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Reasoning through Instructional Analogies

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Abstract

This paper aims to account for students' assessments of the plausibility and applicability of analogical explanations, and individual differences in these assessments, by analyzing properties of students' underlying knowledge systems. We developed a model of explanation and change in explanation focusing on knowledge elements that provide a sense of satisfaction to those judging the explanation. We call these elements "explanatory primitives." In this model, explanations are accepted or rejected on the basis of (a) the individual's convictions concerning particular explanatory primitives and (b) the fit of these primitives to current circumstances. Data are drawn from clinical interviews with 3 high school students who worked through a bridging analogies tutoring sequence on the existence of the normal force in mechanics. Methodologically, our work involves fine-grain analysis of process data and explicit principles of empirical accountability; we believe it marks a methodological advance over most previously reported empirical studies of analogical reasoning.

Over the last three decades, analogies and analogical reasoning have attracted the interest of many researchers. A core focus of attention has been how analogies foster understanding in some new situation or domain (the *target*), by comparing it to a more familiar one (the *source*). Analogies have been studied using diverse research methods and from a wide range of perspectives.

Arguably the most influential theory of analogical reasoning is Structure Mapping Theory (SMT) (Gentner, 1983, 1989). SMT suggests that analogies provide insight by mapping between a source and a target that share a common structure. That is, objects and relations between them in one domain (source) are mapped onto similar ones in the other domain (target), leading to inferences about other possible objects and relations in the target. Employing a slightly different perspective, the Multi Constraint Theory (Holyoak & Thagard, 1989) suggests that analogical inference is the result of interplay between three constraints—structural similarity, semantic similarity, and pragmatic importance—which need to be satisfied simultaneously. Both of these theories have been studied extensively through laboratory experiments with humans and also via computational modeling. Other computational models describe analogy-making in terms of parallel processing and stochastic architectures, which are claimed to be closer to the way people reason (e.g., Hofstadter, 1995).

Amid the complexity and diversity of perspectives, most if not all attempt to respond to a key issue concerning the effectiveness of an analogy: how one evaluates the degree to which the candidate transferred knowledge is applicable to the target domain and whether the analogical inference seems plausible. For instance, in an entry on computational modeling in analogical reasoning in the *Encyclopedia of Cognitive Science*, Kokinov and French (2003) define this as a core issue in analogical reasoning, along with mapping, representation building, and retrieval.

Evaluation of plausibility and applicability has been modeled as being mediated chiefly by structural similarity across domains (e.g. Falkenhainer, Forbus, & Gentner, 1986; Forbus & Gentner, 1989) and pragmatic goals (e.g. Holyoak & Thagard, 1989; Keane, 1996). The present study does not contradict these claims but theorizes that there is a much more direct influence of the individual's knowledge about the target domain on the evaluation of the analogical inference, particularly in the case of instructional analogies, where students do not generate the analogy but are required to reason with it. While other cognitive models of analogical reasoning acknowledge that processes of evaluation based on prior knowledge are important (e.g. Falkenhainer et al., 1986; Gentner & Colhoun, 2010), none so far has attempted to model these processes in detail. The current study may be seen, then, as a complement to other models of analogical reasoning: Using a methodology that is significantly different from the studies cited above, the paper extends the literature by examining in detail the ways in which knowledge of the target domain per se plays a role in evaluation of the plausibility and applicability of instructional analogies.

As we will elaborate later in this section, this study attends specifically to differences in prior knowledge among individuals reasoning through the same instructional analogies and the role these differences play in the evaluation of analogical inferences. We will argue throughout this paper that differences among individuals are highly informative in understanding the process of evaluation of instructional analogies that have been or are intended to be used in classrooms, rather than simplified ones for experimental purposes. Recent findings in cognitive modeling of analogical reasoning (Holyoak, Lee, & Lu, 2010) hint at the necessity and potential of our methodological approach. These researchers presented subjects with analogies where source and target were embedded in stories (experiment 3), instead of using synthetic short analogs (experiments 1 & 2), and examined their subjects' judgments regarding analogical causal attributions. We are not concerned here with the specifics of the findings, but rather with a phenomenon that was reported to be present only in the story condition, which is much closer to the classroom-oriented instructional analogies that we study. In summarizing the results from experiment 3, the researchers write that "closer inspection of the data suggested that the overall pattern of means might be masking important individual differences in task performance" (p. 715). In this study, we focus on those important differences between individuals' thinking in situ.

Analogical reasoning in professional science has been studied from many perspectives. Historical accounts of scientific discoveries (Gentner et al., 1997; Nersessian, 1992), studies of authentic scientific work in research laboratories (Dunbar, 1997), and studies of expert scientists' problem-solving (Clement, 1988) all suggest that the generation of analogies and the reasoning stemming from these analogies play a central role in scientific practice, thought, and creativity. Analogy is also a common explanatory device in mathematics and science classrooms and textbooks (Dagher & Cossman, 1992; Glynn & Takahashi, 1998; Richland, Holyoak, & Stigler, 2004; Sarantopoulos, Tsaparlis, & Strong, 2004). Educational researchers who have explored the outcomes of learning with analogies in science have documented its positive influence on students' learning (Chiu & Lin, 2005; Clement, 1993; Dagher, 1995; Duit, 1991; Duit, Roth, Komorek, & Wilbers, 2001; Gilbert, 1989; Glynn & Takahashi, 1998; Treagust, Duit, Joslin, & Lindauer, 1992). However, researchers have also found that learning with analogies can entail the generation of scientific misconceptions (Dagher, 1995; Duit et al., 2001; Harrison & Treagust, 2006; May, Hammer, & Roy, 2006; Spiro, Feltovich, Coulson, & Anderson, 1989; Wong, 1993; Yerrick, Doster, Nugent, Parke, & Crawley, 2003). In our view, the generation of misconceptions in the target domain suggests that the knowledge students bring with them to the learning event is likely to affect the ongoing reasoning induced by these instructional analogies, and, in particular, prior knowledge is likely to affect the assessment of the plausibility and applicability of the analogical inference. With the exception of one study, none of the studies cited above considered the assessment of the plausibility and applicability of the analogical inference.

The exception is John Clement's work on bridging analogies. Clement (1988, 1993, 1998) argued that direct mapping from the source (which he refers to as the anchor) to the target fails when students feel that the source and target are too remote. Clement suggested that in such cases an intermediate analogy called a *bridging analogy* must be presented. Bridging analogies have been successfully tested as a teaching model in classroom experiments using pre-post assessments. Brown and Clement (Brown & Clement, 1989; Clement & Brown, 2008) also examined some process data in a few case studies of tutoring with bridging pedagogy where they compared analogies. These case studies led them to argue that a good instructional analogy *enriches* the learner's representation of the target beyond projecting abstract mapping relations onto preexisting features of the target, and the analogy thereby helps the student to accept the scientific view as reasonable. They hypothesized that some analogies are better at facilitating this process than others. A good analogy according to this view (Brown, 1994; Brown & Clement, 1989; Clement & Brown, 2008) is one where the elements of the source can be seen as *candidates for reality* (Harre, 1972). Candidates for reality mean that elements of the anchor/source can be seen to operate in the target. In fact, these features of the target are simply "not there" for the student before the analogy is presented. Clement and Brown suggest that candidates for reality provide an *explanatory model* of the phenomenon at hand, arguing that students see the features of the anchor/source as operating in the target, but also attribute explanatory power to them by accounting for the way the target system operates (Brown, 1993, 1994; Brown & Clement, 1989; Cheng & Brown, 2010). Clement, Brown, and Zeitsman (Brown & Clement, 1989; Clement, 1998; Clement & Brown, 2008; Clement, Brown, & Zietsman, 1989) use the term "brittle" to describe anchors that do not create a candidate for reality. A *brittle anchor* is an anchor that has a particular feature that cannot exist in the target domain from the learner's point of view. This feature is not a candidate for reality in the target, so it cannot serve as part of a successfully induced explanatory model.

We interpret Clement, Brown and Zeitsman's work described above as concerning the student's assessment of the plausibility of the analogical explanation, and we argue that the learner's knowledge in the target domain directly affects the acceptance or rejection of this explanation. In our view, students inescapably use their knowledge of the target domain to make a judgment as to what qualifies as a candidate for reality. Thus, learning through analogy always involves a bootstrapping process *within* the target domain; that is, learners use their knowledge about the target domain, guided but not determined by instruction, to change their knowledge and understanding of that domain. Based on this hypothesis, we present an analysis of comparative case studies of tutoring with bridging analogies, focusing on differences among individuals in response to the analogical sequence. Unlike Clement and Brown, who compared analogies, we compare *individuals* who reason through the *same* instructional analogical sequence so as to track the differential influence of individual knowledge. Furthermore, we conduct a microgenetic analysis of students' reasoning designed to understand the moment-by-moment structure of individual responses to the instructional sequence and focus on their acceptance or rejection of the proposed analogy. To the best of our knowledge, no empirical study has systematically examined individual differences in response to instructional analogies. Furthermore, with the exception of the Clement and Brown studies, no study has analyzed real-time data. Even Clement and Brown's work, we maintain, did not entail the kind of fine-grained, extensive, systematic, and principled analysis of real-time data that we present here.

Based on our empirical analysis we developed a model of learning with analogies in which properties such as brittleness, candidates for reality, and explanatory models are derivable consequences of aspects of our model, which includes identifying particular elements of knowledge, their properties and configuration, and the processes in which they are involved.

Some of David Brown's other work provides a reference point—with similarities and critical differences—for the model and analysis we present here. Brown (1993) suggested a taxonomy of knowledge to account for the status as an explanatory model provided by candidates for reality in an analogy. He classified knowledge into four categories (the examples are taken from Cheng & Brown, 2010): (1) Verbal-symbolic knowledge—consciously employed generalizations encoded in words and symbols (e.g., electric current only flows when there is a complete circuit); (2) Conscious models—visual images of observable and unobservable elements that are consciously employed and are domain specific (e.g., visualizing a battery connected to wires and light bulbs); (3) Implicit models—tacit or taken-for-granted aspects of conscious models (e.g., in a conscious model of current flow, the current may be tacitly assumed to be like rainwater flowing in a gutter rather than water under pressure in a garden hose); and (4) Core intuitions—domain-general causal intuitions attached to the above three categories of knowledge elements (e.g., some things are "agents" that initiate causal interactions). Brown's main claim was that explanatory models can provide the basis for the reattribution or refocusing of core intuitions, with a consequent readjustment of the intuitive take on the situation. Brown

(1993) hypothesized that verbal symbolic knowledge can be effectively rooted in what he classified as analog representations (that is, conscious models, implicit models, and core intuitions), only after the refocusing of core intuitions, which enables students to make intuitive sense of the propositions and to use them in a flexible way in their own thinking. Chen and Brown (2010) used this taxonomy to infer that students generate explanatory models by associating their verbal symbolic knowledge with implicit models and core intuitions.

Like Brown, we aim to find a basis for understanding brittleness, candidates for reality, and explanatory models in empirically determinable properties of students' underlying knowledge systems. Unlike Brown's taxonomy, we put forward a functional model and not a structural one. We do not, in general, differentiate types of knowledge elements if they satisfy the same function, which is to provide students with a sense of satisfaction when judging an explanation (that is, the explanation seems plausible enough to make sense to them). Unlike Brown, we aim to account systematically for individual differences in response to instructional analogical sequences, and we support our model with a rich and fine-structured empirical analysis of students' reasoning and their reasons for accepting or rejecting the main claims of an instructional analogical sequence.

The *Knowledge in Pieces* perspective on conceptual change (diSessa, 1988, 1993, 1996; diSessa & Sherin, 1998; diSessa & Wagner, 2005) provides a productive basis on which to explore these issues. Knowledge in Pieces (KiP) aims to describe the knowledge transformation that occurs during the learning of scientific concepts. Briefly, it maintains that intuitive knowledge is a rich resource that is critical to student learning and our accounts of it and that a significant part of learning is reorganizing and recontextualizing intuitive knowledge. KiP grew out of the Piagetian constructivist tradition in which reasoning (and hence learning) is conceptualized as a process of interpreting phenomena through existing knowledge. Thus, a major focus of research in this perspective involves modeling individual reasoning processes by explicitly identifying the particular relevant knowledge structures and the mechanisms by which they are reorganized.

Two other assumptions of KiP influenced the empirical design of this study. The first assumption is that an understanding of a scientific concept can be seen only from its use in many different contexts. Possessing a coherent scientific concept thus requires that, functionally, the concept is invariant across contexts in the sense that, although different versions of the concept can be used in different contexts, the effect of these versions is always the same (diSessa & Sherin, 1998). In this study, our empirical assessment of our interviewees' confidence in the plausibility and applicability of the instructional explanation is informed by KiP in that we seek to evaluate invariance across multiple relevant contexts. We assume that if students really understand and "buy" the inference from the instructional analogy, they will also use this explanation spontaneously and fluently in other contexts.

The second relevant assumption of KiP is that to account for the process of learning, the grain size of the analysis should be very small. We used clinical interviews in this study because they can provide a sufficiently fine time-scale unit to see knowledge in action, and they also provide sufficient data to distinguish shades of meaning that discriminate similar but importantly different knowledge elements or uses of them.

Other more specific parts of the KiP perspective are important to our analysis. KiP maintains that the category of knowledge elements called *phenomenological primitives* (*p-prims)* is a key part of intuitive knowledge (diSessa, 1993). People develop p-prims when interacting with the physical world. When activated, p-prims are recognized and evoked as a whole, and they account for people's comfort with certain situations or surprise in others. The latter property, which we call, for brevity, their "self-explanatory" property, thus can directly provide the evaluation of fit and sense of confidence at the core of our analysis here. The theory of p-prims specifies many aspects of their source, their developmental history, their encoding, and the local processing around invocation and use. A large corpus of specific p-prims has been empirically identified (diSessa, 1993). For instance, Ohm's p-prim schematizes the phenomenological experience that more effort leads to more effect, and more resistance leads to less effect. When Ohm's p-prim applies to a situation—consistent with the self-explanatory property of all p-prims—the knower's attitude is that nothing further needs to be explained. We use documented p-prims as well as other accounts of intuitive knowledge (Brown, 1993; Piaget, 1977; Spelke, 1991; Vosniadou & Brewer, 1992) as resources for forms and content of naïve knowledge in mechanics.

The Model

The functional model that we present here was developed bottom-up in an attempt to explain students' reasoning, and in particular individual differences in that reasoning. The model is intended to account systematically for the judgments learners make when they assess the plausibility and applicability of an instructional analogy. We call the theoretical part of our work a model rather than a theory because it has a narrower focus, and it does not compete directly with, for example, theories of conceptual change or of analogical reasoning, although there are connections that we will make.

Evaluation of the success of our work depends on the breadth and detail of empirical tracking. We apply our model to a fairly large corpus of data and with respect to a particular purpose, which is primarily to understand individual differences in acceptability of analogical instructional explanations. Note that ad hoc interpretations of individual events in the data, much less interpretations not aimed at or capable of explaining individual differences, provide little competition to the breadth of interpretive power attempted here. There are some ambiguities and difficulties in particular interpretations, which we discuss and evaluate in the Findings section. However, the overall evaluation of the model should rest in the accountability and extent of the whole corpus of interpretations.

The construct of p-prim provides the starting point for the model. We generalize the p-prim model by focusing only on p-prim's self-explanatory property. We view any explanation as a reduction of a phenomenon to a *selected set of functional knowledge elements*, which we term *explanatory primitives* (Kapon & diSessa, 2010). Explanatory primitives (e-prims) may or may not have any of the many other properties of p-prims, and they are therefore a category of knowledge elements of which p-prims are a sub-category. Like p-prims, e-prims are self-explanatory, unquestioned units of explanation, which students take as simply "the way things are." As with p-prims, the self-explanatory property must be understood in a relative sense. Under probing by a skillful teacher or interviewer, or even through the passage of time, students may begin to question what they take to be more or less certain or even indubitable. However, within the time span of evaluating an explanation, explanatory primitives are likely to be relatively stable.

To summarize, e-prims and p-prims have the same function. When activated, they provide a sense of comfort and obviousness in reaction to certain phenomena, or surprise or rejection in reaction to others. Since p-prims share the function of being self-explanatory with e-prims, any p-prim is by definition an e-prim, but not all e-prims are p-prims. A p-prim has a very specific form of encoding, developmental history, and so on, whereas e-prims, as a class, can be much more diverse, as we further elaborate below.

