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August 1988

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CALCULATING THE J_c , B , T SURFACE FOR NIOBIUM TITANIUM USING A REDUCED-STATE MODEL

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Abstract

This report presents a method for calculating the J_c , B , T surface for commercial niobium titanium superconductor given T_c , B_{c2} and J_c over a range B at a given temperature. The J_c calculated as a function of B and T agrees with measured values of J_c quite well over a range of T from 1.5 K to 7 K and a range of B from 0.1 T to 10 T. The values of J_c given by this theory are good enough to be used to calculate the performance of superconducting magnets at temperatures other than 4.2 K, and the theory can be used to predict the magnetization effects of superconductors over a wide range of temperatures and inductions.

Background

There is often the need to know the J_c of niobium titanium superconductor under conditions different from those measured by the manufacturer. For example, one often knows the critical current (critical current density, J_c) as a function of magnetic induction at the boiling point of helium ($T = 4.22$ K) over a limited range of magnetic inductions (e.g., 2 T to 8 T). Suppose, however, one wants to know the critical current density over a wide range of magnetic inductions (from 0 to B_{c2}) at a temperature of, e.g., 3.0 K. The measured data are often unavailable and costly to obtain. This report presents a method by which one can calculate the critical surface for a commercial niobium titanium given only the upper critical field, H_{c2} , the critical temperature, T_c , a parabolic breakpoint temperature, T' , and some values of the critical current versus magnetic field at the boiling point of helium (or any other temperature).

The Reduced Critical State Method

The method used to calculate the critical surface for Nb-Ti is based on the following assumptions: 1) the critical temperature, T_c , at zero field and the upper critical field, H_{c2} , at zero temperature are functions only of the alloy or chemical composition of the superconductor; 2) the critical current density, J_c , is a function of the metallurgical treatment of the superconductor such as heat treatment and cold work, whereas T_c and H_{c2} are independent of the treatment of the superconductor; 3) the critical current of the superconductor varies linearly with temperature (for commercial Nb-Ti, this is true above T'); and 4) J_c -versus- B is given at one temperature over a range of magnetic fields, or in the absence of a complete J_c -versus- B curve (particularly at low fields), the ratio of $J_c(T, B)/J_c(T, 5 \text{ T})$ is identical for all materials of the same alloy.

The reduced critical temperature, T_{cR} , and the reduced critical induction, B_{cR} , are defined as follows:

$$T_{cR} = \frac{T_c(B)}{T_c(0)} \quad (1)$$

when J_c is zero, and

$$B_{cR} = \frac{B_{c2}(T)}{B_{c2}(0)} = \frac{H_{c2}(T)}{H_{c2}(0)} \quad (2)$$

when J_c is zero.

B_{cR} can be stated in terms of temperature, T_{cR} , using the following relationship:

$$B_{cR} = 1 - (T_{cR})^N \quad (3)$$

where N is some number less than two. Lubell¹ states that $N = 1.7$ fits measured data for niobium titanium over a reasonable temperature range. This value is consistent with WHH theory.² An even better fit can be achieved using a mixed quadratic and linear equation. (See Reference 5)

Figure 1 shows that Equation 3 fits to a measured value of B_{cR} -versus- T_{cR} for niobium titanium. Also included in Figure 1 are commercial samples of niobium titanium tantalum, niobium zirconium, niobium-tin ribbon, and multifilamentary niobium tin in a bronze matrix. The niobium titanium data shown in Figure 1 comes from References 3, 4, 6-9.

Figure 1.

REDUCED UPPER CRITICAL FIELD VERSUS REDUCED CRITICAL TEMPERATURE FOR VARIOUS COMMERCIAL SUPERCONDUCTORS

Superconducting alloys such as Nb-Ti, Nb-Ti-Ta, and Nb-Zr generally have set values of $T_c(0)$. The upper critical induction $B_{c2}(0)$ quoted in the literature, often the value of surface-current critical induction $B_{c3}(0)$, is generally a little high. The value of B_{c2} where the bulk current density goes to zero can be from 0.5 to 1.2 T lower than the highest value of B_{c3} .^{9,10} The critical temperature $T_c(0)$ is a set value when the filament diameter is above 1 μm . With niobium titanium, the value of T_c and B_{c2} are depressed as the filament diameter drops below 0.2 μm .

