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ABSTRACT

Relations are derived to determine the spin and relative intrinsic parity of a fermion, such as the cascade particle, produced in association with a spinless boson by the reaction of a second spinless particle with a polarized proton target. Inequalities are derived, for general spin parity assignments, between P , the polarization of the final fermions, when produced from an unpolarized target; P' , the dependence of the rate of the reaction upon target polarization; and D , the correlation between initial and final polarization. Use is made of Bohr's theorem of R-invariance.

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The development of polarized targets makes possible the unambiguous determination of both the spin and parity of hyperons that decay via parity non-conserving interactions. The method used is an extension of the ideas put forward by Lee and Yang,¹ and by Teutsch, Okubo, and Sudarshan.²

We consider reactions of the type

followed by

where a , b , and c are spinless bosons, p and B are spin- $\frac{1}{2}$ baryons, and Y is a hyperon of unknown spin. Examples of this type of reaction are

and

We choose the axis of spin quantization in the direction

$\hat{n} = \mathbf{p}_a \times \mathbf{p}_b / |\mathbf{p}_a \times \mathbf{p}_b|$ normal to the plane of reaction (1a). The

particle Y has spin J, as yet to be determined. Let the relative population of the level with spin-projection m be I_m . I_m is a function of the energy and angle of reaction (1a), as well as the spin direction of the target proton. Let I_m^0 be the value of I_m at a particular energy and angle (or averaged over a suitable range) when the target proton is unpolarized. The I_m and the I_m^0 , obey the following relations

$$I_m \geq 0 \quad (4)$$

$$\sum_{m=-J}^{m=+J} I_m = 1 \quad (5)$$

The two measurements that are required to make the spin-parity determination are (1) P, the polarization of the Y in (1a) when the target proton is unpolarized; and (2) P', the dependence of the rate of (1a) on the polarization of the proton, without observing any of the final spin states. Both these quantities are to be averaged over the same energy and angular range. More specifically

$$P' = \frac{\text{rate}(m_p = +\frac{1}{2}) - \text{rate}(m_p = -\frac{1}{2})}{\text{rate}(m_p = +\frac{1}{2}) + \text{rate}(m_p = -\frac{1}{2})} \quad (6)$$

where m_p is the projection of the proton spin along the $\hat{n}$ direction. P' is measured using a polarized target.

$$P = \frac{1}{J} \sum m I_m \quad (7)$$

Lee and Yang² have shown that P can be determined using reaction (1b). The expectation value of the cosine of the angle between the momentum of particle B , in the rest frame of the Y , and the direction $\hat{n}$, is given by

$$\langle \hat{n} \cdot \hat{B} \rangle = \frac{\alpha}{2J(J+1)} \sum_m I_m = \frac{\alpha}{2(J+1)} P. \quad (8)$$

α is the usual parameter that shows the amount of parity mixing in reaction (1a). The longitudinal polarization of B , when Y is unpolarized (or averaged over all polarizations of Y), is equal to α , independent of J . Therefore, P can be measured, when $\alpha \neq 0$, according to equation (8)--for any hypothesized J --in the usual type of bubble chamber experiment.

Let the relative intrinsic parities of the particles in (1a) be $\pi_a, \pi_b, \pi_p, \pi_Y$. And let

$$II = \pi_Y \pi_b / \pi_a \pi_p. \quad (9)$$

We will now derive the following inequality

$$|(2JP - \omega P')| \leq 2J-1 \quad (10)$$

where $\omega = II (-1)^{J-\frac{1}{2}}$

$$\omega = +1 \quad \text{for} \quad J^{II} = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \text{ etc.}$$

$$\omega = -1 \quad \text{for} \quad J^{II} = \frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \text{ etc.}$$

