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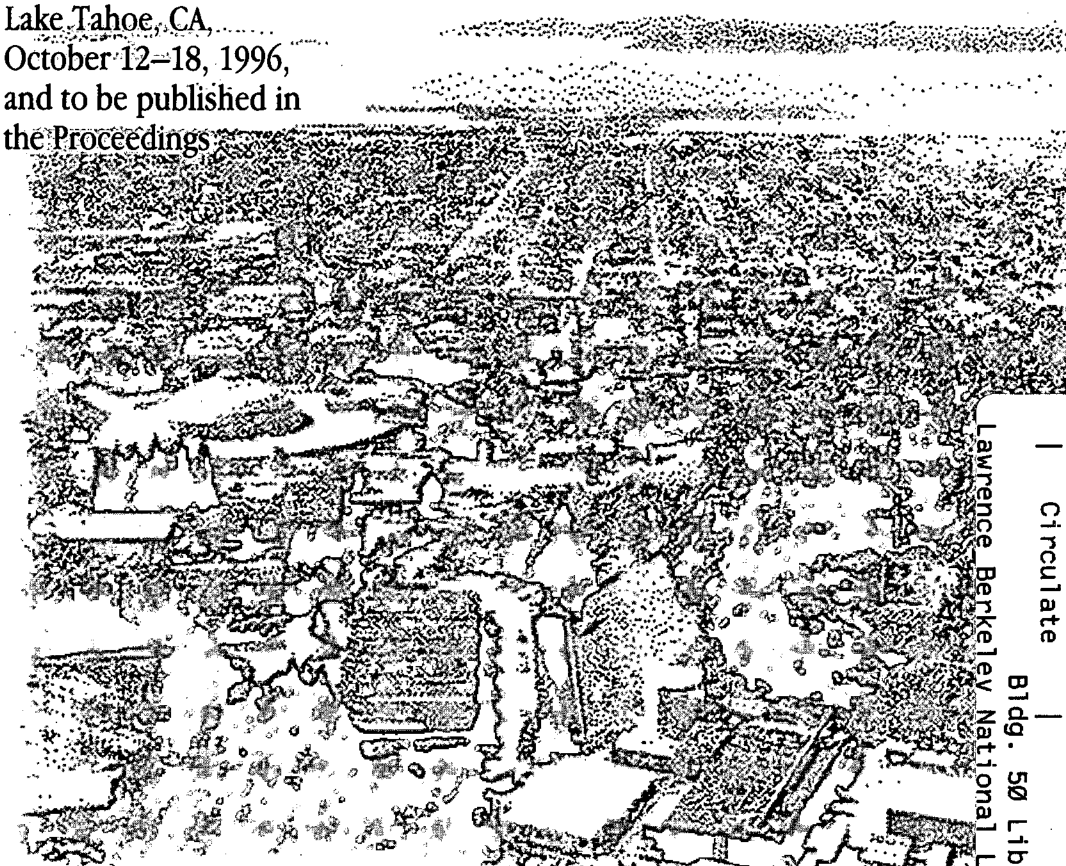

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Emittance in Particle and Radiation Beam Techniques

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Abstract. We discuss the important and diverse role of the phase space area—the emittance—in the advanced techniques involving interaction of particle and radiation beams. For undulator radiation from unbunched beams, the radiation phase space is diluted from the coherent phase space of the single electron radiation. When the undulator radiation is used as a light source, it is important to minimize the dilution by decreasing the beam emittance and matching the phase space distributions of the particle and the radiation beams. For optical stochastic cooling, on the other hand, the phase space should be maximally mismatched for efficient cooling. In the case particles are bunched to a length much shorter than the radiation wavelength, the emittance appears as an intensity enhancement factor. In the operation of free electron lasers, the phase space matching becomes doubly important, once as the dilution factor in the initial stage of energy modulation and then as the radiation efficiency factor at the end where the beam is density modulated. We then discuss some of the beam cooling techniques producing smaller emittances, especially the recent suggestions for relativistic heavy ions in storage rings or electron beams in linacs. These are based on the radiative cooling that occurs when particle beams backscatter powerful laser beams.

