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DESI DR2 Galaxy luminosity functions

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ABSTRACT

We present galaxy luminosity functions (LFs) for the Dark Energy Spectroscopic Instrument (DESI) DR2 Bright Galaxy Survey (BGS) in the g , r , z , and $w1$ bands over $0.002 < z < 0.6$. Our analysis uses updated k -corrections and evolutionary corrections, including new polynomial k -correction fits derived from BGS Year 1 data that supersede earlier GAMA-based prescriptions. Exploiting the statistical power of DESI, we measure LFs to very faint magnitudes, reaching $^{0.1}M_r - 5 \log h \sim -10$. Independent measurements from the North and South survey regions show excellent agreement around the LF knee, but the very small statistical uncertainties reveal that simple analytic forms fail to capture the full LF shape. The bright end departs from a pure exponential decline, while the faint end exhibits complex, non-powerlaw behaviour, including a pronounced upturn at $^{0.1}M_r - 5 \log h \gtrsim -15$, which is stronger for red galaxies than for blue. We show that our LFs are largely complete for galaxies with surface brightness $\mu_{50} < 25$, and that an apparent steepening fainter than -13 is driven primarily by local overdensity and fragmentation of large galaxies. A systematic North–South offset at the brightest magnitudes is traced to red galaxies and may reflect shallower North photometry underestimating extended earlytype profiles, although this remains inconclusive. We therefore also provide LFs based on model Petrosian magnitudes. Redshift splitting reveals small but significant residuals, indicating limitations of a simple global evolutionary model. Using the redshift limits of J. Loveday et al. (2012), we find excellent agreement with GAMA, with substantially reduced statistical errors. These measurements provide a precise reference for studies of environmental and population-dependent LFs and for testing galaxy formation models.

Key words: galaxies: luminosity function, mass function – galaxies: statistics.

1 INTRODUCTION

The galaxy luminosity function (LF) has become one of the most useful statistics for quantifying the galaxy population. Various studies have sought to improve the measurement of the LF, including at faint magnitudes. These studies have been facilitated by new generations of redshift surveys which have surveyed large areas of the sky to sufficiently high redshifts with high com-

pleteness. This has enabled data analysis on large catalogues of galaxies to produce statistically precise results.

Global LFs have been measured with steadily improving precision as redshift and imaging surveys expanded in area, depth, and wavelength coverage, establishing a broadly consistent empirical picture of the local galaxy population while also revealing persistent tensions at both the bright and faint ends. Early optical determinations such as the CfA (J. Huchra et al. 1983) and subsequent local compilations (J. Loveday et al. 1992; R. O. Marzke, J. P. Huchra & M. J. Geller 1994) were followed by the first ‘modern’ large-sample constraints from surveys such as the Las Campanas Redshift Survey (H. Lin et al. 1996; S. A. Shectman et al. 1996) and

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the ESO Slice Project (E. Zucca et al. 1997). By the early 2000s, the very large 2dF Galaxy Redshift Survey and Sloan Digital Sky Survey enabled high signal-to-noise, multiband LFs with well-characterized selection (e.g. S. Folkes et al. 1999; P. Norberg et al. 2002; M. R. Blanton et al. 2003), solidifying the empirical conclusion that a Schechter-like form provides an adequate first-order description over a wide luminosity range, while also emphasizing that the inferred faint-end slope and normalization depend sensitively on the treatment of low-luminosity systems. Outside of the optical bands, all-sky and wide-area surveys such as 2MASS have measured near-infrared LFs which more directly reflect galaxy stellar masses (e.g. S. Cole et al. 2001; C. S. Kochanek et al. 2001).

More recently, global LF work has expanded further from a simple description to focusing on regimes dominated by rare populations. Using SDSS DR6, A. D. Montero-Dorta & F. Prada (2009) reported evidence for a statistically significant bright-end excess, which they associated primarily with the contribution of brightest cluster galaxies (BCGs), reinforcing the notion that the high-luminosity tail can carry an imprint of the densest environments even in nominally global measurements. E. F. Bell et al. (2003), combining SDSS with 2MASS, argued that near-infrared global LFs inferred from 2MASS alone are biased against low-surface brightness galaxies and that correcting this effect yields a steeper faint-end slope and revised K -band luminosity density. Moreover, nearby-universe studies have highlighted that the global faint-end behaviour is not universal across galaxy classes: N. Trentham & R. B. Tully (2002) found markedly different faint-end slopes for early-type versus late-type systems, underscoring that the aggregate global LF reflects the superposition of multiple populations whose relative abundances vary across samples and environments. Taken together with other large-survey results (e.g. P. Norberg et al. 2002; M. R. Blanton et al. 2005; J. Loveday et al. 2012), these empirical findings motivate renewed attention to how specific populations, such as BCGs at the bright end and low-surface-brightness galaxies at the faint end, can shape the global LF and related quantities such as luminosity density and stellar mass density.

Further work sought to capitalize on the large statistical advances that came with 2dFGRS and SDSS by revisiting the pioneering work of B. Binggeli, A. Sandage & G. A. Tammann (1988) and demonstrating that LF shape and normalization vary systematically with galactic environment, reflecting changes in the underlying galaxy population across the cosmic web. Empirically, dense environments such as groups and clusters are associated with enhanced fractions of luminous, red, early-type galaxies, while lower-density field regions host higher proportions of star-forming, late-type systems, and these differences propagate directly into environment-binned LFs. In 2dFGRS, type-dependent and environment-sensitive LF measurements (e.g. S. Folkes et al. 1999; D. S. Madgwick et al. 2002; P. Norberg et al. 2002) established that early-type and late-type LFs differ in both characteristic luminosity and faint-end slope, implying that shifts in the proportions of different galaxy types with environment naturally produce measurable LF variations. Subsequent SDSS analyses similarly emphasized systematic LF dependencies on galaxy colour, morphology, and star-formation activity (M. R. Blanton et al. 2005). More targeted environmental studies have sharpened this view by explicitly comparing field, group, and cluster LFs and by connecting LF variations to physically motivated environment definitions. For example, T.-W. Lan, B. Ménard & H. Mo

(2016) examined LF differences between field and denser systems (groups/clusters), while T. McNaught-Roberts et al. (2014) quantified LF trends with local density, and K. Kraljic et al. (2017) explored LF variations across different cosmic-web classifications. These studies collectively support the empirical conclusion that environment affects both the abundance of luminous galaxies and the representation of faint galaxies, where trends can differ by galaxy type and may be entangled with the detectability of low-surface-brightness populations (as emphasized by E. F. Bell et al. 2003). In parallel, other multiband LF programs such as GAMA (S. P. Driver et al. 2011) have provided a consistent empirical baseline across optical filters (*ugriz*; see J. Loveday et al. 2012), facilitating comparisons of how environment-dependent population shifts manifest with wavelength. More broadly, characterizing LF variation with environment (across density, halo mass, cosmic-web morphology, etc) has become central to linking galaxy demographics to the physics of quenching, merging, and hierarchical growth. Although this paper only focuses on the global LF, the methodology in this paper will be used in a subsequent second paper where we will present density-dependent LFs.

The global LF has also been a useful method for constraining various galaxy formation models. R. G. Bower, A. J. Benson & R. A. Crain (2012) discuss how in semi-analytical galaxy formation models (e.g. A. J. Benson 2012; C. G. Lacey et al. 2016; G. De Lucia et al. 2024) the bright-end is shaped by AGN feedback while the faint end is influenced by SN feedback. Also, N. Trentham & R. B. Tully (2002) find that there is a discrepancy in the faint end slope between simulations and empirical results, with the observed faint end of the LF being less steep than predicted. This result has been confirmed by several studies, (F. Hoyle et al. 2005; Moretti, A. et al. 2015) suggesting that there is further scope to improve galaxy formation simulations.

In addition, the bivariate and conditional LFs are another useful tool to better understand galaxy populations. Given that it is known that the LF varies as a function of intrinsic galaxy properties, prior studies have used the bivariate LF as a means of probing this relationship by examining the luminosity conditional on other factors. For example, bivariate LFs have been constructed for the SDSS r -band and stellar mass, inverse concentration index, morphology, Sérsic index, and other properties (N. M. Ball et al. 2006). In this paper, we use the bivariate LF to investigate the r -band luminosity conditional on other luminosity bands.

In recent years, the Dark Energy Spectroscopic Instrument (DESI) has greatly increased the available data for such studies (DESI Collaboration 2016b, 2022). The Bright Galaxy Survey (BGS) extends fainter than comparable studies (two magnitudes fainter than the SDSS Main Galaxy Survey) with a correspondingly higher median redshift (C. Hahn et al. 2023). This provides us with an opportunity to further constrain the estimate of the LF and explore galaxy evolution. We seek to capitalize on these data to measure the LF in different bands. Additionally, we investigate LFs split by colour.

In Section 2, we describe the input catalogue including details of the redshifts and completeness weights. In Section 3, we discuss the calculation of the absolute magnitudes with the use of k -corrections and e -corrections as well as our choice of LF estimator. Section 4 presents our LF results and we draw our conclusions in Section 5. Throughout this paper, we adopt a standard Lambda cold dark matter (Λ CDM) cosmology with $\Omega_M = 0.313$,

$\Omega_\Lambda = 0.687$ and $H_0 = 100h \text{ kms}^{-1}\text{Mpc}^{-1}$ (Planck Collaboration VI 2020)¹.

2 OVERVIEW OF DESI BGS DATA

In this section, we describe the DESI survey and the BGS data. We provide details of the galaxy catalogue in Section 2.1. In particular, we discuss the LSS catalogues that are described in A. J. Ross et al. (2025). We further describe in Section 2.2 how DESI determines the spectroscopic redshift for each object. In Section 2.3, we describe how we correct for target and redshift incompleteness in the survey. In Section 2.4, we describe the random catalogues that match the A. J. Ross et al. (2025) LSS catalogues.

2.1 Input catalogue

The Dark Energy Spectroscopic Instrument (DESI) is a next-generation Stage IV survey based at Kitt Peak, Arizona as part of the 4-m Mayall Telescope. The primary goal of DESI is to map the large scale galaxy and quasar distribution in order to provide sub-per cent precision measurements of a range of cosmological parameters. This is achieved by observing targets using spectrographs fed by almost 5000 robotically positioned optical fibres across a 3° field of view (J. Guy et al. 2023; E. F. Schlafly et al. 2023; J. H. Silber et al. 2023; T. N. Miller et al. 2024; C. Poppett et al. 2024). As part of this project, there are five main samples of data: Emission Line Galaxies (ELGs), Luminous Red Galaxies (LRGs), Quasars (QSOs), the Milky Way Survey (MWS), and the Bright Galaxy Survey (BGS; DESI Collaboration 2016a).

BGS is the component of DESI that focuses on the mapping of more than 10 million galaxies from $0 < z < 0.6$. BGS Bright is a $r \lesssim 19.5$ magnitude limited sample, while BGS Faint covers the fainter range $19.5 < r < 20.175$ but additionally has a colour-dependent fibre-magnitude limit (C. Hahn et al. 2023) to ensure the redshift measurements have a high success rate. Here, we limit ourselves to the BGS Bright sample. DESI targets, including those in BGS, are selected based on applying photometric criteria to the photometric data taken with the DESI Legacy Surveys, BASS/MzLS in the North and DECaLS in the South (A. Dey et al. 2019). The target selection procedure for BGS is described in detail in O. Ruiz-Macias et al. (2020), while the final target selection choices are summarized in A. D. Myers et al. (2023). As a result of the differing photometry in North and South, the DESI BGS Bright survey is magnitude-limited to $r < 19.54$ in the North ($\text{dec} > 32.375^\circ$) and $r < 19.5$ in the South to achieve the same surface density of targets in both hemispheres. These are extinction-corrected apparent magnitudes which are based on the SFD dust map (D. J. Schlegel, D. P. Finkbeiner & M. Davis 1998). BGS Bright – which will be the focus of this paper – is similar to the depth of the Galaxy And Mass Assembly (GAMA) survey (S. P. Driver et al. 2011), but covers a much larger area of the sky² at 14 000 deg².

The DESI magnitudes are total model magnitudes computed by the Legacy Survey pipeline (A. Dey et al. 2019). Each detected

source is fitted by each of the following seeing convolved model profiles: a point source (PSF), a round exponential (REX), elliptical exponential (EXP), elliptical de Vaucouleurs (DEV) and elliptical Sérsic (SER). The χ^2 of the fit, taking into account a penalty for the number of parameters in the model, is used to determine the best-fitting model. The total magnitude is determined by the integral of the total flux of this best-fitting model. DESI fibre magnitudes are defined as the prediction of the best-fitting model for a 1.5 arcsec diameter fibre aperture and assumed fiducial seeing of 1 arcsec FWHM. We later apply distance modulus, passband and evolutionary corrections to these magnitudes to define rest-frame absolute magnitudes, as discussed in Section 3.1 and 3.2.

In this paper, we make use of DESI DR2 data [otherwise known as Year Three (Y3) data]. This is the data of the main DESI survey collected in the first three years of the survey, with key results released in 2025 (DESI Collaboration et al. 2025a, b, 2026).

Our catalogue contains a total of 7874 903 galaxies spanning an area of 12, 355.08 deg², shown in Fig. 1. In addition, the DESI Y3 catalogue also makes use of data from the *Wide-Field Infrared Survey Explorer* (WISE). WISE is a space-based infrared survey which mapped the whole sky with multiple passes (E. L. Wright et al. 2010). In particular, we use magnitudes from the w1 band centred at 3.4 μm with an angular resolution of 6.1 arcsec, in order to present a w1-band LF.

2.2 Redshifts

In order to determine the redshifts of galaxies from the measured spectra, DESI makes use of Redrock (A. Anand et al. 2024), a template-based classifier which classifies spectra as GALAXY, STAR, or QSO and assigns a redshift to each spectrum based on χ^2 minimization of a linear combination of Principal Component Analysis (PCA) basis spectral templates. Although the PCA-based redshift estimator can occasionally yield unphysical models, such as negative emission lines, Redrock mitigates this issue reasonably well by applying penalties (see A. Anand et al. 2024, for more details). Redrock templates were constructed using eBOSS spectra. Redrock additionally generates a $\Delta\chi^2$ value which is the difference in χ^2 between the best and second-highest likelihood peaks with different redshifts, which acts as a metric of the confidence of the redshift determination.

A large number of spectra were evaluated by visual inspectors to assess the validity of the Redrock algorithm, as detailed in T.-W. Lan et al. (2023). For the Survey Validation (SV), BGS Bright had an average assessed Visual Inspection Quality > 2.5 ³ for 99.6 per cent of galaxies based on 1037 sources, indicating that Redrock returns robust redshifts for the vast majority of objects. Moreover, T.-W. Lan et al. (2023) investigates the use of $\Delta\chi^2$ as a metric for assessing the reliability of the Redrock redshifts. For BGS Bright, the authors find 100 per cent purity for $\Delta\chi^2 > 40$ in the VI sample of 2718 BGS target galaxies, indicating that this is a useful threshold for ensuring that sources have the correct redshifts.

¹For this paper, we choose to work in units in which the exact dependence on h is explicit.

²This excludes low completeness single pass regions around the edge of the survey and a recently added small extension increasing the overlap with the DES survey. If both were included the final area would be closer to 15 000 deg².

³A visual inspection quality of 3 corresponds to a probable classification with at least one secure spectral feature and continuum or many weak spectral features, e.g. spectra with a strong emission line feature and weak Balmer series absorption lines while quality 2 corresponds to a classification with one strong but unidentified spectral feature (T.-W. Lan et al. 2023).

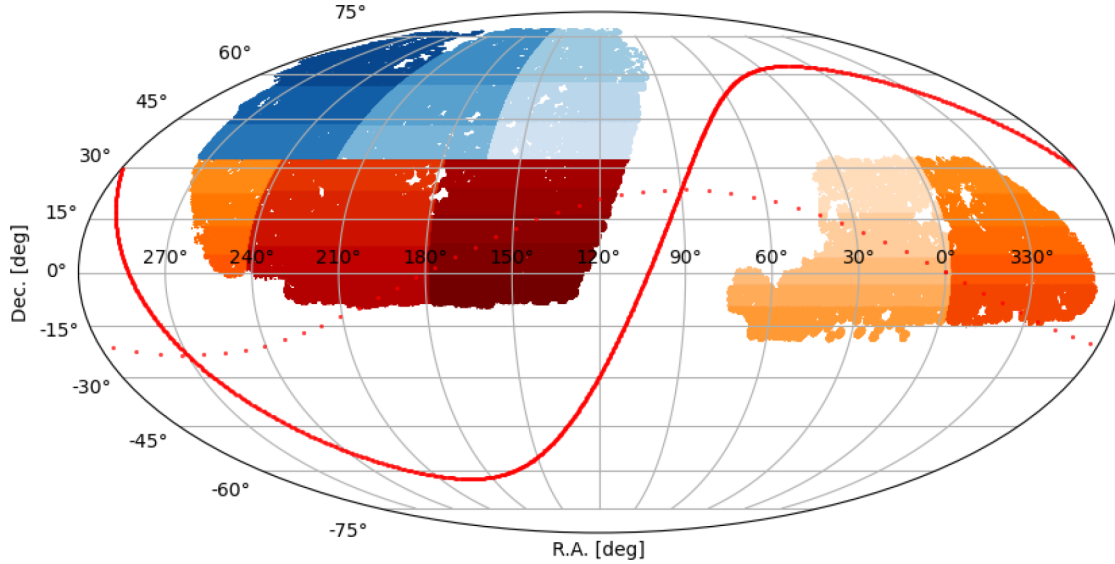


Figure 1. The Y3 DESI Footprint – showing the North (blue) and South (red). The different shades of blue and red highlight the jackknife regions (see Section 3.3 for details). The solid red curve shows the galactic plane while the dotted red curve shows the ecliptic plane.

