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Cost Recovery and Conservation of Residential Water Use by Optimized Block Pricing

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Abstract

A method for pricing residential municipal water with increasing rate structures is developed and applied in this article. The method relies on the water-use functions for each block of a rate structure and on economic and water-supply data to produce a nonlinear programming problem whose objective function is the recovery of water-production costs. Maximum-water use, minimum-water use, cost-recovery, and price constraints were imposed on the objective function to complete the rate-design nonlinear programming problem, whose solution yields the block prices and water-meter charge of a water-rate structure. The water-pricing method is applied to a small municipality –on the order of 90000 residents- whose water-use functions were derived from data collected during the 1988-1991 California drought. The water-pricing method produced a rate structure that complies with specified constraints and recovers water-production costs, demonstrating that it is a useful tool for residential water pricing.

Introduction

Water use during drought. During the period 1988-1991, California suffered the second most severe drought of the 20th Century (Loáiciga et al., 1992; Loáiciga and Leipnik, 1996). Water purveyors throughout California implemented various measures to cope with the low natural supply of water during that period (Lawrence et al., 1994). Two of the most effective measures were water conservation and rising of water prices, which produced reductions in municipal water use on the order of 40%. In the context of municipal residential customers, water conservation consisted primarily of changes in the patterns of water use by households and the installation of low-flow plumbing and water-recycling devices in residences. One of the best documented examples of water-use reduction during the 1988-1991 California drought is that of the Santa Barbara, California, a relatively small city (the 1987 and 1995 populations were 80695 and 93957, respectively) that was severely impacted by the drought (Loáiciga and Renehan, 1997).

Figure 1 shows the total water use (U , in m^3) during (fiscal) years 1988-1989 through 1990-1991 as a function of the average revenue (AR , in $\$ m^{-3}$) in the City of Santa Barbara. The water use and average revenue were normalized to a 1995 population and price-index baselines, and fit with a linear equation:

$$U = -1.166 \times 10^7 AR + 2.509 \times 10^7 \quad (1)$$

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for which the regression coefficient $R^2 = 0.95$. Notice that the simplest statistical model, that is, linear regression, was chosen to relate U and AR . This is consistent with the principle of parsimony, or Occam's Razor (after William of Occam, c. 1285 – c. 1349, English philosopher): "given several possible alternative explanations to an event, the best explanation is the simplest one".