1. INTRODUCTION

Particle beams propagating in free space are characterized by the rms emittance ε_x and the Twiss parameter β_x at the beam waist. The beam size and angle at the waist are given respectively by

$$\Delta x_e = \sqrt{2\pi\varepsilon_x\beta_x} \text{ and } \Delta x'_e = \sqrt{2\pi\varepsilon_x / \beta_x}\,. \tag{1}$$

Here Δx_e is roughly the FWHM value, defined as the rms size times $\sqrt{2\pi}$, and similarly for the angular divergence. The letter x refers to the x-direction, there being the corresponding quantities in the y-direction.

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Coherent radiation beams can be described in a similar way; the quantity corresponding to β_x is the Rayleigh length z_R. Thus the spot size and angle at the waist are respectively given by

$$\Delta x_r = \sqrt{\lambda z_R / 2} \text{ and } \Delta x_r' = \sqrt{\lambda / 2z_R} \;, \tag{2}$$

where λ is the radiation wavelength. The radiation phase space area (corresponding to $2\pi\varepsilon_x$ for the particle beam) is

$$\Delta x_r \Delta x_{r'} = \lambda / 2. \tag{3}$$

The analogy goes further. Thus, the beam envelope at a distance z from the waist for a particle beam is given by

$$\Delta x_e(z) = \sqrt{2\pi\varepsilon_x(\beta_x + z^2 / \beta_x)}\,, \tag{4}$$

and for a radiation beam,

$$\Delta x_r(z) = \sqrt{(\lambda / 2)(z_R + z^2 / z_R)}\;. \tag{5}$$

We will often consider the spontaneous radiation generated by electrons in passing through an undulator (1). A single electron passing through an undulator generates a laser-like beam with a waist in the middle of the undulator with

$$\Delta x_r \sim \sqrt{\lambda L / 8\pi}, \quad \Delta x_{r'} \sim \sqrt{\pi\lambda / L}. \tag{6}$$

Here L is the length of the undulator. The Rayleigh length is therefore $z_R \sim L / \sqrt{8}\pi$. A cautionary remark: Eq. (6) was derived under the assumption that the single-electron undulator radiation can be approximated by the Gaussian TEM_{00} mode, which is not very accurate (2).

The radiation pulse length and bandwidth are, respectively:

$$\ell_r \sim \frac{1}{2} N_u \lambda, \qquad \frac{\Delta\omega}{\omega} \sim \frac{1}{N_u}\;. \tag{7}$$

Note that the product

$$\ell_r \frac{\Delta\omega}{\omega} \sim \lambda / 2 \tag{8}$$

is the same as in the case of the transverse phase space.

2. PHASE SPACE DILUTION OF RADIATION FROM PARTICLE BEAMS

In considering radiation by electron beams, it is convenient to distinguish the unbunched case from that of the bunched case. To define these cases, we introduce the so-called bunching parameter as follows:

$$b = \left| \left\langle e^{ikz_i} \right\rangle \right|, \tag{9}$$

where z_i is the electron location, $k = 2\pi / \lambda$, and λ the radiation wavelength. In this section we consider the case where the bunch length is much longer than the radiation wavelength, and the particles are distributed randomly over the bunch. This is normally the case of particle beams in accelerators and storage rings. This will be referred to as the unbunched case, for which

$$b = 0. \tag{10}$$

The phase space distribution of radiation from an unbunched electron beam is given by a convolution of the distribution of a single electron radiation and the electron beam distribution. Thus the spot size and angular divergence is given by

$$\Delta x = \sqrt{(\Delta x_r)^2 + (\Delta x_e)^2} \ , \ \Delta\phi = \sqrt{(\Delta\phi_r)^2 + (\Delta\phi_e)^2}. \tag{11}$$

This is schematically illustrated in Figure 1.

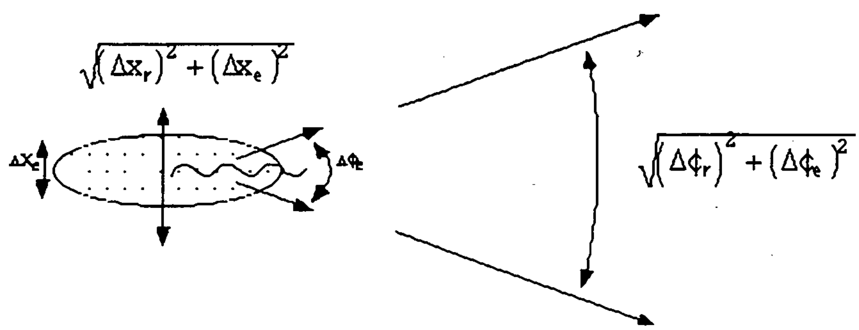

FIGURE 1. The spatial-angular characteristics of undulator radiation by a beam of electrons (unbunched case).

