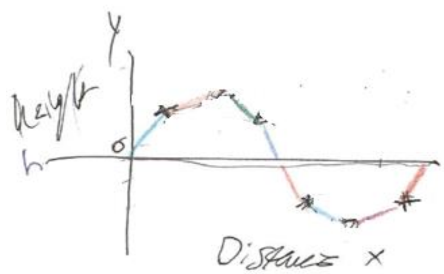


Figure 5.5
Camila's Final Graph for the Octagon Task

Unable to convince one another of their respective positions, Camila suggested they "agree to disagree" and promptly played the next video. Camila's utterance alongside her *browsing* (i.e., playing the video) indicates the VLs were at an impasse and that the video could be used to settle their debate. This reason behind Scripted Pair 1's browsing through playing the video became explicit when Alex reflected that he preferred to work first and then play the video "because we argue, and then see who's right [points to computer]." This marked a shift in Scripted Pair 1's purpose behind their playing the videos. Initially, Alex and Camila turned to the videos when they got stuck and needed help progressing (e.g., their work on Square Task 1), but this shifted over time toward playing the videos to settle their debates.

Browsing through Pausing. The second form of *browsing*, pausing the video, was a pattern of use unique to Unscripted Pair 1, Sarah and Osiris. Throughout Sarah and Osiris's engagement with the videos, they would pause the video to discuss and make progress on their tasks. The reasons behind their pausing of the videos appeared to align with the reasons behind their browsing through playing the videos. As argued above, their playing the video was a means of progressing in their problem solving by gathering ideas about the task and settling confusion to progress on their task. In this section, I explore how Unscripted Pair 1 would watch the videos until they appeared to find a key idea that would help them make progress in their problem solving, and then they would pause the video.

For example, while watching a video of the talent work on the Octagon Task (Figure 5.2), Osiris paused the video because he appeared to have gathered an important insight from the video. Prior to the moment the video was paused, the talent had started discussing the relationship between the change in height of the point on the octagon and its change in distance traveled. This led the talent to begin their graphing process (Figure 5.6). Notice, the talent's initial work on their graph contains a slight mistake. The slope over the first segment and the second segment should be inversely related (i.e., if the slope of the first segment is *m*, then the slope of the second segment would be -*m*), but the talent did not capture this relationship.

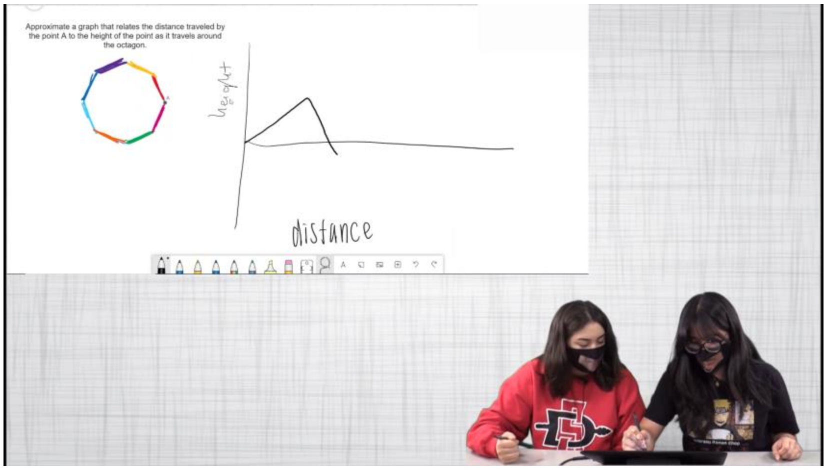


Figure 5.6
The Talent's Work Prior to Osiris Pausing the Video

In the video, the talent quickly notice their mistake and begin to erase their graph. Before they finish erasing or start describing their mistake, Osiris paused the video and argued (correctly) that the talent were making a mistake. Osiris then built upon what he saw in the video, by first correcting the mistake he identified, and then starting his own graphing process (Figure 5.7). Importantly, his graph roughly aligned with the graph the talent had attempted to create (Figure 5.6) and would eventually complete. In sum, Osiris noticed a mistake in the talent's work but also gained an insight from the video that allowed him to progress on his own work. As such, he paused the video and began graphing.

Figure 5.7
Osiris's Initial Progress on the Octagon Task After Pausing the Video

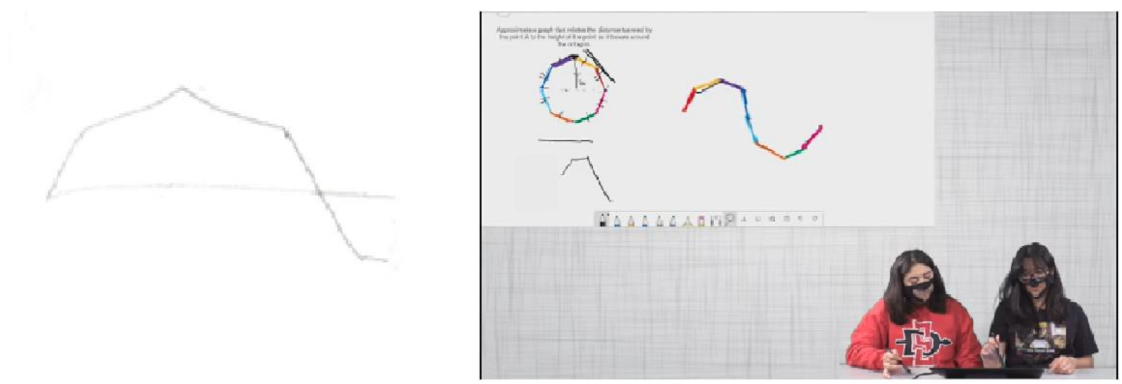
Unscripted Pair 1 frequently made use of pausing the videos. While working on Square Task 1 and watching the corresponding video, Unscripted Pair 1 paused the video two times. Initially, the VLs paused the video for 49 seconds, during which time they made use of the given information in the task statement, as they had seen in the video. Then, after watching the video for another minute, the VLs paused the video again. This time, their discussion lasted over 10 minutes (for further details about this segment see Theme 3b). Similarly, while working on the Octagon Task, Unscripted Pair 1 paused the videos on five separate occasions for segments of 10 minutes, 7 minutes, 30 seconds, 8 minutes, and 30 seconds. Despite the three videos comprising the Octagon Task lasting a total of 18:46, Sarah and Osiris engaged in problem solving for over 26 minutes while the videos were paused. The frequency with which Unscripted Pair 1 paused the videos suggests pausing after they had gathered information from the videos was a common occurrence for the pair.

Summary. The pattern of use of *browsing* indicates a number of key differences between the problem-solving behaviors of Unscripted Pair 1 and Scripted Pair 1. Initially, both pairs browsed through playing the video to get information about the task. For Unscripted Pair 1, this took the form of finding information to begin their task and gathering insights when they were struggling with the tasks. This meaning persisted for Unscripted Pair 1 and was further evidenced in their novel browsing through pausing the videos. Scripted Pair 1, on the other hand, never paused their videos and the meaning behind their playing of the video evolved. By their third research session, Camila and Alex had shifted their *browsing* from playing for insights to playing the videos to settle their debates. These differences, and the evolution in Scripted Pair 1's playing, suggest a key difference in the VLs' problem-solving behaviors: Unscripted Pair 1 primarily used the video to assist their problem solving and Scripted Pair 1 primarily used the videos to determine whose solution was correct.

Theme 1b: Social Interaction

The pattern of use of *social interaction* captured the moments when the videos were playing and the VLs were making comments to each other. In this section, I explore the differences in *social interactions* between Unscripted Pair 1 and Scripted Pair 1. While both pairs engaged in *social interaction*, the nature of their talk while the videos played differed.

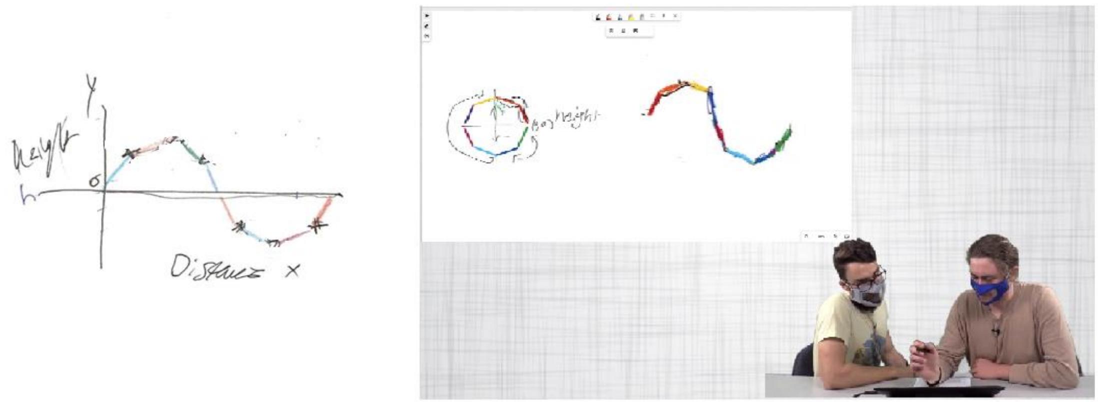
One way in which the *social interaction* of Unscripted Pair 1 and Scripted Pair 1 differed was in Unscripted Pair 1's collaborative celebration versus Scripted Pair 1's feeling of vindication during moments when their solutions and ideas were confirmed by the videos. For example, both pairs struggled with the Octagon Task (Figure 5.2) and went through several iterations of graphs. Eventually, both pairs produced graphs, of varying specificity, that aligned with the general shape of the graph produced by the talent in the videos. For Unscripted Pair 1, Sarah agreed with the general shape Osiris had sketched (Figure 5.8a). Then, when the VLs saw the graph produced by the talent (Figure 5.8b), Osiris said, "Interesting shape [both VLs laugh, and Sarah initiates a fist bump]." Here, Sarah and Osiris's *social interaction* indicates they were happy to have their answer confirmed by the video and their joint hand gesture was a collective form of celebration. Unscripted Pair 1's *social interaction* appeared to be celebratory of their solution and indicated satisfaction in their collective work.



(a) (b)

Figure 5.8
Graph for the Octagon Task: (a) Unscripted Pair 1's and (b) Gisele and Zoe's

In contrast, the VLs in Scripted Pair 1 disagreed about their graph (Figure 5.9a) and only Camila believed it was accurate (for more details about this disagreement, see Theme 3b). Then, while the VLs watched the talent describe why their final graph (Figure 5.9b) is correct, Camila celebrated, "I'm so smart!" Camila reacted with a feeling of vindication in her work. As presented above (see Pattern of Use: Browsing through Playing), Alex and Camila came to see the videos as an arbiter of their disagreements. Camila's reaction was then a natural celebration that her side of the argument was correct. Compared to Unscripted Pair 1's satisfaction with their collective work, the *social interaction* for Scripted Pair 1 appeared to be an excitement over winning an argument.



(a) (b)

Figure 5.9
Graph for the Octagon Task: (a) Scripted Pair 1's and (b) Adrian's

Another form of *social interaction* occurred when the VLs either verbally agreed or disagreed with ideas presented by the talent while a video played. In this section, I provide evidence that Unscripted Pair 1 was actively considering and thinking about the meaning behind the talent's work while Scripted Pair 1 was more passively involved. For example, on Square Task 1 (Figure 5.1), Unscripted Pair 1 produced a final graph that assumed the starting height of the point was half the side length (e.g., given that point *A* starts halfway up a side of length 10m, then the initial height of point *A* is 5m; see Figure 5.10). The talent produced a slightly different graph by assuming the starting height is 0 meters (Figure 5.11). While the talent worked on creating their graph, Sarah engaged in *social interaction* by verbalizing her disagreement with the talent. Sarah said, "They have the same idea, but their starting point is wrong… because now their graph is going negative [points to screen], but I don't think you can really go negative." Here, Sarah believed both graphs captured "the same idea," suggesting she believed the graphs represent the same relationships. Then, Sarah made the claim that the talent's work "is wrong." Together, her indication that the meaning behind the two graphs was the same and her simultaneous disagreement with a feature of the talent's graph suggests she was actively considering and thinking about the ideas expressed in the videos.

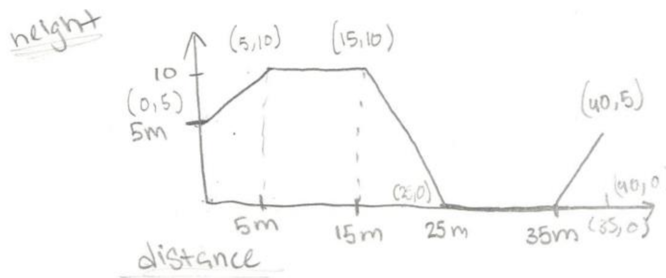


Figure 5.10
Sarah and Osiris's Final Graph for Square Task 1

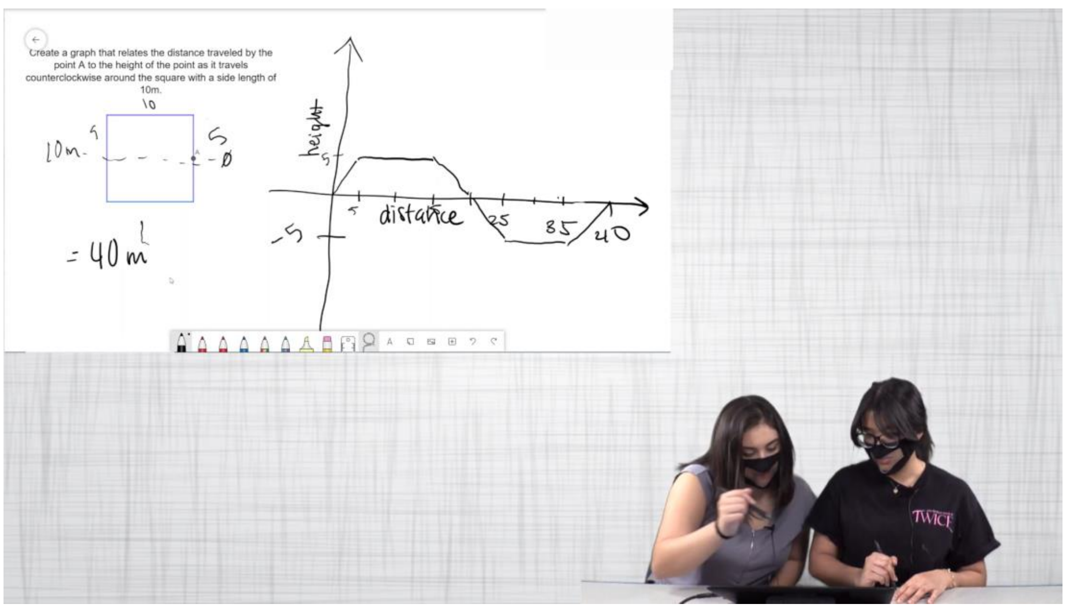


Figure 5.11
The Talent's Final Graph for Square Task 1 from the Unscripted Videos

Similarly, across Square Task 1 and the Octagon Task, Unscripted Pair 1 frequently engaged in *social interaction* by verbalizing their approval of the talent's ideas (e.g., "I agree…," "yeah… hmm," "hmm.. okay," etc.). Both forms of *social interaction*, disagreement and agreement, indicated Unscripted Pair 1 was actively considering the meaning being expressed by the videos.

