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Mehdi Golshani

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EXCITATION OF HIGHLY IONIZED ATOMS BY ELECTRON IMPACT*

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March 10, 1970

ABSTRACT

We use the Coulomb-Born approximation to calculate the total cross section for the electron impact excitation of ionized atoms which have one electron outside their closed shells. When the incident energy is high and the atom is highly ionized, we can use the eikonal and classical approximations in conjunction with the Coulomb-Born approximation. Then, the scattering amplitude reduces to a one-dimensional path integral. The path integral, in turn, can be reduced, to any desired accuracy, to a sum over integrals which can be evaluated analytically. Using this procedure, we have calculated the total cross sections for the heavily ionized atoms Fe^{15+} , Co^{16+} , Ni^{17+} , and Cu^{18+} .

I. INTRODUCTION

Electron impact excitation of various types of atomic ions has been examined in different approximations.¹ Of particular interest have been the Coulomb-Born and the close coupling approximations. By use of the Coulomb-Born approximation, electron impact excitation cross sections have been calculated² for hydrogenic ions by Tully (1960) and Burgess (1961), for He^+ by Tully (1960), for lithium-like ions (Be^+ , C^{3+} , O^{5+} , and Mg^{9+}) by Belly, Tully, and Van Regemorter (1963), for sodium-like ions (Mg^+ , Si^{3+} , and Fe^{15+}) by Belly, Tully, and Van Regemorter (1963); also by Kreuger and Czyzak (1965), and for potassium-like ions (Ca^+) by Van Regemorter (1960, 1961). The close coupling approximation has been used in the calculation of the excitation cross section for He^+ ,³ and for N^{4+} .⁴ Except for the case of $1s \rightarrow 2s$ transition in He^+ , there are no experimental data to compare with these calculations. For He^+ , where experimental data are available, the theoretical threshold behavior is different from the experimental one.

In this paper,⁵ we have used the Coulomb-Born approximation in conjunction with the eikonal and classical approximations. The method is applicable only for the excitation of highly ionized atoms at high incident energies. Thus, we have calculated the total cross sections for the $3s \rightarrow 3p$ and $3p \rightarrow 3d$ excitations of the sodium-like ions Fe^{15+} , Co^{16+} , Ni^{17+} , and Cu^{18+} .

In this calculation, we assume that all electrons are distinguishable, thus, neglecting the exchange amplitude. It is shown by Matt and Massey⁶ that at high incident energies, where the Born

approximation is valid, the exchange amplitude is very small relative to the direct scattering amplitude, and if the transition can occur only through exchange, the cross section will be small and drops rapidly with energy. Since we are working at high energies ($k \gg 1$), we expect the exchange amplitude to be small. However, the exchange amplitude can easily be included in the Coulomb-Born approximation, and this will be the subject of a later paper.

II. THEORY

We consider an atomic ion which has one electron outside its closed shells. In order to calculate the total cross section for the excitation of such ions we use the so-called Coulomb-Born approximation. This approximation differs from the ordinary Born approximation in that plane waves are replaced by Coulomb waves. In this approximation the scattering amplitude takes the form⁷

$$T_{fi}^{C.B.} = \int d^3r X_{\vec{k}}^{(-)*}(\vec{r}) V_{fi}(\vec{r}) X_{\vec{k}}^{(+)}(\vec{r}) \quad (1)$$

where $X_{\vec{k}}^{(+)}$ and $X_{\vec{k}}^{(-)}$ are the Coulomb wave functions representing the incident and scattered electrons respectively. The effective interelectronic interaction V_{fi} is defined by

$$V_{fi}(r) \equiv \int U_f^*(\vec{r}') \frac{1}{|\vec{r} - \vec{r}'|} U_i(\vec{r}') d^3r' , \quad (2)$$

where U_i and U_f are, respectively, the initial and final state wavefunctions of the outermost atomic electron.

