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Publication Date
1965-02-01

UCRL-11975

University of California

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Berkeley, California

AEC Contract No. W-7405-eng-48

COMPARISON BETWEEN PHASE SHIFT AND COMPLEX POTENTIAL DESCRIPTIONS OF ELASTIC SCATTERING

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February 1965

UCRL-11975

Comparison between Phase Shift and Complex Potential Descriptions of Elastic Scattering*

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The equivalence between parameterized phase shift and complex-potential analyses of the elastic scattering of strongly absorbed particles is discussed. Specific comparisons are made using computer analyses of alpha particle scattering data. The relative merits of the two analyses are pointed out.

I. INTRODUCTION

In this paper we want to emphasize the equivalence between parameterized phase-shift (PPS) and complex-potential model (CPM) analyses of the elastic scattering of strongly absorbed[1] "particles", such as deuterons, He^3, alpha-particles, and heavier ions. We make specific comparisons using analyses of alpha-particle scattering data. Then we point out several advantages of the PPS method, which encompasses all analyses[2-6] in which the partial-wave (complex) phase-shifts are explicitly parameterized in the calculation and are not adjusted through an intermediary complex-potential.

The differential cross-section for elastic scattering is

$$\sigma(\theta) = |f(\theta)|^2,$$

with the scattering amplitude, in the absence of spin-dependent interactions, given by

$$f(\theta) = f_c(\theta) + \frac{i}{2k} \sum_{\ell=0}^{\infty} (2\ell + 1)\, e^{2i\sigma_\ell}(1 - \eta_\ell) P_\ell(\cos\theta), \qquad (1)$$

where $f_c(\theta)$ is the Coulomb scattering amplitude, $k = \frac{1}{\hbar}(2\mu E)^{1/2}$, $\sigma_\ell = \arg\Gamma(1 + \ell + in)$, with $n = z_1 z_2 e^2/\hbar v$. One can write

$$\eta_\ell = A_\ell e^{2i\delta_\ell} \qquad (2)$$

so that A_ℓ, the amplitude of the outgoing ℓ^{th} partial wave, and δ_ℓ, its nuclear phase-shift, are real. Thus, $0 \le A_\ell \le 1$, with $A_\ell = 1$ corresponding to no absorption and $A_\ell = 0$, to complete absorption of that partial wave. McIntyre et al.[2] introduced the (arbitrary) parameterization

$$A_\ell = 1 - \left[1 + \exp\frac{\ell - \ell_A}{\Delta\ell_A}\right]^{-1}, \quad \delta_\ell = \delta\left[1 + \exp\frac{\ell - \ell_\delta}{\Delta\ell_\delta}\right]^{-1} \qquad (3)$$

as an improvement over the "sharp cut-off" model (see Figure 1). This form for A_ℓ has been justified by Elton[5], whereas a form of δ_ℓ suggested by Conzett et al.[3] as having qualitative theoretical justification, differs from that of equation (3), particularly for $\ell < \ell_\delta$. However, we shall see, for those cases examined here, that the scattering amplitude is not seriously affected because in the region of ℓ-values for which equation (3) is incorrect, $A_\ell \to 0$. This would not be the case for less strongly absorbed particles, and thus a theoretically proper form of δ_ℓ would be required.

The CPM analysis determines A_ℓ and δ_ℓ by solving the equation satisfied by the radial wave function, $f_\ell(r)$:

$$\left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + k^2 - \frac{2\mu}{\hbar^2} U(r)\right] f_\ell(r) = 0, \qquad (4)$$

where $U(r) = V_c(r) + V_N(r)$, the Coulomb plus the nuclear potential. In order to provide a description of both the elastic scattering and absorptive processes, the latter is taken to be complex: $V_N(r) = -[V(r) + iW(r)]$. Comparison at $r = \infty$ of $f_\ell(r)$ with the solution of equation (4) for $V_N(r) = 0$ then gives A_ℓ and δ_ℓ. Strongly absorbed particles sample only the surface region of the nucleus, so one is justified in questioning the significance of using a parameterized complex-potential to describe the scattering. The PPS analysis parameterizes the A_ℓ and δ_ℓ directly, so it is clear that the two analyses can be equivalent.

II. COMPARISON OF THE ANALYSES

For purposes of comparison, one can make the correspondence between $\underline{\ell}$ and $\underline{r}$, the distance of closest approach for a particle of orbital angular momentum $[\ell(\ell+1)]^{1/2}\hbar$, through the relation

$$kr = n + [n^2 + \ell(\ell+1)]^{1/2}. \tag{5}$$

We indicate this correspondence between coordinate (r) space and orbital angular-momentum (ℓ) space by writing $A_\ell(r)$. The value of $r = R_a$ for which $A_\ell(r) = \frac{1}{2}$ should not be expected to agree with the value of $r = R$ for which $V(R) = \frac{1}{2} V(0)$ in the CPM. Similarly, $\underline{t}$, the interval $\Delta\ell_A$ converted to coordinate space via equation (5) would not necessarily agree with the analogous complex-potential surface-thickness parameter $\underline{a}$. These points have been developed previously[4].

