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UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

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ERRATUM

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In Eq. (3), not all the functions $R_{m,\lambda_1\lambda_2}$ are real, but have a helicity dependent phase. This is due to factors from kinematic singularities, some of which become imaginary in the $t < 0$ region. The calculated upper bound is now applicable to $|\mathrm{Im}\,\rho_{10}|$, which is the transverse polarization of the ρ. Since this cannot be measured from angular distributions of decay products, no comparison with experiment is possible at this time.

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BOUND FOR EFFECTIVE POLARIZATION IN ρ PRODUCTION IN A REGGE-POLE EXCHANGE MODEL*

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ABSTRACT

It is shown that, in the framework of the Regge-pole formalism, an upper bound may be obtained for the parameter $|\mathrm{Re}\ \rho_{10}|$, which describes ρ polarization in the process $\pi^- p \to \rho^0 n$. The bound is clearly violated by experiment at 6 and 8 GeV/c.

In the past year the Regge-pole exchange model has had considerable success in giving a consistent description of a number of charge-exchange, resonance-production, and backward-elastic-scattering processes.[1] However a considerable problem has arisen, in that it has not been possible to fit polarization data for the process $\pi^- p \to \pi^0 n$, using only Regge trajectories associated with established particles.[1,2] The purpose of this work is to investigate whether the above difficulty is present in other processes. In particular, we examine the ρ polarization in the process $\pi^- p \to \rho^0 n$.

The decay distribution of the 2π system in the rest frame of the ρ is given by[3]

$$W(\theta,\phi) = \frac{3}{4\pi}\left(\rho_{00}\cos^2\theta + \rho_{11}\sin^2\theta - \rho_{1-1}\sin^2\theta\cos 2\phi - \sqrt{2}\,\mathrm{Re}\,\rho_{10}\sin 2\theta\cos\phi\right), \tag{1}$$

where the standard angles are taken with respect to the incident beam. The spin density matrix elements $\rho_{mm'}$ can be expressed in terms of the helicity amplitudes for the t-channel process $\pi^-\rho^0 \to \bar{p}n$ by using the crossing relations.[3] One obtains

$$\rho_{mm'} = \frac{\sum\limits_{\lambda\lambda'} F_{m,\lambda\lambda'} F^*_{m',\lambda\lambda'}}{\sum\limits_{m,\lambda\lambda'} |F_{m,\lambda\lambda'}|^2}, \tag{2}$$

where $F_{m,\lambda\lambda'}$ are the t-channel helicity amplitudes, and m, λ, and λ' are the ρ, $\bar{p}$, and n helicities, respectively. The element $\mathrm{Re}\,\rho_{10}$ has a structure very similar to the polarization parameter in $\pi^- p \to \pi^0 n$. It is the interference between two amplitudes differing by one unit of helicity. Hence contributions come only from interference of two different Regge-pole exchanges which have different phases.

For the process $\pi^- p \to \rho^0 n$, G-parity and isospin conservation limit the exchanges to the π, A_1, and A_2 trajectories. We assume the A_1 has $J^P = 1^+$. The t-channel amplitudes for a single Regge-pole exchange may be represented by[4]

$$F_{m,\lambda_1\lambda_2}(x,t) = \frac{\dfrac{1 \pm e^{-i\pi\alpha}}{\sin \pi\alpha}}{\left|\dfrac{1 \pm e^{-i\pi\alpha}}{\sin \pi\alpha}\right|} R_{m,\lambda_1\lambda_2}(t) \left(\frac{1+x}{2}\right)^{|m+(\lambda_1-\lambda_2)|/2}$$

$$\times \left(\frac{1-x}{2}\right)^{|m-(\lambda_1-\lambda_2)|/2} (s/s_0)^{\alpha-M} , \qquad (3)$$

where $R_{m,\lambda_1\lambda_2}(t)$ is the product of the residue function and appropriate kinematic factors, including those obtained from the expansion of the rotation matrix $d^{\alpha}_{m,\lambda_1\lambda_2}(x)$, and may be taken to be real in the absence of intersecting trajectories. The cosine of the t-channel scattering angle is x, and $M = \mathrm{Max}(|m|, |\lambda_1 - \lambda_2|)$. From conservation of parity and G-parity we obtain the following restrictions for $R_{m,\lambda_1\lambda_2}$:

π exchange: For $\lambda_1 \neq \lambda_2$, $R_{m,\lambda_1\lambda_2} = 0$.

