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Electron–positron annihilation in strong magnetic fields

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We study the annihilation of free electrons and positrons in intense magnetic fields. In the case both the electron and the positron are in the lowest energy state the resulting gamma rays emerge predominantly transverse to the field direction; in the situation where one of the particles is in an excited spin state the radiation is predominantly along the field direction. We propose that this could be an explanation of a recently observed 545 keV line from the Crab Nebula.

Models of pulsating neutron stars [1,2] require surface magnetic fields $B = (10^{12} - 10^{13})$ G. These regions are also rich in electrons and positrons and in this article we wish to study free electron–positron annihilation in such intense magnetic fields. The usual annihilation of such particles at rest yields two 511 keV gamma rays. Such lines originating near the surface of a neutron star are expected to be gravitationally red shifted by about 10%, or 35 keV (we shall use the Crab pulsar as an example). We shall show that, depending on the alignment of the magnetic field, higher energy gamma rays are expected and propose this as an explanation of the recently reported observation [3] of a 545 keV line from the Crab pulsar.

Electrons and positrons in magnetic fields arrange themselves into degenerate Landau levels [4, p. 67] with quantum numbers n, m (m is the

orbital angular momentum along the magnetic field). The electron spectrum is

$$E_{n,m,s} = \sqrt{M^2 + (2n + 1 + s) |eB|} . \quad (1)$$

M is the mass of the electron and $s = \pm 1$ depending on whether the electron spin is along or opposite to the magnetic field. In the above, and subsequently, we shall ignore motion along the field lines. The energies of positrons in such fields are obtained from eq. (1) by letting $s \rightarrow -s$ in the right hand side of that equation. For electrons the lowest energy state is the one with $n = 0, s = -1$ while the next highest energy state has quantum numbers $n = 0, s = +1$ or $n = 1, s = -1$;

$$E_{0,m,-1} = M,$$

$$E_{0,m,+1} = \sqrt{M^2 + 2 |eB|} ,$$

$$E_{1,m,-1} = \sqrt{M^2 + 2 |eB|} . \quad (2)$$

The energies of the lowest positron levels are obtained by changing the signs of the spins. The expected 511 keV line results from annihilation

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of an electron in the $(0, m_e, +1)$ state with a positron in the $(0, m_p, -1)$ state. We shall show that along the field direction annihilation higher energy lines from states with parallel spins, i.e. from positrons in the $(0, m_p, \pm 1)$ and electrons in the $(0, m_e, \pm 1)$ predominate.

Electron and positron densities are assumed to be such that the Fermi level, ϵ_F , of at least one of the species satisfies

$$M < \epsilon_F \leq \sqrt{M^2 + 2|eB|}; \quad (3)$$

a more detailed discussion of the requisite densities will be given at the end of this article. We see that annihilations into higher energy gamma rays are possible; we still have to determine the directionality of the different lines. To do this we have to examine the details of the annihilation process.

To lowest order in eB/M^2 the annihilation amplitude is [4, p. 230]

$$\begin{aligned} & A(n_p, m_p, s_p; n_e, m_e, s_e) \\ &= \int d^2P_e d^2P_p u^\dagger(s_p) \psi_{n_p, m_p}^*(P_p) \\ & \quad \times \mathcal{M}(P_p, P_e) \psi_{n_e, m_e}(P_e) u(s_e); \end{aligned} \quad (4)$$

the u 's are two component spinors, $\psi_{n,m}(P)$ is the momentum space wave function for the Landau level (n, m) and

$$\begin{aligned} \mathcal{M}(P_p, P_e) = \frac{\sigma_u}{2M^2} & \left[\sigma \cdot \epsilon_1 (\Pi_e - \Pi_p) \cdot \epsilon_2 \right. \\ & \left. + \sigma \cdot \epsilon_2 (\Pi_e - \Pi_p) \cdot \epsilon_1 \right]; \end{aligned} \quad (5)$$

$\Pi_e = P_e - eA$, $\Pi_p = P_p + eA$, where A is the vector potential responsible for the magnetic field. Looking at the rotation (around the B axis) properties of the wave functions one notes that the terms proportional to Π_e connect a positron with $m = 0$ to an electron with $m = 1$ and the terms involving Π_p connect a positron with $m = 1$ to electrons with $m = 0$. Thus in the annihilation process the orbital angular momentum about the magnetic field changes by one unit, $\Delta l = \pm 1$. For the electron and positron spins aligned the change in the total angular momentum component along the magnetic field, j , satisfies $\Delta j = 0, \pm 2$ while for the spins opposite to each other $\Delta j = \pm 1$. In

the first case both photons can emerge along the field direction, while in the second case there must be a photon momentum component transverse to the field. Equations (4) and (5) give explicit distributions consistent with these arguments. The photon angular distributions are:

$$\begin{aligned} d\Gamma(0, 0, +1; 0, 1, -1) \\ &= C \cos^2(\theta) \sin^2(\theta) d \cos(\theta); \\ d\Gamma(0, 0, +1; 0, 1, +1) &= 2C \cos^4(\theta) d \cos(\theta); \end{aligned} \quad (6)$$

the first set of indices refer to the quantum numbers of the positron while the second to those of the electron. θ is the angle of the emitted photon with respect to the magnetic field while C is a constant. The distribution for an annihilation of positrons with $m = 1$ and electrons with $m = 0$ has the same form. For directions along the magnetic field photons from annihilations with electrons whose spin quantum number $s = +1$ will predominate (in a 30° cone around the magnetic field the intensity of this line will be a factor of 14 larger than that for the line coming from annihilations with electrons in the lowest state). Thus if the magnetic field responsible for the splitting points in our direction, lines from annihilation of electrons and positrons with parallel spins will be much stronger than those from antiparallel spins and we will not see transitions with electrons in the ground state.

As mentioned earlier we would like to suggest that the recently reported [3] 545 keV line from the Crab pulsar could be due to the mechanism presented. With a 10% gravitational red shift added the energy at the annihilation site is really 600 keV. For no gravitational red shift we require a field $B = 6 \times 10^{12}$ G; with a gravitational red shift a field $B = 1.6 \times 10^{13}$ G would be needed. Fields of such magnitude are consistent with those postulated to exist on the surface of neutron stars. Transitions between Landau levels should also occur. These are the same as the cyclotron radiation observed for other systems [5]. For the present case these should be at 35 keV for the no red shift case and 80 keV for a 10% red shift. A simultaneous measurement of annihilation lines

and cyclotron lines will determine both the magnetic field and M_P/R_P of the pulsar.

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