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D. Von–Eiff, S. Haddad, and M.K. Weigel

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AN APPROACH TO THE RELATIVISTIC EXTENDED THOMAS–FERMI EXPANSION FOR GREEN'S FUNCTIONS, PHASE–SPACE DENSITIES AND DENSITIES.†

D. Von–Eiff
Nuclear Science Division
Lawrence Berkeley Laboratory, University of California
Berkeley, California 94720, U.S.A.

S. Haddad and M.K. Weigel
Sektion Physik der Ludwig–Maximilians–Universität München,
Am Coulombwall 1; D–8046 Garching

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We present an extension of the semiclassical Thomas–Fermi model to relativistic systems. These are obtained by application of the gradient expansion scheme on the Wigner transformed Dyson equation. Explicitly we give the expansion of the Green functions, phase–space densities and densities for a system of nucleons in a vector and scalar potential to second order.

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I. Introduction

In recent years the interest in the investigation of nuclear systems has shifted strongly towards a relativistic approach (see, for instance, Refs.1–3). However, only little research utilizing a semiclassical expansion of the relativistic theory , which was sometimes successful in the simplification of nonrelativistic calculations in atomic and nuclear physics (see, for instance, Refs.4–9), has been published, s.Refs. 1, 10–13. Generally the advantage of the semiclassical approach is the avoidance of wave function calculations by utilizing densities, which in many cases makes the calculations easier. Furthermore the gradient expansion may be needed in relativistic transport theories, where Wigner transformed Green functions also occur. In the relativistic case one expects several complications, which make the formalism more difficult[1,11-13]. First one faces relativistic corrections due to the Dirac structure of the approach. Furthermore one has to deal with position dependent Dirac masses, which are absent in the standard nonrelativistic theory. For these reasons one has restricted oneself to the relativistic Thomas–Fermi method in numerical calculations.[1] In the atomic case the semiclassical relativistic extended Thomas–Fermi (RETF) expansion for particles in an external potential was obtained either by a tricky ansatz for the higher order Green functions[14,16] (not suitable for a position and density dependent Dirac mass) or "window" procedures[15] (elimination of the Dirac sea; but tedious resummation procedures enter for the phase–space density), respectively. An alternative procedure was proposed in Ref.11, where one expands the Bloch equation for the propagator $\exp(-\beta H)$ and treats the coupled differential equations in a recursive scheme. However the solution already becomes quite involved in the nonrelativistic case.[6] In this contribution we want to present a pure algebraic method, which resembles more to the standard nonrelativistic scheme as described, for instance, by Grammaticos and Voros[6]. It involves only straightforward but tedious algebraic methods and the residuum calcules. Furthermore it has the advantage that it can be generalized to interacting particles. The paper is organized as follows: In the next section we describe the general G–function expansion by utilizing the Wigner transformed Dyson equation. From the $\hbar$–expansion of the G–function one can obtain by complex integration the phase–space density. Integration over the momentum gives then the density. The energy–density emerges in a similar manner. The explicit expansion for the G–function, which is the key expression for the wanted densities, is calculated up to second order in the third section. The resulting densities are given, in detail, in the appendix.

II. General Theory

The basic quantity for the semiclassical expansion is the Wigner transform of the one–particle propagator, defined as (we use the conventions of Bjorken and Drell[16]; |N> denotes the groundstate)

$$
\begin{aligned}
G(R,p) &= \int d^4r \, \exp(ipr/\hbar)\, G(R+\tfrac{r}{2}, R-\tfrac{r}{2}) \\
&= \int d^4r \, (\exp(ipr/\hbar)(-i)\langle N|T(\Psi(R+\tfrac{r}{2})\bar{\Psi}(R-\tfrac{r}{2}))|N\rangle) \,. \qquad \text{(II.1)}
\end{aligned}
$$

The Green function obeys in the standard space–time representation the Dyson–equation

$$
\{\hbar c(i\gamma^{\mu}\partial_{\mu} - \frac{m_N c}{\hbar})_{12} - \Sigma(1,2)\}\, G(2,1') = \hbar\, c\, \delta(1,1') \;, \qquad \text{(II.2)}
$$

where we sum or integrate over all doubly occurring variables ($1{:=}x_1^{\mu}$, spinor index etc.). After performing the WT one obtains for the Dyson equation in the mixed position–momentum representation the following form ($H \to H - \mu N$):

$$
\begin{aligned}
&\left[c\,\gamma^{\mu}(p_{\mu} + \frac{i\hbar}{2}\frac{\partial}{\partial R'^{\mu}}) + \mu\gamma^0 - m_N c^2 - \Sigma(R,p + \frac{i\hbar}{2}\frac{\partial}{\partial R'})\right]_{12} G_{21'}(R',p' - \frac{i\hbar}{2}\frac{\overleftarrow{\partial}}{\partial R})\Big|_{R=R'}^{p=p'} \\
&= (G^{TF})^{-1}_{12}(R,p)\exp\left[i\,\frac{\hbar}{2}\left[\frac{\overleftarrow{\partial}}{\partial R}\cdot\frac{\overrightarrow{\partial}}{\partial p} - \frac{\overleftarrow{\partial}}{\partial R}\cdot\frac{\overrightarrow{\partial}}{\partial p}\right]\right] G_{21'}(R,p) = \hbar c\, \delta_{11'} \;. \qquad \text{(II.3)}
\end{aligned}
$$