The use of the Coulomb-Born approximation alone is not enough, in general, to reduce the above expression for the scattering amplitude to an analytical form. Further approximations are necessary to accomplish this. In this paper, we follow Chen and Watson⁸ in using the combination of the eikonal and classical approximations. The simultaneous use of these two approximations has the advantage of reducing the expression for the scattering amplitude to a one-dimensional path integral. The

application of the eikonal approximation for the wave functions of the incident and scattered electrons requires⁹

$$k \gg 1, \quad k' \gg 1, \quad (3a)$$

where $\tilde{k}$ and $\tilde{k}'$ are the momenta of the incident and scattered electrons respectively. On the other hand, the validity of the classical theory of elastic scattering requires¹⁰

$$k \ll 4Z, \quad k' \ll 4Z. \quad (3b)$$

Thus, it is for highly ionized atoms and at high incident energies that there is a region of overlap for the validity of these two approximations. This range is defined by

$$1 \ll k \ll 4Z \quad (4a)$$

and

$$1 \ll k' \ll 4Z. \quad (4b)$$

In the eikonal approximation we have

$$\chi_{\tilde{k}}^{\pm}(\tilde{r}) = \frac{1}{(2\pi)^{3/2}} e^{iS_{\tilde{k}}^{\pm}(\tilde{r})}, \quad (5)$$

where the eikonal function S is defined by a path integral along the classical trajectory:

$$S_{\tilde{k}}(r) = \int^r \chi_{\tilde{k}}(s) ds, \quad (6)$$

with

$$\chi_k = \left(k^2 + 2 \frac{Z}{r} \right)^{\frac{1}{2}}. \quad (7)$$

Replacing (5) into (1) we get

$$T_{fi}^{C.B.} = \frac{1}{(2\pi)^3} \int e^{-iS_{k'}^{(-)}(\underline{r})} V_{fi}(\underline{r}) e^{iS_k^{(+)}(\underline{r})} d^3r. \quad (8)$$

Two trajectories pass through each point $\underline{r}$ of the integrand in Eq. (5).

One corresponds to an elastic scattering with the initial momentum $\underline{k}$

and the other corresponds to an elastic scattering with the final

momentum $\underline{k}'$. Chen and Watson⁸ have shown that if the conditions for

the validity of both the eikonal and the classical theory of elastic

scattering are satisfied, then, the main contribution to the integral

comes from points $\underline{r}_0$ at which the two trajectories are tangent.

Furthermore, in this case, the integral reduces to a one-dimensional

path integral. To identify the integration path, we look for points

where pairs of trajectories meet and are tangent. However, to simplify

this calculation, we replace the classical trajectory by a trajectory

which consists of two straight lines intersecting at an angle equal

to the scattering angle. This type of trajectory will be referred to

as a "triangular" trajectory. The triangular trajectories corresponding

to an elastic scattering with the initial momentum $\underline{k}$ and an elastic scattering

with the final momentum $\underline{k}'$ are shown in Figs. 1a and 1b respectively.

The use of this type of trajectory requires the energy to be high and

the scattering angle to be small. At high energies, however, the

scattering amplitude is sharply peaked in the forward direction. The restriction on the scattering angle is, therefore, satisfied whenever the energy is high.

Now, in order to find the integration path, we write the equations of a "triangular" trajectory with the initial momentum $\underline{k}$ and a "triangular" trajectory with the final momentum $\underline{k}'$. Then, we find points where they meet and are tangent. It turns out that the integration path consists of the two dashed lines shown in Figs. 2a and 2b. In Fig. 2a we have trajectories I and II which correspond to elastic scattering with the scattering angles $2\beta_1$ and $2\beta_2$ respectively. Similarly, in Fig. 2b we have trajectories III and IV with the scattering angles $2\beta_3$ and $2\beta_4$ respectively. At each point of these trajectories we have a surface of constant eikonal which is normal to the trajectory at that point. The two principal radii of curvature of the eikonal surface corresponding to trajectory "i" are denoted by R_{1i} and R_{2i} .

