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# A New Design of Differential Space-Time Block Code Allowing Symbol-Wise Decoding

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**Abstract**—For four (or more) transmitters, a new design of differential space-time block code allowing symbol-wise decoding is presented in this letter. The new design not only has the minimum (symbol-wise) decoding complexity as that by Yuen, Guan and Tjhung (YGT) but also yields a lower error rate. While the YGT code uses a specially designed symbol constellation, the new code uses a conventional QAM with a rotation. At a high data rate such as 3bps/Hz, the new design with symbol-wise decoding complexity can even yield a lower error rate than the code by Zhu and Jafarkhani that has the pair-wise decoding complexity.

**Index Terms**—Coding-gain, differential space-time code, symbol-wise decoding.

## I. INTRODUCTION

**D**IFFERENTIAL space-time block codes (DSTBC) are useful for wireless communications with multiple transmitting antennas. With DSTBC, the channel state information (CSI) is not required either at the transmitters or at the receivers, which is important for applications where the CSI changes too fast to be estimated and utilized. The design of DSTBC has attracted the attention of many researchers in recent years. For two transmitters, the design of DSTBC is well established because of the existence of full rate complex orthogonal code. But for more than two transmitters, the design of DSTBC is still an active area of research. For practical use, there is a strong interest to reduce the decoding complexity of DSTBC with as little loss of coding gain as possible. Consider a system of four transmitters with a full diversity DSTBC operating at the data rate (i.e., spectral efficiency)  $r$  bps/Hz. A conventional decoder requires a size- $2^{4r}$  search, a pair-wise decoder requires a size- $2^{2r}$  search, and a symbol-wise decoder needs only a size- $2^r$  search. Although a pair-wise decoder reduces the complexity significantly from a conventional decoder, the complexity difference between a pair-wise decoder and a symbol-wise decoder is still large. For four transmitters, while many designs of pair-wise decodable DSTBC are now available in the literature such as the codes by Zhu and Jafarkhani (ZJ) [1] and Calderbank et al [2], the code

by Yuen, Guan and Tjhung (YGT) [3] seems the only reported DSTBC that is of full rate, full diversity, allows symbol-wise decoding, and suffers no major loss of coding gain from the pair-wise decodable counterparts.

In this letter, we present a new design of DSTBC that allows symbol-wise decoding. The new design differs from the YGT code in the following ways. The YGT code forces the symbol constellation to be constructed in such a way that a symbol-wise decoder becomes near optimal with respect to the given encoder. The resulting symbol constellation consists of symbols falling on the real and imaginary axes, and the constellation spread along one axis is wider than that along the other. See Fig. 3 in [3]. The new design utilizes a conventional QAM constellation and applies a rotation to optimize a “pseudo coding gain (PCD)” governed by the encoder. The decoder of the new design is forced to be symbol-wise in an approximate maximum likelihood fashion. Although the decoder of the new design is not optimal with respect to its given encoder, the higher PCD of the encoder allows the loss of the suboptimal decoder to be more than enough compensated. As shown by simulation, the error rate of the new design is significantly smaller than that of the YGT code at moderately high SNR. While the idea of the new design is simple which innovatively combines those of YGT and ZJ, the new design also shows the highest coding gain among all known DSTBC that allow symbol-wise decoding for four transmitters.

Next, we show the details of the new design as well as its connections to the YGT and ZJ codes. The simulation results are illustrated in Section III.

## II. THE NEW DESIGN

**W**E start with a constellation set  $S = \{s_i, i = 1, 2, \dots, M\}$  in the complex plane where  $M = 2^r$ . We assume that there are  $2l$  transmitting antennas where  $l$  is an integer-power of 2.

If the actual number  $a$  of transmitting antennas satisfies  $2^q < a < 2^{q+1}$ , one should treat the number of transmitting antennas to be  $2^{q+1}$ . The difference  $2^{q+1} - a$  can be treated as the number of channels that have zero gain among total  $2^{q+1}$  nominal channels. The transmit diversity factor remains to be  $a$ .

During the  $k^{\text{th}}$  time block of  $2l$  time slots, we collect  $2m$  symbols randomly from  $S$ . This process should be governed by an outer layer code that is not discussed here. These symbols are denoted by  $S_k = \{s_{k,i}, i = 1, 2, \dots, 2m\}$  where  $m \leq l$ .