In order to demonstrate these considerations more explicitly, we have made PPS analyses of some existing data on the elastic scattering of alpha-particles from A, Cu, and Pb at 18, 40, and 48-MeV, respectively[7-9]. These data had previously been used for a CPM analysis[10]. Figures 2-4 show comparisons between experimental and calculated angular distributions for both PPS and CPM analyses. In all cases the quality of agreement between experiment and calculation is comparable for the two treatments; in some cases one analysis gives better agreement than the other, depending to some extent on what feature of the data is regarded as most significant. The values of the parameters resulting from these analyses are compiled in Table I, together with those resulting from PPS[2] and CPM[11] analyses of data on the scattering of 22 and 40-MeV alpha-particles from Ag. The expected differences between the complex-potential parameters, R and a, and the PPS parameters, R_a and t, are seen. If one now looks at the actual scattering amplitude parameters describing the ℓ-dependence of A_ℓ and δ_ℓ, the equivalence of the two analyses is seen. That is, the ℓ_A and $\Delta\ell_A$ values determined from the separate analyses are in excellent agreement, and the differences in the coordinate space parameters, $\underline{R}_0$ and $\underline{a}$ vs $\underline{R}_a$ and $\underline{t}$, clearly result from their different physical definition. As a further specific illustration, we show in Figure 5 the ℓ-space dependence of δ_ℓ and $T_\ell = 1 - A_\ell^2 (= \sigma_\ell^r/(2\ell+1)\pi\lambda\!\!\!^{-2}$ where σ_ℓ^r is the ℓ^{th}-wave partial reaction cross section) resulting from the two analyses of the Pb scattering data. As remarked above, the large differences between the PPS and CPM values of δ_ℓ for $\ell \leq 18$ do not result in noticeably different scattering amplitudes since $A_\ell \to 0$ for that range of $\underline{\ell}$ values. A quantitative demonstration of this fact is shown in Figure 6. There, along with a plot of T_ℓ determined from the Cu scattering data, we show $\sigma(\theta)/\sigma_R(\theta)$, the ratio to Rutherford scattering, calculated with $\delta_\ell = 0$ for $\ell < \ell'$. With $\ell' = 13$ or 14 only very minor changes are seen at the larger angles. Finally, when $\ell' = 15$, for which $A_{\ell'} \cong 0.1$ $(T_{\ell'} \cong 0.99)$, the angular distribution is severely distorted, again at the larger angles. This emphasizes the sensitivity of the larger momentum-transfer scattering to the values of δ_ℓ in the smaller orbital angular-momentum states, and it indicates that a more justifiable form of δ_ℓ than that given in Eq. (3)

could best be tested by precise large momentum-transfer scattering data. This would be true particularly in instances of weaker absorption, where A_ℓ would not vanish for the smaller values of ℓ.

III. CONCLUSIONS

In summary, PPS and CPM analyses of the elastic scattering of strongly absorbed particles are seen to be equivalent. Some advantages of the PPS treatment are the following:

1. The calculation is simpler than for the CPM, since the numerical solution of Eq. (4) is bypassed. This is particularly advantageous whenever large numbers of partial waves contribute to the scattering.

2. The form of $A_\ell(r)$ gives a clear physical interpretation of the absorption of the incident particles, and we believe that for strong-absorption scattering the determined $A_\ell(r)$ curve is essentially unique. On the other hand, the CPM varies the parameters of $V_N(r)$ to produce this $A_\ell(r)$ curve, and ambiguities in the potentials describing, for example, alpha-particle[12] and deuteron[13] scattering, result from the fact that different potentials can give essentially the same $A_\ell(r)$ and $\delta_\ell(r)$ values. Clearly the value of $V_N(r)$ for $r < r'$, where $A_\ell(r) = 0$ for $r < r'$, can have no effect on the scattering.

3. The PPS analysis described in this paper has found application in the calculation of inelastic scattering of alpha particles, through the model of Austern and Blair[14]. In this model the inelastic scattering amplitudes

are expressed in terms of derivatives of the partial wave amplitudes for elastic scattering, η_ℓ. The equivalence between the Austern-Blair Model and Distorted Wave Born Approximation calculations for inelastic scattering is analogous to the equivalence of PPS and CPM analyses for elastic scattering. Points 1 and 2 mentioned above are equally valid in this case.

4. Eq. (1) for the scattering amplitude is valid, also, in the relativistic region insofar as spin effects are unimportant.