In the case corresponding to Fig. 2a we have

$$d \approx d' = \frac{Z}{E'\theta} \quad (9)$$

where θ is the scattering angle. Similarly, for Fig. 2b we have

$$d' \approx d = \frac{Z}{E\theta} \quad (10)$$

Now, following Chen-Watson treatment,⁸ we get

$$\begin{aligned}
 T_{fi}^{C.B.} = & \int ds \, v_{fi}(r_0) e^{i\phi(s)} \delta(\alpha_{11}; \alpha_{22}) \frac{\pi}{(|\alpha_{11}| |\alpha_{22}|)^{\frac{1}{2}}} \\
 & + \int ds' \, v_{fi}(r'_0) e^{i\phi'(s')} \delta(\alpha'_{11}; \alpha'_{22}) \frac{\pi}{(|\alpha'_{11}| |\alpha'_{22}|)^{\frac{1}{2}}} \quad (11)
 \end{aligned}$$

where the first and second path integrals are along the dashed lines in Figs. 2a and 2b respectively. Here, we have introduced the functions

$$\alpha_{11} = \frac{1}{2} \left(\frac{\chi_k}{R_{12}(k)} - \frac{\chi_{k'}}{R_{11}(k')} \right), \quad (12a)$$

$$\alpha_{22} = \frac{1}{2} \left(\frac{\chi_k}{R_{22}(k)} - \frac{\chi_{k'}}{R_{21}(k')} \right), \quad (12b)$$

$$\alpha'_{11} = \frac{1}{2} \left(\frac{\chi'_k}{R_{13}(k)} - \frac{\chi'_{k'}}{R_{14}(k')} \right), \quad (12c)$$

$$\alpha'_{22} = \frac{1}{2} \left(\frac{\chi'_k}{R_{23}(k)} - \frac{\chi'_{k'}}{R_{24}(k')} \right), \quad (12d)$$

where

$$\delta(\alpha_{11}; \alpha_{22}) = \begin{cases} +i & \text{for } D \equiv \frac{\alpha_{11}}{|\alpha_{11}|} + \frac{\alpha_{22}}{|\alpha_{22}|} = +2 \\ -i & \text{for } D = -2 \\ 1 & \text{for } D = 0 \end{cases} \quad (13)$$

and

$$\delta(\alpha'_{11}; \alpha'_{22}) = \begin{cases} +i & \text{for } D' \equiv \frac{\alpha'_{11}}{|\alpha'_{11}|} + \frac{\alpha'_{22}}{|\alpha'_{22}|} = +2 \\ -i & \text{for } D' = -2 \\ 1 & \text{for } D' = 0 . \end{cases} \quad (14)$$

The arguments of the phase factors $e^{i\phi(s)}$ and $e^{i\phi'(s')}$ are given by

$$\phi(s) = S_k^+(s) - S_{k'}^{(-)}(s) \quad (15a)$$

$$\phi(s') = S_k^{(+)}(s') - S_{k'}^{(-)}(s') . \quad (15b)$$

Substituting (5) into the Schrödinger equation we get

$$|\nabla \tilde{S}_k| = \left(k^2 + 2 \frac{Z}{r} \right)^{\frac{1}{2}} \approx k + \frac{\gamma}{k} \quad (\text{for } k \gg 1) , \quad (16)$$

where $\gamma = Z_k$. To get $S_k^{\pm}$ we integrate along the prescribed path and impose the asymptotic conditions:¹¹

$$S_k^{(+)}(\tilde{r}) \xrightarrow{Z \rightarrow -\infty} k \cdot \tilde{r} - \gamma \ln(kr - k \cdot \tilde{r}) , \quad (17a)$$

$$S_k^{(-)}(\tilde{r}) \xrightarrow{Z \rightarrow +\infty} k \cdot \tilde{r} + \gamma \ln(kr + k \cdot \tilde{r}) . \quad (17b)$$

