Title
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Journal
Journal of Water Resources Planning and Management, 142(1)

ISSN
0733-9496

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Publication Date
2016

DOI
10.1061/(asce)wr.1943-5452.0000558

Peer reviewed

Biogeography-Based Optimization Algorithm for Optimal Operation of Reservoir Systems

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Abstract: The optimal operation of reservoir systems to meet water demand is a complex and nonlinear problem. This paper applies the biogeography-based optimization (BBO) algorithm to solve reservoir operation problems. The BBO algorithm is first verified with the minimization of three mathematical benchmark functions (Sphere, Rosenbrock, and Bukin6). In addition, the BBO algorithm was applied to a single reservoir system and a four-reservoir system. The performance of the BBO algorithm was compared with that of the genetic algorithm (GA) in solving the three optimization problems. The results show that the BBO algorithm minimized the benchmark functions accurately, and outperformed the GA in this respect. In the case of the single-reservoir hydropower optimization problem the BBO reached a near-optimal solution. The values of the objective function averaged 1.228 and 1.746 with the BBO and GA, respectively. The global solution of this problem with the nonlinear programming method equals 1.213. In the four-reservoir system application the BBO converged to 99.94% of the optimal solution in its best-performing history, whereas the GA converged to 97.46% of the optimal solution. The results from the three test problems demonstrated the superior capacity of the BBO to optimize general mathematical problems and the operation of reservoir systems. **DOI:** 10.1061/(ASCE)WR.1943-5452.0000558. *© 2015 American Society of Civil Engineers.*

Author keywords: Biogeography-based optimization; Genetic algorithm; Optimization; Reservoirs; Nonlinear programming.

Introduction

The scarcity of precipitation, whether by climatic variability or climatic change, exerts enormous stress on water resources systems (Bou-Zeid and El-Fadel 2002). That stress is compounded by population growth and the need for enlarged food production. Within this context, it is a high priority to develop effective methods to optimally allocate scarce water resources. Reservoir operation for water and energy production is one such category of methods. Reservoir operation methodology can be categorized into two main groups. The first group consists of classical methods, which include linear programming (LP), dynamic programming (DP), stochastic dynamic programming (SDP), and nonlinear programming (NLP). The second group consists of evolutionary algorithms (EAs), among which the genetic algorithm (GA) is one of the most popular.

An early paper by Revelle et al. (1969) employed LP to derive monthly operation rules from a single reservoir system using linear rules of operation for each month. Mousavi et al. (2005) derived fuzzy operation rules with DP and based on fuzzy logic for the Karun and Dez reservoirs in Iran. Their results showed that the calculated operation rules modeled reservoir uncertainty properly. Karamouz and Houck (1987) compared DP and SDP models used for optimal reservoir operation. They applied DP and SDP to the Gunpowder, Osage, and Blacksmith River reservoirs. Their results showed that DP performed well for reservoirs with medium and large capacity, while SDP proved superior for the operation of small reservoirs. Zhao et al. (2014) applied the improved DP to optimal operation of a hydropower reservoir in China.

Recently, many optimization techniques have been developed and applied in several fields of water resources systems analysis such as reservoir operation (Bozorg Haddad et al. 2011a; Fallah-Mehdipour et al. 2011b, 2012a, 2013a; Taghian et al. 2013; Galelli et al. 2014; Schardong and Simonovic 2015; Mendes et al. 2015), hydrology (Orouji et al. 2013; Cho and Olivera 2012), project management (Bozorg Haddad et al. 2010b; Fallah-Mehdipour et al. 2012b), cultivation rules (Bozorg Haddad et al. 2009; Noory et al. 2012; Fallah-Mehdipour et al. 2013b), pumping scheduling (Bozorg Haddad et al. 2011b), hydraulic structures (Bozorg Haddad et al. 2010a), water distribution networks (Bozorg Haddad et al. 2008a; Fallah-Mehdipour et al. 2011a; Seifollahi-Aghmiuni et al. 2011, 2013; Mala-Jetmarova et al. 2015; Odan et al. 2015), operation of aquifer systems (McPhee and Yeh 2004; Bozorg Haddad and Mariño 2011; Farmer et al. 2015), site selection of infrastructures (Karimi-Hosseini et al. 2011), and algorithmic developments (Shokri et al. 2013). Only a few of these works dealt with the application of the biogeography-based optimization (BBO) algorithm in water resources systems and especially for optimizing the operation of reservoir systems.

EAs have been widely used in reservoir operation. Kumar and Reddy (2006) compared the performance of ant colony optimization (ACO) with GA in the operation of the Hirakud reservoir in India with agricultural, hydropower, and flood control functions. Their results revealed that ACO performance was better than that of GA in terms of accuracy and computational speed. Jothiprakash and Shanthi (2006) derived optimal operation policies using GA

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Note. This manuscript was submitted on November 19, 2014; approved on April 17, 2015; published online on June 11, 2015. Discussion period open until November 11, 2015; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Water Resources Planning and Management***, © ASCE, ISSN 0733-9496/04015034(11)/$25.00.

for the Pechiparai reservoir in India considering different reliabilities. They performed a sensitivity analysis of the GA parameters. The results showed that the amount of crossover is a key factor in improving GA performance. Afshar et al. (2007) employed honeybee mating optimization (HBMO) in optimal operation of the Dez reservoir. They asserted that HBMO approximated the global optimal solution with high accuracy.

Bozorg Haddad et al. (2008b) applied HBMO for operation and design of one reservoir and a multiple reservoir system with the aim of minimizing the costs of operation. They used the NLP method along with HBMO to solve the problem. Their results demonstrated that the NLP method could not solve the problem of multiple reservoir system operation, whereas HBMO converged to the optimal solution. Chang and Chang (2009) conducted a study of the multiobjective operation of the Feitsui and Shihmen two-reservoir system in China using the nondominated sorting genetic algorithm II (NSGA-II). The results showed that the multiobjective operation of these reservoirs reduced water shortages by 10%. Zahraie and Hosseini (2009) implemented optimal operation of the Zayandeh-Rud River reservoir in Iran using GA. After determining optimal outputs, classic and fuzzy regressions were utilized for derivation of operation rules. They used symmetric and asymmetric membership functions in the fuzzy regressions. Their results indicated that the fuzzy model with asymmetric membership function achieved better performance than the two other models. Fallah-Mehdipour et al. (2012a) used genetic programming (GP) and GA in developing real-time operational rules for the Karaj Dam in Iran. They concluded that operational rules achieved with GP improved the objective function value by 12.39% compared to GA. Zhang et al. (2012) used the elite-guide particle swarm optimization (EGPSO) algorithm for optimal operation of a hydropower system of the Yangtze River, China. Bozorg-Haddad et al. (2014a) demonstrated the superiority of EAs, such as the bat algorithm (BA), relative to GA in the optimal operation of reservoirs.

Based on the *no free lunch* theorem it is impossible for one EA to optimally solve all optimizing problems, and an EA can be considered as the most appropriate approach to solve an optimizing problem only when it is developed particularly for that problem (Wolpert and Macready 1997). Hence, new EAs have been introduced to deal with specific optimization engineering problems. BBO is a relatively new algorithm whose high performance has been reported in several fields, e.g., power systems (Jamuna and Swarup 2011), optimization of benchmark functions (Simon et al. 2011), and heat exchanger optimization design (Hadidi and Nazari 2013).