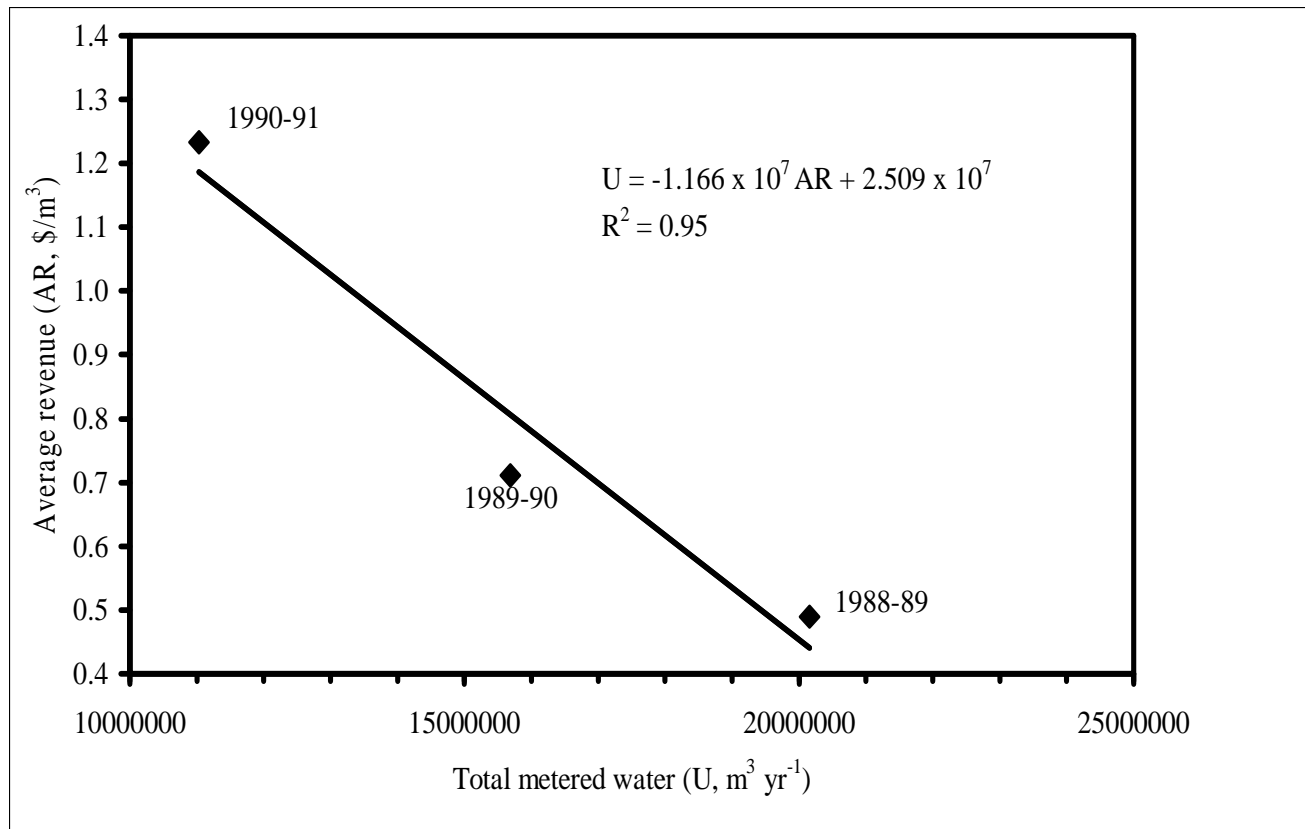


Figure 1. Total metered water use vs. average revenue.

The water use totals shown in Figure 1 include the single-family residential, multi-family residential, commercial, governmental, and agricultural sectors in Santa Barbara. Of these, the single-family residential is the largest sector, and makes up, on the average, about 45% of the total annual water use. At the other extreme, agricultural water use accounts for less than 1% of the total water use. The average revenue shown in Figure 1 equals the total revenue from water sales divided by the total water use by all sectors. The total revenue includes metered-water sales plus fixed water-meter charges excised to all water customers. Notice that the total water use decreased with increasing average revenue. The decline in water use observed in Figure 1, however, was not caused by price increases alone. Conservation was a contributor to diminished water use also. For this reason, it would not be appropriate to call the graph shown in Figure 1 a

demand curve, which describes the change in purchased water caused by changes in water price alone, all other factors kept constant.

Increasing water-rate structures. Figure 2 shows the pre-drought (1987) and post-drought (1995) rate structures for the single-family residential sector in Santa Barbara. The former rate structure was uniform, i.e., each cubic meter of water was priced at \$0.314, regardless of the consumption level. The latter, on the other hand, was increasing, and the price of water increased with consumption. In this second instance, there were three blocks of water. Block 1 priced the first 11.32 m³ of water used during a monthly billing cycle at \$0.742 m⁻³. The second block priced the next 45.31 m³ of water at \$ 1.24 m⁻³. The third block was made of any water in excess of 56.63 m³ (= 11.32 + 45.31) consumed in any month, and was priced at \$ 1.31 m⁻³. In addition, the 1987 and 1995 rate structures included monthly water-meter charges of \$ 4.10 and \$ 5.50, respectively, to each single-family residential water meter.