In order to calculate α_{11} , α_{22} , α'_{11} , and α'_{22} we need to know the principal radii of curvature of the eikonal surfaces corresponding to the trajectories of Figs. 2a and 2b. The procedure for the calculation of these quantities is given by Chen and Watson.¹² For example, if we

choose z_1 axis along the direction of the incident electron and z axis as the axis of symmetry of trajectory II, then, according to the Chen-Watson prescription we have

$$\frac{1}{R_{12}(k)} = \frac{[\partial \theta_c(z,d)]/\partial d}{[\partial |x_1|]/\partial d}, \quad (18a)$$

$$\frac{1}{R_{22}(k)} = \frac{\theta_c(z,d)}{|x_1|}, \quad (18b)$$

where $\tan \theta_c(z,d)$ is the slope of the trajectory, at a point (x,z) , with respect to the z axis. For trajectory II it is given by

$$\theta_c(z,d) = \beta_2 + \beta_2(z), \quad (19)$$

where

$$\beta_2(z) = -d \int_0^z \frac{dU}{dR_0} \frac{dz''}{R_0}, \quad (20)$$

$$\beta_2 = \lim_{z \rightarrow \infty} \beta_2(z); \quad (21)$$

here

$$U = 1 - (1 - \frac{V}{E})^{\frac{1}{2}} \approx \frac{V}{2E} \quad (\text{at high energies}), \quad (22)$$

and

$$R_0 = (d^2 + z''^2)^{\frac{1}{2}}. \quad (23)$$

Explicit expressions for all of the radii of curvature pertaining to this problem are given in Appendix A.

Finally, in order to evaluate

$$V_{fi}(\vec{r}) = \int \psi_{n'\ell'm'}^*(\vec{r}') \frac{1}{|\vec{r} - \vec{r}'|} \psi_{n\ell m}(\vec{r}') d^3r', \quad (24)$$

we approximate the wave functions $\psi_{n'\ell'm'}$ and $\psi_{n\ell m}$ by hydrogenic-type wave functions. Using the expansion

$$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{\lambda, \mu} \frac{4\pi}{2\lambda + 1} \frac{r'^{\lambda}}{r^{\lambda+1}} y_{\lambda}^{\mu*}(\hat{r}) y_{\lambda}^{\mu}(\hat{r}'), \quad (25)$$

and¹³

$$\int d\vec{r}' y_{\ell'}^{m'*}(\hat{r}') y_{\lambda}^{\mu}(\hat{r}') y_{\ell}^m(\hat{r}') = (-)^{m'} \left[\frac{1}{4\pi} (2\ell' + 1)(2\lambda + 1)(2\ell + 1) \right]^{\frac{1}{2}} \begin{pmatrix} \ell' & \lambda & \ell \\ -m' & \mu & m \end{pmatrix} \begin{pmatrix} \ell' & \lambda & \ell \\ 0 & 0 & 0 \end{pmatrix}, \quad (26)$$

we get, after performing some simple integrals,

$$V_{fi}(r_0) = \sum_{\lambda=|\ell-\ell'|}^{\ell+\ell'} \sum_{s=0}^{n-\ell-1} \sum_{t=0}^{n'-\ell'-1} B(n'\ell'm'; n\ell m; \lambda) p_{\nu}^{|\mu|}(\cos \theta) \times \left\{ r_0^{-(\lambda+1)} \left[\frac{\nu!}{c^{\nu+1}} e^{-cr_0} \sum_{p=0}^{\nu} \frac{\nu!}{p!} \frac{r_0^p}{c^{\nu-p+1}} \right] + r_0^{\lambda} \left[e^{-cr_0} \sum_{q=0}^{\nu'} \frac{\nu'!}{q!} \frac{r_0^q}{c^{\nu'-q+1}} \right] \right\}, \quad (27)$$

