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AEC Contract No. W-7405-eng-48

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April 30, 1964

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Introduction

About three years ago there occurred a revival of interest in the S matrix as a framework for the formulation of fundamental subatomic laws. The S matrix was defined by Wheeler in 1939, and the possibility of its role being fundamental was suggested already in 1943 by Heisenberg, who recognized a number of the important advantages over conventional quantum theory and who stressed certain properties of the S matrix that remain central features of current work. The property now generally called "maximal analyticity" was not appreciated in the forties, however, and without this notion S-matrix theory lacked dynamical content. Heisenberg and the other S-matrix students of that period eventually lost interest when they realized they had no way to compute interparticle forces, and more than a decade elapsed before the S matrix was resurrected as a competitor with quantum field theory.

The gradual appreciation of the dynamical content in analyticity occurred during the last half of the fifties and involved many names, major figures being Gell-Mann, Goldberger, Low and Mandelstam. All results at this stage, however, were either motivated by or derived from field theory, and to this day many theorists, including at least some of the aforementioned quartet, believe that even if S-matrix axioms can be found they will simply

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* Prepared for delivery as an invited paper at the Washington meeting of the American Physical Society, April 27-30, 1964.

amount to an alternative statement of field theory. In this view the search for S-matrix axioms is an interesting but academic exercise that is unlikely to increase our understanding of nature. Were I to share such an opinion I should not be taking your time today. I believe that the effort to formulate fundamental laws directly in terms of the S matrix, even if destined only to be partially successful, is opening major new avenues of development that cannot be found through field theory. This belief will be defended in what follows.

It appears that Jack Gunson, Henry Stapp and I independently thought of adding maximal analyticity to the old Heisenberg scheme and of attempting thereby to avoid the use of the field concept. During the past three years Stapp, Gunson and also David Olive have made serious efforts to find a minimal set of S-matrix axioms that will reproduce all properties conjectured on the basis of perturbation field theory. In contrast, my own chief interest has been in "bootstrap" properties that cannot be motivated by a perturbation approach but which have been suggested by experiment. I have been struck, nevertheless, by difficulties encountered in the work of Stapp, Gunson and Olive that hint at a connection between their goal and that of the bootstrappers. I propose today to stress these difficulties--rather than the numerous recent successes of S-matrix axiomaticians--because it is only in the difficulties they have uncovered that distinctions from perturbation field theory are to be found.

It must be added that the opinions I shall present today concerning the difficulties in S-matrix theory are not all shared by Stapp, Gunson and Olive. Even among the small clan of S-matrix enthusiasts, there exist serious differences of outlook.

It is a tragedy that Landau is unable to continue his role in the debate. He was perhaps the first unequivocally to reject the field concept and by 1959 was well aware of the power of combining unitarity with analyticity. Landau at that point, of course, was working with amplitudes both on and off the mass shell while the S matrix is entirely on the shell. Current opinion, which I share, is that taking scattering amplitudes in a meaningful and unique way off the mass shell would be equivalent to field theory; only if such extensions turn out to be meaningless is there likely to be a difference between field theory and S-matrix theory. Landau's opinion today about such questions would be of enormous value.

A Tentative Set of Axioms to Replace Perturbation Field Theory

It is perhaps premature to speak of a consensus having being arrived at in the work of Gunson, Olive and Stapp, but their recent writings contain many common points. In order to achieve all the general properties of the S matrix that are suggested by perturbation field theory, they believe that approximately five axioms suffice. These axioms refer only to the S matrix and its analytic continuation and do not invoke the full apparatus of quantum mechanics, with its state vectors, complete sets of operators, and commutation rules. Little more than the superposition principle is maintained. The only observables are supposed to be particle momenta and spin orientations, before and after collisions. Actually the usual connection by fourier transform with <u>macroscopic</u> space-time must be assumed if one is to connect theory with experiment, but localized space-time functions cannot be formed from momenta constrained to the mass-shell. The sharpest experimental definition allowable is the particle Compton wavelength. By contrast there is no known limit to the accuracy with which momentum can be defined, at least in an infinite universe; the mass-shell momentum-energy continuum is experimentally realizable even though the space-time continuum is not.