Finally, we propose that high-energy p-p and π-p scattering be examined with this PPS analysis, taking care to provide a theoretically proper parameterization of δ_ℓ.[15]

ACKNOWLEDGMENTS

For valuable discussion at various times, we are grateful to Drs. J. S. Blair, A. E. Glassgold, K. R. Greider, and J. G. Vidal. Dr. G. Igo was particularly helpful in making available the phase-shifts from his complex-potential model calculations. One of us (J. A.) wants to thank Dr. A. H. Wapstra for his interest in this work.

TABLE I

Comparison of parameters resulting from PPS and CPM analyses of alpha-particle scattering data

DATA		CPM Analysis						PPS Analysis			
		parameters									
	MeV	V	W	R	a	ℓ_A	$\Delta\ell_A$	ℓ_A	$\Delta\ell_A$	R_a	t
A + He^4	18	100	15	5.4	0.6	(c) 9.7	0.6	(c) 7.5	0.6	6.67	0.33
Cu + He^4	40	49.3	11	6.8	0.5	17.8	0.8	17.0	0.8	7.92	0.30
Pb + He^4	48	25	15	8.1	0.6	21.1	1.3	21.0	1.3	10.16	0.41
(a) Ag + He^4	22	50	20	7.5	0.6	10.5	1.5	10.8	(b) 0.75	9.74	0.33
		150	20	7.5	0.6	11.3	1.3	10.8	0.75	9.74	0.33
(a) Ag + He^4	40	50	20	7.1	0.6	19.3	1.2	19.0	1.1	9.27	0.40
		150	20	7.1	0.6	19.8	1.1	19.0	1.1	9.27	0.40

(a) From references 2 (McIntyre et al.) and 11.

(b) This $\Delta\ell_A$ value resulted from an analysis in which δ_ℓ was zero. This may account for its being appreciably lower than the value determined from the CPM analysis.

(c) The disagreement in ℓ_A values in this case arises from the difference in fits to the experimental data.

REFERENCES

[*] This paper is based on a thesis submitted to the Technical University of Delft, The Netherlands, in partial fulfillment of the Ph. D degree; University of California Lawrence Radiation Laboratory report UCRL-9650, 1961 (unpublished). This work was done under the auspices of the U. S. Atomic Energy Commission.

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1 As used here, absorption encompasses all transitions out of the entrance channel.

2 For alpha-particle scattering: J. S. Blair, Phys. Rev. 95, 1218 (1954), "sharp cut-off" model; J. A. McIntyre, K. H. Wang, and L. C. Becker, Phys. Rev. 117, 1337 (1960), modified "sharp cut-off" model.

3 For heavy-ion scattering: J. Alster and H. E. Conzett, Proceedings of the Second Conference on Reactions between Complex Nuclei, edited by A. Zucker, E. C. Halbert, and F. T. Howard (John Wiley and Sons, Inc., New York 1960), p. 175; J. Alster, Thesis, University of California, Lawrence Radiation Laboratory Report UCRL-9650, 1961 (unpublished); J. A. McIntyre, S. D. Baker, and K. H. Wang, Phys. Rev. 125, 584 (1962); H. E. Conzett, A. Isoya, and E. Hadjimichael, Proceedings of the Third Conference on Reactions

between Complex Nuclei, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, 1963).

4 J. Alster and H. E. Conzett, Phys. Rev. 136, B1024 (1964).

5 Applications to high energy nucleon scattering have been developed by K. R. Greider and A. E. Glassgold, Ann. Phys. (N. Y.) 10, 100 (1960) and L. R. B. Elton, Nucl. Phys. 23, 681 (1961).

6 For a very complete analytical development of PPS analyses of elastic scattering of strongly absorbed particles see W. E. Frahn and R. H. Venter, Ann. Phys. (NY.) 24, 243 (1963); R. H. Venter, Ann. Phys. (N. Y.) 25, 405 (1963).

7 L. Seidlitz, E. Bleuler, and D. J. Tendam, Phys. Rev. 110, 682 (1958).

8 G. Igo, H. E. Wegner, and R. M. Eisberg, Phys. Rev. 101, 1508 (1956).

9 R. E. Ellis and L. Schecter, Phys. Rev. 101, 636 (1956).

10 G. Igo, Phys. Rev. 115, 1665 (1959).

11 W. B. Cheston and A. E. Glassgold, Phys. Rev. 106, 1215 (1957); A. E. Glassgold, Prog. Nucl. Phys., edited by O. R. Frisch (Pergamon Press, London, 1959), Vol. 7, p. 123.

12 G. Igo, Phys. Rev. Letters 1, 72 (1958).

13 C. M. Perey and F. G. Perey, Phys. Rev. 122, 755 (1963); E. C. Halbert, Nucl. Phys. 50, 353 (1964).

[14] N. Austern and J. S. Blair, to be published in Annals of Physics.