where

$$B(n' \ell' m'; n \ell m; \lambda) = (-)^{\epsilon_{\mu}} \begin{pmatrix} \ell' & \lambda & \ell \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell' & \lambda & \ell \\ -m' & \mu & m \end{pmatrix} \quad (28)$$

$$\frac{[(n' + \ell')!]^2}{t!(n' - \ell' - t - 1)!(2\ell' + 1 + t)!} \frac{[(n + \ell)!]^2}{s!(n - \ell - s - 1)!(2\ell + 1 + s)!}$$

$$A_{n' \ell'} A_{n \ell} (-a_{n'})^t (-a_n)^s \left[\frac{1}{(\lambda + |\mu|)!} (\lambda - |\mu|)! (2\ell + 1)(2\ell' + 1) \right]^{\frac{1}{2}} \quad (29)$$

and

$$\nu = \ell + \ell' + s + t + \lambda + 2; \quad \nu' = \ell + \ell' + s + t - \lambda + 1; \quad \mu = m' - m, \quad (30)$$

$$Z' = Z + 1; \quad a_n = 2 \frac{Z'}{n a_0}; \quad a_{n'} = 2 \frac{Z'}{n' a_0}; \quad C = \frac{1}{2}(a_n + a_{n'}), \quad (31)$$

$$\epsilon_{\mu} = \begin{cases} m & \text{for } \mu \geq 0 \\ m' & \text{for } \mu \leq 0 \end{cases} \quad (32)$$

To get the expression for $V'_{fi}(r'_0)$ we make the following substitutions in $V_{fi}(r_0)$:

$$r_0 \rightarrow r'_0, \quad (33a)$$

$$\cos \theta \rightarrow \cos \theta' = \frac{z'}{r'_0}. \quad (33b)$$

III. APPLICATION TO SODIUM-LIKE IONS

As we have already seen, under certain conditions, the scattering amplitude reduces to a path integral:

$$T_{fr}^{C.B.} = \int ds V_{fi}(\vec{r}_0) e^{i\phi(s)} \delta(\alpha_{11}; \alpha_{22}) \frac{\pi}{(|\alpha_{11}| |\alpha_{22}|)^{\frac{1}{2}}} \\ + \int ds V'_{fi}(\vec{r}') e^{i\phi(s')} \delta(\alpha'_{11}; \alpha'_{22}) \frac{\pi}{(|\alpha'_{11}| |\alpha'_{22}|)^{\frac{1}{2}}} ,$$

A numerical evaluation of these integrals showed that with the exception of a phase factor all other factors in each integrand change very slowly. Thus, we can break each integral to a sum over integrals which have the rapidly varying phase factor as their integrand. Each of these integrals is over a small interval. The interval is chosen such that the rest of the factors in the original integrand can be taken effectively constant. By increasing the number of integrals in each sum we can get any desired accuracy. In this calculation, we have chosen a limit of one per cent error.

In looking for highly ionized atoms with one external electron, we found four candidates for which the energy levels are given in Moore-Sitterley's table of atomic energy levels. These are the sodium-like ions Fe^{15+} , Co^{16+} , Ni^{17+} , and Cu^{18+} . For these ions we have calculated the total cross section for the $3s \rightarrow 3p$ and $3p \rightarrow 3d$ transitions. The result is shown in Figs. 3 through 6. In these figures we have plotted total cross section against ϵ , which is the

energy of the incident electron in units of the excitation energy.

In Fig. 7, we have plotted a typical differential cross section. One general feature of these figures is that for any ion the total cross section for the $3p \rightarrow 3d$ transition is smaller than that of the $3s \rightarrow 3p$ transition. Furthermore, the total cross section for the $3p \rightarrow 3d$ transition varies more slowly than that of the $3s \rightarrow 3p$ transition. Another general feature is that the total cross section decreases as the ionic charge increases.