For $\lambda_1 = \lambda_2$, $R_{m,\lambda_1\lambda_2} = -R_{m,-\lambda_1-\lambda_2}$; $R_{m,\lambda_1\lambda_2} = R_{-m,\lambda_1\lambda_2}$.

A_1 exchange: For $\lambda_1 = \lambda_2$, $R_{m,\lambda_1\lambda_2} = 0$.

For $\lambda_1 \neq \lambda_2$, $R_{m,\lambda_1\lambda_2} = -R_{m,-\lambda_1-\lambda_2}$; $R_{m,\lambda_1\lambda_2} = R_{-m,\lambda_1\lambda_2}$.

A_2 exchange: For $m = 0$, $R_{m,\lambda_1\lambda_2} = 0$.

For $m \neq 0$, $R_{m,\lambda_1\lambda_2} = R_{m,-\lambda_1-\lambda_2}$; $R_{m,\lambda_1\lambda_2} = -R_{-m,\lambda_1\lambda_2}$.

(4)

Using constraints (4) in Eq. (3) we consider Eq. (2) for the density-matrix elements. After a small amount of algebra it is evident that the only contribution to $\text{Re}\,\rho_{10}$ is from the interference of the A_1 and A_2 contributions.

We may now proceed to obtain an upper bound for $|\text{Re}\,\rho_{10}|$. Considering only the known contributions to $\pi^- p \to \rho^0 n$ we have

$$\mathrm{Re}\,\rho_{10} = \frac{\mathrm{Re}\left(F^{A_2}_{1,\frac{1}{2}-\frac{1}{2}}\ F^{A_1 *}_{0,\frac{1}{2}-\frac{1}{2}} + F^{A_2}_{1,-\frac{1}{2}\frac{1}{2}}\ F^{A_1 *}_{0,-\frac{1}{2}\frac{1}{2}}\right)}{2\left|F^{A_1}_{1,\frac{1}{2}-\frac{1}{2}} + F^{A_2}_{1,\frac{1}{2}-\frac{1}{2}}\right|^2 + 2\left|F^{A_1}_{1,-\frac{1}{2}\frac{1}{2}} + F^{A_2}_{1,-\frac{1}{2}\frac{1}{2}}\right|^2 + 2\left|F^{A_1}_{0,\frac{1}{2}-\frac{1}{2}}\right|^2 + \Sigma}, \tag{5}$$

where

$$\Sigma = 2\left|F^{A_2}_{1,\frac{1}{2}\frac{1}{2}} + F^{\pi}_{1,\frac{1}{2}\frac{1}{2}}\right|^2 + 2\left|F^{A_2}_{1,-\frac{1}{2}-\frac{1}{2}} + F^{\pi}_{1,-\frac{1}{2}-\frac{1}{2}}\right|^2 + 2\left|F^{\pi}_{0,\frac{1}{2}\frac{1}{2}}\right|^2 .$$

Since Σ is clearly positive definite, we may set it to zero in obtaining an upper bound for $|\mathrm{Re}\,\rho_{10}|$. Defining $\alpha_{A_1} = \alpha_1$, $\alpha_{A_2} = \alpha_2$,

$$R^{A_1}_{0,\frac{1}{2}-\frac{1}{2}}(s/s_0)^{\alpha_1} = F_0, \qquad R^{A_1}_{1,\frac{1}{2}-\frac{1}{2}}(s/s_0)^{\alpha_1 - 1} = F_1, \qquad R^{A_2}_{1,\frac{1}{2}-\frac{1}{2}}(s/s_0)^{\alpha_2 - 1} = F_2,$$

