Attractor Dynamics in Delay Discounting: A Call for Complexity

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Abstract

The outcomes of intertemporal choices indicate that people discount rewards by their delay. These outcomes are well described by discounting functions. However, to fully understand the decision process one needs models describing how the process of decision-making unfolds dynamically over time. Here, we validate a recently published attractor model that extends discounting functions through a description of the dynamics leading to a final choice outcome within and across trials. We focus on the decision dynamics across trials. We derive qualitative predictions for the inter-trial dynamics of sequences of decisions that are unique to this type of model. We test these predictions in a delay discounting game where we sequentially manipulated subjective values of options across all attribute dimensions. Results confirm the model's predictions. We discuss future challenges on integrating attractor models towards a general attractor model of delay discounting to enhance our understanding of the processes underlying delay discounting decisions.

Keywords: decision making; delay discounting; process dynamics, attractor dynamics; hysteresis; neural attractor model

Introduction

Many everyday choices involve options that pose a conflict between immediate, but small gains, and delayed, but larger or more beneficial gains. This conflict occurs on many time scales. For example, you might wonder whether to enjoy spending your money now or saving it for a pension. Or you might be tempted to take the tasty pizza – which is immediately very tasty – instead of the light salad – which might be better for your cardiovascular system in the long-term. In such intertemporal choices (for a review, see Frederick, Loewenstein, & O'Donoghue, 2002), humans discount the offered gain by the delay of delivery. This delay discounting is well described by utility discounting models which assume that the greater the delay in delivery of a reward, the more the utility of a reward is discounted. Hence, these discounting models represent the subjective value of a reward as a function of its delay (see Doyle, 2013 for an overview). While these mathematical models offer a good description of the average outcome of the decision process – the final choice – they mostly leave open how the exact decision process unfolds in time. Decoding this process, though, is necessary in order to fully understand the way decisions are made. To fill this gap, recent developments aim to uncover the process dynamics leading to a final decision in delay discounting (Dai & Busemeyer, 2014; Rodriguez, Turner, & McClure, 2014; Scherbaum et al., 2016). Specifically, the attractor model approach (Figure 1) has recently been proved useful to uncover the process dynamics leading to a final choice outcome on different time scales, that is within and across sequential intertemporal choices (Scherbaum et al., 2016).

In this study, we will use the attractor model and the experimental paradigm as proposed by Scherbaum et al. (2016) to derive and validate qualitative predictions on the inter-trial dynamics of sequences of intertemporal decisions. More detailed, the attractor model of decision making in

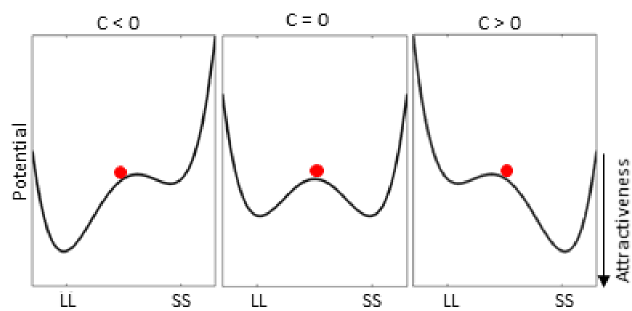

Figure 1: Sketch of possible attractor layouts given different values of the control parameter c. This parameter depends on the relative difference in subjective value (attractiveness) of the options for a subject and hence configures the system for each potential combination of SS and LL: An increase in attractiveness for the LL option results in a negative control parameter which, in turn, increases the depth of the attractor representing the LL option (left panel). In contrast, an increase in attractiveness for the SS option results in a positive control parameter which, in turn, increases the depth for the attractor representing the SS option (right panel). Inherently, the control parameter c is primarily dependent on the values and delays of the presented options, but also on a subject's tendency to discount. Within this potential landscape, the current system state (marked by a red dot) tends to move to the bottom of the potential wells and travels through all intermediate states on its way to a stable final choice.

