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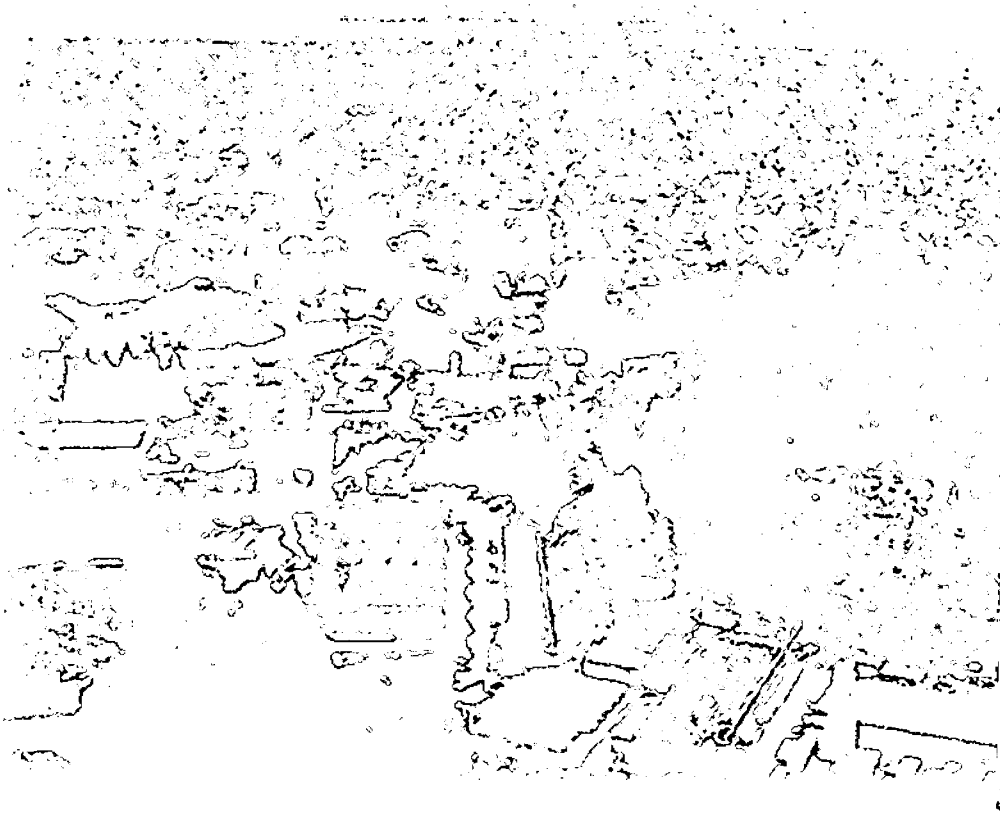

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General Properties of the Self-tuning Domain Wall Approach to the Cosmological Constant Problem

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Abstract

We study the dynamics of brane worlds coupled to a scalar field and gravity, and find that self-tuning of the cosmological constant is generic in theories with at most two branes or a single brane with orbifold boundary conditions. We demonstrate that singularities are generic in the self-tuned solutions compatible with localized gravity on the brane: we show that localized gravity with an infinitely large extra dimension is only consistent with particular fine-tuned values of the brane tension. The number of allowed brane tension values is related to the number of negative stationary points of the scalar bulk potential and, in the case of an oscillatory potential, the brane tension for which gravity is localized without singularities is quantized. We also examine a resolution of the singularities, and find that fine-tuning is generically re-introduced at the singularities in order to retain a static solution. However, we speculate that the presence of additional fields may restore self-tuning.

*J. Robert Oppenheimer fellow.

1 Introduction

There has been renewed interest in Kaluza–Klein theories over the past two years, mainly due to the realization that localization of matter [1] and localization of gravity [2] may drastically change the commonly assumed properties of such models. These theories clearly open new approaches to the cosmological constant problem*, since using extra dimensions the no-go theorem of Weinberg for adjustment mechanisms [3] may be circumvented. A particularly simple scenario has been recently suggested in [4, 5] to at least improve on the cosmological constant problem (see also [6] for an earlier mechanism involving extra dimensions), by proposing a mechanism for the cancellation of all order Standard Model (SM) loop contributions and the leading (tree-level) gravity contribution to the effective 4D cosmological constant,

$$\underbrace{\mathcal{O}(M^4)}_{\text{tree-level}} + \underbrace{\mathcal{O}(T_{br})}_{\text{SM loops}} + \underbrace{\mathcal{O}(T_{br}^2 M^{-4}) + \dots}_{\text{higher order}} \tag{1.1}$$

where M is the fundamental scale of gravity, the Planck scale in the bulk for instance. The mechanism is based on a single 3-brane (to which the SM fields are localized) embedded into a 4+1 dimensional space-time. The essential new ingredient is a bulk scalar field ϕ, which is coupled to the brane tension T_{br} (assumed to be the full loop-corrected SM vacuum energy density). The authors of [4, 5] then showed that one can find static solutions to the classical equations of motion for a vanishing bulk potential of the scalar field, but for arbitrary values of the brane tension T_{br}. However, all the solutions found in [4, 5] which localize gravity involve a naked space-time singularity, which has been interpreted as the boundary of the extra dimension (see also [7] for a discussion on singularity in a brane-world context). Since the bulk is effectively compactified, we have to worry whether the size of the extra dimension is compatible with our phenomenological knowledge of gravity which has been tested up to millimeter distances. The proper distance from the brane to the singularity is found to be $y_c \sim \kappa_5^{-2} T_{br}^{-1}$ where $M_5 = \kappa_5^{-2/3}$ is the 5D Planck scale, which is related to the 4D Planck scale, $M_4 = \kappa_4^{-1}$, by $T_{br} = \kappa_4^2/\kappa_5^4$. If the tension, associated to the vacuum energy of the SM fields, is of the order of the electroweak scale, $T_{br} \sim \text{TeV}^4$, we naturally obtain†

$$M_5 \sim 10^8\,\text{GeV} \quad \text{and} \quad y_c \sim 1\,\text{mm} \tag{1.2}$$

which is phenomenologically safe. Finally, while the SM contribution to the 4D cosmological constant would be of the order of $10^{-64}\,M_4^4$, the self-tuning mechanism cancels this term. However, the next possible term is of the order of $10^{-84}\,M_4^4$, which is still forty orders of magnitude too large. It is worth noticing that the self-tuning mechanism eliminates twenty orders of magnitude and thus should be considered on equal footing to the Randall–Sundrum solution to the hierarchy problem.

The necessity of singularities in the bulk is here only dictated by the phenomenological requirement of localized gravity. However it complements the singularity theorem of Hawking

*The cosmological constant problem concerns the brane cosmological constant that governs the expansion of our universe and it has to be distinguished to the vacuum energy density, or tension, on the brane: in particular it is possible, for a non-vanishing tension, to find solutions to the Einstein equations with a flat Minkowski metric on the brane in which case the brane cosmological constant is zero.

†The fact that the electroweak scale on the brane is five orders of magnitude below the fundamental scale, M_5, is the gauge hierarchy problem in this model and a mechanism, such as low energy supersymmetry for instance, is needed to cancel the quadratic divergences which should bring T_{br} near M_5^4.

and Penrose under which generic initial conditions lead to singular solutions of Einstein's equations [8].

Several works [9] have studied the self-tuning mechanism. In this paper, we examine the general properties of the solutions to the coupled 5D scalar-gravity system in detail. First, we investigate the solutions in the presence of a general bulk potential for the scalar field. The motivation to include a scalar bulk potential are twofold: *(i)* to give a mass to the scalar field and thus evade the cosmological problems associated to a massless scalar field coupled to gravity and/or those associated to an unstabilized extra-dimension; *(ii)* to overcome the singularity problem by considering a more general bulk geometry. We explain that the self-tuning behavior is expected to occur for a generic bulk potential. However, in the generic solutions (except for a vanishing bulk potential) self-tuning does have the more restricted interpretation that there is at least a finite region for the brane tension for which static solutions can be found (but not necessarily for all values of T_{br}). More precisely, Standard Model corrections are expected to occur at the weak scale, whereas self-tuning works up to the larger 5D Planck scale.

After presenting our general results and methods on the solutions in the presence of a generic bulk potential we ask the following question: are there bulk potentials such that the self-tuning solutions avoid naked space-time singularities for a range of values of the brane tension in such a way, that gravity is localized to the brane (so as to reproduce 4D gravity for the observer on the brane). We show that for the 5D scalar-gravity system the only possibility for such solutions is when space-time asymptotes to five dimensional anti-de Sitter space (AdS_5) far from the brane (therefore producing solutions of the sort considered in [10]). After a careful analysis we show that such solutions are always fine-tuned; that is, they occur only for particular isolated values of the brane tension T_{br}. We show that the number of allowed brane tensions is related to the number of negative stationary points of the bulk potential, which appears as a maximal index. In the case of an oscillatory bulk potential, we thus obtain a quantization of the brane tension as in a brane world construction from supergravity [11].

The paper is organized as follows: in Section 2 we present our method for searching for solutions of the coupled scalar-gravity system using the superpotential formalism of [10,12–14]. The advantage of this method is that it results in first order ordinary differential equations, and all the degrees of freedom are transparent without having to make a particular ansatz for the solution. In Section 3 we present a class of exactly solvable systems, given by an exponential bulk potential, which includes all the models presented in [4,5]. We analyze these models in detail and find that all of these solutions necessarily involve a naked singularity or lead to an infinite Planck scale on the brane. We also find that, by relaxing an ansatz for solutions made in [5], the exponential bulk potential is in fact self-tuning. In Section 4 we compare a perturbative and a numerical method to solve the equations for the most general bulk potentials, and present an example for both methods using the exactly solvable cases of Section 3. In Section 5 we present a no-go theorem for self-tuning branes that would localize gravity without singularities. In Section 6 we comment on the effect of resolution of the singularities on self-tuning. We conclude in Section 7.

2 Self-tuning and scalar fields

In this section we demonstrate that the 4D cosmological constant of branes coupled to scalar fields is generically self-tuned to zero, as in [4,5]. We begin with the action for a brane coupled to gravity and a real scalar field*,

$$S = \int d^D x \, \tfrac{1}{2\kappa_D^2} \sqrt{|G|} \left[R - \tfrac{D-2}{D-1} \left(\partial_M \phi \partial^M \phi + V[\phi] \right) \right] - \int d^{D-1} x \, \sqrt{|g|} \, \tfrac{D-2}{(D-1)\kappa_D^2} \, f[\phi] T, \quad (2.1)$$

where G_{MN} is the D-dimensional metric and $g_{\mu\nu}$ is the pullback of the metric onto the flat domain wall at $x^{D-1} \equiv y = 0$. For now we will not be concerned with the origin of the coupling $f[\phi]$ to the brane, and allow it to be arbitrary; T is the non-canonically normalized brane tension and includes standard model vacuum contributions. We look for static solutions with the metric ansatz†,

$$ds^2 = e^{-2A(y)/(D-1)} dx_{D-1}^2 + dy^2. \quad (2.2)$$

The unconventional normalization of the action (2.1) and the warp factor in (2.2) will be convenient in what follows.