Reservoirs are one of the principal resources for water supply in many basins. Optimal operation of reservoir systems can considerably improve regional water management. Reservoir operation is a difficult optimization problem that typically involves nonlinear multiobjective functions and multiple constraints. Water resources analysts are continuously searching for suitable reservoir-operation optimization techniques. EAs are popular algorithms for reservoir operation optimization given their proven computational speed and convergence properties. There have not been previous applications of BBO in water resources management. Due to the fact that powerful EAs have performed successfully in various fields of water resources management, an application of BBO seems appropriate and timely. In so doing, the first detailed results of the aforementioned algorithm are presented to water resources scientists to encourage future novel applications. In addition, this paper compares BBO's and GA's performances in the solution of reservoir operation problems.

Methodology

Biogeography-Based Optimization: Theoretical Principles

Biogeography is the study of the geographical distribution of living organisms. Mathematical biogeographic models attempt to explain how species migrate between habitats, their appearance, adaptation, evolution, and extinction. The habitats that are more suitable places for species settlement have a relatively high habitat suitability index (HSI). The HSI depends on factors such as vegetative cover, precipitation, area, temperature, etc. Variables that determine the quality of habitat are known as suitability index variables (SIV). SIVs are independent variables and HSI is variable dependent on SIVs. Habitats with large values of HSI accommodate more species, and, conversely, a low-HSI habitat supports fewer species. As the number of species in a habitat increases there is a stronger tendency for species to emigrate from the habitat to find new ones with better life-supporting conditions and lower population density than the crowded habitats. Habitats with low population density may attract immigration provided that the habitat has adequate life-supporting characteristics. Fig. 1 illustrates the effect that the number of species has on the immigration rate (λ) and emigration rate (μ).

According to Fig. 1, the maximum rate of immigration to the habitat occurs when there are not species in it. As the number of species in the habitat increases, the rate of immigration decreases. The rate of immigration becomes nil when the number of species in the habitat equals $S_{\max}$. The rate of emigration increases as the number of species in a habitat increases, starting with zero emigration for an empty habitat. The maximal rates of immigration and emigration are identified by I and E, respectively. The equilibrium number of species occurs when the immigration rate and emigration rate are equal to each other. S_0 indicates the equilibrium point.

Letting $n = S_{\max}$, the rates of emigration (μ) and immigration (λ) are expressed in terms of the number S of species in the habitat in the following form:

$$\mu = E\frac{S}{n} \tag{1}$$

$$\lambda = I\left(1 - \frac{S}{n}\right) \tag{2}$$

In the particular condition ($E = I$) then $\mu + \lambda = E = I$.

Applied Principles

Simon (2008) introduced the BBO algorithm utilizing biogeographic concepts. It is assumed that there is a problem to be solved

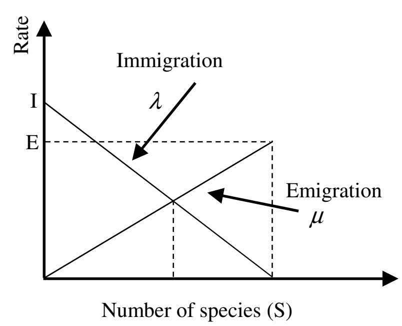

Fig. 1. Species immigration and emigration pattern in a habitat; I and E are maximal immigration and emigration rates, respectively

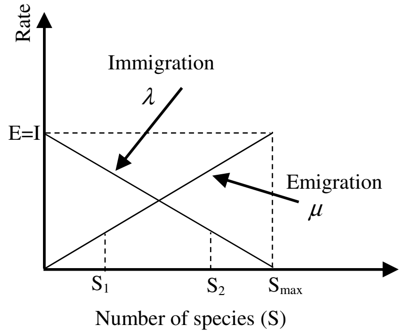

Fig. 2. Comparison of two solutions for one problem; $S1$ has a low HSI, and $S2$ has a high HSI

and a number of possible solutions. Each solution can be considered as a habitat in which the decision variables are the SIVs (habitats act as chromosomes and SIVs are similar to genes in GA). As previously pointed out, the SIVs determine the HSI in a habitat: the greater the HSI, the more suitable the habitat is. In fact, the HSI plays the role of objective function in the BBO algorithm. If for each solution (habitat) there is a specific graph with $E = I$, such as shown in Fig. 2, the number of species (S) has a direct relationship with HSI, in which case can use HSI values instead of S. In Fig. 2, S_1 is a solution with low HSI, and S_2 represents a high-HSI solution. S_1 represents a habitat with few species, while S_2 denotes a habitat with numerous species. The λ_1 associated with S_1 is larger than the λ_2 corresponding to S_2. μ_1 for S_1 is smaller than μ_2 for S_2.

With a specific probability P_{mod}, each solution can be improved by another solution. If the S_i solution is chosen as an improvement, the immigration rate λ is used to modify its SIVs. After selecting the SIVs to be modified, the emigration rate μ relevant to other solutions is used to select the improved solution. SIVs from chosen solutions are randomly replaced with the SIVs of the S_i solution. The appropriate values for μ can be considered arbitrary by using an arithmetic progression between 0 and 1, with the common difference of successive members equal to $1/(N_{\mathrm{pop}} - 1)$, where N_{pop} = population size. After evaluation of μ, λ can be calculated as $\lambda = 1 - \mu$.

In the absence of elitism all solutions are modified at all stages. Yet, the modification amount of any solution is inversely related to its HSI. Selecting the modifier solution is based on a probability proportional to the emigration rate using a roulette wheel for this purpose. Transferring SIVs from one solution to another one is an inferior strategy because it limits the search options within the decision space. Thus, it is recommendable to use the following equation for replacing SIVs:

$$\mathrm{SIV}_{i,k}^{\mathrm{new}} = \mathrm{SIV}_{i,k} + \alpha(\mathrm{SIV}_{j,k} - \mathrm{SIV}_{i,k}) \qquad (3)$$

where $\mathrm{SIV}_{i,k}^{\mathrm{new}}$ = kth modified SIV of the ith solution; $\mathrm{SIV}_{i,k}$ = kth SIV of the ith solution (modified solution); $\mathrm{SIV}_{j,k}$ = kth SIV of the jth solution (modifier solution); and α = parameter between 0 and 1, which is specified by the user.

Serious disasters such as spreading of infectious diseases, natural hazards, and other disasters can rapidly change the HSI of a habitat. Therefore, the condition of a habitat changes from adequate to inadequate, in a manner similar to mutations in GA. The mutation can be exerted on SIVs after migration based on a probability distribution such as the Gaussian distribution or the uniform distribution. Fig. 3 illustrates the BBO algorithm flowchart.

