The N=90 Transitional Nuclei ^{150}Nd and ^{152}Sm Revisited

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Abstract

The purpose of this paper is to show that recent data on the ground–state band and excited states based on the 0_2^+ level in ^{150}Nd and ^{152}Sm, especially the measured B(E2) values, can be well described by including a ΔK=0 coupling between rotational bands. This is contrary to recent statements in the literature. The experimental data are compared with models which have supported the widely differing interpretations of these transitional nuclei. These interpretations include describing excited states as rotational excitations of single–phonon states, the multiphonon 'phase coexistence' picture, and the X(5) critical–point description.

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The N=90 nuclei, ^{150}Nd and ^{152}Sm, are often called transitional nuclei. Lighter isotopes display vibrational–like spectra while heavier isotopes show more rotational–like behavior. In particular, ^{152}Sm has been the focus of many theoretical and experimental investigations. Historically, excited states in this nucleus were described as rotational excitations of single–phonon states (β and γ vibrations) with deformations similar to that of the ground–state band [1–6]. An alternative description reinterpreted the non–yrast states in terms of multi–phonon configurations based on near–spherical shapes [7–11]. This has been called the shape 'phase–coexistence' description. Interest in ^{150}Nd and ^{152}Sm has been heightened with the suggestion that they represent the 'empirical realization of a critical–point description' of a 'first–order phase transition' (denoted as X(5)) from a spherical to an axially deformed shape [12–14].

The issues raised in the interpretation of the excited level structure of these nuclei has spurred several new experimental studies which have yielded extensive new data, including accurate B(E2) values [9,11,14]. It has been claimed that the 'level spacings and B(E2) values cannot be the result of simple mixing of pure rotational bands' [10] and that the multiphonon or X(5) interpretations give a more accurate description. In this paper, we will show that the available data on ^{150}Nd and ^{152}Sm, especially the B(E2) values, can be well described by mixing between rotational bands as discussed by Bohr and Mottelson [15]. Specifically, we will consider the rotational–coupling effects in the ΔK=0 transitions between the ground–state rotational band and the excited K$^\pi$=0$_2^+$ 'β–vibrational' band. A microscopic justification of the parameters we extract for ^{152}Sm is found in the Pairing–Plus–Quadrupole model of Kumar [4,5].

We start by assuming that the structures based on the 0_2^+ states in ^{150}Nd and ^{152}Sm are rotational bands with deformations very similar to those of their respective ground–state bands, as would be expected if the 0_2^+ states were β–vibrational levels. We now follow the prescription found in [15] for describing the effects from mixing the two bands by an effective ΔK=0 coupling. The amplitudes for the E2 transitions between the excited band and ground–state band deviate significantly from the leading–order intensity relation:

$$B(E2; I_i \rightarrow I_f) = < I_i 020 | I_f 0 >^2 M_1^2 \quad (1)$$

where M_1 is the intrinsic matrix element for the transition. This is shown in Fig. 1, where the data points would lie on a horizontal line if Eqn. 1 held. If one allows a coupling between the two bands, the interaction leads to mixed states such that:

$$|\widetilde{0_1} > = \alpha |0_1 > - \beta |0_2 > \quad (2)$$

$$|\widetilde{0_2} > = \alpha |0_2 > + \beta |0_1 > \quad (3)$$

where $|0_1>$, $|0_2>$ ($|\widetilde{0_1}>$, $|\widetilde{0_2}>$) are the unperturbed (perturbed) wavefunctions and $\alpha^2+\beta^2=1$. The amplitude, β, may be written as:

$$\beta = (1 + (R + \sqrt{R^2+1})^2)^{-1/2} \quad (4)$$

with, R = ΔE/2V where ΔE represents the difference in energy between the unperturbed states and V is the interaction matrix element. The interaction matrix element may be expanded [15] as[1]:

$$V = h_1 I(I+1) + h_2 I^2 (I+1)^2 + ... \quad (5)$$

The h_n coefficients are matrix elements related to intrinsic (rather than rotational) operators. Taking the principal term, and in the limit where V$\ll \Delta$E, we find the mixing amplitude may be written as:

$$\beta \approx \frac{V}{\Delta E} \approx \frac{h_1}{\Delta E} I(I+1) = \epsilon_0 I(I+1) \quad (6)$$

With $\beta^2 \ll 1$ then we may rewrite the coupling of the two bands in the form given by Bohr and Mottelson [15]:

$$|\widetilde{0_1} > \approx |0_1 > - \epsilon_0 I(I+1) |0_2 > \quad (7)$$

$$|\widetilde{0_2} > \approx |0_2 > + \epsilon_0 I(I+1) |0_1 > \quad (8)$$

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[1] A constant term in the expansion renormalizes the intrinsic matrix element, M_1.

