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Feedforward Noise Cancellation in an Airduct using Generalized FIR Filter Estimation

Jie Zeng
University of California, San Diego
Dept. of Mechanical and Aerospace Engineering
9500 Gilman Drive
La Jolla, CA 92093-0411, U.S.A
jzeng@ucsd.edu

Raymond de Callafon
University of California, San Diego
Dept. of Mechanical and Aerospace Engineering
9500 Gilman Drive
La Jolla, CA 92093-0411, U.S.A
callafon@ucsd.edu

***Abstract*— A feedforward control algorithm for active noise control based on the recursive estimation of a generalized finite impulse response (FIR) filter is presented in this paper. The feedforward control algorithm is applied to a commercial air ventilation silencer to provide active noise compensation in an airduct. A generalized FIR filter has the same linear parameter structure as a taped delay FIR filter that is favorable for (recursive) estimation purposes. However, the advantage of the generalized FIR filters lies in the possibility to include prior knowledge of system dynamics in the tapped delay line of the filter. By comparison with a conventional FIR filter implementation it is shown that a significant improvement in noise cancellation is obtained.**

I. INTRODUCTION

Active noise control (ANC) can be used for sound reduction and can be particularly effective at lower frequency sound components. ANC allows for much smaller design constraints to achieve sound and noise suppression and has received attention in recent years in many active noise cancellation applications [1], [2], [3], [4]. In this paper we will discuss the design of an ANC algorithm for an ACTA air ventilation silencer that has been depicted in Figure 1. The system is an open-ended airduct located at the Systems Identification and Control Laboratory at the UCSD that will be used as a case study for the ANC algorithm presented in this paper.

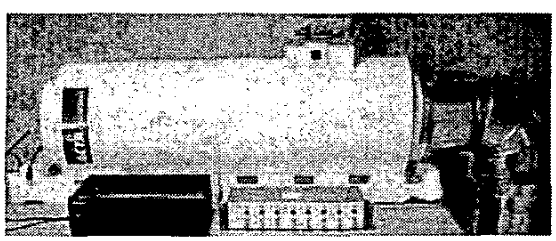

Fig. 1. ACTA airduct silencer located in the System Identification and Control Laboratory at UCSD

The basic principle and idea behind ANC is to cancel sound by a controlled emission of a secondary opposite (out-of-phase) sound signal [5], [6]. Crucial in the active control of sound is the actual algorithm that generates the controlled emission to obtain sound attenuation. Control algorithms for sound cancellation are typically based on feedforward compensation, feedback control or a hybrid form of both [7], [4]. Although feedback control is effective for disturbance attenuation, performance limiting aspects such as large time delays, non-minimum phase behavior and requirements on fast adaptation pose difficult design constraints on creating stabilizing feedback control applications for sound control systems. Successful implementation of feedback sound control can for example be found in specific applications that have been optimized with respect to feedback performance limitations [8].

In most cases the sound disturbance can be measured by a pick-up microphone and feedforward compensation provides a viable alternative to create a controlled emission for sound attenuation. Acoustic coupling and approximation of inverse dynamics limit the possibilities of feedforward compensation but algorithms based on recursive (filtered) Least Mean Squares (LMS) minimization can be quite effective for the estimation and adaptation of feedforward based sound cancellation [9]. In these approaches a linearly parametrized filter such as a finite impulse response (FIR) or linear regression filter are used for the recursive estimation and adaptation of the feedforward compensation.

In this paper a feedforward control algorithm is presented that is based on the recursive estimation of a generalized finite impulse FIR model. Generalized or orthogonal FIR models have been proposed in [10] and exhibit the same linear parametrization as a standard FIR filter. The added advantage of the generalized FIR filters lies in the possibility to include prior knowledge of system dynamics in the tapped delay line of the filter. This can be used to implement more accurate feedforward compensators that have superior performance compared to standard FIR filters.

The paper is outlined as follows. Following the analysis of the feedforward control design in Section II, the framework for the feedforward compensation based on generalized FIR filter estimation is presented in Section III. Section IV illustrated the results on the implementation of a 20th order feedforward compensator enabling a significant reduction of noise in the airduct over the frequency range from 40 till 400 Hz.