[15] H. E. Conzett and E. Hadjimichael, to be published.

FIGURE LEGENDS

Fig. 1 A_ℓ is the amplitude of the outgoing ℓ^{th} partial wave, and δ_ℓ is its nuclear phase shift.

Fig. 2 (A). The solid line is the CPM fit to the elastic scattering of He^4 from A. (Ref. 10). Parameters are: $V = 100$, $W = -15$; $R = 1.17A^{1/3} + 1.36$, $a = 0.6$.

(B). The solid line gives the PPS fit to the same data with the parameters $\ell_A = 7.5$, $\Delta\ell_A = 0.6$, $\delta = 1.2$, $\ell_\delta = 6.5$, $\Delta\ell_\delta = 0.5$. The points give the experimental values.

Fig. 3 (A). Same as 2(A) for He^4 scattered from Cu (Ref. 10). The parameters are $V = -49.3$, $W_0 = -11$, $R = 1.14A^{1/3} + 2.24$, $a = 0.5$.

(B). Same as 2(B). The parameters are: $\ell_A = 17$, $\Delta\ell_A = 0.8$, $\delta = 0.7$, $\ell_\delta = 17$, $\Delta\ell_\delta = 1.0$.

Fig. 4 (A). Same as 2(A) for He^4 scattered from Pb (Ref. 10). The parameters are $V = -25$, $W = -15$, $R = 1.13A^{1/3} + 2.0$, $a = 0.6$.

(B). Same as 2(B). The parameters are: $\ell_A = 21$, $\Delta\ell_A = 1.3$, $\delta = 0.2$, $\ell_\delta = 23$, $\Delta\ell_\delta = 1.4$.

Fig. 5 (A). Plot of δ_ℓ as a function ℓ for the CPM ( ) and PPS(Δ) analyses of the Pb + He^4 elastic scattering data.

(B). Plot of $T_\ell = 1 - A_\ell^2$ as a function of ℓ for the CPM ( ) and PPS(Δ) analyses.

FIG. 6. (A) Plot of $T_\ell = 1 - A_\ell^2$ for the PPS ANALYSIS OF THE Cu + He^4 elastic scattering data