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### A. First Step of Encoding

We construct a  $2m \times 1$  symbol vector  $\mathbf{x}_k$  as follows:

$$\begin{aligned}\mathbf{x}_k &= \begin{bmatrix} \mathbf{x}_{k,1}^T & \mathbf{x}_{k,2}^T \end{bmatrix}^T \\ \mathbf{x}_{k,1} &= \begin{bmatrix} x_{k,1} & \dots & x_{k,m} \end{bmatrix}^T \\ \mathbf{x}_{k,2} &= \begin{bmatrix} x_{k,m+1} & \dots & x_{k,2m} \end{bmatrix}^T \\ \mathbf{x}_{k,i} &= \Re(s_{k,i}) + j\Im(s_{k,i+m}) \\ \mathbf{x}_{k,i+m} &= -\Im(s_{k,i}) + j\Re(s_{k,i+m})\end{aligned}$$

and  $i = 1, 2, \dots, m$ . The mapping from  $S_k$  to  $\mathbf{x}_k$  is the same as a part of the YGT encoder. See the entries in the quasi-orthogonal matrix in Equation (8) in [3].

A key property of the YGT code is also governed by a quasi-orthogonal code first shown in [4]. It is shown in [5] that the quasi-orthogonal code shown in [4] is a member of a large family that all hinge on the  $2 \times 2$  orthogonal code first shown in [6].

### B. Second Step of Encoding

We now construct a  $2l \times 2l$  code matrix  $\mathbf{X}_k$  that is partitioned into four  $l \times l$  submatrices:

$$\begin{aligned}(\mathbf{X}_k)_{1,1} &= \mathbf{Y}_{k,1}; (\mathbf{X}_k)_{1,2} = \mathbf{Y}_{k,2}; \\ (\mathbf{X}_k)_{2,1} &= \mathbf{Y}_{k,2}; (\mathbf{X}_k)_{2,2} = \mathbf{Y}_{k,1}\end{aligned}$$

where  $\mathbf{Y}_{k,1} = (\mathbf{A}_{k,1} + \mathbf{A}_{k,2})/2$  and  $\mathbf{Y}_{k,2} = (\mathbf{A}_{k,1} - \mathbf{A}_{k,2})/2$ . The matrices  $\mathbf{A}_{k,1}$  and  $\mathbf{A}_{k,2}$  are encoded differentially as follows:

$$\begin{aligned}\mathbf{A}_{k,1} &= \frac{1}{\sqrt{\alpha_{k-1,1}}} \mathbf{M}_{k,1} \mathbf{A}_{k-1,1} \\ \mathbf{A}_{k,2} &= \frac{1}{\sqrt{\alpha_{k-1,2}}} \mathbf{M}_{k,2} \mathbf{A}_{k-1,2}\end{aligned}$$

where  $\alpha_{k-1,i} = \det(\mathbf{A}_{k-1,i}^H \mathbf{A}_{k-1,i})$ ,  $\mathbf{M}_{k,1}$  is an orthogonal matrix  $\mathbf{M}(\mathbf{z}_{k,1})$  of the symbol vector  $\mathbf{z}_{k,1} = \mathbf{x}_{k,1} + \mathbf{x}_{k,2}$ , and  $\mathbf{M}_{k,2}$  is an orthogonal matrix  $\mathbf{M}(\mathbf{z}_{k,2})$  of the symbol vector  $\mathbf{z}_{k,2} = \mathbf{x}_{k,1} - \mathbf{x}_{k,2}$ . At the beginning of each frame that is comprised of multiple blocks, we set  $\mathbf{A}_{0,1} = \mathbf{A}_{0,2} = \mathbf{I}$  with  $\mathbf{I}$  being the identity matrix. This part of the encoder resembles the differential encoder by ZJ: see (28)-(31) in [1].

