Title
How to Make the Impossible Seem Possible

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How to Make the Impossible Seem Possible

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Abstract

The mental model theory postulates that reasoners build models of the situations described in premises. A conclusion is possible if it occurs in at least one model; and it is impossible if it occurs in no models. According to the theory, reasoners can cope with what is true, but not with what is false. A computer implementation predicted that certain inferences should yield cognitive illusions, i.e. they have conclusions that should seem highly plausible but that are in reality gross errors. Experiment 1 showed that, as predicted, participants erroneously inferred that impossible situations were possible, and that possible situations were impossible, but they performed well with control problems. Experiment 2 replicated these results, using the same premises for both the illusory and the control inferences: the participants were susceptible both to illusions of possibility and to illusions of impossibility, but they coped with the control problems.

Introduction

Consider the following problem:

Only one of the following premises is true:

There is a 9 in the hand or there is a 10, or both.

There is a Queen in the hand or there is a 10, or both.

There is a 6 in the hand or there is an Ace, or both.

Is it possible that there is a 10 in the hand?

Almost everyone answers, 'yes'. The inference is an illusion of possibility, because in fact it is impossible for there to be a 10 in the hand. (If there were, then two of the premises would be true, but the problem specifies that only one premise is true.) Such illusory inferences are predicted by the theory of mental models. Previous studies have corroborated the occurrence of illusions in inferences leading to necessary conclusions and to probable conclusions (see Johnson-Laird and Savary, 1996). However, an important sort of reasoning in daily life concerns figuring out what is possible. Such 'modal' reasoning has been formalized by logicians in many different calculi (see Hughes and Cresswell, 1996) but it has seldom been studied by psychologists (though cf. Osherson, 1976; Johnson-Laird and Bell 1997). Hence, we examined illusory inferences about possibilities. One advantage of this domain is that the same premises can be used to elicit both illusory inferences and control inferences, and thereby eliminate the hypothesis that illusory premises are somehow too difficult for logically untrained individuals to understand. Our plan in what follows is to outline the theory of mental models, and then to report two studies that confirmed the occurrence of illusions in inferences about what is possible and what is impossible.

The Mental Model Theory of Reasoning

The theory of mental models (see e.g. Johnson-Laird and Byrne, 1991) postulates that reasoning -- deductive or inductive -- is a process in which reasoners represent the truth conditions of premises, and then use this representation together with their semantic and general knowledge to construct mental models of the relevant situations. These models may take the form of visual images, but their critical feature is their structure. The theory's fundamental representational principle concerns truth:

People represent situations by constructing sets of mental models in which each model represents what is true in a true possibility.

In this way, they limit the load on their working memories. The principle is subtle, however, because it applies at two levels. First, individuals represent only true possibilities; second, they represent those literal propositions in the premises -- affirmative or negative -- that are true in the true possibilities. For example, an exclusive disjunction:

There is a king or there is an ace, but not both

has two mental models that represent the true possibilities:

```
King
        Ace
```

where each row denotes a model of an alternative possibility. Each model represents only what is true in a particular possibility. Hence, for the first alternative, the model represents that it is true that there is a king. It is also false that there is a ace, but models do not usually represent this information explicitly. Similarly, for the second alternative, the model represents that there is an ace, but it does not represent that it is false that there is a king. The theory postulates that reasoners make 'mental footnotes' to keep track of this information, but that these footnotes are soon likely to be forgotten. Johnson-Laird and Byrne (1991) used square brackets as a special notation to denote these mental footnotes, but we will forego this notation here.

In contrast to mental models, fully explicit models of the exclusive disjunction represent the false components in each possibility, using negations that are true:

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King   ¬ Ace
¬ King   Ace
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where '¬' denotes negation. These two fully explicit models match the two rows that are true in a truth table of the assertion.

The theory gives an analogous account of all the other major sentential connectives. Thus, a conditional:

If there is a king then there is an ace

has an explicit model of the possibility that both cards are in the hand, but reasoners defer a detailed representation of the case where the antecedent proposition is false, which they represent in a wholly implicit model denoted here by an ellipsis:

King   Ace

. . .