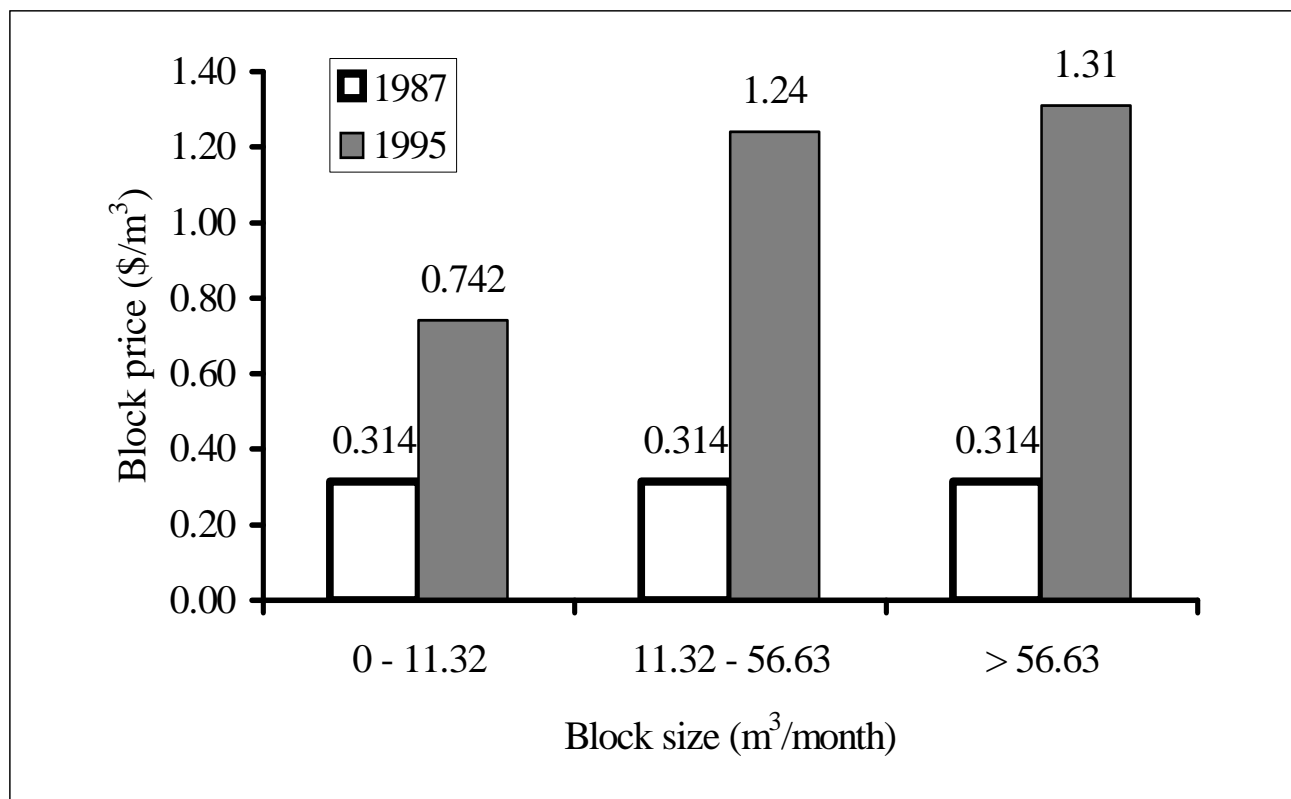


Figure 2. Block prices for the 1987 (pre-drought) and 1995 rate structures (block 1: 0-11.32; block 2: 11.32-56.63; block 3: larger than 56.63 m³/month). The 1987 and 1995 water-meter charges were \$ 4.10 and \$ 5.50, respectively.

The adoption of increasing rate structures to replace flat rates has become commonplace throughout the water-scarce American West over the last three decades. Increasing-rate structures discourage the wasteful use of potable water and attempt to pass the marginal cost of

new water supply to customers. Block-1 water is intended to satisfy basic needs (drinking, food preparation, personal hygiene, household cleaning and basic maintenance, and the like) and is assigned the lowest price. Water price increases as consumption climbs up the block structure, and the high-use blocks (e.g., block 3 in Figure 2) provide water commonly used to satisfy recreational pastimes, for aesthetical landscape maintenance, and other uses not classified as basic.

Study objective. The effectiveness of an increasing rate structure in achieving desired objectives depends on the choice of the decision variables that define it: the number of blocks, block prices, block sizes, and the water-meter charge. The water-use function in each of the blocks of the rate structure is another crucial factor in the design of a rate structure. These functions represent the price-driven consumptive behavior of customers towards water. The design of a suitable rate structure must consider the decision variables and the water-use functions for water, and may optimize one or more of the former to achieve some of the stated objectives (e.g., partial or total cost recovery, affordability of block-1 water, discouragement of the wasteful use of potable water). The objective of this work is to develop a nonlinear programming methodology to design increasing-rate structures in which block prices and water-meter charge are optimized and the block sizes are treated as given. The methodology is illustrated with water-use and rate-structure data for the single-family residential sector collected in Santa Barbara, California, during the 1988-1991 drought. The single-family residential sector is the most stable municipal water-using group and commands a leading portion of the water consumption in non-industrial cities such as Santa Barbara. It is worthy of remark that the treatment of block sizes as fixed quantities does not handicap the water-rate design methodology presented in this work. Block sizes can be varied, in which case the optimization program for rate design undergoes reformulation and is solved anew. This is tantamount to solving a series of rate-design problems that correspond to different sets of block sizes, if so desired. A combination of attractive block sizes, block prices, and water-meter charge is then a candidate for a final water-rate structure. Generally, these candidate rate structures must then undergo public scrutiny and be approved by elected officials, a process that invariably introduces changes in the adopted water rates (Loáiciga and Renehan, 1997; Loáiciga and Leipnik, 2001).

The literature on the effects conservation and pricing on water use is large (see, e.g., Nieswiadomy, 1992; Jordan, 1994, Loáiciga and Renehan, 1997). Much less attention has been given to the optimal design of water-rate structures taking into account block-specific water-use functions and using nonlinear programming (see, however, an alternative approach by Hewitt and Hanemann, 1995). This last focus is pursued in this paper.