Verification of BBO Algorithm with Benchmark Mathematical Functions

Three benchmark mathematical functions are used in this section to test the BBO's ability to find global minima. The benchmark functions are the Sphere, Rosenbrock, and Bukin6 functions described respectively by Eqs. (4)–(6). In fact, functions with unique characteristics, such as the three chosen in this paper, are required. For instance, Sphere is a high-dimensional function, while Bukin6 is a function with many local optimums. The global minimum of the Sphere function is located at the origin of coordinates that is equal to zero. In this study the Sphere function was considered as 20 dimensional. The Rosenbrock and Bukin6 were set as two-dimensional functions. The minimum of the Rosenbrock function is equal to zero and is placed at the point (1, 1). The minimum of the Bukin6 function is also zero, corresponding to the point (−10, 1). Fig. 4 depicts the benchmark functions in three-dimensional space

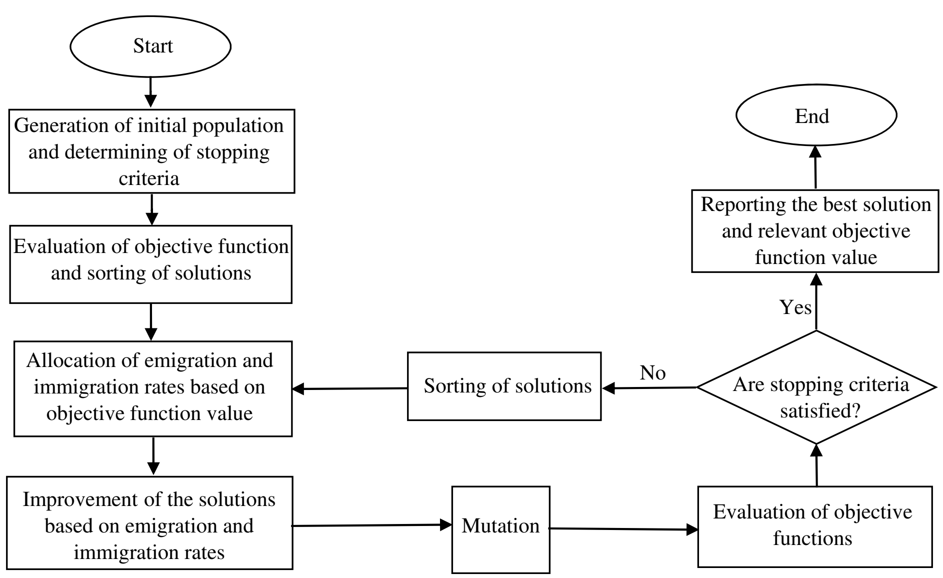

Fig. 3. Flowchart of the BBO algorithm

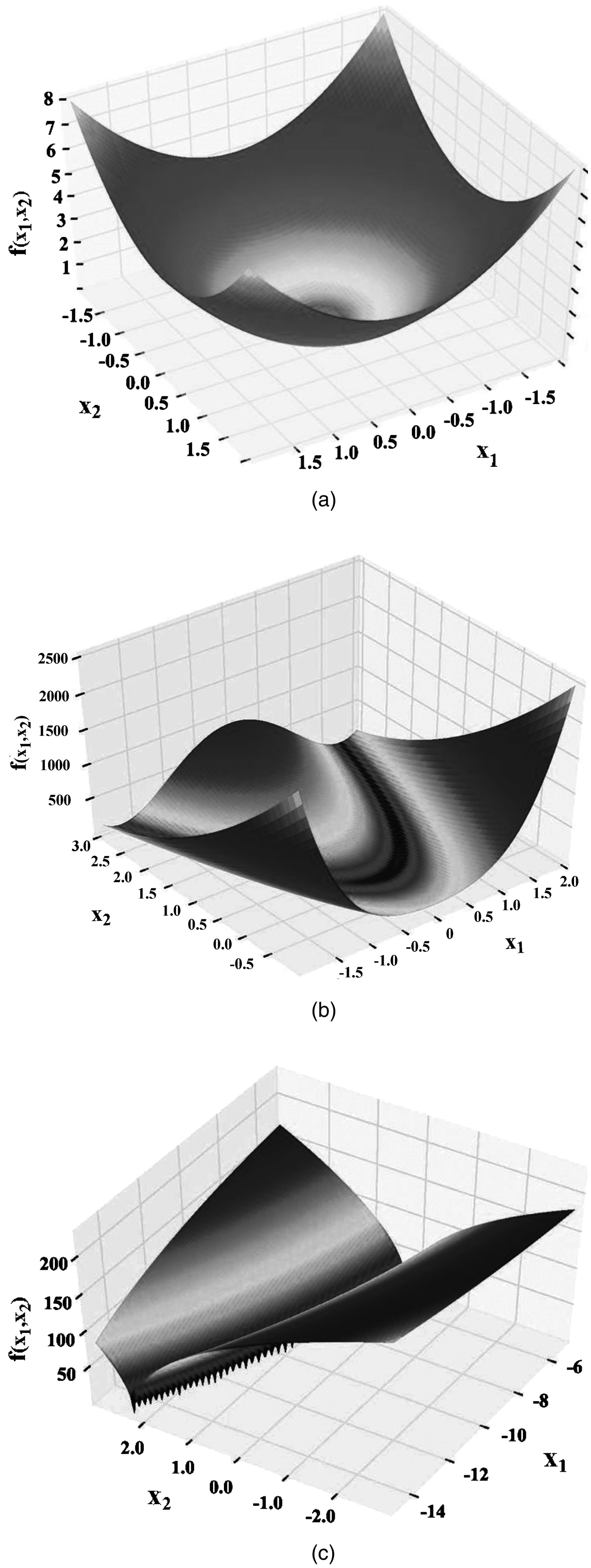

Fig. 4. Functions in three-dimensional space: (a) Sphere; (b) Rosenbrock; (c) Bukin6

$$N = 20 - 5.12 \le x_i \le 5.12 \text{ Sphere } f(x) = \sum_{i=1}^{n} x_i^2 \tag{4}$$

$$N = 2 - 2.048 \le x_i \le 2.048 \text{ Rosenbrock}$$
$$f(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2] \tag{5}$$

$$N = 2 \begin{array}{c} -15 \le x_1 \le -5 \\ -3 \le x_2 \le 3 \end{array} \text{ Bukin6}$$
$$f(x_1, x_2) = 100\sqrt{|x_2 - 0.01x_1^2|} + 0.01|x_1 + 10| \tag{6}$$

The results of BBO for benchmark functions were compared to the results of GA for the same functions. The comparison indicated better performance of BBO than GA in converging to the global optimal solution. The number of objective function evaluations for all three functions and the two-optimization algorithm was approximately 9,000, and the number of starting random populations was 10. The best parameters of BBO and GA were determined by trial and error. Ten runs were carried out for each function in the comparison given that these EAs require random starting populations. The 10 runs, each with a different random population, allow the assessment and comparison of the EAs' solution performances statistically, as shown in the results section. Fig. 5 displays the BBO convergence for the best and worst performance, as well as the BBO and GA convergences at their average performances. The vertical axes of Fig. 5 define the logarithm of the objective functions for the purpose of illustrating the differences between the GA and BBO results more clearly.

Table 1 lists the objective function values for 10 runs of BBO and GA corresponding to the three benchmark functions. The BBO's performance was superior in all runs. In all cases even the worst performance of BBO was better than the best performance of GA. In the majority of situations, BBO reached the global optimum solution with high precision. The best objective function values obtained with BBO for the Sphere, Rosenbrock, and Bukin6 functions were equal to 4.2×10^{-8}, 3.6×10^{-9}, and 3.3×10^{-5}, respectively. The GA converged to 4.6×10^{-2}, 1.1×10^{-5}, and 2.3×10^{-2} for the same functions.

