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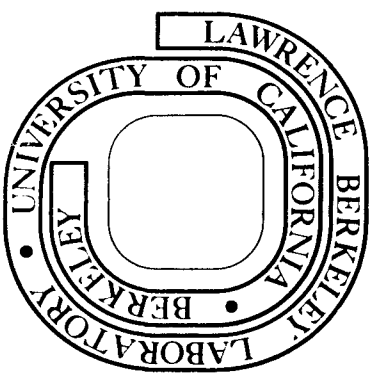

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ANALYSIS OF REACTIONS INDUCED BY POLARIZED PARTICLES*

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The analysis of reactions between particles with spin is usually rather difficult due to the large number of transition matrix elements with different orientations of the particle spins. In terms of these amplitudes the observables form a set of bilinear equations. For an analysis, one important aspect is to determine those subsets that allow a solution. This question has been investigated in detail by Simonius[1] who showed that, as long as experimental errors are neglected, polarization experiments of no higher than second order are needed. Consequently, the data base consists of the cross section $\sigma_o(\theta)$ and of measurements involving one and two particle polarizations. Since all of these data have to be taken at the same energy and angle, this method is generally difficult and probably restricted to a few reactions with a favorable combination of particles.

At relatively low energies and especially in reactions with resonances in the intermediate system, an analysis in terms of (ℓ,s,J) reaction matrix elements is more promising. Penetrability considerations limit the number of these elements, and an R-matrix or S-matrix approach can be used to describe their energy dependence. It is therefore possible to base the analysis on measurements of all types, taken at various energies and angles. Initially, the most important problem is to find those amplitudes $R_i \equiv \langle \ell_i' s_i' J_i^\pi | \, | \ell_i s_i J_i^\pi \rangle$ which are responsible for the major structures of the observables as a function of energy and angle. In the last few years a set of criteria has been derived for such a preliminary analysis[2-4]. Although derived for polarized deuterons, these methods are readily generalized to particles with spin i. I shall devote my talk almost exclusively to this question, leaving the discussion of the full-scale analysis to the next speaker[5].

The natural basis for a preliminary analysis is the set of first order polarization data. Since these measurements involve only one particle polarization, they usually are both the most numerous and the most precise. An effective way to obtain information from the first order observable T_{kq} is to expand the product $\sigma_o(\theta)\cdot T_{kq}(\theta)$ in terms of Legendre polynomials $P_{L,q}(\theta)$, resulting in the expansion coefficients $a_{kq}(L)$ [ref. 4)]. An immediate consequence is the separation of element combinations $R_1 R_2^*$ with equal and opposite parities into different coefficients $a_{kq}(L)$, according to the parity selection rule $\ell_1+\ell_2+L$ = even.

Another important aspect is the fact that a single reaction matrix element R_i can give rise to nonzero coefficients $a_{kq}(L)$ of even rank k and even degree L. For polarized particles with spin $i > 1/2$ both the cross-section $\sigma_o(\theta)$ and the 3 tensor polarization quantities of rank 2 are proportional to $|R_i|^2$. The coefficients $a_{kq}(L)$ are therefore linearly dependent. Dividing them by $a_{oo}(0)$, which is essentially the total cross section, yields the parameters $d_{kq}(L)$ [refs. 2,4]. These are independent of the magnitude of the element R_i and depend only on its spin space. Together with the linear relations they are therefore an important tool for the identification of major amplitudes. In an analysis this is a distinct advantage of rank-2 polarizations over vector polarization data.