For Scripted Pair 1, Camila and Alex, this form of *social interaction* with the videos was not present. While Alex and Camila showed a willingness to agree and disagree amongst themselves, they did not evidence a *social interaction* by explicating a disagreement or agreement while the videos played. On one occasion, Alex appeared to disagree with the graph of the talent, but it quickly became apparent that he was merely attending to surface-level details. In the video, Adrian (the talent from the scripted videos) was explaining his preference for his new graph (Figure 5.12), and Alex *socially interacted* through speaking:

Alex:	I disagree with that graph. Because look, the zero points are at different lengths.
Camila:	I don't think it's supposed to be spot on.
Alex:	[laughing] My bad!

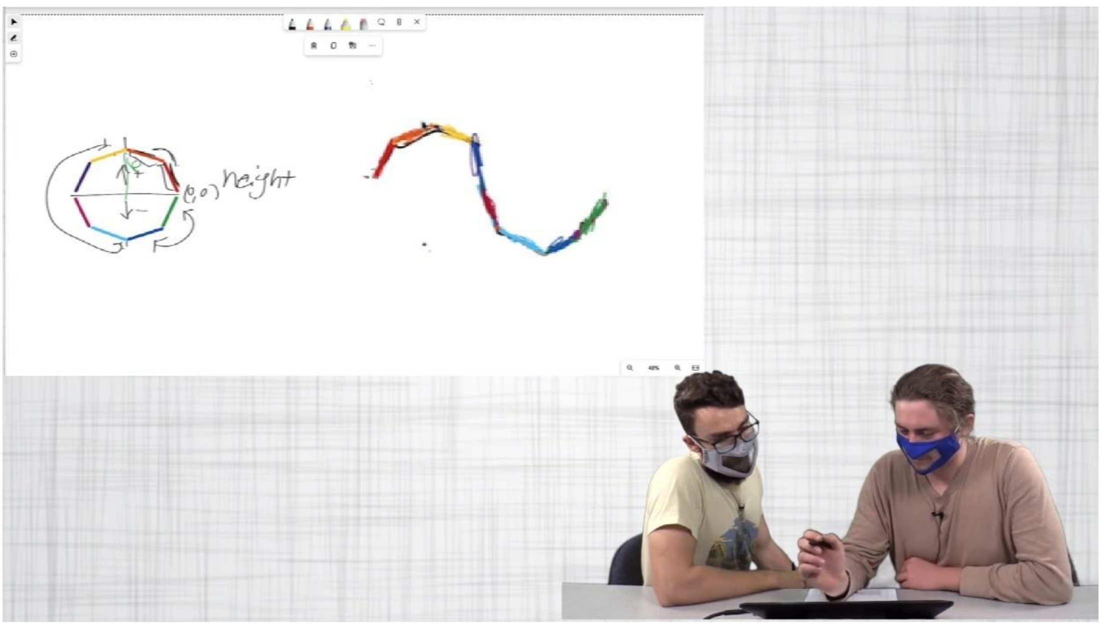


Figure 5.12
Adrian's (the Talent from the Scripted Videos) Graph for the Octagon Task

Initially, Alex indicated he had a disagreement, and he pointed to a specific feature of the talent's graph that he did not think was accurate. In response, Camila dismissed Alex's point by suggesting the talent was approximating. Given Alex did not push back or expand on his disagreement suggests that Camila accurately interpreted Alex's disagreement. Thus, Alex appeared to be attending to the imprecise nature of the talent's graph, and he was disagreeing with surface-level features, as opposed to engaging with the meaning of the talent's graph.

Theme 1c: Information Seeking

The pattern of use of *information seeking* captured moments when the VLs were working while the videos were playing and appeared to be coordinating information from the videos with their own work. Working during the videos included notetaking and progressing on the paired task through the use of information presented by the talent. These forms of *information seeking* were common for Scripted Pair 1, but Unscripted Pair 1 did not engage in this pattern of use of the videos.

In the following, I present two instances when Scripted Pair 1 engaged in *information seeking*. Both evidence problem-solving behaviors of coordinating their work with the talent's work by using information presented within the videos. The first example occurred while Scripted Pair 1 watched the talent work on Square Task 1. While the video played, Camila took several notes, annotated the square by adding quantitative features, and began creating the axis for her eventual graph. In the video, Adrian describes the point's journey:

> Adrian: So, our graph is going to start with 5 meters right here, going up, then we have this **10 meters going stagnant portion of the graph**, and then right here it's gonna be **10 meters going down**, another stagnant portion that will be in the negative part of the graph and then right here it's gonna come back up 5 meters.

Simultaneously, Camila started annotating her square (Figure 5.13a). Her work specifically captured how the talent was using the given side length of 10 meters. Continuing in the video, Adrian begins graphing and describing his graph:

> Adrian: You're gonna start going **in the positive direction for 5 meters**. So, it's gonna go like that [draws a line segment from (0,0) to (5,5)]. And then go about 10 meters where our height is not changing [draws a line from (5,5) to (15,5)]. And then we've got 10 meters going down, and it's gonna cross at 20 because that's an intersection point at 0, and then **go down to -5 for the height**. And then it's gonna go stagnant for 10 meters right there. And then go back to 0 [draws a segment on his graph from (35,-5) to (40,0)].

While this occurred in the video, Camila continued her annotations (Figure 5.13b). Given the similarities between the talent's utterances and Camila's annotations (e.g., Camila's segments labeled -5m aligns with Adrian's "down to -5") suggests that she was seeking out information from the videos and coordinating her work with the information she found relevant.

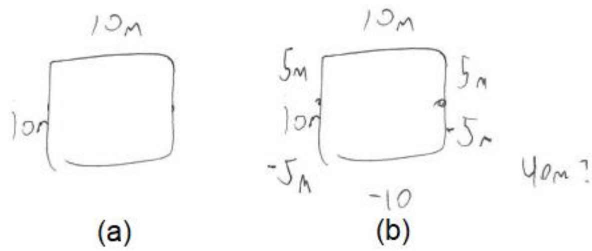


Figure 5.13
Camila's Annotations, Created While Watching the Video

Similarly, during Scripted Pair 1's work on the Octagon Task, Camila corrected her graph while she watched the corresponding video. Prior to watching the video, Camila and Alex created graphs that failed to account for changing changes in height (Figure 5.4). In the video, Adrian makes the same mistake (Figure 5.14) but is eventually guided to a more accurate representation by the teacher:

Teacher: Throughout your graph, we have straight lines throughout it. Now, specifically looking at the red and the orange segments. Is there anything different between these segments? Could you describe for me what is happening for the point that is traveling through red into orange?

Adrian: So, when the point travels through red it is a little bit steeper of a slope. Actually, a good amount steeper. And then here, it's a lot less steep of a slope. So, **our change in height is actually not the same** between these two lines.

Teacher: So you said, these two segments [highlights red and orange on the graph, Figure 5.14] it looks like they are showing the same slope on your graph, but you're saying the slope is different? Do you know what the shape should look like for this graph?

Adrian: Yeah. It would essentially just **look like our octagon**. Except it would kind of have this part be flipped

Teacher: Kind cut and then reflected?

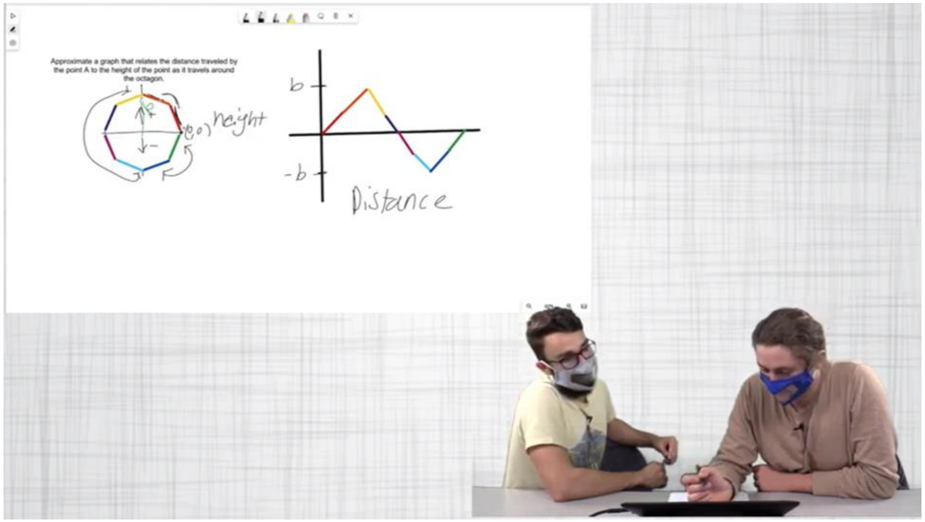


Figure 5.14
Adrian's, the Talent from the Scripted Videos, Initially Graph for the Octagon Task

During this exchange, Camila started working. Initially, she created a sketch of the graph described by Adrian (Figure 5.15). Notice how this sketch captured the octagon shape suggested by the talent. Camila continued with her coordination of work and eventually created her final representation (Figure 5.9a), all while the video was playing.

Figure 5.15
Camila's Sketch of a Graph, Drawn While She Watched the Video

In summary, *information seeking* emerged only for Scripted Pair 1 and resulted in the coordination of the ideas and solutions from the videos with the VLs' annotations and solutions. While it was clear from this pattern of use that Scripted Pair 1 was attending to the videos, it is not clear whether the pair was attending to the meaning of this information (see Theme 3, where the meaning, or lack of meaning, gleaned from the videos is explored). Importantly, the *information seeking* evidenced by Scripted Pair 1 was indicative of how they were using the videos in their problem solving. For this pair, the videos were seen as a source of ideas and solutions that they felt compelled to take notes on or make use of while the video played.

Theme 1d: Environment Configuration

The final subtheme in *patterns of use* of the videos by the VLs is *environment configuration*. This subtheme captures the ways in which the VLs manipulated the videos in such a way that their viewing experiences were influenced. This primarily took form through adjusting the video player by either skipping ahead, rewinding, or watching the videos at twice the original speed (i.e., adjusting the playback speed of the video). Importantly, this theme differentiates itself from *browsing* (i.e., playing and pausing the videos) because the manipulations of environmental configuration altered the way information was coming to the VLs. This is different than, say, playing the video which begins the process of information coming to the VLs. This pattern of use only emerged for Scripted Pair 1.

For example, Scripted Pair 1 configured their environment by rewinding the video to see the final solution again. This occurred after the VLs watched the episode where the talent worked on Square Task 1. As illustrated in Figure 5.13, Camila annotated her square and took down several notes during the video. After the video concluded, she suggested she understood the talent's graph, but then struggled to create her own graph for the task, and asked rhetorically to rewind the video:

> Camila: Yeah, **that makes sense**… It was 5 to 10, right? Wait. Yeah…? No. Hold on. We're good. I forgot where it was [gestures on the square from the starting position to the first corner]. **Can I go back?**

Despite Camila stating that the video "makes sense," she struggled to make use of the talent's work from the video to create her own graph. Consequently, she asked her partner if she could rewind the video, which she subsequently did. Specifically, Camila engaged in an environment configured by manipulating the video to show a static image of the talent's graph (Figure 5.16). With the talent's graph displayed, Camila reproduced the graph (Figure 5.17).

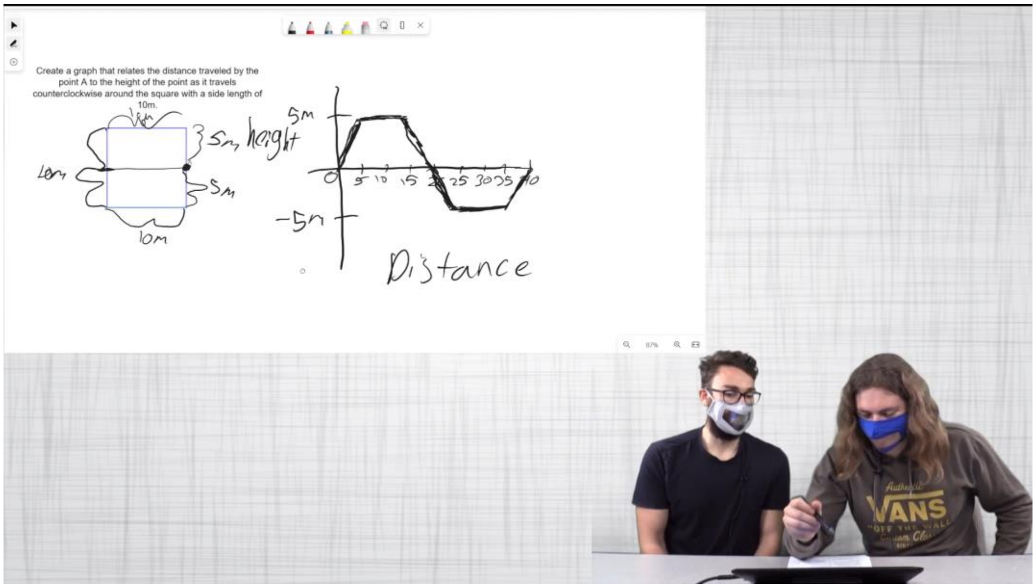


Figure 5.16
Adrian's Graph for Square Task 1, and the Moment Camila Rewound to in the Video

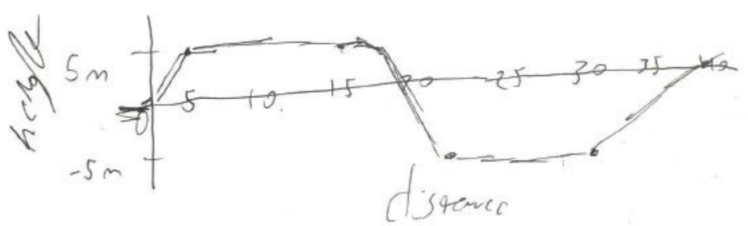


Figure 5.17
Camila's Graph for Square Task 1

Another form of *environment configuration* that emerged for Scripted Pair 1 was watching the videos at twice the playback speed. While the VLs were not explicitly told that they could use this feature, the videos were hosted on YouTube which allows users to control the playback speed as they watch and listen. For Scripted Pair 1, this was initiated by Camila while she and her partner watched the talent complete both Square Task 1 and the Octagon Task. During both occasions, the researcher paused the videos and confirmed that both VLs were comfortable with watching at an adjusted playback speed. Again, this manipulation of the video is considered a form of *environment configuration* because the VLs have adjusted how the video is being presented to them. This form of use indicates that Scripted Pair 1 had an interest in getting through the videos faster. An important feature of the videos is the dialogue between the talent and the teacher. However, the VLs' use of the adjusted playback speed suggests they were not as interested in the dialogue as they were in configuring their environment to get to a solution.

