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Two Potential Mechanisms Underlying the Link between Approximate Number Representations and Symbolic Math in Preschool Children

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Abstract

The approximate number system (ANS) is frequently considered to be a foundation for the acquisition of uniquely human symbolic numerical capabilities. However, the mechanism by which the ANS influences symbolic number representations and mathematical thought remains poorly understood. Here, we tested the relation between ANS acuity, cardinal number knowledge, approximate arithmetic, and symbolic math achievement in a one-year longitudinal investigation of preschoolers' early math abilities. Our results suggest that cardinal number knowledge is an intermediary factor in the relation between ANS acuity and symbolic math achievement. Furthermore, approximate arithmetic performance contributes unique variance to math achievement that is not accounted for by ANS acuity. These findings suggest that there are multiple routes by which the ANS influences math achievement. Therefore, interventions targeting both the precision and manipulability of the ANS may prove to be more beneficial for improving mathematical reasoning compared to interventions targeting only one of these factors.

Keywords: approximate number representations; numerical cognition; math cognition

Introduction

In our information-driven society, math ability is essential for success, particularly in the STEM fields that drive the modern economy. However, within the population there exists large variance in math ability, and low math proficiency is associated with poor health and occupational outcomes (e.g., Parsons & Bynner, 2006). Critically, math ability when a child first enters schooling is the strongest predictor of later math and overall academic achievement (e.g., Duncan, Dowsett, & Claessens, 2007). Many cognitive and socioeconomic factors are known to contribute to individual differences in math achievement (e.g., Bull & Scerif, 2001; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). One of these factors is an evolutionarily ancient system for representing approximate quantities. Although educated humans typically think about number using language, we also possess a system for representing number in an approximate, nonsymbolic fashion. This system, termed the approximate number system (ANS), is not dependent on language or formal schooling and is present in a wide variety of nonhuman species (e.g., Dehaene, 1997; Gallistel & Gelman, 1992; Hubbard et al., 2008).

The ANS is frequently hypothesized to be a cognitive foundation for symbolic math abilities. Lending support to this view is the pervasive finding that the acuity of the ANS, typically measured by an individual's ability to compare two arrays of dots, correlates with symbolic math achievement throughout the lifespan (see Chen & Li, 2013 for review, including failures to find this relation). Importantly, ANS acuity prior to the beginning of formal math instruction is predictive of later math achievement (Libertus, Feigenson, & Halberda, 2013; Mazzocco, Feigenson, & Halberda, 2011; Starr, Libertus, & Brannon, 2013; vanMarle, Chu, Li, & Geary, 2014). These studies suggest that the precision of approximate number representations may contribute to children's acquisition of symbolic math principles and influence symbolic math performance throughout the lifespan.

Despite the many studies documenting a link between ANS acuity and symbolic math, the mechanism underlying this relation remains unclear. One possibility is that the precision of the ANS influences children's acquisition of symbolic number representations. Before children can begin learning symbolic arithmetic and other mathematical operations, they must first learn the meaning of number words and Arabic numerals. While there is debate over the nature of the nonverbal representations that first ground the meaning of number words (e.g., Carey, 2004; Gallistel & Gelman, 1992), it is clear that sometime in early childhood, children map number words onto approximate number representations (Siegler & Opfer, 2003). Perhaps children with more precise internal representations of numerical quantities are at an advantage for forming these mappings, which enables them to master the count list and counting principles earlier than their peers with less precise internal quantity representations (Starr et al., 2013; vanMarle et al., 2014). In support of this view, preschoolers' competence with numerical symbols, and particularly their understanding of cardinality, has been shown to mediate the relation between their ANS acuity and math achievement (vanMarle et al., 2014). Thus, the precision of the ANS may influence the rate and fidelity with which children learn the meaning of numerical symbols. Under this scenario, the ANS may influence math achievement indirectly through its connection to symbolic number representations – the ANS may be influential for grounding the meaning of numerical symbols, but may not directly contribute to symbolic mathematics (see also Lyons & Beilock, 2011).

