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Abstract

Whether different formats of numbers are represented by one or more systems across development is a subject of long-standing interest in the field of numerical cognition, with seemingly contradictory results. Here we examined numerical comparison to test a developmental theory that can reconcile these discrepancies. In Experiment 1, we found numerical understanding progresses through three continuous phases of association between numerical symbols and approximate sense of numerosity. In the youngest age group (prefluent phase), comparing numerals were slower than comparing dot arrays, but became similar (fluent phase) then faster (overlearning phase) with age. Because this developmental change occurred in the order of numeric range 1-9, followed by 10-99 and 100-999, multiple phases co-existed during childhood. Furthermore, results from Experiment 2 indicated that comparing different formats of numbers was affected by ratio even at the highest levels of proficiency, suggesting that the approximate number system is never fully replaced.

Keywords: approximate number system; symbolic numerals; magnitude comparison

Introduction

Numeracy is an important predictor of arithmetic skill and mathematical achievements (Holloway & Ansari, 2009; Libertus, Odic, Feigenson, & Halberda, 2016). Due to the importance of numeracy, a large body of research has focused on how children learn the meanings of symbolic numerals and how representations of symbolic numerals change with age. However, discrepancies in the literature still exist, which mainly center on whether the ability to represent nonsymbolic numerosities (e.g., number of dots) provide the foundation for processing symbolic numerals (e.g., Arabic numerals) across development (Carey & Barner, 2019; Dehaene, 2011).

The purpose of the present study is to reconcile contradictory explanations from previous studies and provide more comprehensive developmental framework on how the relation between symbolic and nonsymbolic number representations changes over time.

A Common System for Symbolic Numerals and Nonsymbolic Numerosities

There is considerable evidence that nonsymbolic numerosities are represented in an inexact way by the approximate number system (ANS) (Dehaene, 2011). Because the ANS produces noisy representations of numbers, discriminability of any two numerosities depends on their ratio. Previous studies using nonsymbolic magnitude comparison task have revealed that ratio effect is present regardless of age, suggesting that the numerosities are processed by the ANS across development (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012). Furthermore, the researchers reported that the ANS is present in newborn infants and non-human species, suggesting that the ANS is an evolutionarily ancient system (Cantlon & Brannon, 2006). Considering the ANS is an intuitive and innate system, researchers have proposed that the ANS provides the foundation of learning symbolic numerals. A large body of studies has shown that comparing Arabic numerals is affected by ratio, which is the hallmark of the ANS (Fazio, Bailey, Thompson, & Siegler, 2014; Moyer & Landauer, 1967; Sekuler & Mierkiewicz, 1977).

Separate Systems for Symbolic and Nonsymbolic Numbers

On the other hand, alternative approach proposes that different formats of numbers are processed by separate systems. Here, we illustrate two different theories from separate system approach.

Separate Number Systems for Children One of the alternative theories proposes that learning the meanings of number words does not involve associating numerical symbols to the ANS, because the ANS cannot provide exactness of symbolic numerals (Carey & Barner, 2019; Carey, Shusterman, Haward, & Distefano, 2017). When children were trained to map the word "ten" with a card that contained 10 objects, they failed to choose a card with 10 objects when 10 were contrasted with different number of objects (Carey et al., 2017). In addition, the effect of ratio was not observable. For example, discriminating 10 from 30 was as bad as discriminating 10 from 15. This result contradicts the ANS mapping hypothesis which predicts that children would be able to recognize 10 objects against different number of objects after training or be influenced by ratios of number sets.