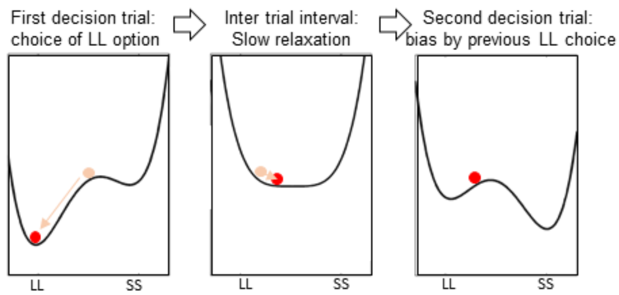

Figure 2: Inter-trial dynamics in the attractor model. Choosing the LL option in a first trial leads to a bias in a second trial due to slow relaxation (e.g. inertia) of the system state during the inter trial interval (ITI, in this study 1.3 seconds).

delay discounting assumes that the depth of the attractors and hence the stability of its end-states is determined by a combination of each option's reward value and delay. By varying either both or one of those properties, the depth of the attractors can be manipulated. To that end, the depth of the attractors represents the relative attractiveness of each option, and that is, the respective subjective (discounted) value, within the attractor model (Figure 1). Hence, the difference in relative attractiveness between the two options determines the systems preference towards either option and is summarized by the *control parameter*, which we will call *c*. Figure 1 depicts three kinds of possible attractor layouts given three prototypical specifications of *c*. The attractors itself picture stable neural representations of the available options. So, the left and right panel of Figure 1 reflect almost exclusive activation of one option's representation, and hence illustrate configurations of the system with a preference towards one option ($c \neq 0$). Accordingly, the special case where $c = 0$ (Figure 1 middle panel) reflects varying amounts of concurrent activation in which the system has not settled into a decision yet, and hence represents a decision in which both options receive an identical input and are thus equally attractive. In this special case scenario, a neutral starting state would keep the system indifferent until slight differences in input (or random noise) tips the system to one side or the other, resulting in a more or less arbitrary decision which was not driven by the systems preference. A major advantage of attractor models is that the decision is not only determined by the current attractor layout, which is in turn determined by the currently offered options, but also through the history of the system's previous decisions (Scherbaum, Dshemuchadse, & Kalis, 2008; Townsend & Busemeyer, 1989). This is due to the genuine assumption that the attractors are formed by the offered options and, hence, these attractors are not present between trials. The inertia of neural systems causes the system to temporally recline in the area where it ended up previously—in the vicinity of the vanished attractor representing the recent choice—and to relax only slowly to the neutral start state under no input. For example, if the model chose one option in a first decision trial, it would remain in the vicinity of this option's attractor in the inter trial interval. In a second decision trial, it would hence start the decision with a bias to the previously chosen option, even if this trial comprises the other option being more attractive (see Figure 2).

Scherbaum et al. (2016) used this premise to predict and validate hysteresis effects (Tuller, Case, Ding, & Kelso, 1994), which are also known as path-dependence, in intertemporal choice. Hysteresis or path-dependence occur, when the decision for one option biases the next decision in favor of the same option (see Figure 2). Hence, in a series of choices in which the initially unchosen option becomes increasingly more attractive (i.e. sequential manipulation of the difference in the relative attractiveness), people stick to the initially chosen option and switch to the now more attractive option much later than they would if their choices were unbiased. However, the sequential manipulation of the difference in the relative attractiveness was merely operationalized by variation of the delay, though the attractor model predicts the same hysteresis effects when the manipulation is realized through a variation of the reward value or even a combination of delay and value. We hence hypothesized that the emergence of hysteresis effects is independent from the attribute dimension which is used to sequentially manipulate the difference in the relative attractiveness between both options (intervals, value difference, or both together).

To provide an insight into hysteresis effects in delay discounting, we applied the same non-verbal delay discounting task as used in the original study. This task redresses the problem that in standard intertemporal choice tasks the sequential manipulation of reward values or delays is simply too obvious (Scherbaum et al., 2016; Scherbaum, Dshemuchadse, Leiberg, & Goschke, 2013). In this task, subjects collect coins of different reward values with an avatar which they move on a checkered playing field by clicking with the computer mouse (Figure 3). The playing field stays constant across trials—except the options which change from trial to trial—and the avatar started each trial from the position of the previously chosen option. The goal is to collect as much reward as possible in the allotted amount of time. In each trial of the task, subjects have to choose between two reward options of different magnitude (small vs. large) at different distances (near vs. far fields). Therefore, this task translates delays into distances, which allows for a more implicit sequential manipulation of the relative attractiveness of options.