A second hypothesis is that the manipulability of ANS representations serves as an intuitive basis for symbolic arithmetic (Barth, La Mont, Lipton, & Spelke, 2005). The ANS enables human infants (McCrink & Wynn, 2004), preschoolers (Barth et al., 2005; Gilmore, McCarthy, & Spelke, 2010), and monkeys (Cantlon & Brannon, 2007) to perform approximate arithmetic operations without the use of symbols. Therefore, the ANS supports both quantity representation and quantity manipulation. Children's approximate arithmetic performance at the beginning of kindergarten is predictive of their symbolic math achievement at the end of the academic year (Gilmore et al., 2010). In addition, approximate arithmetic performance has been found to mediate the relation between ANS acuity and symbolic math achievement in grade-school children (Pinheiro-Chagas et al., 2014). Furthermore, training nonsymbolic arithmetic in adults leads to improvements in their symbolic arithmetic performance (Park & Brannon, 2013; 2014). Thus, it may not be the precision of ANS representations but instead the manipulability of ANS representations that influences symbolic math achievement. As a result, children who are more adept at manipulating approximate quantities in arithmetic operations may also be more adept at symbolic arithmetic because of the overlap in cognitive processes required by both forms of arithmetic.

Critically, the above hypotheses need not be mutually exclusive – it is possible that the ANS contributes to symbolic math through multiple routes, including other possibilities not listed above. For example, the ANS may also influence symbolic math abilities through its representation of numerical order (Lyons & Beilock, 2011), or it may serve as an on-line error detection system, enabling erroneous answers to be rejected in favor of more plausible solutions (Lourenco, Bonny, Fernandez, & Rao, 2012). In addition, the relation between the ANS and symbolic math is not necessarily static, and it is possible that the ANS may contribute to symbolic math abilities in different ways throughout development depending on the child's current level of math knowledge.

In the present research, we tested two possible pathways through which the ANS may influence math abilities in a longitudinal investigation of preschoolers' math achievement. Children were tested at 3.5 years of age with a nonsymbolic numerical comparison task, cardinal number knowledge task, symbolic math test, and general IQ test. One year later, when the same children were approximately 4.5 years of age, we tested them with a nonsymbolic numerical comparison task, nonsymbolic approximate arithmetic task, symbolic math test, and general IQ test. Here we report how each of these measures contributes to math achievement at 4.5 years of age, with a specific focus on whether both the precision and manipulability of the ANS influence math ability via number word knowledge and approximate arithmetic.

Methods

Participants

Data from 97 children were included in the 3.5-year analyses (mean age: 3.61 years, range: 3.50-3.89 years, 49 female). Data from 161 children were included in the 4.5-year analyses (mean age: 4.56 years, range: 4.48-4.85 years, 86 female). Data from an additional 56 children at the 3.5-year visit and 10 children at the 4.5-year visit were excluded due to missing data points. Note that although the same children were tested at 3.5 and 4.5 years of age, a greater proportion of children had missing data points at 3.5 years, resulting in a smaller sample size for analyses that involved both the 3.5- and 4.5-year visit compared to analyses that involved only the 4.5-year visit. The most common cause of missing data points was an inability of our model to settle on a value for w, and this affected a greater number of 3.5-year-olds than 4.5-year-olds.

Procedure

Children came into the lab for two visits each lasting less than one hour at 3.5 and 4.5 years of age. At the first 3.5-year visit, children completed a symbolic math assessment and one session of the nonsymbolic number comparison task. During the second visit, 3.5-year-olds completed an IQ assessment, the counting knowledge task, and a second session of the nonsymbolic number comparison task. The order of the tasks within each session was counterbalanced across participants. At 4.5 years, during the first visit children completed the symbolic math assessment, and one session of the nonsymbolic number comparison task. During the second visit, 4.5-year-olds completed the IQ assessment, a second session of the nonsymbolic number comparison task, and the nonsymbolic approximate arithmetic task. The counting knowledge task was only administered at 3.5 years because the majority of 4.5-year-olds perform at ceiling on this task. The approximate arithmetic task was administered only at 4.5 years because 3.5-year-olds were unable to successfully perform the task. At each visit, parents gave written consent to a protocol approved by the local Institutional Review Board and were compensated monetarily and with a small gift for the child.

Nonsymbolic Numerical Comparison Task On each trial, a touchscreen computer displayed two bounded boxes (8 x 9.5 cm) containing arrays of dots. Children were instructed to touch the box that contained more dots and to make this choice without counting. Arrays contained between 4 and 14 dots, and the numerical ratio between the arrays was 1:2, 2:3, 3:4, or 6:7. To control for non-numerical perceptual cues, the parameters of the arrays varied such that the smaller and larger numerical array each had the larger cumulative surface area on 50% of trials. All of the dots within a single array were homogenous in element size and color, and the color of each array varied randomly from trial to trial. Differential audio-visual feedback was provided after each trial, and children received a small sticker for each correct response to keep them engaged. Children performed practice trials until they made three consecutive correct responses or completed a maximum of ten trials. Children performed 60 test trials in each test session for a total of 120 trials each time point.