Separate Number Systems for Adults Studies on how adults represent symbolic numerals also provide alternative explanations to the ANS mapping theory (Lyons, Ansari, & Beilock, 2012; Marinova, Sasanguie, & Reynvoet, 2018). Among these alternative theories, the symbolic estrangement hypothesis suggests that repeated use of numerical symbols would lead symbols to be estranged from the ANS and represented in exact manner. To test this theory, Lyons et al. (2012)

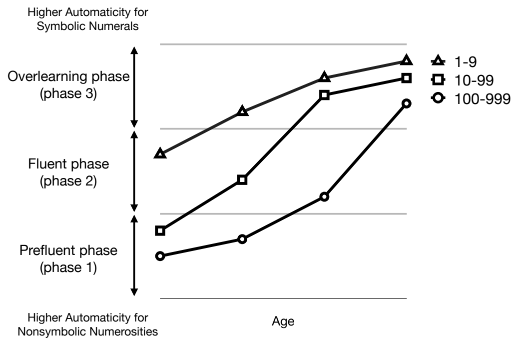

Figure 1: Hypothetical developmental trajectories

asked adults to compare different formats of numbers. The result showed that comparing a dot array with an Arabic numeral took longer than comparing two dot arrays. Worse performance in mixed format comparison task has been thought to be caused by the switch cost between separate systems of symbolic and nonsymbolic numbers.

The Current Studies

Both the common system approach and separate systems approach provide evidence that support corresponding theories. Nonetheless, previous studies have limitations in explaining the reasons for contradicting findings.

We suggest that these seemingly discrepant explanations between theories is not because any of the previous studies provide invalid explanations for local results. Instead, previous studies explain only part of the entire process of acquiring number concepts. Not only do most studies focused on limited range of developmental trajectory, but they did not examine the variability of symbolic numeral proficiency within same age group depending on numeric range.

We propose numerical understanding progresses through three phases of associative learning across development. By logical necessity, continuous progression of acquiring the concept of symbolic numerals entails three qualitative *phases*: processing nonsymbolic numerosities more fluently than symbolic numerals, processing numerosities and numerals with equal fluency, and processing numerals more fluently than numerosities. For convenience, we labeled these phases as prefluent, fluent, and overlearning phases. These phases are useful qualitative description of a continuous quantitative progression, much like 'infant', 'toddler', and 'child' are useful descriptions for periods in a continuous aging process. These phases also map onto the conventional issues of debate.

In prefluent phase (phase 1), approximate sense of discrete number is present, but symbols are meaningless. We predicted that difference in symbolic and nonsymbolic representations would be large in prefluent phase. Fluent phase (phase 2) is when children learn numerical symbols by associating symbols with the quantities they represent. Therefore, acquiring the meanings of symbolic numerals is the process of integrating meaningless symbols into the common system (ANS) that processes nonsymbolic numerosity. In this phase, processing symbolic numerals will be done as automatically as nonsymbolic numerosities. In overlearning phase (phase 3), processing symbolic numerals would become more automatic than processing nonsymbolic numerosities due to the frequent use of symbolic numerals. Previous studies have shown that older children and adults, who have enough experience with symbolic numerals, can compare symbolic numerals faster and more accurately than dot arrays (Fazio et al., 2014; Marinova et al., 2018). Nevertheless, it does not mean different formats of numbers are processed by separate systems. Rather, symbolic numerals are still represented as continuous quantity. For example, comparing Arabic numerals in overlearning phase would be similar to comparing length of lines which can be done quickly and accurately. In this phase, difference between symbolic and nonsymbolic representations would increase again.

Based on these three phases, we also hypothesized that multiple phases would overlap depending on the numeric range. Since learning symbolic numerals require mapping between symbols and the ANS, reaching fluent phase and overlearning phase would be affected by the amount of experience children have with numerals. Considering frequency of symbolic numerals decrease as numerical magnitude increase (Dehaene & Mehler, 1992), association between symbols and the ANS will be done from smaller to larger numerals. Therefore, we predicted that children can be at overlearning phase for smaller quantities while at prefluent phase for larger quantities (Figure 1).