The redshifts returned by Redrock are relative to the solar system barycentre. For the purposes of our analysis, we transform these redshifts into the Cosmic Microwave Background (CMB) rest frame and correct for local peculiar velocities using the CosmicFlows-4 grouped flow field (H. M. Courtois et al. 2023). The effect of these transformations on our luminosity function estimates is small but moderately significant given the very small statistical errors. The bright end of our luminosity functions are dominated by galaxies at $z > 0.1$ where the change in distance modulus is insignificant. At lower redshift, the change in distance modulus for individual galaxies is more significant but the wide sky coverage of DESI combined with variation of the flow field across the sky results in little net change, even for the faint end of the luminosity function.

2.3 Weights

To construct luminosity functions, it is important to correct for incompleteness within the DR2 data set, including systematic effects in the input catalogue, target incompleteness and redshift success rates.

Typically, a systematic weight (WEIGHT_SYS) is used to account for target density fluctuations due to imaging conditions and foregrounds. Specifically, this weight corrects for unphysical correlations of the target density with dust extinction, stellar density, and HI maps (DESI Collaboration, private correspondence). We observe that these weights are close to unity and make a negligible difference to our results. In particular, they have no impact at all on the faint-end of the r -band LF. Their value is in correcting for large scale spatial variations in the completeness, not in determining global quantities such as the LF. As such, we have chosen to ignore this systematic weight in this paper. However, in addition to these systematics, incompleteness at low-surface brightness could be important at the faint end of the galaxy luminosity function. We therefore explicitly investigate this in Section 4.2.

An additional source of incompleteness is targeting incompleteness, which in this case is a correction factor to account for

targets that were not observed. In tiles for which there has only been a single pass, there will be targeting incompleteness as only one object can be targeted per fibre – specifically for each fibre, this target will be within the unique patrol region of each fibre. This means that in regions of high target density, not all targets will be assigned to fibres. Furthermore, fibres cannot be placed arbitrarily close to each other due to the physical and mechanical constraints of the fibre positioners. This limit can also lead to observable targets not being assigned a fibre. The footprint of the survey is determined by the area of sky that is the union of sky reachable by good fibres on observed fields. DESI conducts multiple passes to reduce targeting incompleteness. However, in regions covered by multiple tiles, the targeting incompleteness is reduced but is rarely completely removed. To correct for this incompleteness, a weight, w_{comp} (WEIGHT_COMP), is defined for DR2 in A. J. Ross et al. (2025). In particular, w_{comp} accounts for the observable targets that were not assigned a fibre by assigning their weight to a neighbouring object that was assigned a (working) fibre.

In addition, not all targeted galaxies may receive redshifts (e.g. due to a failure of Redrock to fit a model spectra with confidence). Whilst the DESI LSS catalogues do calculate a redshift completeness weight (WEIGHT_ZFAIL), this weight is designed to correct to a uniform sample (with uniform incompleteness) rather than to a complete sample. As such, this weight is adequate for clustering but less useful for LFs. We choose instead to make our own direct estimate of the redshift completeness weights (w_z).

To do this, we take the full LSS catalogue, that is, the LSS catalogue that contains all potentially observed targets within the footprint (see section 2.3 of the KP3 paper and section 4 of Ross et al. 2025). From this, we then define a ‘zgood’ subset (i.e. the standard LSS catalogue) by applying following selection cuts:

- (i) $\text{DELTA}\chi^2 > 40$.
- (ii) $\text{ZWARN} = 0$
- (iii) $0.002 < z < 0.6$

Here, ZWARN is a bitmask that incorporates various information to indicate if the redshift has problems, such as fibre

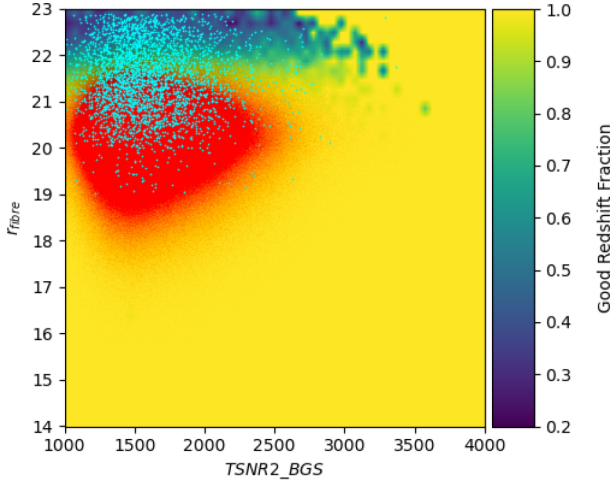


Figure 2. The empirical redshift completeness as a function of the fibre magnitude, r_{fibre} , and the TSNR2_BGS with different galaxy samples over-plotted. TSNR2_BGS is the expected spectral target signal-to-noise ratio for a fiducial BGS source. This plot is for the South region. All the galaxies in this region are plotted as red points with low opacity. The majority fall where the redshift completeness is very high. The cyan points are a subset of faint galaxies with absolute r -band magnitude fainter than -13 (see the discussion of modelling in the incompleteness in this regime in Section 4.2). To define a weight, w_z , to correct for this incompleteness we interpolate the binned incompleteness using the Cloud-In-Cell technique and take its inverse. Note that for empty pixels, we default to a value of 1.

problems, low wavelength coverage, spectrum fitting problems, etc. ZWARN = 0 indicates that there are no apparent problems with the redshift. The criteria above ensure that the redshift is reliable and excludes the redshift range contaminated by stars. In addition, the upper redshift cut ensures that we avoid objects with potentially spuriously fitted spectra. We note that 99.55 per cent of the data is $z < 0.6$, so this excludes relatively few galaxies.

The overall redshift completeness of the survey is simply the number of objects in ‘zgood’ over the number in the full observed catalogue. We calculate the incompleteness in bins of r -fibre magnitude and the template signal to noise squared for BGS (TSNR2_BGS, defined in J. Guy et al. 2023). The TSNR2_BGS is the square of the expected spectral signal-to-noise ratio for a fiducial BGS source for the specific fibre and observation, as a result it is proportional to the effective exposure time for the individual target⁴. Hence, we select r_{fibre} and TSNR2_BGS as variables for this calculation as we expect redshift completeness to decrease for fainter r_{fibre} and smaller TSNR2_BGS.

For the North region of the survey the redshift incompleteness is shown in Fig. 2. We use the Cloud-In-Cell interpolation technique (R. W. Hockney & J. W. Eastwood 1988) to smooth over the bins and define the redshift incompleteness at any arbitrary r_{fibre} and TSNR2_BGS and define its inverse as the redshift completeness weight, w_z .

The total weight applied to the i th galaxy to correct for both targeting and redshift incompleteness is the product of these two weights

$$w_i = w_{\text{comp},i} w_{z,i}. \quad (1)$$

⁴Specifically, the effective exposure time (T_{spec}) is related to TSNR2_BGS as per the following relation: $T_{\text{spec}} = (0.135\text{sec}) \times \text{TSNR2_BGS}^2$ (A. Krolewski et al. 2025).

For the target incompleteness weight, we emphasize that the values are all integers, with a minimum value of 1.0. As such, the median target incompleteness weight is 1.0, with 97.6 per cent of weights ≤ 3.0 .

For the redshift completeness weight, the overall median is 1.0015 with the 10th percentile 1.0000(2) and the 90th percentile 1.0173. If one looks at the faintest objects the weights are more significant. For the 2.33 per cent of objects with $r_{\text{fibre}} > 21.5$, the redshift incompleteness weight median is 1.1626 with the 10th percentile 1.0821 and the 90th percentile 1.6299.

For all objects, the median total weight is 1.0026 with the 10th percentile 1.0000(3) and 90th percentile 2.0074. For the objects that have $r_{\text{fibre}} > 21.5$, the median total weight is 1.2022 with the 10th percentile 1.0852 and 90th percentile 2.5776.

2.4 Random catalogue

To define the selection function of the survey a catalogue of randomly positioned points is generated over the DESI sky footprint with a number density of 2500 objects per square degree, as described in A. J. Ross et al. (2025). Then, only the randoms that are reachable by a good fibre of an observed tile are kept in the catalogue. These randoms are additionally assigned redshifts and other galaxy properties, with the assignment done randomly from the selected galaxy sample. It should be noted that North and South property assignment is done separately from each other to allow for the difference in selection functions. As described in Section 2.3, DESI assigns fibres to obtain the spectra of galaxies. This means that a single pass will suffer from target incompleteness as there will be ‘holes’ in the survey, due to the gaps between petals, holes from where the Guiding, Focusing and Alignment (GFA) cameras are, and malfunctioning fibres. The distribution of the randoms fully takes this into account. We use the total number of randoms to quantify the total area of the sky observed, but also their distribution to map this incompleteness (see figs 2 and 3 in A. J. Ross et al. 2025).

3 METHOD OF GLOBAL LF ESTIMATION

3.1 k -corrections

Typically, absolute magnitudes are corrected by k -corrections to account for band-shifting effects, specifically that the observed passband will map to different rest-frame passbands for galaxies at different redshifts (see D. W. Hogg et al. 2002, for a comprehensive overview of k -corrections). In order to compare the photometric properties of galaxies at different redshifts, we need to transform their photometry to that of a fixed combination of reference frame and filter curve. For ease of comparison, we have chosen this reference frame to be the SDSS r -band (and g -band) filter curves with a reference redshift of $z_{\text{ref}} = 0.1$ as adopted by I. Zehavi et al. (2005) and J. Loveday et al. (2012). Our goal is to model the mean redshift dependent k -correction in each band and its dependence on a single rest-frame colour.

To compute absolute magnitudes and k -corrections FastSpecFit⁵ (FSF) was developed to perform fast spectral synthesis and emission-line fitting of DESI spectra and broadband photometry

⁵The documentation for FastSpecFit can be found at <https://fastspecfit.readthedocs.io/en/latest/>

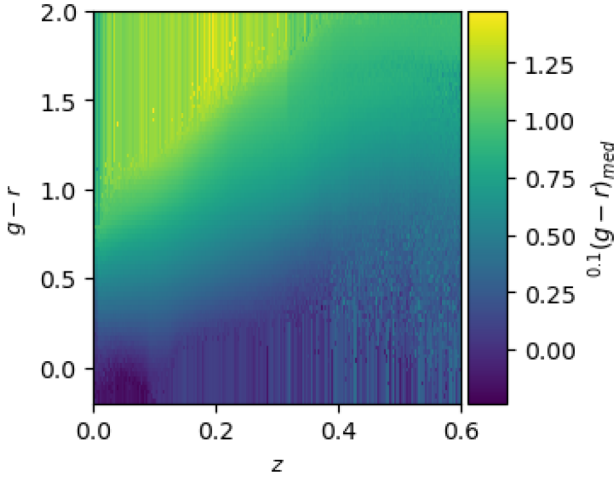


Figure 3. The median rest-frame $^{0.1}(g-r)$ colour of galaxies from the FSF South catalogue in bins of redshift and observer-frame colour. This provides a look-up table to infer the rest-frame colour from the observed properties. A separate lookup table is used for the North.

(J. Moustakas et al. 2023). In particular, FSF works by simultaneously fitting model SEDs to a combination of the broadband photometry and the aperture-corrected DESI spectral photometry. From these model SEDs, FSF absolute magnitudes and k -corrections have been calculated. We require full redshift-dependent k -correction functions so that we can calculate $V_{\max}$ ⁶ as defined in equation (6). Whilst FSF is able to do this for each individual galaxy, for convenience and computational speed, we instead construct polynomial k -corrections fitted to the FSF catalogue of k -corrected magnitudes. Our method is described below.

First, we create a rest-frame colour lookup-table, where we generate a 2D histogram of observer-frame $g-r$ colour against redshift and compute the median rest-frame FSF colour in each pixel (Fig. 3). Using this table, and cloud-in-cell interpolation, each galaxy in the Y3 DESI catalogue is assigned the median rest-frame colour corresponding with its observed colour and redshift. This assigned colour is then used to bin the galaxies into 7 rest-frame colour-bins each containing an equal number of objects.

Each colour-bin is split into a range of redshift-bins and the median FSF k -correction is calculated in each bin⁷. A least-squares polynomial fit is then performed to these median r -band k -corrections to find a 7th-order polynomial for each colour bin. The North and South are modelled separately (second panel of Fig. 4). From this plot, we see that the North and South curves are slightly different reflecting the differences in the North/South photometry. Furthermore, we note that there is a pinch point at

⁶ $V_{\max}$ is the volume within which the galaxy can be re-positioned and still satisfy all the selection criteria to be included in the sample that is being analysed. For instance, there is a maximum redshift to which the galaxy could be relocated before its apparent magnitude is too faint for it to be included in the sample and this depends on the k -correction at this redshift.

⁷The FSF r -band k -corrections as provided have an intentional discontinuity at $z \sim 0.3$ due to switching the reference band. In order to standardize the FSF, k -corrections to be from the BASS/DeCALS r band to the SDSS r -band across all redshifts, we formally use a ‘derived’ FSF k -correction, calculated from the FSF provided absolute magnitude, the apparent magnitude and the redshift (using equation 2). We also use this methodology for the g , z , and $w1$ bands.

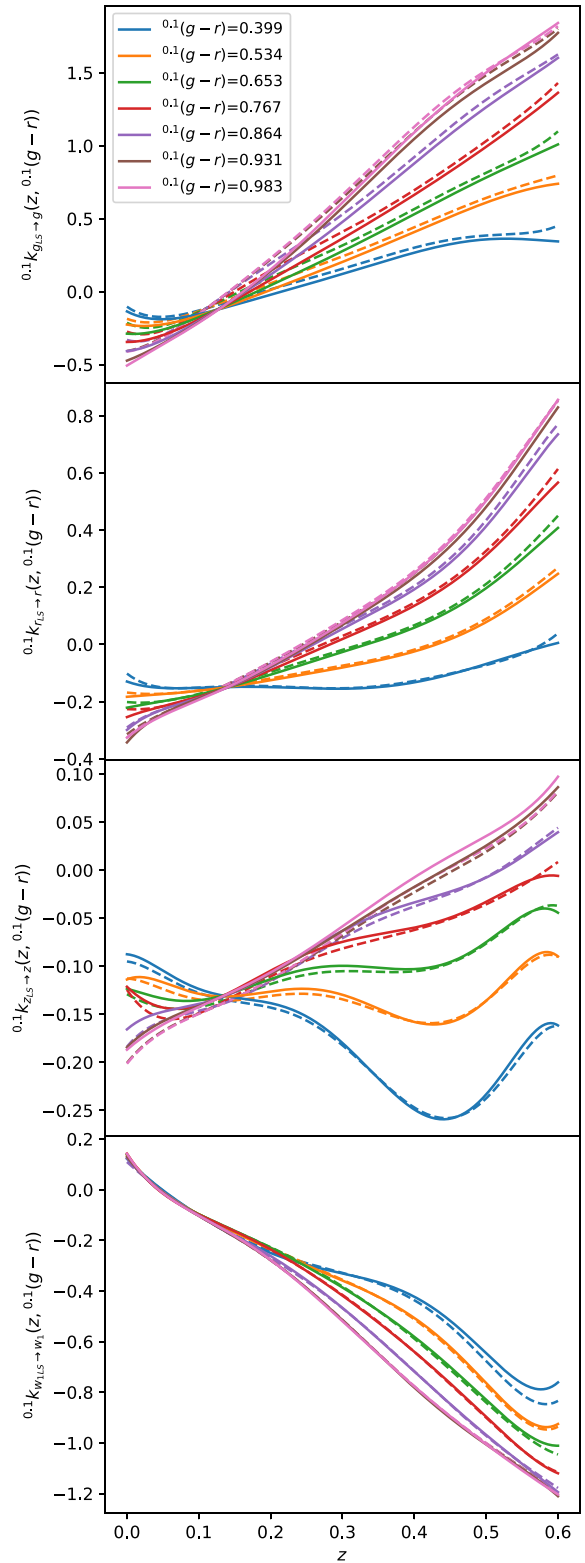


Figure 4. The k -correction polynomials to the SDSS g , r , z , and $WISEw1$ bands with $z_{\text{ref}} = 0.1$ from the respective observer frame DECaLS band (South, solid line), BASS/MzLS band (North, dashed line), and $WISE$ bands. The r , z and $w1$ -band k -corrections are direct fits to the FSF data. For the g band, the r -band polynomials are transformed to the g band using equation (3) and the figure shows polynomial fits to the resulting g -band k -correction.

about $z = 0.14$ which corresponds to the redshift at which the central wavelength of the DESI r -band filter best matches with that of the SDSS r -band filter at a reference redshift $z_{\text{ref}} = 0.1$. This corresponds to the 3.74 per cent shift in the effective wavelength between the SDSS r -band filter (6205.83Å) and the BASS r -band filter (6437.79Å), as $[(1 + 0.141)/(1 + 0.1)] - 1$ is approximately 3.73 per cent.

We note that there is a slight offset in the pinch points for North and South in the g -band in a way that is not apparent in the r , z bands. This is explained by the BASS and DECaLS g -band filters being significantly different, with a greater difference than in any other band. Moreover, the $w1$ -band k -correction is simply the native $w1$ filter shifted to the reference redshift. As such, it has a pinch point consistent with $k_{w1}(z = 0.1) = -2.5 \log(1 + 0.1)$, where we note that the very slight deviation from this can be attributed to the polynomial fits not being constrained to go through this point.

We conducted additional tests to ensure that these k -correction polynomials remain largely invariant to choices in colour and redshift bin size – for example, choosing a larger number of colour-bins for k -correction polynomial fitting has a small but ultimately negligible impact on the LF compared to the jackknife error.