To implement the sequential manipulation of different attributes, we used this task in a modified, two-step procedure: In the first part, we measured the individual amount of discounting (the measurement block). Based on this amount of discounting, we created individually tailored sequences of decision to study hysteresis in the following part (the manipulation block). We expected the hysteresis effect to be present in all variants of sequential manipulations.

Methods

Subjects

43 students (65% female, mean age = 22.98 years) of the Technische Universität Dresden took part in the experiment that lasted approximately 50 minutes. All subjects had normal or corrected to normal vision. Three out of 43 subjects were excluded from any subsequent analysis due to individual discounting behavior in the measurement block not allowing for a sufficient hysteresis manipulation in the manipulation block. Subjects gave informed consent to the study and received a 2.50 € show-up fee and the money they collected within the experiment (Mean = 3.17, SD = 0.39).

Apparatus and Stimuli

The experiment was presented on a 17-inch screen (1280 x 1024 pixels, 85 Hz). As presentation software, we used Psychophysics Toolbox 3 (Brainard, 1997; Pelli, 1997) in Matlab 2010b (the Mathworks Inc.), running on a Windows XP SP2 personal computer. Responses were carried out by moving a high precision computer mouse (Logitech Laser Mouse USB).

Subjects moved an avatar on a playing field divided into 20 x 20 fields (Figure 3). To move the avatar, subjects clicked with the mouse in one of four horizontally or vertically adjacent movement fields, as signaled by a white border surrounding the fields. On each trial two reward options were presented as coins on fields marked with a red border: One reward was near but small, the other reward was far but large. The two options' positions were always chosen so that the first move into one direction decreased the distance to one option but increased the distance to the other option. This way, the first move of the avatar already represented a clear preliminary decision for one option and against the other option.

For both options, a number posed within each coin represented the reward *value* and the horizontal and vertical distance of the reward field to the field of the avatar represented the *distance* of the option. Reward *values* ranged from 1 to 99 credits and *distances* ranged from two to fifteen fields. For better comprehensiveness in the context of intertemporal choice, we maintain in the following the standard description of the time dimension using "soon", "late", "delay", and "interval", although in our scenario time delay is represented by spatial distance. The relation between the two reward values can be characterized as the ratio of the higher and smaller reward value and will be denoted by "difference".

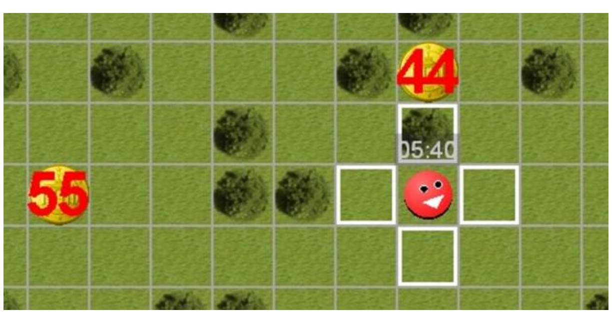

Figure 3: Detail of the dynamic delay discounting paradigm.

Above the avatar (Figure 3) subjects could see the remaining time within one block, as well as below the collected credits in Euro (1 credit = 1/10 € cent), but only in the very moment when either reward was collected.

Procedure

Subjects' task was to collect as much reward as possible within the allotted time limit. In each trial, they had to choose between two reward options (one soon but small, *SS*, one late but large, *LL*; see design). They collected the selected reward by moving their avatar with the mouse across the playing field.

A trial started with an inter trial interval (ITI) of 1.3 seconds. Within this interval, the mouse cursor was locked in the center of the field containing the avatar. After the ITI, the two options were presented. As soon as the two options appeared, participants could click on the adjacent movement fields to move their avatar towards the chosen option (Figure 3). When the avatar reached one option, both options disappeared, the value of the selected option was added on the collected credits, and the next trial started.

The experiment consisted of four blocks, with one block lasting eight minutes. Between blocks, subjects were informed about the credits collected and were instructed to rest briefly before the self-paced start of the next block.

Before the start of the experimental blocks, subjects worked through a test block of two minutes to get used to the virtual environment, handling of the mouse, as well as the range of spatial distances and reward values.

Design

The experiment consisted of four blocks with the first block (*measurement block*) being conceptually different from the three subsequent blocks as its aim was to measure the subjects' individual discounting behavior. In each of the three subsequent blocks (*manipulation blocks*) we realized a unique adaptive hysteresis manipulation constituting an *interval block*, a *value block*, and a *combined block*. Each subject's session started with the measurement block followed by the manipulation blocks. The sequential arrangement of the manipulation blocks was fully varied and balanced between subjects.