Reasoners should make a mental footnote that hands in which a king occurs are exhaustively represented in the explicit model, and so a king cannot occur in the hands represented by the implicit model. Since hands containing an ace are not exhausted in the explicit model, they may, or may not, occur in the hands represented by the implicit model. The mental models for a biconditional, e,g, 'If, and only if, there is a king then there is an ace', are the same, but the footnotes establish that neither kings nor aces can occur in the possibilities represented by the implicit model.

Table 1 summarizes the mental models and the fully explicit models for the major sentential connectives. The fully explicit models correspond one-to-one with the true entries in a truth table for a connective, but mental models represent only those literal propositions in premises -- affirmative or negative -- that are true. If a conclusion holds in all the models of the premises, it is necessary; if it holds in most of the models, it is probable; and if it holds in at least one model, it is possible.

Previous studies have corroborated the predictions of the model theory about inferences yielding both necessary and probabilistic conclusions. In particular, they have shown that the greater the number of models that have to be constructed to draw a necessary conclusion, the harder the

Table 1: The mental models and the fully explicit models for the sentential connectives: the fully explicit models represent the false components of the true possibilities by using negations that are true: '¬' denotes negation and '. . .' denotes a wholly implicit model.

Connective	Mental models		Fully Explicit models	
A and B:	A	B	A	B
A or else B:	A		A	¬ B
		B	¬ A	B
A or B, or both:	A		A	¬ B
		B	¬ A	B
	A	B	A	B
If A then B:	A	B	A	B
	. . .		¬ A	B
			¬ A	¬ B
If, and only if, A then B	A	B	A	B
	.		¬ A	¬ B

task is -- it takes longer and is more prone to error (see e.g. Johnson-Laird and Byrne, 1991). They have corroborated the model theory's prediction of an interaction in reasoning about possibilities and necessities (Johnson-Laird and Bell, 1997): individuals are faster and more accurate in inferring that a situation is possible (one model suffices) than that it is necessary (all models must satisfy the conclusion), but they are faster and more accurate in inferring that a situation is not necessary (one model suffices) that that it is necessary (all models must satisfy the conclusion).

Illusory Inferences About Possibilities

The computer program implementing the theory predicted the occurrence of a novel category of inferences. Their mental models support a conclusion that differs from the correct conclusion supported by fully explicit models, e.g.:

Only one of the following premises is true:

There is a 9 in the hand or there is a 10, or both.

There is a Queen in the hand or there is a 10, or both.

There is a 6 in the hand or there is an Ace, or both.

Because mental models represent only what is true, the mental models for the first premise are as follows:

9	
	10
9	10

The mental models for the second premise are:

Queen	
	10
Queen	10

And the mental models for the third premise are:

6	
	Ace
6	Ace

An exclusive disjunction, as conveyed by 'Only one of the following premises is true', calls for a list of all the true possibilities (see Table 1). Hence, the question:

Is it possible that there is a 10 in the hand?

should be answered affirmatively, because there are four models containing a 10. The response is an illusion of possibility, because the fully explicit models show it to be false. They take into account that when one premise is true, the other two premises are false. Hence, if the first premise is true, there is only the following model:

9	¬ Queen	¬ 10	¬ 6	¬ Ace

If the second premise is true, there is only the model:

¬ 9	Queen	¬ 10	¬ 6	¬ Ace

And if the third premise is true, there are the following models:

¬ 9	¬ Queen	¬ 10	6	¬ Ace
¬ 9	¬ Queen	¬ 10	¬ 6	Ace
¬ 9	¬ Queen	¬ 10	6	Ace

Hence, it is impossible for there to be a 10. A comparable control problem is:

Only one of the following premises is true:

There is a 9 in the hand or there is a Jack, or both.

There is a Queen in the hand or there is a 10, or both.

There is a 6 in the hand or there is an Ace, or both.

Is it possible that there is a 10 in the hand?

The mental models again consist of a list of all the true possibilities. They include models containing a 10, and so the question should be answered, 'yes'. The fully explicit models support the same conclusion, and so it is correct.