The co-existence and competition among ways of thinking about number is also supported by previous studies on number line estimation (Siegler & Opfer, 2003; Siegler & Booth, 2004; Siegler, Thompson, & Opfer, 2009). For example, Siegler and Opfer (2003) presented 0 to 1000 number line and asked participants to place numbers on the number line. The result showed that second and fourth graders represented numbers inaccurately by overestimating small numbers while underestimating larger numbers, indicating logarithmic representation of numbers. However, sixth graders and adults accurately estimated given numbers, showing linear representation. In Siegler and Booth (2004) study, on the other hand, 0 to 100 number line was presented. The result indicated that kindergarteners and first graders showed logarithmic representation of numbers, whereas second graders illustrated linear representation. Taken together, second graders represented numbers in different manners depending on the number range.

The present study was designed to include wide range of age and numbers. Testing three-phase hypothesis required us to examine developmental change across age while testing multiple-phase hypothesis required us to examine variability within age group. Therefore, we tested 4- to 12-year-olds as well as adults to observe three phases. In addition, we included number ranges 1-9, 10-99, and 100-999 to test multiple phase hypothesis. We used magnitude comparison tasks to measure gradual change from prefluent phase to overlearning phase by taking account the noisiness of representations. More importantly, we measured reaction time to directly compare automaticity of processing symbolic and nonsymbolic numbers.

Experiment 1

In Experiment 1, we tested two hypotheses. The first hypothesis was that numerical understanding progresses through three continuous phases – from processing numerosity more automatically than numerals, to processing numerosity and numerals equally automatically, to processing numerals more automatically than numerosities. The second hypothesis was that this understanding of numerals is range-dependent, such that the children with same age could be in as much as all three phases at the same time, with the probability of being in an advanced phase being negatively associated with the magnitude of the numbers. To examine the relative difference between symbolic and nonsymbolic representations of numbers, we used symbolic and nonsymbolic magnitude comparison tasks.

Method

Participants We tested 186 4-year-olds to 12-year-olds and 66 adults in the study. Adults were undergraduate students. Children were tested at a museum or at schools.

Materials and Procedures The experiment was conducted on a 13-inch laptop, using a MATLAB program. Participants were given two comparison tasks. Their tasks were to choose the larger of two quantities. Two stimuli were presented on each side of the screen simultaneously for 2500ms after fixation. Presentation time 2500ms was chosen to be long enough for young children to view 3-digit symbolic numerals but short enough to prevent them from counting dot arrays. Participants could make a response while stimuli were presented. Participants were asked to press 'Q' if they thought the left stimulus was larger and 'P' if they thought the right stimulus was larger. Participants were asked to answer as accurately and as quickly as possible. Feedback was not provided. For children, neutral responses was provided. Reaction times and accuracy were recorded.

Two comparison tasks included dot-dot and numeral-numeral conditions. In dot-dot condition, two sets of dots were presented. In numeral-numeral condition, two Arabic numerals were presented. Twenty-eight trials each for quantities 1-9, 10-99, and 100-999 were presented, yielding 84 trials for each condition. Thus, each participant completed 168 trials. The stimuli of quantities 1-9 consisted of every possible pair ranged from 2 to 9. To equate the ratios between different numeric range, the stimuli of quantities 10-99 were created by multiplying 7 to the stimuli of quantities 1-9. The stimuli of quantities 100-999 were created by multiplying 63 to the stimuli of quantities 1-9. The ratio of large to small numbers ranged from 1.125 to 4.5.

In the dot-dot condition, yellow dots were presented on the left while blue dots were presented on the right side of the screen. Instead of equating total surface areas of dot arrays or sizes of dots, we manipulated left to right ratio of non-numerical features of each pair to check whether participants responded based on number of dots, or on non-numerical features (DeWind, Adams, Platt, & Brannon, 2015; Starr, DeWind, & Brannon, 2017). Ratio of either field area or individual dot size was randomly chosen from three different ratios (2:1, 1:1, and 1:2). Therefore, there were nine possible combinations of non-numerical stimulus features. Field area indicated the area in which the dots were drawn. In the numeral-numeral condition, the physical size of Arabic numerals was equal across trials.