(B) SEE text for details

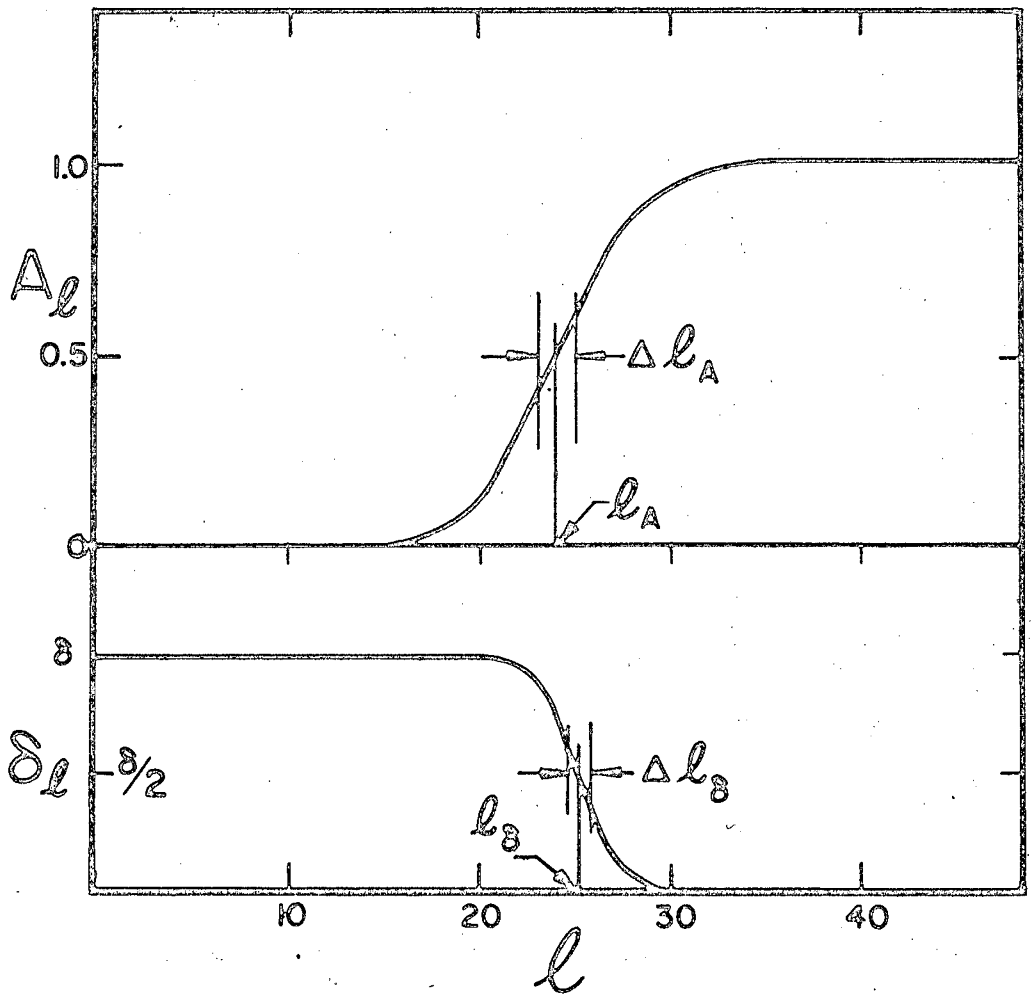

1.0
0.5
0
A_ℓ
$\Delta \ell_A$
ℓ_A
δ
δ_ℓ
δ/2
$\Delta \ell_\delta$
ℓ_δ
10
20
30
40
ℓ

Fig. 1.

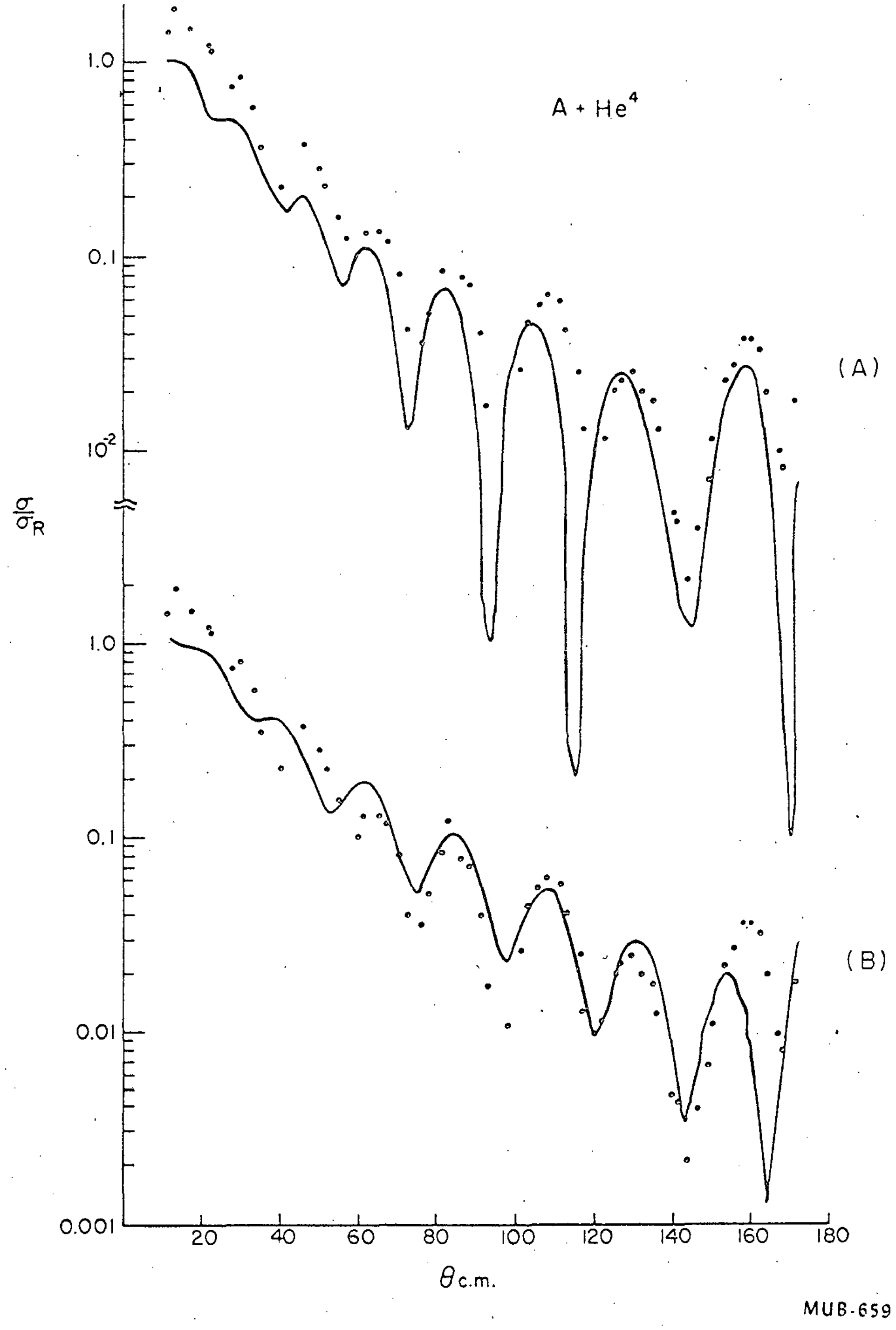

A + He^4
(A)
(B)
1.0
0.1
10^{-2}
1.0
0.1
0.01
0.001
$\frac{\sigma}{\sigma_R}$
20
40
60
80
100
120
140
160
180
θc.m.
MUB-659

Fig. 2.