### C. A Combined Effect

We limit  $\mathbf{M}(\mathbf{z})$  to be a linear matrix function of the vector  $\mathbf{z}$  and of the property:  $\mathbf{M}^H(\mathbf{z})\mathbf{M}(\mathbf{z}) = \|\mathbf{z}\|^2 \mathbf{I}$  with  $\|\mathbf{z}\|^2$  being the squared norm of  $\mathbf{z}$ . For  $l = 2$ ,  $\mathbf{M}(\mathbf{z})$  can be a full-rate Alamouti matrix  $\mathbf{M}(\mathbf{z}) = \begin{bmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{bmatrix}$ , for which  $l = m = 2$ . In this case, there are four transmitters as  $2l = 4$ , and the rate of symbols per time slot is one (i.e., full rate). When  $l > 2$ , there are more than four antennas as  $2l > 4$ , and we have to use fractional-rate (i.e.,  $l > m$ ) orthogonal matrices as shown in [7] and the references therein.

It is easy to verify that

$$\begin{aligned}\mathbf{A}_{k,1}^H \mathbf{A}_{k,1} &= \mathbf{M}_{k,1}^H \mathbf{M}_{k,1} = \|\mathbf{z}_{k,1}\|^2 \mathbf{I} \\ \mathbf{A}_{k,2}^H \mathbf{A}_{k,2} &= \mathbf{M}_{k,2}^H \mathbf{M}_{k,2} = \|\mathbf{z}_{k,2}\|^2 \mathbf{I}\end{aligned}$$

Therefore, we can also write

$$\alpha_{k,i} = \det(\mathbf{A}_{k,i}^H \mathbf{A}_{k,i}) = \det(\mathbf{M}_{k,i}^H \mathbf{M}_{k,i}) = \|\mathbf{z}_{k,i}\|^{2l}$$

After a substitution of the encoder described earlier, it follows that

$$\alpha_{k,1} = \left( \sum_{i=1}^m \left( \begin{array}{c} (\Re(s_{k,i}) - \Im(s_{k,i}))^2 \\ + (\Re(s_{k,i+m}) + \Im(s_{k,i+m}))^2 \end{array} \right) \right)^l \quad (1)$$

$$\alpha_{k,2} = \left( \sum_{i=1}^m \left( \begin{array}{c} (\Re(s_{k,i}) + \Im(s_{k,i}))^2 \\ + (\Re(s_{k,i+m}) - \Im(s_{k,i+m}))^2 \end{array} \right) \right)^l \quad (2)$$

It is important to note that  $(\alpha_{k,i})^{1/l}$  for  $i = 1, 2$  is a linear superposition of such terms:  $(\Re(s_{k,t}) \pm \Im(s_{k,t}))^2$  for  $t = 1, 2, \dots, 2m$ . This property is critical for symbol-wise decoding as shown later.

### D. The Channel Model

It is our convention that each column of the code matrix  $\mathbf{X}_k$  contains symbols transmitted from one antenna, and each row of the code matrix  $\mathbf{X}_k$  contains symbols transmitted during one time slot. We assume a single receiver while the multiple-receivers case can be easily handled as mentioned later. Then, the received signal vector  $\mathbf{r}_k$  during the  $k^{th}$  time block can be written as:  $\mathbf{r}_k = \mathbf{X}_k \mathbf{h} + \mathbf{n}_k$  where  $\mathbf{h}$  is the channel response vector from  $2l$  transmitters to a single receiver, and  $\mathbf{n}_k$  is the noise vector that is assumed to be white, Gaussian, and have the variance  $\sigma^2$ . We assume that  $\mathbf{h}$  is quasi-static, i.e., the change of  $\mathbf{h}$  from one time block to the next is negligible. However,  $\mathbf{h}$  does not need to be constant during a whole time frame in order for the differential code to be used, although our simulation shown later assumes a constant  $\mathbf{h}$  during each frame for the purpose of comparison to other codes. We also assume that all elements in  $\mathbf{h}$  are independent and identically distributed complex Gaussian random variables that change independently from frame to frame.