The range of knowledge constructs that might serve as explanatory primitives can be described along several dimensions that together describe the nature (rather than, say the functional effect) of the knowledge construct. The first dimension is the source and the developmental history of the e-prim—in particular, the experiences from which it was abstracted. This source may be direct experience in the physical world, or, by contrast, it may be language, social interactions, or even extended, explicit instruction. The second dimension is topic; that is, the disciplinary domain of the phenomenon that the e-prim explains (e.g., physics or psychology). The third dimension is encoding, the specific mental-representational form of the e-prim. E-prims differ from p-prims along these three dimensions. E-prims are spread all over the above three-dimensional space. In contrast, p-prims occupy only one locus in this space. The source of p–prims is experiential contact with the physical world. Their topic, as it applies here, is physics or physical phenomena. The encoding of p-prims is not linguistic. They are not, for example, explicit verbally formulated propositions; and they do not compete with comparatively huge constructions, such as the expert encoding of a physical theory.

To illustrate the way in which p-prims are a subcategory of a larger category of knowledge elements, let us examine an e-prim used by our students that we consider categorically not a p-prim. The e-prim in question is *gravity pulls things downwards.* This is a common unit of explanation put forward for the phenomenon of things falling downward. Children hear about gravity, by name, from their elders. Students of the age we study here have most likely heard that Newton was responsible for this idea, and feel a force of positive sanction from its being taught in school. In addition, the idea can be fluently formulated in language, and thus easily accessible to conscious elaboration, analysis, and reflection. In contrast, the very idea that *things tend to fall downward* seems to be learned in infants before about six months of age (Kim & Spelke, 1992), before humans are capable of forming a verbal representation of such an idea. Thus, its source is almost certainly experiential, and its early encoding cannot be verbal in any respect. With additional empirical consideration, one can easily imagine that *things tend to fall downward* could be classified as a p-prim.[1] In sum, on the basis of source (linguistically mediated formal or informal cultural learning vs. direct experience), and on the basis of encoding (at minimum, the capacity to be fluently formulated in language vs. nonlinguistic, probably image schematic encoding) *gravity pulls things downward* is an e-prim, but, unlike *things tend to fall downwards*, it is very unlikely to be a p-prim. This example incidentally shows that distinguishing p-prims from generic e-prims solely from a verbal, propositional gloss of their meanings is difficult or impossible.

P-prims are relevant here mainly in two respects. First, their core property of being self-explanatory inspired the idea of e-prims. Second, as a subset of e-prims, the literature on them constitutes a ready database of examples to use in e-prim analysis. While we comment on the possible status of newly discovered e-prims as p-prims, this is a secondary task. Indeed, given the complex of properties that must be established in order to assert that a knowledge element is a p-prim, our limited data concerning each e-prim can reach only so far.

In developing our model, we chose to introduce the construct of e-prim based on the crucial observation that although the knowledge elements that were identified in the analysis appeared to function as p-prims (they accounted for our interviewees' comfort with certain phenomena and surprise with others), some of them seemed not to have the defining form of p-prims—the particular source, topic, and encoding. On the other hand, to do the work that we intend, such particulars as source and encoding are largely beside the point. The larger, if less specified class, e-prims, which nonetheless has the core self-explanatory property, does the main work that is needed.

Another distinction is between the e-prims that we find here and mental models (e.g., in the Johnson-Laird sense). E-prims, like p-prims, seem to be activated as a whole in thinking. Mental models, on the other hand, are consciously "run" by the thinker over an extended period of time (Norman, 1983). The net result of such complex reasoning, however, can establish causal relations, which is similar to the net result of activating an e-prim.

In our model, explanations are accepted or rejected on the basis of an individual's personal conviction concerning the explanatory primitives that are invoked. We find it useful to think of an individual's system of explanatory primitives in terms of *repertoire* and *priority*. *Repertoire* refers to the individual's particular set of explanatory primitives. The concept of repertoire does not imply that each individual has a completely different set of explanatory primitives. On the contrary, the fact that humans share "neural machinery" and many experiences suggests that there will be significant similarities across individuals. However, a careful examination should reveal differences.

Explanatory primitives are not absolute, and we can expect a student to have a particular degree of confidence in a given e-prim. Drawing on diSessa (1993) we refer to this degree of confidence as the *priority* of the e-prim. DiSessa discusses the dynamics of knowledge systems in terms of *cueing priority* and *reliability priority*. Cueing priority expresses the degree to which a particular p-prim's transition to an active state is affected by other previously activated elements. A high cueing priority means that only a small additional contingent activation is needed over the described context to activate the element in question. Reliability priority expresses the result of the processing of feedback that can reinforce or undo the initial activation. A high reliability with respect to a specified context means that it is unlikely that the p-prim will be rejected by subsequent processing. In our model e-prims that are felt to be certain are said to have high priority, similar to reliability priority. Conversely, those that are plausible but not indubitable are described as having low priority—or they may not be explanatory at all. We propose that a given explanatory primitive, used in a particular situation, exhibits a given *effective* priority based on the interplay between two more refined *priority considerations*: (a) *intrinsic priority*—the degree of inherent confidence in a certain e-prim; and (b) *contextual priority*—the degree of confidence in the applicability of that e-prim in the relevant context. A high intrinsic priority e-prim may fail to provide confidence because of uncertain fit with the context. Given limited data across contexts to differentiate intrinsic from contextual, we may in practice be forced to refer only to the effective (bare) priority of an e-prim observed in a certain context or a narrow range of contexts.

Sometimes two or more e-prims are cued in a situation, and the individual has to decide which is more appropriate. This may emerge in cases where competing e-prims imply different outcomes. A decision, based on contextual reasoning, can lead to an increase in priority of one, and a decrease in priority of the other. In other cases, one e-prim may be seen to explain another and therefore accrues further priority, most likely at the expense of the other. We term the above phenomenon as *negotiating relative priority*. We do not attempt to model the details of negotiating relative priority, except to say that it depends on the fact that the individual needs to sort out both contextual and intrinsic issues. However, our analysis implicates some of the processes involved, as follows. (a) Explanatory hierarchy: One e-prim acquires additional priority at the expense of another since it can explain the other. This principle coheres to some extent with Rottman and Keil's (2011) findings that people assess components of scientific explanations as less important when they (merely) elaborate upon and are incomprehensible without the presence of other components. (b) Consistency: An e-prim acquires additional priority when its implications are consistent with other e-prims. (c) Elimination of conflicts: When two activated e-prims suggest conflicting conclusions, one is preferred over the other based on their relative intrinsic and contextual priorities, and it may gain further priority in comparison. (d) Reclassification: Contextual priority, in particular, may be changed by broadening or narrowing categories or reclassifying objects into existing categories. (e) Recontextualization: Priorities may be changed by resolving alternative framings. Subjects often could frame specific situations in different ways, for example as an agent acting (or failing to act) on another, or as a situation of balanced influences; choosing one frame over another often had decisive and likely enduring effects. These different ways of negotiating priorities are all exemplified in the data analysis.

While priority considerations complicate the model, we believe they are empirically tractable and help us account for when and how particular instructional strategies work. They will also add fine-structure to our understanding of judgments of analogies, including how students differ, how their differences may turn into differing acceptance of an instructional analogical sequence, and how judgments can gradually change. Once again, the assumption is that explanatory primitives, having been learned generally through extensive experience, are relatively stable in their priorities and thus are empirically confirmed by multiple observations of use. However, as mentioned, we conjecture that priorities may shift after consideration and careful relevance judgments. Indeed, we take the shifting of priorities, intrinsic or contextual, as a central locus of learning in instructional settings, including learning based on analogies. Our data contain suggestive examples.

The empirical work in this paper involves a fine-grained analysis of interviews with three high school students. The interviews employed a bridging analogies tutoring sequence on the normal force in physics (mechanics). Our main goal is to provide a knowledge-based account of the students' responses to the instructional sequence. We try to explain different behaviors, specifically individual assessments of the plausibility and applicability of an explanation. We do so by determining and describing the state (repertoire and priorities) and dynamic (negotiating priorities and priority shifts) of the individual's prior knowledge elements that play a role in analogical reasoning. Specifically we determine the explanatory primitives that are being used in particular judgments that subjects make, their priorities, and the dynamic of the priority considerations involved. We do not consider this paper as primarily a theoretical innovation but rather an empirical contribution. The focus of the paper is not p-prims versus e-prims, but rather a new use of a particular perspective on conceptual change (KiP), and a few core ideas within it (self-explanatory knowledge elements and their priorities) to account for individual differences in response to instructional analogies. The model aims to account for obvious and important differences in the way individuals respond to—accept or reject—a suggested analogical explanation. In this regard, we repeat that we are unaware of any published analyses using other views of conceptual change or any existing cognitive models of analogical reasoning that treat both online thinking and individual differences concerning the acceptability of instructional analogies.

Method

Data Collection

The larger corpus from which our analysis was drawn covers one interview with an eighth grader (14 years old) and five interviews with high school students (15-17 years old). None of the students had taken a physics course at school. The interviews lasted between 38 and 52 minutes. All the interviewees were considered to be good students by their teachers. At the time of the interview the high school students were enrolled in a one semester AP chemistry course. All were planning to take honors physics the next semester.

The design of the interview was based on Clement and Brown's instructional sequence concerning the existence of the normal force. However, we made changes and additions to provide better triangulation on the students' particular knowledge elements and the contexts in which they were used.

In the original tutoring sequence, Brown and Clement (1989) aimed to convince students that a table can exert upward force on a book resting on it by clarifying that the table is an elastic entity and therefore can exert a restoring force. The instructional sequence involved the following situations: (a) a book on a table; (b) the book on a hand in comparison to (vs.) the book on the table; (c) the book on a compression spring vs. the book on the table; and (d) the book on a flexible board vs. the book on the table. Clement and Brown (2008) added an extra phase to this sequence: tutoring the molecular model of solids (i.e., molecular bonds as springs). In the final step, the participants were again asked about the book on the table situation.

The present study used the following enriched sequence: (a) a book on a table; (b) the book on a hand vs. the book on the table; (c) the book on a compression spring vs. the book on the table; (d) the book on a flexible board vs. the book on the table; (e) a fishing weight hanging from a soft spring; (f) the fishing weight hanging from a very firm spring (the last two steps were intended to cue the notion that some changes happen at a scale that is too small to be seen); (g) comparing the table and the flexible board in relation to the firm and soft springs; (h) the molecular model of solids—molecular bonds as springs; (i) the book on the table; (j) a fishing weight hanging from a rope (checking invariance across contexts—is the interviewee convinced by the springy or the molecular model to the extent that he/she will use it voluntarily to explain different phenomena in other not-evidently-springy contexts?); (k) the book on the floor (again, checking invariance across contexts); (l) the book on an inclined plane (invariance across contexts); and (m) the book pressed against the wall (invariance across contexts). Situations "k"-"m" were presented to the interviewees to assess strength of conviction and contextual stability only after the interviewees acknowledged the existence of the force of the table on the book and their conviction regarding the molecular model. As such, they were posed to some interviewees only at the end, whereas others, who acknowledged plausibility of the existence of the normal force earlier, saw them both at an early stage as well as at the end. Situation "e" particularly was presented again towards the end to test the interviewees' belief in the molecular model, that is, to see whether they would choose to use the model to predict and explain tension in a rope, as opposed to the original context of compression. We note, generally and in advance of data that subjects' responses were diverse and contextualized across these examples, indicating that they served their purpose well.

The interviewees' interaction with the problem also differed from Clement and Brown's setting. Instead of only talking about the situation or presenting them with pictures, real artifacts were used, allowing interviewees to touch and experience each artifact. This was done to allow us to observe the use of bodily perception in the reasoning process. It turns out that students often use kinesthetic evidence in construing relevant situations.

Framing the Interview

The engagements with students were designed as clinical interviews (diSessa, 2007). The interviewees were told that the goal of the interview was to capture their reasoning processes and not to test their knowledge of physics. The teacher who helped recruit the interviewees was asked to emphasize that the interview was not going to teach the correct view. The interviewer also emphasized this fact. The subjects were told that whether they gave scientifically accurate answers or not, they would be asked to clarify and defend arguments and relate to alternative views. They were asked to maintain their opinions and change them only if they were truly convinced that a change was in order. All the interviews started with a demonstration. The interviewer held a book, let go of it, and the book fell to the floor. She then placed the book on the table and asked the interviewee why the book did not fall to the floor.

Analysis

Interviews with three students—Adam, Jacob, and Sara (pseudonyms)—were selected for detailed analysis. The eighth grader from the original group was eliminated because the interview was conducted in another language, which would have posed complexities in comparison. A second participant was eliminated because it became clear during the interview that the student had taken a summer astrophysics course where she had learned Newtonian mechanics and therefore related to the questions quite differently than other subjects. A final participant was eliminated because she appeared reluctant and uncomfortable with the ground-rules for the interview. This student systematically sought correct answers from the interviewer and seldom displayed her own reasoning.

The aim of the analysis was to identify the knowledge elements that the interviewees used in their extended analogical reasoning process and to determine how this knowledge played out in the reasoning process. The interviews were videotaped and transcribed, including notations of the interviewees' gestures. The transcripts were divided into short thematic episodes (1 to 3 minutes) that reflected consideration of one particular idea. Each episode was divided into smaller units (fraction of minutes) that reflected different aspects of the idea conjectured to be used in the reasoning process. For instance one thematic episode (~1.5 minutes) in one of the interviews dealt with how it might be that a book that is resting on the table will not penetrate the table, despite the fact that gravity pulls the book downward. This episode was further divided into smaller units: assertion based on experience, invocation of relevant knowledge of physics, and exemplification of the assertion. Comparison among interviewees after this phase of analysis highlighted the fact that one of the interviewees reacted very differently to the instructional sequence compared to others. A careful examination suggested that one particular documented p-prim seemed to play a central role in this different reaction and that this difference could not be explained by incorrect or incomplete mapping. This discovery led to a second phase of analysis where the entire data corpus was examined again, this time specifically looking for evidence of p-prims in the students' reasoning. During this analysis two things became clear: (1) there are additional fine-grain differences between the students' responses; (2) p-prims played a role in the students' reasoning, but there were other knowledge elements involved that functioned like p-prims (were treated as relatively self-explanatory) but did not seem to share the same form as p-prims. At this point additional theoretical development took place, resulting in the formulation of the constructs explanatory primitive, intrinsic and contextual priorities. During the third round, the data were searched for explanatory primitives, and the behaviors of the interviewees were constantly cross-examined with respect to the consistent use of particular explanatory primitives, with particular priorities. This allowed us further to refine the theoretical model and to account for subtle differences between interviewees in their reactions to the analogical instructional sequence.

Identifying an Explanatory Primitive

DiSessa (1993) describes a list of 15 heuristics for identifying new p-prims and validating their properties across multiple instances of use. Although these heuristics influenced the empirical work here, a different set of criteria was used to infer the use of a particular e-prim from the interviewees' statements and gestures. There are two reasons for this. First, not every e-prim is a p-prim, and a number of the heuristics are particular to p-prims. The second reason is more fundamental: The purpose of this study was not to discover and describe in detail new types of knowledge, but rather to determine how knowledge, of whatever type, is involved in analogical reasoning. Thus, as mentioned, the construct "explanatory primitive" is functional and does not resolve form or structure. Some of the criteria for inferring an explanatory primitive from the interviewees' expressions were property driven while others were reliability (Merriam, 2002) driven (see explanation below).

We will explain each of the criteria that we employed to identify e-prims and their properties in sequence, followed by an example. All of the examples involve one of the e-prims that were used by our interviewees, the *springiness* p-prim (diSessa, 1993). The transcription conventions used are the following: (a) "//" for a break in speech, typically including a pause, then restart or new direction; (b) "/ … /" for omission of part of the interviewee's or the interviewer's speech; and (c) "[*Italic*]" for interpretive and informational commentary. Times of student contributions are marked in parentheses.

Property-driven criteria. The following list describes the property-driven criteria that were employed to identify an e-prim. These criteria stem from the definitional properties of explanatory primitives.

1. *Functionality* – The function of an e-prim is to help the individual make sense of the world. Operationally this means that the e-prim should line up with the interviewee's attention at the moment;[2] it should be explanatorily useful to the interviewees' goal at that particular moment of reasoning and responsive to the context in which this reasoning takes place.
   Example: Springiness (diSessa, 1993) imputes a *causal connection* between a *deformation* (that which is attended to) and the consequent development of a *restoring force* (inferred from deformation, although this force is not necessarily understood as a Newtonian force). Our interviewees used it to make sense of springs, flexible boards, and other situations that they inferred as involving resistance to deformation.

2. *Obviousness* – The word "primitive" in "explanatory primitive" refers to the unquestioned acceptance of the conclusion that the explanatory primitive suggests. Operationally this means that we look for explicit statements of obviousness or unelaborated confidence and acceptance which interviewees attribute to certain propositions (e.g., "(just) because!").