Summary

Across Scripted Pair 1 and Unscripted Pair 1's work on Square Task 1 and the Octagon Task, their *patterns of use* indicated that the pairs not only physically used the videos differently but that they used the videos for different purposes in their problem solving. Unscripted Pair 1's *patterns of use* of the videos indicated that their problem-solving behaviors were being enhanced by the videos, and Scripted Pair 1's *patterns of use* indicated a set of problem-solving behaviors aimed at finding a solution and having confirmation of their solutions. Through *browsing*, Unscripted Pair 1 evidenced how they were using the videos to gather ideas about how to get started on tasks and generally progress when stuck on their task. Through *social interactions*, Unscripted Pair 1 evidenced how they were engaged with the videos as a source of ideas that could be right or wrong. On the other hand, Scripted Pair 1's *browsing* and *social interactions* suggested the videos served as an arbiter of debates and were the final word on who was right and wrong. Then, through *information seeking* and *environment configuration*, Scripted Pair 1 appeared to use the videos as a source of answers to be reproduced.

Theme 2: Idea Justification

The theme of *idea justification* captures the VLs' problem-solving behavior of providing support for their mathematical claims to their partners while working on novel tasks and engaging with dialogic videos. The Common Core State Standards for Mathematics (2010) state, "One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from" (p. 4). This suggests that the ability to justify a solution is as important as, if not more important than, deriving a solution. Thus, when justifications were presented by the VLs, insight into their mathematical understanding (e.g., their covariational reasoning) was present. Furthermore, given the expectation that the VLs' would explain their work to the researcher, *idea justification* emerged as a problem-solving behavior.

In total, two forms of *idea justification* were identified from the data: (a) *appeal to an authority* and (b) *substantive justification*. The first subtheme, *appeal to an authority*, occurred when a claim is supported by an external source. In their work on proof schemes, Sowder and Harel (1998) identified externally based proofs as a form of reasoning that relied on, for instance, the appeal to authority. For example, suppose one claims, "Math is hard" and they justify this claim by stating, "Because Mike said math is hard." This justification is a direct appeal to the authority of Mike but, beyond that appeal, does not provide support for the original claim. The second and final form of justification identified in the VLs' work was *substantive justifications*. *Substantive justifications* are evidenced through the inclusion of an example, a fact, or deductive/inductive reasoning as support for a claim. Continuing with their work on proof schemes, Sowder and Harel (1998) delineated two additional forms of proof schemes: (a) empirical and (b) analytical. Empirical proof schemes included the use of proof by example in validating a claim. Analytical proof schemes captured the use of axiomatic, or deductive, based reasoning and generalizations to support a claim. Both of these proof schemes are captured by the subtheme of *substantive justifications*. For example, returning to the claim of "Math is hard," a substantive justification could be "Because math requires abstract thinking." The following two subsections explore both forms of justifications identified in the data.

Theme 2a: Appeal to an Authority

The *appeal to an authority* as a form of justification was offered only by Scripted Pair 1. During their work on Square Task 1, they made an appeal to the authority of the talent. During their work on the Octagon Task, they appealed to the authority of the researcher.

Consider Scripted Pair 1's work on Square Task 1 (Figure 5.1). They claimed that the starting height of point *A* was 0 meters. Their justification for this claim came at the end of a lengthy back-and-forth between the VLs. Initially, I present an intermediary justification presented by Camila for her chosen starting height based on the presence of negative values. Then I present evidence that Camila's justification for this intermediary claim relied on an appeal to the authority of the talent.

After Camila completed her graph (Figure 5.18), Alex made the claim that her graph is wrong because "a graph doesn't always have to start at 0." In response, Camila attempted, but struggled, to justify her starting location:

Alex:	Would it be 0 right here [points to the point opposite point *A* on the left side of the square, Figure 5.18]?
Camila:	**That's also 0 because we have this [points at -5 on *y* axis]**. So the -5 would be down here [gestures from the midpoint on the left side of the square to the bottom left corner labeled -5 on Figure 5.18].

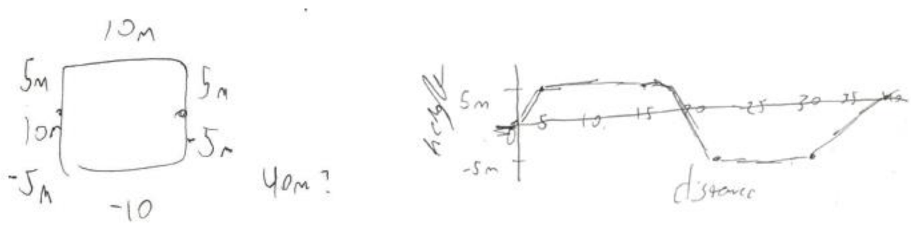


Figure 5.18
Scripted Pair 1's Annotated Square and Graph for Square Task 1

Within this exchange, Alex attempted to explore the implication of calling the starting height of point *A* 0 meters. In response, Camila claimed that "[the height is] 0 because we have [negative values]." Here, Camila has suggested that the presence of the negatives is a justification for the starting height being 0. In response, the researcher asked Scripted Pair 1 a clarifying question about what the VLs thought was positive and what was negative. In response, Camila clarified what is negative and appealed to the authority of the videos as justification for her negative values:

Researcher:	What's negative and what's positive here?
Camila:	The negative is the down and the positive is the up.
Alex:	Below this line [points to midline through square, Figure 5.19] it's negative.
Researcher:	What's negative?

Camila: The number. The height. **Because he [the talent] said it was**…

In clarifying what the VLs were discussing, both Alex and Camila indicated the bottom half of the square represented negative values. Further, Camila clarified that the negative quantity was the height. She then justified this claim by appealing to the authority of the talent when she said, "[the talent] said it was." In sum, Camila first justified her claim that the starting height of point *A* is 0 meters by appealing to the presence of negative values on her square. Then, she justified her negative values by appealing to the authority of the talent. Thus, Camila's justification for her claim that the starting height of point *A* is 0 meters was an appeal to the authority of the talent.

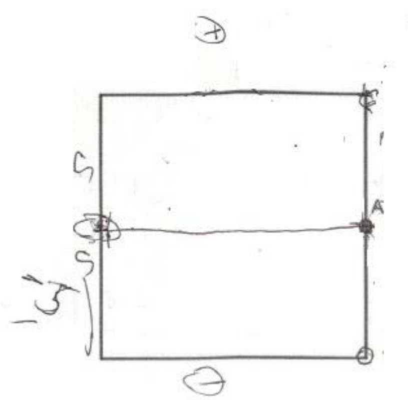


Figure 5.19
Scripted Pair 1's Annotated Square, with a Bisecting Line Drawn Across the Center

As a second example, while working on the Octagon Task (Figure 5.2), Alex presented another justification that relied on an *appeal to an authority*, this time the authority of the researcher. Specifically, Alex provided this justification for the claim that he needed to include points labeled "equal height" on his graph for the Octagon Task (see Figure 5.4b). During their work, Scripted Pair 1 separately created their own graphs (Figure 5.4). While they reflected on their graphs, one difference that stood out to Camila between their graphs was Alex's inclusion of points labeled as "equal height." When pressed by Camila about what this meant and why he included these labels, Alex's justification relied on an appeal to the researcher:

Camila: I'm confused [points to line labeled "equal height" on Alex's graph, Figure 5.4].

Alex: It is just to show, you see how you have these two lines [draws two horizontal line segments on Camila's graph, Figure 5.20]? These are equal, like in height. That's how mine is. **I just wanted to point it out before it was asked.**

Camila: What's the significance of pointing it out?

Alex: Because **he was gonna ask me.**

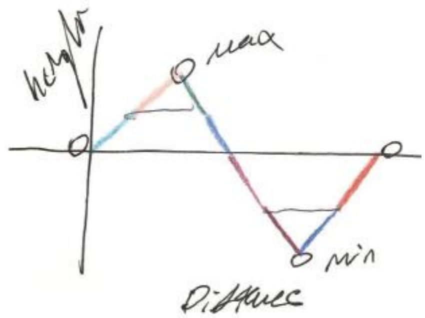


Figure 5.20
Camila's Graph, With Alex's Added Horizontal Lines

When pressed by Camila, Alex again clarified what his lines identified (i.e., points of equal height) and was able to produce similar lines on Camila's graph (Figure 5.20). Alex then justified his inclusion of these lines by stating he thought the researcher "was gonna ask." This is an *appeal to an authority*. Instead of justifying why these points are mathematically significant, Alex justified his graphing as pre-empting a possible line of questioning from the researcher.

Theme 2b: Substantive Justification

A substantive justification is one that includes a support for a claim ranging from the use of an example, a fact, or even deductive reasoning. This final form of justification captures the VLs' attempts to create a justification that leverages mathematical facts or concepts in support of their claims. Importantly, the VLs' *substantive justifications* were not always mathematically correct, and their justifications may not have validity in all mathematical contexts. Instead, these justifications captured the VLs' attempts to create arguments centered on their current mathematical understandings.

For example, Unscripted Pair 1 leveraged a substantive justification in support of the claim that the starting height for point *A* on Square Task 1 is arbitrary and that it is acceptable to have negative heights. While working on Square Task 1, Sarah and Osiris produced a graph (Figure 5.10) that differed from the graph produced by the talent in the video (Figure 5.11). Initially, the VLs struggled with the difference between the graphs and argued that the talent were wrong because Unscripted Pair 1 believed it was incorrect to claim the starting height of point *A* was 0 meters. Further, Sarah claimed, "I don't think you can really go negative," suggesting she did not believe you can have negative heights. Osiris disagreed with Sarah and was able to construct a substantive justification for his idea that the talent's starting height of 0 meters is valid and the presence of negative values is valid:

> Osiris: Actually, I think there's no actual reason, I don't think it matters that much. I don't think it matters at all. **If you were to assume this [points to the starting point on Square Task 1] was 0**, imagine this was 0. **This would be 0 on this side too** [points to midpoint on left side of Square Task 1]. So if they go up 5 [sweeps from the starting point to top right corner], go here 10 [sweeps across top of the square] go down 5 [points to the midpoint on the left side the square] it would be 0 still. Go down -5, **which is not wrong if they're measuring by that [points to starting point] being 0**. Because **there is no exact point that says point *A* has a starting point of 5** [points at task statement].

Osiris claimed that the starting height of point *A* in Square Task 1 was arbitrary, and that calling the starting height 0 implies there will be negative heights. Osiris substantively justified his first claim by pointing out that the problem statement did not provide an initial "starting point of 5" or 0. In fact, Osiris pointed out that this is just an assumption. Finally, he justified the presence of negative heights by describing the point's journey around the square, based on the assumed starting height of 0 meters.

As a second example, Scripted Pair 1 engaged in a series of *substantive justifications* surrounding a disagreement that took place while they worked on the Octagon Task. Specifically, Scripted Pair 1 provided a substantive justification for the claim that the two graphs that Camila created are different (i.e., Figure 5.4a and Figure 5.5 are different). As illustrated previously, Alex and Camila initially created separate graphs that they determined represented the same relationship (Figure 5.4). Explicitly, Camila stated, “We have the same graph.” The VLs then turned to the videos, where an alternative graph was presented by the talent. As a result, Camila changed her mind and created a new graph (for more details, see Theme 1c). Alex challenged Camila and suggested both graphs (Figure 5.4a and Figure 5.5) are the same. In response, Camila justified her claim that her second graph (Figure 5.5) was different from her first (Figure 5.4a) by appealing to the change in the change in height:

> Camila: Do you think the graphs mean the same thing?
>
> Alex: Yeah.
>
> Camila: I don’t… I think that this graph [points to Figure 5.4a] states that all the lines are the same, they have the same height in the end, when that’s not true. This one [points to Side 1 on the octagon, see Figure 5.21] **takes more of a height** because it would be height which is ***y* over distance *x***. So bigger height. And this one [points to Side 2 of the octagon, see Figure 5.21] is a shorter height. And vice versa [points to Side 3 and Side 4, see Figure 5.21].

Explicitly, Camila made the claim that her two graphs were different. Her justification centered on the idea that the change in height of a point traveling over Side 1 of the octagon (see Figure 5.21) is different from the change in height of a point traveling over Side 2 of the octagon. As she phrased it, “[Side 1] takes more of a height [than Side 2].” This was further evidenced when Camila referenced the slope or “[height] over distance” of her graph. Although Camila’s justification is mathematically imprecise (e.g., her references to height and distance should be references to *change* in height and *change* in distance), she attempted to support her claim in a substantive way. Importantly, for her problem-solving behavior, this justification attempted to leverage the covariational reasoning behind the shape in her final graph.

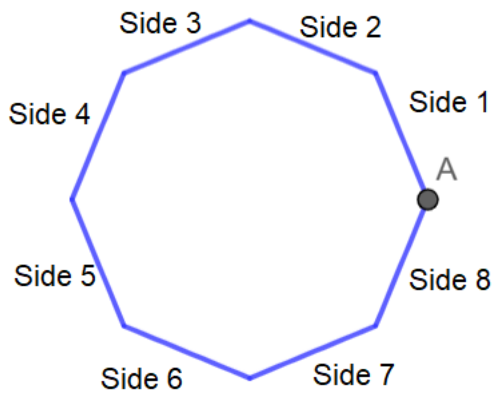


Figure 5.21
Labeled Octagon

Summary

Justifications are an important part of problem solving, and the different ways in which the VLs attempted to create justifications for their claims have implications on their mathematical understandings of those claims. When Unscripted Pair 1 engaged in justification, it was substantive. This means that their claims were supported by examples, facts, or deductive reasoning rooted in their mathematical understanding. This was evidenced by Osiris's ability to reason covariationally about both his graph and the talent's graph. Although Scripted Pair 1 also produced a substantive justification, it was only evidenced during the Octagon Task, and they also produced justifications rooted in an *appeal to an authority*. This latter type of justification is not grounded in mathematical understanding of covariational reasoning. This was evidenced by Camila's struggles to defend her graph and her appeal to the authority of the video.