Reservoir Operation Model

The operation of reservoirs is based on the continuity equation. The continuity equation establishes the water balance in each period of reservoir operation. Eq. (7) expresses the continuity equation for an n-reservoir system

$$S_{(i,t+1)} = S_{(i,t)} + Q_{(i,t)} + M_{(i,j)} \cdot R_{(j,t)} + M_{(i,j)} \cdot Sp_{(j,t)} - \text{Loss}_{(i,t)}$$
$$\text{for } j = 1, \ldots, ni = 1, \ldots, nt = 1, \ldots, T \tag{7}$$

where t = number of given periods; i = reservoir number; $S_{(i,t)}$ and $S_{(i,t+1)}$ = the storages of ith reservoir, respectively, at the beginning and end of period t; $Q_{(i,t)}$ = inflow volume into ith reservoir during period t; $M_{(i,j)}$ = matrix of input-output connectivity among reservoirs; $R_{(j,t)}$ = release volume from jth reservoir during period t; $Sp_{(j,t)}$ = overflow volume from jth reservoir during period t; $\text{Loss}_{(i,t)}$ = evaporation loss from ith reservoir surface during period; n = number of reservoirs; and T = total number of operation periods.

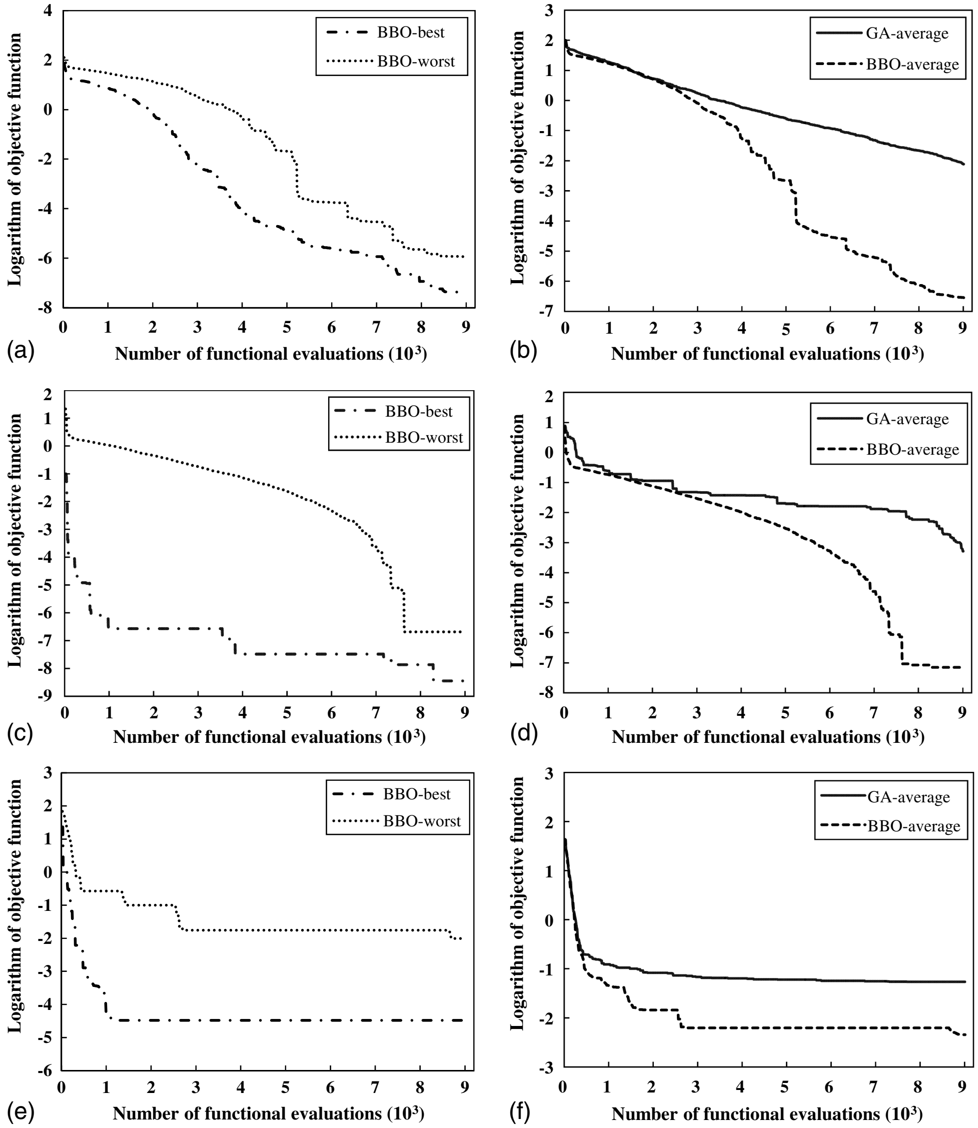

Fig. 5. Best and worst convergence histories of BBO: (a) Sphere; (c) Rosenbrock; (e) Bukin6 function; and average convergence of BBO and GA: (b) Sphere; (d) Rosenbrock; (f) Bukin6 function

Table 1. Values of Objective Function Calculated with BBO and GA in 10 Different Runs for Benchmark Functions

	Function name					
	Sphere		Rosenbrock		Bukin6	
Number of run	GA	BBO	GA	BBO	GA	BBO
1	4.6×10^{-02}	3.0×10^{-7}	2.7×10^{-3}	7.3×10^{-8}	8.6×10^{-2}	3.3×10^{-5}
2	5.1×10^{-2}	4.2×10^{-8}	4.8×10^{-4}	3.6×10^{-9}	4.8×10^{-2}	4.5×10^{-5}
3	1.2×10^{-1}	1.0×10^{-7}	2.4×10^{-4}	2.1×10^{-7}	5.4×10^{-2}	9.9×10^{-3}
4	5.9×10^{-2}	1.1×10^{-6}	1.1×10^{-3}	1.4×10^{-8}	2.3×10^{-2}	7.6×10^{-4}
5	7.5×10^{-2}	1.6×10^{-7}	3.7×10^{-5}	1.9×10^{-8}	4.5×10^{-2}	6.4×10^{-3}
6	1.3×10^{-1}	7.7×10^{-8}	1.1×10^{-5}	3.8×10^{-8}	5.7×10^{-2}	7.2×10^{-3}
7	7.8×10^{-2}	8.8×10^{-8}	1.4×10^{-5}	9.6×10^{-8}	6.7×10^{-2}	8.7×10^{-3}
8	7.8×10^{-2}	4.7×10^{-7}	4.9×10^{-4}	1.7×10^{-7}	6.3×10^{-2}	4.8×10^{-3}
9	8.8×10^{-2}	1.4×10^{-7}	1.4×10^{-4}	5.0×10^{-8}	4.8×10^{-2}	7.2×10^{-3}
10	8.1×10^{-2}	3.5×10^{-7}	5.5×10^{-5}	3.3×10^{-8}	5.2×10^{-2}	3.3×10^{-4}
Best	4.6×10^{-2}	4.2×10^{-8}	1.1×10^{-5}	3.6×10^{-9}	2.3×10^{-2}	3.3×10^{-5}
Worst	1.3×10^{-1}	1.1×10^{-6}	2.7×10^{-3}	2.1×10^{-7}	8.6×10^{-2}	9.9×10^{-3}
Average	8.1×10^{-2}	2.9×10^{-7}	5.2×10^{-4}	7.0×10^{-8}	5.4×10^{-2}	4.6×10^{-3}
Standard deviation	2.7×10^{-2}	3.3×10^{-7}	8.3×10^{-4}	6.8×10^{-8}	1.6×10^{-2}	3.9×10^{-3}
Coefficient of variation	3.4×10^{-1}	1.1×10^{0}	1.6×10^{0}	9.7×10^{-1}	3.0×10^{-1}	8.6×10^{-1}