Arrows pointing to the right imply that the differential operator acts on the quantities to the right etc.

Knowledge of the zeroth–order (Thomas–Fermi) G–function G^{TF} and of the functional dependence of Σ with respect to G is now, in principle, sufficient for the determination of the semiclassical expansion. The relevant expansion of the G–functions, phase–space densities and densities is given by the following scheme ($G(R,p) = G(\vec{R},p)$):

$$
G(R,p) = \hbar^4 \sum_{j=0}^{\infty} \hbar^j\, G^{(j)}(\vec{R},p) \;, \qquad \text{(II.4.1)}
$$

$$n(\vec{R},\vec{p}) = \hbar^3 \sum_{j=o}^{\infty} \hbar^j\, n^{(j)}(\vec{R},\vec{p}) = \hbar^3 \sum_{j=o}^{\infty} \hbar^j \int \frac{dp_o}{2\pi}\, e^{ip_o\eta}\, G^{(j)}(\vec{R},p) \;, \tag{II.4.2}$$

$$n(\vec{R}) = \sum_{j=o}^{\infty} \hbar^j\, n^{(j)}(\vec{R}) = \sum_{j=o}^{\infty} \hbar^j \int \frac{d^3p}{(2\pi)^3}\, n^{(j)}(\vec{R},\vec{p}) \;. \tag{II.4.3}$$

The so–called extended Thomas–Fermi approximation (ETF)[4,6] is restricted to local external potentials, i.e.

$$\Sigma(x,x') = \Sigma\left(\frac{\vec{x}+\vec{x}'}{2}\right) \delta^4(x-x') \;. \tag{II.5}$$

The same structure is encountered in the Hartree–approximation[1,12]. For simplicity we will only treat the case N=Z with a hamiltonian of the standard structure[1,2] ($\Sigma_\mu = \delta_{\mu o} V(\vec{R})$; $c^2M := c^2m_N + \Sigma_s(\vec{R})$):

$$H = c\, \vec{\alpha}\cdot\vec{p} + \beta c^2 M(\vec{R}) + V(\vec{R}) \;. \tag{II.6}$$

($N \neq Z$ would lead to $\mu \to \mu \pm \frac{\nu}{2}$; $\Sigma_o \to \Sigma_o \mp \Sigma_{\tau o}$). The zeroth–order (TF)–solution of Eq.(II.3) corresponds to the LDA–(nuclear matter) solution[1,2,12]:

$$G^{TF}(\vec{R},p) \equiv \hbar^4\, G^{(0)}(\vec{R},p) = \hbar c\, \frac{c\gamma^\mu k_\mu + Mc^2}{c^2k^2 - M^2c^4} =: \hbar c\, \frac{c\gamma^\mu k_\mu + Mc^2}{N} \tag{II.7}$$

with ($\vec{k} \equiv \vec{p}$):

$$ck_o := cp_o + \mu - V(\vec{R}) \;. \tag{II.8}$$

The detailed pole structure is given by ($\epsilon := (c^2\vec{p}^2 + M^2c^4)^{1/2}$)

$$\frac{1}{N} = \left\{ \frac{1}{2\epsilon}\, \frac{1}{ck_o - \epsilon + i\eta\,\mathrm{sgn}(\epsilon-\epsilon_F)} - \frac{1}{2\epsilon}\, \frac{1}{ck_o + \epsilon - i\eta} \right\} \;. \tag{II.9}$$

The poles correspond to the single–particle energies:

$$\hbar\omega(\vec{R},\vec{p}) = V(\vec{R}) \pm \epsilon(\vec{R},\vec{p}) - \mu \;. \tag{II.10}$$

The negative energies describe the energy momentum relation for the antiparticles.