Theme 3: Idea Management

The final theme, *idea management*, captures the ways in which the VLs made use of the ideas presented within the videos. Given the videos were positioned as a tool for the VLs to use in their problem solving, the ways in which the ideas expressed by the talent were attended to and put to use by the VLs was a central feature of their problem-solving behavior. Two subthemes of *idea management* emerged: (a) *repetition* and (b) *negotiation*.

The subtheme of *repetition* captures the ways in which the VLs reproduced an idea, solution, or other work from the videos without evidence of engaging substantively with the meaning of those ideas, solutions, or other works. This reproduction of ideas is a kind of mimicry, where solutions and ideas are repeated but the meaning behind those ideas is not explicitly presented and, when probed, cannot be produced. The subtheme of *negotiation*, on the other hand, captures the ways in which the VLs showed an engagement with the meaning expressed within the videos. This included the use of an idea, solution, or other work presented within the videos by the talent while engaging and extending the associated meaning of the idea, solution, or other work. Engagement and extension of an idea included clear interpretations of an idea, the creation of an argument against an idea, or applying an idea to a novel task. Importantly, what is being evidenced by the VLs' *negotiations* is an understanding of the meaning of an idea.

A critical reader may argue that *idea justification* is simply a form of *idea management*. While this argument can be made, the central difference between these themes was what was being justified or managed. When creating a justification, the VLs were engaged in a process of supporting their own ideas and their own meanings. When the VLs evidenced *idea management*, they were engaged with the ideas and, in the case of *negotiation*, the meaning from the videos.

The following section explores these two subthemes with characteristic examples. This section concludes with a summary exploring the differences between the pairs.

Theme 3a: Repetition

The subtheme of *repetition* captures how ideas presented within the videos were reproduced by the VLs without an apparent engagement with the associated meanings of the ideas. I present evidence that Scripted Pair 1 repeated entire solutions from the videos, whereas Unscripted Pair 1 just repeated specific inscriptions from the talent.

For example, Scripted Pair 1 repeated the talent's solution for Square Task 1. While working on Square Task 1 (Figure 5.1), Camila and Alex struggled to start the task and silently turned to the video (see Theme 1a). After the VLs watched the talent create their graph, Camila attempted to work on her own graph but struggled to make use of the information presented in the video (for more details about her struggle, see Theme 1d). When Camila struggled to start her graph she asked, "Can I go back?" and proceeded to rewind the video. With the video paused and the talent's work on screen (i.e., Figure 5.16 was visible), Camila started looking back and forth between the screen and her task. At the same time as this coordination, Camila started constructing a graph. This process culminated in the recreation of the talent's graph (Figure 5.17). Significantly, when asked by the researcher to explain her graph and her thinking, Camila stated, "This is my graph, and this is my thinking." Despite further probing, Camila had nothing further to add. Camila's response suggests a lack of understanding of her graph. In fact, as argued above (see Theme 2a) Camila lacked an understanding of the assumptions she was making in her solution. Furthermore, at no point was she able to provide an interpretation of what the talent's graph she reproduced meant. Camila's work was thus coded as *repetition* because her explanation lacked an explicit mention of what her graph meant and because she repeated the same graph as the talent.

Another form of *repetition* emerged for both pairs. During their work on Square Task 1 and the Octagon Task, Scripted Pair 1 and Unscripted Pair 1 evidenced the *repetition* of annotations. For example, both pairs replicated the talent's annotations of color coding their graph, but at no point did either pair provide an interpretation or leverage the color coding in any of their explanations. In both scripted and unscripted videos, the talent made regular use of color coding in their graphs to coordinate segments of their shape with their graphs (e.g., Figure 5.22) and leveraged the colors in their explanations. Notice how in both images, the first segment of the graph, starting at the origin, matches the color of the first segment of the shape, starting at point *A*. As a part of the talent's explanation of their initial graph for the Octagon Task in the unscripted videos, Zoe (the talent) explained, "We started off with the red segment. It's increasing in height and in distance… The red and the yellow segments are both increasing in height." Use of the colors became an important part of the problem-solving behavior of the talent, seeing use in the creation, explanation, and description of graphs in both scripted and unscripted videos.

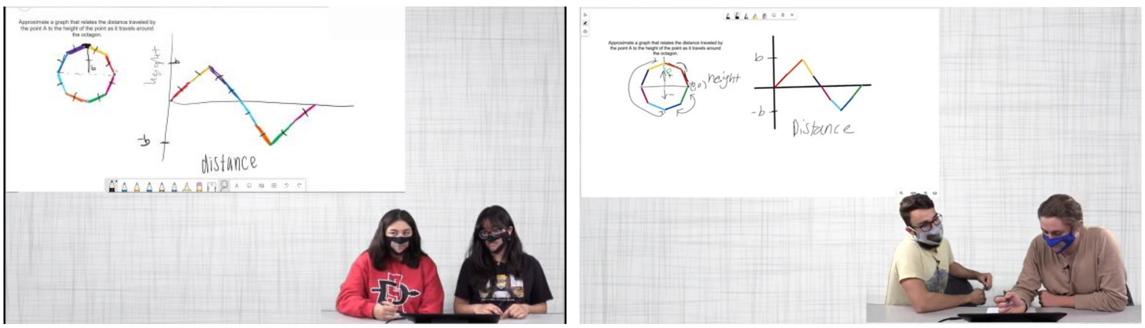


Figure 5.22
Color-Coded Octagon Graph from the Unscripted (Left) and Scripted (Right) Videos

When working on the Octagon Task, both Scripted Pair 1 and Unscripted Pair 1 replicated the use of color coding (Figure 5.23). Interestingly, they did not leverage the colors in any of their explanations. Despite both pairs utilizing the talent's annotations, color coding did not appear in the descriptions of their graphs. Thus, the VLs reproduced the annotations but evidenced none of the underlying meaning of the color coding, which resulted in this use of annotations being coded as *repetition*.

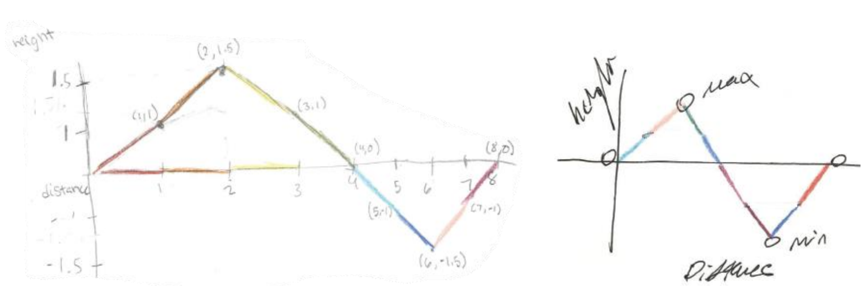


Figure 5.23
Color-Coded Octagon Graph from Unscripted Pair 1 (Left) and Scripted Pair 1 (Right)

Theme 3b: Negotiation

The subtheme of *negotiation* captures the attempts of the VLs to engage with and extend the meaning of the ideas, solutions, and other work presented by the talent within the videos. While both pairs engaged in a *negotiation*, Scripted Pair 1 only evidenced a *negotiation* on the Octagon Task and their *negotiation* process failed to produce a consensus. Unscripted Pair 1, on the other hand, regularly evidenced a *negotiation* with the talent's ideas.

The following section is organized by videos treatment. First, I present a case for the way Unscripted Pair 1 appeared to collaboratively negotiate with the talent's ideas. Then, I characterize the *negotiation* that emerged for Scripted Pair 1. Importantly, the characterization of each pair's *negotiation* process necessitates careful consideration of what happened in the videos. As such, each account presented includes both a description of what the VLs watched the talent do and what the VLs did.

Unscripted Pair 1. Sarah and Osiris readily entered a *negotiation* process with the ideas presented in the videos, as well as with their own ideas. For example, consider their work on Square Task 1. Recall that Sarah and Osiris initially struggled to comprehend the task statement and turned to the videos for ideas (see Theme 1a). In the second segment of the video that the VLs watched (i.e., the segment after the VLs watched a minute of video, paused the video, and then returned to it), the talent begin to describe some of the measurable features of their graph and begin annotating their square (Figure 5.24). In the video, the following exchange occurs:

Zoe:	If we're calling this the midpoint [highlights point *A* on Square Task 1], then this is 5 [labels left and right side of square 5, see figure 5.24] and this is 10 [labels top of the square 10]. The distance from the starting point to the corner is 5, across the top is 10, and down to a horizontal bisecting line is 5.
Gisele:	If you do counterclockwise once, **the total distance would be 40**.
Zoe:	It would be 20 because we only moved halfway.
Gisele:	So, it would go 20 up and then 20 down? Right? Our distance would be 40, but the height goes 20.

Zoe: The **height would only change 5** though. It would go up 5. **You're traveling 5 meters and you're moving up 5 meters. Then you stay at a constant height**…

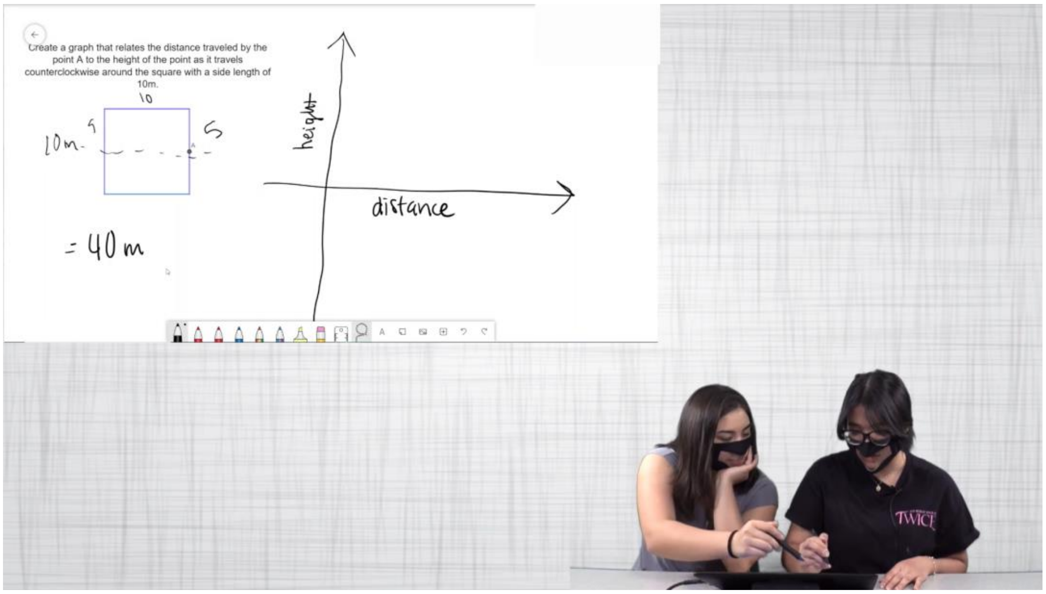


Figure 5.24
The Talent's Annotations in the Unscripted Videos on Square Task 1

Before Zoe continues coordinating the change in height with the change in distance, the VLs paused the video again. Up to this point in the video, the talent have expressed two key concepts that will help them construct their graph: (a) there are limits on both the height and distance of 5 and 40 meters, respectively, and (b) each segment of the square can be used to coordinate the change in height with the change in distance.

After pausing the video, the VLs entered a *negotiation* process. Initially, Sarah and Osiris leveraged the ideas within the video to construct a linear graph:

Sarah: So, kinda **what they're saying** is that's it's 5, 10, this is kinda 5 again [in apparent reference to the first three segments of the square]. If we're going all the way around [does circle gesture over the square], **this would be 5** [points to the upper right corner of the square] 15 [points to the upper left corner of the square] 25 [points to the bottom left corner of the square]… oh yeah so **it would still be 40** [draws an *x*- and *y*- axis].

Osiris: Umm… yeah. How would you graph that though? **Would it be linear?** If it's just a constant motion, it would still be going up one. You know what I mean?

Sarah: Yeah. So… **should we make the 5 meters the *x* or the *y*?** I think we should make it the *y* [marks 5m on her *y*-axis]. It's going to have to reach 40 at some point [marks 40m on the *x*-axis and draws a linear graph starting at *y* = 5 and ending at *x* = 40, Figure 5.25]…

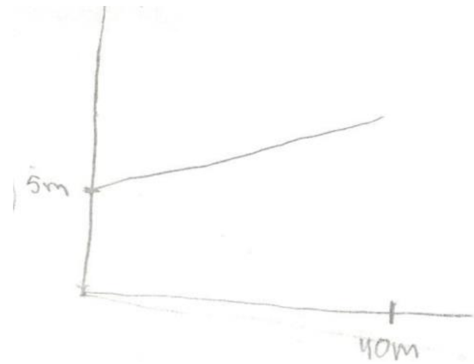


Figure 5.25
Unscripted Pair 1's Linear Graph for Square Task 1

This segment was coded as *negotiation* because of the VLs' use and extension of the talent's ideas. Specifically, Sarah first offered an interpretation of what the talent said, demonstrating engagement with the talent's idea. This included a description of three distances that the talent identified on their square 5, 10, and 5 m (Figure 5.24). Extending the talent's thinking, Sarah then counted off values at the corners of the square until she convinced herself of the talent's claim that the total distance was 40 m. Finally, with the goal of creating a graph, Osiris asked what all of this meant for their graph and posited the graph may be linear. Sarah then attempted to combine all this information. She used the "5 meter" idea to give herself a starting point, the talent's "total distance of 40" idea for her endpoint, and Osiris's linear idea for a general shape. Despite the talent only verbally describing changing quantities in the context of the task, Unscripted Pair 1's *negotiation* resulted in a graph (Figure 5.25).