The simple framework of S-matrix theory and the restricted set of questions that it presumes to answer constitute its chief advantage over quantum field theory. The latter is burdened by a superstructure inherited from classical electromagnetic theory that seems designed to answer a host of experimentally unanswerable questions. Current S-matrix theory goes too far in the other direction, however, because it is not designed to describe experiments where interparticle forces continue to act <u>while</u> momentum measurements are being performed. The known forces that can behave in this way are the long-range interactions of electromagnetism and gravity, a circumstance leading <u>me</u> to believe that in its current form S-matrix theory can at most describe the short-range nuclear interactions. I shall have more to say later about the problem of electromagnetism. For the moment, let me remark only that the difficulty here has been obscured by the concentration on S-matrix properties shared with perturbation field theory. In perturbation theory one cannot easily consider persistent forces.

The first two of the five Gunson-Olive-Stapp S-matrix axioms are clean and non-controversial: (1) Lorentz invariance and (2) decomposition into connected parts. No comment is required about Lorentz invariance, which was emphasized already by Heisenberg in 1943, but the decomposition law is perhaps less familiar. It represents the obvious physical fact that independent, uncorrelated events can occur, and it states that any S-matrix element may be broken into sums of products of "connected parts", each depending on a different and non-overlapping subset of particle momenta and multiplied by the appropriate energy-momentum conservation delta-function. Subsequent S-matrix axioms relate to these connected parts, which do not contain delta functions.

The third axiom is that of the correspondence between particles and poles in connected parts; a connection apparently noticed first by Kramers. Here we already encounter some division of opinion. In the recent work of Olive the pole-particle correspondence is postulated only in physical regions, where it is directly related to the possibility of a causal sequence of macroscopically spaced collisions between stable particles. Poles in unphysical regions, in particular those associated with unstable particles, are then to be deduced from the two axioms still to come. Such a sharp distinction between stable and unstable particles at the axiomatic level disturbs me, however. Physically it is clear that the transition between stability and instability is a smooth one; mathematically the dynamical considerations that predict resonances on the basis of the final two axioms just as well can predict bound states.

To my mind it is more satisfactory to treat all poles on a common basis, regardless of their location. As Gunson has argued, once the possibility of analytic continuation is accepted, <u>any</u> part of the complex momentum space is in principle accessible--through sufficiently accurate measurements in the physical region, followed by extrapolation. You may object that the stable particles necessarily play a special role in S-matrix theory since they define the space in which the S matrix acts. It is unnecessary to speak of such a space, however, if one deals directly with connected parts. It turns out that the residues of all poles in connected parts are factorizable, each factor being itself a connected part for a smaller collection of particles, one of which corresponds to the original pole. As Zwanziger pointed out, if the pole in question corresponds to an unstable particle, one thereby uniquely defines a connected part involving this unstable particle. Connected parts for any collection of particles--stable or unstable--may democratically be defined in such a manner.

Factorizability of residues, by the way, as shown by Stapp and others, seems to be a consequence of the final two axioms. Were factorizability not to emerge, however, the particle-concept itself would be impossible. Here is an example of "bootstrapping" in axiomatics.

And now a difficulty: If the photon has a strictly zero mass, the infra-red phenomenon spoils the simple pole-particle correspondence. Put more simply, the basic notion of an initial or final state with a definite number of particles loses meaning when, regardless of the precision of energy-momentum determinations, the number of low-frequency photons is uncontrollable. This again is a facet of electromagnetism obscured by perturbation field theory, which considers only finite numbers of photons. Some S-matrix theorists believe the infra-red problem to be an inessential difficulty because it has been surmounted in field theory and because the photon, after all, is "just another particle." I do not agree. I believe there is vital significance in this mismatch between electromagnetism and current S-matrix axiomatics. I believe the photon to be an aristrocrat.

