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The Enhanced Honey-Bee Mating Optimization Algorithm for Water Resources Optimization

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Abstract Evolutionary and meta-heuristic algorithms are widely used to solve water resources optimization problems. In this context, the honey bee mating optimization (HBMO) algorithm, inspired by the mating ritual of honey bees, is a reliable and efficient algorithm. The HBMO algorithm is modified in this work leading to the Enhanced HBMO (EHBMO) algorithm. The EHBMO is then applied to solve several unconstrained/constrained mathematical benchmark functions and a multi-reservoir problem. The performance of the EHBMO is compared with those of the elitist genetic algorithm (EGA) and the HBMO algorithm. The results show that the EHBMO achieves a better solution in a smaller number of functional evaluations and with less variance of results about global optima in comparison with the EGA and the HBMO algorithm.

Keywords Enhanced honey-bee mating optimization (EHBMO) · Honey-bee mating optimization (HBMO) · Elitist genetic algorithm (EGA) · Multi-reservoir optimization · Heuristic search

1 Introduction

Reservoir systems must be optimally operated to maximize the efficiency of water use. Several authors have applied classic optimization methods such as linear programming (LP) (Chow and

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Cortez-Rivera 1974) and dynamic programming (DP) (Murray and Yakowitz 1979) to the optimal operation of reservoir systems. Recently, Soleimani et al. (2016) reported the application of stochastic dynamic programming (SDP) for water reservoir operation. However, classic optimization methods have several limitations that constrain their range of applicability (Bozorg-Haddad et al. 2006). The limitations of classic optimization methods have given rise to evolutionary and meta-heuristic algorithms in recent years. Maier et al. (2014) studied the current status and future research directions on evolutionary algorithms applied to water resources problems.

One of the most well-known evolutionary algorithms is the genetic algorithm (GA) developed by Holland (1975). The standard GA begins with a randomly generated population of possible solutions (individuals). After estimating individuals' fitness, some of them are selected as parents according to their fitness values. A new population (or generation) of possible solutions (the children's population) is produced by applying the crossover operator to the parent population and then applying the mutation operator to their offspring. These iterations involving the replacement of parents' population with children's populations of solutions are repeated until stopping criteria are satisfied (Michalewicz 1996). The Elitist version of the GA, that allows the best individual(s) from a generation to carry over to the next one, was introduced by De Jong (1975). Several authors have implemented various types of GAs to water resources optimization (East and Hall 1994; Wardlaw and Sharif 1999; Aboutalebi et al. 2015). Nicklow et al. (2010) provides a review of GAs applied to water resource problems. Also, genetic programming (GP) has been applied to solve reservoir problems by several authors (Fallah-Mehdipour et al. 2012, 2013; Ashofteh et al. 2014, 2015, 2016).

Simulated annealing (SA) was implemented by Tospornsampan et al. (2005) for solving a ten-reservoir optimization problem. Ant colony optimization (ACO) was applied to water resources optimization by Jalali et al. 2006. Ghimire and Reddy (2013) applied the particle swarm optimization (PSO) algorithm to optimize the operation of a reservoir for hydropower production. Asgari et al. (2015) used weed optimization algorithm (WOA) for optimal water reservoir optimization. Bozorg-Haddad et al. (2015a) applied the bat algorithm (BA) to find the optimal operation of reservoir systems. The water cycle algorithm (WCA) was implemented to find optimal operation strategies water reservoir systems by Bozorg-Haddad et al. (2015b). The firefly algorithm (FA) was modified and applied to solve reservoir problems by Garousi-Nejad et al. (2016a), b).

The HBMO algorithm is inspired by the mating ritual of honey bees. It was developed and applied to reservoir operation by Bozorg-Haddad et al. (2006). Several studies have reported the successful application of the HBMO algorithm to solve a variety of problems such as water reservoir operation (Bozorg-Haddad et al. 2008a; Bozorg-Haddad et al. 2010a), water distribution networks (Bozorg-Haddad et al. 2008b; Bozorg-Haddad et al. 2016a; b; Solgi et al. 2015, 2016) and project management (Bozorg-Haddad et al. 2010b). Several of those works have proven the superiority of the HBMO algorithm compared with other algorithms such as the GA, ACO, and PSO.

This study improves the capability of the HBMO by introducing a newly Enhanced HBMO (EHBMO) that relies on a new mating process that replaces the one used in the HBMO algorithm. This change allows the EHBMO to achieve solutions closer to the global optimum with smaller computational effort compared to the HBMO. The performance of the EHBMO algorithm is tested with constrained and unconstrained mathematical optimization problems. The EHBMO is applied to find optimal operation of a multi-reservoir system, also. The next section briefly describes the HBMO algorithm. This is followed by the development of the EHBMO. Lastly, the performance of the EHBMO is compared with those of the elitist GA (EGA) and the HBMO algorithm in solving different well-known benchmark optimization problems.

1.1 The HBMO Algorithm

The flowchart of the HBMO algorithm is shown in Fig. 1 where it is seen that the HBMO employs a simulated annealing (SA) function to choose drones for generating the next generation in the search for a solution (Bozorg-Haddad et al. 2006). The best solution of the HBMO algorithm is known as the queen, which mates with randomly generated drones that are successful in a simulated annealing (SA) function to procreate the next generation. In the mating ritual a drone is first randomly generated. After evaluating its fitness value, its genome is memorized in the queen's spermatheca if the drone succeeds in the SA function according to Eq. (1):

$$MPROB = e^{-\frac{|Qf - f_d|}{Speed}} \tag{1}$$

in which, $MPROB$ = the probability of mating drone d with the queen, Qf = the fitness value for the best solution in the present generation (Queen), f_d = the fitness value of the mating