The equations of motion which follow from the action (2.1) with the ansatz (2.2) are [10,14],

$$A''(y) = \phi'(y)^2 + f[\phi(y)] \, T \, \delta(y), \quad (2.3)$$
$$A'(y)^2 = \phi'(y)^2 - V[\phi(y)], \quad (2.4)$$
$$\phi''(y) - A'(y)\phi'(y) = \tfrac{1}{2} \frac{\partial V[\phi]}{\partial \phi} + \frac{\partial f[\phi]}{\partial \phi} T \, \delta(y), \quad (2.5)$$

where the primes denote derivatives with respect to y. If $\phi(y)$ is monotonic in the bulk then the bulk equations of motion can be written in a first order form [10, 12, 14] introducing an auxiliary field $W[\phi]$,

$$V[\phi] = \left(\frac{\partial W[\phi]}{\partial \phi} \right)^2 - W[\phi]^2, \quad (2.6)$$
$$\phi'(y) = \frac{\partial W[\phi(y)]}{\partial \phi(y)}, \quad (2.7)$$
$$A'(y) = W[\phi(y)]. \quad (2.8)$$

Because of the relation (2.6), $W[\phi]$ will be called *superpotential* even though no supersymmetry is involved. We will use the first order formalism in order to construct solutions in the bulk on both sides of the brane: $W_\pm$, $\phi_\pm$ and $A_\pm$ on the right (+) and left (−) hand side. But the equations of motion (2.6)-(2.8) must then be supplemented by boundary conditions at the brane. The boundary conditions due to the delta-function terms in (2.3) and (2.5) can be written,

$$\Delta \phi' = \Delta \frac{\partial W[\phi(y)]}{\partial \phi(y)} = T \frac{\partial f[\phi(y)]}{\partial \phi(y)} \Big|_{y=0}$$
$$\Delta A' = \Delta W[\phi(y)] = T \, f[\phi(y)] \Big|_{y=0}, \quad (2.9)$$

*Our conventions correspond to a mostly positive Lorentzian signature $(- + \ldots +)$ and the definition of the curvature in terms of the metric is such that a Euclidean sphere has positive curvature. Bulk indices will be denoted by capital Latin indices and brane indices by Greek indices.

†We will not look for non-flat solution on the brane such as de Sitter or anti-de Sitter 4D configurations. When they exist simultaneously with flat Minkowskian solutions, it is a dynamical question to know which solution is preferred by stability. A nice feature of the case with vanishing bulk potential is that the flat solutions are the only 4D maximally symmetric solutions [4].

where ΔF indicates the jump of a discontinuous function F at $y = 0$, $F(0^+) - F(0^-)$. In addition, $\phi(y)$ and $A(y)$ must be continuous across the boundary. Hence, there are four boundary conditions at the brane.

A count of free parameters in the solutions to the equations of motion immediately demonstrates that given $f[\phi(y)]$, the tension T is generically not fine tuned. If we do not impose orbifold boundary conditions in addition to those above, then there are naively six free parameters: one from the solution on each side of the brane of each of equations (2.6)-(2.8). However, one overall constant shift in $A(y)$ is not relevant because $A(y)$ enters into the equations of motion only through its derivatives. That leaves five free parameters and four boundary conditions. There is generically a (finite) line of solutions for a region of scalar-brane couplings $f[\phi]\,T$.

Once again, this is what we mean by self-tuning: Given an arbitrary scalar-brane coupling $f[\phi]$ (possibly satisfying some constraints), there is a range of "brane tensions" T such that static solutions exist. In the generic case, as argued above, there is in fact a continuous set of static solutions for a given boundary condition specified by $f[\phi]$ and T. Furthermore, if $f[\phi] = f'[\phi]$ then the range of T often extends to infinity. The reason is that if $W[\phi] \gg V[\phi]$ asymptotically when W is large, then $W' \simeq W$ there, and T can be chosen arbitrarily large such that $(fT, f'T) \sim (2W, 2W')$.

Orbifold boundary conditions are more constraining. The additional constraints from the orbifold condition are $A(y) = A(-y)$ and $\phi(y) = \phi(-y)$, which implies that $W_+[\phi] = -W_-[\phi]$, where $W_\pm[\phi]$ are the solutions for W on the two sides of the brane. However, continuity of $A(y)$ and $\phi(y)$ is then guaranteed. Hence, in this case there are two free parameters (there would be three but a constant shift in A is not relevant) and two boundary conditions, and there is generically a solution for a region of couplings $f[\phi]\,T$. Thus, there is generically self-tuning in this case, as well.

In the absence of orbifold type boundary conditions, if there are N branes then there are $3N+3-1 = 3N+2$ free parameters and $4N$ constraints, leaving $2-N$ free parameters in the solution for a given set of boundary conditions. Hence, there can be up to two branes without fine-tuning. With orbifold boundary conditions, where image branes are included in N and there is assumed to be a brane at the orbifold fixed point (which contributes 1 to N), there are $3(N-1)/2+3-1 = (3N+1)/2$ free parameters and $4(N-1)/2+2 = 2N$ constraints, leaving $(1-N)/2$ free parameters in the solution. Hence, only if there is a single brane at the orbifold fixed point will self-tuning occur. If the space is compactified on a circle with orbifold boundary conditions, then a parameter count for the case of branes at the two orbifold fixed points demonstrates that a fine-tuning is necessary in this case, as well [10]. Namely, there are two free parameters in the solution but four boundary conditions. In what follows we will concentrate on the case of a single brane coupled to a scalar field.

3 Integrable bulk potentials

In this section we study some special cases which were also partially discussed in [4, 5]. Our discussion of the exact solutions with a vanishing bulk potential is similar, but we will not restrict ourselves to the ansatz $A'[\phi] \propto \phi'$ made in [5]. In agreement with the parameter count in the previous section we will find that there is no fine-tuning in these theories, although some of the exact solutions exhibit non-generic behavior.

Let us first find the exactly solvable models. The challenge is finding a class of solutions to the nonlinear equation (2.6) for a given $V[\phi]$. If (in a region where* $V < 0$) we write $W[\phi]$ and $W'[\phi]$ as†,

$$
\begin{aligned}
W &= \tfrac{1}{2}\sqrt{-V[\phi]}\left(w[\phi] + \frac{1}{w[\phi]}\right), &\quad (3.1)\\
W' &= \tfrac{1}{2}\sqrt{-V[\phi]}\left(w[\phi] - \frac{1}{w[\phi]}\right), &\quad (3.2)
\end{aligned}
$$

then (2.6) is immediately satisfied for any *prepotential* $w[\phi]$. The consistency of (3.1) and (3.2) then translates into a differential equation for $w[\phi]$:

$$
\omega' = \omega - \frac{V'}{2V}\,\omega\,\frac{\omega^2+1}{\omega^2-1} \qquad (3.3)
$$

Exact solutions for $w[\phi]$ can be found when (3.3) is separable, *i.e.* when $V'[\phi]/V[\phi]$ is a constant.

We distinguish three cases:

- a vanishing bulk potential: $V[\phi] = 0$.
- a negative bulk cosmological constant: $V[\phi] = \Lambda < 0$.
- an exponential bulk potential: $V'[\phi]/V[\phi] = \text{const.} \neq 0$.

3.1 Vanishing bulk potential

This case has been extensively studied by refs. [4, 5] but it is a worthwhile exercise to repeat the discussion in terms of a superpotential. The equation for the superpotential can be solved without introducing a prepotential. Indeed, eq. (2.6) becomes simply

$$
W'[\phi]^2 = W[\phi]^2, \qquad (3.4)
$$

with two branches of solutions,

$$
W[\phi] = c\,e^{\epsilon\phi}. \qquad (3.5)
$$

where c is a constant of integration and ϵ is a sign, both of them can take different values on the two sides of the brane (the constants of integration relative to the right (left) hand side of the brane will be denoted with a $+$ $(-)$ subscript). With this form of the superpotential, the eqs. (2.7)-(2.8) can be easily solved as

$$
\begin{aligned}
\phi(y) &= -\epsilon\ln(d - cy) &\quad (3.6)\\
A(y) &= -\ln(d - cy) + e &\quad (3.7)
\end{aligned}
$$

*The case of a positive bulk potential can be studied in a very similar way up to some changes of sign in the equations (3.1)–(3.2).

†From now, we will denote by a prime a derivative of V, f, W or ω with respect to ϕ; or a derivative of ϕ or A with respect to y.

where d and e are some constants of integration that can also differ on the sides of the branes. Moreover, to make sense eq. (3.6) need: $d_+ > 0$ and $d_- > 0$. Thus the continuity requires

$$\phi(0) \equiv \phi_0 = -\epsilon_+ \ln d_+ = -\epsilon_- \ln d_- \tag{3.8}$$

$$A(0) \equiv A_0 = e_+ - \ln d_+ = e_- - \ln d_- \tag{3.9}$$

while the jump equations are

$$\frac{\epsilon_+ c_+}{d_+} - \frac{\epsilon_- c_-}{d_-} = f'[\phi_0]\, T \tag{3.10}$$

$$\frac{c_+}{d_+} - \frac{c_-}{d_-} = f[\phi_0]\, T \tag{3.11}$$

From the expression of the warp factor, we conclude that the Planck scale on the brane, $\kappa_{D-1}^{-2} = \kappa_D^{-2} \int dy\, e^{-(D-3)A/(D-1)}$, is finite *iff* singularities are encountered on both sides of the brane *i.e.* $c_+ > 0$ and $c_- < 0$.

- if $\epsilon_+\epsilon_- = 1$: the consistency of the jump equations requires that $f'[\phi_0] = \epsilon f[\phi_0]$ and then it is possible to find solutions with or without singularities for any value of the brane tension but the singular solutions correspond to $f[\phi_0]\, T > 0$.
- if $\epsilon_+\epsilon_- = -1$: the solutions with singularities exist only if $f[\phi_0]\, T > 0$ and $-1 < f'[\phi_0]/f[\phi_0] < 1$ but do not require any fine-tuning.