The following problem should create an illusion of impossibility:

Only one of the following premises is true:

There is an ace in the hand or there is not a 5, or both.

There is a king in the hand or there is not a 5, or both.

Is it possible that there is a 5 in the hand?

The mental models are as follows:

Ace		¬ 5
	King	¬ 5

which yield the answer 'no'. But the fully explicit models show that this response is an illusion:

Ace	¬ King	5
¬ Ace	King	5

A comparable control problem is based on conjunctions:

Both of the following premises are true:

There is an ace in the hand and there is not a 5.

There is a king in the hand and there is not a 5.

Is it possible that there is a 5 in the hand?

Its mental models support the correct response, 'No'.

In Experiment 1, we gave 20 Princeton students four inferences of each of the four sorts: illusions of possibility and their controls, and illusions of impossibility and their controls. Half the illusory problems were based on disjunctions, and half were based on conditionals, e.g.:

Only one of the following premises is true:

If there is a 9 in the hand then there is a 10.

If there is a Queen in the hand then there is a 10.

If there is a 6 in the hand then there is an Ace.

Is it possible that there is a 10 in the hand?

Likewise, half the controls for the illusions of possibility were based on disjunctions and half were based on conditionals. Half the control problems for illusions of impossibility, however, were based on conjunctions and half were based on conditionals. The 16 problems were presented in a different random order to each participant, and each problem was based on a different set of cards. Participants rated their confidence in each of their answers on a five-point scale (1 = totally unconfident, and 5 = totally confident).

Table 2 presents the results. The participants were correct on 91% of the control inferences, but only on 15% of the illusory inferences. All 20 of the participants were more accurate with the control inferences than with the illusory ones ($p = .5^{20}$, i.e. less than 1 in a million). There was no reliable difference in the confidence ratings for the illusory and control problems. One unexpected finding, however, was that the illusions of impossibility yielded correct conclusion more often than illusions of possibility (Wilcoxon test, $z = 3.697$, $p < .001$). One plausible explanation is that the phrase 'or both' suggested to the participants that both the cards mentioned in the premise were present in the hand.

Table 2: The percentages of correct responses to the four sorts of problems in Experiment 1. The figures in parentheses are the participants' mean confidence in their answers (on a five-point scale).

	Illusions	Controls
Illusions of possibility	1 (4.17)	90 (4.13)
Illusions of impossibility	29 (4.23)	92 (4.35)

In general, the results corroborated the model theory. Nearly everyone made the same illusory responses, which seemed obvious, and yet which were totally wrong. The main weakness of the experiment is that the illusions and control problems depended on different premises, and that half the controls for the illusions of impossibility depended on conjunctions, whereas the corresponding illusions depended on disjunctions. Hence, skeptics could argue that there was something especially difficult about the illusory premises -- that they somehow misled the participants in ways that did not occur with the control premises. Our second experiment was designed to correct these flaws.

In Experiment 2, we used the same premises for the four sorts of inferences: illusions of possibility, their controls with the correct answer 'yes', illusions of impossibility, and their controls with the correct answer 'no'. For example, the following statement:

One of the following premises is true and one is false:

There is a king in the hand, or an ace, or both.

There is a queen in the hand and there is an ace.

was combined on separate trials (with different contents) with the following questions:

Is it possible that there is a queen in the hand and an ace?

Is it possible that there is a king in the hand and an ace?

Is it possible that there is a queen in the hand and not an ace?

Is it possible that there is not a king in the hand and not an ace?

Table 3 summarizes all the problems in Experiment 2, and, as it shows, the mental models of the premises above (see item 3 in the Table) should yield an illusion of possibility to the first question, i.e. the participants should respond 'yes', but the fully explicit models show that it is impossible for there to be both a queen and an ace in the hand. The second question should also elicit the answer 'yes', but this response is supported by the fully explicit models, and so it is correct. Likewise, the third question should elicit an illusion of impossibility, and the fourth question should elicit a correct 'no' answer.

We gave 20 students the 16 inferences in a different random order to each of them. Each problem (unlike the one above) was based on a different set of cards. The participants also rated their confidence in each of their answers.