For the energy one obtains:

$$E = -i\,\mathrm{Tr}\left\{\int d^3R \int \frac{d^4p}{(2\pi\hbar)^4}\, e^{ip_0\eta}\, \gamma^0\, H(\vec{R},\vec{p} + \frac{\hbar}{2i}\frac{\partial}{\partial \vec{R}'})G(\vec{R}',p' - \frac{\hbar}{2i}\frac{\overleftarrow{\partial}}{\partial \vec{R}})\Big|^{p=p'}_{\vec{R}=\vec{R}'}\right\}$$

$$= -i\,\mathrm{Tr}\left\{\int d^3R \int \frac{d^4p}{(2\pi\hbar)^4}\, \gamma^0 (cp_0+\mu)G(\vec{R},\vec{p})\right\} . \qquad \text{(II.11)}$$

The last expression was obtained by use of the Dyson equation (II.3). The kinetic energy is given by:

$$T = -i\,\mathrm{Tr}\int d^3R \int \frac{d^4p}{(2\pi\hbar)^4}\, e^{ip_0\eta}\{[c\vec{\gamma}(\vec{p} + \frac{\hbar}{2i}\frac{\partial}{\partial \vec{R}}) + m_N c^2 - \gamma^0 m_N c^2]G(\vec{R},p)\} \quad . \qquad \text{(II.12)}$$

The interpretation of the integrand as a local kinetic energy density may be misleading, since one actually calculates already in the TF–approximation the expectation value of the free hamiltonian (minus rest mass) in a "free" relativistic Fermi gas with a position and density dependent Dirac mass. For that reason, for instance, is the Dirac energy no monotonic function of the density[17]. A final point is the calculation of the energy density for a mass operator emerging from a self–consistent many–body procedure. In such a case one has to subtract from H the term $1/2(\Sigma_s(\vec{R}) + \gamma^0 V(\vec{R}))$, i.e. the meson contributions.

The $\hbar$–expansion follows from the energy expressions by inserting of the G–function expansion (II.4.1). In the following section we will explicitly give the G–function expansion up to second order, which is the relevant ingredient for the calculation of the different densities, which are obtainable in the next steps by p–integration.

III. Expansion up to second order in $\hbar$

The RETF–expansion of the G–function can be obtained from a straightforward but tedious evolution of the Dyson equation (II.3). With the Wigner operator

$$\overleftrightarrow{\Lambda} := \overleftarrow{\nabla}_R \cdot \overrightarrow{\nabla}_P - \overleftarrow{\nabla}_R \cdot \overrightarrow{\nabla}_P \qquad \text{(III.1)}$$

one gets ($\hbar$=c=1):

$$G^{(1)}(\vec{R},p) = \frac{1}{2i} G^{(0)}(\vec{R},p)\{G^{TF}(\vec{R},p)^{-1} \overleftrightarrow{\Lambda} G^{(0)}(\vec{R},p)\}$$

$$= \frac{1}{N^2}\{\sigma^{\mu i} k_\mu \frac{\partial}{\partial R^i} M - \gamma^0 \sigma^{ji} k_j \frac{\partial}{\partial R^i} V + \sigma^{0i} M \frac{\partial}{\partial R^i} V\} \qquad \text{(III.2)}$$

$$G^{(2)}(\vec{R},p) = G^{(0)}(\vec{R},p)\{\frac{1}{8}[G^{TF}(\vec{R},p)^{-1} \overleftrightarrow{\Lambda}^2 G^{(0)}(\vec{R},p)] + \frac{1}{2i}[G^{TF}(\vec{R},p)^{-1} \overleftrightarrow{\Lambda} G^{(1)}(\vec{R},p)]\}$$

$$= G_s^{(2)}(\vec{R},p) + \gamma^0 G_0^{(2)}(\vec{R},p) - \vec{\gamma}\,\vec{G}^{(2)}(\vec{R},p) + \gamma^0\vec{\gamma}\,\vec{G}_0(\vec{R},p) \qquad \text{(III.3)}$$

with

$$G_s^{(2)}(\vec{R},p) = -\frac{3}{4N^2}\Delta M$$

$$-\frac{1}{N^3}[M^2\Delta M + Mk_0\Delta V + \vec{k}\cdot\vec{\nabla}(\vec{k}\cdot\vec{\nabla}M) + 2M(\vec{\nabla}V)^2$$

$$+ 2M(\vec{\nabla}M)^2 + 2k_0(\vec{\nabla}M\cdot\vec{\nabla}V)]$$

$$-\frac{1}{N^4}[2Mk_0\,\vec{k}\cdot\vec{\nabla}(\vec{k}\cdot\vec{\nabla}V) + 2M^2\vec{k}\cdot\vec{\nabla}(\vec{k}\cdot\vec{\nabla}M) - 2M(\vec{k}\cdot\vec{\nabla}V)^2$$

$$+ 2\epsilon^2M(\vec{\nabla}V)^2 + 2M(\vec{k}\cdot\vec{\nabla}M)^2 + 2M^3(\vec{\nabla}M)^2$$