Results

Responses that took longer than 3 SD of each age group's reaction times (RT) and RTs shorter than 200ms were excluded. Excluding outliers yielded 99.5% of the adults' data and 98.5% of the children's data. For analysis, we fitted a linear multilevel regression model (Gelman & Hill, 2006) with random intercepts by subject using lme4 package in R.

As a preliminary analysis, we first analyzed whether participants' responses in dot-dot condition were based on numerical values or on non-numeric features. Analysis was done after excluding 9 adults whose non-numeric feature data were not available. Their data were included in further analysis. Participants were more likely to solve the dot comparison task based on numerical values (RT: $b = -.09$, $SE = .01$, $p < .001$; accuracy: $b = 1.53$, $SE = .04$, $p < .001$) than dot size (RT: $b = -.02$, $SE = .01$, $p < .01$; accuracy: $b = .41$, $SE = .04$, $p < .001$) or field area (RT: $b = -.05$, $SE = .01$, $p < .001$; accuracy: $b = .29$, $SE = .02$, $p < .001$). This result illustrated that even though participants were influenced by non-numeric features of dot arrays, their responses were mainly based on the quantity of dots.

For the main analysis, we aimed to test three-phase hypothesis and multiple phase hypothesis by analyzing the RTs from correct responses and RTs from all responses. We examined the difference between dot-dot and numeral-numeral condition and how the difference changes over development. To calculate the difference between condition, we first calculated each participant's z-scores of RT in dot-dot and numeral-numeral condition. Then we subtracted z-score of numeral-numeral condition from z-score of dot-dot condition. This value – normalized RT – is given by the formula: (z-score of RT in dot-dot condition) – (z-score of RT in numeral-numeral condition). Negative values of normalized RT indicate that comparing dot arrays were faster than comparing Arabic numerals, whereas positive values indicate Arabic numerals were compared faster than comparing dot arrays. If normalized RT is 0, it indicates RT in dot-dot condition is equal to RT in numeral-numeral condition. Therefore, larger normalized RT means that performance in numeral-numeral condition was relatively better than in dot-dot condition.

We analyzed the effect of age (4 to 19 years) and numeric

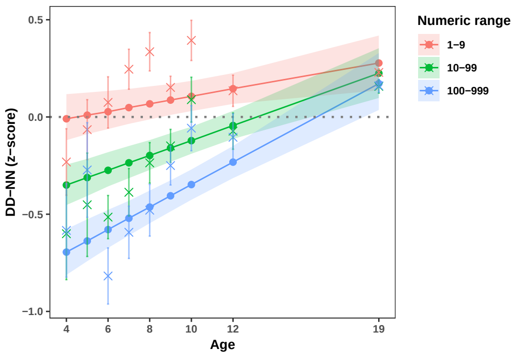

Figure 2: Normalized RT from correct trials by numeric range. Solid lines with 95% confidence intervals represent mean normalized scores generated from 1000 simulations from the fitted model. Raw means and their standard errors are presented with ×.

range (1-9, 10-99, and 100-999) on the normalized RT to investigate if relative automaticity of numerals changes with age and depends on numeric range. We used a mixed linear model including by-subject random intercept (Figure 2). The result showed that normalized RT was larger in older participants (correct trials: b = .30, SE = .05, $p < .001$; all trials: b = .30, SE = .05, $p < .001$), indicating that older participants were relatively better in numeral-numeral condition than in dot-dot condition compared to younger participants. In addition, normalized RT decreased as numeric range increased (correct: b = - .25, SE = .02, $p < .001$; all: b = - .26, SE = .02, $p < .001$). The effect of numeric range illustrates that developmental change occurred gradually from quantity 1-9 to 10-99 to 100-999. In addition, the interaction between age and numeric range was significant (correct: b = .12, SE = .02, $p < .001$; all: b = .14, SE = .02, $p < .001$), indicating the effect of numeric range decreased with age.