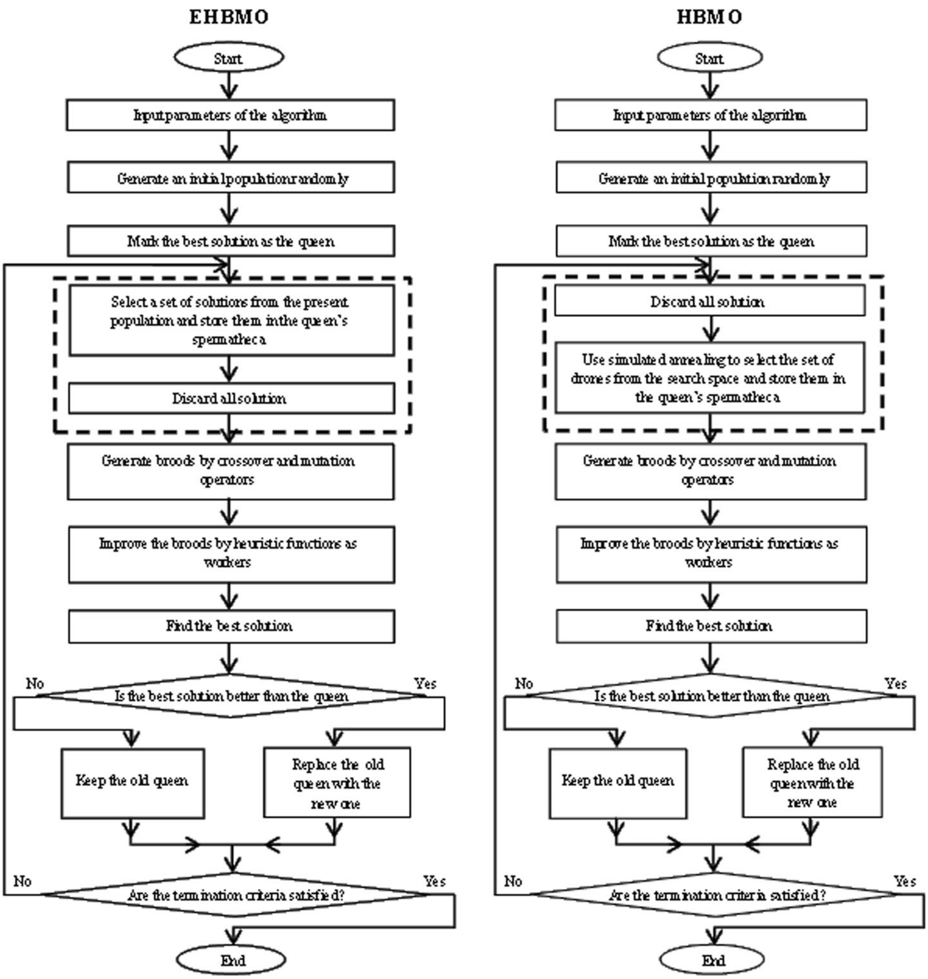

Fig. 1 The flowchart of the EHBMO (left) in comparison with the HBMO (right)

droned, *Speed* = the queen's speed. An uniformly distributed random variable (*PROB*) within [0,1] is generated while *MPROB* is evaluated. If *MPROB* is larger than or equal to *PROB* the drone d is successful in mating with the queen, otherwise it is not. Equation (1) acts as an annealing function. The probability of mating is high when either the queen's speed is high or the fitness of drone $d(f_d)$ is as good as that of the queen (Qf).

This selection method of the HBMO is computationally burdensome because the SA function requires evaluating the fitness values of the drones. In HBMO a drone is randomly generated and its fitness value is evaluated but it may not be successful in mating with the queen. The unsuccessful drones are deleted and another one is generated to be tested. It may be necessary to generate many drones before selecting only one of them from the decision space. This requires a large number of evaluations of the objective function involving unsuccessful drones. It is worthy of notice, also, that the mating process simulated in the HBMO algorithm differs with the actual mating of honey bees in that real queens choose drones from the existing population. These drones inherit their genome from previous generations. However, in the HBMO algorithm drones are randomly generated solutions that are independent of solutions in previous iterations. Therefore, in the present study the EHBMO is modified in a way that the queen only chooses from the existing population instead of the entire decision space.

1.2 The Life Cycle of Honey Bees as a Precursor of the Enhanced HBMO (EHBMO)

There is fossil evidence of honey bees' existence dating back 100 million years ago (Michener and Grimaldi 1988). Honey bees live in well-organized hives. The purpose of such hive is to maximize efficiency by resorting to division of labor. A well-organized hive remains viable except in special circumstances. A colony of bees is a group of bees living together in one bee hive. A honey bee hive consists of a single queen, broods, drones and workers (Moritz and Southwick 1992). Queen and workers are female and drones are male. The queen is generally the main reproductive individual. In other words, the queen is the only bee that can mate with drones and can fertilize the eggs. The primary duty of workers is brood caring and the drones are the fathers of the colony. The queen can lay both fertilized and unfertilized eggs. Fertilized eggs represent female bees (worker or queen) and unfertilized eggs represent drones. So drones are haploid and amplify their mother's genome without alteration of their genetic composition except through mutation. However female bees inherit their genome from both their mothers and fathers. When a new queen is born, it replaces the old queen or it leaves the hive.

To lay fertilized eggs the queen must mate with drones. For this purpose the queen exits from the hive and engages in mating flight around the hive. The queen mates with several drones in each mating flight. In each mating flight the queen usually mates with seven to twenty drones. In each mating the drone's sperm reaches the queen's spermatheca and accumulates there to form the genetic pool of the colony. After the end of the mating flight the queen returns to the hive and starts laying eggs. The successful drones in mating flights die immediately after mating with the queen. In other words, insemination ends with the death of the drone. The unsuccessful drones (those that do not mate with the queen) also die from starvation and exposure because the workers forbid their entry to the hive at the end of the mating season.