Eq. (8) expresses the water losses from a reservoir:

$$\text{Loss}_{(i,t)} = Ev_{(i,t)} \cdot \bar{A}_{(i,t)} \quad \text{for } i = 1, \ldots, n, \ t = 1, \ldots, T \tag{8}$$

$$\bar{A}_{(i,t)} = \frac{A_{(i,t)} + A_{(i,t+1)}}{2} \quad \text{for } i = 1, \ldots, n, \ t = 1, \ldots, T \tag{9}$$

in which $Ev_{(i,t)}$ = net evaporation (evaporation minus precipitation) from ith reservoir surface during period t; $\bar{A}_{(i,t)}$ = average ith reservoir area during period t; $A_{(i,t)}$ and $A_{(i,t+1)} = i$th reservoir areas at the beginning and end, respectively, of period t. The overflow volume or spill from the reservoir is calculated by Eq. (10):

$$SP_{(i,t)} = \begin{cases} S_{(i,t+1)} - S_{\max(i,t)} & \text{if } S_{(i,t+1)} > S_{\max(i,t)} \\ 0 & \text{otherwise} \end{cases}$$
$$\text{for } i = 1, \ldots, n, t = 1, \ldots, T \tag{10}$$

where $S_{\max(i,t)}$ = maximum amount of ith reservoir during period t. Moreover there are constraints imposed on reservoir release and reservoir storage as follows:

$$R_{\min(i,t)} \leq R_{(i,t)} \leq R_{\max(i,t)} \quad \text{for } i = 1, \ldots, n, \ t = 1, \ldots, T \tag{11}$$

$$S_{\min(i,t)} \leq S_{(i,t)} \leq S_{\max(i,t)} \quad \text{for } i = 1, \ldots, n, \ t = 1, \ldots, T \tag{12}$$

$$S_{(i,1)} = S_{(i,T+1)} \quad \text{for} \quad i = 1, \ldots, n \tag{13}$$

in which $R_{\min(i,t)}$ and $R_{\max(i,t)}$ = minimum and maximum, respectively, permissible release of ith reservoir during period t; $S_{\min(i,t)}$ = minimum value of ith reservoir at beginning of period t; $S_{(i,1)}$ = storage of ith reservoir at beginning of operation period; $S_{(i,T+1)}$ = storage of ith reservoir at end of operation period. Eqs. (7)–(13) are used to simulate single-reservoir and multiple-reservoir systems.

Case Study

Single-Reservoir System

The Karun4 reservoir in Iran was considered as the single-reservoir case study. This reservoir was built on the Karun River for hydropower generation. The Karun4 reservoir is located in ChaharMahal-va Bakhtiari province, at the coordinates 31°35′ N latitude and 50°24′ east longitude. The minimum and maximum of reservoir storage are $1,141 \times 10^6$ and $2,190 \times 10^6$ m^3, respectively. In addition, power plant capacity (PPC) is equal to $1,000 \times 10^6$ W. BBO was applied to optimize the operation of the Karun4 reservoir during a 5-year period with monthly time steps from 1996 to 1997 through 2000–2001. The equations governing the operation of the Karun4 reservoir were presented in the previous section. Fig. 6 demonstrates the average volume of inflow and evaporation depth from the surface of the Karun4 reservoir during the 5-year period of operation.

The operation of the Karun4 reservoir was performed for hydropower generation. Power plant productivity was calculated by the following equation:

$$P_{(t)} = g.e. \frac{Rp_{(t)}}{PF \cdot Mul_{(t)}} \cdot \frac{[\bar{H}_{(t)} - Tw_{(t)}]}{1{,}000} \quad \text{for } t = 1, \ldots, T \tag{14}$$

where $P_{(t)}$ = hydropower generation in period t (10^6 W); g = acceleration of gravity (m/s^2); e = efficiency of power plant;

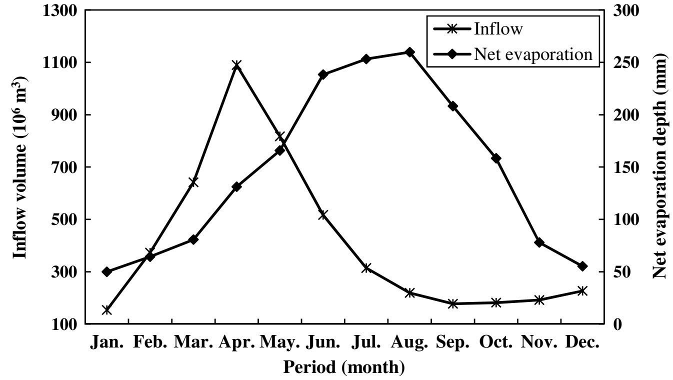

Fig. 6. Monthly average of inflow volume to Karun4 reservoir and monthly average of net evaporation depth

$Rp_{(t)}$ = water release from hydropower plant in period t (10^6 m^3); PF = plant functional coefficient; $Mul_{(t)} = 10^6$ times the number of seconds in period t; $\bar{H}_{(t)}$ = average reservoir water level during period t (m); and $Tw_{(t)}$ = reservoir tailwater level during period t (m). The following constraints and equations were also used:

$$Rps_{(t)} = R_{(t)} - Rp_{(t)} \quad \text{for } t = 1, \ldots, T \tag{15}$$

$$\bar{H}_{(t)} = \frac{H_{(t)} + H_{(t+1)}}{2} \quad \text{for } t = 1, \ldots, T \tag{16}$$

$$0 \leq P_{(t)} \leq PPC \quad \text{for } t = 1, \ldots, T \tag{17}$$

in which $Rps_{(t)}$ = overflow water from plant after hydropower generation in period t; $H_{(t)}$ and $H_{(t+1)}$ = water levels of reservoir at beginning and end, respectively, of period t; and PPC = power plant capacity (10^6 W). The area-storage (A-S) and water level-storage (H-S) equations in the study are defined as follows [area (A) in km^2, water level (H) in m, and storage (S) in 10^6 m^3]:

$$A_{(t)} = a_1 S_{(t)}^3 + a_2 S_{(t)}^2 + a_3 S_{(t)} + a_4 \quad \text{for } t = 1, \ldots, T \tag{18}$$

$$H_{(t)} = b_1 S_{(t)}^3 + b_2 S_{(t)}^2 + b_3 S_{(t)} + b_4 \quad \text{for } t = 1, \ldots, T \tag{19}$$

where a_1, a_2, a_3, and a_4 = constant coefficients of the area-storage equation; and b_1, b_2, b_3, and b_4 = constant coefficients of the water level-storage equation. The following equation expresses the objective function in a single-reservoir system that optimizes hydropower production:

$$\text{Min } Z = \sum_{t=1}^{T} \left[1 - \frac{P_{(t)}}{PPC}\right]^2 \tag{20}$$

where Z = total power deficit (objective function).