When assigning a k -correction based on redshift and observer-frame colour, we use a cubic interpolation scheme between the 7 polynomials. From this, with the BGS catalogues we provide absolute magnitude in the SDSS r -band with reference redshift $z_{\text{ref}} = 0.1$, defined by

$${}^{0.1}M_r - 5 \log_{10} h = m_{r_{\text{LS}}} - 5 \log_{10} \left(\frac{d_L(z)}{h^{-1} \text{Mpc}} \right) - 25 \quad (2)$$

$$- {}^{0.1}k_{r_{\text{LS}} \rightarrow r}(z, {}^{0.1}(g-r)) - {}^{0.1}E(z).$$

Here, the subscript r represents the SDSS r band, while subscript r_{LS} represents the Legacy Survey r -band either from DECaLS in the South or BASS in the North. ${}^{0.1}k_{r_{\text{LS}} \rightarrow r}(z, {}^{0.1}(g-r))$ represents our fitted polynomial k -correction that takes us from Legacy Survey observer frame r -band to the rest-frame SDSS r -band with a reference redshift of $z_{\text{ref}} = 0.1$, where ${}^{0.1}(g-r)$ is the rest-frame colour. $d_L(z)$ is the luminosity distance to the redshift z , determined using our assumed cosmology. Optionally and in addition, an e -correction may be applied in order to account for the intrinsic luminosity evolution of a galaxy over time, which is represented by the ${}^{0.1}E(z)$ term; this is discussed in Section 3.2.

For the $w1$ and z bands, we construct polynomial fits to the median FSF k -corrections in bins of ${}^{0.1}(g-r)$ in just the same way as we do in the r band. To set the g -band absolute magnitude we use

$${}^{0.1}M_g = {}^{0.1}M_r + {}^{0.1}(g-r)_{\text{med}}$$

where ${}^{0.1}(g-r)_{\text{med}}$ comes from the look-up table. This implies the g -band k -correction we use is given by

$${}^{0.1}k_{g_{\text{LS}} \rightarrow g}(z, {}^{0.1}(g-r)) = {}^{0.1}k_{r_{\text{LS}} \rightarrow r}(z, {}^{0.1}(g-r)) + (g-r) - ({}^{0.1}M_g - {}^{0.1}M_r) \quad (3)$$

and its median in colour bins is shown in the top panel of Fig. 4⁸.

We emphasize that whilst this methodology of polynomial fitting is similar to that described in DESI Collaboration (2024),

⁸Along with our luminosity function estimates the polynomial coefficients of our k -correction fits are available in electronic form at <https://icc.dur.ac.uk/data/>

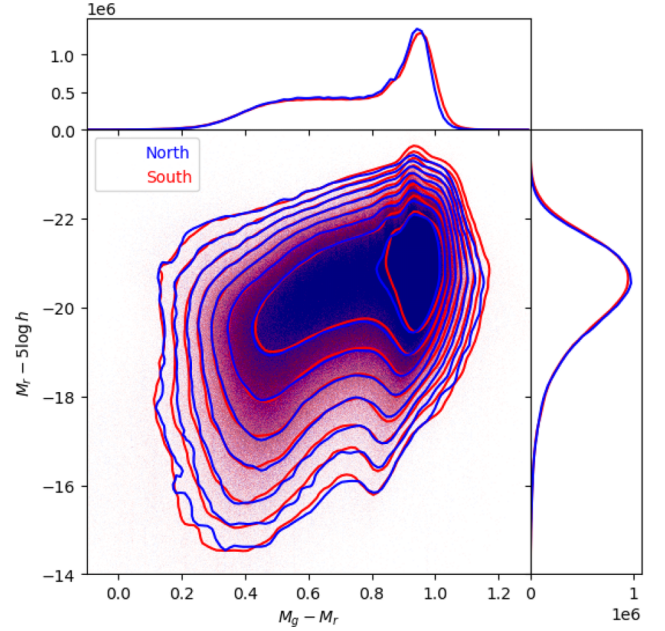


Figure 5. Rest-frame colour versus absolute magnitude distributions in North and South. Contours are plotted representing number density, with each successive contour a factor of 2 larger in number density (starting at 160 objects per bin). The top histogram shows the ${}^{0.1}(g-r)$ colour distribution for North and South. The right histogram shows the r -band absolute magnitude distribution. Both histograms have been normalized by sky-area to adjust for the fact that South is approximately double the size of North.

those k -correction polynomials are based on GAMA DR4 data, while in this paper we have updated the method to make use of a colour lookup table and to directly use DESI BGS galaxies in order to account for the DESI photometry and its differences in the North and South regions.

Fig. 5 presents the ${}^{0.1}(g-r)$ rest-frame colour distribution and the r -band absolute magnitude distribution. We observe that there exists a slight offset in the North and South colour distributions. We attribute this to a possible small error in the red end of one of the filter curves. We note that if we make an empirical correction for this by shifting the r -band magnitudes of the galaxies in the North such that we preserve their ranking in rest-frame colour and then exactly match the cumulative rest-frame colour distribution of the South, then this makes a negligible difference to our subsequent results (see Section 4).

3.2 e-corrections

We follow other authors and implement an additional e -correction in the calculation of absolute magnitude, and hence model the evolution of the galaxy LF as being caused by the evolution of the individual galaxy luminosities. We follow the convention of T. McNaught-Roberts et al. (2014) and model the redshift dependence to first order as

$${}^{z_{\text{ref}}}E(z) = -Q(z - z_{\text{ref}}) \quad (4)$$

with one free parameter Q . Other papers (e.g. J. Loveday et al. 2012) additionally allow the galaxy population to evolve in number density using a second parameter P as

$$P(z) = P(z_0)10^{0.4P(z-z_0)} \quad (5)$$

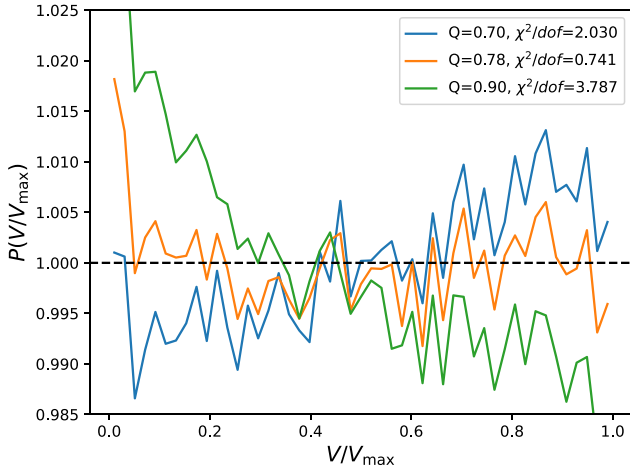


Figure 6. The North $V/V_{\max}$ distribution for $Q_r = 0.78$ (the optimal value we find) and a number of nearby values of Q_r . The reduced χ^2 value is presented for each Q_r -value.

J. Loveday et al. (2012) find that for all bands and colour samples, the estimates of P and Q are strongly anti-correlated. To avoid this degeneracy, we simply assume $P = 0$ (no density evolution). One advantage of doing this is that it allows for a more direct comparison to the results in T. McNaught-Roberts et al. (2014) who also fixed $P = 0$. This was useful in our initial stages when validating our methodology on GAMA data.

In the r -band, to constrain Q_r , we consider the distribution of the ratio of $V/V_{\max}$ – where V is the survey volume up to the observed redshift of the galaxy and $V_{\max}$ is the maximum volume over which the galaxy could be seen, given the observed properties of the galaxy and the apparent magnitude limits of the survey. $V_{\max}$ is defined by

$$V_{\max}(L'_i, k_i) = \frac{4}{3} \pi f_{\text{sky}} [d(z_{\max,i})^3 - d(z_{\min,i})^3], \quad (6)$$

where $z_{\min,i}$ and $z_{\max,i}$ are the minimum and maximum redshifts respectively at which a galaxy could be observed given its r -band luminosity, L' , its colour dependent k -correction, k_i , and the redshift limits of the BGS sample. f_{sky} is the fraction of the sky area over which the North or South region of the survey is defined. For the North, $f_{\text{sky}} = 0.09278$ and for the South, $f_{\text{sky}} = 0.20671$.

If the chosen value of Q_r correctly models the luminosity evolution, then the $V/V_{\max}$ distribution should be uniform, aside from small fluctuations caused by largescale structure. We therefore vary Q_r to minimize the χ^2 difference between the measured and ideally uniform $V/V_{\max}$ distributions, using jackknife errors (see Section 3.3). The resulting $V/V_{\max}$ distributions for several values of Q_r in the North are shown in Fig. 6; we obtain very similar results for the South. From this procedure, we find a global value of $Q_r = 0.78 \pm 0.20$, compared with $Q_r = 0.97 \pm 0.15$ reported by T. McNaught-Roberts et al. (2014). Their differing statistical method and redshift range mean that exact agreement is not expected.

Because DESI BGS is selected in the r -band, only Q_r is required to compute $V/V_{\max}$ for each galaxy. However, Q_r may not fully describe the luminosity evolution in the other bands. We therefore allow the value of Q used when defining absolute magnitudes in the g , z , and $w1$ bands via equation (4) to vary independently. To constrain these parameters, we divide the catalogue into two samples of roughly equal size above and below $z = 0.2$. In each

band, we vary Q , compute the luminosity function in each redshift slice, and determine the value of Q that minimizes the χ^2 difference between the two LFs over a chosen magnitude interval. This interval spans two magnitudes and begins just brightward of the faintest magnitude at which the highredshift sample is complete: $[-19.75, -21.75]$ in the g -band and $[-20.75, -22.75]$ in the z and $w1$ -bands. The resulting bestfit evolution parameters are $[Q_g, Q_z, Q_{w1}] = [0.91 \pm 0.20, 0.45 \pm 0.15, 0.7 \pm 0.15]$.

The evolution of the galaxy luminosity function is undoubtedly more complex than can be captured by a single-parameter model. In reality, the shape of the LF is expected to evolve with luminosity due to changing galaxy populations and the effects of mergers. Nevertheless, this simple parametrisation is sufficient for our purposes: it enables a robust measurement of the luminosity functions at our fiducial redshift of $z = 0.1$.

We additionally consider the idea of splitting the galaxies into two colour classes (red and blue) and finding corresponding Q -values for each population. To define the appropriate colour-cut, we make use of the bimodal nature of the $^{0.1}(g-r)$ rest-frame colour histogram, which has two distinct populations that can each be characterized by Gaussian distributions (see topmost panel of Fig. 5). By fitting Gaussians to each distribution with an Expectation-Maximisation (EM) algorithm, we can split the populations based on the intersection of those two fitted Gaussians. This yields a colour threshold of $^{0.1}(g-r) = 0.75$. Alternatively, we can visually identify a difference in the two populations by splitting the $^{0.1}(g-r)$ histogram into different $^{0.1}M_r - 5 \log h$ absolute magnitude bins as there is a region where the trough of the combined histogram tends to be very similar. This method is consistent with the result found with the EM method. We use the $V/V_{\max}$ method described above and find $Q_{\text{red}} = 0.23 \pm 0.3$ and $Q_{\text{blue}} = 1.59 \pm 0.2$. Our value of Q_{blue} is larger than that of Q_{red} , meaning that the blue population galaxies exhibit more substantial luminosity evolution than the red population. This trend is seen in other papers, notably T. McNaught-Roberts et al. (2014) and J. Loveday et al. (2012).

3.3 LF estimators

With the absolute magnitudes and weights found for each BGS galaxy in our sample, it is now possible to measure the LF. There are numerous different methods to measure the LF (G. Efstathiou, R. S. Ellis & B. A. Peterson 1988; S. Cole 2011). In this paper, we focus on the $V_{\max}$ estimator as outlined in M. Schmidt (1968). We compare this result to other methods in Appendix B, and find that different methods yield extremely similar results. The $V_{\max}$ LF estimator is

$$\phi(L)dL = \sum_{i=1}^N \frac{w_i W(L - L_i)}{V_{\max}(L_i, k_i)}, \quad (7)$$

where $V_{\max}(L_i, k_i)$ is defined in equation (6), w_i represents the combined weight defined in equation (1), and $W(L - L_i)$ represents a binning function

$$W(L - L_i) = \Theta(L_i - L + dL/2) - \Theta(L + dL/2 - L_i) \quad (8)$$

where Θ is the Heaviside step function.

For our bivariate LF, we adapt equation (6) into the following,

$$\Psi(L^r, L^g)dL^r dL^g = \sum_{i=1}^N \frac{w_i W(L^r - L_i^r) W(L^g - L_i^g)}{V_{\max}(L_i^r, k_i)}, \quad (9)$$

where L^r and L^g are the luminosities in the r and g bands respectively.

By weighting each object by the inverse of its maximum detection volume, this method corrects for the issue that intrinsically faint objects are detected only in a small volume, leading to the preferential detection of intrinsically bright objects. Additionally, the $1/V_{\max}$ estimator has the advantages of not assuming a functional form and automatically having the correct normalization. The disadvantage of this estimator is that it assumes all sources follow a uniform spatial distribution. This can result in distortion in the case of overdense or underdense regions (G. Efstathiou et al. 1988). We find that this only affects the r -band LF fainter than $^{0.1}M_r - 5 \log h > -14$ (see Section 4); this is because of the large volume probed by BGS at these magnitudes.

We calculate both Poisson errors and jackknife errors for our LFs. The Poisson error estimate is given by

$$\frac{\Delta\phi(L)}{\phi(L)} = \sqrt{\frac{\sum_i (w_i^2) W(L - L_i)}{(\sum_i w_i W(L - L_i))^2}}. \quad (10)$$

We note that if all weights are unity, then this expression reduces to the standard $1/\sqrt{N(L)}$ term, where $N(L)$ is the number of objects in the luminosity bin.

The jackknife errors are calculated by splitting the footprint of the survey into 3 by 3 ($N = 9$) equal area regions in the North, and 4 by 5 ($N = 20$) equal area regions in the South. We have explicitly checked that using more jackknife regions does not significantly alter our error estimates. We note that the number of jackknife regions used is different for North and South so that the regions are approximately equal in area. The formula for the jackknife error is

$$\text{Var}(x) = \frac{\Delta\phi(L)}{\phi(L)} = \frac{N-1}{N} \sum_{i=1}^N (x_i - \bar{x})^2, \quad (11)$$

where x_i is the value of $\phi(M)$ calculated for the area excluding the i th region (P. Norberg et al. 2009).

4 RESULTS OF GLOBAL LF ESTIMATION

With our methodology now described, we present our main findings. We generate a number of bivariate LFs in the g , r , z , and $w1$ bands (see Fig. 7) as well as the corresponding univariate LFs in those same bands (see Fig. 8). We also present this LFs split by colour.

4.1 Bivariate LFs in g , r , z and $w1$

Our bivariate LFs are particularly useful for better understanding the completeness of the LFs that are presented in this paper. Noting that the BGS Bright survey is limited to $r = 19.5$ in the South and $r = 19.54$ in the North, we use the bivariate LFs to estimate the completeness of the g , z , and $w1$ LFs. Using the g - r bivariate LF as an example, we do this by finding the 5 per cent to 95 per cent values of the distribution of g -band magnitudes in each r -band magnitude bin. Then, we plot the object-weighted least-squares fit regression lines to these percentiles, shown as the dashed pink lines in Fig. 7. As a black curve, we plot the absolute r -band magnitude limit at the chosen minimum redshift limit ($z = 0.002$) for a range of rest frame $^{0.1}(g-r)$ colours at the faint apparent magnitude limit of the sample using equation (2). The intersection of the limit with the 5 per cent regression line gives us an estimate of the completeness limit of the g -band LF, as shown

by the horizontal line in the right-hand side panel. We follow the same process for the z and $w1$ bands.

In addition, the bivariate LF offers a useful way to identify spurious objects. Bins with only a single object are marked by a red dot. As shown in Fig. 7, the r - $w1$ bivariate LF contains spurious peaks in the $w1$ LF. Our analysis shows that these peaks are caused by a small number of galaxies with a low $V_{\max}$ and a high value of w_{comp} which in combination cause such galaxies to have a disproportionate effect on the LF. We visually inspect these galaxies using the Legacy Survey Sky Browser⁹ and confirm that these galaxies are spurious, with fragmentation of large galaxies being a common issue. Typically, each spurious peak in the LF corresponds to one such galaxy. We note that these objects correspond to an unrealistic $^{0.1}(r-w1)$ rest-frame colour. In order to deal with this, we add a colour selection cut which corresponds to a rest-frame colour selection cut of $^{0.1}(r-w1) < 2.25$. This is shown as the dashed red line in the bottom two panels of Fig. 7. This selection cut is only applied to $w1$ -band LFs.

4.2 LFs in g , r , z and $w1$

After generating the bivariate LFs shown in Fig. 7, we present the $1/V_{\max}$ LFs in the g , r , z , and $w1$ bands in Fig. 8¹⁰.

As the Y3 BGS survey has such a large number of galaxies, we note that the statistical errors on the LFs are extremely small. As a consequence no simple functional form provides an adequate fit. This is illustrated by the Schechter functions we plot as fiducial curves in Fig. 8. These fits were performed using the jackknife errors and by minimizing χ^2 over a limited magnitude range. The parameters of these formally poor fits (e.g. $\chi^2/\text{d.f.} = 65.7$ over the full magnitude range for the South r -band LF) are given in Table 1. We strongly emphasize that we do not present these Schechter fits as a useful parametrisation of the LF. While they are not good representations of the data, they are useful for highlighting systematic deviations from the Schechter function form and as fiducial references curves in subsequent plots. In terms of magnitudes, the Schechter function form is

$$\Phi(M) = \frac{\ln 10}{2.5} \Phi^* 10^{0.4(1+\alpha)(M-M^*)} \exp[-10^{0.4(M-M^*)}], \quad (12)$$

where M^* parameterizes the position of the ‘knee’ of the Schechter function, α parameterizes the faint end slope, and Φ^* represents the space density around the ‘knee’.