To characterize age groups by phase of development, we next examined from when normalized RT becomes larger than 0 for each numeric range (1-9, 10-99, 100-999). We used a bootstrap procedure to estimate 95% confidence interval (CI) using boot package in R and examined whether 95% CI included 0 or not. Normalized score in prefluent phase would be significantly smaller than 0 while normalized score significantly larger than 0 would indicate overlearning phase. Normalized score not significantly different from 0 would indicate fluent phase. For bootstrap procedure, we randomly sampled normalized score with replacement based on the experimental data in one simulation. Then, we calculated mean normalized score for each age group and for each numeric range based on the simulated data. Therefore, one simulation yielded total of 27 mean normalized scores. We repeated same procedure 10,000 times and estimated 95% CI (Tian, Braithwaite, & Siegler, 2020).

For numeric range 1-9, normalized RTs from 4-year-olds (correct trials: M = -0.23, 95% CI = [-0.55, 0.10]; all trials: M = -0.18, 95% CI = [-0.55, 0.20]) to 6-year-olds (correct: M = 0.07, 95% CI = [-0.15, 0.36]; all: M = 0.08, 95% CI = [-0.11, 0.29]) were not significantly different from 0. 7-year-olds (correct: M = 0.25, 95% CI = [0.07, 0.46]; all: M = 0.27, 95% CI = [0.09, 0.48]) and older participants were faster at comparing numerals than dot arrays. For numeric range 10-99, comparing numerals become similar with comparing dot arrays from 9-year-olds (correct: M = -0.15, 95% CI = [-0.32, 0.01]; all: M = -0.09, 95% CI = [-0.29, 0.09]) and continued to show similar automaticity until 12-year-olds (correct: M = -0.07, 95% CI = [-0.24, 0.10]; all: M = -0.02, 95% CI = [-0.19, 0.17]). Only adults were faster when comparing numerals than dot arrays (correct: M = 0.16, 95% CI = [0.09, 0.22]; all: M = 0.19, 95% CI = [0.13, 0.26]). For numeric range 100-999, normalized RTs for 10-year-olds (correct: M = -0.05, 95% CI = [-0.27, 0.17]; all: M = -0.00, 95% CI = [-0.23, 0.23]) to 12-year-olds (correct: M = -0.10, 95% CI = [-0.34, 0.11]; all: M = -0.05, 95% CI = [-0.27, 0.15]) were not significantly different from 0. Normalized RT of adults were larger than 0 (correct: M = 0.16, 95% CI = [0.10, 0.22]; all: M = 0.19, 95% CI = [0.13, 0.27]).

Consistent with our prediction, results from normalized RT illustrated that automaticity of processing symbolic numerals is acquired gradually in three phases. In addition, developmental change took place from small to large numeric range, leading to a coexistence of multiple phases during childhood. For numeric range 1-9, 4- to 6-year-olds were already at fluent phase, and 7-year-olds reached overlearning phase. For numeric range 10-99, fluent phase started to occur around 9 years of age. For numeric range 100-999, 10-year-olds reached fluent phase. Only adults were at overlearning phase for numeric range 10-99 and 100-999.

Experiment 2

Results from Experiment 1 indicated that symbolic numerals are gradually mapped to the ANS before being processed much more automatically than would be expected from a purely ANS representation. In Experiment 2, we tested if different formats of numbers are represented by the ANS by employing a mixed format comparison task (Lyons et al., 2012). We were specifically interested in the sensitivity to the ratio for mixed format comparison. Carey et al. (2017) illustrated that children who failed to map number words to nonsymbolic numerosities were not affected by ratio. In this sense, we predicted that participants being sensitive to ratio when solving mixed format comparison task would indicate symbolic and nonsymbolic numbers are represented approximately on a same continuum. In contrast, absence of ratio effect would show that symbolic numerals are processed in a different manner from dot arrays.