Returning to our catalogue, the fourth axiom, as usually stated, associates branch points in connected parts with channel thresholds and defines the nature of each such isolated singularity by giving a formula for the change in a connected part when a single circuit is made around the branch point. The discontinuity formula, long known in a variety of expressions, has been stated by Gunson and Olive in an elegant general rule:

$$T_{ab}(s) - T_{ab}(s_n) = \int_n T_{an}(s)\, T_{nb}(s_n) ,$$

with $S = 1 + T$. The point s_n lies directly below the point s on the next Riemann sheet, reached by a single circuit around the singularity in question. The integral runs over all variables of that channel whose threshold lies at the branch point. Note that T_{ab} is in general not a connected part and contains delta functions. These, however, can be shown to appear in a consistent way on the two sides of the equation; after cancellation of the delta functions there remains a formula involving connected parts only. A definition of the physical sheet and the physical region must accompany the discontinuity formula to make it complete and to guarantee unitarity. These matters have been discussed with care both by Olive and by Stapp. Zwanziger has emphasized that threshold branch points for channels containing unstable particles are described by this same discontinuity formula, so the democratic character of the axioms can be maintained.

It goes without saying that we are in trouble here again with photons. Adding one or several zero-mass particles to a channel fails to displace the threshold, and the unique association of isolated branch points with individual channels is lost. What recipe may replace the discontinuity formula is not known. Unitarity of the S matrix in physical regions follows from the discontinuity formula, so in losing the latter we have lost unitarity. Indeed, looking back over our catalogue it appears that only the axiom of Lorentz invariance has failed to clash with electromagnetism; there is no avoiding the conclusion that the theory presently under consideration describes a world without photons. Fortunately we seem to see a good approximation to such a world if we look only at strongly-interacting particles.

One final axiom remains to complete the S-matrix properties guessed on the basis of perturbation field theory. This fifth axiom postulates that, in addition to particle-poles and threshold branch points, the only other singularities of connected parts are those implied by the analytic continuation of the set of discontinuity formulas. This postulate, which I shall call maximal analyticity of the first degree, has a marked bootstrap aspect, meriting discussion.

The additional singularities are generated through the integration over products of connected parts in the discontinuity formulas. They arise by the "pinching" of combinations of singularities. The simplest type of Landau singularity, as they are called, arises from the pinching of a pair of particle poles, but a pole also may pinch with a threshold branch point or with a Landau singularity; two Landau singularities may pinch with each other, and so on. Axiom #2 starts us off (presumably) with an infinite number of particle poles and certainly axiom #3 gives an infinite number of threshold branch points, so the full set of singularities, even with maximal analyticity, is enormously complicated. In fact the combined set of axioms at this point runs the risk of a contradiction, because we evidently require that analytic continuation in momenta is everywhere possible. Isolated singularities (poles and branch points) cause no trouble in this respect, but what happens if singularities so multiply through the discontinuity formulas as to become dense?

At present it is a matter of faith that such does not happen. This faith has a concrete basis, however, in experience with iterative calculations--where a phenomenon has been observed which I shall call the "Mandelstam progression." Mandelstam discovered that with four-line connected parts (two incoming and two outgoing particles), if you start with the physical-sheet particle poles and threshold branch points and generate Landau singularities by an iterative procedure, there is a systematic tendency for the new singularities from each iteration to be located farther from the physical region than the previous set. Recently Hwa has found this same phenomenon in five-line connected parts. Fluctuations may occur in the progression (anomalous thresholds) but there is every indication that the singularities in a given finite region of the complex momentum space do not continue indefinitely to increase in number. A key requirement of S-matrix theory is to establish that such is really the case.

Recently a quartet of Parisians, Fotiadi, Froissart, Lascoux and Pham, have developed a powerful approach to the Landau singularities which eventually may prove strong enough to answer this question. Alarmingly, the mathematical basis of their new method is homology theory, with which few physicists are familiar at present; but a multi-sheeted Riemann surface in several complex variables is undeniable a matter of topology. Advocates of the S-matrix approach cannot evade this circumstance.

At the risk of being tedious I once again call to your attention the importance to S-matrix theory of the absence of zero-mass particles. The Mandelstam progression has a chance to operate only because, among strongly interacting particles, there are none with vanishing rest mass.

The smallest particle-masses necessarily provide the scale for the spacing of singularities.