II. ACTIVE NOISE CONTROL

A. Feedforward Compensation in an Airduct

Located in the System Identification and Control Laboratory at UCSD, a commercial ACTA silencer for sound control in air ventilation systems is used for the case study of this paper. A photograph of the experiment is given in Figure 1 and a schematic representation of the experimental setup is depicted in Figure 2.

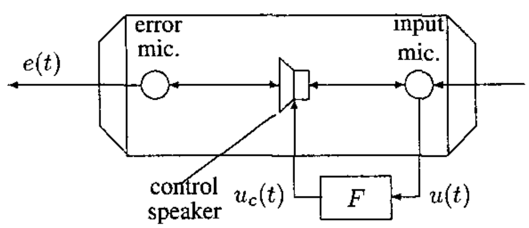

Fig. 2. Schematics of ANC system

As indicated in Figure 2, sound waves from a external noise source are predominantly traveling from right to left and can be measured by the pick-up microphone at the inlet and the error microphone at the outlet. The (amplified) signal $u(t)$ from the input microphone is fed into a feedforward compensator F that controls the signal $u_c(t)$ to the internal speaker for sound compensation. The signal $e(t)$ from the error microphone is used for evaluation of the effectiveness of the ANC system.

B. Analysis of Feedforward Compensation

In order to analyze the design of the feedforward compensator F, consider the block diagram depicted in Figure 3. Following this block diagram, dynamical relationship between signals in the ANC system are characterized by discrete time transfer functions, with $qu(t) = u(t+1)$ indicates a unit step time delay. The spectrum of noise disturbance $u(t)$ at the input microphone is characterized by filtered white noise signal $n(t)$ where $W(q)$ is a (unknown) stable and stable invertible noise filter [11]. The dynamic relationship between the input $u(t)$ and the error $e(t)$ microphone signals is characterized by $H(q)$ whereas $G(q)$ characterizes the relationship between control speaker signal and error $e(t)$ microphone signal. Finally, $G_c(q)$ is used to indicate the acoustic coupling from control speaker signal back to the input $u(t)$ microphone signal that creates a positive feedback loop with the feedforward $F(q)$.

For the analysis we assume in this section that all transfer functions in Figure 3 are stable and known. The error microphone signal $e(t)$ can be described by

$$e(t) = W(q)\left[H(q) + \frac{G(q)F(q)}{1 - G_c(q)F(q)}\right]n(t) \qquad (1)$$

and is a stable transfer function if the positive feedback connection of $F(q)$ and $G_c(q)$ is stable. In case the transfer

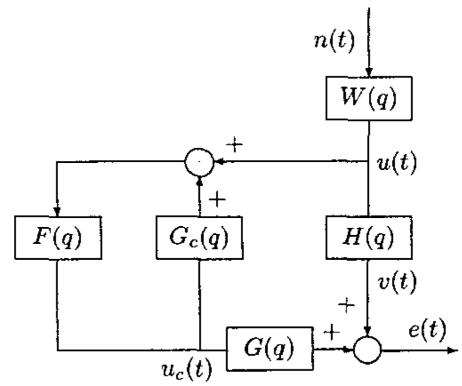

Fig. 3. Block diagram of ANC system with feedforward

functions in Figure 3 are known, perfect feedforward noise cancellation can be obtained in case

$$\begin{aligned} F(q) &= -\frac{H(q)}{G(q) - H(q)G_c(q)} \\ &= \frac{\tilde{F}(q)}{1 + \tilde{F}(q)G_c(q)},\ \tilde{F}(q) := -\frac{H(q)}{G(q)} \end{aligned} \qquad (2)$$

and can be implemented as a feedforward compensator in case $F(q)$ is a stable and causal transfer function. The expression in (2) can be simplified for the situation where the effect of acoustic coupling G_c can be neglected. In that case, the feeforward compensator F can be approximated by

$$F(q) \approx \tilde{F}(q) = -\frac{H(q)}{G(q)} \qquad (3)$$

and for implementation purposes it would be required that $F(q)$ is a causal and stable filter.