We present LFs down to $^{0.1}M_r - 5 \log h \sim -10$, and go even fainter in the other bands. We note that this goes beyond that of comparable large scale surveys in the r band, for example, SDSS reached a faintness of -12 (M. R. Blanton et al. 2005) while the much smaller GAMA survey reached -11 (J. Loveday et al. 2012). Over nearly all this range, our independent estimates from the North and South regions agree well with each other, indicating that the systematic dip below the Schechter function forms at a magnitude of approximately -17 is not caused by systematic errors. However, the faint upturn at -14 , although seen in other work (T.-W. Lan et al. 2016) is probably mainly the result of systematic errors or biases.

⁹<https://www.legacysurvey.org/viewer>

¹⁰The LFs in Fig. 8 differ from the projected LFs in Fig. 7 in the following ways: (1) the Fig. 8 LFs have had the $^{0.1}(r-w1) < 2.25$ selection cut applied as described above; (2) the thickness of the shaded bands in Fig. 8 represents the jackknife error in the LFs, whereas the thickness of the shaded bands in Fig. 7 represents the Poisson error in the LFs.

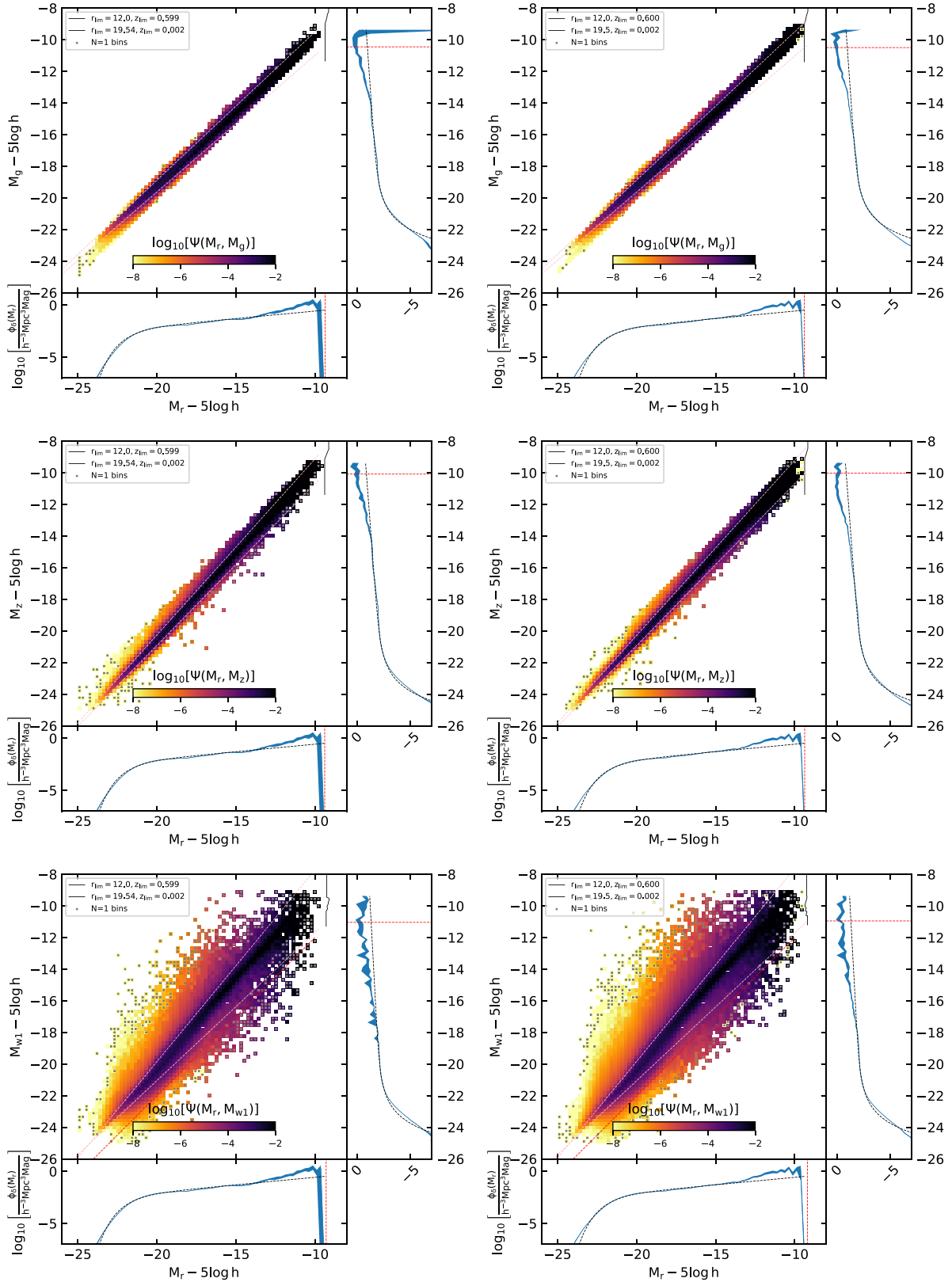


Figure 7. Bivariate LFs for North (left) and South (right) in the g, z and w1 bands. The short solid curves in the top right of each panel represent the magnitude limit of the survey at the chosen $z = 0.002$ limit. The dashed vertical and horizontal lines show the completeness limits for the r -band and g -band luminosity functions. These are based on the intersection of the completeness curves and the 95 percent percentile contour of the bivariate LF. The $r - w1$ bivariate LFs have an additional dashed red line showing a selection cut that is later incorporated to remove spurious objects. Here, all LFs are presented with Poisson errors. The units of the LFs are $h^3 \text{Mpc}^{-3} \text{mag}^{-1}$.

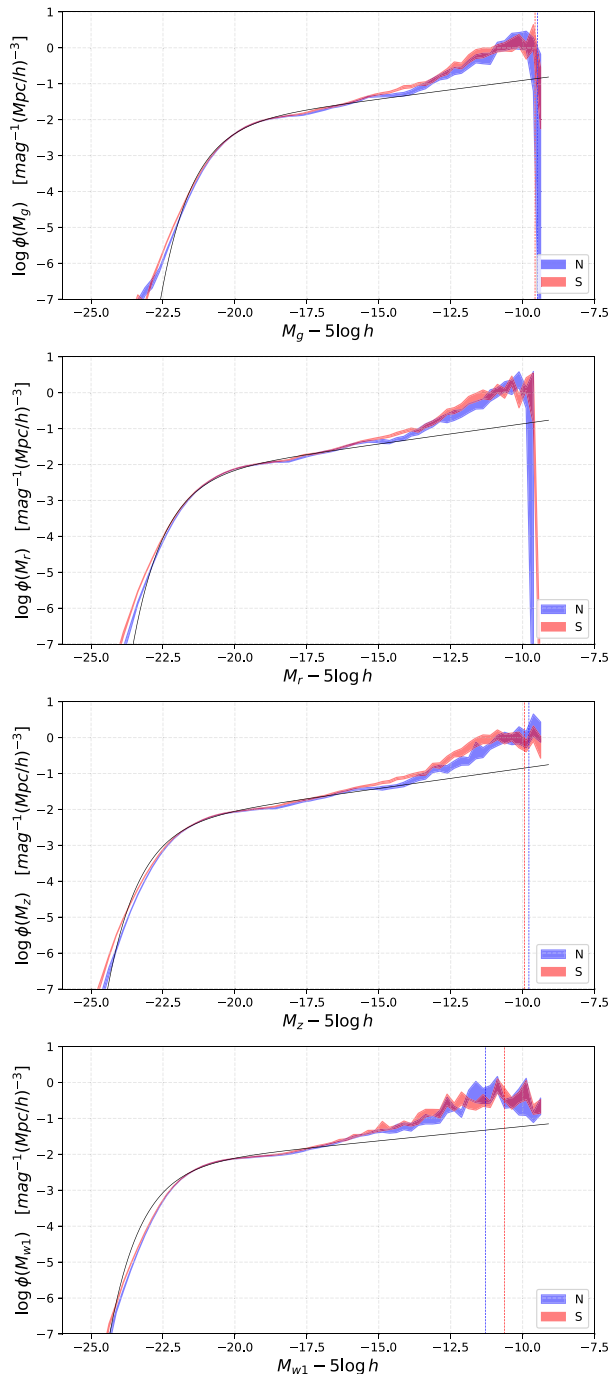


Figure 8. The global $1/V_{\max}$ LF for Y3 data for North and South in the g , r , z , and $w1$ bands. The width of each LF represents the jackknife error. The dashed lines represent the completeness limits derived from the corresponding bivariate LFs. For reference, the smooth solid curves show simple Schechter function fits for each band.

We investigate the impact of the targeting and redshift completeness weights on the shape of the LF by comparing our main results to an alternative LF in which every galaxy is assigned the mean incompleteness weight, $\bar{w} = 1.5275$. This comparison is shown in Fig. 9. For magnitudes brighter than $M_r \approx -16$, the two LFs are indistinguishable, and the discrepancy between the North and South LFs at the bright end remains unchanged. This demonstrates that these features are not artefacts introduced by

the redshift incompleteness weighting. At the faint end, however, the redshift incompleteness corrections are important. We find that the upturn at $M_r > -14$ becomes more pronounced when the individual weights are used. This behaviour arises because intrinsically faint galaxies tend to have lower surface brightness and fainter fibre magnitudes, placing them in the region (see Fig. 2) where the incompleteness corrections are largest.

We also investigated whether the limiting surface brightness of the Legacy Survey photometric data and the explicit fibre magnitude limit in the BGS selection causes our luminosity functions to be incomplete. In the DESI Large Scale Structure analysis, fake galaxy images have been superimposed on the imaging data to gauge the ability of the photometric pipeline to recover them as a function of position on the sky (e.g. see H. Kong et al. 2025). This analysis did not explicitly investigate low surface brightness galaxies. It would be valuable to undertake such an investigation. However, while such an approach can accurately quantify the limits of the data, it cannot directly tell us whether there is a population of galaxies that we are missing. We have therefore used a region of much deeper data from the COSMOS deep drilling fields to directly estimate how many galaxies we are missing. This analysis is presented in Appendix A, together with an assessment of the impact of the BGS fibre magnitude limit.

Fainter than $M_r - 5 \log h \approx -15$, we find a small tail of low-surface brightness objects in the deeper data whose inclusion would boost the faint end of the LF by between 0.5 and 1σ . However, inspection of these images in the Legacy Survey data reveals that about half of them are associated with diffraction spikes or lie within the images of larger galaxies. Consequently, we have chosen not to apply this uncertain correction to our estimates. Separately, based on a simple parametric model of the galaxy surface brightness distribution we find that the fibre-magnitude cut causes a systematic underestimation of the LF which becomes comparable but not larger than the statistical errors fainter than $M_r - 5 \log h \approx -14$. We of course cannot rule out a unseen population of much lower surface brightness galaxies that are not detectable in the deeper data or captured in the assumed Gaussian distribution of our simple parametric model.

In Appendix B, we present the Stepwise Maximum Likelihood (SWML) and $1/V_{\text{dc,max}}$ LFs alongside the $1/V_{\max}$ LFs (see Fig. B1). We find that the upturn is present in the LF in all three of the methods, but it is substantially reduced when using the SWML and $1/V_{\text{dc,max}}$ estimators – which are designed to correct for density fluctuations in the LF. This suggests that most of the faint-end upturn is due to local density fluctuations.

We also investigated if some of the upturn is caused by fragmented galaxy images or other image analysis problems. In Appendix E, we present the details of a visual inspection programme. We find that some of the faintest galaxies are likely to be invalid targets, often features within larger galaxies. However the effect on the LF is modest, restricted to the faint end and does not remove the steepening of the faint end of the LF that we have reported.

We also attempted to use a Convolutional Neural Network to classify and remove these spurious objects. We found that doing so flattened the upturn, but did not completely remove it. As such, it is still possible that the upturn is a real feature in the global LF, but we recognize that further investigation is required to confirm this.

The most comprehensive set of LF estimates previously was that of the GAMA survey (J. Loveday et al. 2012). We have not plotted the Loveday results on Fig. 8 as with such large observational samples the systematic differences coming from differing

Table 1. The Schechter parameters for the global $0.002 < z < 0.6$ LFs, used in Figs 8 and 10 as fiducial reference curves. The meaning of the parameters are defined in equation (12). We do not provide errors for these parameters as they are formally poor fits, as illustrated, for example, in Fig. C2.

	$\log_{10} \Phi^* [h^{-3} \text{Mpc}^3]$	$M^* - 5 \log h$	α
All (South)			
<i>g</i>	−1.91	−19.98	−1.26
<i>r</i>	−2.06	−20.97	−1.28
<i>z</i>	−2.15	−21.88	−1.28
<i>w1</i>	−2.13	−21.78	−1.20

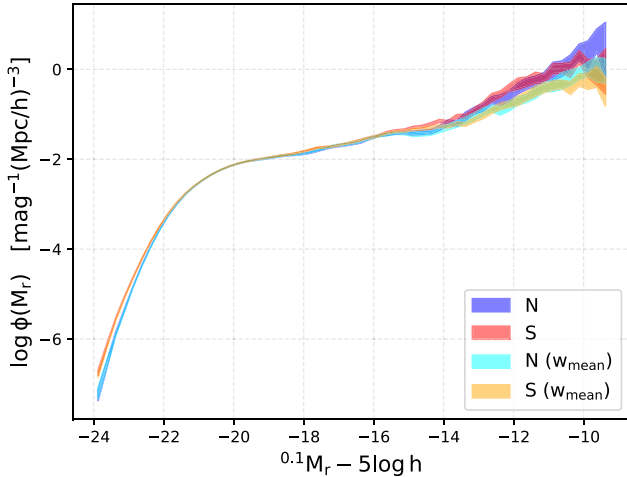


Figure 9. The global $1/V_{\max}$ *r*-band LFs for North and South. The LFs use either the standard weight given in equation (1) where each galaxy has an individual total incompleteness weight, or the mean total weight (w_{mean}) is assigned to all galaxies. The width of each LF represents the jackknife error.

choices of redshift range and evolutionary assumptions dominate over the statistical differences. In Appendix C, we demonstrate that when making the same assumptions we find our estimates are in excellent agreement with those of J. Loveday et al. (2012) albeit with much smaller statistical errors. Also in Appendix C, we show that splitting the sample by redshift reveals small but highly significant residuals that indicate our simple global evolution model, while being accurate around the knee of the LF, fails to capture the more complex evolution that the BGS reveals.

While over most of the magnitude range, we observe that there exists excellent agreement between the North and South LFs we note that there exists a systematic discrepancy between the North and South *g*, *r*, and *z* LFs at the bright-end of the LFs (e.g. $0.1 M_r - 5 \log h < -22.5$). We extensively investigated the cause of this highly statistically significant difference. We first investigated the impact of the previously noted rest-frame colour discrepancy between North and South (as seen in Fig. 5, noting that this distributional difference is consistent with that seen in the FSF catalogue and is not caused by our *k*-correction methodology). To force the colour distributions to be the same, we made an empirical correction by shifting the *r*-band magnitudes of the galaxies in the North such that we preserve their ranking in rest-frame colour and then exactly match the cumulative rest-frame colour distribution of the South. This made a negligible difference to the LFs, showing that whatever is perturbing the rest-frame colour distributions is not responsible for the LF discrepancy.

Instead, we believe that this is at least partially caused by a difference in the light profile fitting between North and South. We investigate the photometric differences between North and South in Appendix D.

In the deep DECaLS photometry of the South, some very luminous early-type galaxies are fit with light profiles with extreme Sérsic profiles ($n_{\text{Sérsic}} > 4$) and hence very extended outer light profiles. Extended wings are not detected in the shallower BASS/MzLS data and so are fitted with less extreme Sérsic profiles and result in fainter magnitudes. Whether this represents a limit of the shallow North data or an issue of the inadequate sky subtraction in the more demanding lower noise South data is unclear. Hence, we conclude that this North-South difference represents a not yet fully understood systematic uncertainty in the bright end of the galaxy LFs.

4.3 LFs split by $^{0.1}(g-r)$ colour

In Fig. 10, we present the global luminosity functions (LFs) split by rest frame colour. To obtain colour-dependent LFs, we separate our sample into red and blue populations using the rest-frame colour-cut $^{0.1}(g-r) = 0.75$, as defined in Section 3.2. We show red and blue $1/V_{\max}$ LFs for each band and note that the red and blue populations are consistently defined by $^{0.1}(g-r) = 0.75$, even when examining the *z* and *w1* bands. For the colour-dependent LFs we show completeness limits that correspond to the global completeness limits found in Fig. 7. We assume that these completeness limits are conservative enough to be applicable to the colour-split LFs.

We observe that the bright-end offset in the absolute magnitudes is predominantly seen in the red population, with a visible difference between the red N and S LFs. The corresponding difference for the blue N and S LFs is minimal. This is consistent with the hypothesis that for the brightest galaxies in the shallower N data, the contribution of the total magnitude of the outer profile is underestimated. For blue galaxies which typically have exponential profiles, very little flux is contributed from large radii. However, for bright red galaxies, which are typically ellipticals with either de Vaucouleur profiles or high-*n* Sérsic profiles, a significant fraction of their light is contributed from their outer profiles. Moreover, as a single profile is fitted across all of the Legacy Survey bands, we expect and see the same offset in the *g*, *r*, and *z* bands. We do not observe an offset between N and S in the *w1*-band. This is explained by the fact that the *w1* PSF is much wider than in the other bands. As such, the PSF convolved profiles do not differ significantly for the typical bright distant galaxy.