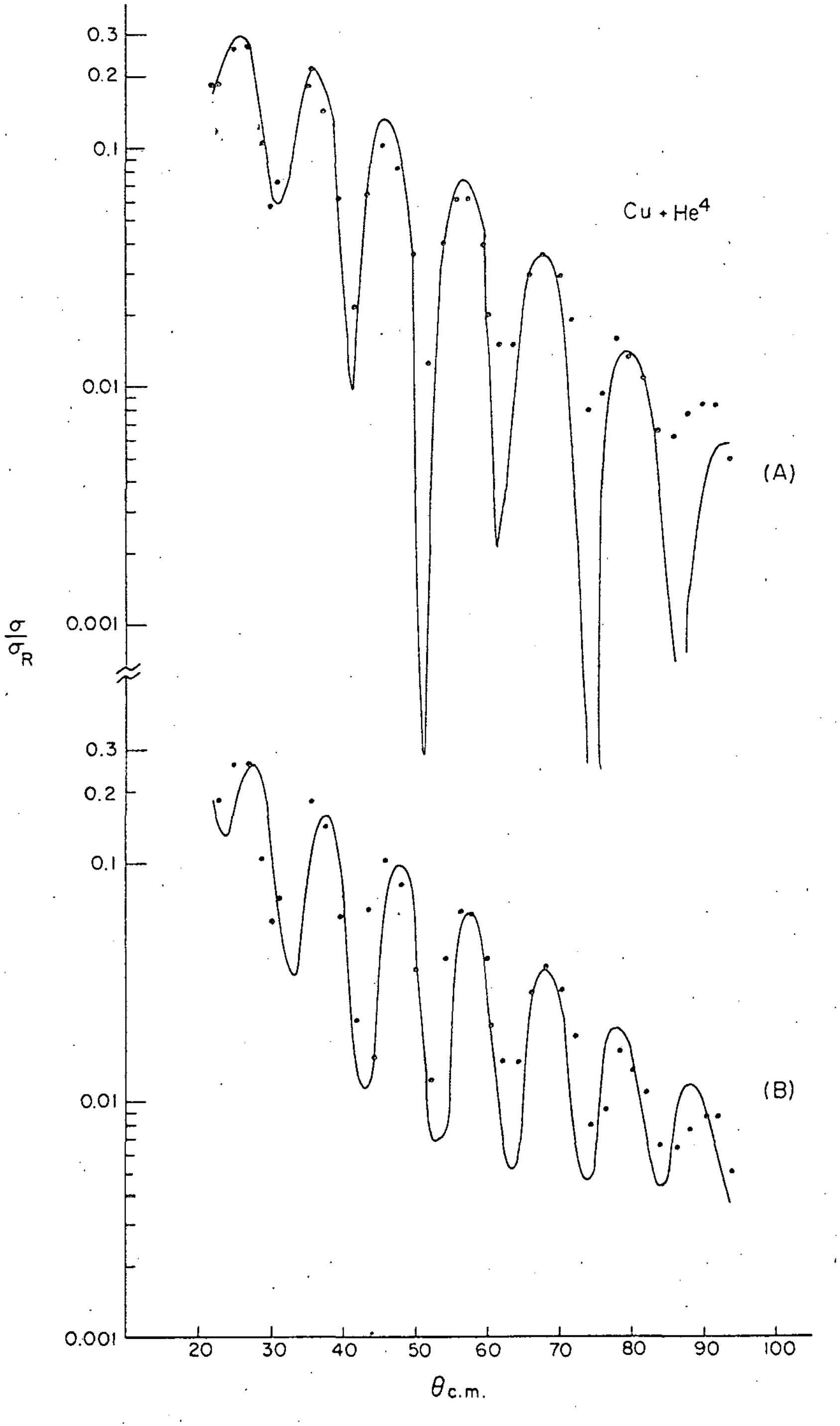

Cu + He^4
(A)
(B)
σ/σ_R
0.3
0.2
0.1
0.01
0.001
$\theta_{c.m.}$
20
30
40
50
60
70
80
90
100
MUB-660

Fig. 3.