In general, the filter $F(q)$ in (2) or (3) is not a causal or stable filter due to the dynamics of $G(q)$ and $H(q)$ that dictate the solution of the feedforward compensator. Therefore, an optimal approximation has to be made to find the best causal and stable feedforward compensator. With (1) the variance of the discrete time error signal $e(t)$ is given by

$$\frac{\lambda}{2\pi}\int_{-\pi}^{\pi} |W(e^{j\omega})|^2 \left|H(e^{j\omega}) + \frac{G(e^{j\omega})F(e^{j\omega})}{1 - G_c(e^{j\omega})F(e^{j\omega})}\right|^2 d\omega$$

where λ denotes the variance of $n(t)$. In case variance minimization of the error microphone signal $e(t)$ is required for ANC, the optimal feedforward controller is found by the minimization

$$\begin{aligned} &\min_\theta \int_{\omega=-\pi}^{\omega=\pi} |L(e^{j\omega},\theta)|^2 d\omega := \min_\theta \|L(q,\theta)\|_2, \\ &L(q,\theta) = W(q)\left[H(q) + \frac{G(q)F(q,\theta)}{1 - G_c(q)F(q,\theta)}\right] \end{aligned} \qquad (4)$$

where the parametrized filter $F(q,\theta)$ is required to be a causal and stable filter. The minimization in (4) can be simplified to

$$\min_{\theta} \int_{\omega=-\pi}^{\omega=\pi} |L(e^{j\omega},\theta)|^2 d\omega := \min_{\theta} \|L(q,\theta)\|_2, \quad L(q,\theta) = W(q)[H(q) + G(q)F(q,\theta)] \tag{5}$$

in case the effect of acoustic coupling G_c can be neglected. The minimization in (4) and (5) are standard 2-norm based feedback control and model matching problems [12], [13] that can be solved in case the dynamics of $W(q)$, $G(q)$, $H(q)$ and $G_c(q)$ are known.

C. Estimation of Feedforward Compensation

In case the mechanical and geometrical properties of the silencer in Figure 2 are fixed, the transfer functions $H(q)$, $G(q)$ and $G_c(q)$ are predetermined, but possibly unknown. It is important to make a distinction between varying dynamics and fixed dynamics in the ANC system for estimation and adaptation purposes. An off-line identification technique can be used to estimate these transfer functions to determine the essential dynamics of the feedforward controller. Subsequently, the spectral contents of the sound disturbance characterized by the (unknown) stable and stably invertible filter $W(q)$ is the only varying component for which adaptation of the feedforward control is required.

Instead of separately estimating the unknown transfer functions and computing the feedforward controller via an adaptive optimization of (4) or (5), a direct estimation of the feedforward compensator can also be performed. For the analysis of the direct estimation of the feedforward compensator we assume that the acoustic coupling G_c can be neglected to simplify the formulae. In that case, the error signal $e(t)$ is given by

$$e(t,\theta) = H(q)u(t) + F(q,\theta)G(q)u(t)$$

and definition of the signals

$$y(t) := H(q)u(t), \;\; u_f(t) := -G(q)u(t) \tag{6}$$

leads to

$$e(t,\theta) = y(t) - F(q,\theta)u_f(t)$$

for which the minimization

$$\min_{\theta} \frac{1}{N}\sum_{t=1}^{N} e(t,\theta) \tag{7}$$

to compute the optimal feedforward filter $F(q,\theta)$ is a standard output error (OE) minimization problem in a prediction error framework [11]. In case the acoustic coupling G_c cannot be neglected, the estimation of the feedforward filter $F(q,\theta)$ has to be considered as a closed-loop identification problem. Using the fact that the input signal $u(t)$ satisfies $\|u\|_2 = |W(q)|^2\lambda$, the minimization of (7) for $\lim_{N\to\infty}$ can be rewritten into the frequency domain expression

$$\min_{\theta} \int_{\pi}^{-\pi} |W(e^{j\omega})|^2 |H(e^{j\omega}) + G(e^{j\omega})F(e^{j\omega},\theta)|^2 \tag{8}$$

using Parceval's theorem [11]. Due to the equivalency of (8) and (5), the same 2-norm objectives for the computation of the optimal feedforward compensator are used.