The VLs then refined their graph by continuing their *negotiation* process with the talent's ideas. Specifically, Sarah expressed hesitation and suggested their linear graph was not accounting for the height of the point. In the following excerpt, read for Sarah's hesitation and her inclusion of the second quantity, height:

Sarah: Wait, our graph would be kinda weird. If we were kind of thinking of it without the height, we can say we start at 5 and we end at 40 [gestures around the square and points to the endpoint of her graph]…

Sarah suggested that their linear graph would be correct if they were “thinking of it without the height.” To incorporate height into their graph, Sarah expanded upon their thinking, made use of and extended the language presented by the talent, and created a new graph (Figure 5.26):

Sarah: If we’re talking about heightwise [moves previous graph aside and draws a new axis] … 0… 5 [labels point (0,5), Figure 5.26a]. So, this would be our starting point. If we’re talking about height, because this is going up [points to the segment of the square between point *A* and the top right corner]. This is staying constant [points to the top side of the square]. This is going down [points to the left side of the square]. This is staying constant [points to the bottom segment of the square]. This is going up [points to the segment of the square between the bottom right corner and point *A*]. So, we go up by 5, we get to 10 [sweeps from point *A* to top right corner]. So**, this would be 5 meters traveled, but at the height of 10 meters** [draws and labels a line segment from (0,5) to (5,10), Figure 5.26b]. Then, **we’re staying constant, but the distance is still increasing** [continues graphing] … then the height is going down by 10, so say it goes all the way down. Then this would be 25 meters [labels x-axis with 25m]. Then, it’s staying constant. This is 35 and then it’s going up by 5 again.

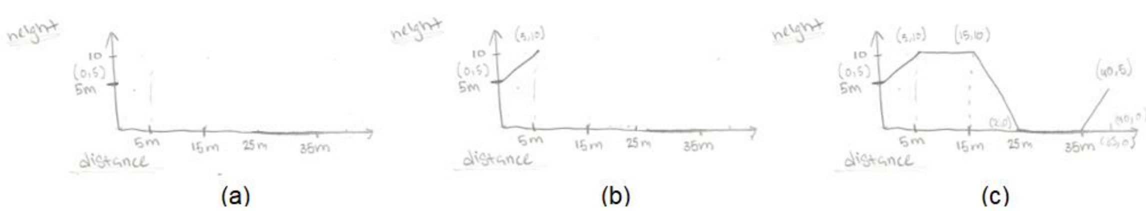


Figure 5.26
Sarah’s Graphing Progress, Culminating in her Graph for Square Task 1

A continuation of the VLs’ *negotiation* was identified in this exchange. Foremost, Sarah engaged with and expanded upon Osiris’s initial graph shape. Sarah positioned her thinking as taking Osiris’s graph and adding the variable of height. This necessitated a new graph. To that end, Sarah described the point’s journey around the square using language that echoed the talent’s language. Recall that the talent began describing this journey moments before the VLs paused the video. In the video, Zoe said, “You’re traveling 5 meters and you’re moving up 5 meters. Then you stay at a constant height." Similarly, Sarah described the point's journey as having "traveled" 5 meters to a new height, followed by a segment of "constant" height. Beyond *repetition* of what the talent had said, Sarah took the talent's coordination of change in the height and change in the distance over the first segment of the square and extended it when she acknowledged "distance is still increasing" over the period of constant height. All this *negotiation* resulted in a solution that the talent had not yet been able to produce, a final graph for Square Task 1 (Figure 5.26c).

The lengthy *negotiation* process that Unscripted Pair 1 evidenced for Square Task 1 was a typical occurrence for the pair. In addition to this *negotiation* process, Sarah and Osiris evidenced six separate *negotiations* on their continued work on Square Task 1 and the Octagon Task.

Scripted Pair 1. Scripted Pair 1 evidenced one instance of *negotiation* with ideas from the videos, but their *negotiation* process was only present for Camila. Alex, on the other hand, disagreed with the ideas from the videos but did not engage in a *negotiation* process with those ideas. In the following, I begin by recapping Scripted Pair 1's journey through the Octagon Task. Then I will describe Camila's *negotiation* with the talent's work from the video. Finally, I conclude by describing the lack of *negotiation* present with Alex's engagement with the ideas from the video.

As described in Patterns of Use: Information Seeking (i.e., Theme 1c), while Scripted Pair 1 watched the talent explore the Octagon Task, Camila and Alex separately started creating graphs (Figure 5.27). After the first video ended, Alex started to describe his graph, which prompted Camila to recreate her graph while Alex spoke (Figure 5.4a). After a brief exchange about the difference between their graphs (for more details, see Theme 2a), the pair agreed with each other's graphs and felt they were saying the same thing.

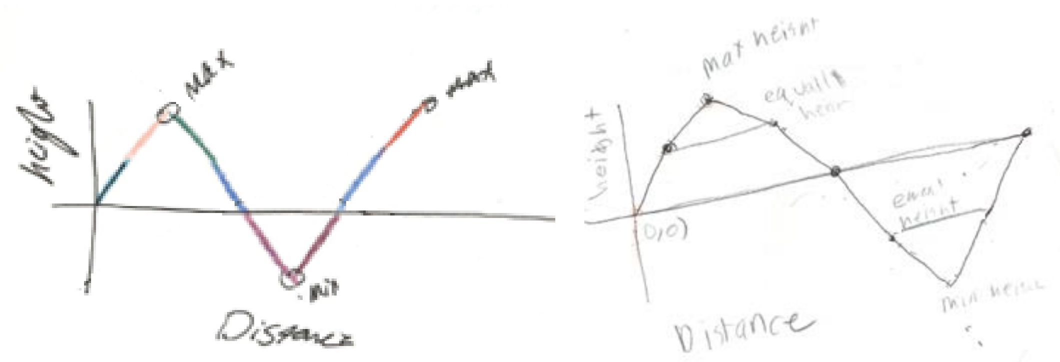


Figure 5.27
Camila's Initial Graph (Left) and Alex's Graph (Right) for the Octagon Task

Scripted Pair 1 then turned to the next video, where the talent begins creating a graph for the Octagon Task. During this video, the talent in the scripted videos, Adrian, creates a graph that closely aligns with the VLs' work (Figure 5.11). Because the talent's graph fails to account for the changing change in heights, the teacher and Adrian work to refine the graph (Figure 5.28):

Teacher: Throughout your graph we have straight lines throughout it. Now, specifically looking at the red and the orange segments. Is there anything different between these segments? Could you describe for me what is happening for the point that is traveling through red into orange?

Adrian: So, when the **point travels through red it is a little bit steeper of a slope**. Actually, a **good amount steeper**. And then here [points to the orange segment of the octagon], it's a lot less steep of a slope. So, **our change in height is actually not the same between these two lines**.

Teacher: So, you said, these two segments [highlights red and orange segments on the graph] it looks like they are showing the same slope on your graph, but you're saying the slope is different. Do you know what the shape should look like for this graph?

Adrian: Yeah. It would essentially just **look like our octagon**. Except it would kind of have this part be flipped.

Teacher: Kind of cut and then reflected?

Figure 5.28
Adrian's Refined Graph for the Octagon Task

While the VLs watched this segment of the video, Camila produced a new graph that aligned with the talent's description and work (Figure 5.5; see Theme 1c for more details about Camila's creation of this graph). At the conclusion of the video, Alex stated he disagreed with Camila's new graph and concluded, "It's [Camila's two graphs, Figure 5.5 and Figure 5.4a] essentially the same thing. It's just one is more complicated." This demonstrated a lack of *negotiation* with the ideas from the videos. Instead of exploring the differences between the graphs that made one more complicated than the other, Alex concluded that the two graphs were close enough and that he preferred the less "complicated" version.

Alex and Camila's disagreement continued when Camila evidenced a *negotiation* with ideas from the video. Camila negotiated with the idea presented by the talent of changing change in height by extending the reasoning to a simplified case. To do this, Camila drew two line segments of equal length, one vertical and one at an angle (Figure 5.29). While gesturing towards her line segments, she argued, "If I say that this line travels the same height as that line, that's wrong." Camila's idea here echoes the talent's observation in the video that the red segment of the octagon is "a good amount steeper" than the orange segment. Adrian (the talent) used this idea to claim that "change in height is actually not the same" over these two portions of the graph. Similarly, Camila drew two line segments with varying steepness and argued the lines do not "travel the same height."

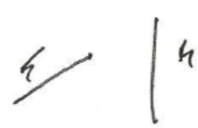


Figure 5.29
Camila's Two Line Segments

In response to Camila's idea, Alex remained steadfast in his disagreement. In the following exchange read for the ways in which Alex did not attend to the ideas or the meaning behind the ideas Camila presented:

Alex: I guess… I disagree completely. Even though it's more slant [makes a roof shape with his hands] it doesn't change the **length** of the actual sides.
Camila: [interrupts Alex] Length would be distance not **height**.
Alex: Because it's a regular octagon…
Camila: That is distance not height.
Alex: That doesn't change the fact that you have a max height and a min height.
Camila: No. you have a max and a min. of course. But **the height is different**.
Alex: **I see no difference**… They're both good enough because they both represent the same idea. They both go back to the same representation. This one [points to Camila's previous graph, Figure 5.4a] is just a lot neater than this one [points to her new graph, Figure 5.5].

This exchange was coded as an apparent lack of *negotiation*. Specifically, Alex appeared to not engage with the meaning of Camila's lines, which I argued above was a result of her own *negotiation* process. In an attempt to argue against the graph and the idea that used two lines reaching different heights (Figure 5.29), Alex attended to another measurable feature of the lines, their length. Camila then attempted to draw his attention back to the height. Instead of engaging with Camila's point, Alex then changed the discussion to maximum and minimum heights. While Alex did attend to another important feature of their graphs (i.e., distance), this feature was unrelated to Camila's point. Once again, Camila attempted to reiterate the idea of "[changes in] height is different," but Alex was unmoved and still believed that the only difference between the graphs was in their neatness.

Eventually, Alex and Camila explicitly "agreed to disagree" and turned to the final video for the Octagon Task. Within this video, the talent makes a final argument for his new graph and carefully describes his graph in terms of segments of fixed changes in distance and changing changes in height. After viewing this segment, Alex claimed he was now convinced by the talent that the new graph was accurate. Despite this claim, Alex appeared to still struggle to describe how the graphs were different:

Alex: After he [the talent] explained it I could see it now… I didn't understand what she [Camila] was saying about the heights and stuff. How this [Side 1 of the octagon, Figure 5.21] is different.

	Until he [points to computer] explained it. Then I was like, okay yeah that makes a lot more sense. **How the distance, well the height** of this [points to the vertex between Side 1 and Side 2 of the octagon] is not the same as the height of this [points to the vertex between side 2 and side 3]. So, it changes. **So, overall, the distance is the same but it's not at the same time, because of the height**… Does that make sense? Did I explain what they [the talent] explained correctly?
Researcher:	What do you think, Camila? Is there anything you would add to that?
Camila:	I think you get it, but your explanation is a little confusing.
Alex:	Yeah.
Camila:	But, yeah. I think that this graph [Figure 5.5] is correct.

Within this exchange, Alex implied he struggled to negotiate with Camila's ideas, but that after he watched the video, he reported that he now understood. Interestingly, as he attempted to engage with the ideas from the videos and the meaning of the graph, he continued to struggle in his differentiation between height and distance. This was most clear when he claimed that "distance is the same but it's not at the same time, because of the height." Additionally, Alex's interpretation fails to mention slope or the change in height. While he accurately described the vertices of the octagon as having different heights, the salient idea from the video is that the change in height between vertices is changing.

Summary

The theme of *idea management* identified contrasting ways the two pairs of VLs made use of ideas presented within the videos in their problem solving. While Unscripted Pair 1 appeared to engage in *idea management* as a collaborative problem-solving behavior that produced solutions with mathematical meaning, Scripted Pair 1 evidenced problem-solving behaviors focused on getting the solutions through their *idea management*.

As described above, Unscripted Pair 1 evidenced a *negotiation* process that became integrated into their collective problem-solving behaviors and that culminated in a solution that made sense to them, regardless of what the video had or had not done yet. This was clear in their joint *negotiation*. As Sarah and Osiris watched the videos, they both attempted to negotiate with the ideas presented by the talent. While working on Square Task 1, Osiris extended the thinking of the talent by conjecturing a shape of the graph. Sarah then continued their problem solving by extending the thinking of the talent by constructing a graph and then refining the graph. Significantly, the talent had not yet produced a graph. Thus, Sarah and Osiris's problem-solving behaviors were not about validating their solutions with the solutions presented by the talent. Instead, Unscripted Pair 1's problem-solving behaviors included *negotiation* of the talent's ideas and culminated in a solution for which the VLs had produced meaning.

In contrast, Scripted Pair 1 appeared focused on getting the correct solution; engaging with and extending the meaning behind the solution was rarely present. Foremost, this was evidenced in Scripted Pair 1's *repetition* of the solution for Square Task 1. Their *repetition* of the graph of Square Task 1 from the video and their lack of explanation for the graph suggests that their problem-solving behavior was focused on finding an answer, and the meaning behind that answer seemed immaterial. The importance placed on the final answer in Scripted Pair 1's problem solving was also evidenced in their lone instance of *negotiation*. During Scripted Pair 1's work on the Octagon Task, only one of the VLs appeared to engage with the meaning expressed in the video. The other VL, Alex, continually disagreed but struggled with the meaning expressed in the video. This struggle persisted through watching subsequent videos and was evidenced in his claim that he understood his mistake. Instead of helping Alex and engaging with his ideas, Camila acknowledged that Alex was still confused but appeared satisfied that he now agreed with her solution. Taken together, Alex's struggle to articulate the meaning of the solution and Camila's complacency with Alex's acknowledgment that the graph was correct, suggests that the primary goal of Scripted Pair 1's problem-solving behaviors was to find the solution, even when negotiating.

Discussion

The goal of this chapter has been to answer the following research question:

> **Research Question**: How do the problem-solving behaviors of vicarious learners engaged in learning to reason covariationally about the sine function differ when

> the dialogic videos that they are viewing and interacting with are scripted versus unscripted?

In answering this question, thematic analysis was used to identify themes in the VLs' uses of the videos to complete Square Task 1 and the Octagon Task. Each of the three identified themes (patterns of video use, *idea justification*, and *idea management*) indicated important features of the VLs' problem-solving behaviors and differences between the treatments' problem-solving behaviors. In the following section, I reflect on the results presented in this chapter, shedding light on a difference in the role that the dialogic videos played in relation to the VLs' apparent goals in problem solving, and I conclude by returning to my hypotheses exploring why these differences are a product of the nature of the dialogue within the videos.

Conclusions

Unscripted Pair 1's problem-solving behaviors appeared to be driven by the goal of *constructing* a solution. Scripted Pair 1's problem solving, on the other hand, appeared to be driven by the goal of *having* a solution. Metaphorically, this difference can be described as enjoying the journey versus enjoying the destination. Unscripted Pair 1 carefully created their solutions and took the time to understand those solutions, which indicated their goal was to construct a solution. Scripted Pair 1 was focused on the solution, and they did not mind how they came to it, which indicated their goal was to have a solution.