   Example: "And then the spring *naturally* wants to be in this position, now, when there's no force pushing down on it. And it pushes back, *it does /…/ Yeah. I'm not sure why, but it does*" (Sara thinking aloud how a spring works, 04:58; emphases added).
3. *Source* – An explanatory primitive derives from a history of successful use. As such, it should adhere to the principle of continuity (diSessa, 1993). Operationally this means that we should be able to identify a familiar experience from which this explanatory primitive could have been abstracted.

   Example: The intuitive notion of springiness develops through our physical experience with springy entities such as trampolines, mattresses, and so on. As one pushes down, one may notice a concomitant increase in resistance. Although it seems similar, we caution that this is only somewhat like the understanding of physical springs acquired in a physics class.

Reliability-driven criteria. The following list describes the reliability-driven criteria that were employed to identify an e-prim. These criteria stem from the methodological requirement that the data should account for the inferences made (Merriam, 2002).

1. *Frequency/stability* (triangulation of expression) – From our experience in analysis, we take it as axiomatic that an explanatory primitive cannot be identified based on only one utterance of the interviewee. There are too many potential contexts of relevance, with unclear boundaries, and too many nuances of meaning to distinguish unambiguously. Operationally this means that we look for repeated use (at least 3-4 instances) during the interview in a variety of contexts.

   Example: Sara used springiness many times during the interview in several contexts where the obviousness of this e-prim was explicitly expressed. Here are a few examples:
   - "They *[the flexible board and the spring]* both bounce back, springy stuff *[gestures bouncing]*" (Sara compares a flexible board and a spring, 17:00).
   - "I don't know *[laughs, and uses her hands to gesture "I do not know"]*, Like that! *[points to the spring]*" (Sara trying to explain to the interviewer the meaning of the word "springy," 17:59; our interpretation is that, while words fail her, she feels the idea is ostensibly obvious.).
   - "Because it *[the wall]* has resistance, and it has the same springiness as the table" (Sara explains to the interviewer why she thinks that a wall exerts force on an object that is pressed against it, 30:00).
2. *Literature* (triangulation of form and content) – It is difficult to fully warrant the definition and use of a particular e-prim in verbal data obtained in the context of a predetermined and extended inquiry. One cannot afford the time or interruption of flow in the inquiry to use typical clinical techniques of probing relevant attention, trying out multiple contexts (for that one element) in comparison, and so on. However, intuitive notions in the domain of early mechanics have been extensively documented (e.g. Brown, 1993; diSessa, 1993; Piaget,

1977; Spelke, 1991; Vosniadou & Brewer, 1992). We use this documentation to triangulate the content and, when we can, the form of the explanatory primitives that we identify in the data. Operationally this means that when an identified e-prim matches a documented p-prim or other documented intuitive principles with respect to all the relevant components of the situations in which we find it used, we consider that our interpretation is on safer ground.

Example: In the last few paragraphs we described some of the content and forms that our interviewees used to express the "normal" causalities that they attributed to springy objects. Springiness is warranted as a good candidate e-prim because its existence and properties were previously documented in dozens of contexts of use with multiple subjects (diSessa, 1993).

Assessing Priority

Priority is the degree of confidence that an interviewee has in an explanatory primitive. The following list describes the criteria for assessing the priority of an explanatory primitive, followed by an example of the use of each criterion for the *springiness* e-prim.

1. *Explicit statement* – The degree of confidence (or lack of confidence) expressed by explicit statements or gestures.

   Example: In the two reactions below, one employs explicit wording that suggests that *springiness* has high priority for the interviewee, whereas the other uses explicit wording that suggests it has low priority for another interviewee.

   - Sara, high priority for *springiness*: "The spring *naturally* wants to be in this position" (Sara, 04:58; emphasis added).
   - Adam, low priority for *springiness*: "But I guess *I don't think it [the spring] really* knows what its natural length is" (Adam showing skepticism toward an idea, 09:58; emphasis added).
2. *Questioning* – The degree of confidence explicitly expressed by questioning the explanatory primitive (low priority) or the systematic lack of such questioning (high priority).

   Example: In the two reactions below, one questions springiness and one does not.

   - Jacob, high priority for *springiness* (lack of questioning): "Yeah. It *[the spring]* wants to return to its original // I'm trying to think of a word to explain that *[pause]* state, I guess. It wants to return to the way it was" (Jacob's reaction to a "child's" explanation that suggests that springs naturally want to return to their natural length, 22:50).
   - Adam, low priority for *springiness* (questioning): "I guess I still don't quite understand *why* it *[the spring]* would return to its normal state *[talking while compressing the spring]*" (Adam, 36:00).
3. *Frequency* - Degree of confidence is implicitly expressed by frequency and fluidity of use. Examples are Sara's and Jacob's frequent explanatory use of springiness, as opposed to Adam's repeatedly expressed doubts.
4. *Prosody* – Degree of confidence may be implicitly expressed by rhythm and intonation of speech (assertive, hesitant, etc.) This criterion was secondary and was never used as a primary means to assess priority.

Intrinsic priority vs. contextual priority. As described in the introduction, the priority of an explanatory primitive is determined by both its intrinsic priority and its contextual priority. Differentiating intrinsic and contextual priority was often empirically difficult since what was reported was (net) confidence that seldom showed, in a single case, whether confidence or the lack thereof came from the underlying principle or from challenging its applicability to the current circumstances. Multiple contexts are needed to distinguish intrinsic from contextual priority. Hence we sometimes use the simple term priority instead of referring to intrinsic or contextual priority when differentiation was not feasible. In the cases where the differentiation was possible, we used the following criteria to differentiate contextual from intrinsic priority:

1. *Contextual statements* – Context dependence may be explicitly expressed in statements such as "this does not apply to X."
   Example: "This isn't a solid we're still talking about?" (Adam's immediate reaction to the idea of molecular bonds in solids as being similar to springs; he is expressing his doubts that the model can apply to "true" solids, 20:50.)
2. *Contextual frequency* – Frequent use of a particular e-prim only in certain contexts but never in others implies contextual boundaries.
   Example: The naïve idea that solid objects have a constant shape, which is documented as the *rigidity* p-prim (diSessa, 1993), was applied by Adam as he reasoned about the table but not when he reasoned about the spring, implying a contextual boundary that was quite strict for him but much less so for others.
3. *The nature of the e-prim* – In the absence of sufficient data in varying contexts, and when the properties of the e-prim have been independently well-documented, as in the case of some p-prims, one may infer context dependence from the content of the e-prim, namely by judging the degree to which the situation matches the meanings of the e-prim. This criterion was secondary and was never used as a primary means of differentiating contextual from intrinsic priority.
   Example: The meaning of the *rigidity* p-prim suggests that it is only applicable to things that appear to be solid bodies.

Negotiating relative priorities. Instances of negotiating relative priorities were identified when more than one explanatory primitive was cued for the same instance, and students proceeded to sort out their preferences. Some cases of negotiating priorities, leading to evident shifts, were instigated by interviewer requests for explanations or justifications.

Findings

The most important aspects of our analysis were to identify plausible explanatory primitives (repertoire), to assess their explanatory power (their different priorities), and to use these data to account for the students' specific behavior, mainly acceptance or rejection of analogically suggested ways of construing the target. This section is comprised of three separate analyses, one for each of the three interviews. The focus of the analysis is the similarities and differences between individuals in terms of the repertoire of explanatory primitives used, the priority attributed to each explanatory primitive, and the shifts in priorities that were implicated in the students' extended considerations. The analysis aims to reveal how differences between individuals' knowledge, that is, their explanatory primitives and their related priorities, can account for (a) different reactions to the analogical explanation, and (b) if and how knowledge changed over the course of the interview.

Two of the interviewees fully accepted the "springy" explanation—the table exerts a force because it is springy—at some point during the interview. The third presented an articulate rejection of this explanation. The analysis of the first interview is used to explain how the e-prim model works, to illustrate the data analysis principles used (criteria are marked in parentheses), and to provide a benchmark against which differences in explanatory primitives and consequent actions of subsequent students can be charted. We identify the repertoire of explanatory primitives that the interviewee cued, assess their priorities, and use them to explain ongoing reasoning. The next two interviews are analyzed comparatively to highlight how the model can explain differences in students' reactions to an instructional analogy. Specifically, we show how different repertoires or priorities lead to different responses.

Jacob: Full Acceptance

At the time of the interview Jacob was in eleventh grade. He had a very positive attitude towards learning science. For example, he said, "I'm really looking forward to taking physics. My dad always raves about it." The 42-minute interview took place after school hours in the school's chemistry/physics lab. Jacob appeared very engaged by the interview. He frequently paused to consider and evaluate ways of construing the situations, and he often examined and interacted with the artifacts.

The existence of a normal force. At first Jacob thought that the table probably does not exert a force: "The particles in the table are more compacted than // are compacted too tightly for the book to get through /…/ I mean it could be repelling it, but it's not apparent, not from just the naked eye" (Jacob, 01:00).

Comparing the content and form of Jacob's reaction to the literature on intuitive knowledge (the *Literature* criterion) suggests two possible explanatory primitives that might play a role in his reasoning in this context: (1) *Two things cannot be in the same place at the same time*. This explanatory primitive is certainly rooted in humans' physical experience (the *Source* criterion); Spelke (1991), for example, documented babies responding to impenetrability at very young ages. (2) The *supporting* p-prim (diSessa, 1993) stipulates that a "strong" or stable underlying object merely keeps overlaying and touching objects from falling. Since Jacob used only the *supporting* argument in his later utterances (the *Frequency/stability* criterion) we tend to think that he used this e-prim, but we cannot completely exclude the use of *two things cannot be in the same place at the same time*. The important point is that both these e-prims do not involve force in any sense (see diSessa, 1993, in the case of *supporting*). With such an explanatory primitive in place, no further explanation is necessary; there is no need for an upward force (*functionality* and *obviousness* criteria). Consistent with this analysis, Jacob expresses a skeptical stance toward the existence of a force ("repelling").

However, Jacob's subsequent experience of holding up his hand while supporting a book initiated a slightly different response, which later resulted in a priority shift. When Jacob was asked whether he thought his hand exerts a force on the book his answer was as follows.

> Hmm *[silently thinking while supporting the book with his hand] /…[the interviewer piled other books on the one Jacob was holding]*/ I'd say it's the

> amount of force for *[stops and think]* my hand to compensate for the attraction of gravity /.../ I'd say, it's attempting to pass through my hand, but that's the force of gravity pulling it to the Earth /…./ and my hand is preventing that from happening. (Jacob, 02:00)

Jacob appeared to transiently cue a different explanatory primitive that is documented in the literature as the *dynamic balance* p-prim (diSessa, 1993). The function of *dynamic balance* is to explain situations of balance and rest when forces are recognized, namely when a pair of forces or directed influences are in conflict and happen to balance each other (*Functionality* criterion). Note that Jacob explicitly mentions the salient force of gravity and his use of "compensate" in relation to this. Jacob is taking the attraction of gravity for granted. In fact, *gravity pulls things downward* was an e-prim that (not surprisingly) all our interviewees expressed at some point with high intrinsic priority. We can see traces of the *supporting* e-prim in Jacob's reasoning in the case of the hand (the hand prevents the book from passing through, and therefore holds it in place – *Frequency/stability* criterion), but the fact that he recognized that a force is involved in the equilibrium situation gave *dynamic balance* a higher relative contextual priority in the case of the hand (negotiating priorities).

Jacob's recognition that his hand exerts a force quite plausibly came from his sensing the effort that his muscles used to maintain the book at rest. Indeed the interviewer had to pile up a few books on his hand before he said that his hand was exerting a force on the book. We can propose *effort entails force* as an e-prim. Effort is a common co-occurring element of situations where force applies. This co-occurrence is documented in the literature. For instance, diSessa (1993) discussed this in the case of the *Ohm's* p-prim and the *force as a mover* p-prim. This co-occurrence is also apparent in everyday familiar language (e.g., "I do not have the strength" could mean "I'm too exhausted to expend effort"). The function of *effort entails force* is to explain how an agent knows that he is exerting a force. We can see further evidence for the connection between *effort entails force* and *dynamic balance* in Jacob's reasoning in the next excerpt (*Frequency/stability* criterion). Here Jacob was asked how the hand prevents the book from falling. He says that he is not sure. Then the interviewer asked him if he *feels* that he exerts a force. Jacob answered affirmatively and immediately started to explain how it is that his hand prevents the book from falling. Here, he uses the expression "winning the battle," which we interpret as expressing "overcoming," which is typically used in conjunction with *dynamic balance* (diSessa, 1993).

> Yeah. To maintain it at this level, it requires a certain amount of force to keep it level. /...[*asked about the direction of the force*]/ It's going upwards while the book is exerting force downwards /…/ right now I'm winning that battle of sorts because I can change the elevation of it / /…/ [*and when the book is at rest*] It's *[both forces are]* equal. (Jacob, 04:30)

The next episode shows how the accrued priority of *dynamic balance* in the case of the hand initiated a priority shift that led Jacob to acknowledge the book on his hand and the book on the table as similar cases. This recognition led him to infer that the table does exert a force on the book, even though it is an inanimate object.

Episode 1 (Jacob, 07:00):

1. Inter: Okay, but tell me, so you just said that your hand exerts a force and

the table does not. What's the difference between the table and the hand?

2. Jacob: Nothing. The table is doing the same thing. It's just not using the same *[pause]*
3. Inter: The same what?
4. Jacob: The table doesn't have muscles, so I didn't assume that it exerted force. But it's doing the exact same idea. It's creating the same product, I guess, but it has // it's not fluctuating. The amount of force it exerts isn't fluctuating right now. *[This is likely an intended contrast to the fact that Jacob emphasized his own role in deciding how much force to apply.]*
5. Inter: How come a table can exert force?
6. Jacob: I'd say *[pauses]*
7. Inter: I mean some people would say a table, it's inanimate. You have a will. The table does not. So how come a table can exert a force *[on the book]*?
8. Jacob: I guess I never really thought about it. I'm not quite sure.
9. Inter: Okay, so think now. Does it make sense?
10. Jacob: Not just from a glance, but I'm sure at a more in-depth level it makes more sense.

Entry 2 and the first two lines in entry 4 in this episode demonstrate that, to Jacob, two explanatory primitives were in conflict in this context: *dynamic balance* (he says there is no difference between his hand and the table; they are doing the same thing—providing a force to balance gravity), and *force requires agency* ("the table doesn't have muscles," so he did not think of the table as providing a force). *Force requires agency* is an intuitive principle that is documented in the literature (e.g. Brown, 1993; diSessa, 1993; Ioannides & Vosniadou, 2002). Its function is to explain the origin of force. *Force requires agency* may originate from physical experience (force is often the result of an action performed by some agent). *Dynamic balance* entailed the existence of the force from the table. Jacob stated that he does not know how the table can exert such a force (entry 8). *Force requires agency* did not lose its high intrinsic priority in Jacob's knowledge system; he did not cease to believe in it. But *dynamic balance* gained higher contextual priority, which led him to conclude that the fact that he cannot identify an agency in the context of the table cannot rule out the existence of the force that the table exerts.[3] Instead, he assumed that there is an agency of which he was unaware, "I'm sure at a more in-depth level it makes more sense" (entry 10). Thus Jacob was led to anticipate some deeper mechanism that somehow affords the ability of the table to apply a force. We ask the reader to keep this anticipation in mind. In terms of Newtonian physics, one can infer that the table pushes on the book merely from the fact that the book pushes on the table. No elaboration on the nature or source of the reaction force is necessary. In this sense, Newton's third law functions as an e-prim for a scientist. But Jacob's thinking is not close to physics in this way, and therefore he needs and expects further justification for the reaction force.

Jacob accepted the idea that the table exerts a force even though he still did not perceive the table to be springy (this is a particularly important point in comparison to other students) and despite the fact that he was aware that he had no idea how an inanimate table can exert a force. The data show that Jacob's acceptance of a normal force was invariant across contexts, even at this early point. For instance, when asked immediately after this episode, Jacob stated that the floor also exerts the same force on a book that is placed on it. "It's *[the floor]* somehow contributing enough force to stop it *[stop the book from reaching the center of the earth]*" (Jacob, 08:00). Jacob uses the same *dynamic balance* explanatory primitive here ("contributing enough force to stop it"). Dynamic balance supplies a higher-priority replacement or extension of *supporting* (it is the force, rather than the mere presence of the floor, that stops the book from penetrating the floor), and as in the table context, Jacob references the same obscurity of the mechanism by which the floor can exerts a force ("somehow"). Nevertheless, Jacob's consistency in using *dynamic balance* across contexts implies a context-stable intrinsic priority.