If we choose $f[\phi] = Ce^{\epsilon\phi}$ for the case $\epsilon = \epsilon_+ = \epsilon_-$ then it is clear that the boundary conditions can be satisfied for any T. There are several important comments to make about this case, which is quite non-generic. First of all, the fact that a specific form had to be chosen for $f[\phi]$ is a result of two non-generic features of the model: First, there is a symmetry $\phi \to \phi + const.$. As a result of this symmetry, one of the free parameters, namely ϕ_0, does not appear on the left hand side of the boundary conditions for the derivatives ϕ' and A'. In addition, it turned out that the left hand sides of the two boundary conditions had the same form up to a sign. As a result, only $f'[\phi]/f[\phi]$ is relevant, and given one solution $(f[\phi]T, f'[\phi]T)$, there is an infinite set of solutions with the *same* $f[\phi]$ and arbitrary T. The fact that T is completely arbitrary is most likely unique to the case of vanishing bulk potential. But the fact that given $f, f' \sim \mathcal{O}(M_5)$, self-tuning occurs for T to within $\mathcal{O}(M_5) \gg \mathcal{O}(M_{EW})$ (where $\mathcal{O}(M_{EW})$ is the expected Standard Model contribution to the tension) is generic, and is what we mean by self-tuning.

Even given the constraint on $f[\phi]$ from the shift symmetry in ϕ in this case, as explained in [4,5] the required exponential form of $f[\phi]$ might be natural from a stringy perspective where ϕ is interpreted as the dilaton.

Furthermore, as pointed out in [5], the solutions with singularities on either side of the brane must be chosen in order to have a finite gravitational coupling (assuming that the spacetime can be cutoff at the singularity in a consistent way). This feature will turn out to be generic except in theories which admit solitonic solutions for fine-tuned boundary conditions *i.e.* for special discrete values of the brane tension.

3.2 Bulk cosmological constant

Even though the case of a (negative) bulk cosmological constant can be studied without introducing a prepotential, we will present our method on this rather simple example before proceeding to the somewhat more complicated case of an exponential bulk potential in the next subsection. We will denote by $V[\phi] = \Lambda < 0$ the cosmological constant. The equations of motion are:

$$\begin{aligned} \frac{d\omega}{d\phi} &= \omega \ , && (3.12) \\ \frac{d\phi}{dy} &= \tfrac{1}{2}\sqrt{-\Lambda}\,\omega(1-\omega^{-2}) \ , && (3.13) \\ \frac{dA}{dy} &= \tfrac{1}{2}\sqrt{-\Lambda}\,\omega(1+\omega^{-2}) \ . && (3.14) \end{aligned}$$

The differential equation for the prepotential can be easily integrated,

$$\omega = c\, e^{\phi} \ , \qquad (3.15)$$

where c is a constant of integration. Plugging this expression for the prepotential into the remaining equations of motion, we obtain,

$$\begin{aligned} \phi(y) &= -\ln(c\,\vartheta(y)) \ , && (3.16) \\ A(y) &= -\ln|\vartheta(y)| + \ln|1-\vartheta^2(y)| + a \ , && (3.17) \end{aligned}$$

where a is a constant of integration and another constant of integration, y_c, also appears in the expression of the function $\vartheta(y)$ defined, on the two different branches of solutions, by

$$\vartheta(y) = -\tanh\tfrac{\sqrt{-\Lambda}}{2}(y-y_c) \quad \text{or} \quad \vartheta(y) = -\coth\tfrac{\sqrt{-\Lambda}}{2}(y-y_c) \ . \qquad (3.18)$$

The nature of the solution depends on the sign of y_c on each side of the brane. On the right (left) hand side, a positive (negative) value of y_c will correspond to a solution involving a singularity at a finite proper distance from the brane, $y = y_c$. Near this singularity, the warp factor, $\exp(-2A/(D-1))$, goes to zero so the singularity appears as an horizon in the bulk. Conversely, if y_c^+, the value of y_c on the right hand side of the brane, is negative (respectively, $y_c^- > 0$), then the transverse dimension will be infinitely large; however, near infinity, the warp factor blows up and the Planck scale on the brane diverges, ruining any phenomenological relevance.

The values of the constants of integration are constrained by the continuity and jump equations (with $\vartheta_\pm = \tanh\sqrt{-\Lambda}y_c^\pm/2$),

$$\begin{aligned} &\phi_0 \equiv -\ln(c_+\vartheta_+) = -\ln(c_-\vartheta_-) \ , && (3.19) \\ &A_0 \equiv -\ln|\vartheta_+| + \ln|1-\vartheta_+^2| + a_+ = -\ln|\vartheta_-| + \ln|1-\vartheta_-^2| + a_- \ , && (3.20) \\ &\tfrac{1}{2}\sqrt{-\Lambda}\left(\tfrac{1+\vartheta_+^2}{\vartheta_+} - \tfrac{1+\vartheta_-^2}{\vartheta_-}\right) = f[\phi_0]\,T \ , && (3.21) \\ &\tfrac{1}{2}\sqrt{-\Lambda}\left(\tfrac{1-\vartheta_+^2}{\vartheta_+} - \tfrac{1-\vartheta_-^2}{\vartheta_-}\right) = f'[\phi_0]\,T \ . && (3.22) \end{aligned}$$

A solution to these equations can be found, provided that $f'[\phi_0] \neq \pm f[\phi_0]$, for any value of the brane tension such that

$$T^2 > \frac{-4\Lambda}{f[\phi_0]^2 - f'[\phi_0]^2} . \qquad (3.23)$$

Moreover, singularities will exist on both sides of the brane *iff*

$$f[\phi_0]T > 0 \quad \text{and} \quad -1 < \frac{f'[\phi_0]}{f[\phi_0]} < 1, \tag{3.24}$$

just as in the case of a vanishing bulk potential.

It is interesting to look at the $\mathbb{Z}_2$ symmetric solution for which there are the additional constraints:

$$y_c^+ = -y_c^-, \quad c_+ = -c_- \quad \text{and} \quad a_+ = a_-. \tag{3.25}$$

The continuity conditions are automatically satisfied but the consistency of the jump equations requires a fine-tuning,

$$f[\phi_0]^2 - f'[\phi_0]^2 = -\frac{4\Lambda}{T^2}. \tag{3.26}$$

As already discussed in the case with a vanishing bulk potential, this fine-tuning is a consequence of the translational symmetry, $\phi \to \phi + const.$, in the theory. As before, because of this shift symmetry we lose the appearance of a free parameter to adjust in the jump equations, which leads to a more restricted set of boundary conditions than for a generic bulk potential $V[\phi]$. The fine-tuning (3.26) is precisely the one appearing in the Randall–Sundrum model when the scalar coupling to the brane is a constant:

$$\Lambda_{bk} = -\frac{D-1}{8(D-2)}\kappa_D^2 T_{br}^2, \tag{3.27}$$

where Λ_{bk} and T_{br} are the canonically normalized quantities [11],

$$\Lambda_{bk} = \frac{D-2}{2(D-1)\kappa_D^2}\Lambda \quad \text{and} \quad T_{br} = \frac{D-2}{(D-1)\kappa_D^2} f[\phi_0]T. \tag{3.28}$$

The solution constructed by Randall and Sundrum that localizes gravity with an infinitely large extra dimension corresponds to a limit of the singular solution where the singularities are pushed to infinity *i.e.* $\vartheta_\pm = \pm 1$. The jump equations then require

$$f'[\phi_0] = 0 \quad \text{and} \quad f[\phi_0]T = 2\sqrt{-\Lambda} \tag{3.29}$$

i.e. the coupling between the brane and the scalar field vanishes and the canonically normalized brane tension, T_{br}, is fine-tuned to (3.27). In this limit, the expressions for the scalar field and the warp factor simply become

$$\phi = \phi_0 \quad \text{and} \quad A = \sqrt{-\Lambda}\,|y|. \tag{3.30}$$

3.3 Exponential bulk potential

The last case that can be solved analytically with our method involves an exponential potential for the scalar field in the bulk. We will concentrate on negative potential while the case of positive potential requires some minimal changes. So the bulk potential will be parametrized by two real numbers a and b:

$$V[\phi] = -a^2 e^{2b\phi}. \tag{3.31}$$

On each side of the brane, the equations of motion are simply:

$$\frac{d\omega}{d\phi} = \omega - b\,\omega\,\frac{\omega^2+1}{\omega^2-1}\,, \tag{3.32}$$

$$\frac{d\phi}{dy} = \tfrac{1}{2}\,a e^{b\phi}\,\omega(1-\omega^{-2})\,, \tag{3.33}$$

$$\frac{dA}{dy} = \tfrac{1}{2}\,a e^{b\phi}\,\omega(1+\omega^{-2})\,. \tag{3.34}$$

The sign of a is not fixed and can be chosen independently on the two sides of the brane, as we will discuss. The first differential equation can be easily solved to express ϕ in terms of ω:

$$e^{-b\phi} = e^{-bc}\,|\omega|^{-b/(1+b)}\left|1+b-(1-b)\omega^2\right|^{b^2/(b^2-1)}\,, \tag{3.35}$$

where c is a constant of integration. This last result can be used to obtain a parametric representation of A and y as functions of ω:

$$y(\omega) = -\tfrac{2}{a}e^{-bc}\int_{\omega_0}^{\omega} d\tilde{\omega}\,\frac{|\tilde{\omega}|^{-b/(1+b)}\left|1+b-(1-b)\tilde{\omega}^2\right|^{b^2/(b^2-1)}}{(1+b-(1-b)\tilde{\omega}^2)}\,, \tag{3.36}$$

$$A(\omega) = A_0 - \int_{\omega_0}^{\omega} d\tilde{\omega}\,\frac{(1+\tilde{\omega}^2)}{\tilde{\omega}\,(1+b-(1-b)\tilde{\omega}^2)}\,. \tag{3.37}$$

On the two sides of the brane, the parameter a can differ by a sign while the initial bound of integration, ω_0, and the constant of integration, c, can take any values compatible with the continuity conditions. A $\mathbb{Z}_2$ symmetric solution will correspond to two different choices of sign for a but the same values for ω_0 and c.