To estimate each child's ANS acuity, we used a psychophysical modeling technique (e.g., Halberda & Feigenson, 2008; Piazza et al., 2010) to calculate a Weber fraction (*w*) based on performance in the nonsymbolic numerical comparison task. The resulting value of *w* represents the noise in each participant's internal ANS representations, such that lower values of *w* correspond to less noise (i.e. higher ANS acuity).

Nonsymbolic Approximate Addition Task On each trial, children viewed an animation that consisted of an array of dots moving behind an occluder box, followed by a second array moving behind the same occluder. This animated arithmetic sequence lasted a total of 2000 ms. Children then saw two boxes containing arrays of dots and were instructed to touch the array that contained the same number of dots as had moved behind the occluder box. Answer choices remained on the screen until a decision was made. Correct and incorrect answer choices differed by a 1:2 or 1:4 ratio, and all choice arrays contained 2, 4, or 8 dots. Individual dot size varied across arrays but was homogenous within each array. Differential audiovisual feedback was provided after each trial, and children were rewarded with a small sticker for correct responses. Children performed practice trials until they made three consecutive correct responses or completed a maximum of ten trials. Children then completed a total of 42 test trials.

Counting Knowledge Task This task was modeled after the Give-a-Number task (Wynn, 1992). The experimenter introduced the child to a dinosaur puppet and asked the child to give the dinosaur a certain number of fish by placing them on a plate in front of the puppet. If the child provided the correct number of fish, the trials progressed in the order 1-3-5-6-6. If the child provided an incorrect number of fish on any trial, the child was asked for N-1 fish. The trials proceeded until the child answered correctly at least twice for N and failed at least twice for N+1, or until the child successfully provided six fish twice. The child's score was equal to this final value of N.

Standardized Assessments Children's mathematical ability was assessed with the Test of Early Mathematics Ability (TEMA-3) (Ginsburg & Baroody, 2003), which consists of a series of verbally administered questions that assess age-appropriate counting ability, number-comparison facility, numeral literacy, and basic calculation skills. To assess general intelligence, children completed two verbal and two nonverbal subtests of the Reynolds Intellectual Assessment Scales (RIAS) (Reynolds & Kamphaus, 2003). A composite IQ score was calculated for each child.

Results

The first series of analyses examined the relation between ANS acuity, as indexed by *w*, counting knowledge, and future symbolic math achievement. First we confirmed that ANS acuity at 3.5 years predicted symbolic math achievement one year later at 4.5 years after controlling for IQ (Figure 1). We performed a linear regression analysis with *w* and IQ at 3.5 years entered as possible predictors of math achievement at 4.5 years. Both *w* and IQ emerged as significant predictors of math achievement (Table 1). In a second regression model, we added counting knowledge as an additional predictor. Consistent with the hypothesis that counting knowledge may mediate the relation between *w* and math achievement, when we added counting knowledge to the model its contribution to TEMA was significant, whereas *w* was no longer a significant predictor. In addition, this model captured a larger proportion of variance than the model without counting knowledge (F = 22.103, $p < .001$). Further, when counting knowledge and IQ were used to predict TEMA, the residuals did not account for significant variation in *w* ($F(1, 82) = 2.998$, $p = .09$). This suggests that ANS acuity is not directly impacting math performance but instead may be acting via its influence on counting proficiency.

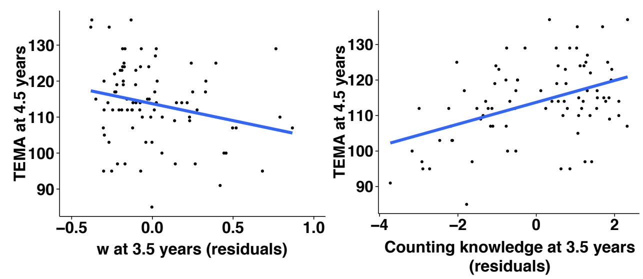

Figure 1. Scatterplots illustrating the relation between *w* at 3.5 years and math achievement at 4.5 years controlling for IQ (left) and between counting knowledge at 3.5 years and math achievement at 4.5 years controlling for both *w* and IQ (right).

Table 1: Regression models predicting math achievement at 4.5 years using measures collected at 3.5 years.