substituting (3) into (5), and setting $\Sigma = 0$ we have

$$|\mathrm{Re}\,\rho_{10}| \leq \left| \frac{\cos\frac{\pi\Delta\alpha}{2}\left|\frac{1 - x^2}{4}\right|^{\frac{1}{2}} F_2 F_0}{(1 + x^2)(F_2^{\,2} + F_1^{\,2}) + 4\sin\frac{\pi\Delta\alpha}{2} F_1 F_2 x + 2F_0^{\,2}\left(\frac{x^2 - 1}{4}\right)} \right|, \tag{6}$$

where $\Delta\alpha = \alpha_2 - \alpha_1$. We first minimize the denominator of (6) by differentiating with respect to F_1, obtaining the condition

$$F_1 = \frac{-2\sin\frac{\pi\Delta\alpha}{2} F_2 x}{1 + x^2},$$

which gives

$$|\mathrm{Re}\ \rho_{10}| \leqslant \left| \frac{\cos\frac{\pi\Delta\alpha}{2}\left|\frac{1-x^2}{4}\right|^{\frac{1}{2}}}{\left|\frac{F_2}{F_0}\right|\left[1+x^2-\frac{4\sin^2\frac{\pi\Delta\alpha}{2}\,x^2}{1+x^2}\right]+2\left|\frac{F_0}{F_2}\right|\left(\frac{x^2-1}{4}\right)} \right| .$$

Finally by minimizing the denominator with respect to $|F_2/F_0|$ and maximizing the resulting expression as a function of $\Delta\alpha$, we finally obtain the nontrivial[5] upper bound for $|\mathrm{Re}\ \rho_{10}|$,

$$|\mathrm{Re}\ \rho_{10}| \leqslant \frac{1}{2\sqrt{2}\,(1+x^2)^{\frac{1}{2}}} . \qquad (7)$$

In Figs. 1 and 2 we compare this bound with the data at 6 and 8 GeV/c.[6,7] The bound is clearly violated, and thus the situation is similar to that in the process $\pi^- p \rightarrow \pi^0 n$. In πp charge exchange the bound for the polarization for known exchanges is identically zero, whereas experimentally the polarization is of order 16% at 5.9 GeV/c and 14% at 11.2 GeV/c.[8] The experimental results for $\mathrm{Re}\ \rho_{10}$ in $\pi^- p \rightarrow \rho^0 n$ are consistent with no variation with energy in the range 2.36 to 8.0 GeV/c.[6,7,9,10,11] It should be stressed that the observed value of $\mathrm{Re}\ \rho_{10} \approx -0.2$ is not to be thought of as small, since from (1) the maximum possible value of $|\mathrm{Re}\ \rho_{10}|$ is $1/(2\sqrt{2})$. Thus the experimental value of $\mathrm{Re}\ \rho_{10}$ represents an effective polarization of 60%. This is important, since it means the terms contributing to

Re ρ_{10} must also be significant in the differential cross section.

It must be emphasized that in obtaining the bound (7), the crucial requirements are:

(A) Only trajectories associated with known particles are used.

(B) The trajectory and residue functions are real below the t-channel threshold. (No other assumptions are needed as to the behavior of $R_{m,\lambda_1\lambda_2}$ and α as function of t to obtain the bound.)

The relaxation of either or both of the above requirements could give agreement with experiment. Condition (A) can be relaxed in a number of ways:

(i) J-plane cuts could possibly give enough contribution to solve the problem.

(ii) Unknown, and presumably lower-lying trajectories having, for instance, π or A_1 quantum numbers would contribute and alter the bound. Any other trajectories having A_2 quantum numbers would not alter our conclusions.

(iii) Direct-channel resonances.

However, the approximate constancy of Re ρ_{10} over a wide range of energy makes (ii) and (iii) implausible.