We investigated how the strength of ratio effect would change in accordance with three phases of numerical development that were proposed in Experiment 1. If development involved mapping symbolic numbers to the ANS, ratio effects would be expected to be stronger in older participants and weaker for large numbers. In contrast, if symbolic numbers become estranged from the ANS, ratio effects would become weaker with age and be stronger for larger numbers. In addition, if learning symbolic numerals go through separate path from the ANS in childhood, strength of the ratio effect would not differ depending on numeric range.

Method

Participants Participants in Experiment 2 were same as in Experiment 1.

Materials and Procedures Participants were given a dot-numeral comparison task. In the task, one of the stimuli was presented in a nonsymbolic format (an array of dots) while the other was presented in a symbolic format (an Arabic numeral). The dots were presented in blue. The side (left or right) containing the dot array was randomized. Except the format of numbers in comparison task, number pairs and procedures in Experiment 2 were identical with Experiment 1.

Results

Responses that took longer than 3 SD of each age group's reaction times and reaction times (RT) shorter than 200ms were excluded. Excluding outliers yielded 96.7% of the adults' data and 97.3% of the children's data. We fitted a linear multilevel regression model with by-subject random intercepts. We analyzed the effect of ratio (1.125 to 4.5), age (4 to 19 years), and numeric range (1-9, 10-99, and 100-999) by using mixed linear model for RT and using generalized mixed linear model with logit link for error rate.

Speed and accuracy improved with age (correct RT: $b = -.41$, $SE = .03$, $p < .001$; all RT: $b = -.37$, $SE = .03$, $p < .001$; error rate: $b = -.37$, $SE = .03$, $p < .001$). RT decreased (correct: $b = -.04$, $SE = .01$, $p < .001$; all: $b = -.05$, $SE = .01$, $p < .001$) and error rate increased ($b = .33$, $SE = .02$, $p < .001$) as the numeric range increased. In addition, RTs and error rates were affected by ratio of stimuli (correct RT: $b = -.06$, $SE = .01$, $p < .001$; all RT: $b = -.05$, $SE = .01$, $p < .001$; error rate: $b = -.35$, $SE = .02$, $p < .001$), thereby exhibiting the hallmark of the ANS. The interaction between ratio and age was not significant for RT (correct: $b = .01$, $SE = .01$, $p = .18$; all: $b = .01$, $SE = .01$, $p = .23$), but was significant for error rate ($b = -.17$, $SE = .02$, $p < .001$). This result indicated that the ratio effect on accuracy became stronger as age of participants increased. Interaction between ratio and numeric range was significant both for RT (correct: $b = .05$, $SE = .01$, $p < .001$; all: $b = .05$, $SE = .01$, $p < .001$) and error rate ($b = .05$, $SE = .02$, $p < .01$), showing ratio effect became weaker for larger numeric range.

To further examine the strength of association between symbolic numerals and nonsymbolic numerosities, we computed each individual's sensitivity to ratio based on the slope of the relation between ratio and error rate. We fitted logistic regression including probability to choose left stimulus as a dependent variable and $\log(n_{\text{left}}) - \log(n_{\text{right}})$ as a predictor. A

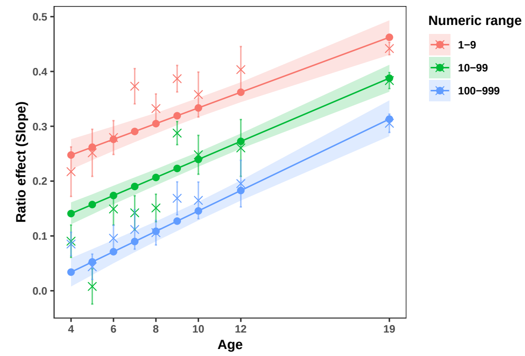

Figure 3: Ratio effects of error rate by age and numeric range. Solid lines with 95% confidence intervals represent mean ratio effects of error rate generated from 1000 simulations from the fitted model. Raw means and their standard errors are presented with ×.

larger value of slope means that participants were more sensitive to ratio between stimuli. We examined the effect of age (4 to 19 years) and numeric range (1-9, 10-99, 100-999) on the ratio effect using mixed linear model including by-subject random intercept (Figure 3).