The proof of the inequality (10) is based on a theorem by A. Bohr.³ Let all the particles in reaction (1a) be described in terms of plane waves with spin quantized along the $\hat{n}$ direction. These wave functions are eigenstates of the operator R , combined inversion and 180° rotation about the n -axis, which commutes with the strong-interaction Hamiltonian. This leads to the selection rule

$$\pi_a \pi_p e^{i\pi m_p} = \pi_b \pi_Y e^{i\pi m}$$

where $m_p = \pm \frac{1}{2}$ is the spin-projection eigenvalue of the target proton. This means that the population I_m of the Y state with spin projection m is contributed entirely by one of the proton spin states and not the other. As a consequence of this, we may write

$$\begin{aligned} \frac{\text{rate}(m_p \rightarrow m)}{\text{rate}(+\frac{1}{2} \rightarrow \text{all } m) + \text{rate}(-\frac{1}{2} \rightarrow \text{all } m)} &= I_m^0 \quad \text{if} \quad II(-1)^{m-m_p} = +1 \\ &= 0 \quad \text{if} \quad II(-1)^{m-m_p} = -1 \end{aligned} \quad (11)$$

from which we confirm that $I_m = I_m^0$ for an unpolarized target. From this and equation (6) we deduce

$$P' = \sum_{-J}^J II(-1)^{m-\frac{1}{2}} I_m^0. \quad (12)$$

For example, in the case $J = \frac{3}{2}$, $II = +1$, we have

$$P' = -(I_{3/2}^{\circ} - I_{-3/2}^{\circ}) + (I_{\frac{1}{2}}^{\circ} - I_{-\frac{1}{2}}^{\circ})$$

whereas from equation (7)

$$P = (I_{3/2}^{\circ} - I_{-3/2}^{\circ}) + \frac{1}{3}(I_{\frac{1}{2}}^{\circ} - I_{-\frac{1}{2}}^{\circ}) .$$

It is now simple to form

$$\begin{aligned} 2JP - \omega P' &= (2J-1)(I_J^{\circ} - I_{-J}^{\circ} + I_{J-1}^{\circ} - I_{-J+1}^{\circ}) \\ &+ (2J-5)(I_{J-2}^{\circ} - I_{-J+2}^{\circ} + I_{J-3}^{\circ} - I_{-J+3}^{\circ}) \\ &+ (2J-9)(\quad) + \dots \quad (13) \end{aligned}$$

The right-hand side of equation (13) cannot have magnitude greater than $(2J-1)$, because of the constraints (4) and (5) on the I_m . This completes the proof.

When $J = \frac{1}{2}$, $2J-1 = 0$, and inequality (10) reduces to the equation

$$P' = II P ,$$

a result that was first derived by Bilenky.⁴

When $J = 3/2$, we have

$$|P' + 3P| \leq 2 \quad \text{for} \quad II = +1$$

$$|P' - 3P| \leq 2 \quad \text{for} \quad II = -1 .$$

The results for spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ are illustrated together in Figure 1. The ordinate is P' , and the abscissa is $\langle n \cdot B \rangle / \alpha = P / (2J+2)$, which is the experimentally measured quantity.

Table I gives the inequalities applying to some of the low spin-parity assignments.

These inequalities are stronger than those derived previously^{1,2} and include them as a limiting case (either for $J \rightarrow \infty$ or for $P' = \pm 1$). Further limits can be deduced by measuring the polarization of Y particles produced from polarized targets. If we define

$$D = \frac{(\text{rate} \times P)(m_p = +\frac{1}{2}) - (\text{rate} \times P)(m_p = -\frac{1}{2})}{\text{rate}(m_p = +\frac{1}{2}) + \text{rate}(m_p = -\frac{1}{2})} \quad (14)$$

where P is defined in equation (7), and can be measured according to equation (8). Arguments similar to those which led to equation (12) show that

$$D = \frac{1}{J} \sum_{m=-J}^J II(-1)^{m-\frac{1}{2}} m I_m^0 \quad (15)$$

When $J = \frac{1}{2}$, $D = II$, i.e., $D = \pm 1$ depending on the relative parity. Note that this is true even if P and P' are zero.