	Model 1		**Model 2**	
R^2	.108		.292	
F-statistics	$F(2, 81) = 6.00$		$F(3, 80) = 12.41$	
p-statistics	$p < .004$		$p < .001$	
Predictor	β_{Adjusted}	*p*	β_{Adjusted}	*p*
ANS Acuity	-.245	.024	-.170	.079
IQ	.216	.045	.097	.322
Cardinal knowledge	--	--	.460	< .001

The next series of analyses examined the relation between *w*, approximate arithmetic performance, and symbolic math achievement all at 4.5 years of age (Figure 2). We began by performing a linear regression analysis to confirm that *w* predicts math performance after controlling for IQ (Table 2). In the next regression model, we added approximate arithmetic performance as another possible predictor. In this model, all predictors contributed significant unique variance, and this model explained significantly more variance in math ability than the model without approximate arithmetic ($F = 9.438$, $p = .003$). However, when approximate arithmetic performance and IQ were regressed on symbolic math achievement, these residuals were significantly correlated with *w* ($F(1, 141) = 4.674$, $p = .03$). Therefore, it appears that approximate arithmetic and ANS acuity each contribute unique variance to symbolic math performance at 4.5 years of age.

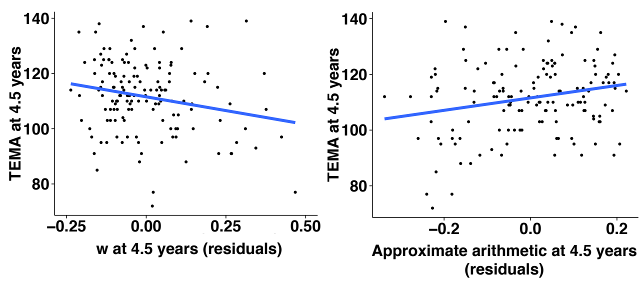

Figure 2. Scatterplots illustrating the relation between *w* and math achievement at 4.5 years controlling for IQ (left) and the relation between approximate arithmetic performance and math achievement at 4.5 years controlling for both *w* and IQ (right).

Table 2: Regression models predicting math achievement using measures collected at 4.5 years of age.

	Model 1		**Model 2**	
R^2	.197		.243	
F-statistics	F(2, 140) = 18.42		F(3, 139) = 16.16	
p-statistics	*p* < .001		*p* < .001	
Predictor	$\beta_{Adjusted}$	*p*	$\beta_{Adjusted}$	*p*
ANS Acuity	-.219	.005	-.171	.03
IQ	.357	< .001	.321	< .001
Approximate Arithmetic	--	--	.234	.003

In a final combined model, we asked whether cardinal number knowledge and approximate arithmetic performance each contribute variance to children's math achievement[1]. This model revealed that both factors make unique contributions math achievement ($F(4, 104) = 17.74$, $R^2 = .383$, $p < .001$; Cardinal knowledge: $\beta = .419$, $p < .001$; Approximate arithmetic: $\beta = .196$, $p = .019$; ANS acuity: $\beta = -.141$, $p = .082$; IQ: $\beta = .129$, $p = .116$), which suggests that both the precision of children's ANS and their ability to manipulate the ANS each impact symbolic math achievement.

[1] *w* values from the 4.5-year visit were used because the values at 3.5 and 4.5 years of age were significantly correlated ($r = .31$, $p < .01$) and using the 4.5-year values enabled us to include a larger proportion of our sample.

Discussion

The goal of the present research was to investigate the mechanisms by which approximate number representations contribute to preschoolers' emerging symbolic math capabilities. We first replicated previous findings (e.g., Libertus, Feigenson, & Halberda, 2011; Starr et al., 2013; vanMarle et al., 2014) that individual differences in the precision of the ANS are related to symbolic math achievement in preschool-aged children. We found that ANS acuity at age 3.5 was predictive of math achievement one year later after controlling for general IQ. Next we turned to quantitative abilities that may serve as intermediary steps between the ANS and symbolic math. We focused on cardinal number knowledge and nonsymbolic approximate arithmetic, which are abilities that have been previously found to correlate with symbolic math performance in young children. Our results suggest that there may be multiple routes by which the ANS contributes to symbolic math achievement. With regards to the precision of the ANS, it appears that ANS acuity may indirectly influence math achievement via children's acquisition of numerical symbols and the cardinality principle. In addition, nonsymbolic approximate addition performance contributes unique variance above and beyond that contributed by ANS precision. Together, these results suggest that both the acuity and manipulability of the ANS influence children's early math performance.