In the EHBMO the queen, broods, and drones represent solutions that are made of genes. Each gene is equivalent to a decision variable expressed as a real value. The best solution is considered as the queen. Broods can be diploid or haploid. The former are made by applying mutation and crossover operators on the queen's genome and drone's, whereas the latter are made by applying mutation on the queen. Brood caring by workers is mapped into the algorithm to improve the broods by applying heuristic functions. A mating flight is mapped into the EHBMO as the queen chooses drones from the present population using a selection method. The genome of each drone that is successful in mating is stored in the queen's spermatheca and this drone is deleted from the population as these drones die after mating. Also, the death of drones at the end of mating season is simulated by destroying all remaining drones after the mating flight in each iteration of the EHBMO algorithm.

1.3 The Enhanced HBMO Algorithm

The EHBMO starts with the random generation of the initial population. The solutions are ranked based on their fitness. Then the fittest (best) solution is marked out as the queen and the other solutions are considered as drones. Some drones (solutions) are chosen from the present population to mate with the queen. The genome of each selected drone is stored in the queen's spermatheca and the drone is deleted from the population so that a drone can only be chosen once. The number of drones selected for mating is equal to the capacity of the queen's spermatheca (SC), which is a predefined parameter. After the selection process, the remaining solutions are deleted. The queen and the solution stored in the queen's spermatheca are used to make the next generation. First, the broods (diploid or haploid) are made. The haploid broods are made by applying mutation on the queen. The diploid broods are made by applying crossover and mutation operators between the queen and the solutions stored in the queen's spermatheca. Then, by applying heuristic functions as workers, an attempt is made to improve the broods. Finally, if the best brood is better than the old queen, the best brood replaces the old queen. Again other solutions of the population are considered as drones and the queen chooses drones from the population to make the next generation. Figure 1 shows the flowchart of the EHBMO algorithm in comparison with the HBMO's.

1.4 Selection

Selection in the EHBMO is the procedure by which SC drones are chosen from the population to mate with the queen. A popular selection approach is proportionate selection (Michalewicz 1996). According to proportionate selection, the probability of a drone being selected is given by Eq. (2):

$$P_d = \frac{\xi(f_d)}{\sum_{d=1}^{ND} \xi(f_d)} \tag{2}$$

in which, P_d= the probability of drone d being selected,$\xi(f_d)$ = the scaled fitness value for drone d, and ND = the total number of drones in the population.

Population diversity and selective pressure are the most important factors in the search process. These factors are inversely related so that increasing one causes reducing another one (Whitley 1989). A high selective pressure may lead to prematurely convergence while a low selective pressure may lead to stagnation (Wardlaw and Sharif 1999). Several scaling functions exist that help balance the effect of selective pressure and population diversity including: linear scaling (Michalewicz 1996), sigma truncation (Michalewicz 1996), power law scaling (Michalewicz 1996), logarithmic scaling (Grefenstette and Baker 1989) and exponential scaling (Grefenstette and Baker 1989). Boltzmann selection is another scaling method that relies on a scaling function$\xi(f_i)$ = exp (f_i/T). It has been indicated that selective pressure to be low (high) when the control parameter T is high (low) (Back 1994). This study proposes a new Boltzmann scaling function as follows:

$$\xi(f_d) = e^{-\frac{Qf - f_d}{Qf - Wf}} \tag{3}$$

in which, Wf = the fitness value of the worst solution in the present generation.

The proposed scaling function is self-regulation and it does not have parameters to be adjusted. This departs from previous scaling methods that require the analyst to set their parameters to regulate the selective pressure. The selective pressure is high (low) when the difference between the best solution and the worst solution in the present generation is low (high). The scaled fitness for the best solution and the worst one are equal to e^0 (1) and e^{-1} (0.368) respectively. Other solutions in the populationare exponentially scaled between 1 and 0.368 based on how close they are to the queen.

The EHBMO selects a drone from the population by first valuating the scaled fitness values of all drones using Eq. (3). Then the probability of selection of each drone is evaluated using Eq. (2). Based on the evaluated probabilities a Roulette Wheel is turned once to select a drone. The selected drone is deleted from the population and is transported to the queen's spermatheca. Again, the probability of selection for each drone is evaluated using Eq. (2) and a new roulette wheel is played based on the new probabilities to select another drone. Deletion of each drone changes the probability of the remaining drones in Eq. (2). This process continuous until the queen's spermatheca is filled.

1.5 Brood Caring by Workers

In the brood caring stage of the EHBMO an attempt is made to improve the generated broods using heuristic functions. For this purpose, a heuristic function is introduced in this study. A heuristic function consists of a procedure to collect and provide information for the search process about the direction to reach a goal. It is clear that the best solution in the population, which is known as the queen, is mostly nearer the optimum rather than other individuals of the population. Also, in the EHBMO successive queens are memorized and compared with each other. Consequently the values of decision variables of the best solution in the present population (new queen) and the result of comparison between it and the best solution of the previous population (old queen) provide valuable information that serves as a guideline to generate new random values for the new generated solutions. In fact, the features of the new queen and changes between two consecutive queens show directions for generating random values of the brood's decision variables that are most likely leading to an improved point. The introduced heuristic function replaces the value of some genes of a brood with new ones that are randomly generated based on the value of the corresponding genes that belong to the brood, the queen of the previous iteration, and the queen of the present population. If $X = (x_1, \ldots, x_n)$ is a brood, $Y = (y_1, \ldots, y_n)$ is the best solution in the present iteration, $Y' = \left(y'_1, \ldots, y'_n\right)$ is the best solution in the previous iteration, and the component x_k from brood X is obtained by substitution, $X' = \left(x_1, \ldots, x'_k, \ldots, x_n\right)$ to produce the brood after brood caring. x'_k is evaluated according to Eq. (4):

$$x'_k = sign(\delta)^2 \times G(y_k, sign(\delta)) + \left(1 - sign(\delta)^2\right) \times \left\{sign(\sigma)^2 \times G(x_k, sign(\sigma)) + \left(1 - sign(\sigma)^2\right) \times x_k\right\} \tag{4}$$