Thus the transverse phase space density is diluted due to the electron beam. Actually, the phase space dilution takes place also in the temporal-spectral domain: the length of the optical pulse and the spectral bandwidth are both broadened from the expression given in Eq. (7) due to the electron beam effect:

$$\ell = \sqrt{\ell_r^2 + \ell_e^2}, \quad \frac{\Delta\omega}{\omega} = \sqrt{\left(\frac{\Delta\omega}{\omega}\right)_r^2 + \left(\frac{\Delta\omega}{\omega}\right)_e^2}, \tag{12}$$

where ℓ_e is the e-beam bunch length, and

$$\left(\frac{\Delta\omega}{\omega}\right)_e = 2\frac{\Delta E_e}{E_e}. \tag{13}$$

The factor 2 is due to the fact that the frequency of the undulator radiation is proportional to E_e^2.

The phase space dilution means that the radiation phase space area in each dimension becomes larger than $\lambda/2$, i.e., the radiation beam becomes partially or completely incoherent depending on the degree of the dilution. More quantitatively, the radiation density in the 6-dimensional phase space, known as the *brightness*, becomes

$$B = N_e B_0 R_\perp R_\parallel , \tag{14}$$

where N_e is the total number of the electrons in the bunch, B_0 is the brightness due to a single electron, and

$$R_\perp = \frac{\lambda/2}{\left((\Delta x_r)^2 + (\Delta x_e)^2\right)\left((\Delta\phi_r)^2 + (\Delta\phi_e)^2\right)}, \tag{15}$$

$$R_\parallel = \frac{\lambda/2}{\sqrt{\left(\ell_r^2 + \ell_e^2\right)}\sqrt{\left(\frac{\Delta\omega}{\omega}\right)_r^2 + \left(\frac{\Delta\omega}{\omega}\right)_e^2}}. \tag{16}$$

We have discussed the phase space properties of the radiation from a beam of particles as if the radiation beam could be treated just as in geometric optics. When the wave nature of the electro-magnetic field is taken into account, the concept of the radiation phase space does not have the usual physical significance. However, the phase space concept, as a mathematical entity behaving similarly to the real phase space of geometric optics, can be justified based on the Wigner distribution function (3),(4).

We now consider two examples in particle-radiation technique, one in which the factor R needs to be maximized and one in which the factor R needs to be minimized.

2.1 Brightness of Synchrotron Radiation Sources

The brightness is an invariant field strength of a radiation source. Considerable progress has been made in recent years in increasing the brightness of synchrotron radiation sources by increasing each factor in Eq (14); the factor N_e by increasing the stored current, the factor $R_\perp$ by reducing the electron beam emittance and choosing the β-function so that the radiation beam phase space is "matched" to that of the electron beam, and the factor $R_{||}$ by reducing the energy spread. For advanced synchrotron radiation sources operating or being built currently at several laboratories around the world, the stored average current is 0.5 to 1 Amperes; the horizontal and the vertical rms emittances are about 0.5×10^{-8} m-rad and 10^{-10} m-rad, respectively; and the e-beam energy spread about a fraction of 10^{-3}. With these values, insertion devices can be built with spectral brightness of about $10^{19} - 10^{20}$ photons/(sec)(0.1% BW)(mm^2)($mrad^2$), permitting experiments with remarkable spectral, angular and spatial resolutions.

2.2 Optical Stochastic Cooling

It is well known that the key parameter in microwave stochastic cooling (5) is the number of particles in a *sample* N_s, i.e., those particles that can be affected by the signal of a given particle. It is given by

$$N_s = \frac{Nc}{\ell \Delta W}, \tag{17}$$

where N is the total number of particles, c is the speed of light, ℓ is the length of the beam, and ΔW is the bandwidth of the cooling system. The signal from other particles in a sample generate noise to the cooling signal of a given particle. Therefore N_s needs to be minimized for faster cooling. This is achieved by employing a broad band microwave system consisting of the pick-up, amplifier and kicker. The entire beam is cooled through the *mixing* process in which particles are continuously reshuffled into different samples. In normal stochastic cooling, the samples are characterized by the longitudinal coordinates, i.e., by the time axis.