Of these four ions that we have considered, only Fe^{15+} has been dealt with in the literature. There are two independent calculations of the total cross section for the $3s \rightarrow 3p$ transition of Fe^{15+} . One calculation is by Belly, Tully, and Van Regemorter; the other calculation is by Kreuger and Czyzak. The latter have also calculated the total cross section for the $3p \rightarrow 3d$ transition of Fe^{15+} . These two calculations are done in the Coulomb-Born approximation and include energies up to $\epsilon = 4$. For these relatively low energies, our procedure is not expected to be valid. However, if we extrapolate our result down to $\epsilon = 4$, we find that the total cross section for the $3s \rightarrow 3p$ excitation of Fe^{15+} as calculated by these two groups, is about fourteen times as large as our result (at $\epsilon = 4$). However, the ratio

$$\frac{\sigma_{\text{tot}}(3s \rightarrow 3p)}{\sigma_{\text{tot}}(3p \rightarrow 3d)}$$

in our calculation agrees with the same ratio in the calculation of Kreuger and Czyzak. We have also compared our result with that of the

Born approximation. At $\epsilon = 4$, the Born approximation cross section for the $3s \rightarrow 3p$ excitation of Fe^{15+} is about six times as large as our result; but, as the energy of the incident electron increases, our result approaches that of the Born approximation. At $\epsilon = 225$ ($k \approx 24$) the Born approximation cross section is about one and a half times as large as our result. At higher energies, the two calculations depart again. However, at very high energies, our result is not expected to be valid due to the restrictions

$$k \ll 4Z, \quad (3b)$$

and

$$k' \ll 4Z. \quad (3b)$$

APPENDIX A

THE PRINCIPAL RADII OF CURVATURE OF THE SURFACES OF CONSTANT EIKONAL

If we denote by R_{1i} and R_{2i} the two principal radii of curvature of a surface of constant eikonal corresponding to trajectory "i," then, following Chen-Watson treatment,¹² we get

$$\frac{1}{R_{21}} = \frac{-e_1 \beta_2 \left[\frac{1}{d'} + \frac{z}{d'} \frac{1}{(z^2 + d'^2)^{1/2}} + \frac{zd'}{(z^2 + d'^2)^{3/2}} \right]}{1 + \frac{1}{2} \beta_2^2 + 2\beta_2 \frac{z}{d'}},$$

$$\frac{1}{R_{22}} = \frac{e_1 \beta_2 \left[1 + \frac{z}{(z^2 + d'^2)^{1/2}} \right]}{d' \left(1 - \frac{1}{2} \beta_2^2 \right) - 2\beta_2 z},$$

$$\frac{1}{R_{11}} = \frac{e_2 \beta_1 \left[\frac{1}{d'} - \frac{z_2}{d'} \frac{1}{(z_2^2 + d'^2)^{1/2}} - \frac{z_2 d'}{(z_2^2 + d'^2)^{3/2}} \right]}{1 + \frac{1}{2} \beta_1^2},$$

$$\frac{1}{R_{21}} = \frac{-e_2 \beta_1 \left[1 - \frac{z_2}{(z_2^2 + d'^2)^{1/2}} \right]}{d' \left(1 - \frac{1}{2} \beta_1^2 \right)},$$

for the case of Fig. (2a), and

$$\frac{1}{R_{13}} = \frac{-e_3 \beta_3 \left[\frac{1}{d} + \frac{z'}{d} \frac{1}{(z'^2 + d^2)^{1/2}} + \frac{z'd}{(z'^2 + d^2)^{3/2}} \right]}{1 + \frac{1}{2} \beta_3^2},$$

$$\frac{1}{R_{23}} = \frac{e_3 \beta_3 \left[1 + \frac{z'}{(z'^2 + d^2)^{1/2}} \right]}{d \left(1 - \frac{1}{2} \beta_3^2 \right)},$$