### E. Pseudo Coding Gain

We partition the received vector  $\mathbf{r}_k$  into two  $l \times 1$  subvectors:  $\mathbf{r}_{k,1}$  and  $\mathbf{r}_{k,2}$ . Similarly,  $\mathbf{h}$  is partitioned into:  $\mathbf{h}_1$  and  $\mathbf{h}_2$ , and  $\mathbf{n}_k$  into  $\mathbf{n}_{k,1}$  and  $\mathbf{n}_{k,2}$ . Then, using the definition of the code matrix  $\mathbf{X}_k$ , it is easy to verify that

$$\begin{aligned}\mathbf{q}_{k,1} &\doteq \mathbf{r}_{k,1} + \mathbf{r}_{k,2} = \mathbf{A}_{k,1}(\mathbf{h}_1 + \mathbf{h}_2) + \mathbf{n}_{k,1} + \mathbf{n}_{k,2} \\ \mathbf{q}_{k,2} &\doteq \mathbf{r}_{k,1} - \mathbf{r}_{k,2} = \mathbf{A}_{k,2}(\mathbf{h}_1 - \mathbf{h}_2) + \mathbf{n}_{k,1} - \mathbf{n}_{k,2}\end{aligned}$$

Note that  $\mathbf{q}_{k,1}$  and  $\mathbf{q}_{k,2}$  are sufficient statistics of the unknown symbols as they are a unitary (up to the factor  $\sqrt{2}$ ) transformation of the original observations  $\mathbf{r}_{k,1}$  and  $\mathbf{r}_{k,2}$ . If the channel response vector  $\mathbf{h}$  is available at the receiver, the matrix  $\mathbf{A}_k \doteq \text{diag}(\mathbf{A}_{k,1}, \mathbf{A}_{k,2})$  serves as an equivalent code matrix. Hence, the minimum value of the following:

$$\begin{aligned}& \left( \det \left( \Delta \mathbf{A}_k^H \mathbf{A}_k \right) \right)^{\frac{1}{2l}} \\ &= \left( \det \left( \Delta \mathbf{A}_{k,1}^H \mathbf{A}_{k,1} \right) \det \left( \Delta \mathbf{A}_{k,2}^H \mathbf{A}_{k,2} \right) \right)^{\frac{1}{2l}} \\ &= (\Delta \alpha_{k,1} \Delta \alpha_{k,2})^{\frac{1}{2l}}\end{aligned} \quad (3)$$

with respect to the differential symbol set  $\Delta S_k = \{s_{k,i} - s_{k,j} \neq 0; i = 1, 2, \dots, 2m; j = 1, 2, \dots, 2m\}$ , would be the coding gain of an optimal coherent decoder. Here, the operator  $\Delta$  implies the use of differential symbols. For example, given that  $\mathbf{A}_k$  is a function of the symbol set  $S_k$ ,  $\Delta \mathbf{A}_k$  is the (same) function of the differential symbol set  $\Delta S_k$ . For this reason, we define a pseudo coding gain as

$$\begin{aligned} G & \doteq \min_k \min_{\substack{s_{k,i} - s_{k,j} \in \Delta S_k \\ 1 \leq j \leq m \\ 1 \leq i \leq m}} (\Delta \alpha_{k,1} \Delta \alpha_{k,2})^{\frac{1}{2l}} \\ & = \min_{\Delta s \in \Delta S} \left| [\Re(\Delta s)]^2 - [\Im(\Delta s)]^2 \right| \end{aligned}$$

for which the symbol constellation is normalized to have unit variance. The pseudo coding gain becomes the exact coding gain of a differential decoder under some conditions as discussed later.

#### F. The Decoder

To derive a symbol-wise differential decoder, we need to exploit the recursion of  $\mathbf{A}_{k,1}$  and  $\mathbf{A}_{k,2}$  in  $\mathbf{q}_{k,1}$  and  $\mathbf{q}_{k,2}$ , which leads to

$$\begin{aligned} \mathbf{q}_{k,1} & = \frac{1}{\sqrt{\alpha_{k-1,1}}} \mathbf{M}_{k,1} \mathbf{q}_{k-1,1} + \mathbf{w}_{k,1} \\ \mathbf{q}_{k,2} & = \frac{1}{\sqrt{\alpha_{k-1,2}}} \mathbf{M}_{k,2} \mathbf{q}_{k-1,2} + \mathbf{w}_{k,2} \end{aligned} \quad (4)$$

where

$$\begin{aligned} \mathbf{w}_{k,1} & = -\frac{1}{\sqrt{\alpha_{k-1,1}}} \mathbf{M}_{k,1} (\mathbf{n}_{k-1,1} + \mathbf{n}_{k-1,2}) + (\mathbf{n}_{k,1} + \mathbf{n}_{k,2}) \\ \mathbf{w}_{k,2} & = -\frac{1}{\sqrt{\alpha_{k-1,2}}} \mathbf{M}_{k,2} (\mathbf{n}_{k-1,1} - \mathbf{n}_{k-1,2}) + (\mathbf{n}_{k,1} - \mathbf{n}_{k,2}) \end{aligned}$$