The colour-split LFs quantify well-known differences in the demographics of red and blue galaxies. At around magnitude -20 red and blue galaxies are equally abundant but their luminosity functions are quite different with red galaxies dominating at

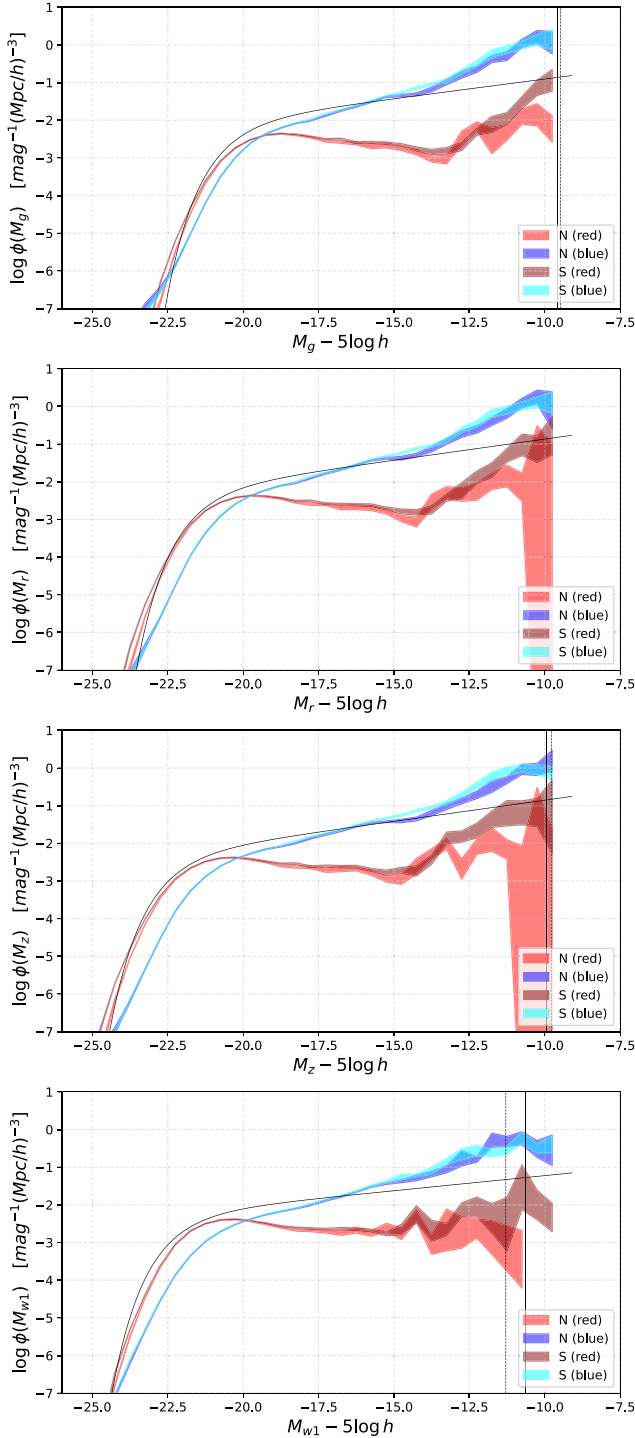


Figure 10. The global $1/V_{\max}$ LF for Y3 data for North and South in the g , r , z and $w1$ bands split by colour. The red LFs represent galaxies with $^{0.1}(g-r) > 0.75$, while the blue LFs are galaxies with $^{0.1}(g-r) < 0.75$. The width of each LF represents the jackknife error. The dashed lines represent the completeness limits derived from the corresponding bivariate LFs. The solid fiducial Schechter curves are reproduced from Fig. 8. Their parameters are given in Table 1.

bright magnitudes and blue galaxies at faint magnitudes. This is true in all bands. The turn up in the luminosity functions at faint magnitudes is most dramatic for red galaxies with the slope of the luminosity function changing sign at around a magnitude of -13 . Despite the very steep faint end of the red-galaxy luminosity function this population is still sub-dominant at the faintest magnitudes we probe. Hence the faint-end turn up in the overall luminosity function is dominated by a corresponding turn up in the faint-end slope of the blue-galaxy luminosity function.

5 CONCLUSIONS

In this paper, we have presented the g , r , z , and $w1$ LFs using the DESI Y3 data. The large number of galaxies in the Y3 BGS dataset mean that we have constrained the errors on the LFs further than prior studies in the literature. Furthermore, these LFs provide useful measurements to very faint absolute magnitudes, extending the LF to extremely faint magnitudes (such as $^{0.1}M_r - 5 \log h \sim -10$). Although an assessment of incompleteness for low surface brightness galaxies indicates that the LF estimate from the BGS is only complete for galaxies with r -band half-light surface brightness brighter than $\mu_{50} > 25$ and that fainter than $^{0.1}M_r - 5 \log h \sim -14$, this impacts the LF estimate at a level comparable to the reported statistical errors.

In order to produce these LFs, we have developed a robust methodology for generating k -corrections for galaxies in order to calculate absolute magnitudes. In addition, we incorporate a e -correction model that uses a single value of Q for all galaxies. We broadly find that this is sufficient enough for our purposes, but note that this does not fully incorporate the entire evolutionary process, as shown by a discrepancy between our $0.002 < z < 0.1$ and $0.002 < z < 0.6$ r -band LFs in Fig. C2. As such, we acknowledge that further work is required to improve this aspect of our model but also note this illustrates the power of the DESI data to probe galaxy evolution.

We verify that our LFs yield similar results when different estimators are used. In particular, we find that the $1/V_{\max}$ LFs yield similar results to the SWML LFs and $1/V_{\text{dc,max}}$ LFs, except at the very faintest absolute magnitudes. Overall, this methodology provides a robust platform for us to further constrain the LFs in the future Y5 DESI data releases.

We find that there are a number of galaxies with spurious properties in the Y3 sample. To begin, we find that there are a number of galaxies with abnormal $^{0.1}(r-w1)$ rest-frame colours. Through visual inspection, this was indicative of a number of bad objects which could be removed with a colour cut. However, further visual inspection reveals that there are a significant number of bad objects at faint r -band absolute magnitudes. This suggests that further work is needed to understand the imaging at the faintest magnitudes. In addition, this acts as an additional constraint on the completeness of our luminosity functions. For example, while our bivariate LFs suggest that our g -band LFs are complete up to fainter than $^{0.1}M_g - 5 \log h = -10$, we suggest that this is a very liberal estimate, and accounting for poor imaging will bring the completeness limit down.

As part of our investigation, we found that there exists a visible disparity between the North and South LFs at the bright-end when using the DESI total magnitudes. When adjusting our methodology to find Petrosian magnitude LFs, we were able to somewhat reduce the bright-end disparity between the LFs. We believe that this difference is explained by the shallower North photometry being unable to properly measure the faint outskirts of objects. As the Petrosian magnitudes account for the Sérsic in-

dex, this explains why they are successful in reducing the bright-end disparity.

The small statistical errors of our estimates result in the LFs not being well fit by simple analytic forms. They exhibit a bright end that is not well fit by an exponential and complex non-powerlaw behaviour faintward of the knee. In particular, we observe the existence of an upturn in the LFs at $^{0.1}M_r - 5 \log h \geq -15$, which is strongest for red galaxies.

DATA ACCESS STATEMENT

All data, unless explicitly stated in text, is sourced from the DESI collaboration. The data used in this analysis will be made public as part of DESI Data Release 2 (details found at <https://data.desi.lbl.gov/doc/releases/> and DESI collaboration in prep.). Documentation of DESI data access is maintained at <https://data.desi.lbl.gov/doc/access/>. The data points for our LFs in Figs 8, 10, B1, and B2 are accessible at <https://icc.dur.ac.uk/data/>.

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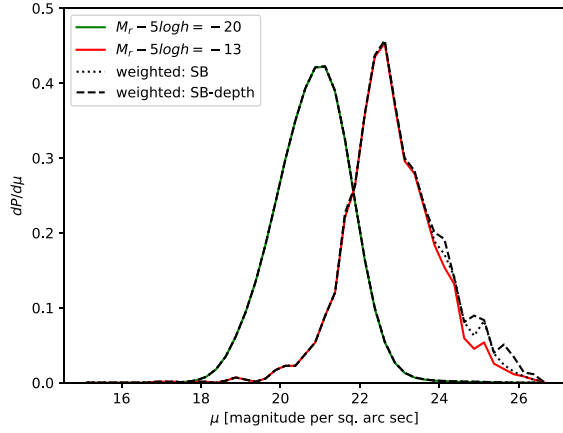


Figure A1. The solid curves show the normalized distributions of half-light surface brightnesses for galaxies from Legacy Survey DR9 (the parent catalogue of the BGS) of absolute magnitude $M_r - 5 \log h = -20$ (green) and -13 (red). The dotted and dashed lines show the effect on these distributions (not normalized) of applying the two up-weighting schemes to correct for incompleteness based on estimates from the deeper COSMOS data.

APPENDIX A: LOW SURFACE BRIGHTNESS INCOMPLETENESS

The median 5σ r -band depth for a fiducial galaxy in the Legacy Survey DR9 data release, that was used to select DESI targets, is 23.9 mag in the South (DECaLS) and 23.5 mag in the North (BASS) (A. Dey et al. 2019). While this is much deeper than 19.5 mag limit of the BGS-BRIGHT sample we are analysing, it is possible that the catalogue is incomplete for low-surface brightness galaxies. To assess this, we have made use of the much deeper COSMOS deep drilling fields (M. L. Graham et al. 2023). These data are comprised of hundreds of additional DECam images that have been analysed with the same Legacy Survey pipeline (A. Dey et al. 2019) to produce much deeper catalogues over tens of square degrees centred on the COSMOS field (N. Scoville et al. 2007). In this analysis, we extract the approximately 20 square deg that has a depth greater than 24.5 mag.

For all the galaxies in the Legacy Survey DR9 catalogue and sources in the deeper COSMOS data, we have computed their mean surface brightness within their elliptical half-light radii (without any correction for MW extinction). The solid curves in Fig. A1 show the normalized surface brightness distributions of Legacy Survey galaxies selected in absolute magnitude bins centred at -20 and -13 . In this comparison, we do not apply the explicit fibre-magnitude limit which is part of the BGS target selection (O. Ruiz-Macias et al. 2020; C. Hahn et al. 2023) as we wish to isolate only the effect of the photometric depth. The distribution for the fainter galaxies peaks lower surface brightness and extends 26 mag per square arcsec. Thus if BGS is incomplete at low surface brightness it will preferential affect the faint end of the galaxy luminosity function.

To assess the incompleteness in BGS, we took all non-PSF sources brighter than $r = 20$ in the COSMOS data and spatially matched them using a 1.5 arcsec search radius to the full DR9 Legacy Survey catalogue from which BGS was selected. The magnitudes of the matching objects agree well with a 1σ (half the 16 to 84 per cent width of the distribution) of 0.13 mag even at magnitudes fainter than the $r = 19.5$ limit of BGS. We matched

to the full Legacy Survey rather than the BGS catalogue as otherwise COSMOS objects close to $r = 19.5$ would be missed in BGS just because of the random scatter in the difference of the two magnitude measurements and the cut at 19.5 imposed in BGS. As a measure of the completeness we tabulated the fraction of matched COSMOS objects as a function of apparent magnitude and surface brightness. The inverse of this can then be used as weight to apply to all BGS galaxies as a function of their apparent magnitude and surface brightness. As the depth of the Legacy Survey has some variation across the sky we also tabulated the completeness as a function of apparent magnitude and surface brightness minus depth. We use this to define an alternative weight to correct for the incompleteness while taking some account of the depth variations.

The dashed and dotted lines in Fig. A1 show the surface brightness distributions after applying these incompleteness corrections. One can see that this makes negligible difference to the distribution at $M_r - 5 \log h = -20$ as essentially no objects with surface brightness brighter than 24 mag per square arcsec are missed by the Legacy Survey. For $M_r - 5 \log h = -13$, one can see that the faint side of the distribution is modestly boosted, especially for the depth-dependent weight. Fig. A3 shows the effect of applying the depth dependent weight on the resulting r -band LF. As expected this correction only affects the faint end of the galaxy luminosity function where it causes a systematic increase in the LF at the level of the 1σ statistical error. While this is not negligible we have chosen to not apply this correction as when we view the Legacy Survey images of the missed objects a significant fraction of them appear to be spurious artefacts. We found 20 per cent were associated with diffraction spikes and 30 per cent were within the image of an already detected larger galaxy (see Manwadkar et al., in preparation, for detailed study of the shredding of larger BGS galaxies).

Another important factor affecting the representation of low surface brightness galaxies in the BGS catalogue is the explicit fibre-magnitude cut that is part of the BGS target selection. Only objects with fibre-magnitude

$$r_{\text{fib}} = \begin{cases} 22.9 + (r - 17.8), & r < 17.8, \\ 22.9, & r \geq 17.8. \end{cases} \quad (\text{A1})$$

(O. Ruiz-Macias et al. 2020; C. Hahn et al. 2023) are included in the BGS.

The only way to estimate the incompleteness beyond this limit is with respect to an assumed model of the intrinsic surface brightness distribution. Here, we construct such a model following the procedure in M. R. Blanton et al. (2005). In Fig. A2, we show the r -band surface brightness distributions of the BGS galaxies with $n_{\text{ext}} \text{Sersic} < 2$ in bins of absolute magnitude. The surface brightness measure we use, μ_{50} , is the mean surface brightness within the ellipse containing 50 per cent of the light. Over the range $-19 < M_r^h < -15$ (where $M_r^h = m_r - 5 \log h$) we fit these data with a Gaussian distribution whose mean and standard deviation are linear functions of magnitude. The best fitting relation is

$$\bar{\mu}_{50} = 20.703 + 0.319(M_r^h + 20.5) \quad (\text{A2})$$

$$\sigma_{\mu} = 1.022 - 0.014(M_r^h + 20.5). \quad (\text{A3})$$

The blue curves in Fig. A2 show this model including its extrapolation to brighter and fainter bins. The selection of only $n_{\text{Sersic}} < 2$ galaxies follows M. R. Blanton et al. (2005) and is motivated by the

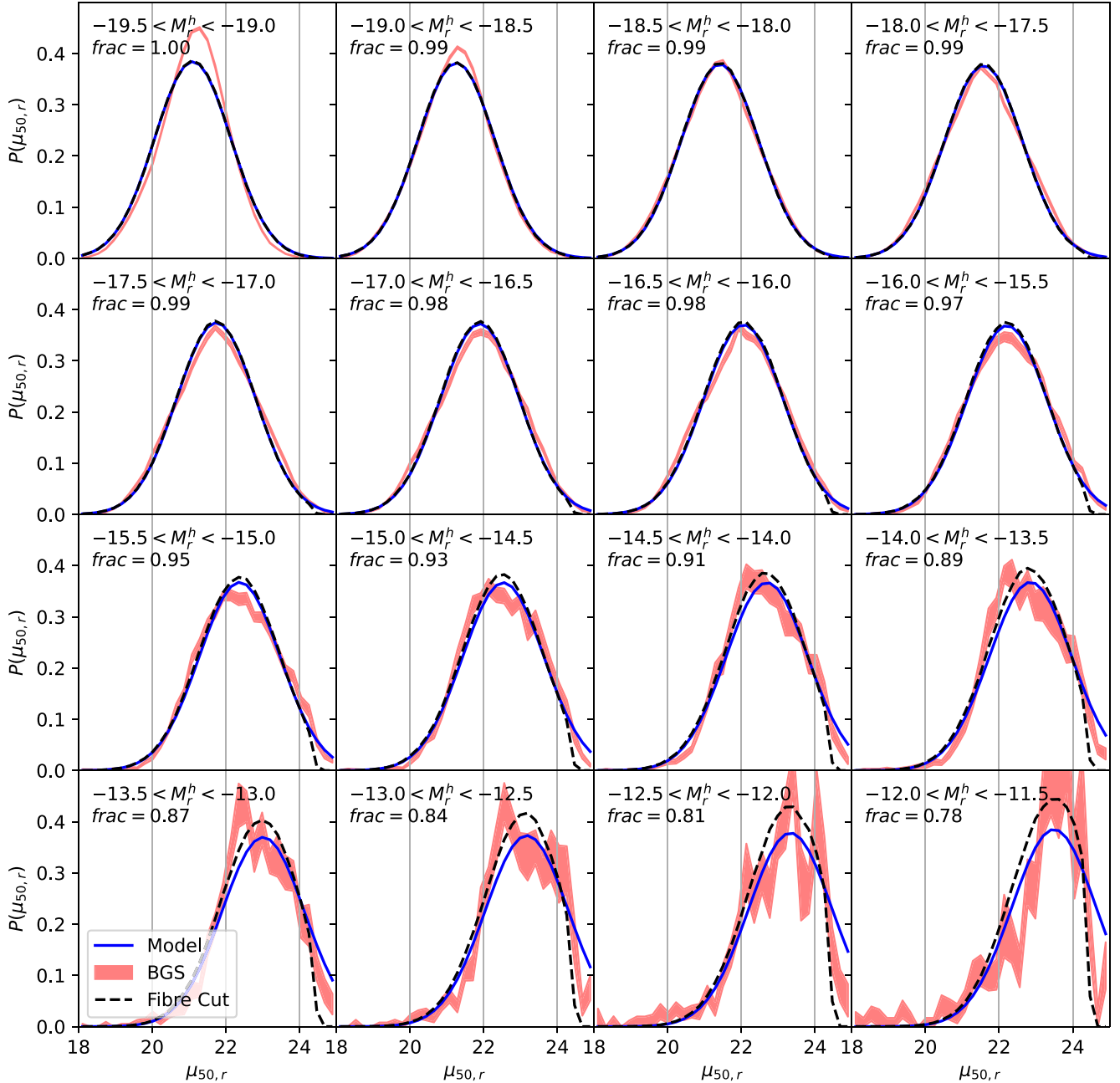


Figure A2. The r -band surface brightness distribution of BGS galaxies in bins of absolute r -band magnitude. The shaded bands show the redshift incompleteness correct estimates from BGS with jackknife errors. The solid curves are a Gaussian model fit to the data in the range $-19 < M_r - 5 \log h < -15$ in which the mean and standard deviation of the Gaussian is assumed to vary linearly with magnitude. The black dashed curves show the affect on these distributions of applying the BGS fibre-magnitude cut. The fraction of galaxies satisfying this selection cut is shown on each panel.

goal of modelling the faint galaxy regime where a large majority of the galaxies are have exponential profiles.