$$+ 4M^2k_0(\vec{\nabla}M\cdot\vec{\nabla}V)]\ , \qquad \text{(III.4.1)}$$

$$G_0^{(2)}(\vec{R},p) = -\frac{1}{4N^2}\Delta V$$

$$-\frac{1}{N^3}[k_0M\,\Delta M + \epsilon^2\Delta V + \vec{k}\cdot\vec{\nabla}(\vec{k}\cdot\vec{\nabla}V) + 2M(\vec{\nabla}M\cdot\vec{\nabla}V)]$$

$$-\frac{2}{N^4}[\epsilon^2\vec{k}\cdot\vec{\nabla}(\vec{k}\cdot\vec{\nabla}V) + Mk_0\vec{k}\cdot\vec{\nabla}(\vec{k}\cdot\vec{\nabla}M) + k_0\epsilon^2(\vec{\nabla}V)^2$$

$$- k_0(\vec{k}\cdot\vec{\nabla}V)^2 + k_0 M^2(\vec{\nabla}M)^2 + k_0(\vec{k}\cdot\vec{\nabla}M)^2$$

$$+ 2M\epsilon^2(\vec{\nabla}M\cdot\vec{\nabla}V)] \ , \qquad \text{(III.4.2)}$$

$$\vec{G}^{(2)}(\vec{R},p) = -\frac{1}{N^3}[M\vec{k}\Delta M + k_0\vec{k}\Delta V + 2\vec{k}(\vec{\nabla}V)^2 - 2\vec{\nabla}V(\vec{k}\cdot\vec{\nabla}V)$$

$$+ 2\vec{\nabla}M(\vec{k}\cdot\vec{\nabla}M)]$$

$$-\frac{2}{N^4}[k_0\vec{k}\vec{k}\cdot\vec{\nabla}(\vec{k}\cdot\vec{\nabla}V) + M\vec{k}\vec{k}\cdot\vec{\nabla}(\vec{k}\cdot\vec{\nabla}M) + \epsilon^2\vec{k}(\vec{\nabla}V)^2$$

$$+ \vec{k}(\vec{k}\cdot\vec{\nabla}V)^2 - 2\vec{k}^2\vec{\nabla}V(\vec{k}\cdot\vec{\nabla}V) + 2\vec{k}^2\vec{\nabla}M(\vec{k}\cdot\vec{\nabla}M)$$

$$+ M^2\vec{k}(\vec{\nabla}M)^2 - \vec{k}(\vec{k}\cdot\vec{\nabla}M)^2 + 2Mk_0\vec{k}(\vec{\nabla}M\cdot\vec{\nabla}V)] \ , \qquad \text{(III.4.3)}$$

$$\vec{G}_0^{(2)}(\vec{R},p) = -\frac{2}{N^3}[\vec{\nabla}M(\vec{k}\cdot\vec{\nabla}V) - \vec{\nabla}V(\vec{k}\cdot\vec{\nabla}M)] \ . \qquad \text{(III.4.4)}$$

The phase–space density is now obtainable from Eqs.(II.4.2,7,9) and (III.2,3) by complex integration. Crucial in this contex is the pole structure of $G^{(0)}$ or N, respectively, which is given in Eq.(II.9). It permits the decomposition of N^{-n} in powers of $(k_0 \mp \epsilon)$. The contributions corresponding to the poles at $k_0 = -\epsilon$ are neglected in the further procedure. The general space density is obtainable by integration over the momentum. Here, one is mainly interested in the scalar and baryonic density, which can be extracted from the general density by performing the corresponding traces. For instance, for the baryon density one obtains ($G^{(1)}$ is traceless):

$$n_B(\vec{R}) = \frac{-i}{(2\pi)^4}\mathrm{Tr}\{\gamma^0\int d^4p\, e^{ip_0\eta}[G^{(0)}(\vec{R},p) + G^{(1)}(\vec{R},p) + G^{(2)}(\vec{R},p)]\} \ , \qquad \text{(III.5)}$$

which yields the following expression for each kind of nucleons ($x_F := \epsilon_F/p_F$)

$$n_B(\vec{R}) = \frac{p_F^3}{3\pi^2} + \frac{1}{24\pi^2}\Big[(-2x - 2\ln\frac{x_F+1}{x_F-1})\Delta V - 2(x_F^2-1)p_F\frac{\Delta M}{M}$$

$$+ (2-x_F^2)\frac{(\vec{\nabla}M)^2}{p_F} - (x_F^2-3)\frac{(\vec{\nabla}V)^2}{p_F} - 2x_F(x_F^2-3)\frac{(\vec{\nabla}V\cdot\vec{\nabla}M)}{M}\Big] \ , \qquad \text{(III.6)}$$

which agrees with Ref.15 for $M^* = m_N$. The other densities and the energy–density can be calculated by similar procedures and are given explicitly in the appendix.