Methodology to obtain water-use functions in each block

The first step to obtain the blocks' water use functions is to assign to each block its share of the total water use. The blocks' shares of the total water use are inferred directly from water-meter records of water delivered to customers (Kennedy/Jenks Consultants, 1995; Loáiciga and Renehan, 1997). The apportionment of used water to blocks is possible only if the block sizes are known, which is the situation dealt with in this work. The choice of block sizes depends on customers' basic water needs, on the capacity of the water-supply system, and on the cultural idiosyncrasies of a community. If pricing and conservation reduce water use during the period of analysis (i.e., during the 1988-1991 drought in this work), one must isolate their individual effects in order to estimate the price vs. water-use function, block by block. This second step is

accomplished by conducting stratified surveys among customers, in which they are asked to estimate the individual degrees of influence that conservation and pricing had in the reduction of their household water use relative to non-drought consumption. Survey responses are used to estimate what the single-family residential water use would have been if price increases exclusively had been implemented. To this end, the average percentage of the water-use reduction attributed to conservation is added to the total (observed) water use. The amount so calculated is then apportioned among the blocks of the rate structure. The percentage of the calculated total water use credited to a block equals the average percentage of the total water use that the block received during a period of record (Loáiciga and Renehan, 1997). The estimated water use in the block is then paired with the corresponding block price that prevailed during the period in which the water use occurred. This procedure is repeated for non-overlapping periods that had different block prices, to yield the water-use function for the block. The latter function is not a demand curve (or demand function). This is so because the demand curve defines the price paid for a quantity of a good sold unrestrictedly, rather than being parceled out in blocks typical of a water-rate structure.

The described methodology was applied to the water-use data of Figure 1 to obtain the block-1, block-2 and block-3 water-use functions shown in Figure 3. The sizes of blocks 1, 2, and 3, are the same as those shown in Figure 2. The points labeled 1, 2, and 3 in each of the water-use functions of Figure 3 denote fiscal years 1988-1989, 1989-1990, and 1990-1991, respectively.

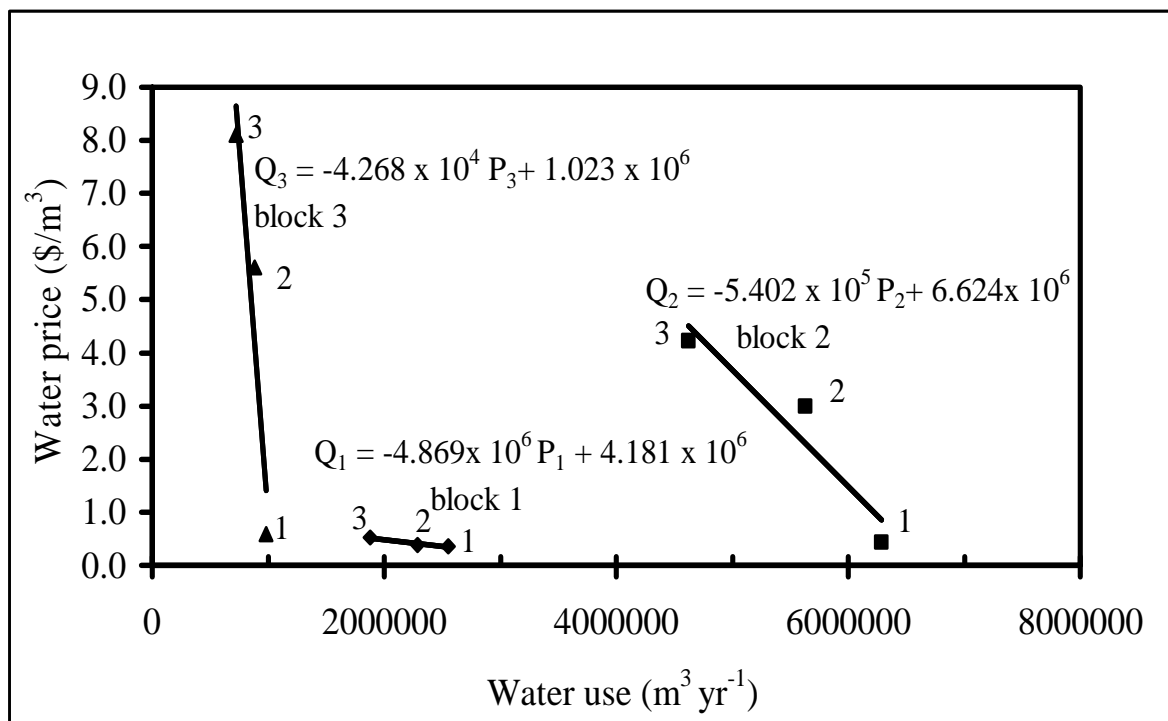


Figure 3. Water-use curves for blocks 1, 2, and 3. Data are presented in Table 1.

Table 1. Use and price data for blocks 1, 2, and 3 (data are plotted in Figure 3).