Recently it was pointed out that the stochastic cooling may be extended to the optical region (6) using the undulators as pick-up and kicker and taking advantage of the recent development in broad band optical amplifiers. In particular, the transit time method appears to be very promising (7).

New effects occur in optical stochastic cooling, since the transverse phase space area of optical beams could be comparable to that of the particle beam. Therefore the sample associated with a given particle must be those particles within the phase space volume of the optical signal of the original particle. Thus Eq. (17) needs to be generalized as (8)

$$N_s = N \ R_\perp R_{||}. \tag{18}$$

We may recover Eq. (17) is a special case of Eq. (18) when

$$R_\perp = 1, \;\; \ell_e >> \ell_r, \;\; \text{and} \;\; (\Delta\omega/\omega)_r >> (\Delta\omega/\omega)_e. \tag{19}$$

In general, the factor $R_\perp R_\parallel$ needs to be minimized in optical stochastic cooling. Therefore, the phase space distributions of the radiation beam and the particle beam need to be mismatched as much as possible. This is exactly opposite to the case of radiation sources considered in the previous subsection.

Realizing that N_s is given by the general expression in Eq. (18), one can contemplate a new way of implementing stochastic cooling. Thus, one may choose the parameters so that

$$\ell_e << \ell_r, \; (\Delta\omega/\omega)_r << (\Delta\omega/\omega)_e. \tag{20}$$

Equation (18) then becomes

$$N_s = NR_\perp \frac{(\Delta\omega/\omega)_r}{(\Delta\omega/\omega)_e}. \tag{21}$$

In this case, the cooling is optimized by choosing the bandwidth of the amplifier as narrow as possible, and the mixing should occur in the energy axis. This regime of stochastic cooling may be referred to as the "energy sampling" as opposed to the "time sampling" in which Eq. (19) is applicable. The energy sampling may be useful for cooling of bunched beams.

3. RADIATION FROM BUNCHED BEAMS

Consider a disc of an electron beam generating undulator radiation. We assume that the disc is so short that the bunching parameter b, defined by Eq. (9), does not vanish, namely, that the beam is bunched. The radiation pattern in this case is schematically illustrated in Figure 2.

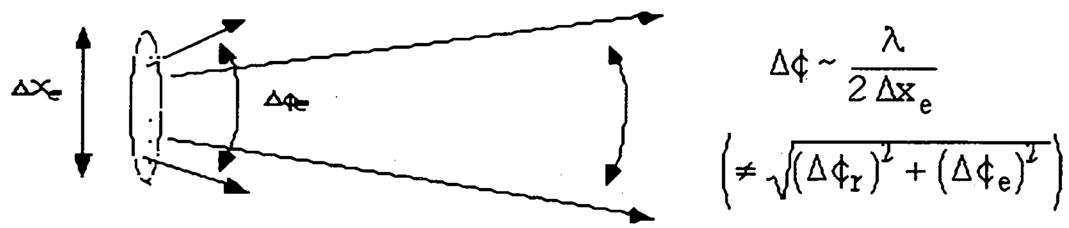

FIGURE 2. Radiation pattern from a bunched electron beam.

It is markedly different from the case of the unbunched beam; the angular divergence is given by the diffraction limit of the transverse size of the beam,

$$\Delta\phi \sim \frac{\lambda}{2\Delta x_e}, \tag{22}$$

rather than the convolution of the angular divergences of the electron beam and the radiation beam. Does the phase space ratio R become irrelevant in this case? No! The phase space ratio is still important, but it enters in a different way; the total energy radiated is given by

$$N_e^2 b^2 U_0 R_\perp , \tag{23}$$

where U_0 is the energy radiated by a single electron. Therefore, the factor $R_\perp$ appears as the radiation efficiency rather than as a phase space dilution factor.

Electron beams in free electron lasers develop a periodic density modulation as schematically shown in Figure 3.