As a final remark we would like to mention that the RETF formalism developed above on the basis of a relativistic field theoretical scheme is the equivalent of the ETF method in nonrelativistic physics (see, for instance, Ref.6). However, due to the more complicated relativistic ingredients caused by the Dirac structure, it involves substantially more effort as in the nonrelativistic case in the numerical many–body treatment, which is presently under investigation. This feature is not unexpected, since also the wave function scheme is much more complicated in the relativistic treatment (see, for instance, Refs.2,18).

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Appendix

α) Phase–space densities:

The Wigner function follows accordingly to Eq.(II.4.2) as:

$$n^{(m)}(\vec{R},\vec{p}) = \frac{1}{(2\pi)} \int dp^0 \exp(ip_0\eta)\, G^{(m)}(\vec{R},p) \ , \qquad \text{(A.1)}$$

The different kinds of phase–space densities for each kind of nucleons (no isospin trace) are defined by ($n_o \equiv n_B$):

$$n_o(\vec{R},\vec{p}) = \mathrm{Tr}(\gamma^0 n(\vec{R},\vec{p})) \ , \qquad \text{(A.2.1)}$$

$$n_s(\vec{R},\vec{p}) = \mathrm{Tr}(n(\vec{R},\vec{p})) \ , \qquad \text{(A.2.2)}$$

$$n_v(\vec{R},\vec{p}) = \mathrm{Tr}(\vec{\gamma}\cdot\vec{p}\; n(\vec{R},\vec{p})) \ . \qquad \text{(A.2.3)}$$

Evaluation of (A.1) and (A.2) leads to ($\Theta(x)$ denotes the step function, $|\vec{p}| \equiv q$ angle averaging over $\vec{R}$ is implied):

$$n_o^{(0)}(\vec{R},\vec{p}) = 2\,\Theta(-\omega(\vec{R},\vec{p})) \ , \qquad \text{(A.3.1)}$$

$$n_o^{(2)}(\vec{R},\vec{p}) = 4\Delta V\Big[\frac{q^2}{36}\frac{1}{(2\epsilon)^2}\frac{\partial^2\delta(\omega)}{\partial\omega^2} + \big(\frac{1}{8}\frac{1}{2\epsilon} - \frac{1}{6}\frac{q^2}{(2\epsilon)^3}\big)\frac{\partial\delta(\omega)}{\partial\omega}$$

$$+ \big(\frac{2}{3}\frac{q^2}{(2\epsilon)^4} - \frac{1}{2}\frac{1}{(2\epsilon)^2}\big)\delta(\omega) + \big(\frac{4}{3}\frac{q^2}{(2\epsilon)^5} - \frac{1}{(2\epsilon)^3}\big)\Theta(-\omega)\Big]$$

$$+ 4M\Delta M\Big[\frac{1}{18}\frac{q^2}{(2\epsilon)^3}\frac{\partial^2\delta(\omega)}{\partial\omega^2} + \big(\frac{1}{4}\frac{1}{(2\epsilon)^2} - \frac{1}{3}\frac{q^2}{(2\epsilon)^4}\big)\frac{\partial\delta(\omega)}{\partial\omega} + \big(\frac{2}{3}\frac{q^2}{(2\epsilon)^5}$$

$$-\frac{1}{2}\frac{1}{(2\epsilon)^3}\big)\delta(\omega)\Big]$$

$$+ 4(\vec{\nabla}V)^2\Big[\big(\frac{1}{24}\frac{1}{(2\epsilon)} - \frac{q^2}{18}\frac{1}{(2\epsilon)^3}\big)\frac{\partial^2\delta(\omega)}{\partial\omega^2} + \big(\frac{1}{3}\frac{q^2}{(2\epsilon)^4} - \frac{1}{4}\frac{1}{(2\epsilon)^2}\big)\frac{\partial\delta(\omega)}{\partial\omega}$$

$$+ \big(\frac{1}{2}\frac{1}{(2\epsilon)^3} - \frac{2}{3}\frac{q^2}{(2\epsilon)^5}\big)\delta(\omega)\Big]$$

$$+\ 4(\vec{\nabla}M)^2\left[\frac{1}{6}\frac{1}{(2\epsilon)^3}\frac{\partial^2\delta(\omega)}{\partial\omega^2}-\frac{1}{(2\epsilon)^4}\frac{\partial\delta(\omega)}{\partial\omega}+\frac{2}{(2\epsilon)^5}\,\delta(\omega)\right](M^2+\frac{1}{3}q^2)$$

$$+\ 4M(\vec{\nabla}V)\cdot(\vec{\nabla}M)\left[\frac{1}{6}\frac{1}{(2\epsilon)^2}\frac{\partial^2\delta(\omega)}{\partial\omega^2}-\frac{1}{(2\epsilon)^3}\frac{\partial\delta(\omega)}{\partial\omega}+\frac{4}{(2\epsilon)^4}\,\delta(\omega)+\frac{8}{(2\epsilon)^5}\,\Theta(-\omega)\right]\ . \tag{A.3.2}$$