Although the above list of five axioms will require further refinement and study, it is plausible from the work of Gunson, Olive, Stapp and especially of Mandelstam that all the significant physical content of perturbation field theory is contained therein. In fact if one wishes to treat a few spin-0 or spin-$\frac{1}{2}$ particle poles as given, with small residues, the same power series expansions apparently can be developed from these axioms as are derived from a Lagrangian with a corresponding set of fields. No further assumptions are needed. We have seen, however, that if the current version of S-matrix theory describes anything it can only be the world of strongly-interacting particles. With electromagnetism turned off, not only does the photon disappear but so do the primary interactions of electrons and muons, which are electromagnetic. Not even the residual weak interactions would be tractable because the electron mass (if not that of the muon) presumably would vanish in the absence of electromagnetism, and electron-neutrino pairs would become just as awkward for the S matrix as are photons. Now, to be restricted to strong interactions is not necessarily a fatal flaw of our theory, but perturbation expansions cannot then be trusted. The content of the theory has to be sought by methods other than power series in coupling constants.

Maximal Analyticity of the Second Degree

Perturbation field theory tolerates the arbitrary insertion of elementary particles of spin 0 and 1/2 , and even of spin-1 if coupled to an appropriately conserved durrent. It has, however, never been established that the perturbation power-series are meaningful, so one cannot infer that our five S-matrix axioms necessarily permit poles corresponding to elementary particles. I refer here to poles whose positions and residues can be arbitrarily assigned without violating the axioms. Perhaps no such poles <u>can</u> be tolerated, in which case there may be no need for further axioms to complete a theory of strong interactions. Perhaps only one set of poles is consistent and that is the one we find in nature. The plausibility of such a conjecture is enhanced by the difficulty of fitting photons or leptons into the S matrix. These are the particles that still appear to us as "elementary". None of the strongly interacting particles has such an appearance.

Despite its attractiveness the conjectured sufficiency of the above five axioms lacks support from the approximation procedures currently used to implement these axioms. What is the basis of these procedures? It is that connected parts in a local region of the complex momentum-space are dominated by "nearby" singularities, the collective effect of distant singularities being representable by boundary conditions. Instead of a series ordered by powers of coupling constants, we have a series of singularities, ordered according to increasing distance from the point of interest. Ignoring all singularities beyond a certain distance leads, through the Cauchy formulae, to an approximate set of integral equations for the connected parts--<u>provided that boundary conditions at infinity are added</u>. These boundary conditions do not seem entirely to be contained in the five axioms.

How are the boundary conditions chosen? If one believes in nuclear democracy, as I do, one chooses the solution to any particular approximate set of equations that causes all poles to be dynamically determined--like the bound states of a potential. This is the so-called "bootstrap" dynamics, and it necessarily leads to the property that all poles are continuable in angular momentum. A converse conjecture has been made by Frautschi and me that an adequate general formulation of the necessary boundary condition is simply to require that all poles be Regge poles. A recent study of perturbation field theory by Gell-Mann, Goldberger, Low and Zachariasen suggested a counterexample to this conjecture, but the matter is not settled, even in the perturbation context. In any event, as emphasized earlier, there is no reason to accept perturbation arguments as necessarily relevant to the S matrix.

Whether or not the uniform requirement of Regge continuation is sufficient, the object of the boundary condition is to eliminate all "unnecessary" poles. For that reason I like to call the sixth requirement "maximal analyticity of the second degree." Let me emphasize the possibility, before leaving this point, that the apparent necessity for a sixth condition may be a consequence of our approximation procedure. In neglecting all singularities beyond a certain distance, an asymptotic requirement implicitly contained in the first five principles may have been lost.

Conclusion

To summarize the current S-matrix picture, which apparently is relevant only to strong interactions, three different although not independent questions can be identified. (1) Can the fifth axiom, maximal analyticity of the first degree, be solidified? The problem here is the propagation of singularities via the discontinuity formula; major progress may require exploitation of homology theory. (2) Can a bootstrap boundary condition, our sixth principle, be found that determines in a democratic fashion all the particle-poles? Continuation in angular momentum is a key consideration here. (3) Can an approximation procedure based on nearest singularities plus the boundary condition be made systematic and then successfully employed to predict the strongly interacting particles.

I should remark parenthetically that my own optimistic feelings about the first two questions are based largely on the qualitative success in the understanding of strong interactions already achieved by crude dynamical applications of the nearest-singularity principle. I can see no reason for this success if a meaning fails to exists for maximal analyticity of first <u>and</u> second degree.