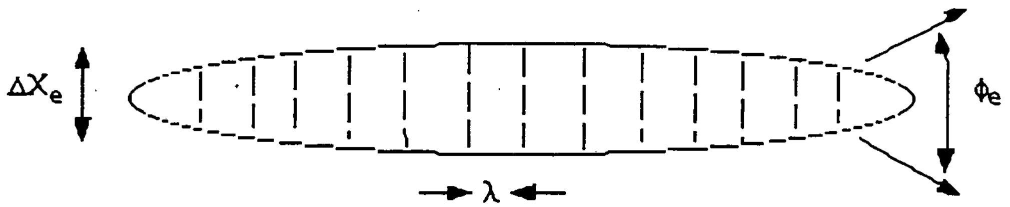

FIGURE 3. A particle bunch with periodic density modulation.

In this case the temporal bandwidth of the radiation is given by that corresponding to the Fourier transform of the bunch length, i.e.,

$$\frac{\Delta\omega}{\omega} \approx \frac{\lambda}{2\ell_e}. \tag{24}$$

Once again, this is quite different from the un-bunched beam case given by the second member of Eq. (12). However, the temporal phase space ratio $R_{||}$ is still important because it enters in the radiation efficiency. Thus, for a periodically modulated beam, the factor $R_\perp$ in Eq. (23) is replaced by a more general expression $R = R_\perp R_{||}$.

In a free electron laser, the electron beam receives a periodic energy modulation with period λ in the beginning part of the undulator. The energy modulation becomes the density modulation after passing through some length of the undulator. The density modulated beam then emits amplified radiation in the final section of the undulator. The ratio R therefore enters twice in the expression of the FEL gain; once as the phase space overlap factor at the beginning and then as the radiation efficiency factor at the end of the undulator. Thus the importance of the beam quality is much more crucial in FELs than in synchrotron radiation devices.

The shortest wavelength record of FELs is 2400 Å achieved at BINP (9). With the recent development of the RF photocathode gun (10), and the advances in the linac technology (11), the beam qualities in the linac have progressed significantly so that it is now possible to contemplate the operation of a self-amplified spontaneous emission for intense, coherent x-rays for 1 Å or shorter wavelengths (12).

4. FUNDAMENTAL LIMIT AND BEAM COOLING TECHNIQUES

4.1 Fundamental Limits

As previously discussed, the minimum photon beam phase space area is $\lambda / 2$. There is a similar fundamental limit for the particle beam arising from quantum mechanics

$$\gamma \Delta x \Delta x' \geq \lambda_c / 2. \quad (25)$$

Here $\lambda_c = \hbar / mc$ is the Compton wavelength corresponding to the particle mass m. For an electron, $\lambda_c \approx 4 \times 10^{-13}$m, and the fundamental limit given by Eq. (25) is many orders of magnitude smaller than that obtained by the RF photocathode techniques, which is about

$$\gamma \Delta x \Delta x' \sim 1 - 2 \times 10^{-6} \mathrm{m-rad / nC}. \quad (26)$$

The particle beam phase space area can be reduced by various non-Liouvillain beam cooling techniques, such as radiative cooling in electron storage rings, electron cooling of heavy ions, ionization cooling, microwave or optical stochastic cooling, channeling, laser cooling of non-relativistic ions, etc. (13). We have already discussed the stochastic cooling in the previous section. The radiative cooling is well-known (14), and is essential for operation of high-brightness electron storage rings in advanced synchrotron radiation facilities.

Recently the radiative cooling method was extended to the case of relativistic heavy ions (15) and to the case of electron linacs (16) by making the particle beams radiate in the presence of high-field laser beams, as shown schematically in Figure 4.

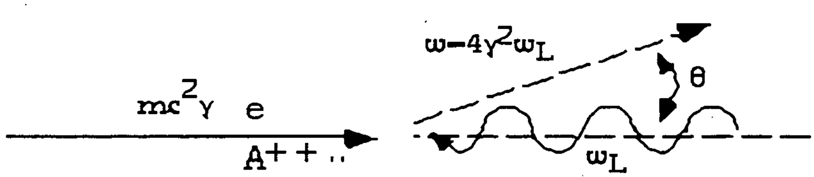

FIGURE 4. Backscattering of a laser beam by a charged particle.