Year	Block 1		Block 2		Block 3	
	Water use (10^6 m^3)	Block price ($\$ \text{ m}^{-3}$)	Water use (10^6 m^3)	Block price ($\$ \text{ m}^{-3}$)	Water use (10^6 m^3)	Block price ($\$ \text{ m}^{-3}$)
(1)	(2)	(3)	(4)	(6)	(6)	(7)
1988-89	2.553	0.360	6.284	0.437	0.982	0.589
1989-90	2.288	0.384	5.631	2.99	0.880	5.61
1990-91	1.878	0.519	4.624	4.23	0.722	8.11

The water-use and price data in each block (shown in Table 1) were fitted with linear equations to obtain the following water-use functions for blocks 1 through 3 (Q_i denotes annual water use in m^3 in the i -th block, and P_i is the i -th block price in $\$ \text{ m}^{-3}$, $i = 1, 2, 3$):

Block 1:

$$Q_1 = -4.869 \times 10^6 P_1 + 4.181 \times 10^6 \quad (2)$$

Block 2:

$$Q_2 = -5.402 \times 10^5 P_2 + 6.624 \times 10^6 \quad (3)$$

Block 3:

$$Q_3 = -4.268 \times 10^4 P_3 + 1.023 \times 10^6 \quad (4)$$

Equations (2), (3), and (4) indicate that if the prices $P_1, P_2, P_3 = 0$, the corresponding annual water uses in blocks 1, 2, and 3 would be 4.181, 6.624, and 1.023 million m^3 , respectively. Actually, the block prices do not drop to zero because of minimum use, revenue recovery, and price constraints that must be met, as shown below.

Rate structure design with fixed blocks

Use-related considerations. The number of blocks (n) and the block sizes are assumed fixed in the methodology for rate structure design, which seeks to determine optimal block prizes and the fixed charge levied on residential water meters. One can solve a series of problems in which

the number of blocks and their sizes are varied, and then choose from among their solutions the one(s) that render the most attractive rate structure(s). Following the results of equations (2)-(4), the water use in the i -th block (Q_i , in $m^3 \text{ yr}^{-1}$) is modeled as a linear function of block price (P_i , $\$ m^{-3}$):

$$Q_i = a_i + b_i P_i \quad i = 1, 2, \dots, n \quad (5)$$

in which a_i and b_i are the intercept and slope coefficients, respectively.

The amount of water use in a block can be at most equal to the number of water meters (N) times the block size (D_i , expressed in $m^3 \text{ yr}^{-1}$, and calculated by multiplying the standard monthly block size by 12):

$$Q_i \leq N \cdot D_i \quad i = 1, 2, \dots, n-1 \quad (6)$$

The maximum use of water in the n -th block is implied by equation (6) and by a constraint on the total available water (Q_{\max} , in $m^3 \text{ yr}^{-1}$) that arises from the treatment capacity of the water system:

$$\sum_{i=1}^n Q_i \leq Q_{\max} \quad (7)$$

The total water use must be at least equal to the minimum use, calculated as the product of a per capita basic use (U , $m^3 \text{ yr}^{-1}$) times the population size (G):

$$\sum_{i=1}^n Q_i \geq U \cdot G \quad (8)$$

Revenue and cost. The revenue (R , in $\$ \text{ yr}^{-1}$) from water sales is the sum of the individual block revenues plus a fixed revenue, in which the latter is equal to the number of water meters times the water-meter charge (F , $\$ \text{ yr}^{-1}$, from which the standard monthly fixed charge is obtained by dividing F by 12):

$$R = N \cdot F + \sum_{i=1}^n Q_i P_i \quad (9)$$

The block prices increase from one block to the next (an increasing block-rate structure):

$$P_{i+1} > P_i \quad i = 1, 2, \dots, n-1 \quad (10)$$

All prices are positive (as well as the fixed charge, F):

$$P_i > 0 \quad i = 1, 2, \dots, n \quad (11)$$

In some instance the block prices (and the fixed charge) may be limited to specified ranges.

The cost of water supply (C , in $\$ \text{ yr}^{-1}$) equals the fixed annual cost (A_F , $\$ \text{ yr}^{-1}$) plus the sum of the variable production costs in the blocks:

$$C = A_F + \sum_{i=1}^n A_{Ci} Q_i \quad (12)$$

in which A_{Ci} , in $\$ m^{-3}$, is the average variable production cost in the i -th block.