To estimate the incompleteness in each absolute magnitude bin caused by the fibre-magnitude limit, we sample from the corresponding model distribution of surface brightness, assign each sampled object an r -band apparent magnitude, r , chosen at random from the corresponding BGS data. This ensures our model sample has the same distance distribution as the BGS data. We then define effective radius r_{50} for each object via the identity

$$\mu_{50} = r + 2.5 \log_{10}(2\pi r_{50}^2).$$

To compute a fibre magnitude we assume each object has an $n_{\text{Sersic}} = 1$ Sérsic profile, convolve with the fiducial 1 arcsec FWHM seeing and compute the magnitude in the 1.5 arcsec diameter fibre aperture. This an appropriate choice of Sérsic profile for faint galaxies as the majority of such galaxies have profiles that are well fit by exponential profiles. The distributions of surface brightness of the objects that satisfy the BGS fibre-magnitude cut are shown by the black dashed lines.

The fraction in the label on each panel is the fraction of the model galaxies satisfying the selection criteria. While the agree-

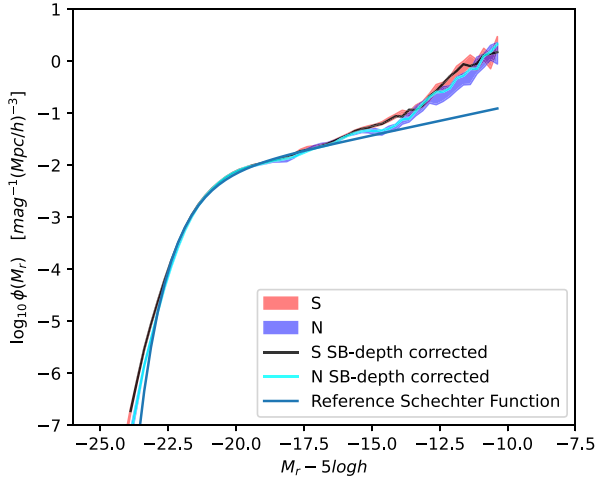


Figure A3. A comparison the $1/V_{\max}$ r -band LF before and after correcting for low surface brightness incompleteness. The red and blue shaded regions show the standard/uncorrected 1σ error interval for the South and North data, respectively. The black and cyan lines show the corresponding estimates after applying the correction estimated from the COSMOS deep drilling fields. The correction factor used here was tabulated as a function of apparent magnitude and surface brightness minus survey depth.

ment between the data and this simple model is not perfect, we see no evidence that there are fewer low surface brightness objects in the BGS data than predicted by this model. Hence, the incompleteness fractions shown in each panel are reasonable estimates of the incompleteness caused by the fibre-magnitude limit.

Faintward of $M_r - 5 \log h = -14$, the level of incompleteness indicated by this model becomes comparable, but does not exceed, the statistical errors of our estimated LFs. The incompleteness revealed by the deeper COSMOS deep drilling data should not be considered as addition to this as the model galaxies deselected by the fibre-magnitude cut could represent galaxies detected by COSMOS.

In summary, this investigation revealed only a modest level of incompleteness due to the depth of the Legacy Survey imaging and fibre-magnitude cut. However, the latter estimate is model dependent and so does not preclude an additional population of much lower surface brightness objects that could only be detected in data deeper than that of the COSMOS deep drilling fields. Our estimate should be taken as the galaxy luminosity function for galaxies with half-light surface brightnesses greater than approximately 25 mag per square arcsec.

APPENDIX B: COMPARISON OF LF METHODS

To verify the robustness of the $1/V_{\max}$ LF method described in Section 3.3, we compare to two other LF estimators. Specifically, we compare with the Stepwise Maximum Likelihood (SWML) estimator of G. Efstathiou et al. (1988) and a density-corrected $V_{\max}$ method ($V_{\text{dc,max}}$) estimator, which is based on a method from S. Cole (2011) described in more detail below. From this, we can confirm that brighter than -14 , our LFs are largely invariant of the estimator used. Fainter than -14 , the turn up of the LF is significantly reduced using the SWML and $V_{\text{dc,max}}$ estimators.

The SWML method does not require the assumption of a functional form of the LF [compared to other methods such as

the Sandage, Tammann & Yahil (STY) estimator]. Moreover, the SWML estimator is unbiased by density fluctuations if one assumes that the LF shape is independent of density (unlike the $V_{\max}$ method, which can be biased by density fluctuations). In our second paper, Moore et al. (in preparation), we show that the LF does depend on density, however, we expect the bias caused by the violation of this assumption to be small. In the past, the SWML estimator has been preferred for data sets with smaller sample sizes as it acts to factor out the effect of density perturbations on the LF.

Broadly, likelihood methods consider the probabilities of observing a galaxy at redshift z_i and magnitude M_i within a magnitude-limited survey. This can be used to construct a likelihood function as follows.

Given the LF, $\Phi(L)$, the probability of observing a galaxy of luminosity L_i at redshift z_i is

$$p_i = \frac{\Phi(L_i)}{\int_{L_{\min}(z_i)}^{L_{\max}(z_i)} \Phi(L) dL}, \quad (\text{B1})$$

giving an overall likelihood for the galaxy sample of

$$\mathcal{L} = \prod_i p_i. \quad (\text{B2})$$

For the SWML method, $\Phi(M)$ is expressed as a set of variables Φ_j in equally spaced magnitude bins whose values are then varied using an iterative procedure to maximize the likelihood. This yields (see G. Efstathiou et al. 1988) the estimator

$$\Phi(M_k) \Delta M = \frac{\sum_{i=1}^N W(M_i - M_k)}{V_k^{\text{SWML}}} \quad (\text{B3})$$

where

$$V_k^{\text{SWML}} = \sum_{i=1}^N \frac{H(M_k - M_{\text{faint}}(z_i, k_i))}{\sum_{j=1}^{N_p} \phi_j H(M_j - M_{\text{faint}}(z_i, k_i)) \Delta M}, \quad (\text{B4})$$

and the binning functions are

$$W(x) = \begin{cases} 1, & \text{if } -\frac{\Delta M}{2} \leq x \leq \frac{\Delta M}{2} \\ 0, & \text{otherwise,} \end{cases} \quad (\text{B5})$$

and

$$H(x) = \begin{cases} 0, & \text{if } x \leq -\frac{\Delta M}{2} \\ \frac{x}{\Delta M} + \frac{1}{2}, & \text{if } -\frac{\Delta M}{2} \leq x \leq \frac{\Delta M}{2} \\ 1, & \text{if } x \geq \frac{\Delta M}{2}. \end{cases} \quad (\text{B6})$$

Here, $M_{\text{faint}}(z_i, k_i)$ represents the faint absolute magnitude limit of the survey computed at the redshift z_i and with the k -correction k_i of galaxy i . We adopt a slight modification where the function $H(M_j)$, which returns the fraction of magnitude bin j that overlaps with the observed absolute magnitude range, has been generalized to also include a bright magnitude limit.

The density-corrected $V_{\max}$ estimator (hereafter, named $V_{\text{dc,max}}$) is based on analysis presented in S. Cole (2011). It is another maximum likelihood iterative method that seeks to correct for density fluctuations in the LF. In this method, an effective or density corrected volume

$$V_{\text{dc,max},i} = \int_{z_{\min,i}}^{z_{\max,i}} \Delta(z) \frac{dV}{dz} dz = \sum_j \Delta V \Delta_j G(V_j) \quad (\text{B7})$$

is calculated for each galaxy, where the sum is over the volume bins, the overdensity parameter is set to $\Delta_j = 1$ in the first itera-

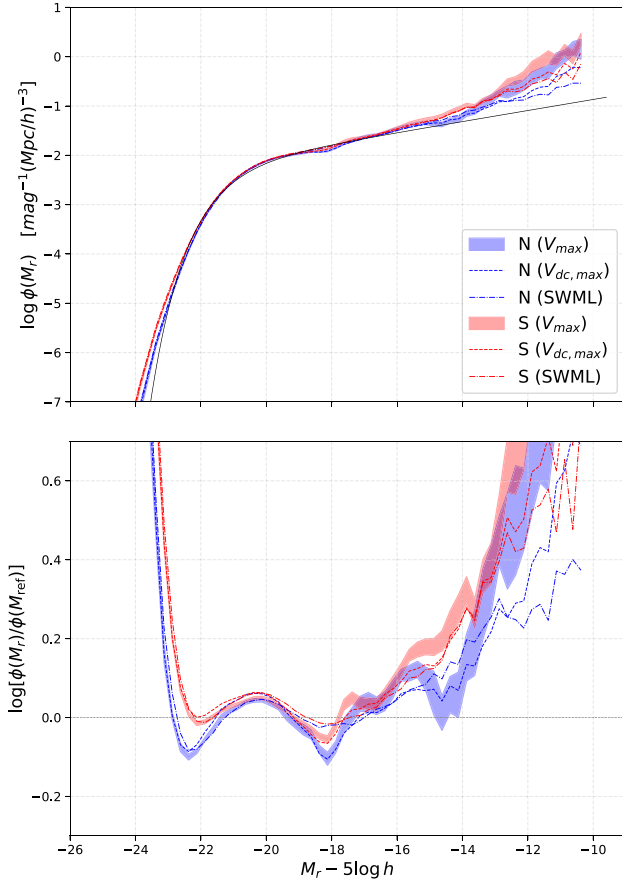


Figure B1. Top: A comparison of the different LF estimators ($1/V_{\max}$, $1/V_{\text{dc}, \max}$, SWML) in the North (blue) and South (red). The black curve shows the r -band fiducial Schechter function from Table 1. Bottom: The ratio of each of the LFs estimates to fiducial Schechter function.

tion, and G is a binning function

$$G(V_j) = \begin{cases} 0, & \text{if } V_j - \Delta V/2 > V_{\max, i} \\ 1, & \text{if } \frac{\min(V_j + \Delta V/2, V_{\max, i}) - \max(V_j - \Delta V/2, V_{\min, i})}{\Delta V} \\ 0, & \text{if } V_j + \Delta V/2 < V_{\min, i} \end{cases} \quad (\text{B8})$$

For subsequent iterations, the estimate of the overdensity is updated using

$$\Delta_j = \frac{N_j}{N_{\text{exp}, j}}, \quad (\text{B9})$$

where N_j is number of galaxies in volume bin j and

$$N_{\text{exp}, j} = \sum_i \frac{H(V_{\min, i} < V_j < V_{\max, i}) \Delta V}{\Delta_i V_{\max, i}}. \quad (\text{B10})$$

Here, $N_{\text{exp}, j}$ is the expectation value for the number of galaxies that we would expect in each volume element if they were uniformly distributed in space given their individual $V_{\min, i}$ and $V_{\max, i}$.

From this, the LF for each iteration may be calculated as

$$\phi = \frac{1}{\Delta M} \sum_i \frac{1}{V_{\text{dc}, \max}} \Theta(M_i; \Delta M), \quad (\text{B11})$$

where Θ is the Heaviside step function. We additionally enforce for each iteration the constraint $\langle \Delta_j \rangle = 1$.

The SWML and $V_{\text{dc}, \max}$ estimators both remove the effect of density fluctuations by making the assumption that the proba-

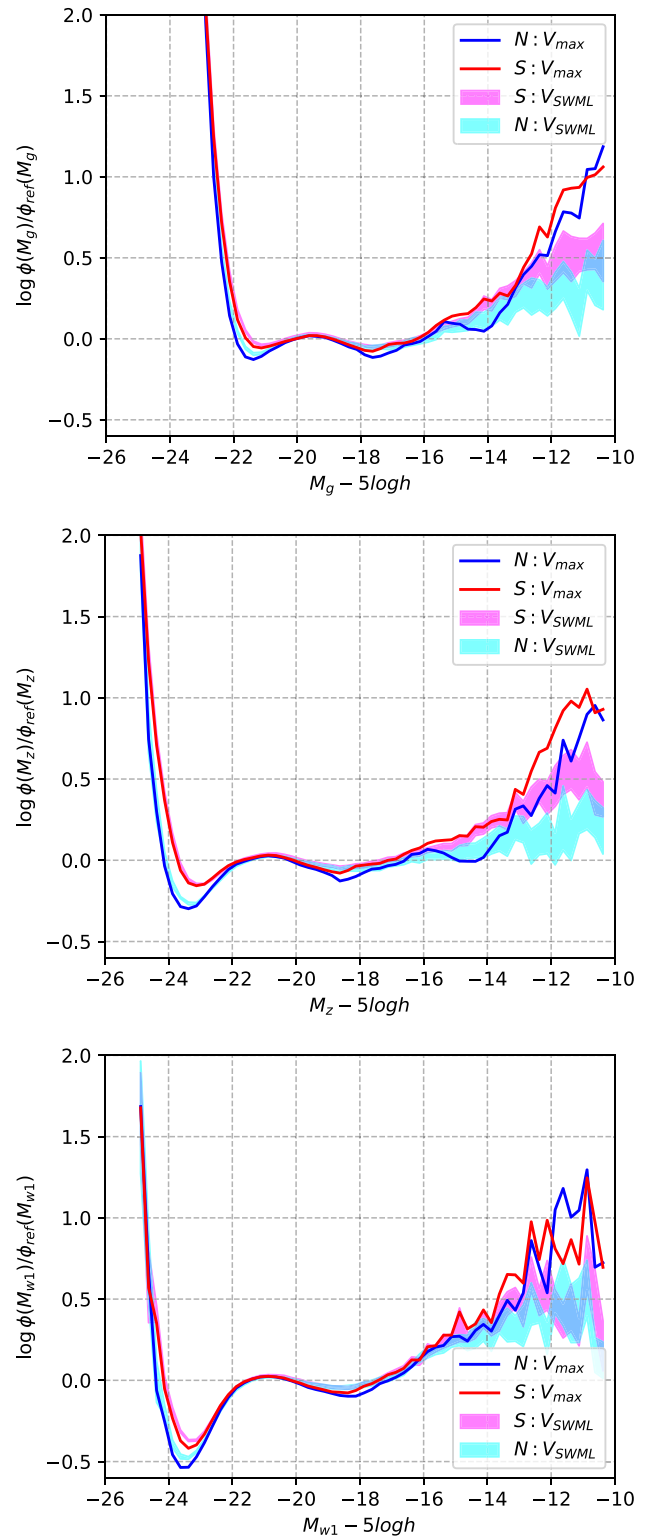


Figure B2. SWML estimates of the g , z , and $w1$ luminosity functions. All estimates are shown relative to their fiducial Schechter function give in Table 1. The solid lines show the standard $1/V_{\max}$ estimates (North red and South blue) while the shaded regions show the corresponding SWML estimates and their jackknife errors.

bility $P(L, z)$ of having a galaxy of luminosity L at redshift z is factorizable as the product of the local density and the shape of the LF. The key difference is that the SWML method starts with the conditional probability distribution $P(L|z)$ and just solves for the shape of the LF, while the $V_{\text{dc}, \text{max}}$ method starts with the joint $P(L, z)$ and solves for both the shape of the LF and the run of density with redshift.

The SWML method only determines the shape of the LF and not its overall amplitude. To constrain this we have normalized each LF to have the same luminosity density as the corresponding $1/V_{\text{max}}$ LF estimate.

For each of the three estimators and for North and South separately, Fig. B1 shows the LF estimates relative to the corresponding fiducial Schechter function from Table 1. Some small systematic discrepancies exist between the different estimators but most of the features we have previously highlighted remain robust. The bright end of the luminosity functions agree extremely well. The dip below the fiducial Schechter function around -17 persists but the turn up around -14 is greatly reduced and consequently likely a result of a density fluctuation.

For the other bands we compare SWML estimates computed using

$$\phi(L)dL = \sum_{i=1}^N \frac{w_i W(L - L_i)}{V_i^{\text{SWML}}}, \quad (\text{B12})$$

with the corresponding $1/V_{\text{max}}$ estimates in Fig. B2. Here V_i^{SWML} for galaxy i is simply the value given by equation (B4) for the r -band magnitude bin in which galaxy i falls. As in the r -band we see good agreement with all features in the $1/V_{\text{max}}$ estimates except for at the very faintest magnitudes where in all bands the $1/V_{\text{max}}$ has a sharper upturn.

APPENDIX C: TESTS EVOLUTION AND COMPARISON TO GAMA

We would expect our LF estimates to agree well with those of GAMA (J. Loveday et al. 2012), although with much smaller statistical errors. In Section 4, we did not compare our LF estimates with the Loveday LF estimates as J. Loveday et al. (2012) make a number of different assumptions. Most importantly, the Loveday LFs are defined for a more limited redshift sample (from $0.002 < z < 0.1$) with no evolution ($P = 0$; $Q = 0$). Fig. C1 presents the DESI Y1 BGS LFs in North and South for $0.002 < z < 0.1$ and $Q = 0$. We observe good agreement between the DESI LFs and the Loveday GAMA LFs in the g , r , and z bands within the GAMA errors over a wide magnitude range. Residual differences at the very bright end are probably due to the different choice of magnitude. GAMA uses Petrosian magnitudes while DESI uses total magnitudes derived from model fits. The effect of magnitude choice on the DESI LFs is discussed in Appendix D. Differences fainter than -14 are influenced by local density fluctuations at the very faintest magnitudes some spurious objects in the DESI catalogue (see Section 4).