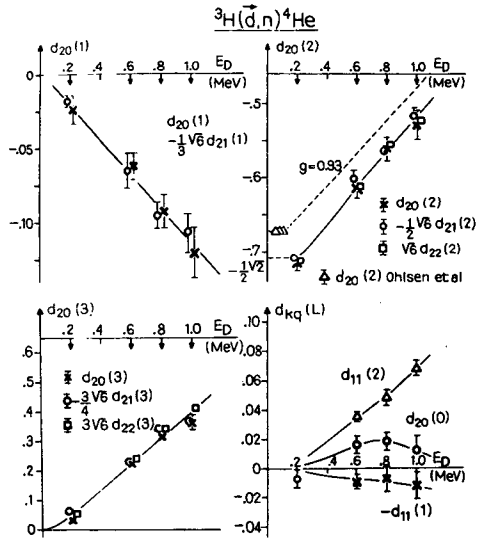

Fig. 1

The approximation that only one or two amplitudes are nonzero can be successful in identifying major elements even in the presence of other sizeable amplitudes[2,4]. Using this assumption, simple explicit criteria can be derived in many cases. Alternately, computer codes can be used, which calculate the formulae for the observables in terms of a given set of matrix elements[6,7]. Useful criteria can thus be derived 1) from the numerical values of the parameters $d_{oo}(L)$ and $d_{2q}(L)$ for isolated resonances, 2) from linear relations between the quantities $a_{kq}(L)$ for the same polarized particle by varying one of the parameters k,q and L, 3) from linear relations between the same observable, measured for different reaction partners. An example for the first two cases are levels induced by S-wave particles. The coefficient $d_{k0}(L)$ is given by

$$d_{k0}(L) = \delta(k,L)\,\delta(k,\text{even})\,\hat{i}\hat{L}(-)^{i+I+2J+s'}(2J+1)(2\ell'+1)\begin{pmatrix}\ell' & \ell' & L\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}\ell' & \ell' & L\\ J & J & s'\end{Bmatrix}\begin{Bmatrix}i & i & L\\ J & J & I\end{Bmatrix}$$

with $\hat{i} \equiv (2i+1)^{1/2}$. For orders $q > 0$ the linear relation is then (1)

$$d_{k0}(L) = \frac{(-)^q}{2}\sqrt{\frac{(L+q)!}{(L-q)!}}\,d_{kq}(L). \qquad (2)$$

The most probable case, k = 2, yields[2]

$$d_{20}(2) = -\frac{1}{2}\sqrt{6}\,d_{21}(2) = \sqrt{6}\,d_{22}(2). \qquad (3)$$

Figure 1 shows data for the $^3He(\vec{d},p)^4He$ reaction[8] above the $3/2^+$ level at 107 keV. Excellent agreement is found for eq. (3) and other relations [4], while the ideal value $d_{20}(2) = -1/2\sqrt{2}$ is approached closely.

For resonances induced by particles with angular momentum $\ell > 0$, the magnitudes and relations between the coefficients $d_{kq}(L)$ are best derived using a computer code. A simple indicator for the spin space of the dominant element R_i is the coefficient

$$d_{k0}(0) = \delta(k,\text{even})\,(-)^{i+I+J+2s}(2\ell+1)(2s+1)\,\hat{i}\hat{k}\begin{pmatrix}\ell & \ell & k\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}\ell & \ell & k\\ s & s & J\end{Bmatrix}\begin{Bmatrix}i & i & k\\ s & s & I\end{Bmatrix}. \qquad (4)$$

In fig. 2 some of the $^3He(\vec{d},p)^4He$ data of Grüebler et al.[9] are shown. The value $d_{20}(0) = -1/7\sqrt{2}$ derived from eq. (4) for the proposed $7/2^+$ d-wave level[2,4] is in good agreement with the data near 9 MeV. Other coefficients also agree well with computer generated values, indicated by horizontal lines.