It can be shown that the composite noise vectors  $\mathbf{w}_{k,1}$  and  $\mathbf{w}_{k,2}$  are each white, Gaussian and have variances

$$\begin{aligned} \sigma_{k,1}^2 & = 2\sigma^2 \left( 1 + \frac{\alpha_{k,1}}{\alpha_{k-1,1}} \right) \\ \sigma_{k,2}^2 & = 2\sigma^2 \left( 1 + \frac{\alpha_{k,2}}{\alpha_{k-1,2}} \right) \end{aligned} \quad (5)$$

respectively. Furthermore,  $\mathbf{w}_{k,1}$  and  $\mathbf{w}_{k,2}$  are independent of each other under the Gaussian assumption of  $\mathbf{n}_k$ .

If  $\alpha_{k,1} = \alpha_{k,2} = \text{const}$  hence  $\sigma_{k,1}^2 = \sigma_{k,2}^2 = \text{const}$ , the optimal (i.e., maximum likelihood) differential decoder of the  $k^{\text{th}}$  block is given by

$$\min_{S_k} J \doteq \min_{S_k} (J_1 + J_2)$$

where

$$J_i = \left\| \mathbf{q}_{k,i} - \frac{1}{\sqrt{\alpha_{k-1,i}}} \mathbf{M}_{k,i} \mathbf{q}_{k-1,i} \right\|^2$$

for  $i = 1, 2$ . In this case, the pseudo coding gain  $G$  becomes the exact coding gain [8]. If  $\alpha_{k,1} = \alpha_{k,2} \neq \text{const}$  (i.e., they are dependent on the symbols in the  $k^{\text{th}}$  block), the above detection is near optimal, and  $G$  may be treated as a near coding gain. In fact, under the YGT constellation [3],  $\alpha_{k,1} =$

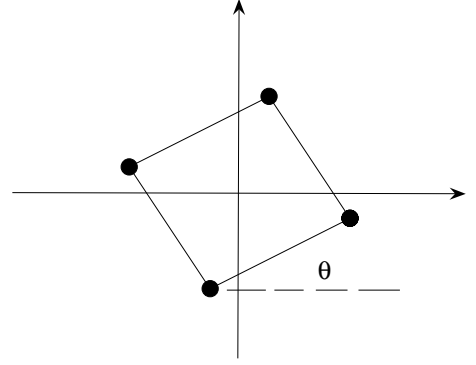


Fig. 1. A 4-QAM constellation with a rotation angle.

$\alpha_{k,2} \neq \text{const}$  and the above detection becomes equivalent to the YGT decoder.

If  $\alpha_{k,1} \neq \alpha_{k,2}$ , then  $\min_{S_k} (J_1 + J_2)$  is only an approximation of the maximum likelihood detection, and  $G$  becomes only a guide of the actual coding gain. Unfortunately, the maximum of  $G$  over all possible constellations is achieved when  $\alpha_{k,1} \neq \alpha_{k,2}$ . This implies the existence of tradeoff between the value of  $G$  and the optimality of  $\min_{S_k} (J_1 + J_2)$ . The best tradeoff should yield the highest actual coding gain. The YGT code in [3] chooses one extreme of the tradeoff where  $\min_{S_k} (J_1 + J_2)$  is forced to be optimal conditionally upon a chosen set of constellations. This penalizes the value of  $G$ . The best value of  $G$  under  $\alpha_{k,1} = \alpha_{k,2} \neq \text{const}$  can be shown to be 1.33 for 2 bps/Hz and 0.49 for 3 bps/Hz, which is consistent with Fig. 2 in [3]. But for the 4-QAM and 8-QAM constellations (see Figs. 1 and 2) with the rotation angle  $\theta \doteq 0.5 \tan^{-1}(0.5) \approx 13.28^\circ$ , the  $G$  values can be shown to be 1.79 for 2 bps/Hz and 0.60 for 3 bps/Hz. These values are significantly larger than those for the YGT constellation. The rotation angle  $13.28^\circ$  is optimal in maximizing  $G$  under the QAM constellation, which is also shown in [9]. For example, referring to Fig. 2, one can verify that at  $\theta \doteq 0.5 \tan^{-1}(0.5)$ ,