Once the B_{c2} -versus- T_c curve has been determined at zero critical current, a large part of the critical surface can be calculated, provided one knows the critical current versus magnetic induction at a temperature T_0 that is less than T_c . The critical surface for Nb-Ti can be divided into two regions. In the region above a temperature T' ($T' \sim 0.4 T_c$ for Nb-Ti), J_c has a linear dependence with temperature. At temperatures below T' , the value of J_c has a parabolic dependence with temperature (see Figure 2).

The current density in the superconductor at a given temperature T greater than T' and a magnetic induction B can be calculated using the following equation:

$$J_c(T, B) = J_c(T_0, B) + \frac{dJ_c}{dT} (T - T_0) \quad (4)$$

where T_0 is the temperature ($T_0 > T'$) for which a known curve of J -versus- B is given and where

$$\frac{dJ_c(B)}{dT} = \frac{J_c(T_0, B)}{T_0 - T^*} \quad (5)$$

and where from Equation 3

$$T^* = (1 - B_{cR})^{1/N} T_c(0) \quad (6)$$

Equation 4 is valid only over the range of magnetic induction B for which $J(T_0, B)$ is given. If the value of $J(T, B)$ calculated using Equation 4 is negative, then $J(T, B)$ is zero. The key elements for calculating an accurate value of $J(T, B)$ are the values of $B_{c2}(0)$ and $T_c(0)$ used for the material and the values of $J(T_0, B)$. Given values of $J(T_0, B)$ at more than one temperature, one can improve the values of $B_{c2}(0)$ and $T_c(0)$ used in Equations 3 and 6.

It has been observed that at temperatures below about 3.5 K there is a different temperature dependence for J_c .^{10,11} A parabolic temperature dependence can be used for J_c at temperatures below T' (~ 3.5 K). The parabola should have the property that $dJ_c/dT = 0$ at $T = 0$ and dJ_c/dT must equal the linear dJ_c/dT at $T = T'$. (T' is the parabolic breakpoint temperature.) The other boundary condition is that the J_c -versus- T function must be continuous from $T = 0$ to $T = T^*$ (see Figure 2).

If one looks at the case where T_0 is greater than T' and T is less than T' , the following set of equations now applies:

$$J_c(T, B) = AA + CCT^2 \quad (7)$$

where

$$AA = \frac{dJ_c}{dT} \left(\frac{T'}{2} - T^* \right) \quad (8a)$$

and

$$CC = \frac{1}{2T'} \frac{dJ_c}{dT} \quad (8b)$$

where T^* is determined by Equation 6, and dJ_c/dT is determined by Equation 5 because T_0 is above T' . When T is greater than T' and T_0 is greater than T' , Equation 4 is used as before. (See Figure 2 for a definition of AA , T' , T^* and dJ_c/dT .) For the case when T_0 is less than T' , the reader should consult Reference 5 for the proper equations to describe this case.

Figure 2.
**LINEAR PLUS PARABOLIC TEMPERATURE
DEPENDENCE OF CRITICAL CURRENT DENSITY
IN NIOBIUM TITANIUM**

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Equation 4 or 7 can be used to calculate $J_c(T, B)$ provided one has a value of $J_c(T_0, B)$. Many times one has a value of $J_c(T_0, B)$ over limited values of B . The extension of J_c at a higher field is quite easy because the J_c function is nearly linear with magnetic induction B . The following relationship can be used to estimate the critical current density given $J_c(B)$ at a B greater than 0.7 B_{c2} :

$$J_c(T_0, B) = J_c(T_0, B_0) \frac{B_{c2}(T_0) - B(T_0)}{B_{c2}(T_0) - B_0(T_0)} \quad (9)$$

where T_0 is the temperature at which measured values $J_c(B, T_0)$ are given; B_0 is the highest magnetic induction at which the critical current density is known; $B_{c2}(T_0)$ is the critical magnetic induction at temperature T_0 ; and $J_c(T_0, B_0)$ is the critical current density at temperature T_0 and magnetic induction B_0 .