If two trajectories collide below threshold, condition (B) is invalid and the analysis used to obtain our bound breaks down. A model of this type, having complex residue and trajectory functions, has been postulated to explain the $\pi^- p \to \pi^0 n$ polarization.[12]

Finally we make the following points. Violation of the bound is particularly serious in view of its looseness; i.e., in obtaining the bound we set $\Sigma = 0$ in (5), whereas in general we can expect this quantity to be nonvanishing and so reduce the bound. Even though a bound cannot be obtained in the noncharge-exchange production, due to the additional $I = 0$ exchanges, our result means that a Regge fit to these processes will require $I = 1$ contributions in addition to the π, A_1, and A_2 exchanges.

The same analysis can be used for the process $\pi^- p \to \omega n$, where we replace A_2, A_1, and π by ρ, $B'(2^-)$, and $B(1^+)$, respectively. Lack of accurate data at high energy does not permit us to come to any conclusions at present.

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FOOTNOTES AND REFERENCES

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† Miller Postdoctoral Fellow of the University of California, Berkeley.

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FIGURE LEGENDS

Fig. 1. Comparison of bound and data for $|\mathrm{Re}\ \rho_{10}|$ at 6 GeV/c.

Fig. 2. Comparison of bound and data for $|\mathrm{Re}\ \rho_{10}|$ at 8 GeV/c.