It should be noted that the signals in (6) to estimate the feedforward filter $F(q,\theta)$ are easily obtained by performing a series of two experiments. The first experiment is done without a feedforward compensator, making $e(t) = H(q)u(t) \triangleq y(t)$ where $e(t)$ is the signal measured at the error microphone. The input signal $u_f(t)$ can be obtained by applying the measured input microphone signal $u(t)$ from this experiment to the control speaker in a second experiment that is done without a sound disturbance. In that situation $e(t) = G(q)u(t) \triangleq -u_f(t)$. In Section III-B it will be shown that these experiments can be combined by using a filtered input signal $u_f(t)$ based on an estimated model $\hat{G}(q)$ of $G(q)$.

In general, the OE minimization of (7) is a non-linear optimization but reduces to a convex optimization problem in case $F(q,\theta)$ is linear in the parameter θ. Linearity in the parameter θ is also favorable for on-line recursive estimation of the filter and can be achieved by using a FIR filter parametrization

$$F(q,\theta) = D_0 + \sum_{k=0}^{N} \theta_k q^{-k} \tag{9}$$

for the feedforward compensator $F(q,\theta)$. D_0 is a (possible) direct feedthrough term of $F(q,\theta)$. A FIR filter parametrization also guarantees the causality and stability of the feedforward compensator for implementation purposes. One drawback of the FIR filter is the accuracy: many parameters θ_k are required to approximate an optimal feedforward controller with lightly damped resonance modes. To improve these aspects, generalized FIR filters can be used.

III. FEEDFORWARD DESIGN WITH GENERALIZED FIR

A. Generalized FIR Filter

Filter estimation using FIR models converge to optimal and unbiased feedforward compensators irrespective of the coloring of the noise as indicated in (8). However, a FIR filter is usually too simple to model the dynamics of a complex sound control system with many resonance modes. As a result, many tapped delay coefficients of the FIR filter are required to approximate the optimal feedforward compensator.

To improve the approximation properties of the feedforward compensator in ANC, the linear combination of tapped delay functions q^{-1} in the FIR filter of (9) are generalized to

$$F(q,\theta) = D_0 + \sum_{k=0}^{N} \theta_k V_k(q)$$

where $V_k(q)$ are generalized (orthonormal) basis functions [10] that may contain knowledge on system dynamics.

For details on the construction of the functions $V_k(q)$ one is referred to [10]. A short overview of the properties is given here. Let (A, B) be the state matrix and input matrix of an input balanced realization with a McMillan degree $n > 0$, and with $rank(B) = m$. Then matrices (C, D) can be constructed according to

$$\begin{aligned} C &= UB^*(I_n + A^*)^{-1}(I_n + A) \\ D &= U[B^*(I_n + A^*)^{-1}B - I_m] \end{aligned}$$

where $U \in \mathbb{R}^{m\times m}$ is any unitary matrix. This yields a square $m \times m$ inner transfer function $P(q) = D + C(qI - A)^{-1}B$, where (A, B, C, D) is a minimal balanced realization.

As $P(q)$ is a analytic outside and on the unit circle, it has a Laurent series expansion

$$P(q) = \sum_{k=0}^{\infty} P_k q^{-k}$$

which yields a set of orthonormal functions P_k [10]. Orthonormality of the set P_k can be seen by z-transformation of P_k :

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} P_i(e^{j\omega})P_k^T(e^{-j\omega})d\omega = \begin{cases} I & i = k \\ 0 & i \neq k \end{cases}$$

Subsequently, define $V_0(q) := (qI - A)^{-1}B$ and

$$V_k(q) = (qI - A)^{-1}BP^k(q) = V_0(q)P^k(q) \qquad (10)$$

then a generalized FIR filter can be constructed that consists of a linear combination $\sum_{k=0}^{N}\theta_k V_k(q)$ of the basis functions $V_k(q)$. This yields a generalized FIR filter

$$F(q) = q^{-n_k}\left[D_0 + \sum_{k=0}^{N}\theta_k V_0(q)P^k(q)\right] \qquad (11)$$

that also incorporates a (possible) delay time of n_k time steps in the feedforward compensator. A block diagram of the generalized FIR filter $F(q)$ in (11) is depicted in Figure 4 and it can be seen that it exhibits the same tapped delay line structure found in a conventional FIR filter, with the advantage of more general basis functions $V_k(q)$.