$$n_s^{(0)}(\vec{R},\vec{p})=2\,\frac{M}{\epsilon}\,\Theta(-\,\omega(\vec{R},\vec{p}))\ , \tag{A.4.1}$$

$$n_s^{(2)}(\vec{R},\vec{p})=4M\Delta V\left[\frac{q^2}{18}\frac{1}{(2\epsilon)^3}\frac{\partial^2\delta(\omega)}{\partial\omega^2}+(\frac{1}{4}\frac{1}{(2\epsilon)^2}-\frac{1}{3}\frac{q^2}{(2\epsilon)^4})\frac{\partial\delta(\omega)}{\partial\omega}+\right.$$

$$\left.+\ (\frac{2}{3}\frac{q^2}{(2\epsilon)^5}-\frac{1}{2}\frac{1}{(2\epsilon)^3})\delta(\omega)\right]+$$

$$+\ 4\Delta M\left[\frac{1}{9}\frac{q^2M^2}{(2\,\epsilon\,)^4}\frac{\partial^2\delta(\omega)}{\partial\omega^2}+(\frac{1}{6}\frac{q^2}{(2\epsilon)^3}+\frac{M^2}{2}\frac{1}{(2\epsilon)^3}-\frac{4}{3}\frac{q^2M^2}{(2\,\epsilon\,)^5})\frac{\partial\delta(\omega)}{\partial\omega}+\right.$$

$$+\ (\frac{3}{4}\frac{1}{(2\epsilon)^2}-\frac{q^2}{(2\epsilon)^4}-\frac{3M^2}{(2\,\epsilon\,)^4}+\frac{20}{3}\frac{q^2M^2}{(2\,\epsilon\,)^6})\delta(\omega)+$$

$$\left.+\ (\frac{3}{2}\frac{1}{(2\epsilon)^3}-\frac{2q^2}{(2\epsilon)^5}-\frac{6M^2}{(2\,\epsilon\,)^5}+\frac{40}{3}\frac{q^2M^2}{(2\,\epsilon\,)^7})\Theta(-\omega)\right]+$$

$$+\ 4(\vec{\nabla}M)\cdot(\vec{\nabla}V)\left[\frac{M^2}{3}\frac{1}{(2\epsilon)^3}\frac{\partial^2\delta(\omega)}{\partial\omega^2}+(\frac{1}{2}\frac{1}{(2\epsilon)^2}-\frac{2M^2}{(2\,\epsilon\,)^4})\frac{\partial\delta(\omega)}{\partial\omega}+\right.$$

$$\left.+\ (\frac{4M^2}{(2\,\epsilon\,)^5}-\frac{1}{(2\epsilon)^3})\delta(\omega)\right]+$$

$$+\ 4M(\vec{\nabla}V)^2\left[(\frac{2}{9}\frac{q^2}{(2\epsilon)^4}+\frac{1}{3}\frac{M^2}{(2\,\epsilon\,)^4})\frac{\partial^2\delta(\omega)}{\partial\omega^2}+\frac{4}{3}\frac{q^2}{(2\epsilon)^5}\frac{\partial\delta(\omega)}{\partial\omega}+\right.$$

$$+\ (-\frac{20}{3}\frac{q^2}{(2\epsilon)^6}-\frac{1}{(2\epsilon)^4})\delta(\omega)+$$

$$\left.+(-\frac{40}{3}\frac{q^2}{(2\epsilon)^7}-\frac{2}{(2\epsilon)^5})\Theta(-\omega)\right]+$$

$$+\ 4M(\vec{\nabla}M)^2\left[(+\frac{1}{9}\frac{q^2}{(2\epsilon)^4}+\frac{1}{3}\frac{M^2}{(2\,\epsilon\,)^4})\frac{\partial^2\delta(\omega)}{\partial\omega^2}+(\frac{1}{(2\epsilon)^3}-\frac{4}{3}\frac{q^2}{(2\epsilon)^5}-\frac{4M^2}{(2\,\epsilon\,)^5})\frac{\partial\delta(\omega)}{\partial\omega}+\right.$$

$$+\left(\frac{20}{3}\frac{q^2}{(2\epsilon)^6}+\frac{20M^2}{(2\epsilon)^6}-\frac{6}{(2\epsilon)^4}\right)\delta(\omega)+$$

$$+\left(\frac{40}{3}\frac{q^2}{(2\epsilon)^7}+\frac{40M^2}{(2\epsilon)^7}-\frac{12}{(2\epsilon)^5}\right)\Theta(-\omega)\Big] \quad . \tag{A.4.2}$$