Four-Reservoir System

This problem was introduced and solved by Chow and Cortes-Rivera (1974). This problem is a hypothetical example of reservoir operation of a four-reservoir system to maximize benefits during the operation period (12 months). Data required for modeling the system, such as inflows and reservoir storage, are available in Murray and Yakowitz (1979). The connectivity matrix M [Eq. (7)] for this problem is

$$M = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (21)$$

This matrix describes the manner in which releases from upstream reservoirs accrue to downstream ones. Fig. 7 depicts the four-reservoir system.

In the multiple-reservoir problem used in this study (which is a benchmark problem for testing optimization schemes for reservoir systems) the simulating of the operation of reservoirs is carried out using Eqs. (7)–(13) with the assumption that there are no overflows and losses. The objective function for the multiple-reservoir system is the maximization of system benefits:

$$\text{Max } B = \sum_{i=1}^{n}\sum_{t=1}^{T} b_{(i,t)} \cdot R_{(i,t)} \quad (22)$$

where B = objective function (entire profit during operation period); and $b_{(i,t)}$ = benefit related to the ith reservoir in period t. There are constraints imposed on release and reservoir storages that form an optimization problem in conjunction with the objective function [Eq. (22)]. Constraints on the volume releases are directly entered into the optimization algorithm, while constraints on reservoir storage are appended as penalty functions. The following three penalty functions on excess reservoir storage are added in each period:

$$P1_{(i,t)} = \begin{cases} K_1[S_{(i,T+1)} - S_{(i,\text{target})}]^2 & \text{if } S_{(i,T+1)} < S_{(i,\text{target})} \\ 0 & \text{otherwise} \end{cases}$$
$$\text{for } i = 1, \ldots, n, \quad t = 1, \ldots, T \quad (23)$$

$$P2_{(i,t)} = \begin{cases} K_2[S_{\min(i,t)} - S_{(i,t+1)}]^2 & \text{if } S_{(i,t+1)} < S_{\min(i,t)} \\ 0 & \text{otherwise} \end{cases}$$
$$\text{for } i = 1, \ldots, n, \quad t = 1, \ldots, T \quad (24)$$

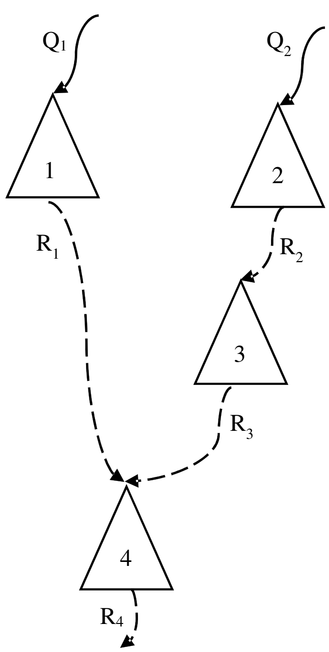

Fig. 7. Schematic of four-reservoir system

$$P3_{(i,t)} = \begin{cases} K_3[S_{(i,t+1)} - S_{\max(i,t)}]^2 & \text{if } S_{(i,t+1)} < S_{\min(i,t)} \\ 0 & \text{otherwise} \end{cases}$$
$$\text{for } i = 1, \ldots, n, \quad t = 1, \ldots, T \quad (25)$$

in which $S_{(i,\text{target})}$ = target volume of ith reservoir at end of operation period $[S_{(i,\text{target})} = S_{(i,1)}]$; $P1_{(i,t)}$, $P2_{(i,t)}$, and $P3_{(i,t)}$ = penalty functions related to not meeting the target storage at the end of the operation period, reservoir storage being less than the minimum storage, and reservoir storage exceeding the maximum storage, respectively; K_1, K_2, and K_3 = constants of the penalty functions. The penalty constants of K_1, K_2, and K_3 were considered equal to 60, 40, and 40, respectively. Therefore, the objective functions modified with the penalty functions is

$$\text{Max } B = \sum_{i=1}^{n}\sum_{t=1}^{T}[b_{(i,t)} \cdot R_{(i,t)} - [P1_{(i,t)} + P2_{(i,t)} + P3_{(i,t)}] \quad (26)$$

Results and Discussions

The results obtained for the optimization of the single-reservoir and multiple-reservoir systems are presented in the next two sections.

Results for Single-Reservoir System Operation

The optimal operation of Karun4 was solved for using the BBO method, and compared with solutions calculated with GA and NLP. BBO and GA were programmed with the *MATLAB 7.11.0* software. The NLP method was implemented in *Lingo 11.0*. The GA method was compared to BBO to assess the relative performance of these two EAs. The NLP method was implemented to test BBO's ability to converge to the global optimum.

A trial-and-error technique and sensitivity analysis were used to determine the parameters of BBO and GA. The selected parameters of each algorithm are listed in Table 2. Due to the use of the random generator in BBO and GA several solutions must be obtained with an initial random population to test the performance of these EAs over a number of initial populations. Then different runs were used for BBO and for GA to assess and compare their solution performances with the 10 runs statistically.

The objective function value of the NLP method, considered to be the global optimum, equaled 1.213 with *Lingo* [obtained after 16 h of processing time with an Intel Core i7 (2.93 GHz) processor

Table 2. Characteristics of BBO and GA Used in Reservoir System Problems

Algorithm	Parameter	Single reservoir system	Multiple reservoir system
BBO	NFE[a]	70,000	500,000
	Mutation rate	0.05	0.05
	Mutation function	Gaussian	Gaussian
	Selection function	Roulette wheel	Roulette wheel
	α	1.00	0.40
GA	NFE	70,000	500,000
	Mutation rate	0.05	0.06
	Mutation function	Uniform	Uniform
	Selection function	Roulette wheel	Roulette wheel
	Crossover fraction	0.60	0.70
	Crossover function	Two-point crossover	Two-point crossover

[a]Number of functional evaluation.

Table 3. Value of Objective Function for 10 Different Runs Calculated with BBO and GA for Karun4 Reservoir

Number of run	GA	BBO	NLP
1	1.673	1.232	1.213
2	1.549	1.239	
3	1.865	1.227	
4	1.752	1.235	
5	1.987	1.223	
6	1.753	1.223	
7	1.931	1.227	
8	1.570	1.223	
9	1.842	1.229	
10	1.535	1.225	
Best	1.535	1.223	—
Worst	1.987	1.239	—
Average	1.746	1.228	—
Standard deviation	0.162	0.005	—
Coefficient of variation	0.093	0.004	—

computer]. BBO converged to an optimal objective function value equal to 1.223 after 70,000 evaluations of the objective function in its best-performing run. The processing of the BBO algorithm lasted less than two minutes in each run. The GA's best-performing value of the objective function among the 10 runs equaled 1.535. Table 3 lists the results for the solution of the single-reservoir problems. According to the obtained results (Table 3), the variation regarding objective function values obtained with the BBO method in 10 different runs is insignificant and close to zero. The closeness of results from BBO is testimony to the convergence accuracy and precision of BBO. The coefficient of variation of the objective function values obtained with BBO in 10 runs was 23.25 times smaller than GA's.