$$\begin{aligned} G & = \left| (\Re(\Delta s_{AB}))^2 - (\Im(\Delta s_{AB}))^2 \right| \\ & = \left| (\Re(\Delta s_{BD}))^2 - (\Im(\Delta s_{BD}))^2 \right| \\ & = \left| (\Re(\Delta s_{CD}))^2 - (\Im(\Delta s_{CD}))^2 \right| \\ & \approx 0.60 \end{aligned}$$

The square-shaped 8-QAM constellation shown in Fig. 2 is not the only 8-QAM constellation that achieves  $G = 0.60$ . In fact, a rectangular-shaped 8-QAM that has 8 symbols on the corners of 3 consecutive squares also has the same  $G$  value under the rotation angle  $13.28^\circ$ . But the corresponding  $\alpha_{k,1}$  and  $\alpha_{k,2}$  are statistically less close to each other, which makes  $\min_{S_k} (J_1 + J_2)$  less optimal. Consequently, the actual coding gain using the rectangular-shaped 8-QAM is not as good as using the square-shaped 8-QAM, which has been confirmed by our simulation.

Note that in general,  $\alpha_{k,1}$  and  $\alpha_{k,2}$  are random variables under different  $k$ . One way to characterize the difference between them is to use statistical distributions. It can be

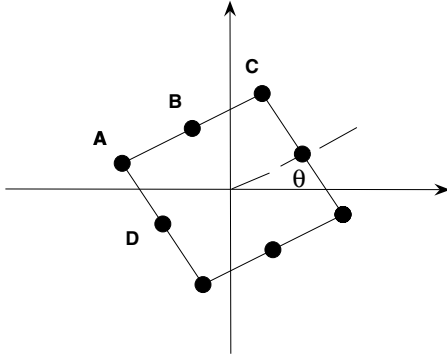


Fig. 2. An 8-QAM constellation with a rotation angle. Here, A, B, C and D indicate the locations of four symbols. The difference between the two symbols at A and B is denoted by  $\Delta s_{AB}$ .

shown that under a random selection of symbols of zero mean,  $\alpha_{k,1}$  and  $\alpha_{k,2}$  have the same distribution. But what is more important for the optimality of  $\min_{S_k}(J_1 + J_2)$  is the difference between  $\sigma_{k,1}^2$  and  $\sigma_{k,2}^2$ . This difference would be small if  $\alpha_{k,1}$  and  $\alpha_{k,2}$  do not change much as  $k$  changes. From the expressions of  $\alpha_{k,1}$  and  $\alpha_{k,2}$  in terms of the original symbols, we can see that if  $m$  is large then  $\alpha_{k,1}$  and  $\alpha_{k,2}$  should be stable and hence  $\sigma_{k,1}^2$  and  $\sigma_{k,2}^2$  should be close to each other. In our simulation, we have observed that even for  $m = 2$ , the distributions of  $\sigma_{k,1}^2$  and  $\sigma_{k,2}^2$  tend to concentrate toward a common constant. This makes  $\min_{S_k}(J_1 + J_2)$  not a bad approximation of the maximum likelihood detection. However, it is worth stressing that the optimality  $\min_{S_k}(J_1 + J_2)$  always depends on the symbol constellation.