Most critical current density measurements are made using short sample measurements. The flaws in this method include: 1) the size effects where there are large variations in B across the filaments, and 2) the measurements do not reflect true J_c because there is "sausaging" of the filaments. A well made superconductor will have a true J_c which is close to that measured by short sample methods (at inductions above 1 T). The only reliable method for measuring J_c at low field is magnetization. (See Reference 5)

A number of niobium titanium superconductors were examined to determine the ratio of low-field J_c to the J_c at 5 T. Figure 3 shows the reduced current density [$J_c(B)$ divided by $J_c(5T)$] at 4.2 K. The conductors shown in Figure 3 include Nb-Ti made in the years between 1969 and 1987.^{3,6,7,12-14} Conductors A through G and I had J_c calculated from short-sample measurements. Conductors H through Q had J_c calculated from magnetization. The scatter in the data in Figure 3 is as large as ± 16 percent. It is believed that J_c at H_{c1} (about 0.011 T) is about 12 times the J_c at 5 T. The author found the results given in Figure 3 to be surprising, yet investigations of samples of Nb-Ti as far back as 1965 yield results which fall on the curve given in Figure 3.¹⁵

Figure 3.
REDUCED CRITICAL CURRENT DENSITY
VERSUS MAGNETIC INDUCTION
FOR VARIOUS SAMPLES OF NIOBIUM TITANIUM

Comparison of Calculations with Measurements

Figure 4 shows the calculated value of J_c as a function of B at various temperatures for the superconductor used in the Hampshire et al. measurements.³ The calculations were based on measurements at 4.22 K given in the black squares shown in Figure 4. The other points shown in Figure 4 are measured values. From Figure 4 one can see that the measured points agree with the calculated curves within a few percent. Figure 5 shows a comparison with calculations and measurements for a modern Nb-Ti conductor.¹⁰ The curves were fit to the point in the filled circles. One can see in Figure 5 that the measured data points fit quite well with the calculated values of J_c .

Figure 4.
CRITICAL CURRENT DENSITY VERSUS
TEMPERATURE AND MAGNETIC INDUCTION
LINEAR PLUS PARABOLIC TEMPERATURE FIT

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Figure 5.
CRITICAL CURRENT DENSITY VERSUS
TEMPERATURE AND MAGNETIC INDUCTION
LINEAR PLUS PARABOLIC TEMPERATURE FIT AT 4.24 K
Nb-46.5Ti

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Reference 5 shows other comparisons between calculated J_c and measured J_c . A good fit between measured and calculated values of J_c can be obtained for a variety of niobium titanium alloys from 44 w% titanium to 53 w% titanium.^{6,9}

The relationship shown in Figure 3 suggests that one can calculate the entire J_c , B, T surface for niobium titanium if one knows the value of J_c at 4.22 K and 5 T. (This is often the specified value of J_c .) Figure 6 shows the calculated J_c , B, T surface for Nb-Ti with a $J_c(5.0 \text{ T}, 4.2 \text{ K}) = 3000 \text{ A mm}^{-2}$. The calculated surface shown in Figure 6 is probably accurate to ± 10 percent over much of the range. The type of calculation shown in Figure 6 may be the only reasonable way to estimate the J_c of superconductor at low fields such as at injection into a machine like the SSC.

The theory given in Equation 4 permits one to calculate the temperature dependence of the magnetization of niobium titanium. At 4.22 K the theory calculates for Nb-46.5 Ti ($T_c = 9.35$, $H_{c2} = 13.6 \text{ T}$) the following:

$$\frac{1}{J_c} \frac{dJ_c}{dT} \sim 0.201 \text{ K}^{-1}$$

at a magnetic induction of 0.3 T, where T^* is nearly equal to $T_c(0)$. Measurements of the temperature dependence of the width of magnetization curves were made by Ghosh of Brookhaven National Laboratory.¹³ These measurements consistently showed that dJ_c/dT divided by J_c was consistently 0.2 to 0.22 K^{-1} at magnetic inductions of 0.0 to 0.3 T. Magnet measurements by W. B. Sampson et al.¹⁶ in 1978 indicate that the temperature dependence at 4.0 K is such that dJ_c/dT divided by J_c is from 0.200 to 0.208 K^{-1} at an injection magnetic induction of 0.375 T in the CBA magnets.

Figure 6. Critical Surface of Niobium Titanium in Three Dimensions Based on $J_c = 3000 \text{ A mm}^{-2}$ at 5 T and 4.2 K

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