Unscripted Pair 1's process of constructing a solution was evidenced in their problem-solving behaviors of using the videos to gather ideas about how to start the tasks and to gain insights when stuck. During their work on both Square Task 1 and the Octagon Task, Unscripted Pair 1 expressed confusion about their tasks and then started the videos. This turn to the videos suggested they saw the videos as a resource. Then, Sarah and Osiris would watch the videos until they felt comfortable working on the novel tasks, as indicated by their pausing the videos and subsequent work on the tasks. Significantly, this pair would pause the videos before the talent concluded their work. Thus, when the videos were paused, Sarah and

Osiris had to negotiate with the ideas, and the meaning of those ideas, presented within the videos and by their partner. Finally, Unscripted Pair 1's process of constructing a solution was evidenced in their problem-solving behavior of creating *substantive justifications* and their lack of an *appeal to an authority*. Through the creation of *substantive justifications*, Sarah and Osiris demonstrated an understanding of their work. Beyond constructing a solution that was true because the videos said it was (i.e., an *appeal to an authority*), the solutions that Sarah and Osiris found could be supported mathematically.

Scripted Pair 1's problem-solving behaviors suggest that they were driven by a different goal than Unscripted Pair 1. Namely, to have a solution rather than to construct a solution. Camila and Alex indicated two related problem-solving behaviors in their work on Square Task 1 and the Octagon Task: they used the videos as a source of answers and to settle their debates. Using the videos as a source of answers was indicated across their *patterns of use*, the justifications they created, and the way they managed ideas from the videos. Within their *patterns of use*, Camila and Alex celebrated being vindicated by the videos and got through the videos as fast as possible (as indicated by their adjusted play speed and their lack of pausing). These *patterns of use* demonstrated a lack of engagement with the meaning being developed with the videos. This was further evidenced in the pair's justification through an *appeal to an authority*. This form of justification showed Camila and Alex had a lack of understanding of the solutions they had produced. Lastly, during their *idea management*, the *repetition* of solutions indicated the videos were seen as a source of answers, and that the meaning behind these answers was less important.

The use of the videos to settle their debates also suggested that Scripted Pair 1 was primarily interested in having a solution. Through their *patterns of use*, Scripted Pair 1 explicitly turned to the videos to settle their debates and vocally celebrated when the videos vindicated their solutions. Similarly, the pair's appeal to the authority of the videos as a form of justification suggests the videos and the ideas presented within them were above reproach. If the videos concluded something, then the VLs took it as truth. This was further evidenced in the conclusion of Scripted Pair 1's *negotiation* process. Despite a lengthy disagreement, where Camila evidenced *negotiation* with the talent's ideas, Alex could not be convinced until he watched the video. Even after watching the video and Alex claiming he understood why Camila was correct, he struggled to articulate why Camila's solution was accurate. Instead of continuing the conversation and pushing her partner to a deeper understanding, Camila appeared satisfied that Alex agreed with her, that the debate was settled, and that they had a solution.

Revisiting my Hypotheses

In conclusion, I return to my hypotheses for differences between my treatments' problem-solving behaviors in light of these findings. This section begins with a consideration of two possible moderating variables and concludes with two hypotheses.

Foremost, it is possible that the differences observed between Scripted Pair 1 and Unscripted Pair 1's problem-solving behaviors could be due to differences in the personalities and past mathematical experiences of the pairs. Schoenfeld (2016) identified four categories that can influence one's problem-solving behaviors: (a) knowledge, (b) heuristics, (c) metacognition, and (d) beliefs. As such, it is possible that the individual personalities of the VLs' influenced their metacognition and their beliefs, and it is inevitable that the VLs' past mathematical experience influenced the knowledge and heuristics they had upon entering this study.

While I acknowledge these unavoidable differences between the VLs, I believe a plausible explanation for the differences observed between the pair's problem-solving behaviors is in their experiencing the videos as containing *authoritative voices* versus *internally persuasive voices*. Bakhtin (1981) describes these contrasting ideas as the way in which a word or idea expressed by another person is experienced by the listener. If that word or idea must be attended to, accepted, and not negotiated with, then that word is a part of an authoritative voice (Bakhtin, 1981). An internally persuasive voice, on the other hand, is open for negotiation. It can be argued with and reasoned against. According to Morson (2004), the prototypical example of an authoritative voice in education is that of a teacher who cannot be questioned and provides justifications for formulas like “because some dead guy said so” (p. 319). In contrast, a teacher whose goal is to foster internally persuasive voices in a mathematics classroom would likely encourage students to share their thinking and critique the reasoning of others. For Morson (2004), fostering internally persuasive voices in educational settings is about teaching students how to learn, as opposed to teaching students what to learn. When students learn how to learn they are prepared to be better future students and future citizens.

I argue that the differences between Scripted Pair 1 and Unscripted Pair 1’s problem-solving behaviors are a product of their experiences of the voices within the videos as either authoritative or internally persuasive. Scripted Pair 1 appeared to experience the scripted dialogue of their videos as containing primarily authoritative voices, and Unscripted Pair 1 experienced the authentic and unscripted dialogue of their videos as containing internally persuasive voices. This was evidenced primarily by Scripted Pair 1’s use of the videos to have a solution. As I have argued, this pair saw the videos as having the final say in who was right, they used the videos as a source of answers for *repetition*, and they showed little *negotiation* with the ideas from the videos. This positions the unscripted videos as akin to the unimpeachable teacher teaching students what to learn. On the other hand, Unscripted Pair 1 frequently negotiated with the videos. This was evidenced in their *patterns of use*, which showed a willingness to pause and verbalize disagreements. This was also evidenced in their *idea management*, where the pair frequently evidenced *negotiation* with the ideas from the videos.

I believe the difference between Unscripted Pair 1’s experience of internally persuasive voices and Scripted Pair 1’s experience of authoritative voices is a product of the negotiation within their respective video types. Recall, a purported benefit of vicarious learning is the opportunity to model one’s behavior after the direct participants of a dialogue (Braaksma et al., 2004; Craig et al., 2000; Chi et al., 2017; Kuhn & Modrek, 2021; Mayes et al., 2002; Rummel & Spada 2005; Schunk et al., 1989; Twyford & Craig, 2017). As such, it is possible that the VLs were modeling their negotiation, or lack thereof, from their respective videos.

There are two features of the negotiations within the dialogue that differ across the video types: (a) the presence of pushback and (b) the authenticity of the dialogue's negotiation. Foremost, the unscripted dialogue contained negotiation in the form of pushback. As the talent engaged in unscripted dialogue, their resolutions were a product of a constant exchange of ideas where the talent frequently disagreed and argued against one another. Thus, the VLs viewing the unscripted videos were able to model their behavior off the talent's negotiation processes as evidenced by their problem-solving behaviors. In contrast, the talent enacting the scripted dialogue engaged with and made use of new ideas from the teacher, but this process did not contain any kind of pushback on the new ideas. Again, the talent's behaviors appeared to serve as a model for the VLs. In the same way as the talent in the scripted video did not push back against the ideas from the teacher, the VLs viewing the scripted videos did not push back on the ideas from the videos.

Furthermore, the negotiation between the talent in the unscripted videos was authentic, and the negotiation between the talent and the teacher in the scripted videos was not. The authentic interactions between the talent in the unscripted videos resulted in a set of videos that captured students' experiencing internally persuasive voices. In other words, the talent's co-constructive interactions in the unscripted videos suggested they were frequently engaged in a negotiation process and were likely experiencing one another's words and ideas as open for negotiation (i.e., as internally persuasive). The inauthentic dialogue in the scripted videos, on the other hand, contained authoritative voices. After all, the talent in the scripted videos did not negotiate with the script and instead reproduced the words and ideas he was instructed to produce. This suggests the unscripted dialogic videos contained the talent's experiences of internally persuasive voices and the scripted dialogic videos contained the talent's experiences of authoritative voices. If this difference was perceptible, then this difference could explain the differences in the VLs' experiences of internally persuasive versus authoritative voices.

Another reason for the hypothesized difference between the VLs' experiences with the videos was that the nature of the dialogue would be perceptible to the VLs. If, as I had hypothesized, the scripted mistakes were perceptibly inauthentic to the VLs, then I expected the problem-solving behaviors of the VLs viewing the scripted videos to involve less engagement with the ideas from the videos. The logic of this conclusion was that if the VLs sense that an idea is not truly believed and is perceived as inauthentic, then the VLs have little incentive to engage with the faux idea. Instead of considering and debating the validity of every idea presented by the talent in the scripted videos, the VLs could simply sit back and wait until a final solution was presented.

I believe this was borne out in the data. Firstly, this was evidenced by Scripted Pair 1's lack of engagement with faux ideas. As illustrated by their *patterns of use*, instead of pausing the videos and engaging with ideas as they emerged (as Unscripted Pair 1 did), Scripted Pair 1 watched videos from beginning to end at twice the playback speed. Using the videos in this way did not give the VLs time to consider and debate every idea presented by the talent. In fact, it barely gave them time to attend to the meaning of the solutions they did attend to (e.g., Camila's replicated solution for Square Task 1, see Theme 3a). Secondly, Scripted Pair 1 appeared to sit back and wait for the final solution. This was most prevalent in their use the videos to settle their debates. Instead of pausing the videos and considering different ideas in relation to their solutions, Scripted Pair 1 would watch the entire video and then determine, based on the final solution in the video, which one of them was correct. It is as if Scripted Pair 1 perceived that the dialogue of the videos they viewed was inauthentic, and this perception guided their use. They knew the videos would contain a correct solution at the end, so they attempted to get to the end as quickly as possible. They sensed when an idea was believed by the talent, so they only attended to those solutions.

Chapter 6: Conclusion

Within this dissertation study, two forms of dialogic instructional mathematics videos were created and shown to four pairs of students. The first set of dialogic videos contained authentic unscripted dialogue between two undergraduate students engaged in a sequence of tasks that emphasized covariational reasoning and culminated in the construction of a graph of the sine function. Based on the emergent struggles and alternative conceptions presented by the students in the unscripted videos, a second set of videos was created. Within this second set of videos, a student and a teacher enacted a scripted dialogue that included the same task sequence and presented the same struggles and alternative conceptions contained in the unscripted videos. Each form of video was shown to two pairs of students. The students viewing the videos are referred to as vicarious learners (VLs) because of their position as indirect participants in the dialogue of the videos. The VLs were paired and randomly assigned to view the unscripted or scripted videos over five research sessions. In the research sessions, the VLs engaged with and used the videos to work on their own set of tasks that built toward the construction of the sine function.

The overarching goal of this dissertation study is to contribute to the field by illuminating the differences between the VLs who viewed scripted versus unscripted dialogic instructional videos. In Chapter 4, I examined the orientations of VLs toward the students in the videos (i.e., the talent). In Chapter 5, I examined the problem-solving behaviors of the VLs as they used the videos to learn to reason covariationally and ultimately construct a graph of the sine function. These results are theoretically and methodologically significant. In this chapter, I review these results and elaborate on their significance and implications. Additionally, I present some limitations of this study. In conclusion, I will discuss areas of future research.

Summary of the Findings

Two research questions guided this study. The first question examined the orientation of VLs toward the talent and the emergent differences in those orientations based on the nature of the dialogue of the videos the VLs viewed. The second question examined the VLs' problem-solving behaviors as they engaged in learning to reason covariationally through the use of scripted versus unscripted dialogic instructional videos that culminated in the construction of the sine function. In this section, I present the results of each research question in turn.

Answering Research Question 1

In answering Research Question 1, two orientations toward the talent emerged: quasi-collaborative and distanced. In the following section, I begin by providing an overview of the context of this study, then I attend to the differences between the orientations, and I conclude with a comparison of shows of emotions between the treatments of this study.

Overview

Recall, the analysis of this question was performed on four pairs of VLs recruited from college algebra courses. Two pairs of VLs were assigned to view the unscripted dialogic videos (for convenience, these pairs are referred to as Unscripted Pair 1 and Unscripted Pair 2). Two pairs were also assigned to view the scripted dialogic videos (these pairs are referred to as Scripted Pair 1 and Scripted Pair 2). Each pair viewed videos and worked on tasks that accompanied the videos over five research sessions. During the first phase of analysis, orientations toward the talent were identified. During the second phase, shows of emotions during these identified orientations were analyzed and coded.

Orientations

Foremost, a difference emerged between the VLs viewing scripted and unscripted videos based on two kinds of orientation: quasi-collaborative and distanced. The VLs viewing the unscripted videos tended to express a quasi-collaborative orientation toward the talent. Building off the work of Lobato and Walker (2019), five categories emerged that constitute the orientation of quasi-collaboration. Together, these categories portray the VLs acting as if they were in a collaborative group with the talent. The VLs' categories of quasi-collaboration included *coordinating activity with the talent*, *acting as if the talent could engage with their work*, *understanding the talent's mathematical personalities*, *being in a community of learners*, and *responding directly to the talent*. The most prominent categories of a quasi-collaborative orientation for the VLs' viewing unscripted videos were *coordinating activity with the talent* and *responding directly to the talent*.

In contrast, the VLs viewing the scripted videos tended to express a distanced orientation toward the talent. Two categories that evidenced a distanced orientation were identified. These categories were *seeing the talent as a source of answers* and *evaluating the talent* and their solutions. These forms of a distanced orientation suggest that the VLs viewing the scripted videos were cognitively distanced through their positioning of the talent as experts to learn from and the VLs' position as passive observers of the dialogue and the talent.

Shows of Emotions

The data were also analyzed for indicators of the VLs exhibiting emotion in relation to their engagements that evidenced an orientation toward the talent. Emotions (such as delight or frustration) were coded following Evans et al.'s (2006) textual analysis. A difference was found between the shows of emotions of the VLs viewing unscripted versus scripted videos. Unscripted Pair 1 and Unscripted Pair 2 evidenced consistent shows of emotions across all forms of quasi-collaborative orientations, while Scripted Pair 2 evidenced no emotions and Scripted Pair 1 rarely evidenced a show of emotions. Furthermore, the VLs evidenced few shows of emotions across engagements that evidence a distanced orientation.

Another difference between the VLs viewing the unscripted versus the scripted videos was in the nature of their shows of emotions. The VLs viewing the unscripted videos evidenced emotions ranging from excitement to be on the same page with the talent to an appreciation for seeing the talent struggle and having the ideas from the talent to work from. In contrast, Scripted Pair 1's few shows of emotions (e.g., happiness) appeared in celebration of their work being confirmed by the talent.