The elasticity (springiness) of solids. Jacob was presented with the flexible board ("flexible table") before he saw the compression spring. When asked if the flexible table exerted a force on the fishing weight he said:

> Yes, but not enough to maintain its original shape, so it's preventing the fishing weight from completely destroying it or altering its shape, I guess, but it's not // it doesn't have enough strength, I guess. No, that's too vague. It doesn't exert enough force to maintain an even *[flat]* surface. (Jacob, 10:00)

Jacob was beginning, vaguely, to provide a model of a restoring force. He posited that the flexible board might, but does not, exert enough force to maintain an entirely flat surface. What he is missing is a sense of process over time; the board might not initially, but gradually would develop a sufficient opposing force during flexing. Instead, he focuses on the first stage, where the board "does not have enough strength." When asked to compare the flexible board to the table, he did not see them as similar. Instead, he pointed out differences. "Well, the mass // This *[table]* is a lot thicker, whereas it's *[flexible board]* very, very thin" (Jacob, 11:00).

Exploring the spring—from reaction force to springiness. Exploring the compression spring helped Jacob to articulate his ideas about elasticity further. At first Jacob pushed aside the suggestion of force when asked if the spring exerts a force on the fishing weight that is placed on it, and instead he uses the previously cued *supporting* p-prim: "Not necessarily exerts a force, but resists a force that the gravity is exerting on the fishing weight" (Jacob, 21:00). But after playing with the compression spring, pressing it between two fingers, the idea that indeed the spring exerts a force regained priority: "Yeah *[the spring exerts a force]*, but I'd say it's in an attempt to resist the force that I'm exerting on it" (Jacob, 21:20).

This pattern repeats some aspects of Jacob's previous reasoning about whether his hand exerted a force on the book that was placed on it. In both cases it seemed clear that Jacob felt the strain of his muscles and afterward concluded that he exerted a force, using *effort entails force,* which seems to have high intrinsic priority for him. In both cases an entity resisted an applied force. In the case of the book on the hand, the book was pushing against the resisting hand, and Jacob felt that his hand, in the role of "resisting," exerted a force. Thus the connection between resistance and the existence of a resisting force was established. In the case of the compression spring, Jacob was aware that his fingers were exerting a force against the resistance of the compression spring, and Jacob connected the spring's resistance with its applying a resisting force (in contrast to mere blocking). We infer that the idea that resistance can or must entail a force was seeded in the prior situation (book pushing on resisting hand), and carried into this one (spring resists hand's push). Note the natural progression: Supporting, early in the interview, becomes a somewhat more agentive "resisting," which becomes an actual exertion of force.

At this point Jacob readily said that the spring exerts the same kind of a force on the fishing weight that it exerts on his hand. When asked about a mechanism by which this force is exerted, his response was: "It's a resistance of some kind, but I'm not sure /…/ Resistance to the pressure I'm putting on it" (Jacob, 22:00).

One way of interpreting the above instance is that Jacob might have cued a new e-prim—the *springiness* p-prim (diSessa, 1993). *Springiness* imputes a causal connection between a deformation and the consequent development of a restoring force. The suggestion that Jacob cued *springiness* is further supported by his reaction to the following anthropomorphic metaphor, which the interviewer offered him at this point. "Once a small child told me that the spring wants—of course it doesn't have a will, but—it wants to go back to its original length." Jacob was asked if the metaphor made sense to him. Jacob's answer revealed that the agentive attribute that this metaphor cued helped him to articulate his idea of *springiness*. Note also that the sequence from supporting, to resisting, to resisting by applying a force is here extended one more step. The force has a purpose, to return to its original length ("position").

> Yeah. It *[the spring]* wants to return to its original // I'm trying to think of a word to explain that *[pause]* state, I guess. It wants to return to the way it was. /…/ I feel the resistance that it's – the resistance to the force I'm exerting on it. /…/ Yeah *[the resistance is a force]*, I'd say it's // as it *[the spring]* compresses, the force increases or the force *[that the spring exerts]* is trying to increase outwards to return to its original position. (Jacob, 22:50)

Jacob's resonance with the anthropomorphic metaphor suggests that *springiness* has high explanatory power in his view. This is supported by the invariance across contexts that Jacob demonstrated regarding *springiness* hereafter. He used *springiness* spontaneously and fluently (the *obviousness* criterion) in several contexts. For example, his explanation of a fishing weight hanging from a weak and evidently stretching spring was that the spring exerts an inward force on the objects that are connected to its edges "because the spring wants to achieve its original position" (Jacob, 27:50). This idea was further developed when Jacob explained the same situation, but with a stronger spring: "It's an attempt to maintain or re-achieve its original state" (Jacob, 29:50). In the following quote Jacob applies *springiness* as the basis of the similarity between the flexible board and the compression spring.

> As you compress the spring, it wants to go back *[gestures outward]* // not go back. It wants to have the force exerted outwards to regain its original state. The same thing happens with the *[holds the flexible board]* /…/. When you push it down, you're applying a force to it, but when you let go, the force, when it's removed, it returns to its original position as opposed to *[your]* pushing and then having it stay exactly where it was. (Jacob, 30:40)

Springiness acquires contextual priority in the case of the table. Jacob was asked to compare the behavior of the soft and the firm springs. This was done to see whether he believed that the firm spring was actually stretching when the fishing weight was hung from it; the stretching of the firm spring was not obvious, as it was with the soft spring. Jacob's reaction was, "Possibly, but it's too minute to see with the naked *[eye]*" (Jacob, 28:50).

The above excerpt may suggest that the comparison between the soft and the firm spring cued another explanatory primitive that *some changes may be invisible*. The e-prim *some changes may be invisible* was referred to a few times in Jacob's reasoning (*Frequency/stability* criterion). A conflicting intuitive idea is documented in the literature—*things tend to be as they appear*. For instance, researchers found that children's naive conceptualization of Earth is as a flat entity. This conceptualization is based on their everyday experience (Vosniadou & Brewer, 1992) . We believe that *things tend to be as they appear* is an explanatory primitive that has very high intrinsic priority. However, we also think that as humans mature they learn selectively to "turn it off" in certain contexts. In such cases, *some changes may be invisible* acquires high contextual priority and overcomes *things tend to be as they appear.* It is easy to imagine the source and functionality of *some changes may be invisible*. Humans encounter many examples of changes that are invisible to the naked eye. One example is physical growth when it is measured on a daily basis. However it should be noted that this e-prim is unusual in form. It does not, in itself, provide an explanation, but rather paves the way for explanations that involve identifying particular effects that are not apparent but are plausible. Humans learn to recognize multiple occasions where, "on second thought," there is actually some important but invisible action taking place. This developmental process is described in Piaget's studies on causality (Piaget, 1977, p. 137-138).

The next excerpt shows how the cueing of *some changes may be invisible* changed the contextual priority of *springiness* with respect to the rigid table. It allowed Jacob to grasp the relation between the table and the flexible board as similar to the relation between the strong and weak spring. That is, he perceived the apparently rigid table as invisibly (but necessarily) springy.

> The springs have the same sort of goal, to return to their initial state, and the same could probably be said of the table. This one *[pushing on the flexible board and slowly talking as if thinking aloud]* just has less resistance to the weight, whereas this one *[the table]* has more. So when we do that *[puts the fishing weight on the flexible board]*, you can see it's a visible change, and it's because *[talking slowly again]* the weight is exerting more force than the makeshift table *[pushing the flexible board]* can resist, but this table *[puts the weight on the rigid table]* has // I'm not sure exactly. I mean, it's noticeably thicker." (Jacob, 33:00) *[The last two sentences express lingering doubt, but see below.]*

When the interviewer asked if the table deforms in any way, he explicitly expressed his relative confidence, relying on *some changes may be invisible*.

> Yeah. I mean, it's not visible; it's not noticeable to the naked eye. But I'm sure, like, on a maybe subatomic level, on an atomic level, very, very small, there's a change in the composition of the table. Not a change in it, but the atoms in the table might be compressing. But their resistance to the weight is stronger than that of this *[the flexible board]*. (Jacob, 34:00)

In the above excerpt Jacob autonomously develops an explanatory model for the way in which the table, in virtue of being springy, can actually exert the force. His conviction that the table is springy—although the deformation of the table is not apparent to the naked eye—together with his knowledge about the target (the table is composed of atoms) leads him to conclude that the springy compression may be at the atomic level and therefore is not apparent to the naked eye.

Jacob's cueing of *springiness* and attributing high priority to it, together with his cueing of *some changes may be invisible,* which gave *springiness* the contextual priority in the case of the rigid table, allowed him to consider the spring and the table as similarly springy. His knowledge about the target, namely that the table is composed of atoms and that changes at the atomic scale cannot be observed by the naked eye, resonated with *some changes may be invisible*. This allowed him to express *springiness* using terms relevant to the target domain (i.e., "the atoms in the table might be compressing") and bootstrap a new understanding of the target domain. This appears to be a clear example of one of the main claims of this paper: the role of knowledge of the target domain, per se, is crucial if not determinant in accepting analogies.

Molecular bonds as springs. When the scientific representation of the molecular model of solids (molecular bonds as elastic springs) was suggested to Jacob, he immediately accepted it. This is not surprising since he developed most aspects of the model by himself during the guided tutoring sequence. In the following excerpt, Jacob presents an articulate explanation for the origin of the normal force that the table exerts.

> The bonds between the molecules are strong enough to resist the force and // I wouldn't say compensated because it's not like the table is coming up out, but the bonds are strong enough that when the force is applied, there's not a noticeable change /…/ Then the force exerted by the "springs" *[gestures the quotes when saying "springs"]* and the force exerted by the weight are roughly // are fairly similar, so that the force being generated from the table can compensate for the force coming downwards by the weight. (Jacob, 37:00)

This explanation integrates the explanatory primitives that Jacob employed in his previous reasoning: *springiness* (the bonds, in a deformation, "resist" the applied force), *some changes may be invisible* ("not a noticeable change" in the deformation), and *dynamic balance* ("the force being generated from the table can compensate for the force coming downwards by the weight").

Overall, in the context of the phenomena discussed, *springiness* now takes on high relative priority for Jacob. *Springiness* explained an aspect of *dynamic balance* (the source of the countering force) similar to the way *dynamic balance* explained *supporting*. *Springiness* provided an agency for the force that *dynamic balance* entails, satisfying the *force requires agency* explanatory primitive. The consistency that Jacob found between the explanatory primitives gave the molecular model high explanatory power in Jacob's view. Indeed, Jacob demonstrated full invariance across contexts of this model as far as we explored it. He used it in different contexts without any prompting from the interviewer. For instance in the following excerpt he explains why he thinks a rope from which a fishing weight is hung exerts an upward force on the weight via the molecular model.

> As the weight's trying to pull it downward, the weight is trying to get as far down as possible. The rope is resisting it /…/ Its bonds are strong enough that it can compensate for the weight and the force being exerted by the weight /…*[two minutes later]*/ This spring *[points to the compression spring]* and the table are similar, but the rope and this spring *[holds the stretching spring and stretches it]* are similar. (Jacob, 38:00)

Notice the traces of *springiness* ("resisting") and *dynamic balance* ("compensate") in the above excerpt. The elements from the source at this point fully enriched the target, with the molecular model supplying the missing piece in the explanatory model ("Its bonds are strong enough that it can compensate"). Jacob came to see the molecular bonds in the solid as similar to elastic springs, and this new understanding was bootstrapped in the target domain. Note that in the last sentence Jacob further elaborates his notion of springy bonds: In the case of the rope the springs are stretching, and in the case of the table they are compressed.

Jacob: Discussion

The following discussion summarizes the repertoire of explanatory primitives and the priority considerations that Jacob employed and their effect on his response to the instructional sequence. Quite a few explanatory primitives appeared in Jacob's reasoning during the process of the guided analogical reasoning: (1) *supporting*, (2) *dynamic balance*, (3) *springiness*, (4) *effort entails force*, (5) *force requires agency*, (6) *some changes may be invisible*, (7) *gravity pulls things downward*, and (8) *two things cannot be in the same place at the same time*. Note that only the first three e-prims are also previously documented p-prims. Some, like *two things cannot be in the same place at the same time*, may be unidentified p-prims, although this assumption requires further validation. However others are unlikely to be p-prims for various reasons. *Some changes may be invisible* is most likely a consciously and reflectively accessible idea (p-prims are not encoded that way); *gravity pulls things downward* is almost certainly learned on a cultural and explicitly linguistic basis, as well as being consciously and reflectively accessible.

Jacob's dynamic reasoning was guided by priority considerations regarding the explanatory primitives that were cued. *Supporting* had high contextual priority in the case of the books that were placed on the inanimate table. In contrast, *dynamic balance,* which entails the existence of force and therefore some agent exerting the force, seemed irrelevant to the table as it was initially construed. Books placed on the animated hand provided a different framing. Using *gravity pulls things downward*, Jacob interpreted the pressure of the books on his hand as the force of gravity. He then interpreted his muscular effort as an opposing force using *effort entails force*. The "battle" between these two forces to maintain the book balance cued and gave *dynamic balance* high contextual priority in the case of the hand. *Dynamic balance* further gained relative priority over *supporting*, because *supporting* was explained by *dynamic balance*. This acquired priority allowed Jacob to regard the books in his hand and books on the table as similar.

Jacob's attribution of high priority to *dynamic balance* in the context of the table was not trivial. It competed with another high priority explanatory primitive, *force requires agency.* Inanimate objects are usually regarded as non-agentive. Thus, for Jacob to accept that the table exerts force entailed recognition that inanimate objects can be viewed as agentive. This could only have been achieved because *dynamic balance* acquired high priority for Jacob. Jacob did not discard *force requires agency.* Instead, he changed its role in the argument. At first when Jacob could not identify an agent he assumed that there was no force. As *dynamic balance* gained contextual priority in the case of the table, Jacob began to accept the existence of a force that the table exerts on the book even though he was still unable to identify the agent applying it. He managed this discrepancy with *force requires agency* by assuming that the agent exists, even though he was unable to identify it at this point. Cueing *springiness* supplied the agency by which the force predicted by *dynamic balance* was exerted—springs and tables "want" to return to their natural positions. This increased the relative priority of *springiness* similar to the way in which *dynamic balance* gained priority over *supporting* by explaining it. Moreover, since *springiness* seemed to make all the above explanatory primitives consistent with one another, its relative priority may have been strengthened even more. Thus, striving for consistency and hierarchy (one e-prim explains another) appears to be a mechanism that drives the negotiation of priorities.

Cueing *some changes may be invisible* supplied *springiness* with contextual priority in the case of the rigid table; it came to be seen as "fitting." Jacob's knowledge about the target (the table is made of tiny atoms whose displacements might be invisible), the contextual priority of *some changes may be invisible*, and the contextual priority he attributed at this point to *springiness* allowed him to bootstrap an understanding of the table as springy in virtue of the compressed atoms. This acquired insight resonated with the molecular model (molecular bonds as springs), which was eventually suggested to him, leading to quick and invariant acceptance. Jacob confidently accepted the model, and applied it frequently and on his own initiative in different contexts (floor, rope).

Sara: Full Acceptance

Sara was in the tenth grade at the time of the interview. The 38-minute interview took place after school hours in the school's career center. Like Jacob, she invested a lot of thought in the discussion. She often hesitated in choosing between competing arguments, and she also changed her mind often in reaction to considerations. She was open and willing to consider counter-suggestions. However, when finally convinced, her defense was direct and strong. Like Jacob, she accepted the explanation suggested by the analogical sequence, but only after extended consideration. The specifics of her acceptance, though, were different from Jacob's. Such nuances highlight the role played by the individual repertoire of explanatory primitives and their dynamic priorities in an individual's ongoing analogical reasoning.

When dynamic balance is not cued. Like Jacob, Sara initially rejected the idea that a table can exert a force:

Episode 2 (Sara, 02:02):

1. Inter: Now it *[the book]* doesn't fall to the floor, and I ask you, why?
2. Sara: Because there's a table in between the floor and the book.
3. Inter: So what does the table do?
4. Sara: Elevate it?
5. Inter: So the table elevates it, okay. Do you think that the table exerts any force on this book? *[Sara shakes her head, "no"].* Okay, why?
6. Sara: 'Cause when you put the book down, the book didn't bounce, so

it's not coming back up.

Like Jacob in the early stage of his reasoning, Sara used the *supporting* p-prim. Her first statement identifies the reason that the book does not fall as the simple fact that the table supports ("elevates") it. Consistent with *supporting*, Sara does not impute any active role to the table, denying that elevating involves an agentive-typical effect like causing movement ("bouncing").