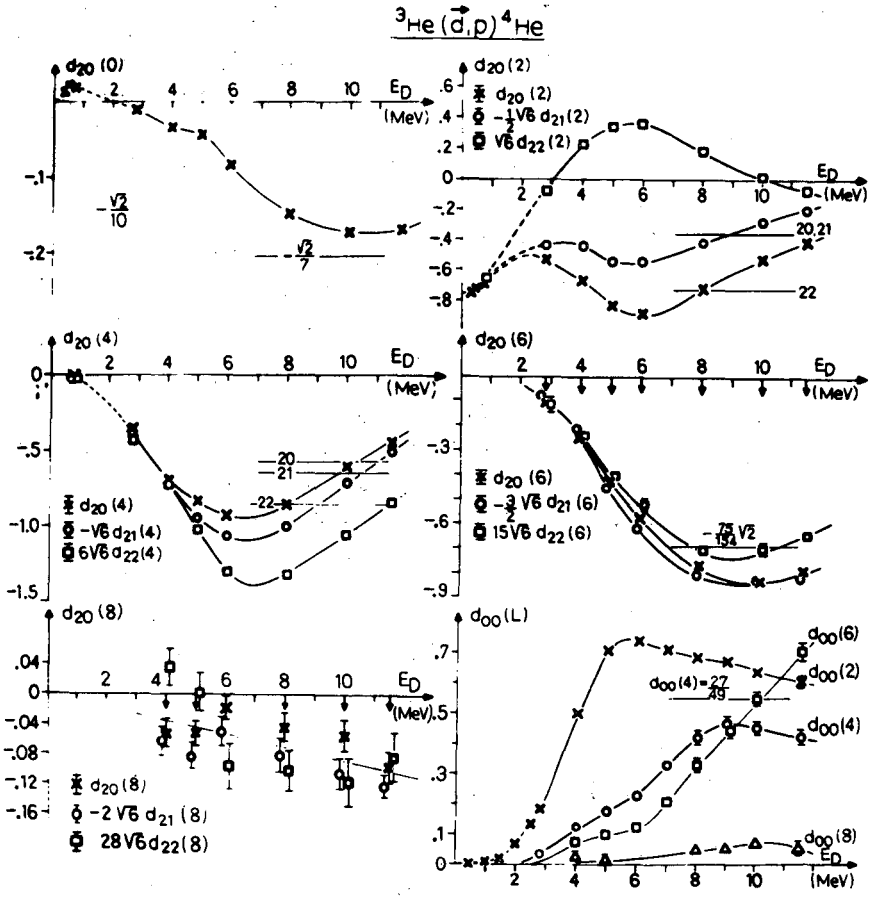

Fig. 2

For isolated resonances with only small competing amplitudes,

$$T_{k0}(0^\circ) = \delta(k,\text{even})(-)^{i+I+\rho}(2s+1)\hat{i}\hat{k}\begin{pmatrix} s & s & k \\ \rho & -\rho & 0 \end{pmatrix}\begin{Bmatrix} s & s & k \\ i & i & I \end{Bmatrix} \qquad (5)$$

also provides a good criterion for reactions in which the parameter $\rho \leqslant s, s', J$ can assume only one value[2].

An important example for the second case pertains to the overlap of two elements $R_1 R_2^*$, that satisfy the conditions

$$(\ell_1'+\ell_2'), (J_1+J_2) \geqslant \Lambda = \ell_1+\ell_2+k \ . \qquad (6)$$

For $q > 0$ the coefficients then obey the relation

$$d_{k0}(\Lambda) = \frac{(-)^q}{2}\ \frac{\Lambda!}{(\Lambda-q)!}\sqrt{\frac{(k+q)!}{k!}\,\frac{(k-q)!}{k!}}\ d_{kq}(\Lambda) \ . \qquad (7)$$

Conditions (6) select preferentially elements with $J > \ell$. Thus eqs. (7) assist primarily in the identification of levels such as the $7/2^+$ d-wave state in fig. 2 or of the overlaps shown in fig. 3 [ref. 4]. Relations (7) also provide a quantitative means to determine the highest significant orbital angular momentum ℓ_{max}. It is precisely the set of elements that satisfy conditions (6) which gives rise to the coefficients $a_{kq}(\Lambda)$ of highest degree $\Lambda = 2\ell_{max}+k$. Equation (7) can thus be used effectively to limit the set of matrix elements necessary. The absence of coefficients with L=9 and the agreement found for L=7 and 8 (figs. 2 and 3) limit the angular momenta in ^{3}He($\vec{d}$,p)^{4}He below 12 MeV to $\ell \leqslant 3$.