In the measurement block, reward values ranged from 11 to 99 and distances from three to 15. That was given by orthogonally varying the intervals (1, 4, 8, and 12 fields), the differences (20, 50, 70, 80, 88, 93, 97, and 99%), and the delay of the sooner option (2 and 3 fields). Additionally, the reward values of the late option were randomly chosen from a discrete uniform distribution between 55 and 99 credits. The combination of 8 differences, 2 distances of the SS options and 4 intervals between the SS and the LL option yielded a complete set of 64 trials. We generated 5 such sets, with a randomized order of trials within each set. The measurement block's time limit ensured that subjects could work through the complete design matrix, that is one of those 5 sets, at least one time.

To realize the adaptive hysteresis manipulation, we calculated the subjects' individual discounting curve from which we adaptively derived trials compatible to the respective hysteresis manipulation (see Results). The structure of an adaptive hysteresis manipulation is to sequentially change subjects' preference from the SS towards the LL option, or vice versa. In our adaptive trial sequences, we aimed to change subjects' preference in 12 steps as indicated by the differences between the subjective value ratio (SS/LL) in the trials and the indifference points (-0.3000, -0.2455, -0.1909, -0.1364, -0.0818, -0.0273, 0.0273, 0.0818, 0.1364, 0.1909, 0.2455, 0.3000), that is the *manipulation points*.[1] It is imperative that a negative manipulation point indicates a preference for the LL option and a positive manipulation point a preference for the SS option. Furthermore, it applies that the higher the absolute manipulation point, the more distinct are the relative attractiveness of both options. Hence, a manipulation point of zero represents no preference, that is the indifference point. Please note that the interpretation of the manipulation point is analog to the interpretation of the control parameter.

We then applied this manipulation in three different sub-blocks. First, in the interval block we consecutively increased or decreased the delay of the LL option to the avatar while keeping all other factors constant within the sequence. For each sequence the delay of the sooner option and the reward value of the late option were randomly chosen from discrete uniform distributions between 2 and 3 fields, and 55 and 99 credits, respectively. The reward value of the sooner option was randomly drawn from the uniform distribution between subjects' two indifference points at the intervals 6 and 7. Furthermore, we varied the direction of these sequences (*direction* = ascending or descending) and created eight sequences for each direction. This resulted in 16 possible sequences, and hence 192 trials.

Second, in the value block we consecutively increased or decreased the reward value of SS option while keeping all other factors constant within the sequence. Again, for each sequence, the delay of the sooner option and the reward value of the late option were randomly chosen. The delay of the LL option to the avatar was drawn randomly between all intervals at which subjects' indifference point was positioned in such a way that all 12 manipulation points were valid, that is, did not exceed 1 or fall below a value of 0. For each trial within the sequence, the reward values of the SS option were then calculated. Again, we also varied the direction of these sequences and created eight sequences

[1] For instance, a subject's indifference point at interval 1 is 0.8 (see Figure 4). Given a manipulation point of -0.3, the respective manipulated trial must yield a subjective value ratio (SS/LL) of 0.5 at an interval of 1. The same logic applies over all manipulation points and intervals.

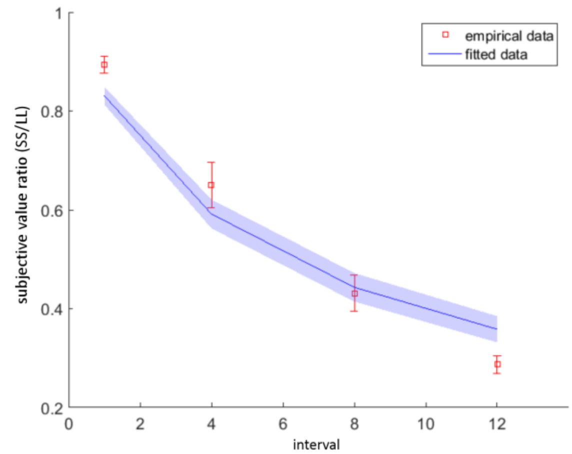

Figure 4: Subjects' indifference points in the measurement block, depicting the decrease in subjective value of the late-large option as a function of intervals between the two options. Indifference points are the subjective value ratio (SS/LL) at which subjects chose indifferently between the two options, i.e., the probability of choosing LL over SS is 50%. Note: Error bars indicate standard errors. The curve displays the fitted hyperbolic functions.

for each direction. This resulted in 16 possible sequences, and hence 192 trials.