In Fig. C1, we plot the same fiducial Schechter functions from Table 1 as we did in Fig. 8. This enables one to see that there are small shifts in the LF estimates that makes those plotted here in better agreement with J. Loveday et al. (2012) than those plotted in Fig. 8. The reason for this is not due to the different choice evolution parameter Q . With these redshift restricted samples, $0.002 < z < 0.1$, the LF estimates are very insensitive to the choice of Q and we see no discernable difference between esti-

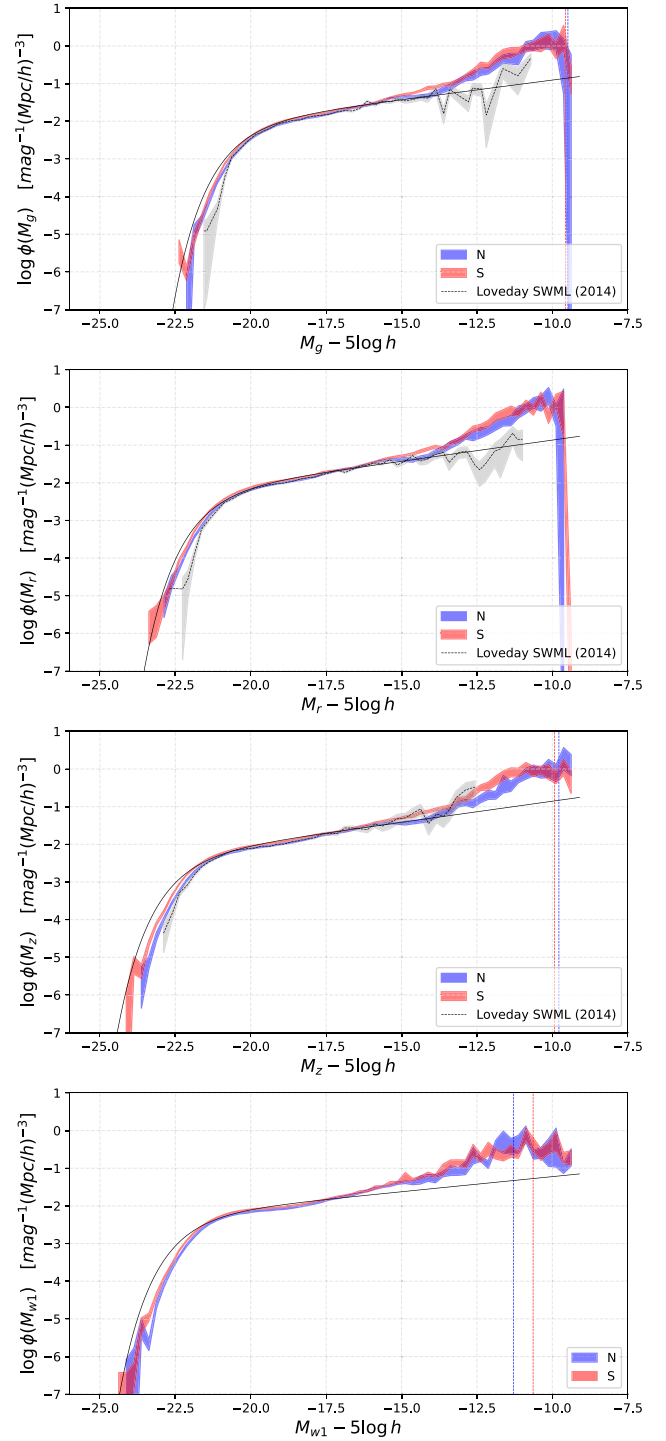


Figure C1. The global $1/V_{\text{max}}$ LF for Y3 data for North and South in the r , g , z and $w1$ bands in the range $0.002 < z < 0.1$. The absolute magnitudes are calculated with $Q = 0$. The width of each LF represents the jackknife error. The dashed lines represent the completeness limits derived from the corresponding bivariate LFs. Schechter functions are the corresponding LFs from Fig. 8.

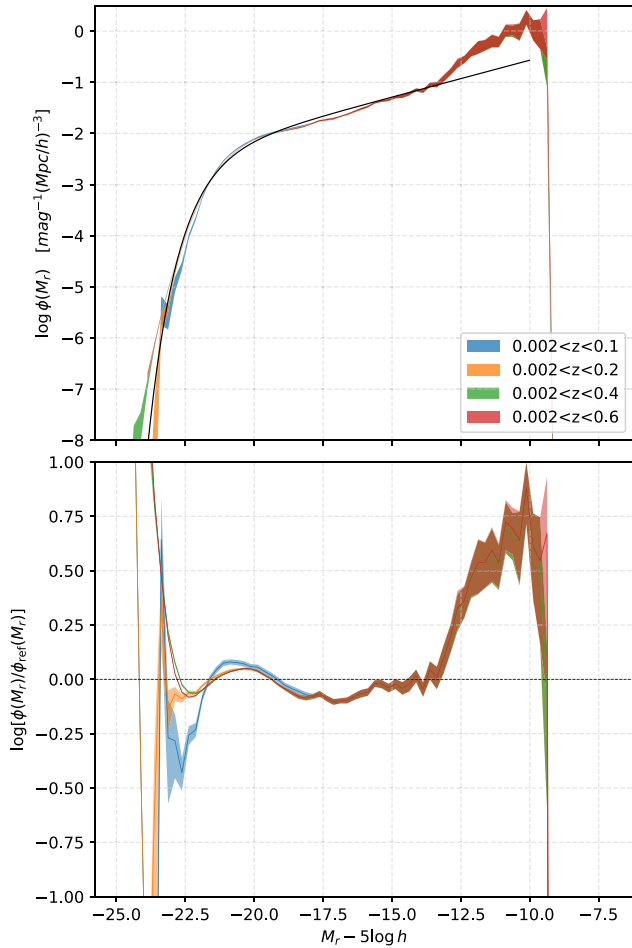


Figure C2. Top: The $1/V_{\max}$ total magnitude LFs for different redshift ranges in the South, using $Q = 0.78$. Bottom: The ratio plot of each LF to the r -band Schechter function in Fig. 8.

mates with $Q = 0$ and 0.78 . Instead the small differences between the LF estimates in Figs C1 and 8 are due to the failure of the simple evolution model to fully capture the evolution over the larger redshift range $0.002 < z < 0.6$. We investigate this further below.

In Fig. C2, we plot LF estimates using our standard $Q = 0.78$ evolutionary correction for samples with a range of different upper redshift limits between $z_{\max} = 0.1$ and $z_{\max} = 0.6$. The faint parts of the luminosity function necessarily agree perfectly as there are no faint galaxies in the sample above redshift $z = 0.1$. The luminosity functions also agree very well around the knee of the luminosity function. This is to be expected as the sample we have used to constrain Q is dominated by galaxies around the knee of the luminosity function. Brightward of the knee we begin to see statistically significant differences in the estimates. This is particularly clear in the lower panel which shows the ratio of these estimates to our fiducial r -band Schechter function from Table 1. These differences indicate that a more detailed model of galaxy evolution is required to model evolution of the LF to the precision warranted by the small statistical errors of the DESI data. However, such modelling would also have to take account of subtle systematics that might exist in the photometric modelling

as the angular size and surface brightness of the sources decrease with redshift.

APPENDIX D: PHOTOMETRY DIFFERENCE IN NORTH AND SOUTH

As discussed in Section 4, one concern with our LFs is that there is a discrepancy between the North and South r -band LFs at the bright end. First, we consider the possibility that there may be an inherent discrepancy in the FSF k -correction fits. To do this, we make use of the overlap region in the DESI survey – that is, the region where there exists both North and South photometry data for the same objects. This region is shown in Fig. D1.

In Fig. D2, we compare the apparent magnitudes (g and r) in BASS and DECaLS. As the effective photometric pass bands are not the same for BASS and DECaLS we expect some systematic differences between the two magnitudes. This can clearly be seen in the g -band where there is a fairly constant offset in the median of 0.059 mag. The r -band filters are much more similar and here the corresponding median offset is 0.023 mag. Because of these differences it is better to compare their absolute magnitudes using the k -corrections described in Section 3.1 which take the passbands into account and convert to a consistent SDSS $z_{\text{ref}} = 0.1$ rest-frame passband.

For blue galaxies, the two magnitudes agree well over the full absolute magnitude range with a median magnitude offset of only 0.01 . However, for the red galaxies, we only see good agreement between the two magnitudes for galaxies fainter than $^{0.1}M_r - 5 \log h = -21$. At bright magnitudes we see an increasing offset that grows to 0.1 magnitudes at $^{0.1}M_r - 5 \log h = -23$ mag. We note that there exist a sufficient number of blue galaxies at these bright magnitudes to demonstrate that this does not hold for blue galaxies.

We have seen in Fig. 10 the discrepancy between the North and South luminosity functions is confined to the bright end of the red galaxy luminosity function. The offset that we have found above is in the sense that could help explain this discrepancy. To investigate this quantitatively, we have assigned new magnitudes to the red galaxies in the North by shifting them by the magnitude-dependent median difference found in the overlap region such

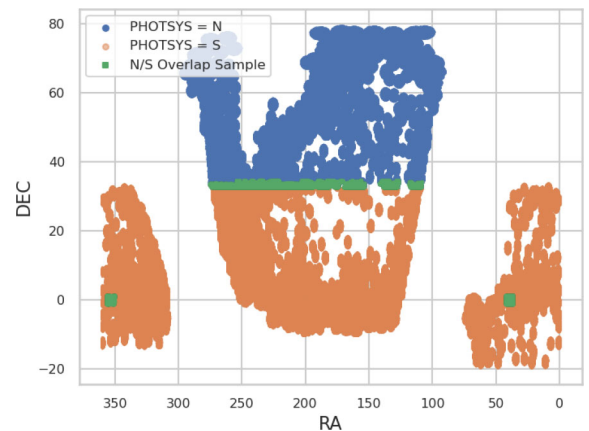


Figure D1. A plot showing the North and South regions of the Y1 DESI survey. The green region represents an overlap region – the area where objects received both North and South photometry fluxes (based on BASS/MzLS and DECaLS).

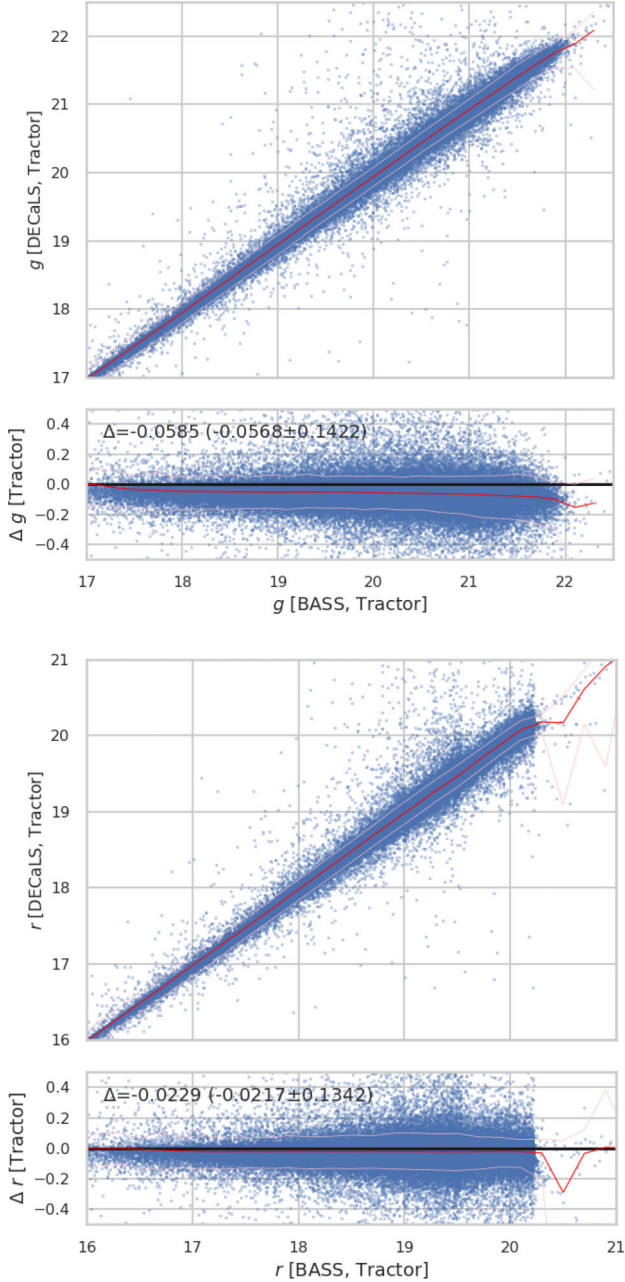


Figure D2. Comparison of BASS and DECaLS apparent magnitudes. The upper pair of panels are for the g -band and the lower pair for the r -band apparent magnitudes. In each case, the the lower sub-panel shows the difference (DeCaLS–BASS) versus the BASS magnitude. In all panels, the red lines shows the median and the light lines the 5th and 95th percentiles of the distributions. The values in the lower plots are the median across the whole sample and, in brackets, the mean and standard deviation of the difference.

that

$$M_{r,i,\text{new}}^{\text{BASS}} = M_{r,i,\text{old}}^{\text{BASS}} + \langle M_r^{\text{DECaLS}} - M_r^{\text{BASS}} \rangle_{M_{r,i,\text{old}}^{\text{BASS}}} \quad (\text{D1})$$

The effect of this adjustment on the r -band luminosity function can be seen Fig. D4. The adjustment makes a notable difference to the LF at bright magnitudes, bringing the North LF far closer to that of the South. While we do not expect this adjustment to fully correct the LF since it only takes account of the median offset

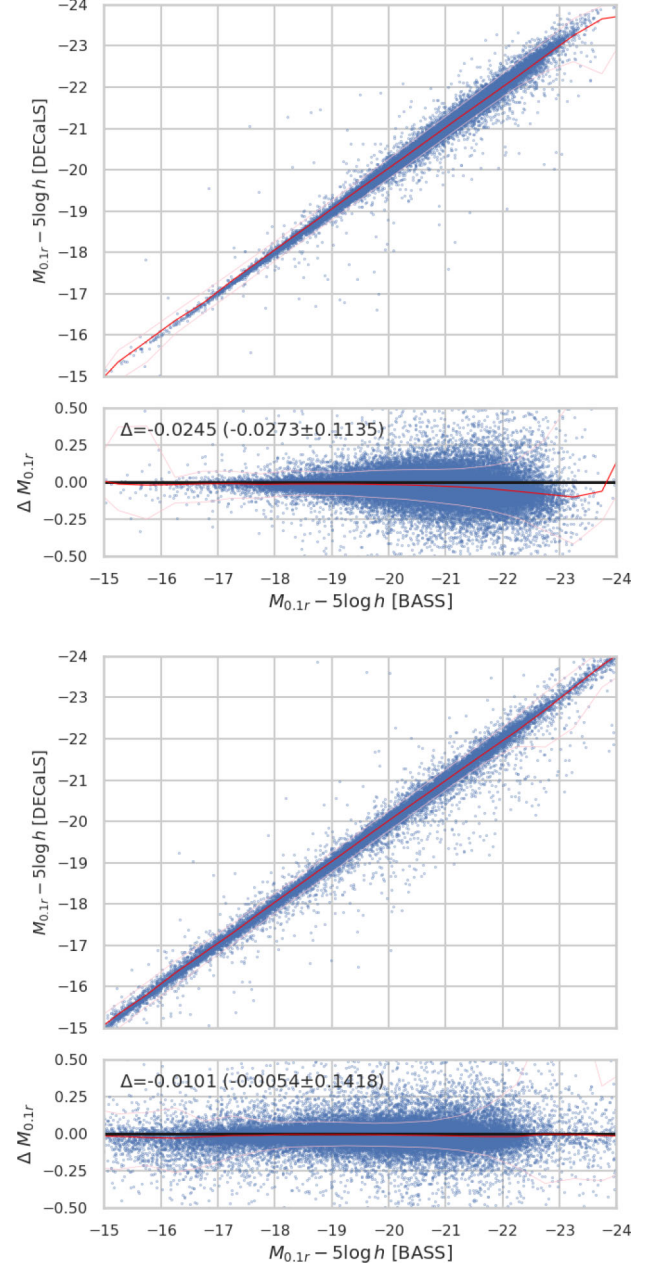


Figure D3. Comparison of BASS and DECaLS r -band absolute magnitudes k -corrected to the SDSS $z_{\text{ref}} = 0.1$ passband for red and blue galaxies. The upper pair of panels is for the red galaxies $[^{0.1}(g-r) > 0.75]$ and the lower pair for blue galaxies $[^{0.1}(g-r) < 0.75]$. The lower panel in each pair shows the residual (DECaLS–BASS) versus the BASS magnitude. In all panels, the red lines show the median and the light lines the 5th and 95th percentiles of the distributions. The values in the lower plots are the median across the whole sample and, in brackets, the mean and standard deviation of the difference.

and not the cause of the scatter, it nevertheless indicates that the difference in the bright ends of the r -band LFs is consistent with brighter red galaxies being systematically assigned different magnitudes dependent on whether the North or South photometry is used. It does not explain why this is the case or why we see similar offsets in other bands and so we have investigated other properties of the bright red galaxies.