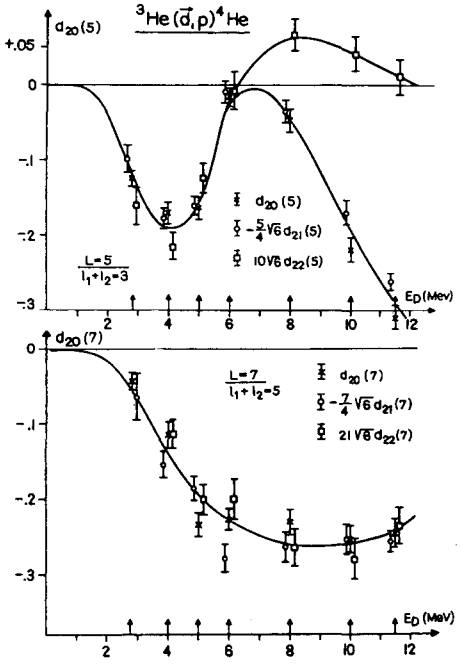

Fig. 3

Other useful restrictions on the size of the element set can be obtained from structures that appear in only one or in several coefficients $a_{kq}(L)$ of different degree L. Similarly, the presence or absence of a structure in observables of different rank k may lead to restrictions[4] due to the triangular condition for the triad (s_1, s_2, k).

An example for the third type of criteria are the relations between the vector polarizations of the different particles in the reaction A(b,c)D with spins I,i,i',I' respectively. If in each case a structure is caused by the overlap of the large elements R_1 and R_2, and if the same coordinate system is used for all observables[4], the relation in the incoming channel is[3]

$$\frac{d_{11}(L)}{d_{0011}(L)} = \frac{\hat{i}}{\hat{I}}\,\delta(s_1',s_2')\,(-)^{s_1-s_2}\,\frac{\begin{Bmatrix} i & i & 1 \\ s_1 & s_2 & I \end{Bmatrix}}{\begin{Bmatrix} I & I & 1 \\ s_1 & s_2 & i \end{Bmatrix}} \quad . \tag{8}$$

The formula for the quantities $d^{11}(L)/d^{0011}(L)$ in the outgoing channel is obtained by interchanging primed and unprimed quantities. Figure 4 shows that the L=2 coefficients in the ^{3}He(d,p)^{4}He reaction below 700 keV are clearly due to the overlap of two elements with channel spins $s_i = 3/2$. The other combinations would require a different relative sign.

If the same structure appears in polarizations of both the incoming and outgoing channel, the element combination responsible must obey both conditions $s_1=s_2$ and $s_1'=s_2'$. The ratio $a_p(L)/a_{p'}(L)$ is then given by[10]

$a_p(L)$ =	$a_{11}(L)$	$a_{0011}(L)$
$a_{p'}(L)=$ / $a^{11}(L)$	$-2\,\frac{K(i,I,s)}{K(i',I',s')}A$	$-2\,\frac{K(I,i,s)}{K(i',I',s')}A$
$a^{0011}(L)$	$-2\,\frac{K(i,I,s)}{K(I',i',s')}A$	$-2\,\frac{K(I,i,s)}{K(I',i',s')}A$

with

$$K(j,J,s) = \frac{j(j+1)+s(s+1)-J(J+1)}{s(s+1)\sqrt{j(j+1)}} \tag{9}$$

and

$$A = \frac{l_1(l_1+1)-l_2(l_2+1)-J_1(J_1+1)+J_2(J_2+1)}{l_1'(l_1'+1)-l_2'(l_2'+1)-J_1(J_1+1)+J_2(J_2+1)} \tag{10}$$