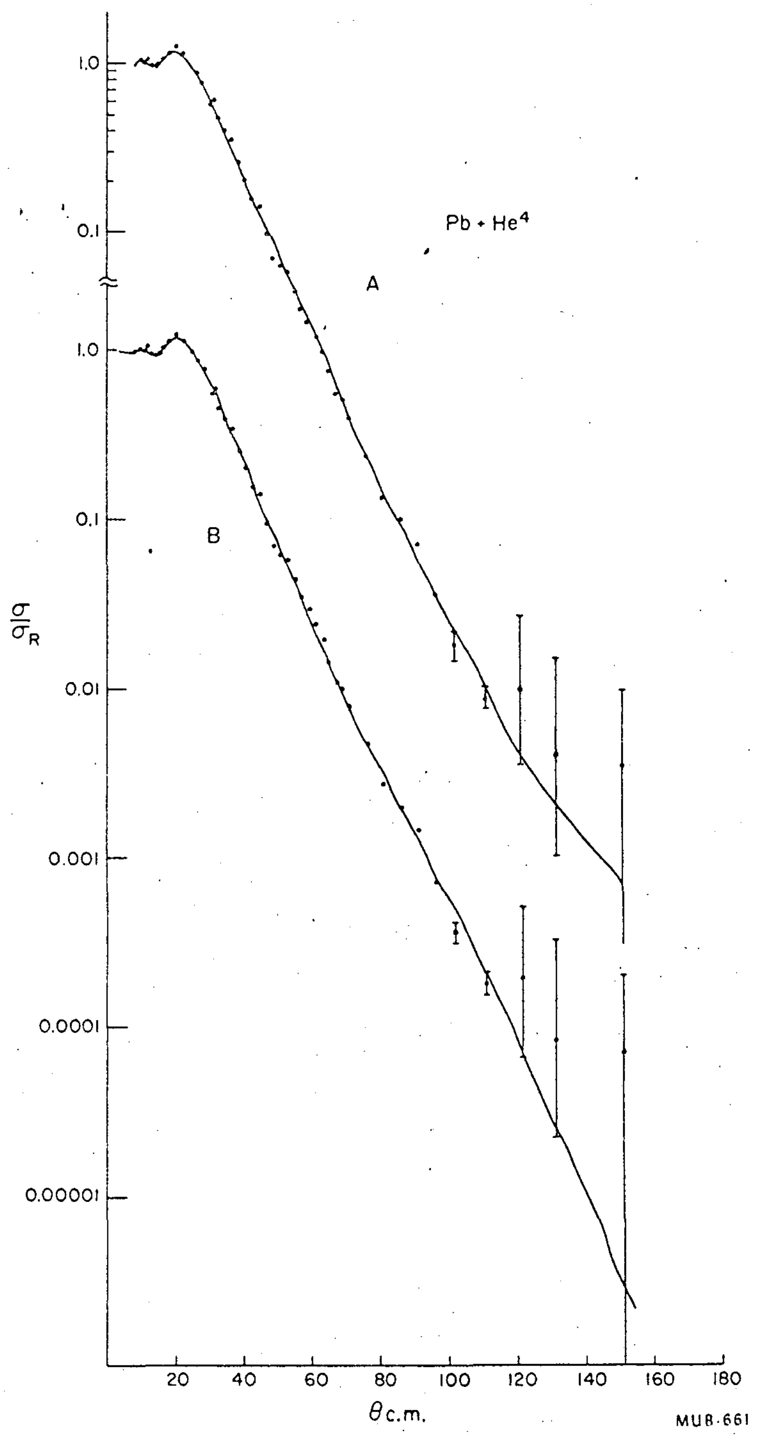
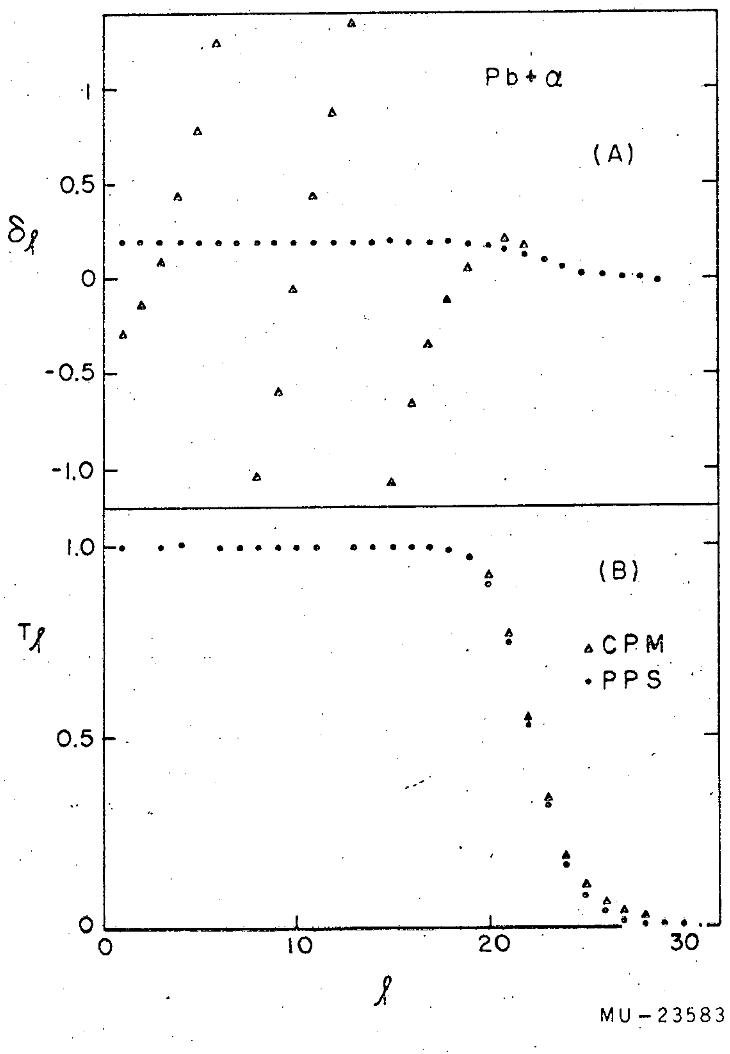