The objective function and constraints. The objective function of the block-pricing problem is to minimize the difference between revenue and cost. This is consistent with the average-cost pricing criterion applied by many (non-private) municipal water purveyors, specially in California, whereby fixed and variable water-supply costs are recovered through water sales:

$$Z = \min_{\text{w.r.t. } P_i, F} N \cdot F + \sum_{i=1}^n Q_i P_i - \left(A_F + \sum_{i=1}^n A_{Ci} Q_i \right) \quad (13)$$

Other objectives functions have been suggested to this author, some of them involving the consumer surplus. Although alternative objective functions may be appealing from a theoretical standpoint, average-cost pricing has indisputable relevance to the modus operandi of water purveyors and was adopted in this study.

The revenue equals or exceeds the cost (a constraint that ensures recovery of the cost of water supply):

$$N \cdot F + \sum_{i=1}^n Q_i P_i \geq A_F + \sum_{i=1}^n A_{Ci} Q_i \quad (14)$$

The use of constraint (14) in association with the objective function (13) leads to equality of revenue and cost in the solution of the rate-design nonlinear programming problem, as we shall in the results sections below.

Substitution of the water-use equation (5) into equation (13) produces the final formulation of the block-pricing objective function:

$$Z = \min_{\text{w.r.t. } P_i, F} \sum_{i=1}^n b_i P_i^2 + \sum_{i=1}^n (a_i - A_{Ci} b_i) P_i + N \cdot F - A_F - \sum_{i=1}^n A_{Ci} a_i \quad (15)$$

which is a quadratic function of the prices P_i and depends linearly on the water-meter charge F .

The objective function is subject to the revenue constraint (equation (14)), which is transformed by substituting equation (5) in it to produce:

$$\sum_{i=1}^n b_i P_i^2 + \sum_{i=1}^n (a_i - A_{Ci} b_i) P_i + N \cdot F \geq - \left(A_F + \sum_{i=1}^n A_{Ci} a_i \right) \quad (16)$$

Equation (5) is used in the maximum block water use, the total water use, and the basic use constraints (equations (6)-(8), respectively), which, together with the price constraints (10)-(12) are added to the objective function (15) and the revenue constraint (16) to produce a nonlinear programming problem with a quadratic function and linear and nonlinear constraints whose solution yields the optimal block prices and the fixed charge.

A rate-design example, results, and discussion

The previous model of rate-structure design was applied to data collected in the period 1987-1995 in the City of Santa Barbara, California. Table 2 shows the single-family block parameter values (i.e., water-use coefficients, number of single-family residential water meters, block sizes, and average variable costs). Table 3 contains the remainder of the water and economic data used in this example. Solution and results of the rate-design problem were obtained with Microsoft Excel's Solver software (Microsoft Corp., Redmond, Washington, 2003).

Columns (2), (3), (4), and (5) of Table 4 contain the water prices, water uses, revenues from water sales, and variable costs of water supply, respectively, in blocks 1, 2, and 3 obtained from the solution of the nonlinear programming problem. The optimal prices in blocks 1, 2, and 3 were 0.404, 0.970, and 2.00 dollars per m^3 , respectively. The optimal monthly water-meter charge to be applied to each single-family water meter equaled \$ 5.83 (reported as a footnote in

Table 4). All of the block 1 available water was used ($Q_1 = 2.214 \times 10^6 \text{ m}^3 \text{ yr}^{-1}$, or 23.9% of a total annual water use equal to $9.252 \times 10^6 \text{ m}^3$), while the block 2 and 3 water uses were less than their available maxima. Block-2 water constituted 65.9 % (or $6.100 \times 10^6 \text{ m}^3$) of the total annual water use, while block-3 water –the most expensive– made up 10.2% (or $0.938 \times 10^6 \text{ m}^3$) of it. Block 2 had the largest contribution to the (variable) revenue accruing from metered water sales: \$ 5.917 million annually, or 68.1 % of the total variable revenue (= \$ $8.683 \times 10^6 \text{ yr}^{-1}$). In spite of its low share of the total water use, block-3 water sales accounted for 21.5% (or \$ 1.871 million) of the variable revenue because of its relatively larger price.

Table 2. Single-family block parameter values (for the City of Santa Barbara, 1995).

Block	Parameter			
	Intercept coeff.	Slope coeff.	Block size	Average variable cost ^(A)
i	a_i	b_i	D_i	A_{Ci}
	($\text{m}^3 \text{ yr}^{-1}$)	($\text{m}^3 \text{ m}^3 \text{ yr}^{-1} \text{ \$}^{-1}$)	($\text{m}^3 \text{ yr}^{-1}$ each water meter)	\$ m^{-3}
(1)	(2)	(3)	(4)	(5)
1	4.181×10^6	-4.869×10^6	$11.3 \times 12 = 135.6$	0.30
2	6.624×10^6	-5.402×10^5	$45.3 \times 12 = 543.6$	0.40
3	1.023×10^6	-4.268×10^4	Open block ^(B)	0.45

^A From Loáiciga and Renehan (1997).