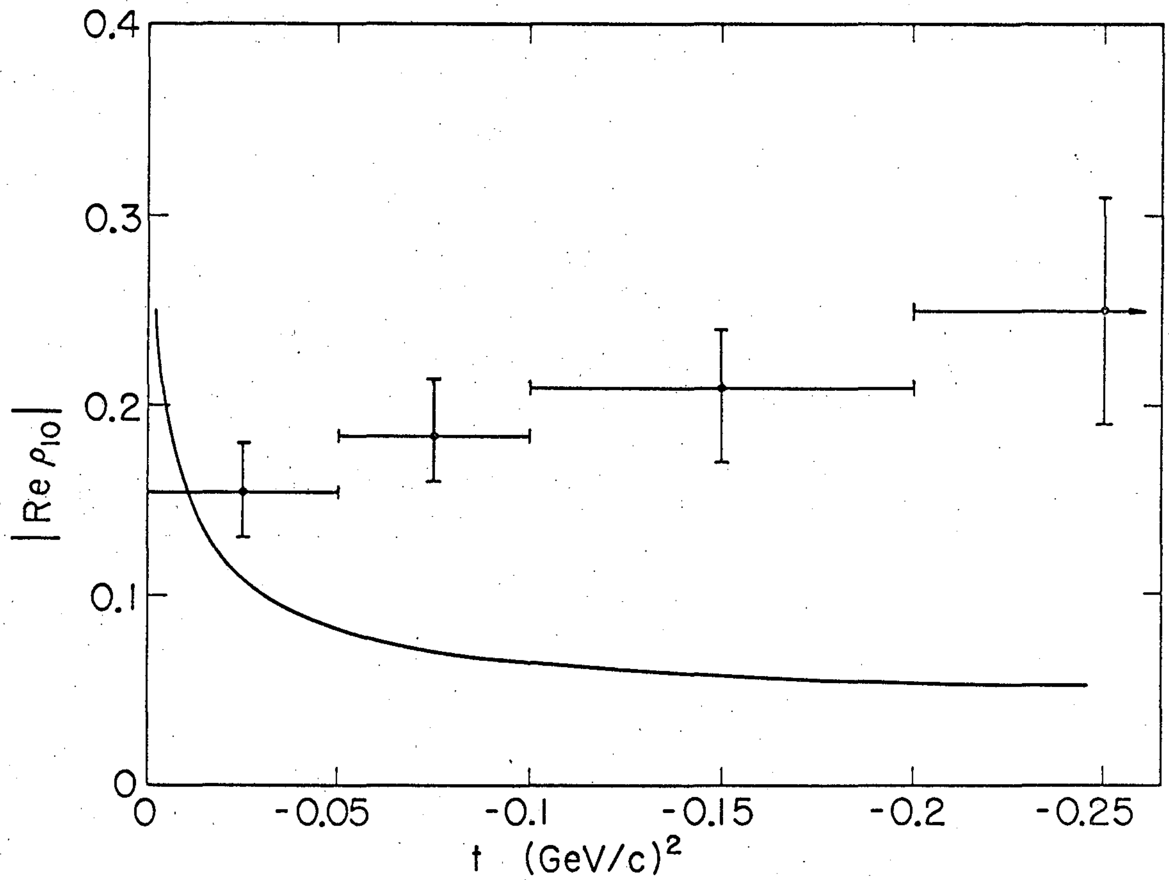

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Fig. 1

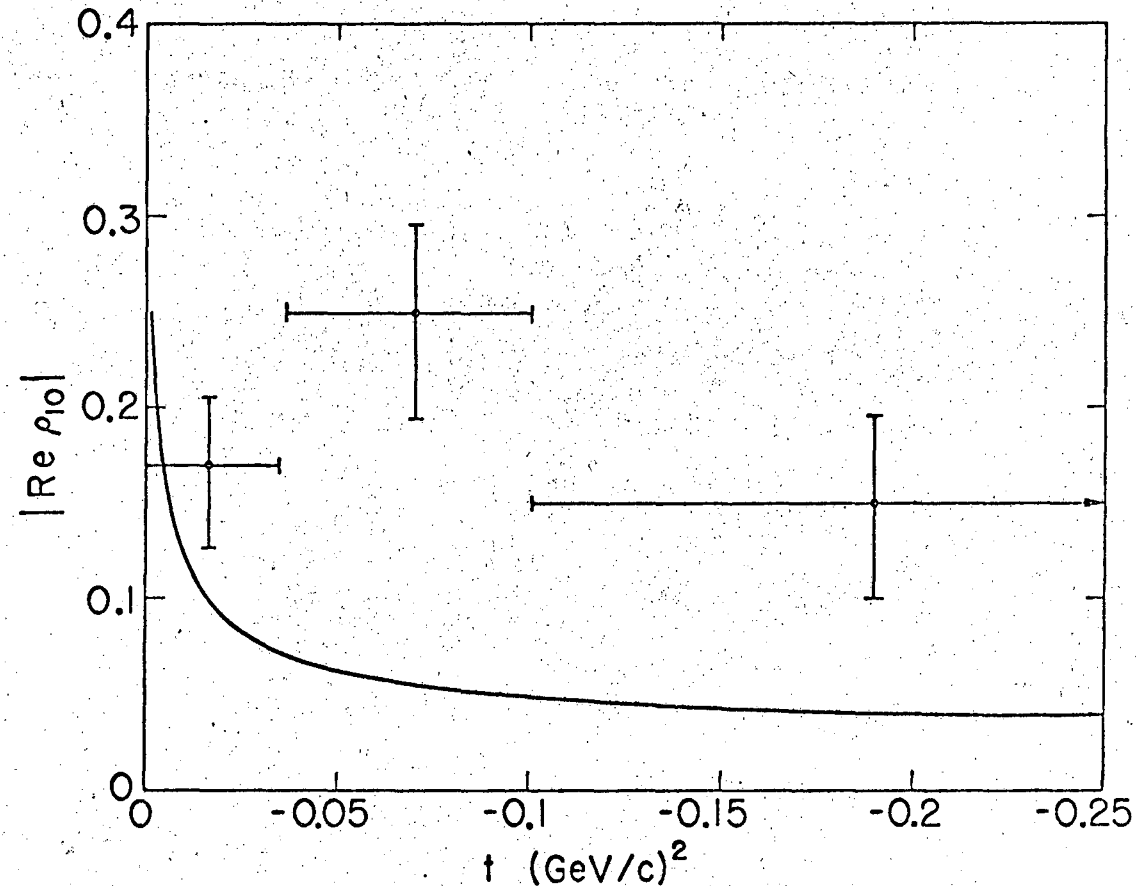

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Fig. 2