Taken together, the consistency of emotions shown by Unscripted Pair 1 and Unscripted Pair 2 and the nature of their emotions suggest that their orientations were emotionally involved. In contrast, the VLs viewing the scripted videos appeared to be emotionally distanced from the talent.

Summary

In sum, the VLs viewing unscripted dialogic videos consistently evidenced an emotionally-involved quasi-collaborative orientation toward the talent. In contrast, the VLs viewing the scripted videos tended to evidence a cognitively- and emotionally-distanced orientation toward the talent which Mckendree et al. (1998) called epistemic detachment.

Answering Research Question 2

In answering Research Question 2, thematic analysis was used to identify problem-solving behaviors as VLs learned to reason covariationally through the use of scripted versus unscripted dialogic instructional videos that culminated in the construction of the sine function. One pair of VLs was selected from both video treatments (Unscripted Pair 1 and Scripted Pair 1). Their work on two important tasks (Square Task 1 and the Octagon Task) was identified for analysis of problem-solving behaviors. In total, three themes were identified: (a) *patterns of use*, (b) *idea justification*, and (c) *idea management*. In the following, an overview of each theme is presented, along with the emergent differences between the problem-solving behaviors of the VLs viewing and using scripted versus unscripted videos.

Patterns of Use

The theme of *patterns of use* centers on the VLs' problem-solving behaviors while they played or manipulated the videos. Yoon et al.'s (2022) four categories of patterns of use were adapted in the creation of four subthemes: (a) *browsing*, (b) *social interaction*, (c) *information seeking*, and (d) *environment configuration*. *Browsing* included playing the videos for Scripted Pair 1 and playing and pausing the videos for Unscripted Pair 1. Through an evolution in their playing, Scripted Pair 1 evidenced the use of the videos to settle their debates. Unscripted Pair

1, on the other hand, appeared to use the video, through playing and pausing, to gather information and insights into the tasks. The next subtheme, *social interaction* included comments between the VLs about the videos while the videos played. Scripted Pair 1 engaged in minimal *social interactions*, and their few comments expressed vindication upon having their solution confirmed by the videos. In contrast, Unscripted Pair 1 made comments celebrating their joint success, discussed when they had a disagreement with the talent, and acknowledged when they agreed with the talent. The final two subthemes, *information seeking* and *environment configuration*, were only evidenced by Scripted Pair 1. Through Scripted Pair 1's *information seeking*, they appeared to coordinate ideas and solutions from the videos with their own annotations and solutions (e.g., taking notes from the video and then reproducing the talent's solution). In Scripted Pair 1's *environment configuration* they manipulated the videos by rewinding and playing on an adjusted playback speed of times two.

In sum, there were important differences in the *patterns of use* of the videos between Scripted Pair 1 and Unscripted Pair 1. Specifically, Scripted Pair 1's played the videos to settle debates, they socially interacted to celebrate being vindicated, they information sought to coordinate solutions with the video, and they environment configured to get through the videos quickly and find final answers. In other words, Scripted Pair 1's problem-solving behaviors focused on seeking and validating their solutions through the use of the video. In contrast, Unscripted Pair 1's pausing and playing appeared to show how the videos were a source of information they could leverage in their work, and their *social interaction* evidenced problem solving with the ideas from the videos as a resource that could be right or wrong. In other words, Unscripted Pair 1's use of the videos indicated the videos were enhancing their problem solving.

Idea Justification

The second theme, *idea justification*, identified how the VLs engaged in the problem-solving behavior of providing support for their mathematical claims while working on novel tasks and engaging with the dialogic videos. Two subthemes were identified: (a) *appeal to an* *authority* and (b) *substantive justifications*. While both pairs engaged in *substantive justifications*, Scripted Pair 1 only did so on the Octagon Task and uniquely created justifications that relied on appeals to an authority. Through their use of *substantive justifications*, both pairs attempted to use an example, fact, or deductive reasoning grounded in covariational reasoning to support their ideas. Their use of *substantive justifications* evidenced a conceptual understanding of their claims. On the other hand, through appeals to an authority, Scripted Pair 1 provided support for their claims based on the external authority of the videos and the researcher. These appeals to an authority were not grounded in a mathematical understanding of covariational reasoning and evidenced a belief in the claim's validity rooted in an external authority.

Idea Management

The final theme, *idea management*, captured how ideas, solutions, and meanings expressed by the talent in the videos were used by the VLs. Two subthemes were identified: (a) *repetition* and (b) *negotiation*. The VLs evidenced *repetition* when they reproduced an idea or a solution from the talent, but not the meaning of the idea or solution. Both pairs engaged in the *repetition* of annotations from the videos, but only Scripted Pair 1 engaged in the *repetition* of a solution from the video.

Negotiations, on the other hand, showed how ideas and solutions were not only used but how the meanings behind those ideas and solutions were engaged with. This was evidenced when an idea was explicitly interpreted and extended by applying the idea and solution beyond its use in the videos. Unscripted Pair 1 engaged in frequent collaborative *negotiations* with the talent. For example, they frequently extend the talent's ideas by completing tasks prior to the talent but with the language and ways of reasoning presented by the talent. In contrast, Scripted Pair 1 engaged in one episode that evidenced *negotiation* through the extension of an idea from the talent. Furthermore, Scripted Pair 1's lone *negotiation* culminated in an exchange that suggested only one of the VLs from the pair was negotiating. Despite the lack of negation by both VLs in Scripted Pair 1, the pair was content to have an agreement on the final solution, even if they did not have an agreement on the meaning of the solution.

Summary

Reflecting on the results, I concluded that there was an overarching difference in the goal of the pairs evidenced through their use of the videos in their problem solving. Scripted Pair 1 had a goal of *having* a solution and Unscripted Pair 1 had a goal of *constructing* a solution. Scripted Pair 1 evidenced their goal of having a solution through their use of the videos to settle debates, their appeal to the videos as a justification for a claim, and their *repetition* of solutions from the videos. On the other hand, Unscripted Pair 1 evidenced their goal of *constructing* solutions through their use of the videos as a source of insight for progressing on the tasks and their frequent *negotiation* with ideas from the videos.

Significance and Implications

Having summarized the results of this dissertation study, I now turn to the significance of these findings. In the following section, I elaborate on several theoretical and methodological significances of this study. I conclude with a set of implications for video designers.

Theoretical

This dissertation study is the first investigation, based on my review of the literature, that compares vicarious learning across the conditions of viewing unscripted versus scripted dialogic videos. To date, a major emphasis within the field of vicarious learning has been on studying the comparative effectiveness of monologic versus dialogic videos (e.g., Chi, Kang, et al., 2017; Cox, et al., 1999; Craig et al., 2009; Muldner et al., 2014; Muller et al., 2007). This has been used to make claims about both the effectiveness of dialogic instructional videos and to determine important features of dialogic videos. This study extends the latter work by subjecting an important feature of dialogic videos to a comparative investigation, namely the feature of scripted versus unscripted dialogue. Through this comparison, qualitative differences were identified between the two conditions. The first difference emerged in the orientations of the VLs toward the talent in the videos, and the second difference arose from the problem-solving behavior of the VLs as they used the videos to learn to reason covariationally about the sine function.

This study also contributes to the literature regarding the VLs' orientations toward the talent in dialogic videos. Within this dissertation study, VLs took on both orientations previously identified in the literature (i.e., epistemic detachment and emotionally-involved quasi-collaboration). Significantly, the VLs' orientation was connected to the nature of the dialogue in the videos. VLs viewing the scripted videos had an epistemically-detached orientation, and VLs viewing the unscripted videos had an emotionally-involved quasi-collaborative orientation. Recall, as a part of the Scottish Vicarious Learning Project (SVLP), McKendree et al (1998) first conceived of VLs as epistemically-detached spectators. The SVLP hypothesized that orienting toward dialogic videos as epistemically detached could be beneficial in two ways. Foremost, vicarious learning was conjectured to lessen the cognitive load, allowing VLs to better focus on the substance of the argument being made in the video and better take on the perspective of someone else. Secondly, VLs were conjectured to benefit from not having to worry about being exposed emotionally through struggling with content publicly. However, Lobato and Walker (2019) demonstrated that Grade 9 students using unscripted dialogic videos exhibited an emotionally-involved quasi-collaborative orientation rather than an epistemically detached orientation. Through an emotionally-involved quasi-collaborative stance, their VLs were able to take on the perspective of the talent in the video and use the videos to advance their mathematical thinking. This dissertation study demonstrates that VLs do not strictly experience epistemic detachment or emotionally-involved quasi-collaboration. Instead, the VLs of this study at times had engagements that evidenced both forms of orientation. Importantly, the tendency was for the VLs viewing the unscripted videos to take a stance of emotionally-involved quasi-collaboration, while the tendency was for the VLs viewing the scripted videos to take a stance of epistemic detachment.

Another contribution of this study is that it speaks to the relationship between the age of VLs and their orientation toward the talent in dialogic videos. Specifically, Lobato and Walker (2019) found that their Grade 9 students evidenced an emotionally-involved quasi-collaborative orientation toward the talent in their unscripted dialogic videos. However, they raised the question of whether “undergraduates, with their increasing social and cognitive maturity, may be less inclined to invest emotionally in the talent who appear in the videos” (p. 197). In fact, the undergraduates VLs in this study did experience frequent displays of emotions during expressions of their orientation toward the talent, as long as they were in the treatment that used the unscripted dialogic videos. Evidence of a quasi-collaborative orientation and of displays of emotion seemed more related to the treatment’s use of unscripted versus scripted videos rather than the age of the VLs. That said, there was one interesting difference between the high school VLs in the Lobato and Walker study and the undergraduates in the unscripted video treatment in this study. In the Lobato and Walker study, there was more evidence of the category *being in a community of learners with the talent*, and the emotions associated with this category seemed more intense. For example, the VLs in the Lobato and Walker study talked about the pain and isolation of feeling confused in math class: “Sometimes you feel like you are the only one [confused] and you’re like the alien” (p. 192). However, this feeling seemed to be mitigated when *being part of a community of learners with the talent* in the videos: “When I get really confused, I get isolated, like I’m the only one, but then knowing that she’s [one of the talent’s] confused too …we’re both confused” (p. 192). In contrast, the undergraduate VLs viewing the unscripted videos of this study expressed *being in a community of learners with the talent* as beneficial to their understanding, more engaging, and with less intense emotions. For example, Osiris stated, “Actual students on the screen doing [math] and getting it wrong or right, I feel like it’s giving me a layer of deeper understanding … It makes you feel like you’re more involved.” Furthermore, the VLs of this study displayed less intense emotions, such as an appreciation for seeing the talent’s relatable struggle and a sense of relief when the talent resolved their difficulties. This difference between the Grade 9 VLs of Lobato and Walker's study and the undergraduate VLs of this dissertation study suggests the age of the VLs may be influencing the engagements that evidence orientations toward the talent, and the intensity of the VLs' emotional involvement with the talent.

Methodological

This study makes two methodological contributions. The first contribution is to the coding scheme developed by Lobato and Walker (2019) to study the orientation of high school VLs' toward students in unscripted dialogic videos. In this study, I extended Lobato and Walker's quasi-collaborative coding scheme to the investigation of the orientations of undergraduate VLs and to VLs viewing scripted dialogic videos. This involved the expansion of one a priori code and the induction of three new codes. This resulted in a coding scheme that can be used in future studies at the undergraduate level and in future comparative studies.

The second methodological contribution of this study is the themes in the VLs' problem-solving behaviors. In identifying themes, I brought together codes that were adapted from previous research on the use of monologic videos with induced codes for under-researched areas of *idea justification* and *idea management* when using dialogic videos to engage in mathematical problem solving. Weinberg and Thomas (2018) identified a lack of inquiry into how online mathematics videos are used for learning. Thus, the adaptation of Yoon et al.'s (2022) categories of patterns of use serves as an important step in identifying how students' problem solving is influenced by their use of instructional videos, broadly, and dialogic videos, specifically. Furthermore, the induced codes for *idea justification* and *idea management* play an important role in identifying new mathematical behaviors modeled by the talent. As such, these themes can be used in future inquiries into the growing field of dialogic mathematics videos. For example, my identified themes can be used to assess the problem-solving behaviors of VLs viewing dialogic video content aimed at the creation of justifications (e.g., proof writing).

Video Design

The evidence presented in this study suggests that unscripted dialogic videos may be more fruitful for VLs' problem-solving behavior and their orientations than scripted dialogic videos. This has a number of implications for designers of instructional videos. While there are a number of affordances designers of scripted dialogic videos can take advantage of, the affordances may fall short of the shortcomings. One affordance is that scripted videos are quicker to produce. By scripting the dialogue, the video designer can ensure a coherent flow of information, eliminating the need for extensive post-production editing. Additionally, the video developer has more control over the environment (e.g., specific solutions and ideas can precisely be presented). This allows for a more focused video that aligns with specific learning objectives.

However, this study suggests that VLs can readily discern the inauthenticity of the learning exhibited by the scripted talent, as evidenced by the scripted video viewers' epistemic detachment. Furthermore, the VLs appeared to be less engaged with the mathematical meanings expressed within the scripted videos and focused more on solutions, as evidenced by their problem-solving behavior. A central design goal of dialogic instructional videos is to stand apart from traditional, monologic, videos through an emphasis on conceptual understanding, as opposed to procedural-focused content. Ultimately, if scripted dialogic videos have perceptibly inauthentic learning and the viewers fail to attend to the meaning expressed within the scripts, then the viewers may be served just as well viewing monologues.

The unscripted dialogic videos tell a different story for video producers. The findings of this study suggest that unscripted dialogic videos better serve viewers to engage with and develop meaning for ideas and solutions presented by authentic students in videos. One affordance of unscripted dialogue is the opportunity to capture authentic interactions and learning. This allows viewers to observe real-life conversations and problem-solving behaviors that can be relatable and applicable to the viewers' experiences. Additionally, unscripted dialogic videos allow for the emergence of unpredictable alternative conceptions or difficulties not identified during the design of lessons for filming. For example, while filming the unscripted dialogic videos of this study a difficulty emerged in the talent's units for their first graph. This confusion led to an impromptu task that necessitated the same units on both axes. Significantly, this deviation planted an important seed for their eventual use of radians as the units for both dependent and independent variables of the sine function. Moreover, the VLs also expressed this confusion and were able to leverage the confusion and resolution in the videos for their own progression. Through the authentic struggle of the talent in the unscripted videos, a novel difficulty was captured and resolved within the videos.