Fig. 8 depicts the convergence of the GA and BBO in 70,000 evaluations of the objective function. In addition to reaching better objective function values than GA, the BBO convergence is superior to that of GA. The results depicted in Fig. 8 establish conclusively that the BBO convergence history is better than that of GA over the entire range of functional evaluations (70,000 total number of functional evaluations). Figs. 9–11 illustrate, respectively, the amount of release from the reservoir, generated power, and variation of reservoir storage during the operation period. The graphs of these figures show that the output variables from BBO are very close; in fact, they are almost identical to the NLP results. GA, on the contrary, exhibited differences with NLP results in several months. Fig. 10, specifically, shows substantial divergence of the

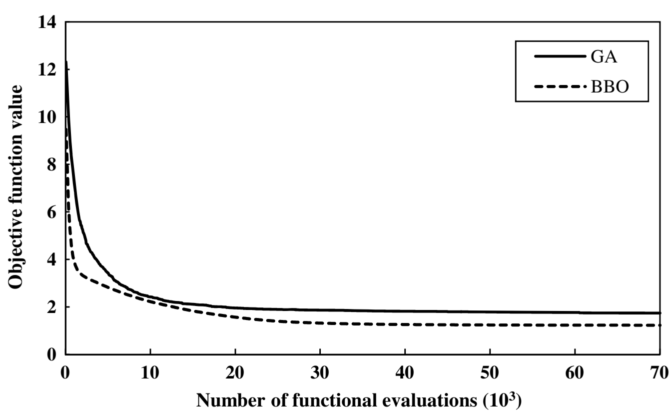

Fig. 8. Average convergence histories of BBO and GA in solving the optimization of Karun4 reservoir's operation

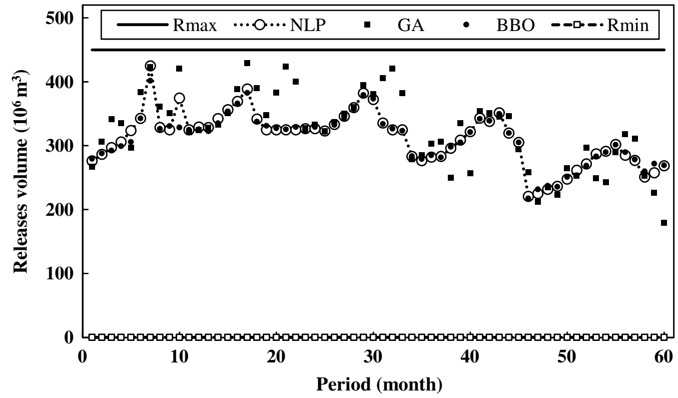

Fig. 9. Amount of release during Karun4 reservoir operation period

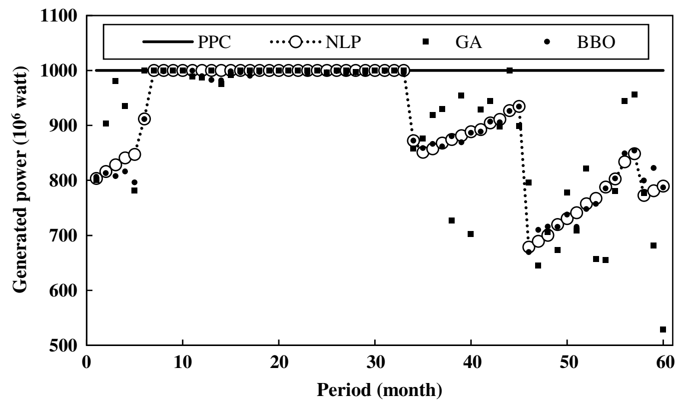

Fig. 10. Power generated during Karun4 operation period

generated power calculated with GA and NLP after the 34th month of operation.

Results for Multiple-Reservoir System Operation

Chow and Cortes-Rivera (1974) solved the four-reservoir problem by means of LP and reported its optimal solution to be equal to 308.26. Murray and Yakowitz (1979) reported an optimal solution equal to 308.23 using the differential dynamic programming (DDP) method. Bozorg Haddad et al. (2011a) solved this problem using HBMO, considering 220 populations in the HBMO and using 5,000 iterations (approximately 1 billion evaluations).

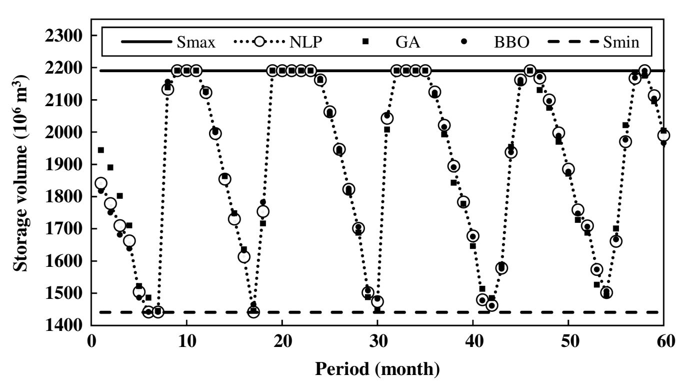

Fig. 11. Variation of storage volume during Karun4 reservoir operation period

Table 4. Value of Objective Function for 10 Runs Calculated with BBO and GA for Four-Reservoir System

Number of run	GA	BBO	Global
1	300.42	308.00	308.29
2	298.89	308.02	
3	300.09	308.12	
4	300.47	307.56	
5	298.46	307.11	
6	300.00	307.88	
7	299.22	307.57	
8	299.87	308.08	
9	299.20	308.00	
10	300.35	306.55	
Best	300.47	308.12	—
Worst	298.46	306.55	—
Average	299.70	307.69	—
Standard deviation	0.7060	0.5107	—
Coefficient of variation	0.0024	0.0017	—

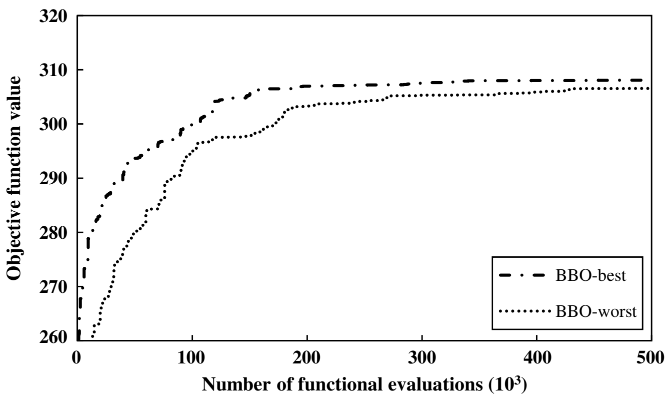

Fig. 12. BBO algorithm convergence in its best and worst performance histories for four-reservoir system

They calculated 307.50 as the average of the objective function value in 10 runs of the HBMO algorithm. The best and the worst reported solution during these 10 runs were 308.07 and 306.71, respectively. Moreover, Bozorg Haddad et al. (2011a) solved this problem using the LP method with the *Lingo 8.0* software. They reported the global optimal value of the objective function equal to 308.29 with NLP. Bozorg Haddad et al. (2014b) used the water cycle algorithm (WCA) to optimize the operation of this reservoir system. Their results yielded 306.920 as the optimal value of the objective function of this problem using WCA. The optimal value calculated with BBO in this study was 308.29.