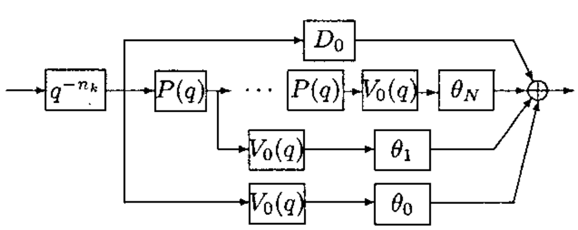

Fig. 4. Basic structure of generalized FIR filter

An important property and advantage of the generalized FIR filter is that knowledge of the (desired) dynamical behaviour can be incorporated in the basis function $V_k(q)$. Without any knowledge of desired dynamic behavior, the trivial choice of $V_k(q) = q^{-k}$ reduces the generalized FIR filter to the conventional FIR filter. If a more elaborate choice for the basis function $V_k(q)$ is incorporated, then (11) can exhibit better approximation properties for a much smaller number of parameters N than used in a conventional FIR filter. Consequently, the accuracy of the optimal feedforward controller will substantially increase. In the next section we will elaborate on the choice of the basis function $V_k(q)$ and the use of the generalized FIR filter in the role of ANC based on feedforward compensation.

B. Construction of Feedforward Controller

Continuing the line of reasoning mentioned in Section II-C where the effect of the acoustic coupling $G_c(q)$ was assumed to be negligible, the parametrization of the generalized FIR filter in (11) will be used in the OE minimization of (7). As the generalized FIR filter is linear in the parameters, convexity of the OE minimization is maintained and an on-line recursive estimation techniques can be used to estimate and adapt the feedforward controller for ANC purposes. For the construction of the feedforward controller $F(q)$ we make a distinction between an initialization step and the recursive estimation of the filter. Both are discussed in more details below.

1) Initialization: To initialize the on-line adaptation of the generalized FIR filter, the signals $y(t)$ and $u_f(t)$ in (6) have to be available to perform the OE-minimization. With no feedforward controller in place, the signal $y(t)$ is readily available via

$$y(t) := H(q)u(t) \qquad (12)$$

Because $G(q)$ is fixed once the mechanical and geometrical properties of the ANC system in Figure 2 are fixed, an initial off-line estimation can be used to estimate a model for $G(q)$ to construct the filtered input signal $u_f(t)$. The use of an estimated transfer function for filtering purposes is common practice in most filtered least mean squares algorithms [9]. Similar approaches are also found in identification algorithms that provide unbiased estimates of models on the basis of closed-loop experimental data.

Estimation of a model $\hat{G}(q)$ can be done by performing an experiment using the control speaker signal $u_c(t)$ as excitation signal and the error microphone signal $e(t)$ as output signal. Construction of the prediction error

$$\varepsilon(t, \beta) = e(t) - G(q, \beta)u_c(t)$$

and a minimization

$$\hat{G}(q) = G(q, \hat{\beta}),\ \hat{\beta} = \min_{\beta}\frac{1}{N}\sum_{t=0}^{N}\varepsilon^2(t, \beta) \qquad (13)$$

yields a model $\hat{G}(q)$ for filtering purposes. Since $\hat{G}(q)$ is used for filtering purposes only, a high order model can be estimated to provide an accurate reconstruction of the filtered input signal via

$$\hat{u}_f(t) := \hat{G}(q)u(t) \qquad (14)$$

To facilitate the use of the generalized FIR filter, a choice have to be made for the basis function $V_k(q)$ in (10). A low order model for the basis function will suffice, as the generalized FIR model will be expanded on the basis of $V_k(q)$ to improve the accuracy of the feedforward compensator. As part of the initialization of the feedforward compensator, a low order IIR model $\hat{F}(q)$ of the feedforward filter $F(q)$ can be estimated with the initial signals available from (12), (14) and the OE-minimization

$$\hat{F}(q) = F(q, \hat{\theta}),\ \hat{\theta} = \min_{\theta} \frac{1}{N} \sum_{t=0}^{N} \varepsilon^2(t, \theta) \qquad (15)$$

of the prediction error

$$e(t, \theta) = y(t) - F(q, \theta)\hat{u}_f(t)$$

where $\hat{u}_f(t)$ is given in (14). An input balanced state space realization of the low order model $\hat{F}(q)$ is used to construct the basis function $V_k(q)$ in (10).