This mixing will affect the E2 transition strengths between the bands such that they obey the relationship:

$$B(E2; I_i \rightarrow I_f) = < I_i 020 | I_f 0 >^2 \, (M_1 + M_2(I_i(I_i+1) - I_f(I_f+1)))^2 \tag{9}$$

where M_2 is the contribution to the transition matrix element which we attribute to the ΔK=0 coupling and is given by:

$$M_2 = \sqrt{\frac{5}{16\pi}} \epsilon_0 e Q_0 \tag{10}$$

The data in Fig. 1 are consistent with the generalized intensity rule given by Eqn. 9. The fact that the data in Fig. 1 can be fitted with a straight line immediately reveals that mixing between these two rotational bands can provide a consistent picture of the transition strengths between the two bands. Deviations from a straight line would indicate effects that we have not included such as unequal quadrupole moments of the two bands or multiple band mixing. We can extract values of M_1 and M_2 from the data and these are also given in Fig. 1. Using the values of M_2 extracted from the data in Fig. 1, and the quadrupole moments as deduced from the B(E2; $2_1^+ \rightarrow 0_1^+$) values for the ground–state bands [11,14], from Eqn. 10 we estimate values of $\epsilon_0 \approx$ –0.012(3) and $\approx$ –0.007(1) for ^{150}Nd and ^{152}Sm, respectively. Note, Eqn. 9 is only valid if the mixing amplitude, β, is sufficiently small and we find that this approximation begins to break down for I$\geq$6. The data points in Fig. 1 do not reach beyond this limit.

The band mixing also implies corrections to the E2 matrix elements within the ground–state band which can be written as:

$$B(E2; I_i \rightarrow I_f) = \frac{5}{16\pi} e^2 Q_0^2 < I_i 020 | I_f 0 >^2 \, (1 + \alpha(I_i(I_i+1) + I_f(I_f+1))^2 \tag{11}$$

where the α is given by:

$$\alpha = -\sqrt{\frac{16\pi}{5}} \frac{\epsilon_0 M_1}{e Q_0} \tag{12}$$

The parameter α is usually called the stretching parameter since it is a measure of the increasing deformation of the ground–state rotational band due to mixing with the $K^\pi = 0_2^+$

band. Using the values of the matrix element M_1 (from Eqn. 9 and Fig. 1), the mixing amplitude, ϵ_0 (from Eqn. 10), and the measured quadrupole moments from the ground–state–band B(E2; $2_1^+ \rightarrow 0_1^+$) values, we find values of $\alpha \approx 2.3(7)\times 10^{-3}$ and $\alpha \approx 1.1(2)\times 10^{-3}$ for ^{150}Nd and ^{152}Sm, respectively. We can now see how well these values compare with those extracted from the measured ground–state–band B(E2) values. This is shown in Table I. The second column gives the experimentally measured B(E2) values for transitions up to the I=6 state in the ground–state bands of ^{150}Nd and ^{152}Sm. In the third column we present the B(E2) values calculated assuming that the quadrupole moment throughout the band is a constant and has the same value as that extracted from the B(E2; $2_1^+ \rightarrow 0_1^+$) value (Q_0=5.23(4) eb and Q_0=5.90(6) eb for ^{150}Nd and ^{152}Sm, respectively). In the fourth column, we give the value of the parameter α required by Eqn. 11 to reproduce the measured values. The values of α extracted in this way are in fair agreement with the value as determined from Eqn. 12. It is interesting to note that the change in the quadrupole moment in ^{152}Sm which results from the mixing is consistent with the measured isomer shift between the 2_1^+ and 0_1^+ levels [16].

We now look at the effect of the mixing on the state energies. In general it is possible to express the energies of states in a rotational band in terms of an expansion of the form:

$$E = AI(I+1) + BI^2(I+1)^2 + ... \tag{13}$$

In Fig. 2, we show such fits for the ground–state bands in ^{150}Nd and ^{152}Sm including only the first two principal terms (the extracted values of A and B are shown). It is well known that, for nuclei in this region, including higher terms in the expansion does not give a rapid convergence and the expansion coefficients are rather poorly defined. (The energy can be expanded in powers of the rotational frequency rather than angular momentum [17] and such an expansion typically has a more rapid convergence). Within our approximations, the mixing should give rise to a correction of the energies in the ground–state band proportional to $I^2(I+1)^2$. The constant of proportionality (the change in the expansion parameter, B) will be approximately $\Delta B \approx -\epsilon_0^2(E(0_2^+)-E(0_1^+))$, yielding values of $\Delta B \approx$–97(34) eV and $\Delta B \approx$–