^B There is no formal size in block 3 on a meter-by-meter basis, even though a practical limit could arise if blocks 1 and 2 use their total available allocations. In this instance, the block-3 available water equals the annual available water (Q_{max}) minus the block 1 and 2 total allocations.

Table 5 provides overall results from the rate-design problem. It is seen in column (2) that the (fixed) water-meter charge raised \$ 1.143 million or 11.6 % of the total annual revenue (= \$ 9.826 million), the remainder being variable revenue from metered water sales. It is also evident from columns (3) and (6) that the total annual revenue equaled the total annual cost, so that the rate design produced complete cost recovery. From columns (5) and (6), the variable cost (= \$ 3.526 million) was 35.9 % of the total annual cost. Most of the total cost of water supply (i.e., 64.1%) arose from fixed expenditures (e.g., infrastructure upkeep and repayment, administrative and labor payroll), while variable costs stem from items tied up to the level of water production (treatment supplies, pumping costs, casual contracted labor, and the like).

Table 3. General data for the single-family rate design example (for the City of Santa Barbara, 1995).

	Water meters	Population	Basic use	Fixed Cost	Available water ^(A)
(1)	(2)	(3)	(4)	(5)	(6)
Dimensions	# of meters	# of people	m ³ yr ⁻¹	\$ yr ⁻¹	m ³ yr ⁻¹
Symbol	N	G	U	A _F	Q _{max}
Value	16326	93957	34.31	6.3 x 10 ⁶	25.4 x 10 ⁶
Source	Kennedy/Jenks Consultants (1995)	Loáiciga and Renehan (1997)	Tchobanoglous and Schroeder (1987)	Loáiciga and Renehan (1997)	City of Santa Barbara (1998)

^A Maximum water supply for the single-family residential factor, equals 45% of the total water-treatment capacity.

Figure 4 shows the 1995 (post-drought) rate structure imposed by the City of Santa Barbara and our optimized rate structure. The optimized rate structure was designed based strictly on water use, revenue, cost, and system-size considerations as explained above. The 1995 rate structure, on the other hand, was the result of cost, revenue and political considerations. For example, block-3 users include an influential group of residents who lobbied to avoid substantial differences between the block-2 and block-3 prices, thus avoiding the larger contribution to revenue that the optimal structure assigned to block-3 users. Organized constituencies in the predominantly block-2 and block-3 user groups lobbied against block-1 prices that they considered too low, tantamount to a subsidy that ultimately would have to be paid by them. In addition, the City of Santa Barbara does not fully recover water-production costs from water sales and meter charges. This is so because it has at its disposal additional revenues that it earns from utility user fees and from returns on investment of a water fund that has been built over the years (Loáiciga and Renehan, 1997). Moreover, considerations to water pricing in other sectors (multi-family residential, commercial, governmental) may also have been factored in the setting the 1995 single-family rate structure in the City of Santa Barbara (Kennedy/Jenks Consultants, 1995).

Table 4. Optimized single-family block prices, use, variable revenue, and variable cost (1995 conditions).

Block	Item ^(A)			
	Price (\$ m ⁻³)	Water use (m ³ yr ⁻¹)	Revenue ^(B) (\$ yr ⁻¹)	Variable cost (\$ yr ⁻¹)
(1)	(2)	(3)	(4)	(5)
1	0.404	2.214 x 10 ⁶	0.8944 x 10 ⁶	0.6641 x 10 ⁶
2	0.970	6.100 x 10 ⁶	5.197 x 10 ⁶	2.440 x 10 ⁶
3	2.00	0.938 x 10 ⁶	1.871 x 10 ⁶	0.4220 x 10 ⁶
	SUM =	9.252 x 10 ⁶	8.683 x 10 ⁶	3.526 x 10 ⁶

^A The monthly water-meter charge equaled = \$ 5.83, applied to each single-family water meter.

^B This is variable revenue, which accrues from metered water sales and does not include revenue from the fixed charge excised to each water meter.

Table 5. General results for the single-family rate design example.

Total water	Fixed revenue	Total revenue ^(A)	Fixed cost	Variable cost	Total cost ^(A)
(1)	(2)	(3)	(4)	(5)	(6)
m ³ yr ⁻¹	\$ yr ⁻¹	\$ yr ⁻¹	\$ yr ⁻¹	\$ yr ⁻¹	\$ yr ⁻¹
9.252 x 10 ⁶	1.143 x 10 ⁶	9.826 x 10 ⁶	6.300 x 10 ⁶	3.526 x 10 ⁶	9.826 x 10 ⁶

^A Equals the sum of the fixed revenue in column (2) plus the sum of the block revenues from column (4), Table 4.