These three questions are tied together by asymptotic considerations. A finite number of Mandelstam-type iterations produces an acceptably finite density of singularities; the difficult aspect of question #1 therefore is to show that asymptotically the singularities keep moving to greater and greater distances. If and when the asymptotic behavior of this progression becomes understood, question #2 may disappear; that is, it may turn out to be unnecessary to add a pole-determining boundary condition. In any event an understanding of the most distant singularities should clarify whether dynamical calculations can in fact be based on an ordering of singularities according to distance.

In closing I have three remarks about electromagnetism. First of all, we need not be distraught because the currently-defined S matrix is too limited to describe this most familiar of the interactions. All physical theories of the past have been limited to special ranges of phenomena and have been replaced in time by broader theories. It is probably hopeless at present to construct a complete theory; the problem is to identify those areas of nature than can meaningfully be approximated as separate. Strong interactions appear to constitute such a subdivision. Second, it has already developed in practice that, given the strong interaction S matrix, a recipe can be found for adding electromagnetic perturbations of finite order in the fine structure constant. What remains obscure is the handling of persistent electromagnetic effects or, if you like, infinite numbers of soft photons. In fact, Zwanziger and Weinberg have shown that for reactions which can be characterized approximately as involving a finite number of real photons, the special properties of electromagnetism usually associated in field theory with gauge invariance follow automatically in momentum space from Lorentz invariance and the zero photon mass. Here perhaps is an indication that a concept broader than the S matrix, but still based on the momentum-energy continuum rather than the space-time continuum, eventually will encompass particles of zero mass.

Finally, let me point out the logical incompleteness of current S-matrix theory in its failure to provide the mechanism by which particle momenta are to be experimentally measured. The actual determination of momentum, as well as its definition, requires a course-grained macroscopic space-time measurement that never can be described through the present conception of the S matrix. In practice such measurements always depend on electromagnetic interactions; and a little thought suggests it is impossible, in principle, to perform a momentum determination without employing the weak long-range forces characteristic of electromagnetism. The zero-mass photon, together with the small magnitude of the fine structure constant, makes it possible for one isolated system to observe another, and thereby plays a role that cannot be filled by any of the strongly interacting particles.

If this view is correct the photon mass and the fine structure constant are interlocked with the theory of measurement itself, perhaps even with the meaning of macroscopic space-time, and their values never will be explained purely by dynamical considerations. In contrast the parameters of strong interactions, having no connection with the measurement process, have a chance of being determined through dynamics. My survey today has described the continuing attempt to formulate a purely dynamical theory of the strong interactions.

You may have been struck by the absence from this survey of symmetry considerations, apart from Lorentz invariance. This was not an oversight but represents a widely held conviction that arbitrarily postulated symmetries have no more place in the basic theoretical structure than do arbitrarily postulated particles. The presence in strong interactions of SU_2 and partial SU_3 symmetries, as well as time-reversal and parity, cannot be denied; but neither, for example, can the existence of the pion and the nucleon as especially stable particles. Confusion about such questions arises because in special limited applications of S-matrix theory the existence of certain symmetries and particles is often added to the list of basic principles. There is room, however, to hope that all strongly interacting particles <u>and</u> symmetries ultimately will emerge together as bootstrap consequences of the five or six principles we have discussed here today.

My final remark is directed to a question raised at the beginning: What can the S-matrix approach teach us that cannot just as well be learned from field theory? Perhaps nothing. Perhaps a future field theory will somehow dispense with Lagrangians and describe a nuclear democracy; but then how will this field theory recognize the distinction between electromagnetic and nuclear interactions? The original idea behind field theory, after all, was that every interaction is like electromagnetism. The absence of a classical limit for quantum fields associated with massive particles is ignored in the properties assigned to these fields. Conversely the assignment of a non-zero mass to the photon seems perfectly allowable in field theory.

S-matrix theory in contrast, permits no doubt that the zero mass of the photon gives this particle a distinguished status, outside the dynamical bootstrap. Furthermore, with the emphasis on physical observability, one becomes sensitive to a possible connection between the unusual photon properties and the basic requirement underlying all of physics that one isolated system be capable of observing another. We are approaching the time when this requirement must searchingly be examined. I do not see how it <u>can</u> be examined in any framework that fails to rest squarely on physical measurements themselves.

When I am told that S-matrix theory destroys the unity of physics by placing electromagnetism in a separate category from nuclear interactions, I do not know what to say. Without such a separation, there would <u>be</u> no physics.

Thank you.