34(7) eV for ^{150}Nd and ^{152}Sm, respectively. We should also see a correction to the energies of the 0_2^+–band equal in magnitude but opposite in sign. The data do not show such shifts and our simple two–band–mixing picture fails. As can be seen in Fig. 2 (from the plot for the 0_2^+ band in ^{152}Sm which is known to much higher angular momentum than the analogous band in ^{150}Nd), the 0_2^+–band is clearly rotational like, but has a moment of inertia 10–15% larger than the ground–state band. It is not so surprising that additional physics is required to explain the exact state energies since we know that the parameters in the energy expansion of Eqn. 13 are very sensitive to deformation and pairing effects. However, the 'phase coexistence' picture (which regards the states based on the 0_2^+ levels as near spherical multiphonon excitations) and the X(5) critical point description, fare considerably worse in their estimate of the level spacings of the excited states (see below).

In Fig. 3 we compare the experimental values for the interband E2–transition strengths in ^{152}Sm (which over the years has received far more theoretical attention than ^{150}Nd) with calculated values from the Pairing–Plus–Quadrupole (PPQ) model [4,5], the Interacting Boson Approximation (IBA) [9], the Geometric Collective Model (GCM) [10], and the X(5) critical–point description [12,13]. In Fig. 4 we compare the experimental level scheme with those calculated from each of these theoretical approaches. In order to get a more quantitative comparison of theory and experiment we follow the prescription in [6] and deduce a figure of merit, χ^2_ν, for each model where:

$$\chi^2_\nu = \frac{1}{N-\nu-1}\sum\left(\frac{B(E2)_{expt} - B(E2)_{model}}{\sigma_{expt}}\right)^2 \qquad (14)$$

In this expression, σ_{expt} is the experimental uncertainty in the measured B(E2) value, N is the number of data points (we use a total of 10 data points corresponding to transtions between the states as shown in Fig. 4), and ν is the number of free parameters for each model. We have used values of ν=2, 2, 4, and 1 for the PPQ, IBA, GCM, and X(5) models, respectively, as described in the relevant papers [4,5,9,10,12,13]. In Table II we compare the experimental and calculated B(E2) strengths, give the deviation (which we define as D=$\frac{B(E2)_{expt}-B(E2)_{model}}{\sigma_{expt}}$), and present the resulting figures of merit. The PPQ model

most closely reproduces the inter- and intra-band B(E2) values and provides a reasonable microscopic justification for the parameters we have extracted from the band–mixing analysis as can be seen in Fig. 3. The IBA and GCM have evident problems in accurately reproducing all the B(E2) values especially for some of the interband transitions. The X(5) description does the worst job, particularly in reproducing the interband B(E2) values.

From Fig. 4, it can be seen that there is a large variation between different model predictions of the state energies. The PPQ best reproduces the experimental state energies. The IBA and X(5) models are able to reproduce the yrast energies and the position of the excited 0_2^+ state reasonably well. However, the GCM, IBA, and X(5) descriptions fail to reproduce the energies of the 2_2^+ and 4_2^+ states.

The differences between the theoretical models can be understood in terms of their derived potentials. The PPQ model describes the excited states as a rotational band based on a β vibration with a deformation very similar to that of the ground–state band. The 0_2^+ β–vibrational band–head is confined within the same deformed minimum as the ground–state. The IBA and GCM approaches have been used to justify the shape 'phase coexistence' picture. The yrast states are described as a deformed rotational structure while the excited states are more spherical and are regarded as multiphonon excitations built on the 0_2^+ state. The wavefunction of this state is not confined to the deformed minimum and is spread over a range of β deformation. The X(5) critical point description approximates the potential as a square well and its one free parameter (energy scale) is adjusted to reproduce the yrast energies. From the discussions above, it is clear that the 'rotational–like' description (PPQ) does well in reproducing both the transition strengths and state energies. The 'phonon–like' descriptions (IBA, GCM, and X(5)) are generally worse in describing the interband transition strengths and are especially poor in reproducing the intraband energy spacings of the excited states.

In summary, we have shown that the available data on the ground–state band and excited states based on the 0_2^+ state in ^{150}Nd and ^{152}Sm, including measured B(E2) values, can be well described by a coupling between rotational bands as expected if the 0_2^+ is predominantly a β

vibration. This is contrary to recent statements in the literature. A microscopic justification of the parameters extracted is found in the Pairing–Plus–Quadrupole model. While it is likely that 0_2^+ is not a pure β–vibration (see, ref. [18] for a recent review of 0_2^+ states in deformed nuclei and their characterization as β vibrations), describing the level sequence based on this state as rotational, and including an effective ΔK=0 coupling to the ground–state band, reproduces salient features rather well and provides the best presently available description of states in these transitional nuclei.