Table 2 lists the BBO and GA parameters used in the optimization of the four-reservoir system. Ten runs were performed using BBO and using the GA. The results revealed that BBO converged to 99.94% of the global optimal solution in its best performance. GA converged to 97.46% of the optimal solution in its best performance. The average values of the objective function calculated with BBO and GA in 10 runs were 307.69 and 299.70, respectively. Table 4 summarizes the calculated results from the 10 runs of BBO and GA.

The results of Table 4 show that, in addition to a suitable performance of BBO in reaching the global optimal solution, the standard deviation of the objective function value in 10 different runs of the BBO algorithm equaled 0.51. The standard deviation of the objective function value calculated with GA was equal to 0.70, which is 1.37 times larger than that of BBO in 10 runs. The average

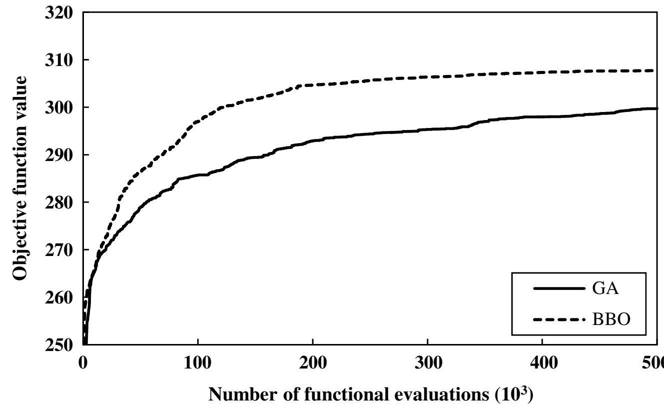

Fig. 13. BBO algorithm and GA average convergence performance for four-reservoir system

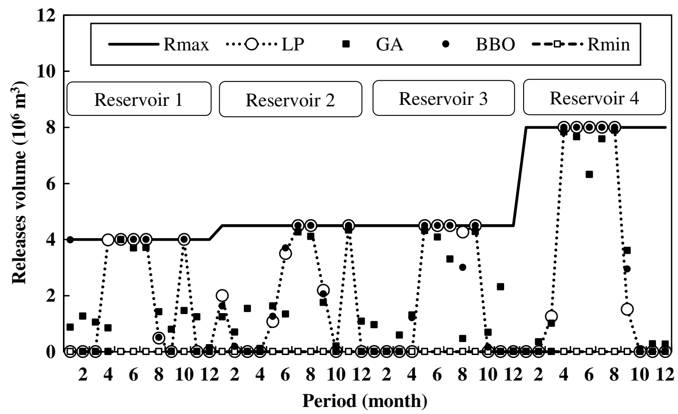

Fig. 14. Monthly release of four-reservoir system for best-performing histories of BBO algorithm and GA

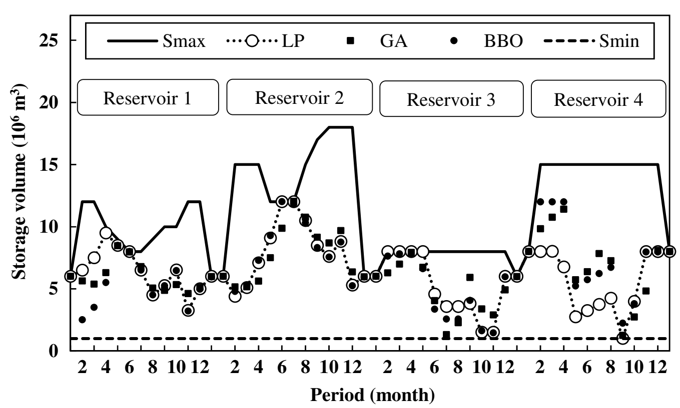

Fig. 15. Variation in storage volume of four-reservoir system for best-performing histories of BBO algorithm and GA

and worst values of the objective function in 10 runs of BBO were 99.80 and 99.43% of the global optimal solution, respectively. The GA's average and worst performances in 10 runs were 97.21 and 96.81% of the global optimal solution, respectively. Fig. 12 illustrates the BBO's best and worst convergence histories. Fig. 13 displays the BBO's and GA's average histories in 10 runs. Fig. 14 depicts the calculated total release from the four-reservoir system

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using the best-performing histories of the BBO and GA algorithms. Fig. 15 displays the variation of reservoir storage in the four-reservoir system corresponding to the best performance histories of the BBO and GA algorithms.

Concluding Remarks

This study evaluated the BBO algorithm's ability to solve for the optimal operating of single-reservoir and multiple-reservoir systems. Results from the two case studies indicate the following:

1. Closeness of the best performance to the global optimal solution: Concerning the operation of the Karun4 reservoir, the objective function value obtained from the NLP method was equal to 1.213. Objective function values of BBO and GA for this problem were 1.223 and 1.535, respectively. The calculated results show the superiority of BBO compared to GA. In the case of the four-reservoir system, BBO converged accurately to the global optimal solution, reaching 99.94% of the optimal solution in its best-performing history, while GA reached 97.46% of the optimal solution in its best-performing history. It is pivotal to consider that the proposed reservoir optimization examples address short-term and small-scale problems. Clearly BBO can be applied to long-term (e.g., 50 years) operation of a multireservoir system at a national or global scale, with a significant gain in system performance, benefits, and cost reduction.
2. Comparison of the average performance of BBO with GA: In the single-reservoir system, the average values of the objective function with BBO and GA equaled 1.228 and 1.746, respectively. Given the global optimum of 1.213 calculated with NLP it is concluded that BBO exhibited the best average performance and converged very close to the global optimum solution. Based on the average-performance criterion concerning the four-reservoir system BBO reached 99.80% of the optimal solution, whereas GA reached 97.21% of the optimal solution.
3. Variation of the objective function in different runs: Considering the operation of the Karun4 reservoir, the standard deviation of BBO results was lower than that of GA in 10 runs. The standard deviations of BBO and GA were 0.005 and 0.162, respectively. Similar to the Karun4 reservoir case, for the operation of four-reservoir systems the standard deviations of solutions from 10 runs equaled 0.51 and 0.70 for the BBO algorithm and GA, respectively.

Based on three performance criteria outlined in this section BBO exhibited all-around better performance than GA in the problems considered. In addition, BBO has another advantage over GA; namely, it is easier to set parameters: Although BBO and GA employ approximately a similar number of parameters, the mechanism of parameter-tuning for BBO is easier and faster. Owing to the fact that BBO is not sensitive to minor changes in parameters, the process of trial and error becomes simpler.

Overall superiority of BBO over GA stems from relying on a simpler algorithmic structure because of the former's easier mechanism for setting its parameters, making BBO an efficient, relatively user-friendly optimization method, with high potential for solving reservoir operation problems.

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