Different kinds of solutions can be obtained depending on the value of b and of the range of integration for the variable $\tilde{\omega}$:

- $b < -1$: in that case, $dy/d\omega$ has an integrable singularity at $\omega = +\infty$ but a non-integrable singularity at $\omega = 0$ while the singularities of $dA/d\omega$ are both non-integrable. So we can find solution with or without bulk singularity at finite proper distance.
  - Solutions without singularity will be given by (3.36)-(3.37) where on the right (left) hand side of the brane, a has to be chosen positive (negative) and ω_0 is a negative initial bound of integration. The parameter ω will range from ω_0 to 0^-. It is easy to find the asymptotic behavior of this solution for large $|y|$, *i.e.* ω near 0^-:

    $$A \underset{|y|\sim\infty}{\sim} -\ln|y|\,, \tag{3.38}$$

    from where it becomes evident that the singularity free solutions do not localize gravity because the Planck scale on the brane diverges.
  - Solutions with singularity will still be given by eqs. (3.36)-(3.37) with a positive (negative) parameter a on the right (left) hand side of the brane. And the range of integration goes from a positive initial value, ω_0, to $+\infty$. In that case, y reaches a finite value y_c while the warp factor goes like:

    $$A \underset{y\sim y_c}{\sim} -\ln|y-y_c|\,, \tag{3.39}$$

which indicates that the singularity is a horizon where the metric on the brane vanishes. The behavior of the warp factor near the singularity insures that the Planck scale on the brane, $\kappa_{D-1}^{-2} = \kappa_D^{-2} \int dy\, e^{-(D-3)A/(D-1)}$, is finite.

- $-1 < b < 1$: in that case the singularities of $dy/d\omega$ at $\omega = \pm\infty$, $\omega = \pm\sqrt{(1+b)/(1-b)}$ and $\omega = 0$ are integrable while those appearing in $dA/d\omega$ are non-integrable. All the solution are singular with a horizon or a curvature singularity.
- $1 < b$: this case is quite similar to the first case because the singularity of $dy/d\omega$ at $\omega = 0$ is integrable but the singularity at $\omega = +\infty$ is non-integrable while the two singularities of $dA/d\omega$ are both non-integrable. So we can construct solutions with or without singularity.
  - Solutions without singularity will be given by (3.36)-(3.37) where on the right (left) hand side of the brane, a has to be chosen negative (positive) and ω_0 is a positive initial bound of integration. The parameter ω will range from ω_0 to $+\infty$. It is easy to find the asymptotic behavior of this solution for large $|y|$, *i.e.* ω near $+\infty$:

$$A \underset{|y|\sim\infty}{\sim} -\ln|y| \,, \tag{3.40}$$

    from where, once again, it becomes evident that the singularity free solution does not localize gravity.
  - Solutions with singularity will still be given by eqs. (3.36)-(3.37) with a negative (positive) parameter a on the right (left) hand side of the brane. And the range of integration goes from a negative initial value, ω_0, to 0^-. In that case, y reaches a finite value y_c while the warp factor goes like:

$$A \underset{y\sim y_c}{\sim} -\ln|y - y_c| \,, \tag{3.41}$$

    which indicates that the singularity is a horizon where the metric on the brane vanishes.
- $b = \pm 1$: in these two cases, we can construct singularity free solutions with a blowing up warp factor as well as singular solutions with a horizon at a finite proper distance.

Figure 1 illustrates the different types of solutions.

The main result of the analysis of these integrable bulk potentials is that we find two kinds of solutions: *(i)* solutions with a horizon in the bulk at a finite proper distance from the brane and with a finite lower dimensional Planck scale; *(ii)* solutions without singularity at a finite proper distance but associated to a bulk geometry that decouples gravity on the brane. Both kinds of solutions do not require any fine-tuning and can be constructed for a range of brane tension, T. However it seems impossible to find a singularity free solution that localizes gravity on the brane, unless the brane tension is fine-tuned as in the Randall–Sundrum model. This point surely deserves further scrutiny which the next sections will be devoted to.

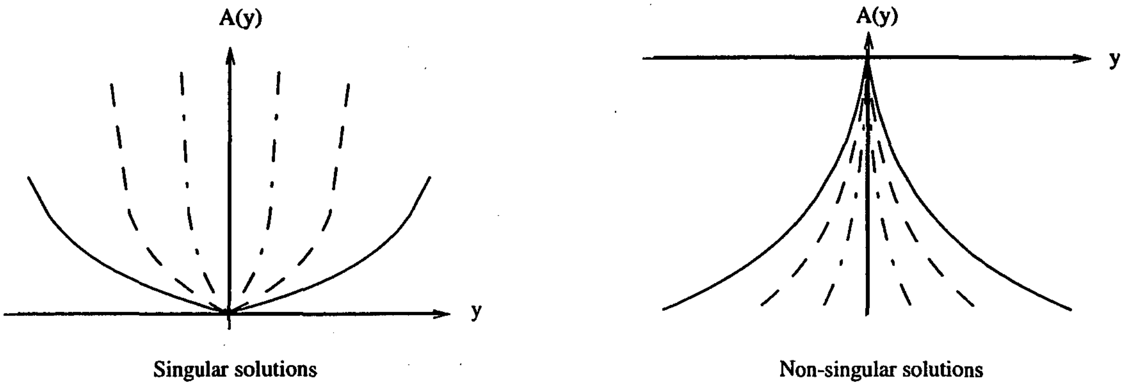

Figure 1: Shapes of the warp factor for singular and non-singular solutions to the equations of motion with an exponential bulk potential. We have drawn $\mathbb{Z}_2$ symmetric solutions for different values of the tension on the brane: when the tension increases, the horizon becomes closer and closer to the brane for the singular solutions while the warp factor, $e^{-2A/(D-1)}$, blows up faster and faster for the the singularity-free solutions. In both cases, the jump conditions require that the scalar coupling to the brane satisfies: $|(df/d\phi)/f| < 1$ on the brane.

4 Perturbative and numerical methods

In this section we give two methods for finding approximate solutions to the coupled equations of motion in the bulk that satisfy the boundary conditions at the brane. First, we present a perturbative method, which we then show to break down for the interesting case of perturbing around a solution with localized gravity. Then we give a systematic numerical method for solving the equations.

4.1 A perturbative method

As emphasized in the previous section, the key to finding the self-tuned solutions is to utilize the fact that one can transform the superpotential without changing the bulk potential for the scalar field. Thus, one has an additional degree of freedom if one picks a different superpotential function on the two sides of the brane, $W_+[\phi]$ and $W_-[\phi]$. Let us now assume that we have found a static solution to the equations of motion $\phi_0(y)$ and $A_0(y)$, and assume that we have chosen one of the integration constants such that $A_0(0) = 0$. This is a solution for a particular value T of the brane tension. In order to find the solution obtained by perturbing the brane tension as $T \to T + \delta T$, we first need to find how one can change the superpotential W around $W_0[\phi]$ so as to leave the bulk potential unchanged:

$$V[\phi] = W_0'[\phi]^2 - W_0[\phi]^2, \tag{4.1}$$

which should be invariant under $W \to W + \delta W$. Linearizing (4.1) around W_0 we obtain a differential equation for δW of the form

$$\frac{\delta W'}{\delta W} = \frac{W_0}{W_0'}, \tag{4.2}$$

which is solved by

$$\delta W[\phi] = C \exp \int \frac{W_0[\phi]}{W_0'[\phi]} d\phi, \tag{4.3}$$

where the arbitrary constant C yields the extra degree of freedom needed to find a solution for any value of T. The superpotential variation can be expressed in terms of the unperturbed background solution using the equations of motion $d\phi_0/W_0'[\phi_0] = dy$ and $W_0[\phi_0] = dA_0/dy$, we obtain

$$\delta W[\phi(y)] = C \exp \int dy \frac{dA_0}{dy} = C\, e^{A_0(y)}. \tag{4.4}$$

From (4.3) we can see that the change in the derivative of the superpotential is given by

$$\delta W'[\phi(y)] = C \frac{A_0'(y)}{\phi_0'(y)} e^{A_0(y)}. \tag{4.5}$$

In order to satisfy the jump equations (2.9) for the perturbed tension $T + \delta T$, we need to choose C differently on the two sides of the brane. The values of $C_\pm$ are then given by

$$C_\pm = \frac{f'[\phi_0] - f[\phi_0] \frac{A_{0\mp}'(0)}{\phi_{0\mp}'(0)}}{\Delta \frac{A_0'}{\phi_0'}} \delta T, \tag{4.6}$$

where $\Delta\, A_0'/\phi_0'$ denotes the jump in A_0'/ϕ_0' at $y = 0$. Once $C_\pm$ is determined from (4.6), we can simply integrate the equations

$$\delta\phi'(y) = \delta W'[\phi_0(y)], \quad \delta A'(y) = \delta W[\phi_0(y)] \tag{4.7}$$

to obtain the perturbed solutions $\phi_0(y) + \delta\phi(y)$ and $A_0(y) + \delta A(y)$. This method always results in a perturbed solution. However, in the most interesting case, when the unperturbed solution asymptotes to AdS space thereby localizing gravity to the brane (that is for $A(y) \sim (D-1)|y|/R_{AdS}$ for large values of y), one can easily see that the perturbative method presented here always breaks down. This can be seen by inspecting (4.4), which shows that in this case $\delta W[\phi(y)] \propto e^{(D-1)|y|/R_{AdS}}$. Therefore the perturbed values of $\delta\phi$ and δA grow exponentially, and thus the linearized approximation breaks down. We will examine the case of localized gravity in detail in Section 5. But first we give a numerical method that can be used for any choice of the bulk potential.

4.2 A numerical method

Next we present a method for solving the system (2.6)-(2.8) numerically for any brane tension and potential in the bulk. Thus the input functions are the bulk potential $V[\phi]$, the brane tension T, the coupling of the scalar to the brane determined by the function $f[\phi]$, and in addition we can pick the value of the scalar field at the brane $\phi(0) \equiv \phi_0$ arbitrarily. In order to find a numerical solution to these equations, one has to first make sure that the boundary conditions that one imposes do satisfy the jump equations (2.9). Our strategy is the following: we first determine the superpotential functions $W_+[\phi]$ and $W_-[\phi]$ to the left and the right of the brane numerically such that the boundary conditions arising from the coupling to the brane are satisfied. This can be done by noting that once ϕ_0 is fixed, the jump equations are just given by

$$W_+ - W_- = f_0 T, \quad W'_+ - W'_- = f'_0 T, \tag{4.8}$$

where $W_\pm$ refers to the values of the superpotential functions to the right and left of the brane at ϕ_0, $f_0 = f[\phi_0]$, etc. In addition, the superpotential functions must be such that they reproduce the correct value of the bulk potential at the brane:

$$W'^2_+ - W^2_+ = V_0, \quad W'^2_- - W^2_- = V_0, \tag{4.9}$$

where $V_0 = V[\phi_0]$. Eqs. (4.8) and (4.9) together are enough to determine the values of both $W_\pm$ and $W'_\pm$ at the branes. They are given by the expressions:

$$\begin{aligned} W_\pm &= \pm\frac{1}{2} f_0 T + \frac{1}{2} f'_0 \sqrt{T^2 + \frac{4V_0}{f'^2_0 - f^2_0}} \\ W'_\pm &= \pm\frac{1}{2} f'_0 T + \frac{1}{2} f_0 \sqrt{T^2 + \frac{4V_0}{f'^2_0 - f^2_0}}. \end{aligned} \tag{4.10}$$

Due to the quadratic nature of equations (4.8) and (4.9) there is a second solution, where the signs in front of the square roots in (4.10) are both simultaneously flipped. Once the value of $W_\pm$ and $W'_\pm$ are fixed, one can numerically integrate the equation*

$$W'_\pm[\phi] = \mathrm{sgn}(W'_\pm)\sqrt{V[\phi] + W_\pm[\phi]^2} \tag{4.11}$$

to obtain the superpotential functions to the left and the right of the brane that satisfy all boundary conditions. Once $W_\pm[\phi]$ are numerically known, we can simply integrate the equations

$$\phi'(y) = W'[\phi(y)], \quad A'(y) = W[\phi(y)] \tag{4.12}$$

to the left and the right of the brane to obtain the numerical solutions for $\phi(y)$ and $A(y)$. We will show an example for this below for the case when the exact solution is known, and compare the two results.