where

$$G(a,b) = \frac{1+b}{2} \times RND(ub_k, a) + \frac{1-b}{2} \times RND(a, lb_k) \tag{5}$$

$$\sigma = y_k - x_k \tag{6}$$

$$\delta = y_k - y_k^{'} \tag{7}$$

in which, x_k= the value of the brood's k-th component before substitution;$x_k^{'}$= the value of the brood's k-th component after substitution;y_k= the value of the best solution's k-th component in the present iteration;$y_k^{'}$= the value of the best solution's k-th component in the previous iteration;$RND(a, b)$= a random value between a and b;$sign(a)$= returns the sign of the number a (sign function) that can be equal to 1, −1 or 0; lb_k= the feasible lower value of component k; and ub_k= the feasible upper value of component k. Thereafter, the functions $sign(a)$ and $G(a,b)$ are evaluated and are substituted in Eq. (4) as follows:

$$x_k^{'} = \begin{cases} RND(ub_k, y_k) & \text{if} \quad \delta > 0 \\ RND(y_k, lb_k) & \text{if} \quad \delta < 0 \\ RND(ub_k, x_k) & \text{if} \quad \delta = 0 \quad and \quad \sigma > 0 \\ RND(x_k, lb_k) & \text{if} \quad \delta = 0 \quad and \quad \sigma < 0 \\ x_k & \text{if} \quad \delta = 0 \quad and \quad \sigma = 0 \end{cases} \tag{8}$$

According to Eqs. (8) if the gene of the best solution (y_k) in the present iteration is larger than that of the best solution in the previous iteration $\left(y_k^{'}\right)$, a random value between y_k and ub_kreplaces the gene of the brood. Conversely if y_k is less than $y_k^{'}$, $x_k^{'}$ is made equal to a random value between lb_k andy_k. When y_k and $y_k^{'}$ are the same ($\delta = 0$), $x_k^{'}$is determined based on the result of the comparison between the present value of the brood's gene and that of the corresponding gene of the best solution in the present iteration (y_k). If y_k is larger than x_k,$x_k^{'}$is made equal to a random value between x_k and ub_k. If y_k is less than x_k, $x_k^{'}$is made equal to a random value between lb_k and x_k. Otherwise ify_k,$y_k^{'}$ and $x_k$are the same, the value of the brood's gene is not changed.

1.6 Crossover Operators

The crossover operator generates new offspring by exchanging some genes between the queen and the drone. Michalewicz (1996) have described several methods of crossover including: (1) one-point crossover; (2) two-point crossover; and (3) uniform crossover.

1.7 Mutation Operators

The mutation operator replaces randomly some genes of an offspring. Two methods of mutation for real-value representations are uniform mutation and non-uniform mutation. Uniform mutation permits a value that is randomly generated within the feasible range of valuesto replacethevalue of a gene. The algorithmic search for an optimum becomes more localized as a run of the algorithm progresses when implementing on non-uniform mutation (Michalewicz 1996).

1.8 Differences between the HBMO and EHBMO

- In the mating process of the HBMO drones are generated randomly among the decision space. However in the EHBMO drones are selected only from the previous population.
- In the HBMO selection of drones is performed based on the SA function. But the EHBMO uses proportionate selection and roulette wheel. Hence the developed scaling function plays a key role.
- The HBMO eliminates the present population (of current solutions) before selection of the drones. The EHBMO, instead, first selects the drones among the present population and then eliminates the remaining.
- The EHMBO, unlike the HBMO, takes advantage of a developed heuristic function in the brood carrying stage.

1.9 Case Studies

The benchmark mathematical optimization problems and a well-known multi-reservoir optimization problem are used in this work to examine the performance of the EHBMO algorithm. The elitist GA (EGA) and the HBMO algorithm are also implemented to solve these problems and to compare their results with those of the EHBMO. The brood caring stage is removed in the EHBMO (WBC) and its results are compared to the EHBMO with the mathematical functions. This permits assessing how the brood caring stage and the introduced heuristic function affect the EHBMO. The global optima of all problems are calculated with the Lingo software to investigate the accuracy of the solutions obtained with the presented algorithms. The parameters of all the algorithms are listed in Table 1. The values of the parameters were determined by performing sensitivity analysis. In this way a combination of parameters is considered and the algorithm is run several times. The algorithm is run several times, each time with a different combination of parameters. The results of different runs are compared and the best parameters values are chosen. According to the Table 1 it is recommended that for the EHBMO a population size of at least 180 be selected. The capacity of the spermatheca is an important parameter in the EHBMO that ensures population diversity. It is therefore recommended that in the EHBMO the population size and the capacity of spermatheca be selected such that the capacity of the spermatheca equal about 14% of the number of populations.

Table 1 Parameters of the EGA, HBMO, and EHBMO employed to solve the test problems

Algorithm	Parameter	Goldstein-Price	Shubert	Constrained	Four-reservoir
GA	Number of populations	100	100	100	100
	Probability of crossover (%)	40	50	50	20
	Number of iterations	1,200	4,500	60,500	15,000
HBMO	Number of populations	201	201	201	220
	Capacity of the spermatheca	50	50	50	10
	Number of iterations	100	100	10,000	5,000
EHBMO	Number of populations	211	211	181	211
	Capacity of the spermatheca	30	30	25	30
	Number of iterations	100	100	1,000	4,000
EHBMO (WBC)	Number of populations	211	211	181	-
	Capacity of the spermatheca	30	30	25	-
	Number of iterations	200	200	5,000	-

1.10 Unconstrained Mathematical Benchmark Functions

The first unconstrained problem is the Goldstein-Price function (Goldstein and Price 1971). This function is given by Eq. (9). The second unconstrained problem is Shubert's function (Hennart 1982). Shubert's function is given by Eq. (10).