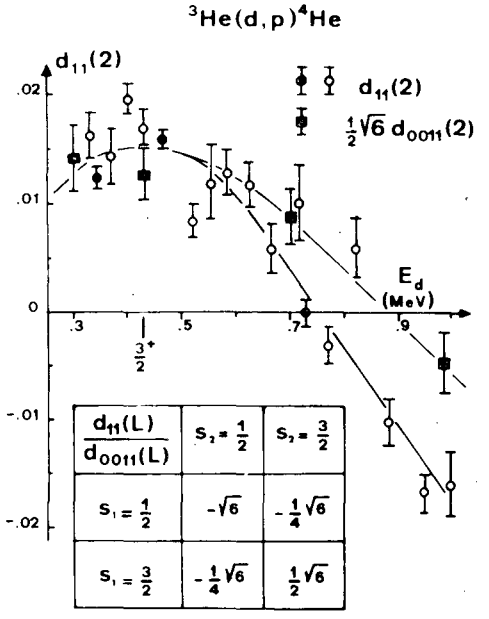

Fig. 4

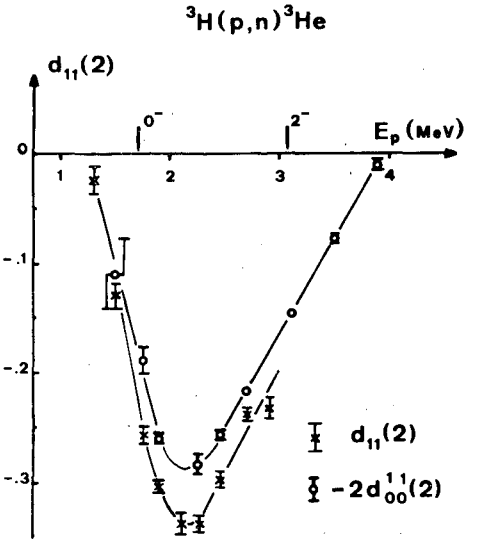

Fig. 5

Since these ratios depend on the detailed spin structure of both elements, they are best suited to confirm assignments already made. In fig. 5 data for the ^{3}H(p,n)^{3}He reaction[11] show that the large overlapping 0^- and 2^- P-wave elements must belong to the class for which A=1 and thus $\ell_i=\ell_i'$.

With the methods outlined here, one or more tentative sets of major elements can be derived with a modest effort. Such sets can then be used as a first guess in an analysis. A careful preliminary analysis, accounting for all major structures, should come reasonably close to a solution. Indeed a preliminary evaluation[4] of ^{3}He(d,p)^{4}He data up to 12 MeV agrees very well with the R-matrix fit by Dodder and Hale[12]. A similar effort by the Zürich group[13] on the ^{6}Li(d,α)^{4}He reaction also shows good agreement with the level structure proposed for ^{8}Be.

A recent discussion of experimental data has shown that it is often the less conspicuous features of an angular distribution that give the least ambiguous information[4]. In order to exhaust the information content of an angular distribution, measurements should therefore be taken at as many angles as possible, even at the cost of less statistics for the individual point. This holds especially for first order polarization experiments, since a successful analysis depends largely on the constraints imposed by these data. The more difficult second order experiments are usually neither numerous nor precise enough to restrict the solution space sufficiently. Together with exhaustive first order data, however, they may provide the independent information needed to arrive at a solution.

In view of the difficulties encountered in an analysis of reactions, an effort should be made to investigate every useful indicator. One possibility is the evaluation of the data in the transverse coordinate system S^T, with the z-axis perpendicular to the scattering plane. The conditions of parity conservation give a simple geometric interpretation to the physical limits of spin-1 polarizations, the somewhat neglected Lakin cone[14,15]. Other interesting quantities also have a simple form. Thus in the system S^T, Johnson's[16] parameter $T_{22} - \sqrt{3/2}\, T_{20}$, which is

sensitive to tensor interaction, corresponds to the efficiency $(A_{xx}-A_{yy})^T$. It should also be noted that for spin 1, it is the alignment direction perpendicular to the scattering plane that occasionally leads to values of unity for the analyzing power A_{yy}. Reaching a maximum possible value imposes linear conditions on some transition matrix elements. If a set of experimental data near such a critical point (E,θ) is introduced into the analysis, these linear conditions will be imposed directly on the set of bilinear equations. Since this will considerably reduce the solution space, it may be advantageous to measure the efficiency A_{yy} directly and use it also in the analysis.

Finally, the discussion here shows that in the full-scale analysis it may be useful to compare the fitted values to both the data points and the expansion coefficients. While it is obvious that the actual data points should be used to obtain the fit, the differences in the expansion coefficients are more likely to yield indications on how to improve the solution.

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+ On leave of absence from the University of Basel, Switzerland

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