Our finding that cardinal knowledge is an intermediary step between the ANS and symbolic math is consistent with prior work that has identified cardinal knowledge as a key mediator of the relation between ANS acuity and math achievement in preschool-aged children (Chu, vanMarle, & Geary, 2015; vanMarle et al., 2014; see Schneider, Beeres, Coban, & Merz, 2016 for meta-analysis). In this light, the finding that symbolic numerical comparison is a stronger predictor of symbolic math achievement than nonsymbolic numerical comparison (Schneider et al., 2016) is not necessarily a refutation of the idea that the ANS contributes to symbolic math. Rather, cardinal representations of symbolic numbers may serve as stepping-stones between approximate number representations and symbolic mathematical operations. When children begin to form mappings between numerical symbols and approximate quantities, the ease with which these mappings are formed and the quality of these mappings may be influenced by the precision of children's ANS. Once this mapping is formed and children become proficient with numerical symbols, the precision of the ANS itself may not directly impact math achievement.

In the present study we focused on children's understanding of cardinality. Therefore, although our results point towards the importance of cardinal number knowledge for early math proficiency, it remains to be seen how other forms of symbolic number knowledge, including ordinality, may relate to math ability. Indeed, symbolic numerical ordering ability has been found to mediate the relation between ANS acuity and symbolic math achievement in adults (Lyons & Beilock, 2011). However, a recent training study with adults found that training adults on a symbolic numeral ordering task did not produce gains in symbolic arithmetic ability (Park & Brannon, 2014). Additional work is therefore needed to investigate how the ANS supports symbolic number representations throughout the lifespan, particularly after children have mastered cardinality, and how this relation contributes to symbolic math achievement.

The present results also provide support for a second route by which the ANS influences math ability. Beyond basic quantity representation, the ANS also supports arithmetic operations in infants, preschool children, and monkeys, all of whom have no understanding of symbolic arithmetic (Barth et al., 2005; Cantlon & Brannon, 2007; McCrink & Wynn, 2004). This capacity may provide an intuitive basis for the acquisition of symbolic arithmetic principles. Consistent with this view, we found that approximate arithmetic ability in 4.5-year-olds was a significant predictor of performance on the TEMA, which is primarily composed of symbolic numerical problems. Further, approximate arithmetic ability predicted unique variance in TEMA scores that was not accounted for by ANS acuity. As evidenced by a recent series of training studies, the link between approximate and symbolic arithmetic extends into adulthood: training adults on a nonsymbolic arithmetic task similar to that employed here led to significant gains in their symbolic arithmetic performance. (Park & Brannon, 2013; 2014). Critically, training on other types of tasks, including nonsymbolic numerical comparison, did not transfer to symbolic arithmetic. Although both nonsymbolic numerical comparison and approximate arithmetic tasks require representing approximate numerical quantities, approximate arithmetic additionally requires the manipulation of those quantities. This manipulation component appears to be a critical factor in the relation between the ANS and symbolic math.

The majority of studies relating the approximate number system to symbolic math have focused on individual differences in the acuity of approximate number representations. However, the manipulability of these representations may be a second mechanism by which the ANS influences symbolic math. Approximate arithmetic may share a cognitive foundation with symbolic arithmetic, making it an attractive target for potential interventions to remediate deficits in math proficiency. A priority for future studies should therefore be to adapt these training paradigms for children and determine if improving children's approximate arithmetic abilities also produces benefits for symbolic math. In addition, the relation between the precision and manipulability of the ANS remains relatively unexplored and is deserving of further study.

The ANS endows young children with a robust sense of quantity prior to beginning formal mathematics training. Although many studies have provided evidence for a correlation between the fidelity of the ANS and symbolic math achievement, there remain key open questions concerning the mechanisms underlying this relation and whether this correlation is indicative of a causal relation. In the present study, we identified two potential pathways through which the ANS influences preschoolers' early math proficiency. The first role for the ANS may occur when children are beginning to learn the meaning of number words, at which point approximate number representations may serve as an anchor for acquiring symbolic number representations. Therefore, the precision of the ANS may influence children's facility with mapping numerical symbols to approximate magnitudes. A second role for the ANS may stem from its ability to support arithmetic operations. The shared demand for manipulating quantities may form a conceptual bridge between nonsymbolic and symbolic arithmetic. The present results therefore suggest a nuanced relation between approximate number representations and symbolic math achievement in which multiple features of the ANS contribute to the emergence of symbolic math ability in young children. In light of these results, interventions designed to target one or both of these pathways may be differentially beneficial for children depending on their level of symbolic number knowledge and mathematical proficiency.

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