Third, in the combined block we consolidated the former manipulations and varied both the delay of the LL option to the avatar and the reward value of the SS option in such a way that the manipulation points consecutively increased or decreased within the sequence. Again, for each sequence the delay of the sooner option and the reward value of the late option were randomly chosen. For each trial within the sequence, the delay of the LL option was randomly chosen from the set of intervals in which the respective manipulation point was valid. The reward values of the SS option were then calculated. Again, we also varied the direction of these sequences and created eight sequences for each direction. This resulted in 16 possible sequences, and hence 192 trials.

In sum, we applied a 2 (direction: ascending, descending) x 3 (manipulation type: interval, value, combined) full factorial within-subjects design.

Results

On average, subjects completed 134 trials (*SD* = 23) in the measurement block. Hence, subjects ran through at least two out of five sets of 64 trials. The aim of the measurement block was to measure subjects' individual discounting behavior indicated by subjects' indifference points as depicted by Figure 4. As an estimate of the indifference point, the point of inflection of a logistic function was fitted to the individual choices as a function of increasing value

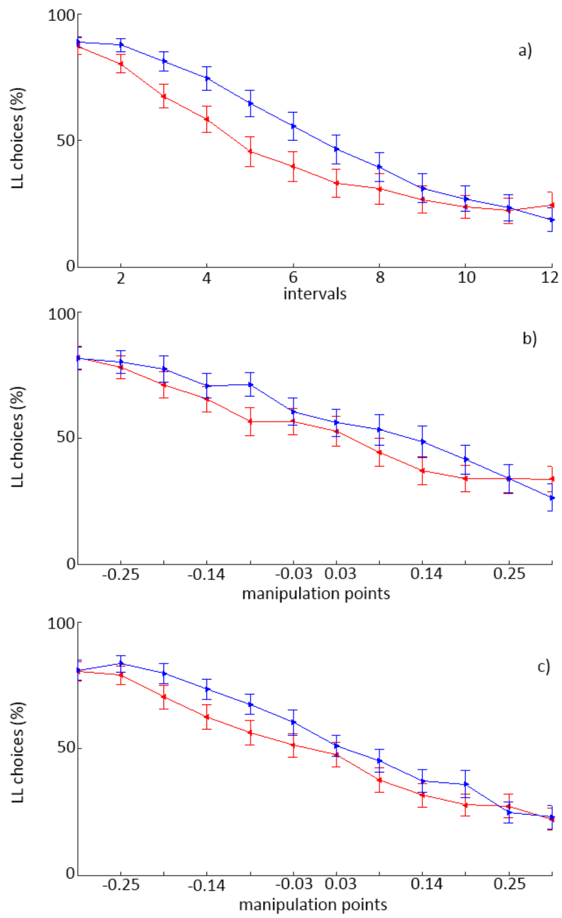

Figure 5: Average hysteresis plots between manipulation types. Plots depict subjects' mean response pattern over intervals (panel a) or manipulation points generated by variation of rewards only (panel b) and a combination of rewards and intervals (panel c). Note: Error bars indicate standard errors. The separate colors indicate whether mean responses were derived from ascending or descending sequences. The blue line represents descending sequences (LL→SS). The red line represents ascending sequences (SS→LL).

differences was determined.[2] To evaluate subjects' discounting behavior in one parameter, we extracted the *k*-parameter by fitting a hyperbolic function to each subject's indifference points over the different intervals. Data revealed an average *k*-parameter of the hyperbolic discounting curve with $M(SD) = 0.23(0.19)$, bootstrapped 95% $CI = [0.18, 0.30]$, indicating a very strong discounting behavior. The hyperbolic model had a good fitting performance over all subjects, indicated by a high average R^2, $M(SD) = .87(.10)$.[3]

In the manipulation blocks, subjects completed 387 trials ($SD = 67$) on average. Hence, on average, subjects ran through 32 hysteresis sequences ($SD = 6$), consisting 16 ascending ($SD = 3$) and 16 descending ($SD = 3$) sequences. The SS option was chosen in 48.37% ($SD = 22.19$) of the trials, indicating only a slight decision bias which was not predicted by the model.