$$\frac{1}{R_{14}} = \frac{e_4 \beta_4 \left[\frac{1}{d} - \frac{z_3}{d} \frac{1}{(z_3^2 + d^2)^{1/2}} - \frac{z_3 d}{(z_3^2 + d^2)^{3/2}} \right]}{1 + \frac{1}{2} \beta_4^2 - 2\beta_4 \frac{z_3}{d}},$$

$$\frac{1}{R_{24}} = \frac{-e_4 \beta_4 \left[1 - \frac{z_3}{(z_3^2 + d^2)^{1/2}} \right]}{d \left(1 - \frac{1}{2} \beta_4^2 \right) + 2\beta_4 z_3},$$

for the case of Fig. (2b). Here we have defined

$$e_1 = \frac{\chi_1}{|\chi_1|},$$

$$e_2 = \frac{\chi'_1}{|\chi'_1|},$$

$$e_3 = \frac{\chi'_3}{|\chi'_3|},$$

$$e_4 = \frac{\chi_4}{|\chi_4|},$$

where

$$\chi_1 = -2\beta_2 z + d' \left(1 - \frac{1}{2} \beta_2^2 \right) ,$$

$$\chi'_1 = d' \left(1 - \frac{1}{2} \beta_1^2 \right) ,$$

$$\chi'_3 = d \left(1 - \frac{1}{2} \beta_3^2 \right) ,$$

$$\chi_4 = 2\beta_4 z_3 + d \left(1 - \frac{1}{2} \beta_4^2 \right) .$$

Here, z_2 is related to z by a rotation about the origin through an angle $\beta_2 - \beta_1$. Similarly, z_3 is related to z' by a rotation through an angle $\beta_3 - \beta_4$:

$$z_2 = z - (\beta_2 - \beta_1)d' ,$$

$$z_3 = z' + (\beta_3 - \beta_4)d ,$$

where

$$d' = \frac{Z}{E'\theta} ,$$

$$d = \frac{Z}{E\theta} ,$$

$$\beta_1 = -\frac{\theta}{2} ,$$

$$\beta_2 = -\frac{E'}{E} \frac{\theta}{2} ,$$

$$\beta_3 = -\frac{\theta}{2} ,$$

$$\beta_4 = -\frac{E}{E'} \frac{\theta}{2} .$$

ACKNOWLEDGMENT

I am grateful to Professor K. M. Watson for suggesting this problem and for his continued advice and encouragement.

FOOTNOTES AND REFERENCES

- * This work was supported in part by the U.S. Atomic Energy Commission.
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FIGURE CAPTIONS

- Fig. 1a. Triangular trajectory with the initial momentum $\underline{k}$.
- Fig. 1b. Triangular trajectory with the final momentum $\underline{k}'$.
- Fig. 2a. Trajectories I and II have a common part along $\underline{k}$.
- Fig. 2b. Trajectories III and IV have a common part along $\underline{k}'$.
- Fig. 3. Total cross sections for the $3s \rightarrow 3p$ (curve 1) and $3p \rightarrow 3d$ (curve 2) excitations of Fe^{15+} .
- Fig. 4. Total cross sections for the $3s \rightarrow 3p$ (curve 1) and $3p \rightarrow 3d$ (curve 2) excitations of Co^{16+} .
- Fig. 5. Total cross sections for the $3s \rightarrow 3p$ (curve 1) and $3p \rightarrow 3d$ (curve 2) excitations of Ni^{17+} .
- Fig. 6. Total cross sections for the $3s \rightarrow 3p$ (curve 1) and $3p \rightarrow 3d$ (curve 2) excitations of Cu^{18+} .
- Fig. 7. Differential cross sections for the $3s \rightarrow 3p$ (curve 1) and $3p \rightarrow 3d$ (curve 2) excitations of Fe^{15+} ($\epsilon = 50$).

Fig. 1a.

Fig. 1b.

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Fig. 2a.

Fig. 2b.

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Fig. 3.

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Fig. 4.

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Fig. 5.

Fig. 6.

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Fig. 7.

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