When $J = \frac{3}{2}$, we can solve equations (5), (7), (12) and (15) simultaneously for the four I_m^0 , then apply (4) to each of them to obtain the inequalities, when $II = +1$,

$$|3P - P'| \leq 1 - 3D \quad (16)$$

$$|P + P'| \leq 1 + D \quad (17)$$

TABLE I: Relations between P and P'

	II = +1	II = -1
$J = \frac{1}{2}$	$P' = P$	$P' = -P$
$J = \frac{3}{2}$	$ P' + 3P \leq 2$	$ P' - 3P \leq 2$
$J = \frac{5}{2}$	$ P' - 5P \leq 4$	$ P' + 5P \leq 4$
$J = \frac{7}{2}$	$ P' + 7P \leq 6$	$ P' - 7P \leq 6$
	etc.	

from which one can derive the weaker inequalities,

$|P| \leq \frac{1}{2}(1 - D)$, $-1 \leq D \leq +\frac{1}{3}$, and equation (10). When $II = -1$, the appropriate relations are obtained from (16) and (17) with the signs of P' and D reversed.

The generalizations of (16) and (17) to arbitrary J and II are

$$|JP + (J-1)\omega P'| \leq J-1 + J\omega D \quad (18)$$

and

$$|P - \omega P'| \leq 1 - \omega D. \quad (19)$$

From (18) and (19) one can derive weaker inequalities. By adding (19) to twice (18) one derives, for $J \neq \frac{1}{2}$,

$$|P + \omega P'| \leq 1 + \omega D. \quad (20)$$

This means that, independent of ω , for all spin parity assignments except $J = \frac{1}{2}$,

$$|P \pm P'| \leq 1 \pm D. \quad (21)$$

Since (21) is always true, no new information is yielded by use of it.

The stronger inequality is obtained by the choice of sign conforming to relation (19).

Other inequalities implied by (18) and (19) are

$$\omega D + \frac{J-1}{J} \leq 0 \quad (22)$$

since the right-hand side of (18) must be positive.

$$(2J-1)|P| \leq 2(J-1) + \omega D \quad (23)$$

is obtained by adding (18) to (J-1)times (19).

Relation (10) can be derived by adding (19) to J times (18).

Inequalities involving D are presented for the lowest J values in Table II.

Recently Ademollo and Gatto⁵ and Peshkin⁶ have treated the problem of spin-parity determination in reactions similar to (1a, b), and making use of R invariance. These treatments do not consider the case of polarized targets. M. K. Gaillard⁷ has treated the application of polarized targets to the determination of spins and parities of particle resonances.

The author wishes to thank Professor O. Chamberlain for suggesting this problem, and Dr. H. P. Stapp for reading the manuscript.

TABLE II: Relations involving P, P', and D

	II = +1	II = -1
$J = \frac{1}{2}$	D = +1	D = -1
$J = \frac{3}{2}$	$ 3P - P' \leq 1 - 3D$ $-1 \leq D \leq +\frac{1}{3}$ $ P \leq \frac{1}{2} (1 - D)$	$ 3P + P' \leq 1 + 3D$ $-\frac{1}{3} \leq D \leq +1$ $ P \leq \frac{1}{2} (1 + D)$
$J = \frac{5}{2}$	$ 5P + 3P' \leq 3 + 5D$ $-\frac{3}{5} \leq D \leq +1$ $ P \leq \frac{1}{4} (3 + D)$	$ 5P - 3P' \leq 3 - 5D$ $-1 \leq D \leq +\frac{3}{5}$ $ P \leq \frac{1}{4} (3 - D)$
$J = \frac{7}{2}$	$ 7P - 5P' \leq 5 - 7D$ $-1 \leq D \leq +\frac{5}{7}$ $ P \leq \frac{1}{6} (5 - D)$	$ 7P + 5P' \leq 5 + 7D$ $-\frac{5}{7} \leq D \leq +1$ $ P \leq \frac{1}{6} (5 + D)$

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FIGURE CAPTIONS

Fig. 1. Relations between P and P' for spin $\frac{1}{2}$ and $\frac{3}{2}$. The abscissa is the experimentally measured

$$\langle \hat{n} \cdot \hat{B} \rangle / \alpha = P / (2J + 2).$$

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Fig. 1.