The core prediction of the model was that subjects show identical hysteresis effects irrespective of the specific hysteresis manipulation, that is, whether the sequential manipulation of the attractiveness of both options was realized through varying intervals, differences or a combination of both. Figure 5 depicts the hysteresis effect for each manipulation type. The plots indicate that the hysteresis effects are very similar between manipulation types, but show the qualitatively best pattern for the interval manipulation (Figure 5, panel a). In order to test model's predictions, we conducted a two-factorial Repeated Measures ANOVA (direction x manipulation type) on subjects' mean choice. As expected, we solely found a main effect of direction ($F(1,39) = 17.44$, $p < .001$, $\eta^2 = 0.31$), indicating that hysteresis emerged irrespectively of manipulation types. Thus, neither the main effect of manipulation type ($F(2,78) = 3.03$, $p = .054$, $\eta^2 = 0.07$) nor the interaction ($F(2,78) = 1.22$, $p = .302$, $\eta^2 = 0.03$) were statistically significant. In order to focus the analysis on the hysteresis effect, that is, eliminating the variance of the absolute level of LL choices, we summarized hysteresis effects into one hysteresis parameter. The hysteresis parameter was given by calculating the differences between subjects' mean choice in ascending and descending hysteresis sequences for each manipulation type. An additional one-factorial Bayesian Repeated Measures ANOVA on the hysteresis parameter revealed that the data show substantial evidence in favor of the null hypothesis ($BF_{01} = 4.64$) claiming that the hysteresis effect does not vary systematically between all three manipulation types. Therefore, we consider the predictions of the model as confirmed.

Discussion

In this study, we tested predictions of the attractor model of delay discounting in a recent developed non-verbal delay discounting paradigm. Our results validated the model in such a way that its predictions concerning hysteresis effect in delay discounting were confirmed. Specifically, when sequentially varying the attractiveness of both options from a very strong preference towards the SS option to a very strong preference towards the LL option, and vice versa, hysteresis effects occur irrespectively of how the

[2] The fitting of the logistic regression model was performed using the StixBox mathematical toolbox by Anders Holtsberg (http://www.maths.lth.se/matstat/stixbox/). The fit was based on the model $\log\left[\frac{p}{1-p}\right] = Xb$, where p is the probability that the choice is 1 (SS) and not 0 (LL), X represents value differences, and b represents the point estimates for the logistic function.

[3] The fit of the hyperbolic function was based on minimizing the summed squared errors (*SSE*). R^2 is defined as the ratio of the sum of squares of the regression (*SSR*) and the total sum of squares (*SST*). Since *SST* is defined as *SSR+SSE*, R^2 is defines by *1-SSE/SST*.

attractiveness of any option is varied within the sequence. Therefore, the current study both replicated and added empirical evidence for the validity of the attractor model of delay discounting.

One might object that the predictions of the model were merely derived through a qualitative, argumentative manner. This is obviously true, but not a weakness of the current study. First, concerning the interval manipulation, it was already shown that the exact same predictions can be derived by means of computational simulation based on a competitive neural-network (Scherbaum et al., 2016), hence running a computational simulation with the same model would not provide any new information. Second, and this point is genuine, the model does not allow for reasonable separate simulations of all manipulation types. This is due to the fact that the model merely uses subjective values for each option. The emergence of those subjective values, however, is not covered within the model.

Leaving the emergence of subjective values open the model proves to be useful for predicting intra- and inter-trial dynamics in delay discounting, when a specific discounting function is already given, but it does not explain the emergence of discounting functions. This gap has also been argued for recently by others, reasoning that intertemporal choice consists of two processes (Rodriguez et al., 2014): First, the process of delay discounting, and second, the process of choice. This gap between the two processes could be closed by connectionist models, which have already been used to explain how different discounting functions emerge by linking discounting behavior with aspects of self-control (Scherbaum, Dshemuchadse, & Goschke, 2012).

The two models provide insights into the dynamics of delay discounting and the dynamics of choice, respectively. Integrating these two models into one general connectionist model of delay discounting could provide insights into the interacting process dynamics of preference (delay discounting) and choice. Such an integration could therefore enhance our understanding of the processes underlying delay discounting decisions and, hence, complement our knowledge about decision outcomes.

Acknowledgments

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