$$\begin{gathered}\textit{Minimize} \quad f(x_1,x_2) = \left[1+(x_1+x_2+1)^2\times\left(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2\right)\right]\times\\ \left[30+(2x_1-3x_2)^2\times\left(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2\right)\right]\\ -2\le x_1,x_2\le 2\end{gathered} \tag{9}$$

$$\begin{gathered}\textit{Minimize} \quad f(x_1,x_2)=\left\{\sum_{j=1}^{5} j\times \mathrm{Cos}((j+1)\times x_1+j)\right\}\times\left\{\sum_{j=1}^{5} j\times \mathrm{Cos}((j+1)\times x_2+j)\right\}\\ -10\le x_1,x_2\le 10\end{gathered} \tag{10}$$

1.11 Constrained Mathematical Benchmark Function

The EHBMO was applied to solve a two-variable nonlinear, constrained, programming problem to further test its capabilities. The objective function and constraints of the constrained problem are given by Eqs. (11)–(13). Bozorg-Haddad et al. (2006) solved this function for an allowable decision variable between 0 and 6. In the present study the degree of difficulty of this problem was increased by expanding the decision space.

$$\begin{gathered}\textit{Minimize} \quad f(x_1,x_2)=\left(x_1^2+x_2-11\right)^2+\left(x_1+x_2^2-7\right)^2\\ -6\le x_1,x_2\le 6\end{gathered} \tag{11}$$

Subject to:

$$g_1(x)=5.062-x_1^2-(x_2-2.5)^2\ge 0 \tag{12}$$

$$g_2(x)=(x_1-0.05)^2+(x_2-2.5)^2-4.83688798\ge 0 \tag{13}$$

1.12 Multi Reservoir Optimization

A four-reservoir optimization problem was solved to further test the performance of the EHBMO in solving water resources problems with a large decision space. This problem was introduced by Chow and Cortez-Rivera (1974) (see also, Murray and Yakowitz 1979). Although this problem can be solved with classic methods, it is challenging for evolutionary and meta-heuristic algorithms and this feature makes it a reasonable choice for testing the newly developed algorithm. Recently, several investigators examined the performance of different types of evolutionary and meta-heuristic algorithms to solve this problem (Bozorg-Haddad et al. 2010a; Bozorg-Haddad et al. 2015a; b; Garousi-Nejad et al. 2016a).

Releases from the reservoirs are used to generate hydropower and to satisfy irrigation water demand in the four-reservoir system. The objective function is given by Eq. (14) that satisfies the constraints stated in Eqs. (15) through (21).

$$Maximize \quad OF = \sum_{n=1}^{N} \sum_{t=1}^{T} BNF(n,t) \times R(n,t) \tag{14}$$

Subject to:

Reservoir water balance:

$$S(n,t+1) = S(n,t) + I(n,t) + MR(n,t) \quad t = 1,...,T \quad , \quad n = 1,...,N \tag{15}$$

Constraints on reservoir releases:

$$R\min(n,t) \le R(n,t) \le R\max(n,t) \quad t = 1,...,T \quad , \quad n = 1,...,N \tag{16}$$

Constraints on reservoir storage:

$$S\min(n,t) \le S(n,t) \le S\max(n,t) \quad t = 1,...,T \quad , \quad n = 1,...,N \tag{17}$$

Constraints on initial storage at each reservoir:

$$S(n,1) = \mathrm{Sin}it(n) \quad n = 1,...,N \tag{18}$$

Constraints on ending storage:

$$S(n,T+1) = \mathrm{Star}(n) \quad n = 1,...,N \tag{19}$$

$$M = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & -1 \end{bmatrix} \tag{20}$$

$$MR = M \times R \tag{21}$$

in which, n = the counter of the reservoir number; N = the total number of reservoirs; t = the counter of operation period; T = the total number of operation periods; $BNF(n, t)$= benefit per unit of release of reservoir n in period t; $R(n, t)$= the release of reservoir n in period t; $S(n, t)$= the storage of reservoir n at the start of period t; $I(n, t)$= the inflow to reservoir n in period t; M = a $N \times N$matrix of indices of reservoir connectivity; R = a $N \times T$matrix of indices of reservoirs' releases;MR = a $N \times T$matrix that equals the multiplication of matrix $M$by matrix R; R min (n, t)= the minimum release of reservoir n in period t; R max (n, t)= the maximum release of reservoir n in period t; Smin(n, t)= the minimum storage of reservoir n in period t; Smax(n, t)= the maximum storage of reservoir n in period t; *Sinit*(n)= the initial storage of reservoir n; *Star*(n)= the target ending storage of reservoir n.

The values of R min $(n, t)$for all reservoirs and all periods are equal to 0.005. The values of R max $(n, t)$for all periods for reservoirs 1, 2, 3, and 4 are equal to 4.0, 4.5, 4.5, and 8.0 respectively. Both *Sinit*(n) and *Star*(n) for reservoirs 1, 2, 3, and 4 are equal to 6.0, 6.0, 6.0, and 8.0, respectively. Other data of the four-reservoir problem including inflows, reservoir storages, and additional details required for modeling the system are available in Murray and Yakowitz (1979).

2 Results

The results of 10 independent runs of the EGA, HBMO, and EHBMO are listed in Table 2. Figure 2 shows the convergence curve of the EHBMO and the progression traces of the best run of all the algorithms.