$$n_v^{(0)}(\vec{R},\vec{p}) = 2\,\frac{q^2}{\epsilon}\,\Theta(-\,\omega(\vec{R},\vec{p})) \quad , \tag{A.5.1}$$

$$n_v^{(2)}(\vec{R},\vec{p}) = 4\Delta V\Big[\frac{q^4}{18}\frac{1}{(2\epsilon)^3}\frac{\partial^2\delta(\omega)}{\partial\omega^2}+\left(\frac{1}{4}\frac{q^2}{(2\epsilon)^2}-\frac{q^4}{3}\frac{1}{(2\epsilon)^4}\right)\frac{\partial\delta(\omega)}{\partial\omega}+$$

$$\left(\frac{2}{3}\frac{q^4}{(2\epsilon)^5}-\frac{1}{2}\frac{q^2}{(2\epsilon)^3}\right)\delta(\omega)\Big]$$

$$+\;4M\Delta M\Big[\frac{1}{9}\frac{q^4}{(2\epsilon)^4}\frac{\partial^2\delta(\omega)}{\partial\omega^2}+\left(\frac{1}{2}\frac{q^2}{(2\epsilon)^3}-\frac{4}{3}\frac{q^4}{(2\epsilon)^5}\right)\frac{\partial\delta(\omega)}{\partial\omega}+\left(\frac{20}{3}\frac{q^4}{(2\epsilon)^6}-\frac{3q^2}{(2\epsilon)^4}\right)\delta(\omega)+$$

$$+\left(\frac{40}{3}\frac{q^4}{(2\epsilon)^7}-\frac{6q^2}{(2\epsilon)^5}\right)\Theta(-\omega)\Big]+$$

$$+\;4M(\vec{\nabla}M\cdot\vec{\nabla}V)\Big[\frac{q^2}{3}\frac{1}{(2\epsilon)^3}\frac{\partial^2\delta(\omega)}{\partial\omega^2}-2q^2\frac{1}{(2\epsilon)^4}\frac{\partial\delta(\omega)}{\partial\omega}+\frac{4q^2}{(2\epsilon)^5}\,\delta(\omega)\Big]$$

$$+\;4(\vec{\nabla}V)^2\Big[\left(-\frac{1}{9}\frac{q^4}{(2\epsilon)^4}+\frac{1}{12}\frac{q^2}{(2\epsilon)^2}\right)\frac{\partial^2\delta(\omega)}{\partial\omega^2}+\left(-\frac{1}{3}\frac{q^2}{(2\epsilon)^3}+\frac{4}{3}\frac{q^4}{(2\epsilon)^5}\right)\frac{\partial\delta(\omega)}{\partial\omega}$$

$$+\left(-\frac{20}{3}\frac{q^4}{(2\epsilon)^6}+\frac{q^2}{(2\epsilon)^4}\right)\delta(\omega)$$

$$+\left(-\frac{40}{3}\frac{q^4}{(2\epsilon)^7}+\frac{2q^2}{(2\epsilon)^5}\right)\Theta(-\omega)\Big]$$

$$+\;4(\vec{\nabla}M)^2\Big[\left(\frac{q^4}{9}\frac{1}{(2\epsilon)^4}+\frac{q^2M^2}{3}\frac{1}{(2\epsilon)^4}\right)\frac{\partial^2\delta(\omega)}{\partial\omega^2}+\left(\frac{q^2}{3}\frac{1}{(2\epsilon)^3}-\frac{4}{3}\frac{q^4}{(2\epsilon)^5}-4q^2M^2\frac{1}{(2\epsilon)^5}\right)\frac{\partial\delta(\omega)}{\partial\omega}+$$

$$+\left(\frac{20}{3}\frac{q^4}{(2\epsilon)^6}+20q^2M^2\frac{1}{(2\epsilon)^6}-\frac{2q^2}{(2\epsilon)^4}\right)\delta(\omega)$$

$$+\left(\frac{40}{3}\frac{q^4}{(2\epsilon)^7}+40q^2M^2\frac{1}{(2\epsilon)^7}-\frac{4q^2}{(2\epsilon)^5}\right)\Theta(-\omega)\Big] \quad . \tag{A.5.2}$$

β) Densities:

The densities emerge from the phase–space densities by momentum integration (see Eq.(II.4.3)). It follows (The baryon density n_o is given in expression (III.6)):

$$n_s^{(0)}(\vec{R}) = \frac{1}{2\pi^2}\left[M p_F\, \epsilon_F - \frac{M^3}{2}\ln(\frac{x_F+1}{x_F-1})\right] , \qquad (A.6.1)$$

$$n_s^{(2)}(\vec{R}) = \frac{1}{24\pi^2}\left[-\Delta V\,\frac{2M}{p_F} + \Delta M(3\ln(\frac{x_F+1}{x_F-1}) - 2x_F)\right.$$

$$\left. - \frac{(\vec{\nabla}M)^2}{M}\, x_F(x_F^2+2) - (\vec{\nabla}M)\cdot(\vec{\nabla}V)\,\frac{2}{p_F}(2+x_F^2) - \frac{(\vec{\nabla}V)^2}{M}\, x_F(x_F^2+1)\right] . \qquad (A.6.2)$$

$$n_v^{(0)}(\vec{R}) = \frac{1}{8\pi^2}\left[5p_F^3\epsilon_F - 3p_F\epsilon_F^3 + \frac{3}{2}M^4\ln(\frac{x_F+1}{x_F-1})\right] , \qquad (A.7.2)$$

$$n_v^{(2)}(\vec{R}) = \frac{1}{24\pi^2}\left[-2M\Delta M\ln(\frac{x_F+1}{x_F-1}) - 4p_F\Delta V + (3x - \ln(\frac{x_F+1}{x_F-1}))(\vec{\nabla}V)^2\right.$$

$$\left. + (3x_F - \frac{1}{2}\ln(\frac{x_F+1}{x_F-1}))(\vec{\nabla}M)^2 + 6\,\frac{M}{p_F}\,(\vec{\nabla}M)\cdot(\vec{\nabla}V)\right] . \qquad (A.7.3)$$

γ) Energy density ($m \equiv m_N$):

$$e^{(0,2)}(\vec{R}) = n_v^{(0,2)}(\vec{R}) + [(1-\frac{\lambda}{2})\,\Sigma_s(\vec{R}) + m]\, n_s^{(0,2)}(\vec{R})$$

$$+ (1-\frac{\lambda}{2})\, V(\vec{R})\, n_o^{(0,2)}(\vec{R})$$

$$= n_v^{(0,2)}(\vec{R}) + [(1-\frac{\lambda}{2})\, M + \frac{\lambda}{2} m]\, n_s^{(0,2)}(\vec{R}) + (1-\frac{\lambda}{2})\, V n_o^{(0,2)}(\vec{R}) ,$$

$$(A.8)$$

$\lambda = 0$ corresponds to a purely external potential; $\lambda = 1$ describes the case, if meson contributions are included (see text).

δ) Kinetic energy density:

$$\tau^{(0)}(\vec{R}) = \frac{1}{8\pi^2}\Big[2p_F\epsilon_F^3 - M^2(5-4\frac{m}{M})p_F\epsilon_F + M^4(3-4\frac{m}{M})\ln(\frac{p_F+\epsilon_F}{M})\Big] - mn_o^{(0)}(\vec{R}) \ , \qquad (A.8.1)$$

$$\tau^{(2)}(\vec{R}) = \frac{1}{24\pi^2}\Big\{\Big[(3-(1+x_F^2)\frac{m}{M})x_F - (3-x_F^2)\frac{m}{p_F} - \ln(\frac{x_F+1}{x_F-1})\Big](\vec{\nabla}V)^2$$

$$+ 2m\Big[x_F - \frac{p_F}{m}(2-(1-x_F^2)\frac{m}{M}) + \ln(\frac{x_F+1}{x_F-1})\Big]\Delta V$$

$$- \Big[\frac{2m}{p_F}(2+x_F^2) + 2x_F\frac{m}{M}(3-x_F^2) + 6\frac{p_F}{M}(1-x_F^2)\Big](\vec{\nabla}M)\cdot(\vec{\nabla}V)$$

$$- \Big[\frac{m}{p_F}(2-x_F^2) - \Big[3-2\frac{m}{M}\Big]x_F + \frac{m}{M}x_F^3 + \frac{1}{2}\ln(\frac{x_F+1}{x_F-1})\Big](\vec{\nabla}M)^2$$

$$- 2m\Big[x_F + \frac{p_F}{M}(1-x_F^2) - (3-2\frac{M}{m})\frac{1}{2}\ln(\frac{x_F+1}{x_F-1})\Big]\Delta M\Big\} \ . \qquad (A.8.2)$$

By use of the definition of the Fermi momentum $V(\vec{R})$ and its derivatives can be eliminated by means of $n_o(\vec{R})$ and M and their derivatives. For $M = m$ the relativistic atomic expansion is recovered,[15] which reduces to the nonrelativistic limit for p_F , $V(\vec{R}) << m$. In principle one can also eliminate M (i.e. Σ_s) in favour of $n_s(\vec{R})$. However such a procedure is not applicable for a general relativistic mass operator, since its Dirac structure is more complicated as the general density structure[2,12].

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LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
INFORMATION RESOURCES DEPARTMENT
BERKELEY, CALIFORNIA 94720