It is critical to show that regardless of its optimality,  $\min_{S_k}(J_1 + J_2)$  can be always decomposed into independent symbol-wise decoders. Recall

$$J_i = \|\mathbf{q}_{k,i} - \frac{1}{\sqrt{\alpha_{k-1,i}}} \mathbf{M}_{k,i} \mathbf{q}_{k-1,i}\|^2$$

for  $i = 1, 2$ . Then, it follows that for  $i = 1, 2$ :

$$J_i = \|\mathbf{q}_{k,i}\|^2 - \frac{2}{\sqrt{\alpha_{k-1,i}}} \Re(\mathbf{M}_{k,i} \mathbf{q}_{k-1,i} \mathbf{q}_{k,i}^H) + \frac{(\alpha_{k,i})^{1/l}}{\alpha_{k-1,i}} \|\mathbf{q}_{k-1,i}\|^2 \quad (6)$$

From (1) and (2), we know that  $\alpha_{k,i}^{1/l}$  is a linear superposition of such terms of  $(\Re(s_{k,t}) \pm \Im(s_{k,t}))^2$  for  $t = 1, 2, \dots, 2m$ . We also know that  $\mathbf{M}_{k,i}$  is also a linear superposition of terms of individual symbols  $s_{k,t}$  for  $t = 1, 2, \dots, 2m$ . Therefore,  $J_1 + J_2$  can be straightforwardly decomposed into a sum of symbol-wise cost functions, and each cost function can be minimized independently.

If there are multiple receiving antennas, there is a cost function  $J_1 + J_2$  for each receiving antenna, and the sum of all these cost functions is then equivalent to (exactly if  $\alpha_{k,1} = \alpha_{k,2} = \text{const}$ , nearly if  $\alpha_{k,1} = \alpha_{k,2} \neq \text{const}$ , or approximately if  $\alpha_{k,1} \neq \alpha_{k,2}$ ) the likelihood function of

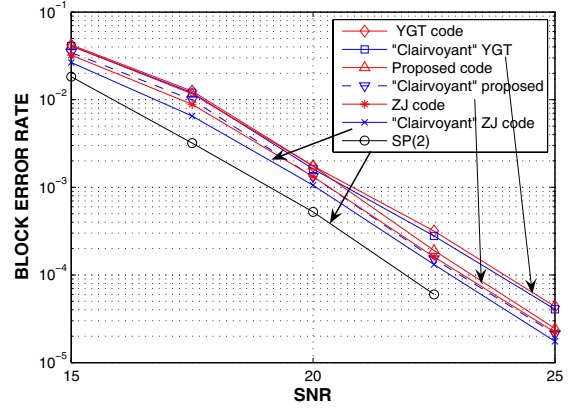


Fig. 3. Block error rates for 2 bps/Hz. In descending order of error rates at SNR=25dB, shown here are the YGT code, clairvoyant YGT, proposed code, its clairvoyant version, the ZJ code, clairvoyant ZJ, and the SP(2) code. The difference between the proposed code and the ZJ code is almost negligible. Note that SP(2) uses a search of size-256, the ZJ code uses a search of size-16, and the proposed code and the YGT code use a search of size-4.

the total received signals. The total cost can be similarly decomposed into symbol-wise cost functions. The total diversity becomes the product of the number of transmitters and the number of receivers.

### III. SIMULATION

We assume four transmitting antennas and one receiving antenna. Each frame consists of 64 blocks and each block has 4 symbols. The channel response (fading) factors are constant during each frame but vary from frame to frame as independent complex Gaussian random variables. The noise is white Gaussian. The SNR of the received signals is computed by  $E(\|\mathbf{X}_k \mathbf{h}\|^2) / E(\|\mathbf{n}_k\|^2)$ .

Fig. 3 shows the block error rates of four codes versus SNR for 2 bps/Hz. The first is the YGT code and its clairvoyant version. The clairvoyant version is such that the symbols at block  $k-1$  are perfectly known when the symbols at block  $k$  are detected. The second is the proposed code and its clairvoyant version. Both the YGT code and the proposed code are decoded in the symbol-wise fashion. The constellation used for the proposed code is the rotated 4-QAM as shown in Fig. 1 with  $\theta = 13.28^\circ$ . The third code is the ZJ code and its clairvoyant version. This code uses pair-wise decoder. The fourth code is the SP(2) code that is decoded by a program provided by authors of [10]. For the 2bps/Hz case, the SP(2) code requires a size-256 search, the ZJ code needs a size-16 search, and both the YGT code and the proposed code need only a size-4 search. But the difference among the coding gains of the four codes, even with different decoding complexities, is not very large. It is important to note however that the proposed code has a consistently better coding gain than the YGT code.