2) Recursive estimation: With a known feedforward $F(q, \theta_{k-1})$ already in place, the signal $y(t)$ can be generated via

$$y(t) := H(q)u(t) = e(t) + F(q, \theta_{k-1})u_f(t) \qquad (16)$$

and requires measurement of the error microphone signal $e(t)$, and the filtered input signal $u_f(t) = G(q)u(t)$ that can be simulated by (14). With the signal $y(t)$ in (16), $\hat{u}_f(t)$ in (14) and the basis function $V_k(q)$ in (10) found by the initialization in (15), a recursive minimization of the feedforward filter is done via a standard recursive least squares minimization

$$\theta_k = \min_{\theta} \frac{1}{k} \sum_{t=0}^{k} \lambda(t)(y(t) - F(q, \theta)\hat{u}_f(t))^2 \qquad (17)$$

where $F(q, \theta)$ is linearly parametrized according to (11) and $\lambda(t)$ indicates an exponential forgetting factor on the data [11]. As the feedforward filter is based on the generalized FIR model, the input $\hat{u}_f(t)$ is also filtered by the tapped delay line of basis functions. Since the filter is linear in the parameters, recursive computational techniques can be used to update the parameter θ_k.

IV. IMPLEMENTATION OF FEEDFORWARD ANC

Upon initilization and calibration of the feedforward controller, a 18th order ARX model $\hat{G}(q)$ of $G(q)$ was estimated in order to be able to create the filtered input digital $\hat{u}_f(t)$ in (14). The filtered input signal $\hat{u}_f(t)$ and the observed error microphone signal $y(t)$ sampled at 2.56kHz were used to estimate a low (4th) order OE model to create the basis function $V_k(q)$ in (10) for the generalized FIR filter parametrization of the feedforward controller. During the estimation of the low order model $\hat{F}(q)$ also an estimate of the expected time delay n_k in (11) was performed and was found to be $n_k = 9$.

After initialization, the information of the filter $\hat{G}(q)$, the basis function $V_k(q)$ and the time delay n_k was used to perform a recursive estimation of the generalized FIR filter based feedforward compensator $F(q)$. For practical and implementation purposes, only $N = 5$ parameters in the generalized FIR filter of (11) were estimated. With a 4th order basis function $V_k(q)$ this amounts to a 20th order generalized feedforward filter. For comparison of the performance of the 20th order generalized FIR filter based feedforward compensator, also $N = 20$ parameters of a conventional FIR filter in (9) were estimated.

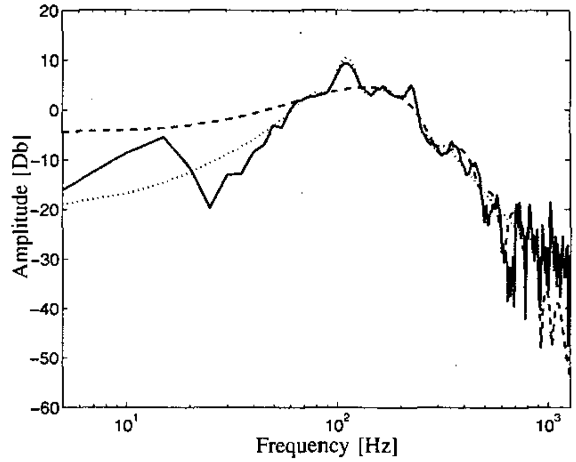

Fig. 5. Spectral analysis of ideal feedforward controller (solid), approximation using FIR filter estimation (dashed) and approximation using generalized FIR filter estimation (dotted)

To compare the results of the two feedforward compensators, in Figure 5 an amplitude Bode plot of the estimated ideal feedforward controller, the conventional FIR approximation and the generalized FIR approximation are given. The amplitude Bode plot of the ideal feedforward controller of (3) was found during initialization by a spectral analysis using the filtered input signal $\hat{u}_f(t)$ and the observed error microphone signal $y(t)$.