2.1 Unconstrained Mathematical Benchmark Functions

The global minimum values of the Goldstein-Price function the Shubert function are equal to 3.00000000 and −186.73090000, respectively. It is seen in Table 2 that the performance of the EHBMO is better than those of the EGA and HBMO in solving the unconstrained problems. In both unconstrained problems, The EHBMO's convergence rate is superior to those of the GA and HBMO in addition to yielding a better objective function value. The EHBMO achieved a closer solution to the global optimum than the other algorithms, while the number of functional evaluations with the EHBMO is less than those of the other algorithms and the value of the *CV* of the EHBMO is smaller (better) than those of the EGA and HBMO. Table 2 shows that the values of *ANFE* of the HBMO algorithm employed to solve the Goldstein-Price and the Shubert function are respectively five and 20 times larger than those of the EHBMO while the best solutions of the EHBMO are better than those of the HBMO. Figure 2 shows that the EHBMO achieved a better solution with fewer objective functional evaluations than the other algorithms. For the Goldstein-Price function, the values of x_1 and $x_2$of the best solution obtained with the EHBMO are equal to −1.00 and 4.62×10^{-6} respectively. For the Shubert function, the values of x_1 and x_2 of the best solution obtained with the EHBMO are equal to 5.482845 and −7.708321 respectively, which is one of several global optima of this function.

2.2 Constrained Mathematical Benchmark Function

The global optimum of this problem using nonlinear programming (NLP) of Lingo 11 after 288 iterations is 10.16858. The value of x_1 and $x_2$corresponding to the answer of Lingo 11 are −2.187390 and 3.026615, respectively.

The results of Table 2 establish that the best solution of the EHBMO after 1000 iterations equals 10.16859. The value of x_1 and x_2 related to the best solution obtained with the EHBMO are −2.187986 and 3.024136, respectively. It is seen in Table 2 that the value of the *CV*of the EHBMO after 1000 iterations is smaller (better) than those of the EGA and HBMO after 60,500 and 10,000 iterations respectively. Although the number of iterations of the EGA and HBMO is respectively 60 and 10 times larger than that of the EHBMO, the best solution of the EHBMO after 1000 iterations is closer to the global optimum than those achieved with the EGA and HBMO. The worst solution obtained with the EHBMO after 1000 iterations is better than the best solutions obtained with the EGA and HBMO. It is noticeable that the EHBMO achieved a better solution while its *ANFE* value is approximately equal to only 20% of those associated with the EGA and HBMO (Table 2). Also, in Fig. 2, it is seen that the EHBMO achieved a better solution with fewer objective functional evaluations than those required by the other algorithms.

2.3 Effect of the Brood Carrying Stage on the EHBMO's Performance

In addition to the EHBMO, the results of the EHBMO without brood caring (WBC) are also shown in Table 2. It is seen in Table 2 that the performance of the EHBMO with brood caring

Table 2 The results of 10 independent runs of the EGA, HBMO, and EHBMO

Problem	Algorithm	Best	Ave	Worst	SD	CV	ANFE	MNFE
Goldstein-Price function	EGA	3.00014410	3.00137296	3.00395976	1.5×10^{-3}	5.0×10^{-4}	118,801	118,801
	HBMO	3.00001856	3.00030142	3.00125367	3.9×10^{-4}	1.3×10^{-4}	103,281.6	83,128
	EHBMO	3.00000001	3.00000028	3.00000206	5.9×10^{-7}	1.9×10^{-7}	21,211	21,211
	EHBMO (WBC)	3.00000598	3.00004217	3.00014251	4.2×10^{-5}	1.4×10^{-5}	42,211	42,211
Shubert function	EGA	-186.73063000	-186.72760000	-186.71927000	3.1×10^{-3}	-1.7×10^{-5}	445,501	445,501
	HBMO	-186.73068000	-186.72905000	-186.72467000	1.8×10^{-3}	-9.9×10^{-6}	411,330	365,984
	EHBMO	-186.73091000	-186.73088000	-186.73079000	3.1×10^{-5}	-1.7×10^{-7}	21,211	21,211
	EHBMO (WBC)	-186.73090000	-186.73077000	-186.73047000	1.4×10^{-4}	-7.6×10^{-7}	42,211	42,211
Constrained problem	EGA	10.1789	10.3441	10.51271	0.15	0.014	5,989,501	5,989,501
	HBMO	10.17372	10.31677	10.36931	0.06	0.006	5,818,210	5,679,087
	EHBMO	10.16859	10.16875	10.16959	2.9×10^{-4}	2.9×10^{-5}	180,181	180,181
	EHBMO (WBC)	10.16859	10.16972	10.17678	2.4×10^{-3}	2.4×10^{-4}	900,181	900,181
Four-reservoir system	HBMO[a]	308.07	307.50	306.71	0.417	0.0010	-	-
	GA[b]	282.90	278.37	272.62	3.30	0.0118	-	-
	GA[c]	300.47	299.70	298.46	0.75	0.0020	-	-
	BA[b]	308.20	307.84	307.12	0.35	0.0011	500,000	-
	WCA[c]	306.92	304.92	302.38	1.89	0.0060	-	-
	FA[d]	306.35	305.51	304.72	0.6647	0.00185	500,050	-
	EGA	295.16	286.36	273.76	7.06	0.0247	1,485,999	1,485,999
	HBMO	308.20	306.92	305.41	1.07	0.0035	1,296,393	1,295,178
	EHBMO	308.24	308.08	307.21	0.321	0.0010	840,211	840,211

Best, *Ave*, and *Worst* the best, average, and worst values of the objective functions obtained over 10 runs, respectively

SD standard division of the objective function values obtained over 10 runs, *CV* coefficient of variation of the objective function values obtained over 10 runs, *ANFE* the average of the number of objective function evaluations executed to achieve the optimal solution over 10 runs, *MNFE* the minimum of the number of objective function evaluations executed to achieve the optimal solution over 10 runs

[a] Reported by Bozorg-Haddad et al. (2010a))

[b] Reported by Bozorg-Haddad et al. (2015a)

[c] Reported by Bozorg-Haddad et al. (2015b)

[d] Reported by Garousi-Nejad et al. (2016a)