Pb + He^4
A
B
1.0
0.1
1.0
0.1
0.01
0.001
0.0001
0.00001
σ/σ_R
20
40
60
80
100
120
140
160
180
θc.m.
MUB-661

Fig. 4.


Pb + α
(A)
δ_ℓ
1
0.5
0
-0.5
-1.0
(B)
T_ℓ
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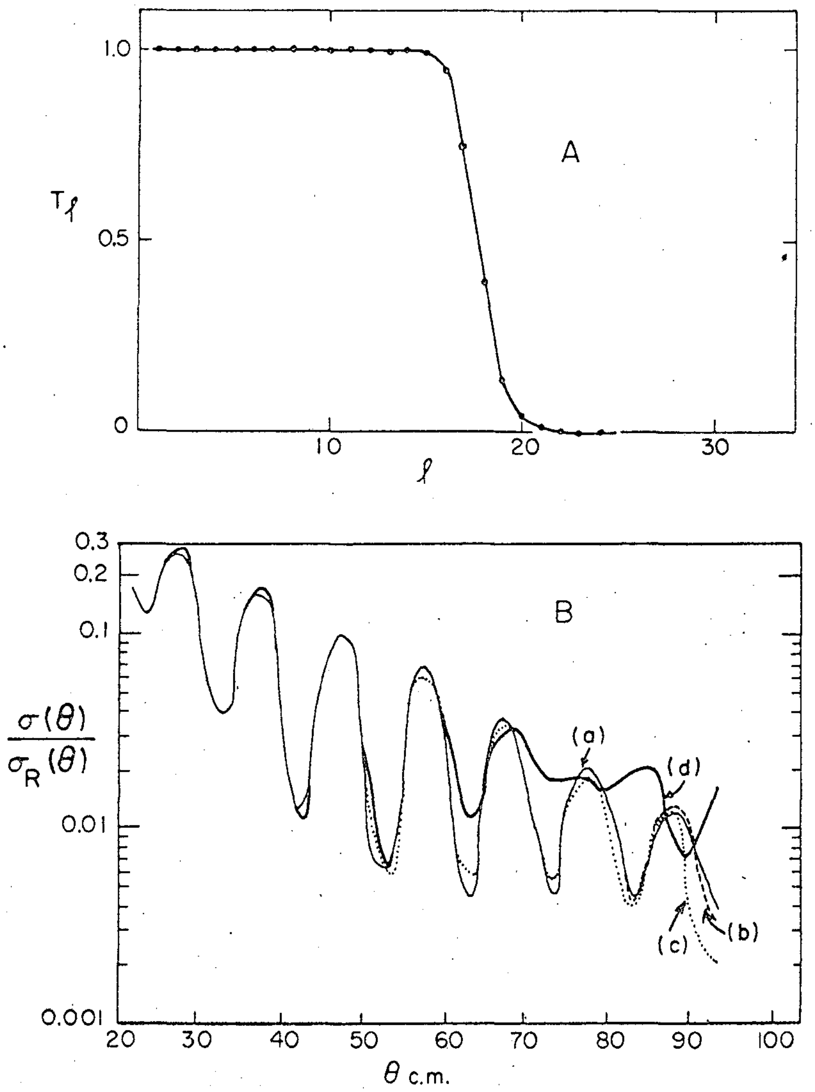
0.5
0
△ CPM
• PPS
0
10
20
30
ℓ
MU-23583

Fig. 5.



MU-23584

Fig. 6.