The percentages of correct responses to each of the problems are shown in Table 3, and the results are summarized in Table 4. The participants were correct on 79% of the control inferences, but only on 22% of the illusory inferences. 19 out of the 20 participants were more accurate with the control inferences than with the illusory ones, and one participant yielded the opposite results (Sign test, $p < .0005$). There was no reliable difference in the confidence ratings for the illusory and control problems. As in Experiment 1, the illusions of impossibility were less

Table 3: The problems in Experiment 2, their mental and their fully explicit models. Each pair of premises was prefaced by: 'One of the following premises is true and one is false'. The questions concerned possibilities, and the table shows the percentage of correct answers to each of them.

Form of premises and questions	Mental models	Fully explicit models
1. If king then ace	K A	¬K Q ¬A
If queen then ace	Q A	K ¬Q ¬A
	. . .	
king and ace?	5 (Illusion of possibility)	
king and not queen?	75 (Control 'yes' response)	
king and not ace?	25 (Illusion of impossibility)	
not queen and not king?	80 (Control 'no' response)	
2. If king then not ace.	K ¬A	¬K Q A
If queen then not ace.	Q ¬A	K ¬Q A
king and not ace?	0 (Illusion of possibility)	
queen and not king?	95 (Control 'yes' response)	
king and ace?	50 (Illusion of impossibility)	
not king and not queen?	80 (Control 'no' response)	
3. King or ace, or both.	K	K ¬Q A
Queen and ace.	A	K Q ¬A
	K A	K ¬Q ¬A
	Q A	¬K ¬Q A
queen and ace?	30 (Illusion of possibility)	
king and ace?	95 (Control 'yes' response)	
queen and not ace?	40 (Illusion of impossibility)	
not king and not ace?	95 (Control 'no' response)	
4. Not king or not ace,	¬K	¬K Q A
or neither.	¬A	¬K ¬Q A
Not queen and not ace.	¬K ¬A	K Q ¬A
	¬Q ¬A	¬K Q ¬A
not queen and not ace?	15 (Illusion of possibility)	
not king and not ace?	75 (Control 'yes' response)	
not a queen and an ace?	55 (Illusion of impossibility)	
king and ace?	90 (Control 'no' response)	

effective than the illusions of possibility (Wilcoxon test, $z = 3.03$, $p < .005$). In this case, the potential explanation that we mentioned earlier is inappropriate. Instead, if a question has one conjunct represented in a mental model, people are more likely to infer that the situation is possible.

Table 4: The percentages of correct responses to the problems in Experiment 2. The figures in parentheses are the mean confidence ratings (on a five-point scale).

	Illusions	Controls
Illusions of possibility	8 (4.41)	85 (4.25)
Illusions of impossibility	36 (3.99)	73 (4.23)

Conclusions

Our experiments have confirmed that illusory inferences occur in modal reasoning, i.e. reasoning about possibilities. Previous results have shown that illusions also occur in inferences yielding logically necessary conclusions and probabilistic conclusions (Johnson-Laird and Savary, 1996). One skeptical reaction to these earlier studies was that the premises eliciting illusions must be too complicated for logically-untrained individuals to understand -- even though, as we have argued, the participants were equally confident in drawing correct conclusions to comparable control problems. The results of Experiment 2 are a decisive refutation of the skeptical view, because the same premises were used to elicit both the illusions and control inferences. All that differs are the particular questions, and, as Table 3 shows, there is no overall difference between the questions eliciting illusions and those eliciting control responses.

Reasoners are susceptible to both illusions of possibility and illusions of impossibility. These illusions are predicted by the model theory, but they refute current theories of reasoning based on formal rules of inference (see e.g. Osherson, 1976; Rips, 1994), which contain only rules that yield valid conclusions. These theories therefore have no way to explain the fact that reasoners draw one and the same invalid conclusion to certain premises. The illusions seem to arise because reasoners can cope with truth, but not with falsity. To rely only on truth is a sensible adaptation to limited processing capacity. In some cases, however, it leads reasoners into the illusion that they grasp a set of possibilities that is in fact beyond them.

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