The next episode demonstrates how the cueing of different explanatory primitives leads to a different reasoning pattern. In episode 3 we can see that, unlike Jacob, who concluded that his hand exerts an upward force on the book only after several books were piled on his hand, Sara immediately said that her hand exerted a force on the book. However, as episode 3 suggests, for Sara, unlike Jacob, the hand example did not cue the *dynamic balance* p-prim. As a consequence, she was not willing to consider the hand and the table as similar. Instead, she pursued the inanimate/animate difference as a central distinction between the hand and table.

Episode 3 (Sara, 02:28):

1. Inter: /…/ Is your hand exerting force on the book?
2. Sara: Yes.
3. Inter: Okay. How can you know that your hand is exerting force on the book?
4. Sara: Because.
5. Inter: Because what?
6. Sara: Because I'm lifting it, and I'm using the muscles in my arm to lift it. But the table doesn't have muscles, and it's not moving. /…/ the table is inanimate so // *[unfinished sentence]*

It seems from entry 6 in Episode 3 that in the context of a table versus a hand, Sara uses *force requires agency*; hence inanimate objects cannot exert a force. Sara's use of the *force requires agency* e-prim is based on articulate use of distinctive properties of animate and inanimate objects ("But the table doesn't have muscles, and it's not moving"). The notion that an inanimate object cannot be an agent seems to have high intrinsic priority in Sara's knowledge system. Unlike Jacob, Sara did not cue the integrated trio *effort entails force*, *gravity pulls things downwards*, and *dynamic balance* in relation to the table at this point. Note that, in line 6, her reference to the muscles in her arm is not, in our interpretation, a reference to her muscular effort per se, but rather to the (agentive) activity that involves her muscles in lifting the book, in contrast to the lack of this ability in the inanimate table (*Functionality* criterion). She "just knew" that her hand exerted a force on the book based on her unquestioned ability to lift the book. Note that the interviewer did not have to pile up books on her hand prior to this contention, as she had to do for Jacob. Direct and high priority use of the idea that humans can supply forces is supported also by Sara's explicit differentiation between animate (herself) and inanimate (the table) objects. For Sara, the mere fact of herself (as an agent) holding something up entailed a force. In contrast, Jacob seemed to require stimulated attention to a mediating conception, effort.

Dynamic balance ("compensating," in Jacob's words) provided a sufficiently attractive possibility for Jacob to be willing to entertain the existence of a force from the table, even if at that point he was well aware that he could not identify the agency responsible for that force. Sara, who did not cue *dynamic balance*, continued to emphasize the lack of agency by remarking on the table's lack of muscles and the lack of motion of the table.[4] With a stronger sense of *force* *requires agency*, and no evident sense, up to this point, of the power of *dynamic balance*, Sara continued to deny that the table was similar to the hand. In her opinion, it could not exert a force. Hence, individual differences between Jacob and Sara in the cued repertoire of e-prims resulted in clearly different reactions to the instructional sequence. The notable case in point is cueing or not cueing and using *dynamic balance*.

Springiness vs. dynamic balance. The compression spring immediately cued *springiness* for Sara. She was certain that the spring exerted a force on the fishing weight that was placed on it, and supplied the following explanation.

> 'Cause it takes energy from the books to push it down. And then the spring naturally wants to be in this position, now, when there's no force pushing down on it. And it pushes back, it does /…/ Yeah. I'm not sure why, but it does." (Sara, 04:58)

Sara's model of the spring involved energy, whereas Jacob's did not. DiSessa (1983) describes a possible richer-than-usual encoding of *springiness* as involving both causality (deformation-restoring force-rebound) and energy flow. Yet Sara did not use the energy flow to explain why the spring pushes back (she might have said, "it needs to release the energy"). Note especially the agentive talk that crept in and the explicit expression of obviousness: "the spring naturally wants to be in this position."

When she was presented with the flexible board, Sara was not sure at first that it exerted a force on the weight. After she touched it, however, she changed her mind: "Because if it wasn't exerting a force, it would // the weight would just fall right through. Right?" (Sara, 07:10). This seems to be the start of a *dynamic balance* argument. Without a second force to compensate, balance, or resist, the force of gravity would just cause motion. Sara now appears to be in the position Jacob found himself earlier in the sequence. We cannot completely disregard unpredictable factors of attention here, however, Sara's physical interaction with the board appeared to change her mind and convince her that a force was involved. That acknowledgement of a force was necessary to find relevance for *dynamic balance*.

At this point, when Sara was asked again about the rigid table, she changed her mind, and considered it reasonable enough to entertain the notion that a rigid table exerts a force. However, she was not quite sure why.

> Maybe it *[the rigid table]* does *[exert a force on the book]*. Maybe it does! *[surprised tone]* Just 'cause it's thick and it's just more weight on the legs. /…/ If there is more weight on the table, there's more gravity pulling it down. Right? (Sara, 07:50)

In the above excerpt, Sara was surprised by her own change of opinion. She was willing, now, to consider that the rigid table exerts a force. Unlike Jacob, she did not explicitly use *dynamic balance* to justify this move, which suggest that *dynamic balance* still had low contextual priority for her. But she did briefly mention *gravity pulls thing downward*, which suggests that force considerations are starting to seem more relevant. She does, in fact, refer to the massiveness of the table, which in some way allows it to resist the effect of the book by applying a force.[5]

A few moments later Sara was asked what caused her to change her previous assumption that an inanimate table cannot exert a force. She proceeded to explain.

Episode 4 (Sara, 09:03):

1. Inter: Okay, so you say now yes. /…/ How can it *[the table]* exert a force on this weight *[fishing weight placed on the rigid table]*?
2. Sara: I don't know.
3. Inter: So what made you change your mind? /…/
4. Sara: On the spring it can exert a force.
5. Inter: So the spring can exert a force and it's –
6. Sara: Inanimate.

Episode 4 shows that Sara still could not figure out how the table exerts a force, but the notion that it can, an idea that at first she completely dismissed, suddenly became reasonable. She was certain, based on the intrinsic priority that she attributed to *springiness*, that the spring can exert a force, even though it is inanimate. She became aware of this conflict only after the interviewer pointed it out. Sara resolved the conflict by recognizing the spring as an exception to her initial contention that inanimate objects cannot exert a force. There are two possible explanations for Sara's new take on the table. The first is that the considerations of *springiness* broke through Sara's categorical reluctance to confer force-providing status to inanimate objects. Hence, *springiness* acquired higher relative priority relative to *force requires agency* by the fact that she broadened her category of agentive objects to include some inanimate objects. An alternative explanation is that these two elements (*springiness*, entailing a force via deformation, and *only animate objects can provide a force*) are not completely coordinated in her knowledge system. Both may be encoded, and she moves between the two. In this view the table was re-classified as one of these exceptions to *only animate objects can provide a force*, without the need to broaden the meaning of agency.

Although at this point Sara believed that the table exerts a force on the book or fishing weight when placed on it, she was still fairly far from Jacob's position when he was at this point in the sequence. She cued *dynamic balance* in the flexible board case, but the intrinsic and contextual priorities of *dynamic balance* were not as strong for her. She did not use *dynamic balance* to explain why the rigid table exerts a force. Instead, she referred to the massiveness of the table, as though agency could be seen in its size (Ioannides & Vosniadou, 2002; Piaget & Garcia, 1974). Indeed, later, she contradicted *dynamic balance* by saying that the table might exert a smaller force than gravity.

When Sara was later asked how the table could exert a force, her answer (next excerpt) suggested that the distinction between the ability of inanimate and animate objects to exert a force still had strong priority in her reasoning.

> I don't think the table itself is exerting a force, but I think that there's a force that is there that the table can use, or something like that. Gravity comes from the Earth so maybe there's some other type of force that the table uses to push it back. (Sara, 15:00)

Springiness can explain how a table may be seen to be agentive. This is precisely how it worked for Jacob. However, it seems that Sara was still trying to find a way to avoid a direct agentive attribution to the table. She was inclined to think that the table *uses* a force somehow, instead of generating or exerting a force, relying on *gravity pulls things downwards* to point to the Earth as an example of a "hidden" agent. Although *dynamic balance* was vaguely cued with the flexible board, its low priority resulted in its absence from most of Sara's reasoning. (The criteria of *Frequency*, *Explicit statement*, *Questioning*, and *Prosody* are used here to infer lower priority.)

Springiness as a high priority p-prim. When Sara was asked to compare the compression spring to the flexible board she said: "They both bounce back, springy stuff *[gestures bouncing]*" (Sara, 17:00). She was, in fact, the one who introduced the term "springy" into the discussion. When asked for its meaning, her first reaction was: "I don't know *[laughs, and uses her hands to gesture "I do not know"]*, Like that! *[points to the spring]*" (Sara, 17:59). This reaction is consistent with the non-verbal basis of p-prims. *Springiness* is recognized in a behavior, and it is not propositionally encoded. However, of course, articulate consideration can be pursued. When further pressed, Sara generated a more articulate explanation of springiness, integrating her former ideas about energy: "The ability for it to reflect or bounce back or whatever energy is put into it." (Sara, 18:20)

Explaining dynamic balance via springiness. Sara was sure that a stretched spring exerts a force on a fishing weight that is hung from it.

> The weight has the gravity, and it's pulling down *[touches the fishing weight and pulling on it],* and the spring is pulling up. It has to be equal because if it was different, it adjusts. The spring can adjust itself to make itself equal *[...]* 'Cause it *[the spring]* wants to be in the natural position." (Sara, 19:50)

Sara is cueing *dynamic balance* here, in a much more articulated manner than previously, referring explicitly to the forces that are in conflict. Sara explains *dynamic balance* with *springiness*. She explains how the spring pulls the weight up by referring to its agentive nature, "it wants to be in the natural position." Finally, what Sara says is consistent with her putting together the dynamic process of adjustment—"if it was different, it adjusts"—which Jacob did not initially do. We would say that here Sarah has a nearly scientific model of the generation of a force to counteract another via elastic deformation.

The table as a springy entity. As with Jacob, the comparison of the firm and soft springs cued *some changes may be invisible*. With Sara, too, this e-prim led to a full acceptance of the table as a springy entity.

Episode 5 (Sara, 26:00):

1. Inter: If you look at the way that this weight affected this *[soft]* spring and the way it affected that *[firm]* spring.
2. Sara: I think it affected them in the same way, just that one *[points to the soft spring]* was more affected than this one.
3. Inter: Okay, so what about … both tables *[the ordinary table, and the table made out of the flexible board]*?
4. Sara: They *[the table and the flexible board]* are still affected in the same way, but this one *[flexible board]* is more affected because it *[pauses to think]*

doesn't have as much resistance as this one *[ordinary table]*.

5. Inter: So this one *[table]* resists more, and so you think it's possible that it's a little bit // do you think it's a little bit deformed when this weight is on it?
6. Sara: Maybe, on a really small scale.

Sara seems to accept, tentatively at first, the rigid table as a springy entity. The acknowledgement of the springy property of rigid objects was context-stable from this point on. In the case of an object that pushes against a wall, Sara was certain that the wall exerts a force on the object "because it has resistance, and it has the same springiness as the table, only there's no gravity pushing on an object straight on the table. You'd have to use other kinds of force, like human force pushing something" (Sara, 30:00). Another example is a fishing weight hanging from a rope, as follows.

> The rope // is like // *[stops to think after every word]* a spring. It has a lot of fibers woven together /…/ The rope can match the energy force exerted from the fishing weight now, but if there was a lot more weight on it, it would probably break." (Sara, 30:59)

The above excerpt is an instance of developing a mental model of the rope as springy. Like Jacob, Sara bootstrapped new insights regarding the applicability and manifestation of *springiness* and *dynamic balance* in the target domain using her knowledge of this domain (the rope has "a lot of fibers woven together"). She presented an articulated consideration of increasingly greater reaction force as deformation increases, leading to a necessary (dynamic) balance at some point, at which point the object does not move.

Molecular bonds as springs. Like Jacob, Sara needed only a minimal suggestion in the direction of the molecular model before she approved of it and appropriated its details. When she reflected on the molecular model as it applied to the table, she seemed to think that it embodied the essence of the springy attribute: "It acts like a trampoline. It just makes a little indent where the gravity is, and then, when it leaves, it springs back" (Sara, 33:10).

Sara already had a strong conviction that the table was springy. The model only gave this idea further support. This strong conviction was apparent in her fluid and confident use of the model in many different contexts. For instance, when explaining how the rope exerts a force, Sara made the drawing in Figure 1, and gave the following explanation while drawing.

> "Well, the rope and table are all solids /…/ So *[quietly starts to draw]*, it's kind of like /…/ This is a rope *[talks and continue to draw]* /…/ And the fishing weight /…/ And this would be like one strand *[works on her drawing and refers to it]* /…/ It just springs with this *[mimicking the stretching and contracting movement of a spring]* /…/ It's like a spring." *[She points to the spring and talks confidently while putting the last touches on her drawing.]* (Sara, 34:00)

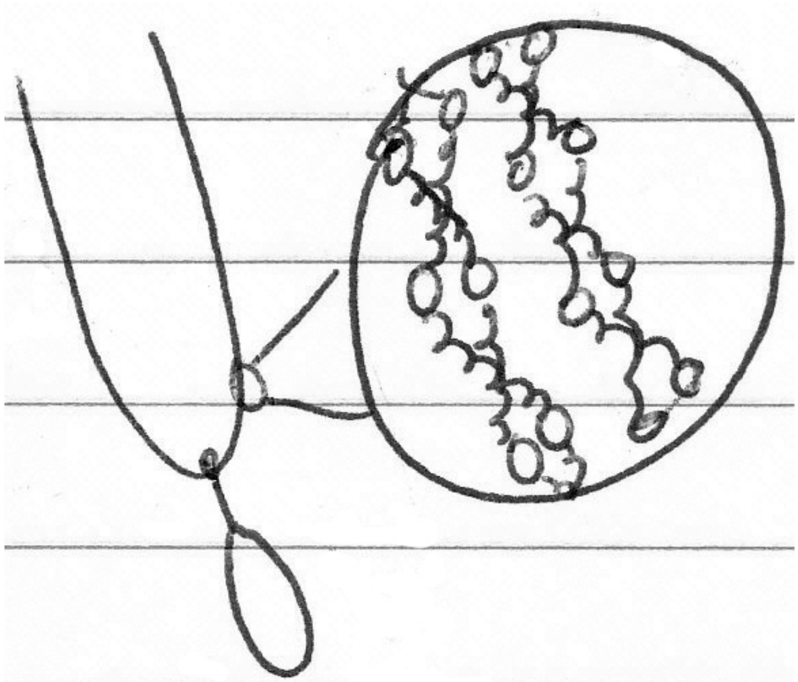

Figure 1. Sara's representation of molecular bonding as springs in the rope.

Unlike Jacob, Sara is not using the words "molecular bonds," but as Figure 1 shows, she depicts the chemical bonds literally as little springs.

Sara: Discussion

As with Jacob, Sara's reasoning demonstrated a number of explanatory primitives: (1) *supporting*, (2) *dynamic balance*, (3) *springiness*, (4) *force requires agency* (inanimate objects cannot exert a force), (5) *some changes may be invisible*, and (6) *gravity pulls things downward*. Again, note that only the first three are unambiguously p-prims, and the last two are almost certainly not. The general similarity between Sara's and Jacob's repertoire is not surprising given the general commonality of human experience and the fact that both are reasoning through the same instructional analogical sequence. However, careful examination of their moment-by-moment reasoning reveals considerable differences in their reasoning that can be explained by the cued repertoire (Sara did not cue, at particular stages, some of the e-prims that Jacob did) and by differences in the priorities they assigned to the explanatory primitives that they both cued.

The force requires agency e-prim, based on the distinction between the properties of animate and inanimate objects, had a significantly higher intrinsic priority for Sara than for Jacob. This explains why in the case of the book on the hand, Sara focused on muscles (as an index to agentive action) whereas Jacob focused on the "compensating force" (*dynamic balance*). This also explains why Sara was only willing to consider the inanimate table as an agent that can exert a force much later than Jacob, after the "breakthrough" in acknowledging that the spring was an example of an inanimate object that can exert a force.