4.3 An example for the perturbative method

In this Section we test how well the perturbative method described in 4.1 works. We will compare the analytic solution of the model with a vanishing bulk potential to the perturbed solution around a different analytic solution. The main conclusions are as expected: the perturbative method works well far from the singularities. However it gets worse as we approach the singularity itself, and does not capture the essential feature of the self-tuning solutions: whereas in the self-tuning mechanism, for a fixed value of the scalar field on the brane, the place of the singularity adjusts itself with respect to the value of the brane tension, here the

*We will see in Section 5 that, unless the brane tension is fine-tuned, the superpotential, $W[\phi]$, will be a monotonic function of ϕ and thus $W'_\pm[\phi]$ will keep the sign of $W'_\pm$.

singularity of the perturbed solution remains at the same place where the singularity of the unperturbed solution was. Therefore, we conclude that this method is not very efficient in capturing the basic properties of the self-tuning solutions.

The example we consider is the vanishing bulk potential discussed in 3.1, with the choice $\epsilon_+ = 1, \epsilon_- = -1$, and we choose $f[\phi] = e^{\phi/2}$, $\phi_0 = 1$, and for the unperturbed solution we choose $T = 1$, while we pick $\delta T = 0.1$ for the perturbed solution. The analytic solution is given by (3.6)–(3.7), with

$$d_\pm = e^{\mp\phi_0}, \quad c_+ = \frac{3}{4}Te^{-\phi_0/2}, \quad c_- = -\frac{1}{4}Te^{3\phi_0/2}, \quad e_\pm = \mp\phi_0, \tag{4.13}$$

where A has been normalized to zero on the brane. The perturbed solution from (4.7) is given by

$$\delta\phi_+(y) = -\frac{\delta T}{T}\ln\left(\frac{d_+ - c_+y}{d_+}\right), \quad \delta\phi_-(y) = \frac{\delta T}{T}\ln\left(\frac{d_- - c_-y}{d_-}\right). \tag{4.14}$$

where $\delta\phi_\pm$ has been normalized to zero on the brane. The perturbed solution obtained for $T = 1, \delta T = 0.1$ compared to the exact solution for $T = 1.1$ can be seen in Fig. 2. As mentioned above, the perturbative solution nicely follows the exact solution away from the singularity, but deviates from it close to the singularity, in particular the place of the singularity is incorrectly predicted to coincide with the singularity of the unperturbed solution.

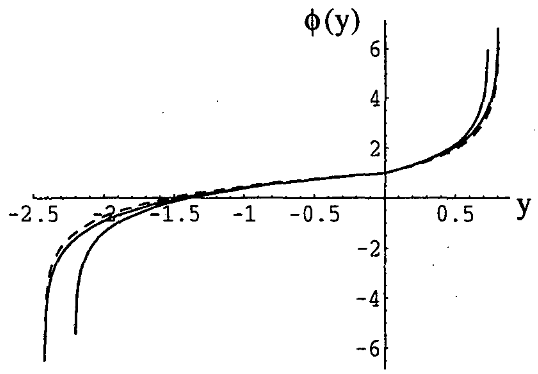

Figure 2: The exact solution versus the perturbed solution for the case of a vanishing bulk potential. The curve that blows up at smaller values of y corresponds to the exact solution. The initial unperturbed solution is given by the dashed curve. The singularity of the perturbed solution appears at the same distance from the brane as the singularity of the initial solution; the shift of the singularity with a variation of the tension is missing.

4.4 An example for the numerical method

We have shown above, that the perturbative method in general does not do a good job in finding the solutions, since it becomes unreliable close to the singularities. However, the numerical solution should not have these problems. Indeed, we analyze the same example as above (the case with vanishing bulk potential) using the numerical method, and find that the exact and numerical curves are virtually indistinguishable. Therefore, we suggest that in order to analyze potentials for which no exact solutions can be found, one should use the numerical method rather than the method based on perturbations.

We are looking for a numerical solution to the case analyzed perturbatively above, that is vanishing bulk potential, $f[\phi] = e^{\phi/2}$, $\phi_0 = 1$, $T = 1$ and $\epsilon_+ = 1, \epsilon_- = -1$. From Eqs. (3.4) and (3.8) we find the starting values of the superpotential to the left and the right of the brane:

$$W_+ = \frac{3}{4} e^{\phi_0/2} T, \quad W_- = -\frac{1}{4} e^{\phi_0/2} T. \tag{4.15}$$

Numerically integrating the equation

$$W'_\pm[\phi] = \pm W[\phi] \tag{4.16}$$

with the boundary conditions $W_\pm[\phi_0] = W_\pm$ one obtains the numerical values for $W_\pm(\phi)$. Finally, the values for $\phi(y)$ can be obtained by numerically inverting the integral

$$y = \int_{\phi_0}^{\phi} \frac{d\phi}{W'(\phi)} \tag{4.17}$$

to the left and right of the brane. The numerical solution obtained this way overlayed on the exact solution of Section 2 can be seen in Fig. 3. One can see that the two curves are virtually indistinguishable, suggesting that the numerical method works very well around the singularities, and should be the preferred method of looking for solution in the absence of exact solutions.

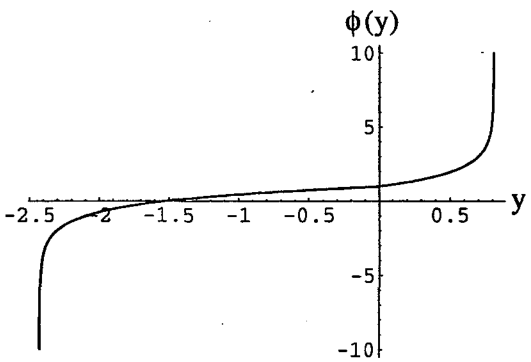

Figure 3: The numerical solution overlayed on the exact solution for the case of a vanishing bulk potential. The fact that the two curves are indistinguishable shows that the numerical method works extremely well even close to the singularities.

5 Localized gravity without singularities

5.1 A no-go theorem

We would like to reexamine in this section the count of free parameters versus fine-tuning parameters needed to preserve a static Poincaré invariance on the brane, with the restriction that the solution corresponds to an infinitely large extra-dimension (without singularities) in the bulk and localizes gravity on the brane.

Consider the general D-dimensional background preserving $Poincaré_{D-1}$

$$ds^2 = e^{-2A(y)/(D-1)} dx_{D-1}^2 + dy^2 \ . \tag{5.1}$$

The graviton zero-mode is localized on the brane* precisely when the effective Planck scale is finite on the brane [14]. In terms of the warp factor, this condition is:

$$\frac{1}{\kappa_{D-1}^2} = \frac{1}{\kappa_D^2} \int dy \ e^{-(D-3)A(y)/(D-1)} \ < \ \infty \ . \tag{5.2}$$

It is convenient to introduce a transverse coordinate z for which the bulk metric is conformally flat:

$$ds^2 = \Omega^2(z) \left(dx_{D-1}^2 + dz^2 \right) \ , \tag{5.3}$$

where the conformal factor, Ω, is related to the warp factor, $e^{-2A(y(z))/(D-1)}$, by the two identities:

$$\Omega(z) = e^{-A(y)/(D-1)} \quad \text{and} \quad \Omega^2(z) dz^2 = dy^2 \ . \tag{5.4}$$

Then the condition (5.2) is equivalent to having a massless normalizable bound state, which is interpreted as the graviton on the brane:

$$\psi_0 \propto \Omega^{(D-2)/2} \quad \text{with} \quad \int dz \ |\psi_0|^2 < \infty \ . \tag{5.5}$$

Let us assume that the behavior of ψ_0 at infinity is a power law:

$$\psi_0 \underset{z \sim \infty}{\propto} z^{-\alpha} \ . \tag{5.6}$$

The localization of gravity (5.5) then requires: $\alpha > 1/2$.

In our study, the value of the parameter α is constrained by the fact that the background is created by a scalar field coupled to gravity. From the equations of motion, we easily deduce that A has to satisfy: $d^2A/dy^2 \geq 0$, which translates, in the z coordinate, in a lower bound on the value of α:

$$\alpha \geq \frac{D-2}{2} \ . \tag{5.7}$$

Furthermore it is worth noticing that an upper bound on α comes by the requirement of a geometry without singularity at a finite proper distance†. Indeed the proper distance from the brane to infinity is given: $l_\infty = \int dz \, \Omega$ that diverges *iff*:

$$\alpha \leq \frac{D-2}{2} \ . \tag{5.8}$$

*We do not consider the recently proposed possibility that gravity might be quasi-localized to the brane [16], since in those models the $A'' > 0$ condition is not satisfied, therefore it is not possible to generate those backgrounds from a single scalar field.

†For a power law conformal factor, the curvature always vanishes at infinity. However quadratic invariants such as $R_{M_1 M_2 M_3 M_4} R^{M_1 M_2 M_3 M_4}$ will be singular at infinity as soon as $\alpha > (D-2)/2$.

So the only background for the scalar field coupled to gravity that localizes gravity without singularity (with an infinitely large extra dimension) is asymptotic, at infinity, to the horizon of an anti-de Sitter space, as in the RS model, and corresponds to $\alpha = (D-2)/2$. In that case, the warp factor is exponentially decreasing with the proper distance to the brane:

$$A \underset{|y|\sim\infty}{\sim} (D-1)|y|/R_{AdS} \,. \tag{5.9}$$

The aim of this section is to show that such a background necessarily requires a fine-tuning between the brane and the bulk. This is not to say that there is no self-tuning in these models, only that the nonsingular solutions require fine-tuning.