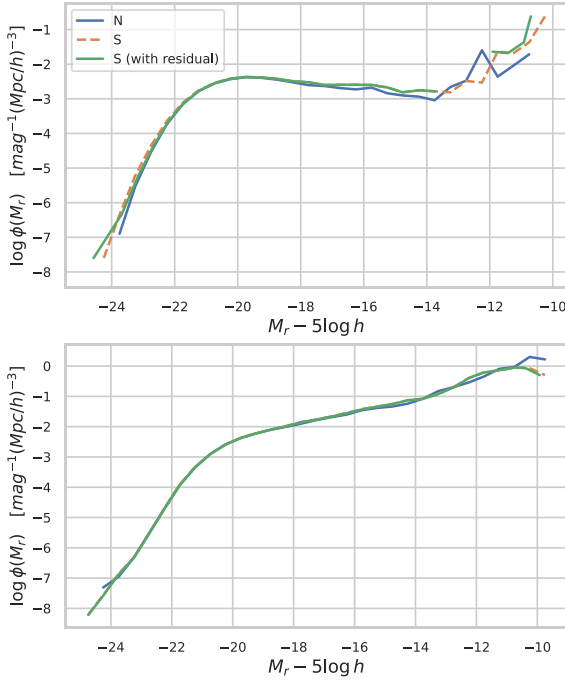


Figure D4. Top: a plot of the North and South r -band LFs for red galaxies. Here, each galaxy in the South has its $^{0.1}M_r - 5 \log h$ changed by the residual of the BASS/DECaLS $^{0.1}M_r - 5 \log h$ for that magnitude bin. This yields the South LF ‘with residual’ offset. Bottom: The same LFs but for the blue galaxies.

Bright red galaxies are typically early-type galaxies with light profiles that are of the de Vaucouleurs form (Sérsic profile with index $n = 4$) or have even higher Sérsic indices. It is therefore interesting to see if the distribution of Sérsic index for bright galaxies is consistent in the North and South regions. Fig. D5 compares normalized histograms of the distribution of Sérsic index and the corresponding cumulative fractions, $F(< n)$, for galaxies brighter than $^{0.1}M_r - 5 \log h < -22$, which is the magnitude range where the North and South luminosity functions disagree. The first thing we see is that the spikes at $n = 1$ (exponential profile) and $n = 4$ (de Vaucouleurs profile) are smaller in the South than the North. Our understanding of this is that the TRACTOR photometric pipeline only adopts a Sérsic profile if the $\Delta\chi^2$ between its fit and the fit of the simpler exponential/de Vaucouleurs profile is above a threshold that warrants the inclusion of the extra parameter. As the South/DECaLS data are deeper than the North/BASS data the typical signal-to-noise is greater and this threshold will be passed more often in the South. This is not a concern as presumably the value of n found should still be close to the $n = 1$, or $n = 4$ values. However, we also see a difference in the distributions at values of $n > 4$.

The maximum value of the Sérsic index that is considered is $n = 6$. In the South, we see there are more galaxies fit with this maximum allowed value than in the North. Moreover this difference is not just restricted to distortions in the distribution close to $n = 6$ as the cumulative distribution shows the difference persists to $n < 4$. One possibility is that in the deeper South data provide more information in the low surface brightness wings of the most extended galaxies and this favours larger n than is warranted in the noisier North data. One way to investigate this would be to compare fixed aperture magnitudes rather model fits

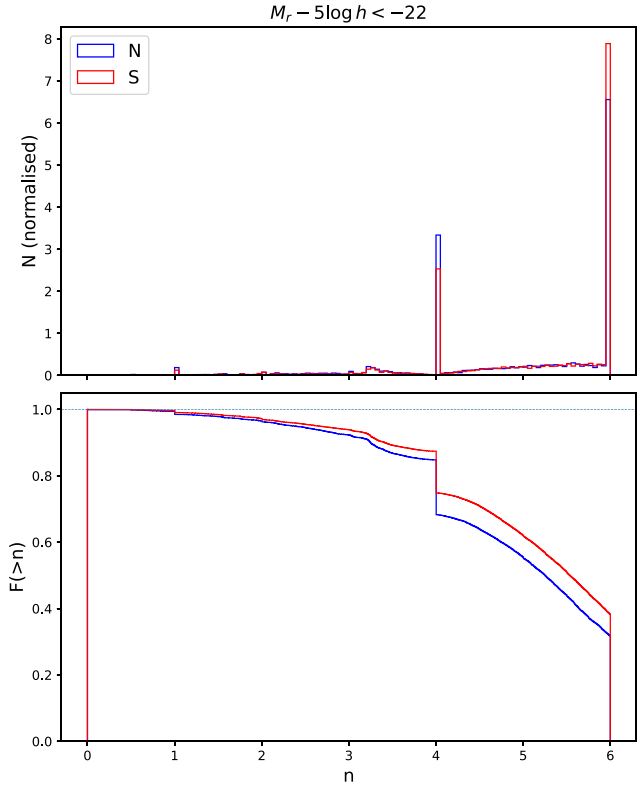


Figure D5. Top: the normalized distribution of the Sérsic index. Bottom: The cumulative distribution plots of the Sérsic index, n , for galaxies brighter than $^{0.1}M_r - 5 \log h < -22$ in the North and South.

but these are not available. As an alternative, we have computed Petrosian magnitudes from the model fits (M. R. Blanton et al. 2001). Petrosian magnitudes are aperture magnitudes defined within an aperture that scales with the scale of the light profile of the galaxy and are independent of the light distribution beyond this radius.

We do not have the light profiles to compute true Petrosian magnitudes and so instead we have computed Petrosian magnitudes from the 2D elliptical Sérsic fits. The radial coordinate of these model profiles, denoted r , is the semi-major axis of the ellipse. For a Sérsic profile, the ratio of the Petrosian flux to the total flux is given by

$$\frac{F_p}{F_{\text{tot}}} = \frac{\int_0^{r_p} \exp(-r^{1/n}) r dr}{\int_0^\infty \exp(-r^{1/n}) r' dr'}, \quad (\text{D2})$$

where the Petrosian radius, r_p , is the root of equation

$$\eta(r) = \frac{r^2 \exp(-r^{1/n})}{2 \int_0^r \exp(-r'^{1/n}) r' dr'} = 0.2. \quad (\text{D3})$$

As with true Petrosian magnitudes, we believe these magnitudes should be less sensitive to the outer profile of the galaxy. As such, we hypothesize that North and South LFs based on Petrosian absolute magnitudes will converge at the bright-end.

To estimate the LF using these Petrosian magnitudes, we must define a new sample that is complete with respect to Petrosian magnitude limits. As the Petrosian magnitudes are all fainter than the corresponding total magnitude some objects will be fainter than the BGS Bright target selection faint magnitude

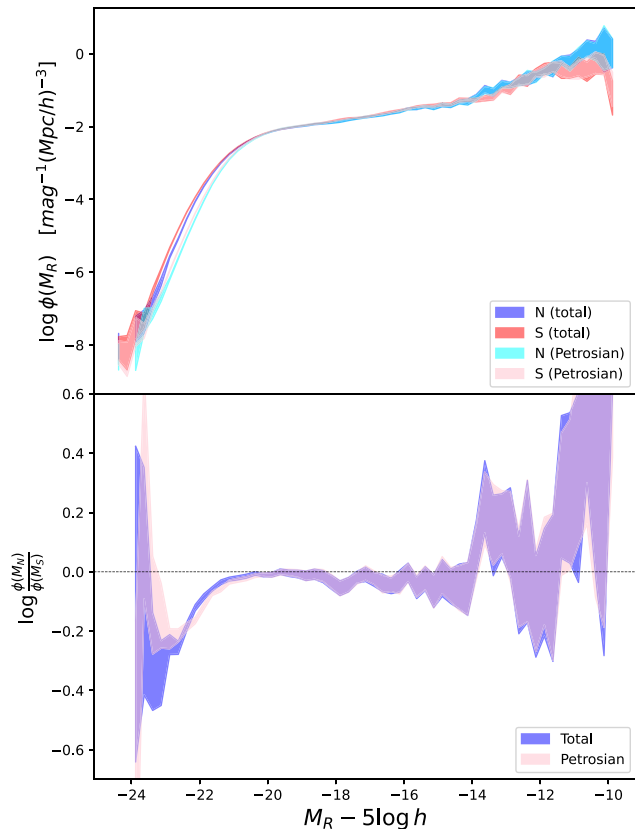


Figure D6. Top: The total magnitude r -band LFs plotted alongside the Petrosian magnitude LFs for samples selected within the range $0.002 < z < 0.6$. All absolute magnitudes are calculated with $Q = 0.78$. Bottom: the ratio the North and South LFs. In blue, is shown the ratio for the total magnitude LFs and in pink those for the Petrosian magnitudes. The width of each band represents the jackknife error estimate.

limit. Hence, we discard objects by applying $r_{\text{petro}} < 19.5$ for the South, and $r_{\text{petro}} < 19.54$ for the North. We additionally recompute all required values (such as the V_{max} values) as detailed above.

Fig. D6 presents these Petrosian-magnitude LFs alongside those that use the standard TRACTOR total magnitudes. Faintward of magnitude -20 the two types of luminosity function agree extremely well. This is to be expected as we have seen that this part of the LF is dominated by blue galaxies which are typically late-type galaxies with exponential light profiles. As such light profiles have very little flux outside the Petrosian radius the LFs will necessarily agree very well. In contrast, at the bright end the Petrosian LFs are shifted faintward consistent with this part of the luminosity function being dominated by red late-type galaxies with extended light profiles for which there is significant flux beyond the Petrosian radius. Also of interest is that the Petrosian LFs are in moderately better agreement in North and South but as can be seen in the lower panel of Fig. D6 this change does not fully remove the discrepancy between the bright ends of the North and South LFs. Nevertheless this analysis does suggest that the biggest systematic uncertainty in the bright end of the galaxy luminosity function is related to quantifying the low surface brightness outskirts of bright red early-type galaxies. We would expect the difference in North and South LFs to decrease further if true Petrosian magnitudes were used rather than our

model Petrosian magnitudes which are still somewhat influenced by the fits to the outer profile.

Whilst this analysis could be taken to indicate that the disparity in the LFs at the bright end is due to flux being missed in the North data due to erroneous Sérsic profile fitting, we caution against this interpretation. We note the alternative possibility that the very high Sérsic indices found in the South may be an indication that the formally smaller photometric errors make the profile fitting more susceptible to other systematic errors such as residual errors in the sky subtraction. Without further investigation, we are unable to say which LF is the correct one. Instead, we interpret the difference as the level of systematic uncertainty that remains in the estimate of the bright end of the galaxy luminosity function.

APPENDIX E: IMAGING VISUAL INSPECTION

Our LF estimates reach extremely faint magnitudes across all bands. Whilst we have outlined the methodology for estimating the completeness of the g , z , and $w1$ bands using the bivariate LFs, we caution that this is a theoretical completeness limit that does not consider other factors that may introduce error in the faint-end of the LF (including for the r -band LF). In order to better understand the completeness of our data, we conduct a visual inspection programme of the DESI Y3 galaxies using the Legacy Survey Sky Browser. In particular, we are interested as to whether the galaxies at very faint r -band absolute magnitudes are real galaxies with accurate fluxes. We also used this investigation to inform the choice of the $r - w1 < 2.25$ colour cut we applied to remove problematic galaxies from the the to the sample used to estimate the $w1$ LF.

The Legacy Survey Sky Browser is a viewing tool that allows us to visually inspect the imaging from various surveys across the whole sky. This viewer usefully contains the Legacy Survey DR9 images in the $ugriz$ bands as well as the unWISE W1/W2 NEO7 images, allowing for quick visual comparisons of selected galaxies. In addition, these images can be overlaid with DESI information, including the DESI Bright-time targets and the DESI footprint and fibres. We constructed a script to extract two sets of Sky Browser DR9 images each comprising of 20 random objects within each magnitude bin. The script compiles the images and overlays details such as the RA, DEC, z , and TARGETID. From this, visual inspection may quickly be conducted for each magnitude bin.

We then define two categories: ‘good’ and ‘bad’. Good galaxies are characterized by being visible, likely galaxies. Typically they will be the dominant object targeted by the fibre, and although there may be other similar objects or features in the image that could conceivably affect the flux extraction, we are confident that the object targeted is a galaxy. Bad galaxies on the other hand are objects that are unlikely to be galaxies, or are galaxies that are obviously incorrectly modelled. The vast majority of bad galaxies are shredded components of much larger galaxies. This issue and ways of correcting the galaxy photometry are investigated further in Manwadkar et al (in preparation).

Table E1 shows our classification results for both sample A and B which were classified by different members of the team. We investigate the impact of these problem targets on the r -band LF, by weighting by the fraction of galaxies classified as good in each magnitude bin. The result of this is shown in Fig. E1 which shows both $1/V_{\text{max}}$ and SWML LF estimates. The confirmed fraction agrees well between the two samples at absolute magnitudes

Table E1. Table showing the visual classification of two independent sets of randomly selected objects (A & B) in bins of r -band magnitude. The two sets were classified by different members of the team. The columns M_r^{faint} and M_r^{bright} define the $M_r - 5 \log h$ range. Columns 3 & 4 (5 & 6) indicate how many galaxies in the sample A (B) appear to be well modelled galaxies (Good) and how many appear problematic (Bad).

M_r^{faint}	M_r^{bright}	Good (A)	Bad (A)	Good (B)	Bad (B)
-10.25	-10.5	7	13	6	14
-10.5	-10.75	8	12	3	17
-10.75	-11.0	12	13	11	9
-11.0	-11.25	13	7	10	10
-11.25	-11.5	15	5	8	12
-11.5	-11.75	9	11	5	15
-11.75	-12.0	15	5	11	9
-12.0	-12.25	10	10	10	10
-12.25	-12.5	13	7	8	12
-12.5	-12.75	18	2	8	12
-12.75	-13.0	12	8	10	10
-13	-13.25	12	8	11	9
-13.25	-13.5	15	5	12	8
-13.5	-13.75	14	6	15	5
-13.75	-14.0	15	5	17	3
-14.0	-14.25	12	8	17	3
-14.25	-14.5	16	4	15	5
-14.5	-14.75	15	5	17	3
-14.75	-15.0	14	6	16	4
-15.0	-15.25	17	3	18	2
-15.25	-15.5	14	6	20	0
-15.5	-15.75	18	2	17	3
-15.75	-16.0	18	2	18	2
-16.0	-16.25	18	2	15	5
-16.25	-16.5	20	0	19	1
-16.5	-16.75	19	1	20	0
-16.75	-17.0	20	0	20	0
-17.0	-17.25	20	0	19	1
-17.25	-17.5	20	0	20	0
-17.5	-17.75	19	1	20	0
-17.75	-18.0	20	0	20	0
-18.0	-18.25	19	1	20	0
-18.25	-18.5	20	0	19	1

brighter than -14 . At fainter magnitudes the confirmed fraction is slightly lower in sample B than sample A. This difference is as much down to the subjective nature of the visual classification as it is to the small sample size. Nevertheless it is clear by both estimates that shredded galaxies are contributing significantly to the upturn we have observed in the faint end of the luminosity function. In the case of the SWML LF estimate, which also

reduces the strength of the faint end upturn by correcting for local density fluctuations, the estimate corrected using sample B shows no upturn. Hence we conclude that within the accuracy of our estimates the faint upturn in the $1/V_{\text{max}}$ LF is caused by a combination of local overdensity combined with galaxies that are photometrically shredded to produce spurious faint targets.

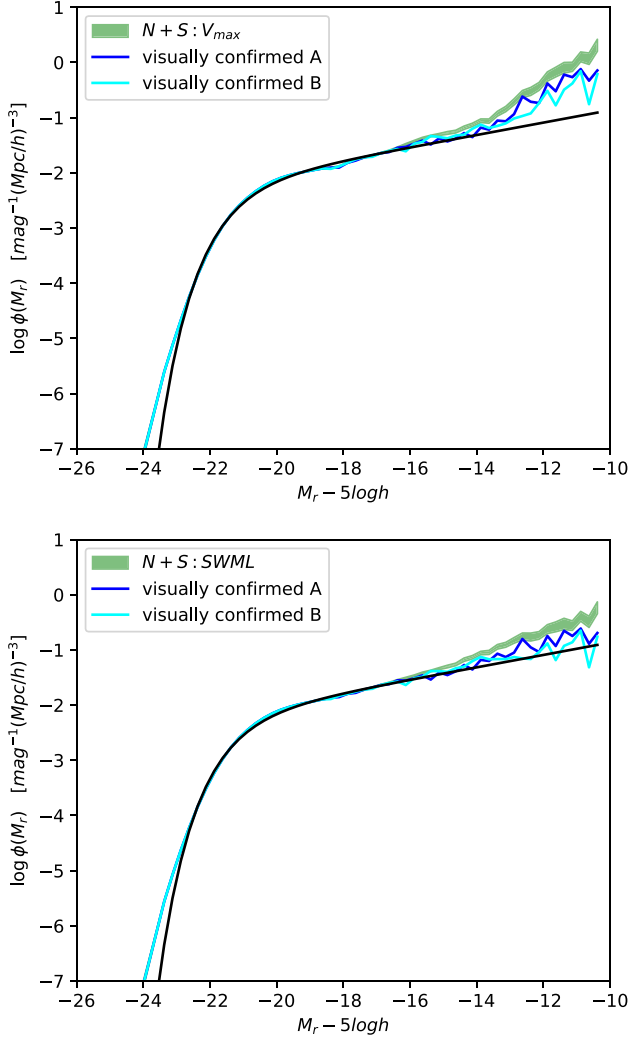


Figure E1. The area weighted average of the North and South $1/V_{\max}$ (top) and SWML (bottom) LFs are shown by the shaded band. The width of the band shows the jackknife $\pm 1\sigma$ error range. The effect on this LF of weighting by the visually confirmed fraction in sample A and B are shown by the lines as indicated in the legend.

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