Sara attributed high contextual priority (in the case of a spring) and significant intrinsic priority to the *springiness* p-prim. Jacob, too, prioritized *springiness*. However the intrinsic priority that Sara attributed to *springiness* seemed to be higher than Jacob's. Her use of *springiness* was more common and frequent. However, the main difference between Sara's and Jacob's reasoning was the different priorities they attributed to *dynamic balance* and *force requires agency*. Jacob cued *dynamic balance* much earlier than Sara and attributed higher intrinsic and contextual priority to it, and he also attributed higher relative priority to it than to *supporting*; he accepted the plausibility that the table exerts a force to participate in a dynamic balance while he was still explicitly aware that he could not point to an agent. Sara, on the other hand, agreed to consider the table as capable of exerting a force only after several steps. First, she acknowledged that the spring was an example of an inanimate object that could exert a force. Second, she recognized the spring to be an exception to her previous view of agency, which forced her to expand, at least to some extent, her prior definition for agency, or at least expand the list of exceptions to this categorical denial of force without a living agent. For both of them *springiness* was taken to entail a necessary agency behind the force exerted by the table. The expanding of the category, or the reclassification of objects into an existing category seem to be the mechanisms that drove Sara's negotiation of priorities, where *springiness* acquired contextual priority at the expense of *force requires agency* in the context of the table, resulting in the dismissal of *supporting* that initially guided her reasoning.

Both Sara and Jacob fully accepted the molecular model of solids. It supported and strengthened their previously acquired conviction concerning the springy property of solids. Both bootstrapped an understanding of *springiness* as a candidate for reality and as an explanatory model based on their knowledge of the target domain. For Jacob the model explicated the agency by which the table can exert the force (agency in the elastic molecular bonds). He perceived molecular bonds to *behave like* small springs. For Sara the molecular model provided a yet more detailed view of her initially comfortable take on *springiness* at a macro level.

Adam: Reservations

Adam was in the tenth grade when the interview took place. He was very engaged in the analogical inquiry sequence. During the 51-minute interview, Adam constantly played with the artifacts. He observed them from different perspectives before and while talking. Unlike the other five interviewees, Adam did not leave the interview with a confident understanding of how the table actually supports the book. Explaining this dramatic difference will be a primary aim of our analysis.

> It's kind of interesting to see that I can't even // I have a grasp on it but// /…/ It's interesting to see that a high schooler can't fully explain why a book doesn't just fall through the desk *[laughs]*. It's definitely a good thing to think about. (Adam, 50:00)

Supporting. Adam started a little differently than Sara and Jacob: "I guess *[touches the book gently]* matter can't inhabit the same two spots at once? /…/ It can't be where the table is." (Adam, 01:59) His language suggests that he was relying mainly on *two things cannot be in the same place at the same time* (Spelke, 1991) rather than (bare) *supporting*.

At this point Adam asked the interviewer for some hints, and she told him that other people mentioned several different arguments, such as that the table prevents the book from falling or that the table exerts a force. Adam found the "preventing from falling" argument more reasonable, which we interpret as a purer example of cueing *supporting*: "It's not really exerting force, but it prevents the force of this book from moving any farther with its own" (Adam, 03:00). An instant later he acknowledged that gravity pulls the book downward, but at the same time he dismissed the idea that the table exerts an upward force on the book: "I guess it doesn't really exert a force upward, but so much as it prevents the book's force" (Adam, 04:00).

Adam is suggesting that in order to counter the effect of a force, one does not need to apply another force. Rather the force or its effect can be directly blocked (diSessa, Gillespie, & Esterly, 2004). The *supporting* p-prim encodes this causality in the context where the blocked force is gravity (diSessa, 1993). Adam used *supporting* with increased priority since it explained the previously activated e-prim *two things cannot be in the same place at the same time*. Supporting, that is, blocking or forestalling the effect of the force of gravity explained impenetrability. Indeed, from that moment and for the duration of the interview, he stopped using *two things cannot be in the same place at the same time.* This is an additional example of how forming an explanatory hierarchy drives the negotiation of priorities.

Force requires agency vs. dynamic balance. Adam sensed that his hands exerted a force on a book when it was placed on them. He tried to hold the book in one hand as well, probably to feel the effort of his muscles better. That act suggests that he found effective cueing for *effort entails force*. He immediately accepted his sensing of effort as proof that his hands exerted a force on the book. This experience, in turn, seemed to cue *dynamic balance*: "'Cause I have to push upward the entire time to keep it from falling down *[lifting the book]*" (Adam, 04:40).

The next excerpt demonstrates that, similar to Jacob's case, *dynamic balance* allowed Adam to see that there are grounds to consider the hand and the table as similar. However, unlike Jacob, Adam could not convince himself that this was genuinely a valid resemblance. He based this lack of conviction on the *force requires agency* explanatory primitive.

> I guess they *[table and hand]* should be the same. I can't quite figure out how the table would push upwards, though /…/ *[smiles]* I guess there's really nothing except it's my muscles are holding it up *[demonstrating the movement with his hands]*, so that's why I think that's easier for me to comprehend, because I can see that I can push things and move things *[demonstrating both movements with his hands]*. But I don't see how a table would be able to do that. Yeah. (Adam, 05:00)

Although like Jacob, Adam seemed to have cued *dynamic balance* in the case of the book pressing on his hand, he assigned it a lower contextual priority than Jacob did in the case of the table. By contrast he assigned *force requires agency* higher contextual priority. The above excerpt demonstrates that for Adam, if he can identify no particular means for the exertion of a force, there is no force. Note, again, that in Newtonian physics, the principle of action and reaction sanctions a reaction force without naming a means. Jacob's high priority for *dynamic balance* appeared to do the same for him. Jacob thus treated *dynamic balance* as secure (higher priority), in the face of more detailed concerns that still needed to be worked out. Although Adam did cue *dynamic balance*, he rejected it based on its low relative priority with respect to *force requires agency* in the context of the table.

Understanding how a spring works. Adam articulated a detailed explanation of how the compression spring works.

> Okay, so it was compressed because of the kinetic energy *[pronouncing "kinetic" slowly, as if uncertain about the meaning]* of the weight pushing down because of the gravity, so they're attracted towards the Earth /…/ And therefore, this *[holds and compresses the spring]* compresses to absorb the energy, and then, once you remove that force, it returns back to its lowest energy state *[moving his hand upwards]*, which is just like that *[points towards the non compressed spring]*. (Adam, 06:10)

His model for the energy transformation was more elaborate than Sara's, but like Sara, Adam did not connect energy considerations directly to the force. Adam's slow-paced and thoughtful construction of his description and his concomitant gestures were striking. Adam appeared to be constructing a mental model of the energy flow in a spring as opposed to activating an e-prim. We base this interpretation on the observation that Adam consciously ran this model, developing it gradually on the run. His gestures also reflect, in our view, the runnability of this model. The net result of this complex reasoning is a causal relation that explains the behavior of a spring.

Adam concluded, in the end, that the spring exerts a force. But he concluded this not by activating *springiness*, or by running the mental model for springs, but from the direct sensation of this force when he compressed the spring between his fingers: "I think, if you push down on it *[takes the spring, places it on the table and compresses it with his hand]*, you can feel that it's exerting a force upwards, so I would presume that it is" (Adam. 07:30). Like Jacob at this point in the sequence, Adam interpreted the resistance of the spring to his own pressing as the force exerted by the spring.

Springiness. Unlike Jacob, the anthropomorphic metaphor (the spring wants to return to its original length) resonated at best weakly for Adam: "It kind of does in a way *[make sense]*. But I guess I don't think it *[the spring]* really knows what its natural length is. *[He compresses the spring.]* It just has an equal reaction to what you put in" (Adam, 09:58). This contrasts markedly with both Jacob and Sara, who quickly appropriated the idea that springs "want" to return to their rest lengths, and presumably, therefore, "know" them.

While Adam seemed to develop an adequate model of the energy flow in the spring earlier, the above excerpt suggests that he did not activate a particular form of the spring's causality (the tendency to return to the rest state, i.e. *springiness*). As mentioned, other students had or relatively easily acquired *springiness* in that form and with higher priority than Adam. Thus, he understood some version of action and reaction but did not see a springy object as one that necessarily returns to its natural length. One might argue that Adam's rejection stems from an epistemological stance against anthropomorphic explanations. We disagree. He did not reject the actual act of anthropomorphism: "It kind of does in a way." What he did not understand is specifically *how* the spring knows its natural length: "I guess I don't think it *[the spring]* really knows what its natural length is." In this last statement, Adam is appropriating anthropomorphism for his own purposes, to express his belief that springs might or might not "know" their rest length, but that, in fact, they do not know. Moreover, later in the interview he explicitly used anthropomorphic language without any prompt from the interviewer (molecular bonds "want to"….).

Adam was able, after some thought, to construct a model of energy flow for the spring. However, this does not necessarily imply that *springiness* was a high intrinsic priority e-prim for him. We think that he constructed this explanation as a mental model during the interview as he thought about the spring. Later he used it to explain the behavior of the flexible board. When presented with a weight that was placed on the flexible board, Adam first spent a minute lifting and placing the weight on the board and pushing the board with his fingers, trying to figure out how it worked. Then, he started to explain how the flexible board behaves using the energy-flow model that he developed earlier. Note in the following excerpt the dynamics of the construction: the careful consideration, the running of the model, the questioning, and the use of scientific knowledge learned in school.

> I think it *[the flexible board]* must be *[exerting a force on the fishing weight]*. Because, well, it's bending down. So it's absorbing some of the weight's energy. So the gravity is pulling down on the weight. I guess it's *[gravity is]* pulling down on everything /…/. So, the gravity is pulling down on that *[pointing at the weight]*, which transfers energy into this *[holding the flexible board]*, so it's stored as // I think it kind of switches the kinds of energy, so it goes from more I guess *[lifts the weight and then put it back on the board]*, if that's kinetic energy? *[He whispers "kinetic energy"*[6] *as if he is unsure whether this is a correct use of the term.]* I don't know. And then it kind of stores it as, I don't know, maybe call it mechanical energy?" (Adam, 11:00)

The above excerpt reveals that at this point Adam perceived the flexible board and the spring as similar in the sense that both can bend and both can absorb energy as they deform; then they can both reverse the same energy flow. His mental model also included a somewhat vague connection between the restoring force and the stored energy, a connection that in its full articulation resembles a more expert-like mental model of springs (diSessa, 1983). Sara elaborated this connection only towards the end of the interview, and Jacob did not mention it at all. Recall Adam's unproblematic use of *gravity pulls things downwards* in the above excerpt as opposed to his careful construction of the model of springs. This, too, might suggest that *springiness* does not function as a high intrinsic priority e-prim for him but functions more in the form of a dynamic mental model. Note that a mental model could be considered to have high explanatory power (high priority) or low explanatory power (low priority). We prefer to describe Adam's description of the spring as a dynamic mental model rather than (merely) an instance of cueing an e-prim because Adams generated it as he thought about spring and flexible boards (rather than cueing a complete existing construct), and he continued to use it by "running" it over time at each use.

When the interviewer asked about the direction and magnitude of the force that the flexible board exerts, Adam added *dynamic balance* to his reasoning:

> I think it *[the force that the flexible board exerts on the fishing weight]* must be close to equal or else it would still be moving. 'Cause if this *[the weight]* was greater than the force that that *[the board]* was exerting, then it *[fishing weight]* would sink down farther, as if it *[the board]* weren't there. And if this force *[pointing to the board]* was greater, then it would just spring it up into the air. (Adam, 12:30)

Here Adam made canonical use of *dynamic balance* in conjunction with the *overcoming* p-prim, with which *dynamic balance* is frequently associated (diSessa, 1993). *Overcoming* entails that if the engaged forces are of different strengths, one overcomes the other and "gets its way." In this case, gravity's greater strength would cause the weight to "sink down further," and the springy force's greater strength would "spring it *[the fishing weight]* up into the air." In the earlier excerpt (Adam, 11:00), Adam referred to the spring and the flexible board themselves as the agents that absorb energy ("Because, well, it's bending down. So it's absorbing some of the weight's energy.") This acknowledgement, together with his physical sense of the force that both exert, led Adam to expand the agency property he attributed to springs and flexible boards, perceiving them now as agents that can exert a force. Finding agency in the situations allowed *dynamic balance* to regain its contextual priority in the case of springs and flexible boards, satisfying Adam's intrinsically high prioritized e-prim, *force requires agency*.

At this point the interviewer wondered out loud why the balance argument (one force cancels the effect of the other) that seemed to work so well for the flexible board and the spring was not valid in Adam's view in the case of the table. Adam answer was: "Well, this *[pushing the flexible board]* is flexible so it doesn't seem to me as if this *[pushing the ordinary table]* has a way to exert force" (Adam, 13:30).

The above excerpt suggests that for Adam, the capability to deform was a crucial aspect of the agent of a restoring force. Again, the intrinsic priority of *force requires agency* dominated Adam's reasoning. Without evidence for an agent that is capable of exerting a force, Adam could not accept the existence of a force. Without visible deformation in the table, the idea that it could exert a force simply seemed implausible.

One could argue that the notion that an object that is deformed and therefore exerts a force is an expression of Adam's own version of *springiness*. Although we cannot exclude this possibility, we think that the data suggest that the causal relation between the deformation and the restoring force was not initially part of Adam's knowledge system. Rather, it was a reasoned conclusion he reached during the interview, as he constructed a mental model of springy things. This contention is supported by Adam's unproblematic use of other e-prims, as opposed to his articulated and thoughtful construction of *springiness*. It is also supported by his earlier, firm rejection of the assertion that a spring "wants" to regain its natural length, in contrast to the resonance that this idea evoked in all the other interviewees. Regardless of the status of *springiness* in Adam's knowledge system (an e-prim or a constructed mental model that happens not to seem explanatory of the relevant phenomena), it had low contextual priority in the case of the table, in contrast to the other interviewees. Whatever his version of *springiness* is, we can say confidently that it seems to have limited applicability (limited contextual priority).

Rigidity. Like the other interviewees, when presented with the comparison of a firmer spring to a softer spring, Adam cued the e-prim *some changes may be invisible*. This allowed him to see a potential parallel between the relation between the soft and firm spring and the relation between the flexible board and the table. Unlike the other interviewees, the parallel did not convince him that this relation was justified in the case of the table, as shown in the following episode.

Episode 6 (Adam, 17:50):

1. Inter: So could you see any resemblance between the relation between this spring *[stretching the soft spring]* or that spring *[stretching the*

hard spring] and this table *[pressing the flexible board]* and this table *[knocking on the table]*?

2. Adam: Yeah. *[He laughs.]* I guess I still have trouble with this table *[pressing both hands against the rigid table]* because I don't see how it would absorb it. Because I see the means of absorption by like that *[stretching the soft spring],* and I can see that this can flex *[pressing the flexible board]* and then return the energy when you remove the force. And I can see, I mean, the same on that *[pointing to the firm spring]* with a larger scale. But I still have trouble seeing that this *[pressing against the table with both hands]* would really exert force. Hmm.
3. Inter: So you think // is your trouble then from /…/ I mean, do you see the table as now maybe deformed in some manner that you can't see?
4. Adam: I guess maybe it's like, ha ?? *[pushing with his thumbs hard on the table and watching closely, thinking silently]* It's just like visually obvious to me with these *[taking the hard spring and bending it]* that they're absorbing it, and with this *[leaning back and holding the table]*, I guess maybe like when you see something like a tennis ball bouncing or something like that, if you have seen like slow motion pictures and it shows that it compresses a ton, then it returns and bounces up. *[He is demonstrating the compression and returning with both hands while talking]* But like, maybe that happens with the table, but it's not visible. Or, I don't know. Hmm.

The discussion above appears to demonstrate that springiness had very low contextual priority for Adam in the case of the table (*Questioning*, *Prosody*, *Explicit statement* criteria). As Adam says in entry 2, "absorption" is a result of a deformation, and he simply could not accept that tables deform. This is true even though the analogy with the soft and stiff springs made him aware that some changes can be invisible. We think that the table cued a conflicting e-prim for Adam that was not evidently cued by the other students, the *rigidity* p-prim (diSessa, 1993). Rigidity is a property of certain materials in naïve thinking. The property is also categorical in most instances; objects can be completely rigid. To a physicist, rigidity has a much lower priority, and in fact nothing can possibly be categorically rigid. Instead, objects are always more or less deformable. For Adam *rigidity* had high contextual priority in the case of what he perceived to be solid objects. A strong indicator of this high priority is his exclusion of the alternative (springy) analysis based on its conflict with rigidity ("I see the means of absorption *[in the springs and flexible boards]*/…/ But I still have trouble seeing *[it in the case of the table]*"). Adam was prone to, and eventually settled on, classifying the table as categorically rigid. Thus, the non-deforming table could not exert a restoring force.[7] A similar case of a subject's denying *springiness* recalcitrantly in the case of apparently rigid objects (glass, steel) is documented in diSessa (1983).