Unscripted videos take more time to film because post-production is an involved process of trimming less productive ways of thinking, Furthermore, the authentic language of the unscripted students' is not always precise and can leave questions unanswered. Despite this, I believe the findings of this study support the use of unscripted dialogic videos over scripted dialogic videos because of the benefits to the VLs' orientation and to their problem-solving behaviors.

Study Limitations

One limitation of this study was in the composition of Unscripted Pair 2. During one of their last research sessions, one of the VLs from this pair, Becca, revealed that she had taken mathematics courses at the undergraduate level up through integral calculus. As a part of the study design, the talent were recruited from college pre-calculus courses, and the VLs were recruited from college algebra courses. This sampling was purposeful. The expectation was that the talent and the VLs would have similar mathematical abilities, but that the talent would be at a slightly more advanced level. Having similarly skilled talent and VLs has been found to be advantageous to VLs' attentiveness toward the talent's ideas (Chi, 2013). Given Becca's experience (i.e., her familiarity with concepts in derivative calculus), she was frequently able to demonstrate more advanced covariational reasoning than the talent. This skill difference frequently precluded Becca from engaging in problems she found novel, making the analysis of this pair's problem-solving behaviors difficult. Additionally, it is possible Becca's additional mathematical preparation influenced her orientation toward the talent. Recall, Becca was frustrated with the pace of the talent during an exchange coded as *being in a community of learners with the talent*. It is possible that Becca's frustration with the pace of the videos was produced, in part, by her mathematical experience.

A second limitation can be seen in the differences in the design of the two forms of instructional videos. While the videos are positioned as differing merely along the axis of the nature of the dialogue, another difference is the video participants. Specifically, there is an on-screen teacher in the scripted videos, whereas the teacher in the unscripted videos is off-screen and talks a lot less. Chi (2013) reported that VLs viewing videos with multiple on-screen participants attended more to the words and actions of similarly skilled students than the words and actions of other students or teachers. As such, it is possible that the presence of more students resulted in more attention to and engagement with the unscripted videos. Importantly, for both forms of instructional videos, new ideas and solutions came from the students. Thus, the same ideas and solutions came from the students in both videos. The teacher within the scripted videos served to guide the discussion and guide the work of the talent on the tasks back to the script when the talent went off-script. Furthermore, the unscripted dialogic videos contained two women, and the scripted dialogic videos contained two men. It is possible that gender dynamics played a role in the ways in which the VLs engaged with their respective videos. Ding et al. (2011) found that mixed-gendered pairs engaged in computer-supported collaborative learning were less likely to co-construct knowledge with their partner than single-gendered pairs. Thus, it is possible that the VLs' experiences of quasi-collaboration were influenced by the genders of the talent.

Another limitation of this study stems from the small sample size. While the findings reported in both Chapter 4 and Chapter 5 suggest that the unscripted dialogue positively influenced the VLs' orientations and problem solving, the small sample size limits the generalizability of these findings. Furthermore, this study is exploratory in nature, suggesting further work is needed before conclusions can be made about the creation of unscripted over scripted dialogic videos.

Finally, differences in problem-solving behaviors were constrained to a qualitative analysis of problem solving as it occurred with the use of videos. This study is limited in its ability to connect those differences to measures of learning outcomes.

Future Research

One avenue of future research pertains to the VLs' interlocutions (i.e., the ways in which the VLs engaged with one another as partners in a dialogue). Foremost, one purported benefit of vicarious learning is that the students directly participating in a dialogue serve as a model for the viewers. Thus, a question emerges as to how the talent serve as a model for how to participate in a dialogue by influencing VLs' interlocutions and if the influence on VLs' interlocutions differs when the dialogue of the videos is scripted versus unscripted. One relevant finding from this dissertation study was the induced category of orientation of *evaluating the talent*. As interlocutors, the pairs of VLs may similarly engage in this form of interlocution with one another. Because the VLs viewing scripted videos engaged in many evaluations of the talent, it is possible they would similarly be engaged in more evaluative interlocutions. Future work can be done to probe the relationship between the ways the VLs orient toward the talent in the videos and the interlocutions they engage in as collaborative interlocutors.

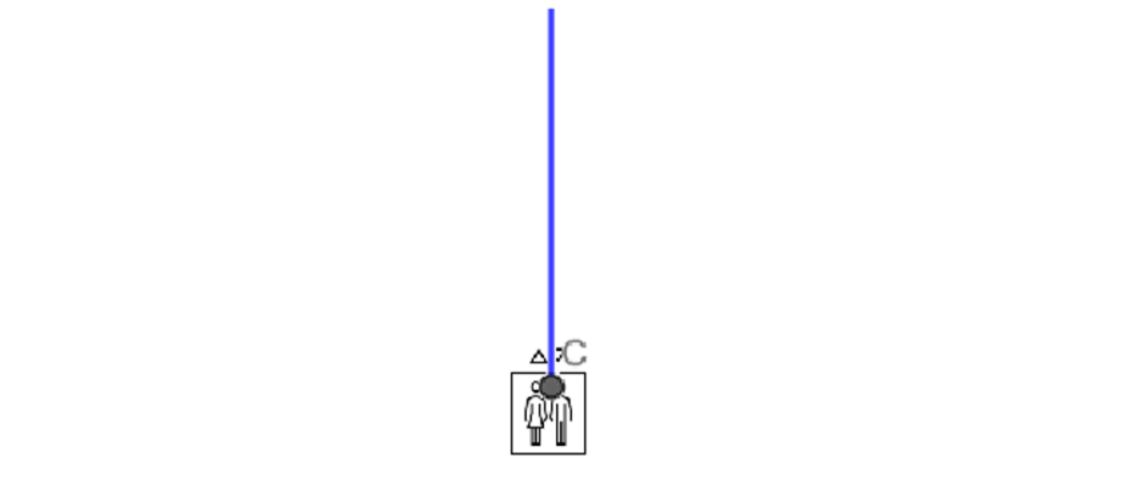
Another important continuation of this work pertains to the emergent difference in the justifications and ways of managing ideas from the videos. A question emerges as to how VLs' justifications and management of ideas would differ when the mathematical content of the videos centers on justifications and management of ideas. Specifically, if a series of dialogic instructional videos covering introductory to proof content (e.g., the introduction to logic through sets and set operations) was created, would VLs' justifications still rely on appeals to authority, or would the videos serve as a model for the production of mathematically rich justifications?

One final area of future research would explore a connection between VLs' mechanisms of learning or their problem-solving behaviors and what was learned. To date, a number of researchers have reported on the positive effect vicarious learning has on student learning outcomes (e.g., Chi, Kang, et al., 2017; Cox, et al., 1999; Craig et al., 2009; Muldner et al., 2014; Muller et al., 2007), but there remains a need for further inquiry into how that learning occurs. Furthermore, future inquiries can place a comparative lens on how learning occurs between viewers of scripted and unscripted dialogic videos.

Appendix A: Paired Tasks

Elevator Task 1:

Create a graph that relates the distance traveled by the point C to the height of the point as it travels up the elevator shaft with a height of 10m.



Elevator Task 2:

Create a graph that relates the distance traveled by the point C to the height of the point as it travels up and down the elevator shaft with a height of 10m.

Elevator Task 3:

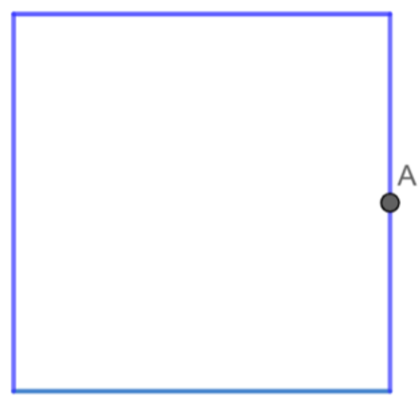
Create a graph that relates the distance traveled by the point C to the height of the point as it travels up and back down the elevator shaft with a height of 27m.

Elevator Task 4:

Create a graph that relates the distance traveled by the point C to the height of the point as it travels up and back down the elevator shaft with an unknown height.

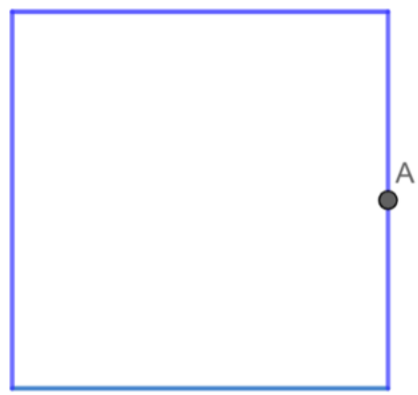
Square Task 1:

Create a graph that relates the distance traveled by the point A to the height of the point as it travels counterclockwise around the square with a side length of 10m.



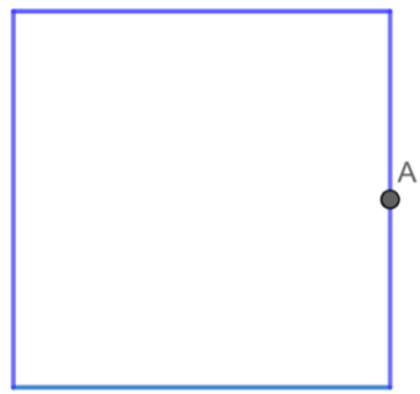
Square Task 2:

Create a graph that relates the distance traveled by the point A to the height of the point as it travels counterclockwise around the square with a side length of 7m.



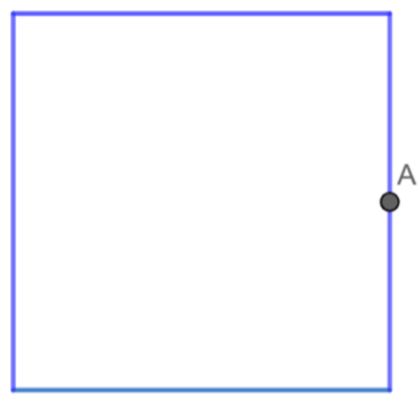
Square Task 3:

Create a graph that relates the distance traveled by the point A to the height of the point as it travels counterclockwise around the square with a side length of 1m.



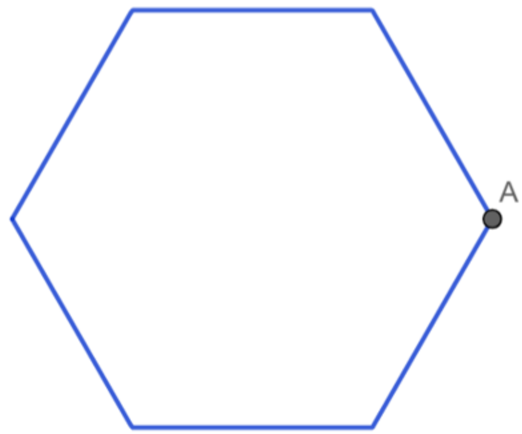
Square Task 4:

Create a graph that relates the distance traveled by the point A to the height of the point as it travels counterclockwise around the square with an unknown side length.



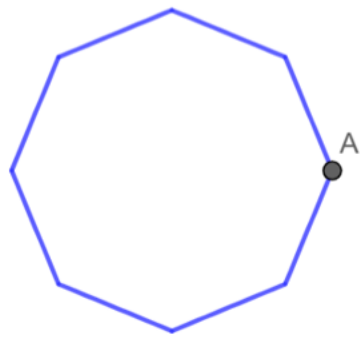
Hexagon Task:

Create a graph that relates the distance traveled by the point A to the height of the point as it travels counterclockwise around the regular.



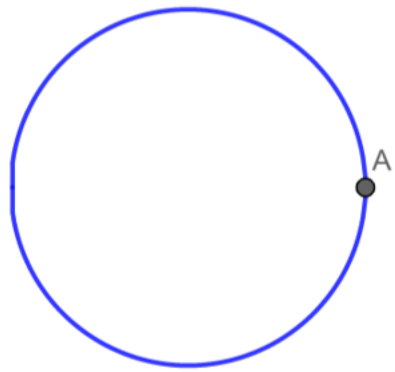
Octagon Task:

Create a graph that relates the distance traveled by the point A to the height of the point as it travels counterclockwise around a regular octagon.



Circle Task:

Create a graph that relates the distance traveled by the point A to the height of the point as it travels counterclockwise around a circle.



Radian Exploration:

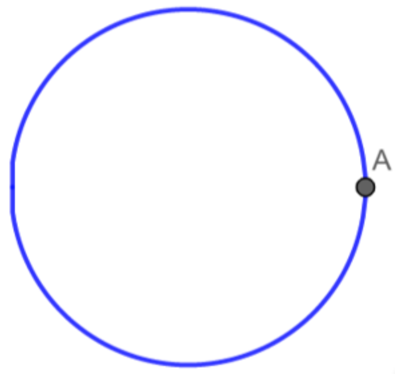
Open the following link:

https://www.geogebra.org/m/wcfupehr#material/htu34her

Construct a definition of a radian:

The Circle Task Revisited:

Create a graph that relates the distance traveled by the point A to the height of the point as it travels counterclockwise around a circle.



Appendix B: Research Session Protocol

Provide two copies of the task sheet and pull up the accompanying video:

Tasks and Accompanying videos:

Task	Accompanying Video(s)
Elevator Task 1	Lesson 1 Episode 1
	Lesson 1 Episode 2
Elevator Task 2	Lesson 1 Episode 3
	Lesson 1 Episode 4
Elevator Task 3	Lesson 1 Episode 5
Elevator Task 4	
Square Task 1	Lesson 2 Episode 1
Square Task 2	Lesson 2 Episode 2
	Lesson 2 Episode 3
Square Task 3	Lesson 2 Episode 4
Square Task 4	Lesson 2 Episode 5
The Hexagon Task	Lesson 3 Episode 1 Lesson 3 Episode 2 Lesson 3 Episode 3
The Octagon Task	Lesson 3 Episode 4 Lesson 3 Episode 5 Lesson 3 Episode 6
The Circle Task	Lesson 4 Episode 1 Lesson 4 Episode 2 Lesson 4 Episode 3 Lesson 4 Episode 4
Radian Exploration	Lesson 5 Episode 1 Lesson 5 Episode 2
The Circle Task Revisited	Lesson 6 Episode 1 Lesson 6 Episode 2 Lesson 6 Episode 3 Lesson 6 Episode 4 Lesson 6 Episode 5

- Can you read the task aloud?
- Does that question make sense?
- As a reminder, you have options. You can either work on the task and then watch the videos, watch the videos and then work on the task, or any other combination of working and watching the videos. It is up to you.
- Please call me over when you're done.

After they complete the task and watch the video:

- Do you have any reflections?
- Can you explain your thinking on the task to me?
    - If only one student explains their solution, ask the other student to explain their thinking in their own words.

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