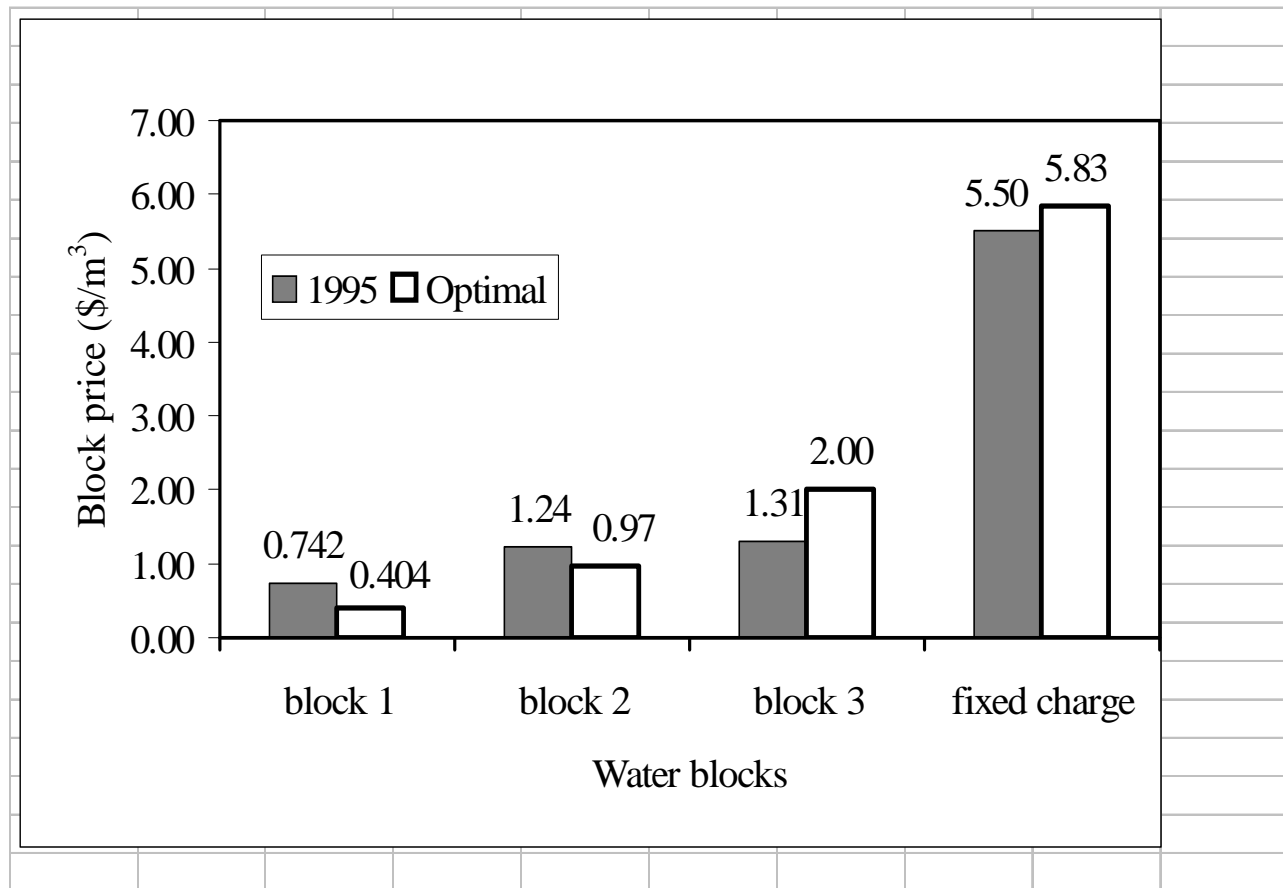


Figure 4. Rate structures in the City of Santa Barbara (1995) and that obtained with the nonlinear optimization model (optimal). The sizes of blocks 1, 2, and 3 are the same as those in Figure 2. The fixed charge is the monthly fee applied to each water meter.

Conclusion

The nonlinear programming method for water-rate structure design developed in this work incorporates all the relevant water-use and water-supply factors that enter in rate design for residential customers: a block structure, size and capacity of the water system, water-production costs, revenue stream and cost recovery, and block by block water-use functions. The nonlinear programming method has a quadratic objective function and linear and quadratic constraints, and is easily coded in the Microsoft Excel spreadsheet and solved with its Solver software. The application of the method for water-rate structure design showed that, in spite of the discrepancies between the contexts leading to the optimized water prices and the water prices approved by an elected city council, the two rate structures exhibit remarkable similarities in the range of the blocks' prices and the closeness of the water-meter charges. Evidently, the nonlinear programming method developed in this work holds promise as a screening tool in the search for residential water-rate structures.

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