We think that the idea of a restoring force, as entailed by the other students' take on *springiness*, was not part of Adam's initial set of explanatory primitives. Rather, he constructed it during the interview. But even if *springiness* was part of his repertoire, it had very low contextual priority in the case of the table. Adam could not apply it to the table because the table cued a strong explanatory primitive, *rigidity*, which contradicted the core of the restoring force mechanism; namely, an object can exert a restoring force only in virtue of "absorbing," which seems to require deformation.[8] This interpretation is supported by Adam's reaction to the molecular model (molecular bonds as springs). His immediate reaction was surprise: "This isn't a solid we're still talking about?" (Adam, 20:50). Describing the "solid" as made up of bending springs did not make sense to him. Indeed, *springiness* is the diametrical opposite of the essence of *rigidity*. This is a case where competing e-prims seem mutually exclusive in a given context. Adam simply could not consider *springiness* to be valid in the context of categorically rigid objects.

An alternative interpretation would be that, for Adam, *things are as they appear* and not *rigidity* was cued in the context of the table, where there was no visible deformation. So *things are as they appear* conflicted with *springiness* in the case of the table. Our judgment, however, is that *rigidity* seems more applicable to the specific aspects of Adam's attention that he revealed in the context of the table, compared to *things are as they appear* (*Functionality* criterion). Adam was attentive to the fact that the table is solid; he did not use "solid" in the context of the spring and the flexible board. This is reinforced by his surprised reaction to the molecular model as a possible representation of the table. "This isn't a solid we're still talking about?" (Adam, 20:50). Beyond the details concerning which explanatory primitives are used and their nature, a major point stands out. The particular explanatory primitives that are cued and the dynamics of their priorities influence students' judgment regarding the plausibility and applicability of the analogical inference.

Springiness. Adam did not reject the molecular model because he did not understand it; he seemed to have an excellent understanding of it.

> So I guess it's reasonable to presume that they *[bonds]* could absorb energy. 'Cause otherwise, if it can't happen on a microscopic scale, there's no way that this *[holding the compression spring in his hand and compressing it]* would be possible if the bonds couldn't move. (Adam, 21:00)

That is to say, molecular bonds have to deform microscopically to allow a macroscopic deformation in a spring. Adam correctly mapped the springs in the model onto the chemical bonds and understood how he could use the molecular model to explain the behavior of springy objects. But notice his use of "presume" as opposed to "assume," and his use of the hedging "could" rather than "will"; he was not sure that this conclusion was correct (*Explicit statement* criterion). The second sentence in the above excerpt reveals that, in his view, the molecular model could explain how a spring behaves, but a spring cannot explain how the molecular model behaves. That is, *springiness* does not have autonomous explanatory power; it is not a (high priority) e-prim or mental model.

When asked specifically to explain using the molecular model, he was able to generate an excellent explanation for the existence of a normal force in the table. But, under continuing consideration, he explicitly questioned the model, and, in his actions, made clear it did not have high explanatory power in his view, as we discuss below.

Adam never used the molecular model on his own initiative to explain reaction forces, despite several requests for explanations in different circumstances. He did not do so when asked later about the force a table might exert, nor did he in the case of pushing on a wall, or in the context of a weight suspended on a rope. *Dynamic balance*, for him, governed all the following situations (*Frequency* criterion):

- Book on the table: “I think it must exert some sort of force to prevent this from just falling through it” (Adam, 23:00).
- The force a wall exerts when pushed upon: “It would be pushing outward, so it would be, I guess it would have to be an equal force, ‘cause otherwise the wall would be moving” (Adam, 25:30).
- Why the rope exerts a force on a fishing weight that hangs on it: “It must be exerting a force on the weight to prevent it from just falling” (Adam, 29:50).

Adam appeared to accept the molecular model enough to conclude that agency could exist in rigid objects. This gave *dynamic balance* some contextual priority in the case of the table, the wall, and the rope. For instance, he used the *dynamic balance* argument regarding the seemingly rigid bodies only after the molecular model was presented. However Adam did not use the restoring force argument directly for these rigid objects. When he was asked how the floor can exert a force, Adam suggested that the carpet behaves like a spring since it is flexible: “‘Cause *[bending down and pressing the book to the floor]* it’s just pushing on the carpet. If you notice, when you push on it, it normally returns just back to how it is” (Adam, 24:00).

So the carpet was classified, as were springs and the flexible board, as a non-rigid object that deforms and is therefore capable of exerting a restoring force, in accordance with Adam’s elaborated model for the spring. Although Adam understood the molecular model, he did not consider it to be valid in the case of truly rigid objects. Indeed, about 15 minutes later, when Adam reexamined the compression spring, he gave more detailed voice to his doubts, articulating his previous claim that the molecular model can explain the behavior of a spring, but a spring cannot explain molecular scale behavior.

> Like, I can see that we all obviously accept gravity, but I don’t quite // I guess I still don’t quite understand why it *[the spring]* would return to its normal state *[talking while compressing the spring]*. I don’t quite understand like *[presses the compression spring between his hands]*, how the atoms would know that they would return to that shape. (Adam, 36:00)

Adam explicitly contrasted here his unquestioned belief in gravity (“we all obviously accept gravity”) as an explanatory primitive with his attitude toward *springiness* (*Explicit statement* criterion). For him, *springiness* presented a low priority explanation, worth presenting only if explicitly asked.

Toward the end of the interview Adam further articulated his doubts about the molecular model.

> Well, I guess if you believe this model that atoms // that the bonds return // like want to return to equilibrium, then it’s easier for me to understand that just something on a larger scale would have the same properties. (Adam, 42:00)

Adam admitted here that the model entails agency in the bonds (they “want to”; note the anthropomorphism), which is essential according to his view on the existence of a force. However he also explicitly expressed his doubts regarding the initial assumptions upon which the model is based, namely that molecular bonds behave like small springs. He says, “I guess if you believe this model….” In our view, Adam did not fully believe it.

Adam: Discussion

Of all our interviewees, we found Adam the most intriguing because of his articulate rejection of the suggested analogical reasoning. This distinguished him from the other interviewees and raises the question of how to explain this difference.

In the context of our interview Adam presented the following explanatory primitives: (1) *supporting*, (2) *overcoming*, (3) *dynamic balance*, (4) *rigidity*, (5) *springiness* (questionable), (6) *things are as they appear* (questionable), (7) *effort entails force*, (8) *force requires agency*, (9) *some changes may be invisible*, (10) *gravity pulls things downwards*, and (11) *two things cannot be in the same place at the same time*. Note that only about half of the e-prims that Adam cued were clearly p-prims, and several were clearly not p-prims. Although Adam's repertoire of explanatory primitives at first glance seems similar to Jacob's and Sara's, a careful examination reveals that he also cued different e-prims (e.g., *rigidity*). His use of *rigidity* versus their use of *springiness* presented the most obvious difference.

While the dynamics of his negotiating priorities in some cases resembled those of Sara and Jacob, frequently they were markedly different. *Springines*s has been identified as a competing p-prim to *rigidity* (diSessa, 1993), so it is not surprising to find that a high priority for one co-occurs with a low priority for the other. The data suggest not only that Adam assigned a much lower contextual priority to *springiness* in the case of the table, but they suggest that his intrinsic priority for *springiness* was low as well. Instead of coming in with or developing a high priority for *springiness* as a p-prim, he constructed a low-priority mental model of springy objects during the interview. He started by describing the energy flow and much later derived a restoring force from it. His model suggests that deformation in elastic objects results in a restoring force, which he explained with energy transformation and restoration. He was able to apply this model to explain the force that any observed-to-be-deformable object exerts (spring, flexible board, and carpet). However, having a model of how a spring behaves does not imply that Adam had an intuitive grasp of *springiness*, at least not in the same form that the other interviewees presented. In fact, his rejection of the fact that springs might "know" their rest length, his skeptical reaction to the molecular model, together with his consistent reluctance to use the molecular model to explain relevant phenomena all suggest that he did not find *springiness* explanatory.

Two dominant explanatory primitives governed Adam's reasoning during the interview: *force requires agency*, and *rigidity*. Like Adam, Sara also prioritized *force requires agency*. She concluded firmly that the table exerts a force on the book only after she perceived the table as springy. However, unlike Adam, Sara attributed high intrinsic priority to *springiness* that was evident in her frequent and unquestioning use of this e-prim, which was not apparent in Adam's reasoning. Like Sara and unlike Jacob, Adam needed to identify an agency explicitly in order to conclude the existence of a force. However, Adam classified the table as categorically rigid. This immediately forced on *springiness* a very low contextual priority. In terms of negotiation of priorities, Adam activated conflicting e-prims and the mechanism that drove the negotiation of priorities was to eliminate conflicts, and deny *springiness* in rigid contexts.

What would convince Adam that the table can deform elastically? One possibility would be to show him direct data that it deforms, if only microscopically. This might lower the contextual priority of *rigidity*. To make precisely this point, Minstrell (1982) used a laser beam that is reflected at slightly different angles from the table when heavy objects are placed on it. A non-empirical approach would be to explain how springy molecular forces develop from purely distance-related electrostatic forces.

Adam understood the meaning of the molecular model of solids. He was aware that it offered the possibility of agency in the table: Elastic molecular bonds can exert a restoring force. However, he was not convinced that the bonds could really be elastic in the sense that the term applies to springs. It did not seem plausible to him that the bonds can "know" their initial state in order to return to it. In contrast to the other students, the tendency of deformed objects to return to their natural shape was not something that Adam was willing to take for granted. We interpret this unwillingness as a demonstration of the low intrinsic priority that he assigned to *springiness*.

The high priority that Adam attributed to *force requires agency* allowed him to conclude, after he was presented with the molecular model of solids, that the table might exert a force. The notion that the table might exert a force has the earmarks of a mid-priority idea. Adam made it clear that this was only a possibility, hardly a certainty. In fact, he identified a cogent logical fallacy in the argument. Asserting that a molecular bond is like a spring makes the explanation circular since the model is precisely intended to explain how springs (and other elastic materials) work.

Adam left the interview feeling that he did not understand how the table could exert a force, even though he was able to generate explanations for this force using the molecular model. The explanatory primitives model gives a clear explanation of his position. First, the application of the molecular model of solids to the table was challenged by Adam's unusually strong contextual priority for the *rigidity* primitive. Second, Adam appeared to have, at best, a variant of the usual *springiness* explanatory primitive, and most certainly did not attribute high priority to it. In contrast, Sara and Jacob, who embraced the molecular model based on high intrinsic and later contextual priority for *springiness*, demonstrated commitment to *springiness* before being exposed to the model.[9] In fact, they filled in the model fluently from a preliminary sketch, and adopted it instantly, and without needing priming or coaxing across multiple situations.

Less central to Adam's reasoning, but constituting further evidence for the role of explanatory primitives in individual reasoning, is the role of *dynamic balance* in recognizing the book on the hand and the book on the table as similar. Experiencing the book on the hand cued *dynamic balance* for Adam and Jacob, whereas it did not for Sara. *Dynamic balance* allowed Adam and Jacob to consider the book on the table and the book on the hand to be similar in a very particular respect. Both Adam and Jacob were aware that they could not identify a particular agency. However, the contextual priority of *dynamic balance* overcame the contextual priority of *force requires agency* for Jacob in the case of the table, and he concluded that the table exerts a force on the book, postponing the task of identifying the actual agent. In contrast Adam's reasoning reflects a different pattern. The contextual priority of *force requires agency* overcame the contextual priority of *dynamic balance*, which led Adam to reject the similarity that he acknowledged at first, and to assume that the table does not exert a force on the book. He did so specifically because he could not identify an agent responsible for this force. Sara, who had not used *dynamic balance* at all up to this point (book on the table vs. book on the hand) and who had used only *force require agency,* did not even consider the possibility that the table and the hand could be similar.

General Discussion

Summary of the Findings

The comparison of the three students' reasoning as they engaged in an instructional analogical sequence on the existence of a normal force supports our primary hypothesis that observable differences in an individual's explanatory primitive repertoire, and the priority attached to them, led to observable differences in the dynamics and ultimate effectiveness of the sequence. We model confidence in a specific explanatory primitive as an expression of a pair of priority attributes: intrinsic and contextual priorities. The level of confidence instilled by a particular e-prim (intrinsic priority) is moderated by the degree to which it seems to fit present circumstances (contextual priority). We model the dynamics of reasoning as negotiating priorities where one e-prim gains further priority relative to another at a particular time and in a particular context during the reasoning process. This can happen in a variety of ways, as suggested by our analyses.

We examined the participants' thinking processes in detail as they engaged in analogical reasoning. We identified several explanatory primitives that played a role in our participants' reasoning as induced by the instructional analogical sequence. At first glance, the students may have seemed to apply a similar set of explanatory primitives to the instructional sequence. That would not be surprising, considering the degree of commonality we can expect with respect to basic and familiar physical experiences. Careful examination, however, revealed differences between the students' repertoire of explanatory primitives and the priorities involved. These differences played out in substantially different learning trajectories.

1. Differences in the presence or absence of a particular e-prim in individuals' cued set of explanatory primitives led to different reactions to the instructional analogy sequence. Consider for instance the absence of *dynamic balance* in Sara's take on the book on the hand as opposed to Jacob's and Adam's immediate cueing of *dynamic balance*. Jacob and Adam based their consideration of the book on the hand and the book on the table as possibly similar situations based on this e-prim, whereas Sara refused to even consider this similarity as plausible, basing her rejection on the inanimate-animate difference between the table and the hand. Another example is the absence of *rigidity* in Jacob's and Sara's reasoning and its presence in Adam's. *Rigidity* partly accounts for Adam's rejection of the springy description of the table and the molecular model of solids, which Sara and Jacob fully accepted and appropriated.
2. Differences in the priorities attached to specific explanatory primitives resulted in different reactions to the instructional sequence. The empirical differentiation between intrinsic and contextual priority in most cases could not be fully achieved since they are entangled in individual cases. Thus we discuss differences in priorities in general and refer to specific priorities only when we think a clear differentiation can be made empirically.
    a. Intrinsic priority: (i) All the interviewees were committed to *force requires agency*, but Sara presented an additional commitment to a particular aspect, the difference between inanimate and animate objects. Consider Sara's focus on the animate quality of the hand that holds the book in contrast to Jacob's and Adam's focus on the function of the hand. Sarah connected agents directly to the ability to support, without the intermediary of a force. In contrast, Jacob's and Adam's attention to the function of the hand—both were

consummately aware of their muscular effort—suggested that forces are involved. Thus Sara rejected the suggested similarity between the book on the hand and the book on the table cases. (ii) *Effort entails force* and even more *gravity pulls things downwards* had a strong intrinsic priority for Jacob and Adam. The moment that they felt their muscular effort and the pressure of the books on their hands they assumed that forces were involved. (iii) *Dynamic balance* seemed to have high intrinsic priority for all our interviewees. However, Jacob assigned it the highest priority in that, the moment it was cued, he disregarded competing primitives. (iv) *Springiness* seemed to have a very high intrinsic priority in Sara's reasoning, which was evident in her abundant and obvious reliance on this e-prim. The cueing of *springiness* was what led Sara to consider the possibility that the table might be exerting a force on the book, since deformed springy objects are inanimate things that exert a restoring force as they "try" to return to their original shape. In contrast, consider the low intrinsic and contextual priority that Adam assigned to *springiness*. This low intrinsic priority was evident in his constant questioning of this e-prim. (How would springs or chemical bonds "know" their natural length?) This is partly why the comparison of the table to a compression spring and the molecular model of solids made no sense to him although he clearly understood the relational transfer that the analogy suggested. In contrast, Jacob and Sara who assigned high priority to *springiness* developed most aspects of the molecular model of solids by themselves, and fully appropriated it.

b. Contextual priority: (i) *Dynamic balance* was evoked only when the interviewees first recognized that forces were definitely involved in a context. Sometimes situations were framed in different ways. That is, interviewees paid attention to systematically different features of situations from one moment to the next, such as balancing forces or, alternatively, impenetrability. (ii) *Rigidity* in its essence is a very contextual e-prim. (iii) *Springiness* acquired contextual priority for Jacob in the case of the table after he cued *some changes can be invisible*.

c. Negotiating priority: The data suggested several occasions where comparisons between explanatory primitives led to a change in individual reasoning. (i) Jacob replaced *supporting* with *dynamic balance* to explain why the table exerts a force. *Dynamic balance* gained further priority over *supporting* due to its ability to explain *supporting*, and *supporting* disappeared from his considerations. Similar considerations of *hierarchy* led Adam to prioritize *supporting* over *things cannot be in the same place at the same time*. (ii) Sometimes competing explanatory primitives entailed very different trajectories of reasoning for the interviewees who balanced priorities differently. For instance, after the book on the hand was presented and before the book was placed on the compression spring, both Jacob and Adam considered *dynamic balance* and *force requires agency* as applicable. However, the explanatory primitives competed since *dynamic balance* requires a force, while neither could identify an agent for this force in the case of the table. Based on Adam's strong intrinsic priority for *force requires agency*, he dismissed the relevance of *dynamic balance*. Jacob, on the other hand, was more committed to *dynamic balance* as soon as he recognized that forces were involved. So he chose to consistently pursue *dynamic balance*, even though he was aware that he could not identify an agent. The above are instances where the negotiation of priorities involved resolving alternative framing. (iii) *Springiness* acquired high intrinsic priority in Jacob's reasoning since it resolved an acknowledged conflict between two competing highly prioritized e-prims: *force requires agency* and *dynamic balance*. *Springiness* suggested the agent that can apply the force in the case of the table, and thus supported Jacob's sense of consistency. (iv) Sara negotiated the contextual priority of springiness in the case of the table through the reclassification of the initially-seen-as inanimate springs as agents.