TABLES

TABLE I. Comparison between the measured B(E2)'s in the ground–state band, the B(E2)'s extracted assuming a constant quadrupole moment and no mixing (α=0), and the values of α required to reproduce the experimental values.

^{150}Nd	B(E2)$_{EXPT}$ (eb)^2	B(E2)$_{Q_0=5.23(4)}$ (eb)^2	α ($\times 10^3$)
2→0	0.544(9)	0.544(9)	–
4→2	0.862(9)	0.778(12)	2.0(2)
6→4	0.994(9)	0.857(13)	1.2(1)
^{152}Sm		B(E2)$_{Q_0=5.90(6)}$ (eb)^2	
2→0	0.693(14)	0.693(14)	–
4→2	1.007(14)	0.990(20)	0.4(3)
6→4	1.180(24)	1.091(22)	0.7(2)

TABLE II. Comparison between the B(E2) values (in W.u.) in ^{152}Sm and the predictions of the different models. The deviation, D_{model}, between the model prediction and experimental value is given for each transition and is defined as: $D=\frac{B(E2)_{expt}-B(E2)_{model}}{\sigma_{expt}}$, where σ_{expt} is the uncertainty on the experimental B(E2) value. A figure of merit, χ^2_ν, is also given where ν is the number of free parameters for each model (ν=2, 2, 4, 1 for the IBA, PPQ, GCM, and X(5), respectively – see text). The figure of merit is defined as $\chi^2_\nu = \frac{1}{N-\nu-1}\sum D^2$, where N is the number of data points.

Transition	EXPT	IBA	D_{IBA}	PPQ	D_{PPQ}	GCM	D_{GCM}	X(5)	$D_{X(5)}$
$2_1 \rightarrow 0_1$	144(3)	144	0	134	3.33	138	2	144	0
$4_1 \rightarrow 2_1$	209(3)	216	2.33	206	1.00	208	0.33	228	6.33
$2_2 \rightarrow 0_2$	111(27)	89	0.81	154	1.59	93	0.67	114	0.11
$4_2 \rightarrow 2_2$	204(38)	140	1.68	230	0.68	163	1.08	173	0.82
$0_2 \rightarrow 2_1$	32.7(22)	55	10.14	43	4.68	44	5.14	91	26.5
$2_2 \rightarrow 0_1$	0.92(8)	0.1	10.25	0.7	2.75	3	26.0	3	26.0
$2_2 \rightarrow 2_1$	5.5(5)	10	9.00	6.0	1.00	9	7.00	13	15.0
$2_2 \rightarrow 4_1$	19.0(18)	20	0.56	29	5.56	25	3.33	52	18.3
$4_2 \rightarrow 2_1$	0.7(2)	0.1	3.00	0.04	3.30	2	6.50	1	1.50
$4_2 \rightarrow 4_1$	5.4(13)	8	2.00	5.6	0.15	9	2.77	9	2.77
			χ^2_2=44.4		χ^2_2=12.5		χ^2_4=163.6		χ^2_1=248.6

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FIGURES

FIG. 1. Plots of M = $(B(E2;I_i \rightarrow I_f)/\langle I_i 020|I_f 0\rangle^2)^{1/2}$ as functions of $I_f(I_f+1)$–$I_i(I_i+1)$. The linear fits to the data are shown by the straight lines and the values of the parameters M_1 and M_2 (see Eqn. 9) are given.

FIG. 2. Plots of the excitation energy versus I(I+1) for states in the ground state bands (open circles) in ^{150}Nd and ^{152}Sm. The solid lines are fits including all the levels in the bands. The parameters A and B (see Eqn. 13) from these fits are also given (in keV). For ^{152}Sm we also show the excited states (open squares) based on the 0_2^+ state (the β–vibrational band) and the result of a similar fit to this sequence.

FIG. 3. Plots of $I_f(I_f+1)$–$I_i(I_i+1)$ versus M (defined in the caption of Fig. 1) for ^{152}Sm. The experimental points are shown as solid circles, the solid line is the fit to the data using Eqn. 9. Each panel then compares these values against predictions from the various theoretical calculations of the PPQ model [4,5] (top left), the IBA [9] (top right), the GCM [10] (bottom left), and the X(5) description [12,13] (bottom right).

FIG. 4. Comparison between the experimental partial level scheme for ^{152}Sm and those calculated from the theoretical models discussed in the text. The widths of the arrows are proportional to the B(E2) strengths of the transitions.