The previous asymptotic behavior has a nice interpretation in terms of the superpotential, $W[\phi]$. According to the equations of motion, the fact that A is asymptotically linear means that ϕ becomes constant and we will denote by ϕ_c^- and ϕ_c^+ the asymptotic values of ϕ at $y = -\infty$ and $y = +\infty$ respectively.

The equation

$$\frac{\partial \phi}{\partial y} = \frac{dW}{d\phi}, \tag{5.10}$$

is similar to an RGE with W' playing the role of the β-function. In order for ϕ to approach a constant (fixed point) at infinity, the β-function $dW/d\phi$ must have zeroes. In other words, in order to extend the range of the transverse coordinate from $y = -\infty$ to $y = +\infty$, the values ϕ_c^- and ϕ_c^+ at infinity must be some roots of $dW/d\phi$:

$$\left.\frac{dW}{d\phi}\right|_{\phi_c^-} = 0 \quad \text{and} \quad \left.\frac{dW}{d\phi}\right|_{\phi_c^+} = 0 \,. \tag{5.11}$$

Furthermore, at infinity, A has to be linearly *increasing* in $|y|$; otherwise the conformal infinity of AdS would be reached without localized gravity [11]. Given that

$$\frac{\partial A}{\partial y} = W[\phi], \tag{5.12}$$

we conclude that ϕ_c^- and ϕ_c^+ must satisfy,

$$W[\phi_c^-] < 0 \quad \text{and} \quad W[\phi_c^+] > 0. \tag{5.13}$$

Finally, ϕ_c^- and ϕ_c^+ must be dynamically reached at $y = -\infty$ and $y = +\infty$, which according to (5.10) is possible *iff*:

$$\left.\frac{d^2W}{d\phi^2}\right|_{\phi_c^-} > 0 \quad \text{and} \quad \left.\frac{d^2W}{d\phi^2}\right|_{\phi_c^+} < 0, \tag{5.14}$$

or at least a similar condition for the first non-vanishing higher order derivatives at ϕ_c^- and ϕ_c^+.

Pictorially, the previous conditions are summarized in figure 4.

Finally, the last equation of motion that relates the superpotential, W, to the scalar potential in the bulk, V, partially fixes the possible values of ϕ_c^- and ϕ_c^+. Indeed this differential equation evaluated at infinity gives:

$$V[\phi_c^-] = -W[\phi_c^-]^2 < 0 \quad \text{and} \quad V[\phi_c^+] = -W[\phi_c^+]^2 < 0. \tag{5.15}$$

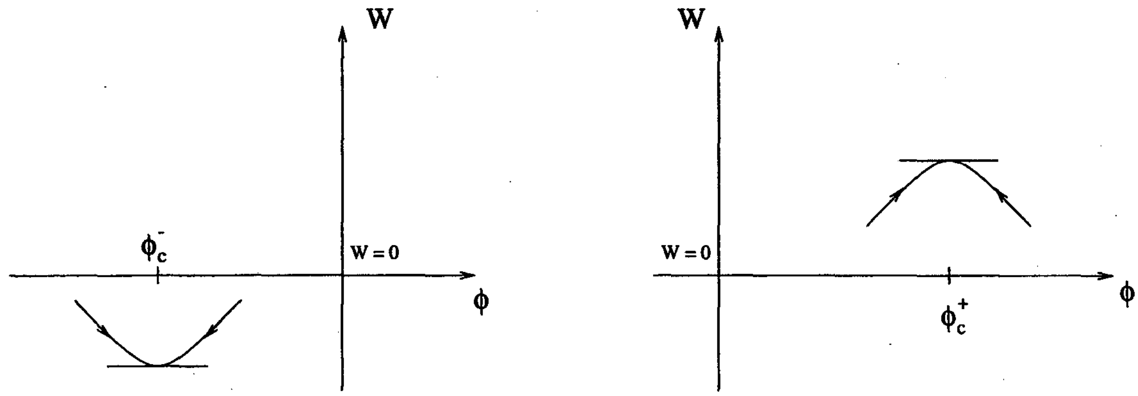

Figure 4: Asymptotic behaviors of $W[\phi]$ leading to a singularity free bulk geometry localizing gravity on the brane. The absence of singularities is equivalent to the conditions: ${W'}_c^- = {W'}_c^+ = 0$; the dynamics of the equations of motion require ${W''}_c^- > 0$ and ${W''}_c^+ < 0$, while the localization of gravity requires $W_c^- < 0$ and $W_c^+ > 0$.

while a differentiation with respect to ϕ gives:

$$\frac{dV}{d\phi} = 2\frac{dW}{d\phi}\left(\frac{d^2W}{d\phi^2} - W\right) \; ; \quad \text{thus} \quad \frac{dV}{d\phi}_{|\phi_c^-} = 0 \text{ and } \frac{dV}{d\phi}_{|\phi_c^+} = 0. \tag{5.16}$$

More information on W can be obtained by considering higher order derivatives of the differential equation between W and V. Indeed it is easy to prove by induction the following relation:

$$V^{(n)} = \sum_{k=1}^{n} 2\binom{n-1}{k-1} W^{(k)}\left(W^{(n-k+2)} - W^{(n-k)}\right) \tag{5.17}$$

where $V^{(n)}$ denotes the n^{th} order derivative of V and similarly for $W^{(n)}$; in addition, we will denote by ${W^{(n)}}_c^{\pm}$ the values of $W^{(n)}$ at $\phi = \phi_c^{\pm}$. At the second order, by evaluating (5.17) at $\phi = \phi_c^{\pm}$, we obtain a quadratic equation for ${W^{(2)}}_c^{\pm}$:

$$2{W^{(2)}}_c^{\pm 2} - 2W_c^{\pm}{W^{(2)}}_c^{\pm} - {V^{(2)}}_c^{\pm} = 0 \; . \tag{5.18}$$

The superpotential will be real-valued provided that:

$${V^{(2)}}_c^{\pm} > V_c^{\pm}/2 \; , \tag{5.19}$$

and then there are four different branches at each asymptotic point:

$$W_c^{\pm} = \epsilon_1\sqrt{-V_c^{\pm}} \; , \tag{5.20}$$

$${W^{(2)}}_c^{\pm} = \epsilon_1\frac{\sqrt{-V_c^{\pm}}}{2} + \epsilon_2\frac{\sqrt{2{V^{(2)}}_c^{\pm} - V_c^{\pm}}}{2} \; , \tag{5.21}$$

with $\epsilon_1 = \pm\epsilon_2 = \pm 1$.

However the compatibility between the gravity localization requirement (5.13) and the dynamics of the differential equation (5.14) can be fulfilled *iff*

$$V^{(2)}{}^{\pm}_{c} \geq 0 \,, \tag{5.22}$$

and only one out of the four branches is retained:

$$\phi_c^- : \quad W_c^- = -\sqrt{-V_c^-} \quad W^{(2)}{}^{-}_{c} = -\frac{\sqrt{-V_c^-}}{2} + \frac{\sqrt{2V^{(2)}{}^{-}_{c} - V_c^-}}{2} \,, \tag{5.23}$$

$$\phi_c^+ : \quad W_c^+ = \sqrt{-V_c^+} \quad W^{(2)}{}^{+}_{c} = \frac{\sqrt{-V_c^+}}{2} - \frac{\sqrt{2V^{(2)}{}^{+}_{c} - V_c^+}}{2} \,. \tag{5.24}$$

At higher order, the relation (5.17) becomes linear in $W^{(n)}{}^{\pm}_{c}$ and allows to compute $W^{(n)}{}^{\pm}_{c}$ recursively in terms of lower derivatives:

$$W^{(n)}{}^{\pm}_{c} = \frac{V^{(n)}{}^{\pm}_{c} - \sum_{k=3}^{n-1} 2\binom{n-1}{k-1} W^{(k)}{}^{\pm}_{c} \left(W^{(n-k+2)}{}^{\pm}_{c} - W^{(n-k)}{}^{\pm}_{c} \right) + 2(n-1) W^{(2)}{}^{\pm}_{c} W^{(n-2)}{}^{\pm}_{c}}{2\left(n W^{(2)}{}^{\pm}_{c} - W_c^{\pm} \right)} \,. \tag{5.25}$$

Of course, for this expression to make sense for any integer $n > 2$, it is important that no solution to the equation $nW^{(2)}{}^{\pm}_{c} - W_c^{\pm} = 0$ can be found. However, the roots of this equation would satisfy:

$$\frac{V^{(2)}{}^{\pm}_{c}}{V_c^{\pm}} = \frac{2}{n}\left(1 - \frac{1}{n}\right) \,, \tag{5.26}$$

which is incompatible with the physical requirements (5.15) and (5.22) of localized gravity.

At this stage, it is worth noticing that the uniqueness of a superpotential W reaching a given asymptotic point follows from our requirements of a localized gravity without singularity. Otherwise, a scalar field V in the bulk could be constructed such that the equation $nW^{(2)}{}^{\pm}_{c} - W_c^{\pm} = 0$ has some solution in which case $W^{(n)}{}^{\pm}_{c}$ would not be determined, leading to a continuum of solutions.

Finally, the whole expression of W reaching the asymptotic points $\phi_c^{\pm}$ can be uniquely reconstructed from its derivatives through a Taylor expansion:

$$y < 0 : \quad W_-[\phi] = \sum_{n=0}^{\infty} \frac{1}{n!} W^{(n)}{}^{-}_{c} (\phi - \phi_c^-)^n \,, \tag{5.27}$$

$$y > 0 : \quad W_+[\phi] = \sum_{n=0}^{\infty} \frac{1}{n!} W^{(n)}{}^{+}_{c} (\phi - \phi_c^+)^n \,. \tag{5.28}$$

From the expression of the superpotential, we can solve the equations of motion on the two sides of the brane. And the jump conditions are given by:

$$\frac{W_+[\phi_0] - W_-[\phi_0]}{f[\phi_0]} = \frac{W_+'[\phi_0] - W_-'[\phi_0]}{f'[\phi_0]} = T \,. \tag{5.29}$$

The first equation will fix the value, ϕ_0, of the scalar field on the brane while the second one fine-tunes the value of the brane tension. Generically, we will obtain only discrete values of ϕ_0.