It can be observed from the frequency plots in Figure 5 that the generalized FIR filter gives a better approximation of the spectral estimate of the ideal feedforward compensator. The performance of the generalized FIR filter is also confirmed by the estimates of the spectral contents of the microphone error signal $e(t)$ plotted in Figure 6. The spectral content of the error microphone signal has been reduced significantly by both the FIR and the generalized FIR filters in the frequency range from 40 till 400Hz. However, the generalized FIR filter does a much better job than the conventional FIR filter s indicated by the lower spectral content of the signal $e(t)$ in that frequency range.

A final conformation of the performance of the ANC has been depicted in Figure 7. The significant reduction of the error microphone signal can be observed from the time traces and the norm of the signals (displayed on the right Figure 7).

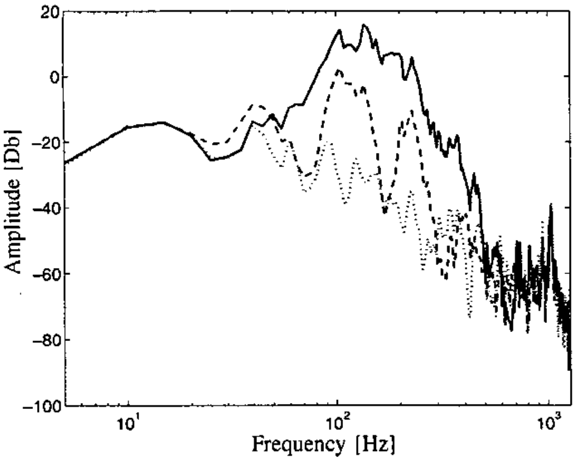

Fig. 6. Estimate of spectral contents of error microphone signal $e(t)$ without ANC (solid), with ANC using 20th order FIR filter (dashed) and with ANC using 20th order generalized FIR filter (dotted)

The experimental data indicates the effectiveness of the generalized FIR filter for feedforward sound compensation.

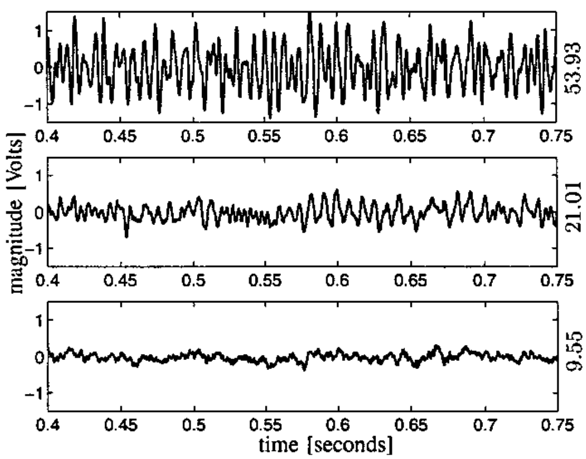

Fig. 7. Evaluation of error microphone signal before ANC (top), with ANC using 20th order FIR filter (middle) and with ANC using 20th order generalized FIR filter (bottom)

V. CONCLUSIONS

In this paper a new methodology has been proposed for the active noise control in an airduct using generalized FIR filters. A generalized FIR filter has the same linear parameter structure as a taped delay FIR filter that is favorable for (recursive) estimation purposes. The advantage is to be able to include prior knowledge of system dynamics in the tapped delay line of the filter for better accuracy of the feedforward filter.

The approach in this paper is illustrated on a commercial airduct silencer that can be implemented in an air conditioning system. The feedforwared filter is estimated via recursive filtered least squares techniques. The design is compared with a conventional FIR filter method and evaluated on the basis of an experimental data from the active noise cancellation experiment. Comparison indicates that generalized FIR filter yields a better approximation of the desired feedforward compensation and provide significant improvement in active sound suppression.

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