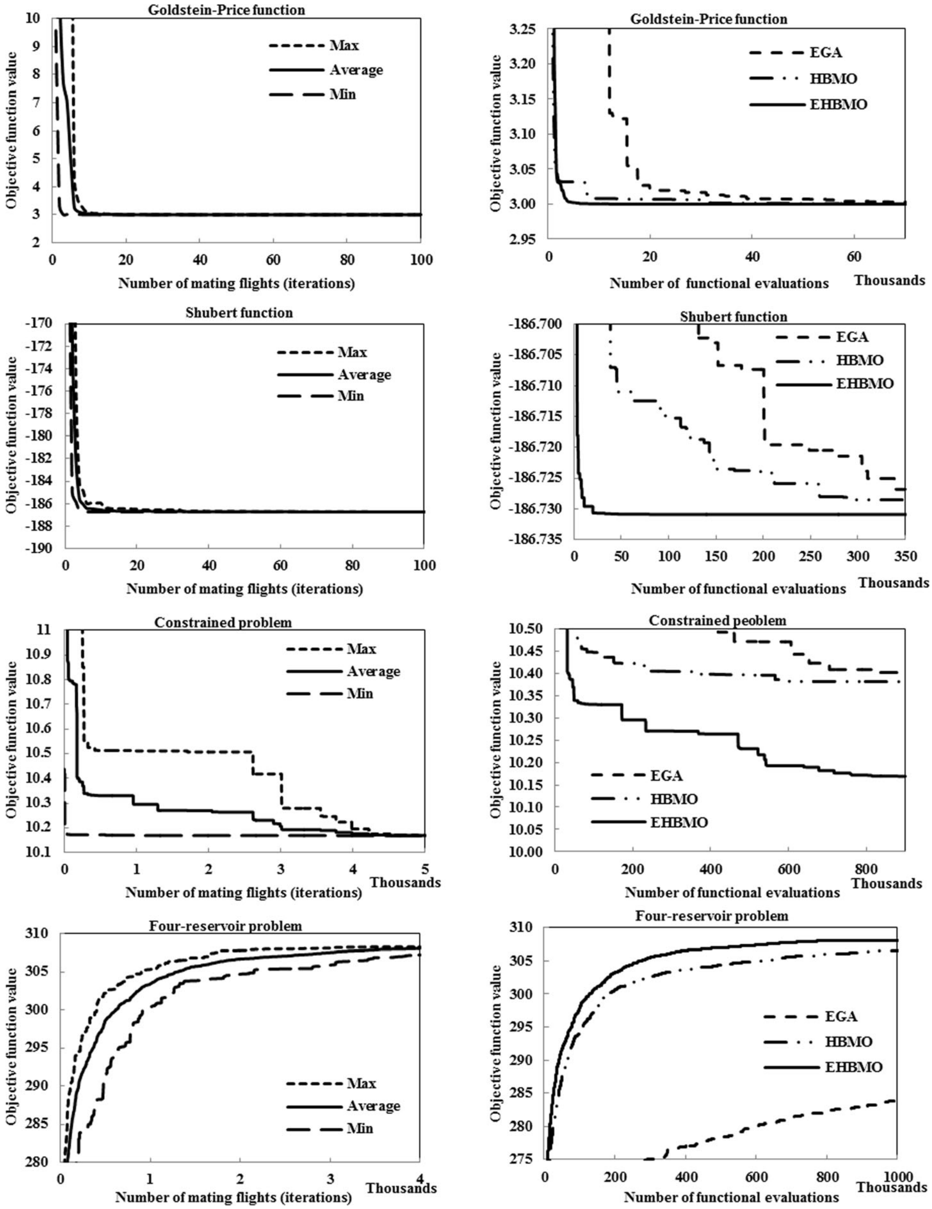

Fig. 2 Convergence curve of the EHBMO (left) and the progression traces of the best run of all the algorithms (right)

is better than that of the EHBMO (WBC). The EHBMO achieved a better solution than the EHBMO (WBC) while the number of iterations and the number of functional evaluations executed by the EHBMO is two times less than that of the EHBMO (WBC). Therefore, the introduced heuristic function and its application in the brood caring stage improve the capability of the EHBMO. Furthermore, it is seen in Table 2 that the performance of the EHBMO (WBC) is also better than those of the EGA and HBMO. This demonstrates that even without brood carrying the EHBMO has a better performance in comparison to its competitors.

2.4 Multi Reservoir Optimization

The four-reservoir problem has been previously solved by several investigators with a variety of optimization methods. Chow and Cortez-Rivera applied Linear Programming (LP) to solve this problem and they reported that the LP solution is equal to 308.26. The best solution achieved by Murray and Yakowitz (1979) using differential dynamic programming (DDP) was equal to 308.23. Bozorg-Haddad et al. (2010a) solved this problem with Lingo 8 software and reported that the global optimum of the four-reservoir problem is equal to 308.29. This problem has already been solved with several evolutionary and meta-heuristic algorithms (Bozorg-Haddad et al. 2010a; Bozorg-Haddad et al. 2015a; b; Garousi-Nejad et al. 2016a). The results reported in previous studies are listed in Table 2. Moreover, Table 2 lists the results of 10 independent runs of the EGA, HBMO, and EHBMO for the four-reservoir problem obtained in this study.

The EHBMO achieved a closer solution to the global optimum in comparison to other algorithms while the number of functional evaluations done by the EHBMO is smaller than those of the EGA and HBMO. According to Table 2, the best solution obtained with the EHBMO equals 308.24 that is 0.016% different from the global optimal solution of the problem. It is also seen in Table 2 that the value of the *SD* for the EHBMO is smallest than those of the other algorithms. The average value of objective function obtained with the EHBMO exhibits 0.068% difference with the global optimal solution of the problem and the small coefficient of variation (*CV*) obtained with the EHBMO illustrates the high accuracy of the EHBMO and its capacity to reach near-optimal global solutions of the reservoir problem. Bozorg-Haddad et al. (2010a) solved this problem with the HBMO algorithm. They reported that the best solution obtained with the HBMO was equal to 308.24 after a long processing time (65,000 iterations). The number of populations was equal to 220, the number of iterations was 65,000, and the number of functional evaluations of the HBMO was estimated to equal about 14 million. The EHBMO has achieved the same solution with less than one million (840,211) objective function evaluations, which is approximately equal to only 6% of the number of functional evaluations executed by the HBMO algorithm.

