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Response to "Comment on 'Dynamics of zonal flow saturation in strong collisionless drift wave turbulence' " [Phys. Plasmas 11, 1744 (2004)]

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The Comment of Krommes is addressed. It is shown that all of the substantive claims therein are incorrect. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1669399]

We would like to thank J. A. Krommes for his careful reading of our paper [Kim and Diamond (2002)] (KD)[1] and for the points about the spectral symmetry and τ scaling in his comments (2003). We, however, find that the latter is largely concerned with minor details, which are not at all pertinent to the goals of the analysis performed by KD; the aim of KD was to study the excitation of nonaxisymmetric modes and its effect on Kelvin–Helmholtz instability through a basic order of estimate. Also, all of his substantive claims are incorrect.

First, in the cold ion limit, the nonlinear growth rate of generalized Kelvin–Helmholtz (GKH) mode obtained by Kim and Diamond[1] [KD's Eq. (13), or Krommes' Eq. (1)], and the equivalent expression derived by Krommes and Kim[2] [Krommes' Eq. (2)] become identical, modulo notation for response function in the case when the background turbulence is isotropic. This can be easily seen by expressing

$$|\hat{z}\cdot(\mathbf{q}\times\mathbf{k})|^2(\mathbf{q}\cdot\mathbf{k}),$$

in Krommes' Eq. (2) as

$$(q_x k_y - q_y k_x)[(q_x^2 - q_y^2)k_x k_y + q_x q_y (k_y^2 - k_x^2)]. \tag{1}$$

For isotropic turbulence, which was a modeling assumption in the interest of simplicity in KD, the last term in Eq. (1) vanishes since $k_y \sim k_x$. Thus, $|\hat{z}\cdot(\mathbf{q}\times\mathbf{k})|^2(\mathbf{q}\cdot\mathbf{k}) \sim (q_x k_y - q_y k_x)(q_x^2 - q_y^2)k_x k_y$. Then, by changing $q_x \to p$ and $q_y \to q$, Krommes' Eq. (2) becomes identical to the growth rate given by KD's Eq. (13) [Krommes' Eq. (1)], except for the difference in the response function. Though the disputed term might quantitatively contribute, depending on the details of the modulation process, the impact of this contribution upon the estimates presented in KD is *insignificant*. Indeed, the pattern of convective cell generation depends on the structure of the underlying turbulence, as shown by many authors in the past.[3] We do agree, though, that the 1/2 factor in front of this diagonal term in KD should be corrected to unity.

Second, in the case of nonzero ion temperature, the main equations of KD were systematically derived by taking moments of gyrokinetic equation, and by taking the finite Larmor radius (FLR) effect to be small (i.e., $\rho_i^2 k^2 = \tau \rho_s^2 k^2 \ll 1$). Specifically, KD kept the FLR effect to first order in the potential equation and ignored it in the temperature equation. It *is* self-consistent to keep only the leading-order term for the evolution of temperature itself, since the effect of temperature appears as first-order FLR correction in the potential equation for GKH. Instead of showing this derivation, we shall show below that the same set of the equations, which was in fact used by Rogers *et al.*,[4] can also be obtained from Dorland and Hammett (DH),[5] which Krommes referred to in his Comment. We start with DH's Eq. (56), which takes the following form:

$$\frac{\partial \tilde{n}}{\partial t} + [\psi, \tilde{n}] + \frac{\tau}{2}[\nabla^2 \psi, T] = 0, \tag{2}$$

in 2D slab geometry by ignoring linear terms. Here, $\tilde{n} = n_1/n_0$ and T are linear perturbations of guiding center density and temperature. Density and temperature are normalized by background density n_0, and ion temperature T_{i0}, respectively, and the length by ρ_s. $\psi \simeq (1 - \tau k^2 \rho_s^2/2)\phi$ is the gyrophase averaged potential. To obtain the expression for $\tilde{n}$ (or n_1), we use DH's Eq. (27) in DH's Eq. (5), which gives

$$\left(1 - \frac{b_0}{2}\right)\frac{n_1}{n_0} = \frac{b_0}{2}\frac{T_{1i}}{T_{i0}} + b_0 \frac{e\phi}{T_{i0}} + \frac{e}{T_{e0}}(\phi - \langle\phi\rangle), \tag{3}$$

to first order in $b_0 = \tau \rho_s^2 k^2$ (i.e., FLR effect). Thus, by measuring ϕ by T_{e0}/e, length by ρ_s, and temperature by T_{i0}, we obtain, to first order in FLR

$$\begin{aligned}\tilde{n} &= \phi - \langle\phi\rangle - \nabla^2\left[\phi + \frac{\tau}{2}T + \frac{\tau}{2}(\phi - \langle\phi\rangle)\right] \\ &= \psi - \langle\psi\rangle - \nabla^2\left[\psi + \frac{\tau}{2}T + \tau(\psi - \langle\psi\rangle)\right].\end{aligned} \tag{4}$$

On the other hand, to leading order, DH's Eq. (60) simplifies to

$$\frac{\partial T}{\partial t} + [\psi, T] = \frac{\partial T}{\partial t} + [\phi, T] = 0. \tag{5}$$

Note that Eqs. (2), (4), and (5) are identical to Eq. (1) in Rogers *et al.*[4] For GKH (or CC), $\tilde{n} = -\nabla^2[\psi + \tau/2T] = -\nabla^2[\phi + \tau/2T]$, by putting $\phi = \langle\phi\rangle$ to ensure nonadiabatic electron response for GKH. Therefore, Eq. (2) recovers, for example, KD's Eq. (44) to first order in b_0. Note that KD used different notation for ψ, and also that the Laplacian of KD's Eq. (28) is used to obtain KD's Eq. (44). Krommes' Eq. (7b) for temperature contains a first-order FLR effect, which is not necessary for the reason mentioned above. That is, the finite ion temperature effect on potential is first order in FLR effects, as can be clearly seen from Krommes'

Eq. (7a). Thus, to first order in b_0, it is self-consistent to keep only the leading-order term in the temperature equation. It is, of course, an interesting question as to how to extend the analysis in KD[1] and also in Rogers *et al*.[4] to retain higher-order corrections. However, this is a technical detail which is outside the scope of KD.[1]

Third, for the purpose of understanding the basic physics without being lost in a forest of formalism, KD used the conservation of N_k (i.e., wave-kinetic equation) with a simple quasilinear response to relate the ITG pressure perturbation to ϕ. Note that up to first order in τ, both potential enstrophy and pressure are conserved, separately, as indicated in KD.

Finally, the wave-kinetic equation involves linear frequency (and also nonlinear frequency shift), as shown by Smolyakov and Diamond,[6] for instance. Thus, the modulation of the wave-kinetic equation naturally involves the modulation of linear frequency. Note that the modulation of nonlinear frequency is higher order in fluctuation level. Therefore, it is *not* incorrect to modulate the frequency (as done by KD). In fact, the modulation of the frequency is the only channel through which the effect of zonal temperature feeds back on the wave population.

In summary, except for a factor of 1/2 in front of the diagonal Reynolds stress term in KD which should be corrected to unity, all other criticisms raised by Krommes are unfounded. In particular, there is *no* conceptual error in KD.

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