We now consider the quaternion code by Calderbank et al shown in [2]. This code requires size-16 search for 2bps/Hz. The block error rate of the code is not available in [2], but is shown in Fig. 4 for 2bps/Hz - courtesy of S. Das of [2]. We see that when SNR=20dB, the block error rate is about  $9 \times 10^{-4}$  which is better than all above codes except the SP(2).

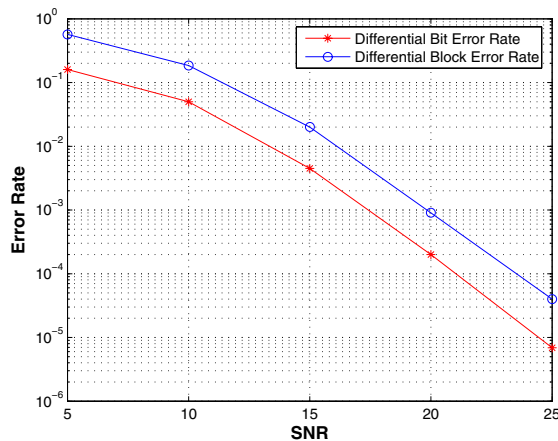


Fig. 4. Bit error rate and block error rate of the quaternion code by Calderbank et al for 2 bps/Hz - Courtesy of S. Das of [2].

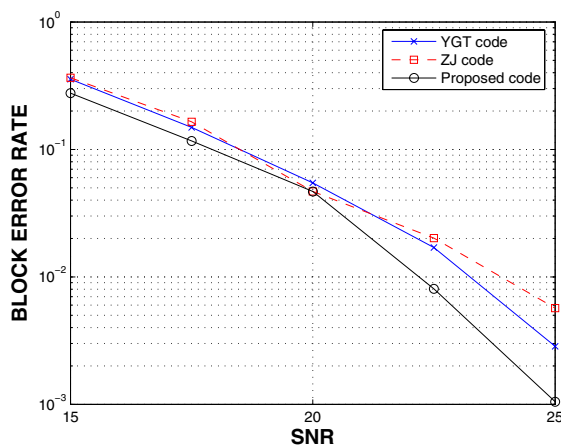


Fig. 5. Block error rates for 3 bps/Hz. The proposed code and the YGT code use a search of size-8, and the ZJ code uses a search of size-64. In this case, the proposed code outperforms both the YGT code and the ZJ code by a significant margin at high SNR.

But when SNR=25dB, the block error rate is about  $4 \times 10^{-5}$  which is nearly the same as the YGT code but worse than all other codes. It appears that the slope of the error curve of the quaternion code is not as steep as others.

Fig. 5 compares the block error rates of the proposed code, the YGT code and the ZJ code for the 3bps/Hz case. For 3bps/Hz, both the proposed code and the YGT code use a size-8 search, and the ZJ code requires a size-64 search. The constellation used for the proposed code is shown in Fig. 2 with the rotation angle  $\theta = 13.28^\circ$ . The constellation used for the ZJ code is the same as Fig. 2 but with  $\theta = \pi/4$  - courtesy of Y. Zhu of [1]. The ZJ code with  $\theta = 13.28^\circ$  did not perform as well as with  $\theta = \pi/4$ . The 8-PSK-constellation was also considered for the ZJ code. But it yielded poorer results. We

see that the block error rate of the proposed code is about one third of that of the YGT code at SNR=25dB. We also see that the ZJ code becomes even worse than the YGT code at SNR=25dB. This is somehow different from the result shown in Fig. 5 in [3].

Finally, we note that only at 1 bps/Hz, the ZJ decoder was shown to be the exact maximum likelihood decoder. For this reason, it seems no surprise that at a higher data rate such as 3 bps/Hz, the ZJ code can perform worse than the proposed code and the YGT code. A bit error rate comparison of the ZJ code and the proposed code was also conducted at 3 bps/Hz, which yielded a similar conclusion.

#### IV. CONCLUSION

Symbol-wise decodable DSTBC (differential space-time block codes) are important for practical applications in wireless communications with multiple transmitters. For four transmitters, the YGT code [3] is symbol-wise decodable and has a coding gain close to the ZJ code [1] that requires pair-wise decoding [1]. The code proposed in this paper is also symbol-wise decodable and outperforms the YGT code by a significant margin. The new code results from a fusion of the YGT code and the ZJ code.

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