Altogether, these observations suggest that the negotiation of priorities includes at least the following processes: (1) reacting to explanatory hierarchy; (2) changes that achieve consistency; (3) changes that eliminate conflict; (4) changing priorities by broadening categories, or reclassifying objects into existing categories; (5) recontextualizing situations—that is, selecting from among different frames of interpretation, with consequences for priorities.

We posit that in addition to the nature of the analogy, the learner's prior knowledge of the target plays a significant and direct role in the acceptance or rejection of an instructional analogy. According to our model, the decision whether an element activated by the source can operate in the target depends on the explanatory primitives that the source evokes and their judged applicability to the target. E-prims are empirically determinable functional elements of an individual learner's knowledge system. They may be evoked by the anchor; however, their ultimate relevance is evaluated using knowledge about the target: Do these e-prims have high contextual priority in the target? The term *brittleness* thus describes a judgment by an individual learner that the activated explanatory primitives simply do not apply to the target; they have low contextual priority. In this view, bridging analogies can evoke explanatory primitives that were not evoked initially by the source. These new explanatory primitives may cause a shifting of priorities in small clusters of e-prims in the process of negotiating priorities. For instance in Jacob's case, *some changes may be invisible* facilitated the use of *springiness* in new situations, and *dynamic balance* was substituted as a "better" replacement for *supporting* since it explained it.

Our model describes the role of explanatory primitives in individuals' judgments of plausibility and applicability of an instructional analogical sequence and accounts for: (a) students' acceptance, rejection, or ambiguous attitudes toward various contentions and explanations based on their individual repertoire of explanatory primitives; (b) differences in students' reactions, which are traceable to the different priorities assigned to explanatory primitives; and (c) shifts in judgments based on cueing different explanatory primitives in different circumstances, or shifting the relative priority through the negotiation of priorities.

At a higher level, our data makes evident a distinct and rich fine-structure in students' differing paths of reasoning. We believe this is substantially beyond the level of detail provided in previous accounts of student process data concerning analogical reasoning (Brown & Clement, 1989; Cheng & Brown, 2010; Clement & Brown, 2008), and we elaborate this point in detail later in this section.

Caveats. We acknowledge that the empirical resolution of priority (low, medium, high) is somewhat low, and that the empirical disambiguation of intrinsic from contextual priorities is sometimes difficult. However, the reasons for the latter difficulty are comprehensible: students report net confidence and seldom directly report contextual effects. A clinical focus on particular e-prims, outside of the constraints of a specified instructional sequence, may be useful in this regard. The dynamics of negotiating priorities as we currently model it employs quite plausible (and empirically at least somewhat supported) mechanisms, such as if an e-prim explains another, it accrues additional priority. The analysis suggested five mechanisms. It is likely that more might be involved, and this could be a topic for future work. We think that eventually, computational models should be able to take this line further.

A methodological concern is the influence of the interviewer. There are certainly micro-interactional issues in the analysis that are worth future consideration. However, clinical interviews (or any other kind of interviews) are prominent in contemporary analyses of learning. So are analyses of student knowledge in the context of other interactions, for example, in classroom discussion. There are continuing analyses that show what kind of interactive influences exist, and also what kind of influences are not there, or which are easily detectable (e.g., diSessa, 2007). In this respect our work is within, but not beyond the current state of the art concerning taking explicitly the effects of social interaction into account. Furthermore, we feel that the prime interactional influence is evident in the analysis. We tracked student reactions—and differences in student reactions—in a more or less fixed sequence of interventions. Variations in sequencing were both noted and discussed, albeit briefly. The interviewer maintained a consistent "devil's advocate" position throughout, and there were many times that interviewees rejected ideas proposed by the interviewer, or they introduced ones that were not even hinted at by the interviewer. When interactional issues seemed strong (reluctance to display personal reactions), we removed one interviewee from the pool.

Relation to Other Theories

This section discusses the contribution of this study in relation to cognitive models of analogical reasoning, p-prim theory, and prior work on bridging analogies.

Relations to cognitive models of analogical reasoning. Our model explicates the process of the evaluation of analogical inference by modeling individuals' underlying knowledge systems. Existing theories of analogical reasoning acknowledge the importance of prior knowledge in the process of evaluation (e.g. Falkenhainer et al., 1986; Gentner & Colhoun, 2010); however they do not model it in detail. In fact, Gentner (2010) refers to the understanding of analogical causal inference as an open challenge. Moreover, as far as we know, prior cognitive studies of analogical reasoning (e.g., Gentner, 1983, 2010; Hofstadter, 1995; Holyoak et al., 2010; Holyoak & Thagard, 1995; Keane, 1996) have not been concerned with systematically matching elements and processes in the models with fine-grained empirical analysis of online complex thinking that we employed here, let alone attending to the differences between individuals in this process. We are unaware of attempts to track systematically and explain the kind of individual thinking that we observed in our data.

As we stated in the introduction, we believe that our model complements rather than contradicts existing theories of analogical reasoning. We saw some behaviors that are predicted by existing theories of analogical reasoning. In particular, we saw traces of mapping in the students' descriptions of similarities between source and target. Indeed, nothing in what we said could or was intended to rule out that mapping might provide an alignment of relational structure (Gentner & Markman, 1997) guiding further considerations. However, our analysis strongly suggests that whether the learner accepted the source and target as essentially similar in the ways the instructional analogy intended to portray depends not only on structural similarities (Gentner, 1983, 1989) and the learner's pragmatic goals (Holyoak & Thagard, 1989, 1995), but also on the learner's prior knowledge. Specifically, this acceptance depended on the explanatory primitives that were evoked by the source and their applicability, which was judged in the target domain based on the learner's prior knowledge of that domain. For instance, Adam saw the intended mapping between flexible things and the "rigid" table (objects, parts of objects, motions, etc.). He just did not believe some projected descriptions (e.g., the development of a springy force) applied to the table; they were excluded by his conviction that the table was, in fact, rigid. Colhoun and Gentner (2009) speculate that the process of causal evaluation may be "outsourced" to post-analogical processes. The evaluation of the activated e-prims and the negotiation of their priorities in the target domain might be an explication of this "outsourcing."

The next connection concerns the difficulty that the kind of bootstrapping we observed in understanding the target domain poses for pure structure mapping, even augmented by causal analysis. Although most of the analysis here dealt with judgments of plausibility and the applicability of the analogical explanation, we presented some data related to the construction of new knowledge in this instructional analogical sequence. These observations are more aligned with the "transfer in pieces" perspective (Wagner, 2006) as opposed to perspectives that regard transfer as transmission of a "whole" abstract knowledge structure across situations (e.g. Fuchs, Fuchs, Hamlett, & Appleton, 2002; Gentner, Loewenstein, & Thompson, 2003; Gick & Holyoak, 1980, 1983). The transfer in pieces perspective describes the process of transfer as "incremental growth, systematization, and organization of knowledge resources that only gradually extend the span of situations in which a concept is perceived as applicable" (Wagner, 2006, p. 10). Consider for instance Jacob's autonomously developed explanatory model for the way in which the springy table can actually exert the force, the process by which he developed most aspects of the molecular model of solids by himself during the guided tutoring sequence, and the manner in which he appropriated the model and used it to explain other phenomena. Jacob's explanations presented clear traces of the explanatory primitives that he evoked and considered in earlier phases of his reasoning. In addition, he expressed these e-prims differently in the context of the target domain than in the context of the source domain, and this process was not linear in the sense that Jacob integrated only a few previously cued e-prims into this construct and they were not integrated in the order in which they were cued. In our view the above bootstrapping process is a characteristic illustration of a transfer in pieces.

Relations to p-prim theory. E-prims and p-prims serve as explanatory "atoms" that have the same self-explanatory function—they account for one's comfort or surprise in certain situations, based on activation and priorities. P-prims are a highly specified category of e-prims, where the form (source, topic, and encoding) and local processing are restricted by p-prim theory. In contrast, the form of e-prims is more loosely defined to include other kinds of "explanatory atoms." E-prims encompass a larger class of knowledge elements whose explanatory function is the definitional inclusion criterion, but whose form—the particular source, topic, and encoding—is not; p-prims are a subcategory of the category of e-prims.

Let us examine the ontological status of some of the particular e-prims identified in the analysis. Six out of the 11 e-prims that were reported were not previously identified p-prims: (1) *things are as they appear*, (2) *effort entails force*, (3) *force requires agency*, (4) *some changes may be invisible*, (5) *gravity pulls things downwards*, and (6) *two things cannot be in the same place at the same time*. Some of these e-prims might turn out to be p-prims after further empirical evaluation, which is beyond the scope of the present study. A good example of an e-prim that may also be a p-prim is the e-prim *two things cannot be in the same place at the same time*. However, some of the e-prims that our interviewees used are categorically not p-prims, since they have evident characteristics that are not compatible with the form of p-prims (e.g. verbal encoding). Note that both p-prims and non-p-prim e-prims were empirically necessary for our analyses. On the basis of the analytical observation that not all e-prims are p-prims, and the empirical observation that non-p-prim e-prims as well as p-prims had to be included in the analysis to account fully for the focal phenomenon, which is individual differences in response to instructional analogies, the concept of e-prims needed to be introduced. We do not see the fact that there are e-prims that are not p-prims as the principal contribution of this paper. The main contribution, in our view, is that e-prims (with p-prims as a subset) can account for important phenomena concerning individual differences in judging analogical instructional treatments. There is no precedent for this in prior work on p-prims. In fact, no prior p-prim work considered learning from analogies.

Past p-prim work made a substantial empirical contribution to the present work in making available a relatively large collection of reasonably thoroughly documented p-prims (a-fortiori, e-prims) available for use in analysis. E-prims and their priorities are the primary theoretical constructs that are involved in this new work, but these constructs make no significant revision to the theory of p-prims per se. The concepts of primitives in general and priorities in particular from p-prim theory inspired the present theoretical model. Our analysis adds and attempts to elaborate how individual priorities change. We termed this process negotiating priorities, and we identified and formulated several principles that govern this negotiation. While prior work (diSessa, 1993) implicated shifting of priorities as a primary locus of learning, no process data was analyzed to highlight, exemplify and demonstrate the mechanisms involved.

Relations to prior work on bridging analogies. Brown and Clement have taken several approaches and introduced several theoretical constructs and ways of construing the unfolding of bridging analogies. Perhaps the closest approach to ours centers on the idea that the source provides "elements" (Brown & Clement, 1989; Clement & Brown, 2008) to the target. In the case of successful analogies, elements from the source can be seen to "operate in the target," which provides a "candidate for reality." In the case of persistently unsuccessful analogies, the source is said to be "brittle"; elements from the source cannot be seen to operate in the target.

Our differences with this approach are both theoretical and methodological. Their "elements" are not unambiguously identified as knowledge elements. Instead, what is said about "elements" is consistent with their being literal features of the source, such as springs, or any unrestricted relational features. Our equivalent to "element," the e-prim, is an unambiguously persistent knowledge element encoded in students' minds. It is not, technically, "in the source," but is evoked by it and then is judged as relevant to the target. E-prims have an explicit definition involving a well-developed idea (from the theory of p-prims): namely, the property of being self-explanatory. Accumulating an explicit list of e-prims, unlike "elements," is a high-priority pursuit in our work. In addition, e-prims have two important and empirically tractable parameters, intrinsic and contextual priorities, which have no equivalent in "elements." These properties are absolutely crucial to our tracking and explaining the main focus, here, individual differences (although Brown and Clement mark individual differences, they do not try systematically to track and explain them with explicitly comparative data). With regard to the success or failure of analogies, our theory also relies on the construct of priorities. If an analogy does not evoke a sufficiently high priority e-prim or set of e-prims in the target, which has more priority than other spontaneously activated e-prims, then it fails. The constructs of "candidate for reality," and "brittle" are thus unnecessary, if not tautological, since they seem to us mere descriptions of behavior.

Methodologically, we used a near-exhaustive analysis of moment-by-moment thinking across a full corpus of several students thinking about the same instructional sequence. There is no equivalent in Brown and Clement's work. In addition, we announce and adhere to an explicit set of accountability principles for interpreting data to determine e-prims and their priorities. Again, there is no equivalent in Brown and Clement's work. Finally, the theoretical connection to p-prims in our work brings in what proved to be immensely helpful, an extensive prior empirical body of work identifying many particular e-prims (qua p-prims). "Elements," in Brown and Clement's work makes little or no connection to this corpus, and has developed no equivalent one.

In the introduction, we mentioned Brown's taxonomy of knowledge types (Brown, 1993; Cheng & Brown, 2010)—including verbal-symbolic knowledge, conscious models, implicit models, and core intuitions—which might in some respects constitute a competitor to the model we offer here. In the following, we review and make a few additional points to elaborate on commonalities and differences with our own point of view.

First, in general, the broader Knowledge in Pieces framework on which our work is based seeks to identify and specify different forms of knowledge. This is an important commonality. On the other hand, we do not find the descriptions of those particular four types clear and evidently cogent. Aesthetics for "good theory" are rather varied, so we only mark, rather than seek to defend extensively, that judgment. What is much more evident, however, is that we provided explicit and defensible principles of empirical interpretation for our determinations of e-prims, which is lacking (and we feel is needed) in this facet of Brown's work. In addition, our work uses a much larger corpus of data to analyze systematically, compared to the empirical work in Cheng and Brown (2010).

Even if Brown's taxonomy turns out to be clear and empirically well-defended, it is not evident that it can track details in individual differences. The locus of individual differences, which we have identified in the repertoire and priorities of e-prims is missing. More evidently, Brown does not seek to do that job.

With respect to the dynamics of change, Brown offers the idea of "refocusing core intuitions." Again, we find the idea vague, and not clearly applicable in detail to process data. We can also report that preliminary attempts on our part to sort our data into Brown's categories proved challenging. That judgment aside, our competitor, negotiating priorities, is also in our judgment not yet mature or complete. But we did track changes in our process data and, in doing so, identified several distinct types of negotiating priorities. We feel this puts our model ahead in terms of precision and empirical accountability.

In closing this discussion, however, we underscore the obvious fact that Brown and Clement's work has been enormously influential in defining our own—from defining a paradigm of instruction, to identifying core phenomena to explain. Even where we have taken distinct directions, their work has provided landmarks from which to navigate.

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Footnotes

1 DiSessa (1993) makes an abbreviated argument that a related idea *unsupported things fall* is indeed a p-prim.

2. The e-prim's features should be among those attended to or derivable from what is attended to.

3. One can imagine the suspension of belief in the *force requires agency* e-prim *in this context* to be a step in a process of either generally reducing the priority of that e-prim or changing the meaning of "agency." We discuss these considerations later in light of the other students' reasoning. Note, also, that Jacob's state here is interestingly complex. One could gloss it as "gradually increasing the confidence he has in *dynamic balance*." However, in this case, this potential increase in confidence is entangled specifically with the current state and future trajectory of another e-prim, *force requires agency*. In general, we believe our model provides important and empirically tractible refinements over more global assessments and descriptions, such as "becoming comfortable with an idea," or "gaining more skill in using it."

4. Self-motion is an early index to "alive" in Piagetian studies (Piaget & Garcia, 1974), and we interpret Sara's reference to it to be a common consideration for the more sophisticated notion of agency.

5. In a later discussion, we substantiate via the existing literature the fact that size or weight can convey strength or capacity.

6. Adam heard the term kinetic energy in chemistry class.

7. Methodological note: This is a case where intrinsic and contextual priorities seem definitively distinct. In cases where Adam thinks that springiness applies, he uses it fluently. In the case of rigid objects, he doubts it applies. The contextual range of the rigidity p-prim explains the boundary.

8. Adam's belief that a deformation is a requisite condition for energy transfer is correct according to Newtonian physics.

9. This is a particularly clear case methodologically where the properties of a knowledge element important to the acceptance of a model can be seen in contexts other than the model, before the model has been presented.