4.2 Radiative Cooling

Radiative cooling occurs when particle beams are forced to emit spontaneous radiation. The equilibrium normalized emittance is

$$\gamma \Delta y \Delta y' \sim \beta_y \frac{u}{mc^2 \gamma^2}, \quad (27)$$

where β_y is the average β-function in the y-direction and u is the characteristic energy of the radiated photons. Equation (27) is valid when there is no dispersion and the excitation of the betatron oscillation is solely due to the angular deviation of the emitted photons from the direction of the particle momentum. In case there is a dispersion of an average magnitude η_x, the equilibrium emittance is

$$\gamma \Delta x \Delta x' \sim \frac{\eta_x^2}{\beta_x} \frac{u}{mc^2}. \tag{28}$$

In an electron storage ring where the emission is dominated by the radiation from bending magnets, $u \sim \hbar c \gamma^3 / \rho$. The dispersion is normally non vanishing in the horizontal (x) direction but vanishes in the vertical (y) direction. Therefore

$$\gamma \Delta x \Delta x' \sim \lambda\!\!\!^{-}_e \gamma^3 \eta_x^2 / \beta_x \rho \ , \tag{29}$$

$$\gamma \Delta y \Delta y' \sim \lambda\!\!\!^{-}_e \ \beta_y \gamma / \rho. \tag{30}$$

Here $\lambda\!\!\!^{-}_e$ is the electron's Compton wavelength. Equations (29) and (30) are applicable for the horizontal direction (the bending plane) and the vertical direction, respectively. The horizontal emittance is many orders of magnitude larger than the fundamental limit. The vertical emittance is much smaller, but the limit is hard to achieve due to the coupling of the horizontal motion to the vertical direction.

4.3 Radiative Laser Cooling in Relativistic Heavy Ion Rings

Here the laser beam is backscattered by relativistic heavy ions in the dispersion-free straight sections (15). The average energy of the scattered photons is

$$u \sim 2\gamma^2 \hbar \omega_L, \tag{31}$$

where ω_L is the laser frequency. By choosing the laser frequency to be in resonance with one of the transition frequencies of the heavy ion, the interaction cross section is increased from the Thomson cross section $\sim r_e^2$ by a factor $(\lambda^* / r_e)(\omega_L / \Delta\omega_L)$. Here λ^* is the transition wavelength in the ion's rest frame, and $\Delta\omega_L$ is the bandwidth of the laser which is chosen to cover the full Doppler broadening of the ion beam. This scheme is different from the usual laser cooling, which depends on the line shape of the individual ions, and thus employs narrow bandwidth lasers. The limiting equilibrium emittance in this scheme is obtained by inserting Eq. (31) in Eq. (27):

$$\gamma \Delta y \Delta y' \approx \lambda\!\!\!^{-}_M k_L \beta_y. \tag{32}$$

Here $\lambda\!\!\!^{-}_M$ is the Compton wavelength of the heavy ion. This is a very small emittance.

Explicit examples for RHIC and LHC indicate that the heavy ion beams can be cooled efficiently in this way, and in the process generate diffraction limited x-rays or γ-rays.

4.4 Radiative Laser Cooling in Electron Linacs

With a sufficiently intense laser pulse, a high energy electron beam can be cooled significantly during a single collision with the laser pulse. The radiative process here may be viewed either as the Thomson backscattering or as the undulator radiation. The process here is similar to the one discussed in Section 4.2 except that the beam parameters change substantially during the laser-electron beam collision. It can be applied to the cooling of the electron beams in the linac (16).

The required laser pulse energy W_L to reduce the electron energy from E_o to $E << E_o$ is

$$W_L = \frac{mc^2}{4E} \frac{\Sigma}{\sigma_{Th}}. \tag{33}$$

Here Σ is the cross-sectional area of the laser beam and σ_{Th} is the Thomson cross section. The achievable emittance is similar to Eq. (32):

$$\gamma \Delta x \Delta x' = \lambda\!\!\!^{-}_e \frac{K}{4} k_L \beta_x . \tag{34}$$

Here K is the deflection parameter of the electron trajectory in the laser field. [In Eq. (34), K is assumed to be of order unity or less. If $K >> 1$, then Eq. (34) needs to be multiplied by K^2 to account for the dispersion generated by the laser field itself.] Explicit examples show that this could be an attractive cooling method either for the TeV linear colliders or for high brightness electron beams for x-ray SASE.

5. SUMMARY

In this paper we have summarized the important role of emittance and phase space in particle and radiation beam techniques, emphasizing the physical pictures rather than mathematical rigor. We have also discussed some cooling techniques made feasible with the recent advent in the high power laser technology (17).

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