Indeed, if there exist a continuum interval of solutions for ϕ_0, then the jump equation becomes a differential equation that can be integrated on this interval and we obtain that $f[\phi]$ has to be proportional to $W_+[\phi] - W_-[\phi]$, but in that case there is only one possible value for the brane tension[‡] In all the other cases, given two asymptotic points, $\phi_c^{\pm}$, we will obtain discrete solutions for ϕ_0 and T. Moreover, using different critical asymptotic points, we usually find different values of the brane tension and the number of values of T that allows an infinitely large extra dimension with localized gravity is related to the number of critical points of the bulk potential V satisfying (5.15), (5.16) and (5.22):

$$n_T \leq (3n_C - 2)n_S, \tag{5.30}$$

where n_T stands for the number of values of T such that gravity is localized without a singularity, while n_C is the number of critical points of V as defined above. The multiplicity factor comes from the fact that the scalar field can asymptote either the same critical point or two adjacent ones on the two sides of the brane. The upper bound has been semi-quantitatively corrected by taking into account the average number, n_S, of solutions to the jump equation for ϕ_0. It may also happen that some values of T are degenerate. We will explicitly describe an example in the next subsection.

It is worth noticing that in the case of an oscillatory bulk potential, we will obtain an infinite number of discrete values for the brane tension that is as quantized.

To complete the proof of the no-go theorem presented above, we need to show that there is no loophole in the above argument due to the fact that we have used the superpotential formalism. The subtlety that one might worry about is that in the proof above we have implicitly assumed that ϕ is monotonic, by writing the second order equations (2.3)-(2.4) in terms of first order equations involving W. In particular, if ϕ is not monotonic (that is if $\phi' = 0$ at a finite value of y) one does not have a globally defined superpotential function $W(\phi)$, but instead one must define separate superpotential functions W_i for the regions between y_i and y_{i+1}, where $\phi'(y_i) = \phi'(y_{i+1}) = 0$. For these superpotentials that are not globally defined it is then possible to have $W'(\phi_*) = 0$ without satisfying $V'(\phi_*) = 0$. However, it is impossible to continue the solution beyond ϕ_*. This by itself however may not be a problem, as long as at ϕ_* one is smoothly switching over to another branch of W. Of course this switch-over can only happen at a point where $\phi' = 0$, since otherwise ϕ is monotonic, and one can solve the equations in terms of the superpotential, which is well-defined around ϕ_*. Next we show that such possibilities do not get around the no-go theorem presented above. The reason is that in order to have localized gravity with an infinitely large extra dimension, we need $\phi' \to 0, \phi'' \to 0$ for $|y| \to \infty$, and therefore we find from the second order equation (2.5) that $V' \to 0$. Thus the critical points at infinity must belong to an "ordinary branch" described above, where W can be continued at both sides of ϕ_c. However, once we are on an "ordinary branch" which can be globally defined, all the critical points will actually happen at $V' = 0$. Since the only possibility for switching over to another branch is at $W' = 0$, and at those points $V' = 0$, one can never switch off the ordinary branch, and therefore one can not circumvent the no-go theorem by gluing non-monotonic ϕ's together.

[‡] Whereas the non-canonically normalized brane tension, T, is fine-tuned, the physical brane tension, $T_{br} \propto f[\phi_0]T$, is not fine-tuned since the value of ϕ_0 can vary continuously. Whether this is a solution to the cosmological constant problem or not, beside the fine-tuning requires on $f[\phi]$, depends whether SM loops will modify T or T_{br}. This issue deserves further analysis in future work.

5.2 A numerical example

In this section we demonstrate the ideas of the previous sections in a numerical example. The bulk potential in this example (Fig. 5) is

$$V(\phi) = -\phi^6 + 11\phi^4 - 7\phi^2 + 1, \tag{5.31}$$

which is generated by the superpotential $W[\phi] = \phi^3 - \phi$.

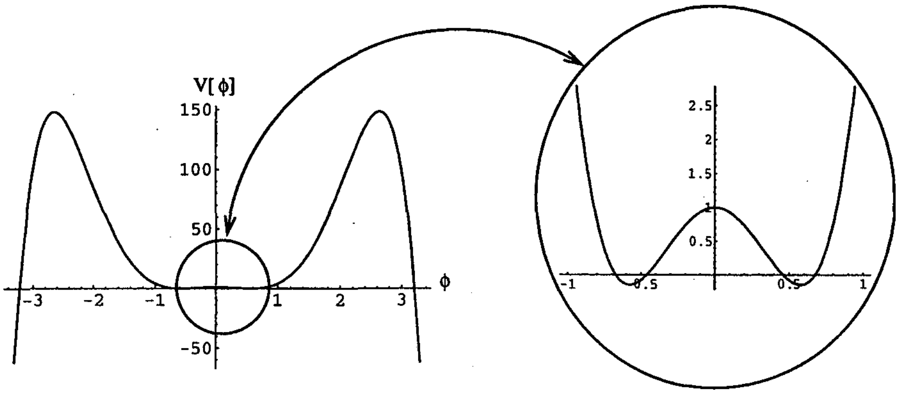

Figure 5: The scalar bulk potential (5.31). The physical interesting region, near the two negative stationary points, is emphasized.

As in some of our previous illustrative examples the potential (5.31) is unbounded from below, and so the theory might be unstable to quantum fluctuations. However, we will only be concerned with static solutions and for our purposes any instabilities will not be relevant.

As emphasized in Figure 5, the bulk potential (5.31) has two negative stationary points at $\phi = \pm 1/\sqrt{3}$. According to our no-go theorem, there are isolated superpotentials solving the equations of motion with critical points at $\pm 1/\sqrt{3}$. Here these superpotentials are very simple because from (5.25) only a finite number of derivatives are non-vanishing thus the superpotentials are polynomial and take the form

$$W[\phi] = \phi^3 - \phi \quad \text{reaches } \phi_c = \pm 1/\sqrt{3} \text{ on the negative (positive) branch,} \tag{5.32}$$

$$W[\phi] = -\phi^3 + \phi \quad \text{reaches } \phi_c = \pm 1/\sqrt{3} \text{ on the positive (negative) branch.} \tag{5.33}$$

Depending on which critical point we want to asymptote at infinity, we can construct four types of solutions that localizes gravity

- for $(\phi_c^+ = 1/\sqrt{3}, \phi_c^- = 1/\sqrt{3})$: the solution is

$$\phi(y) = \tfrac{1}{\sqrt{3}} \tanh(\sqrt{3}|y - y_c^\pm|) \tag{5.34}$$

$$A(y) = \tfrac{1}{18} \tanh^2(\sqrt{3}(y - y_c^\pm)) + \tfrac{2}{9} \ln\cosh(\sqrt{3}(y - y_c^\pm)) + a_\pm \tag{5.35}$$

  where $y_c^\pm$ and $a_\pm$ are four constants of integration to be determined by the continuity and jump conditions. The continuity conditions imply that $y_c^+ = -y_c^-$ and $a_+ = a_-$. The jump equations will depend on the precise form of $f[\phi]$. Except in the degenerate case where $f[\phi] \propto W_+[\phi] - W_-[\phi]$ that has been discussed in a footnote on page 20, we will generically obtain a finite number of discrete values for ϕ_0. For instance in the case of an exponential coupling, $f[\phi] = a\, e^{b\phi}$, the values of ϕ_0 will satisfy

$$\frac{3\phi_0^2 - 1}{\phi_0(\phi_0^2 - 1)} = b\,, \tag{5.36}$$

which admits three solutions whatever the value of b is. And thus there exist three values for the brane tension that will lead to a localized gravity.

- for $(\phi_c^+ = 1/\sqrt{3}, \phi_c^- = -1/\sqrt{3})$ or $(\phi_c^+ = -1/\sqrt{3}, \phi_c^- = 1/\sqrt{3})$: in those cases, there is no discontinuity in the superpotential and the brane tension has to vanish.
- for $(\phi_c^+ = -1/\sqrt{3}, \phi_c^- = -1/\sqrt{3})$: this case is analogous to the first one and, for an exponential coupling, three values for the brane tension are possible and they are just the opposite of the ones obtained in the first case.

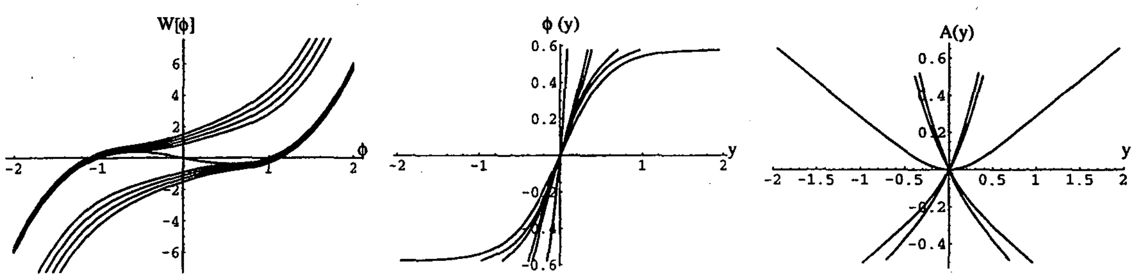

Figure 6: Solutions for $W[\phi]$, $\phi(y)$ and $A(y)$ in the bulk potential (5.31). There is a unique, fine-tuned, regular solution which approximates the Randall-Sundrum solution for large $|y|$. All other solutions are singular.

Besides the previous fine-tuned solutions with an infinitely large extra dimension and localized gravity, all other solutions will either decouple gravity on the brane or involve a horizon at a finite distance. We would like now to examine numerically the self-tuning of these singular solutions. To do that we scan for solutions to (2.6) by fixing the value of $W[\phi]$ at $\phi = 0$. The solutions are plotted in Fig. 6. Note that in agreement with the no-go theorem of the previous section there is a unique solution in this branch of solutions with stationary points at the positions of the minima of $V[\phi]$. As a result of the uniqueness of the solitonic solution, there are no solutions (in this branch) with $0 < |W[0]| < W[\phi_*] \simeq .385$, where ϕ_* is the value of the field at the position of the local minimum of $V[\phi]$, in this case $\phi_* \simeq -.577$. Also in agreement with the results of previous sections, the solitonic solution is the unique regular solution for $\phi(y)$ and $A(y)$, which in this case approximates the Randall-Sundrum solution for large $|y|$.