Reservoir releases and reservoirstorage variations are graphed in Fig. 3, respectively, for all reservoirs. The maximum and minimum were met by the optimized releases and storages for all reservoirs. Additionally, reservoir storage is the same at the beginning and end of the operation period and it is equal to the target storage value for all reservoirs.

2.5 Concluding Remarks

The HBMO algorithm is an optimization algorithm inspired by the mating ritual of honey bees. Although in recent years the good performance of the HBMO algorithm was shown in various studies, it imposes a large computational burden to model the mating ritual of honey bees. In the present study a modified version of the HBMO algorithm was developed and called the EHBMO. The EHBMO was designed to reduce the computational demands of the HBMO algorithm while retaining its strengths. The performance of the developed algorithm was evaluated with unconstrained and constrained mathematical benchmark functions and a multi reservoir system. The performance of the EHBMO was compared to those of the EGA and the HBMO algorithm in terms of the convergence to global optima and of the variance of results about global optima. The results showed that the EHBMO algorithm is an improvement over the HBMO algorithm. In all the solved problems the number of functional evaluations executed by the EHBMO was significantly less than those required by the EGA and the

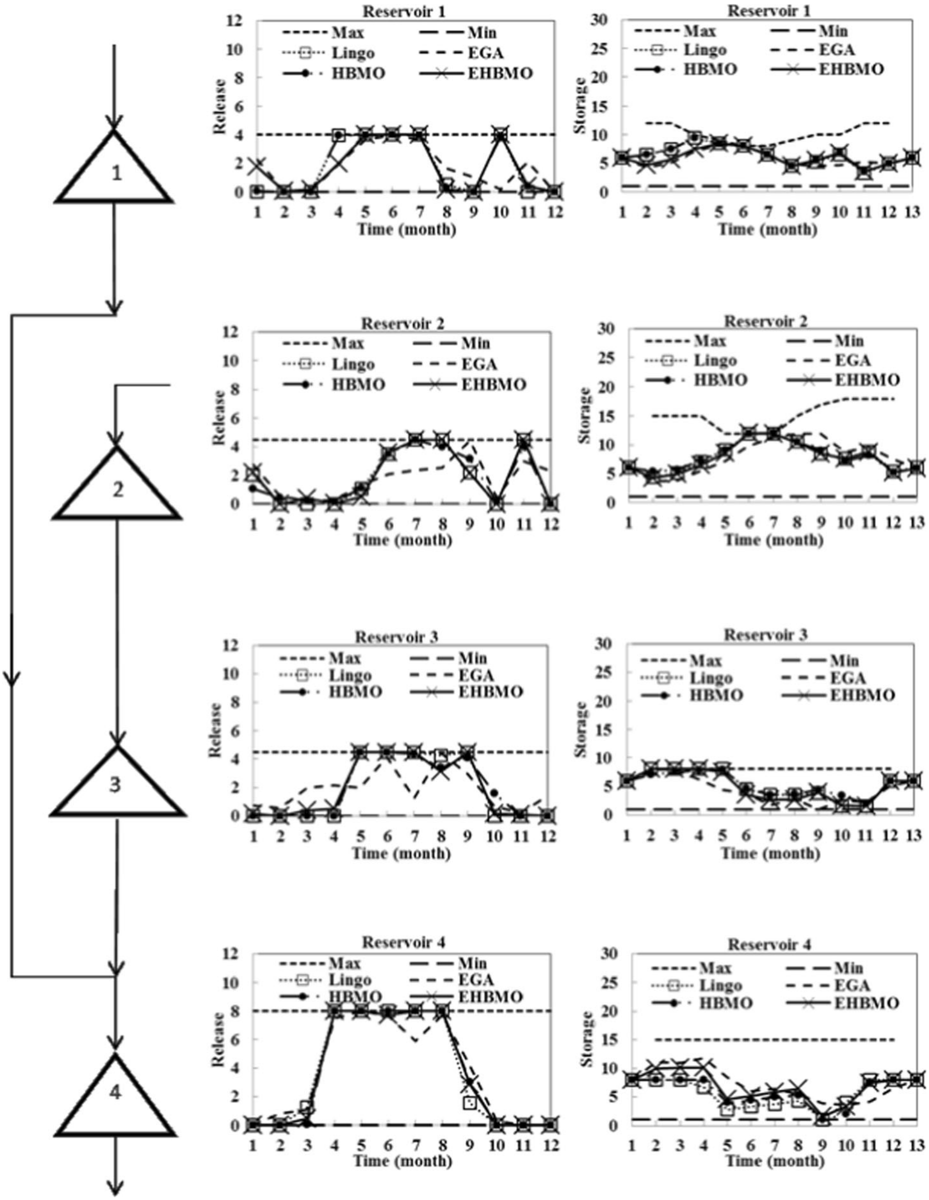

Fig. 3 Reservoir schematic, reservoir releases and reservoir storages obtained with LP, GA, HBMO, and EHBMO for the four-reservoir system

HBMO algorithm while the EHBMO achieved a better solution with smaller coefficient of variation in comparison to the other algorithms. The EHBMO achieved 99.984% of the global optimum in the multi reservoir optimization problem with a number of functional evaluations of that is significantly smaller than those of its competitors, which proves the efficiency and applicability of the EHBMO algorithm in solving such problems. Lastly, although the performance of the EHBMO was examined with several problems in this study, the performance of the EHBMO deserves further scrutiny with other types of problems with larger decision spaces and higher complexity than those of this study's test problems.

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