The result showed that ratio effect of error rate became stronger as age increased ($b = .47$, $SE = .04$, $p < .001$) and became weaker as numeric range increased ($b = -.40$, $SE = .02$, $p < .001$). Interaction between age and numeric range was significant ($b = .05$, $SE = .02$, $p < .05$), indicating that developmental change was clearer in larger numeric range. Taken together, results from individual's ratio effect supported the hypothesis that symbolic numbers become more strongly associated with the ANS with age and experience.

General Discussion

The purpose of the present study was to propose a developmental framework that can reconcile seemingly incompatible results from previous studies. Even though the existing theories provide evidence for part of the developmental trajectory, they could not explain the cause of the contradictory findings. For example, in contrast with the results that seem to support separate number systems for children, numerous studies revealed that children could associate number words to the ANS when the task does not require children to understand exact meanings of the number words (Gunderson, Spaepen, & Levine, 2015; Odic, Le Corre, & Halberda, 2015; Wagner & Johnson, 2011). Similarly, even though separate systems approach proposes adults show switch cost between different formats of numbers (Lyons et al., 2012; Marinova et al., 2018), adults are influence by ratios when comparing two Arabic numerals (Moyer & Landauer, 1967). Nevertheless, the common system approach does not explain through which process symbolic numeral representation becomes more precise than nonsymbolic numerosity representation.

We predicted that results from previous studies can be explained by three phases of associative learning between numerical symbols and the ANS. We examined variability across age groups to test if acquisition of number concept develop in three phases. In addition, we examined variability within age group to test if multiple phases overlap depending on the numeric range.

Results from Experiment 1 showed that normalized RT was affected by numeric range, showing that mapping between symbolic numbers and the ANS occurs gradually from smaller to larger numbers, leading to the coexistence of multiple phases during development. Children could process symbolic numerals in a similar way with dot arrays from age 4 for numeric range 1-9, age 9 for numeric range 10-99, and age 10 for numeric range 100-999. This difference between numeric range indicates that even if children became faster in comparing numerals from small numeric range, they were still faster in comparing dot arrays from larger numeric range. Furthermore, results from Experiment 2 indicated that different formats of numbers are continued to be represented by a common system even after symbolic numerals become more automatically processed than nonsymbolic numerosities.

Multiple phases or representations of numerical concepts have also been investigated in other contexts. For example, studies on number line estimation illustrated that second graders showed linear representation for 0 to 100 range of number line while showing logarithmic representation for 0 to 1000 range of number line (Siegler & Booth, 2004; Siegler & Opfer, 2003). Furthermore, studies using Stroop task to measure automaticity of processing symbolic numerals also showed that children's performance is affected by numeric range (Mussolin & Noël, 2007). Mussolin and Noël (2007) asked second to fourth graders to compare physical size of two Arabic numerals when the relative difference of numeral magnitudes were either congruent or incongruent with physical size difference. The results showed that for second graders, interference of numeral magnitude was significant for 1-digit numerals, but the effect was weaker for 2-digit numerals smaller than 50 and not significant for 2-digit numerals larger than 50. In contrast, third and fourth graders were equally interfered by numeral magnitude regardless of numeric range.

In conclusion, our results indicate that the ANS provides a foundation for children's first learning of symbolic numerals and that symbolic numerals are continued to be represented by the ANS in adulthood. The results supported our hypotheses that numerical development go through three continuous phases. In addition, developmental process occurred not in the way of mastering the previous phase and then moving onto the next phase. Instead, multiple phases overlapped depending on numeric range.

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