In order to determine the range of boundary conditions at the brane which could be satisfied, we note that except for the solitonic solution, all other solutions span an infinite range in ϕ. As noted in Section 4.2 the boundary conditions can be expressed in terms of $W[\phi]$ and $W'[\phi]$ at the boundary. Hence, if there is a space-filling range of solutions $(W[\phi], W'[\phi])$ then no fine-tuning is necessary in order to have a static solution with arbitrary boundary conditions in that range. In other words, a given $(2W[\phi_0], 2W'[\phi_0])$ can be equated with a certain (orbifold) boundary condition $(f[\phi_0]\,T, f'[\phi_0]\,T)$. Then, given one such $(f[\phi_0]\,T, f'[\phi_0]\,T)$ there is a continuous set

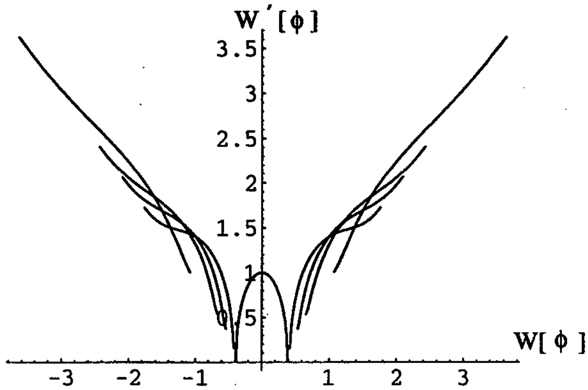

Figure 7: Parametric plot of several solutions for $(W[\phi], W'[\phi])$, which specify the boundary conditions at the wall.

of solutions around that point. That means that for a given $(f[\phi_0], f'[\phi_0])$ in a certain range of f'/f there is a range of tensions T for which there is a solution; hence the theory is self-tuning. Fig. 7 is a parametric plot of $W[\phi]$ versus $W'[\phi]$ for several solutions of $W[\phi]$. It is intended to illustrate the fact that there is a space-filling region for $W > .385$. Given any boundary condition parametrized by $f[\phi]$ and T, T can be rescaled by an amount given by the intersection of the set of solutions $(2W, 2W')$ with a line through the origin and $(fT, f'T)$. As is generically expected, one can see from Figure 7 that if f, f' are $\mathcal{O}(1) = \mathcal{O}(M_5)$, then T can be rescaled by $\mathcal{O}(M_5)$ without eliminating a solution. Hence we have demonstrated the self-tuning of this model. However, the caveat is that because of the isolated solitonic solution, there is a fine-tuned region near $W, W' \sim \mathcal{O}(1)$. If f is $\mathcal{O}(1)$ at some matching point ϕ_0, and T is $\mathcal{O}(M_{EW}/M_5) \ll 1$ then a fine tuning is reintroduced because of the uniqueness of the solitonic solution. There is still a large region of parameter space where the theory is self-tuning, but whether that region is natural or not requires exploration. This phenomenon is a result of the existence of solitonic solutions, and is nongeneric. Furthermore, if we do not require orbifold boundary conditions, then once again self-tuning is natural. Note also that asymptotically the parametric plots of W vs. W' include the line $W = W'$. This is generic in any region where the solutions satisfy $W[\phi] \gg V[\phi]$. This also implies that for any solution $f \sim f'$, there is a very large range for T for which there are solutions. This behavior for large values of W is common, and extends the range of T over which self-tuning occurs possibly to $\pm\infty$ if $f[\phi] = f'[\phi]$, which may be natural from a stringy perspective [4, 5].

6 Resolution of singularities and fine-tuning

In this section we reconsider the resolution of singularities as proposed in [15]. As we have seen, the case of zero bulk potential, which is the case studied in [15], is quite non-generic. There is a shift symmetry in the scalar field which makes the boundary conditions more constraining, and two of the boundary conditions turn out to have the same form. Hence, we study the generic case, and propose a new resolution of the singularity which may restore self-tuning.

The idea of [15] is to add a brane at each of the singularities such that the equations of motion, or boundary conditions, are satisfied there, as well. As pointed out in [15] the singularities contribute to the effective 4D energy density when integrated over the extra dimension, and the contribution of the singularities is essential for vanishing of the 4D cosmological constant.

However, addition of branes at the singularities adds new boundary conditions. In order to answer the question of whether or not the theory is self-tuning we must better understand the continuation past the singularities. In [15] it is proposed that the spacetime is either periodically continued or cut-off at the singularity. In the case that the spacetime is cut-off at the singularities on each side, there are generically two new boundary conditions at each singularity, but no additional free parameters. Hence, without orbifold boundary conditions there are $3 + 3 - 1 = 5$ free parameters and $4 + 2 + 2 = 8$ boundary conditions, and the system is overconstrained. If one imposes orbifold boundary conditions, then there are only half as many additional boundary conditions ($2 + 2 = 4$ boundary conditions in all), but only $3 - 1 = 2$ free parameters. In either case the system is overconstrained. Hence, a fine-tuning is required. Although in the absence of a bulk potential the situation is modified because of the non-generic features mentioned above, in that case a fine-tuning at each of the singularities is required, as well.

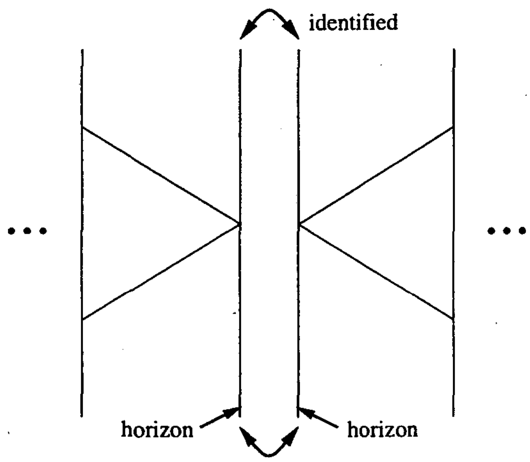

Figure 8: Penrose diagram for causally disconnected regions glued together at the resolved singularity.

Although the details of any continuation of the spacetime beyond the singularity will depend on quantum gravitational dynamics, we propose another scenario which does slightly better than the cut-off scenario, although fine-tuning will still be required. The singularities are generically horizons (*c.f.* Sections 3.3) because the warp factor $e^{-2A(y)/(D-1)}$ vanishes there. Hence, light does not cross the horizon and we can imagine a scenario in which there are bulk fields living beyond the singularity, out of causal contact with our universe. The Penrose diagram for this scenario is illustrated in Figure 8. However, the bulk fields across the horizon would provide an additional three free parameters which one might hope would help avoid fine-tuning. More precisely, following the counting in Section 2, if there are orbifold boundary conditions, then there are $3 + 3 - 1 = 5$ free parameters from the bulk fields on both sides of the singularity, but $2 + 4 = 6$ boundary conditions. Hence, although the system is less constrained than the case in which the spacetime is cut-off at the singularity, one fine-tuning is still required.

One might suspect that additional fields would add additional degrees of freedom which could help in self-tuning. For example, the addition of a scalar field with second derivatives in its equation of motion would contribute two additional free parameters on each side of a brane, but only two boundary conditions (continuity and change in the derivative at the brane). Hence in a system of branes there are net, at least naively, two additional free parameters from the extra scalar field, which could be used to restore self-tuning at the singularities or perhaps even produce nonsingular solutions. However, a more detailed analysis is required in this case.

7 Conclusions

We have studied brane worlds coupled to a scalar field and have found that self-tuning is a generic feature of these models. In these models the dynamics of the scalar field provides additional degrees of freedom, which generically alleviates the need for fine-tuning of static solutions. We have reexamined the exactly solvable models, two of which were studied previously [4,5], and have found that those case are more constrained than the generic case because of a shift symmetry in the scalar field in these models. Still, these theories are self-tuning, including the case of an exponential potential. Whereas in [5] a fine-tuning was necessary in this case due to a particular ansatz for the scalar and graviton fields, we showed that the more general solution is not fine-tuned, in agreement with a counting of free parameters in these models. We demonstrated that singularities in the self-tuned solutions are generic if gravity is to be localized, and we presented a no-go theorem to this effect. We provided perturbative and numerical techniques in order to calculate the self-tuned solutions, and we illustrated the major points of the paper via several numerical examples. Finally, we pointed out that the fine-tuning that is required in order to resolve the singularities in the spirit of [15] for the case of zero bulk potential is generic.

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References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B429**, 263 (1998) [hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B436** (1998) 257 [hep-ph/9804398].

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [hep-ph/9905221]; Phys. Rev. Lett. **83**, 4690 (1999) [hep-th/9906064].

[3] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989) and references therein. For some recent proposals for a 4D adjustment mechanism, see also V. A. Rubakov, Phys. Rev. **D61**, 061501 (2000) [hep-ph/9911305]; A. Hebecker and C. Wetterich, hep-ph/0003287.

[4] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, hep-th/0001197.

[5] S. Kachru, M. Schulz and E. Silverstein, hep-th/0001206; hep-th/0002121.

[6] V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. **125B**, 139 (1983).

[7] S. S. Gubser, hep-th/0002160; C. Gomez, B. Janssen and P. Silva, hep-th/0003002.

[8] S. W. Hawking and R. Penrose, Proc. Roy. Soc. Lond. **A314**, 529 (1970).

[9] D. Youm, hep-th/0002147; S. P. de Alwis, hep-th/0002174; J. Chen, M. A. Luty and E. Ponton, hep-th/0003067; I. Low and A. Zee, hep-th/0004124; S. P. de Alwis, A. T. Flournoy and N. Irges, hep-th/0004125.

[10] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, hep-th/9909134.

[11] C. Grojean, J. Cline and G. Servant, hep-th/9910081; C. Grojean, Phys. Lett. **B479**, 273 (2000) [hep-th/0002130].

[12] A. Brandhuber and K.Sfetsos, JHEP **9910**, 013 (1999) [hep-th/9908116]; I. Bakas and K. Sfetsos, hep-th/9909041; K. Behrndt and M. Cvetič, hep-th/9909058; Phys. Rev. **D61**, 101901 (2000) [hep-th/0001159]; K. Skenderis and P. K. Townsend, Phys. Lett. **B468**, 46 (1999) [hep-th/9909070]; A. Chamblin and G. W. Gibbons, Phys. Rev. Lett. **84**, 1090 (2000) [hep-th/9909130]; R. Kallosh and A. Linde, JHEP **0002**, 005 (2000) [hep-th/0001071].

[13] M. Gremm, hep-th/9912060; hep-th/0002040.

[14] C. Csáki, J. Erlich, T. J. Hollowood and Y. Shirman, hep-th/0001033.

[15] S. Förste, Z. Lalak, S. Lavignac and H. P. Nilles, hep-th/0002164.

[16] R. Gregory, V. A. Rubakov and S. M. Sibiryakov, hep-th/0002072; C. Csáki, J. Erlich and T. J. Hollowood, hep-th/0002161; G. Dvali, G. Gabadadze and M. Porrati, hep-th/0002190; E. Witten, hep-ph/0002297.

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