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Unitary multichannel sidewise dispersion calculation of the nucleon anomalous magnetic moments*

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In sidewise dispersion calculations of the anomalous magnetic moments of the nucleon, the intermediate πN strong states involved are the S_{11} and P_{11}, both of which are highly inelastic at low energies. These inelastic effects have not previously been included in a systematic manner consistent with unitarity in order to ensure a real absorptive part. We present a multichannel ND^{-1} formalism for the strong scattering and photoproduction which gives a unitary result for the anomalous moments. The strong form factor is given simply in terms of D^{-1}. Explicit calculations demonstrate that contributions to the dispersion integrals far beyond $W \sim 1500$ MeV will be necessary in determining μ.

I. INTRODUCTION

The sidewise dispersion relation was introduced by Bincer[1] to calculate the anomalous magnetic moment of the nucleon. The sidewise form factors $F_2(W)$ describe an off-mass-shell nucleon of mass squared W^2 going to γN. The anomalous magnetic moment μ is $F_2(m)$. Bincer considered only the contribution of the πN intermediate state to the absorptive part, Fig. 1(d). Subsequent calculations[2-4] have shown the importance of energies up to 1.5 GeV in the πN intermediate state. Since the relevant $I=\frac{1}{2}$, $J=\frac{1}{2}$ πN partial waves, S_{11} and P_{11}, are highly inelastic, it is important to consider the inelastic intermediate states. These states are related by unitarity and must be treated in a consistent manner in order to obtain a real absorptive part, which has not previously been done. Note that this reduces to a trivial problem if the amplitudes can be approximated by a Breit-Wigner form. However, this is clearly not the case for these partial waves at the energies in question. In this paper we present a general, consistent calculation of the absorptive contribution using the unitary multichannel ND^{-1} formalism that has been previously applied to low-energy hadron scattering[5] and to photoproduction.[6]

In doing so, we must also calculate the sidewise form factor $K_{\pi NN}(W)$ for the off-shell nucleon $N(W) \to \pi + N$.[1] The result is remarkably direct in the ND^{-1} formalism. In addition, as shown by Suura and Simmons,[7] the ratio g_A/g_V can be obtained from this calculation.

Thus we are concerned with four processes in which multichannel unitarity must be used. The same intermediate strong channels occur in each as shown in Fig. 1: (a) the strong scattering amplitude, (b) the strong nucleon form factor, (c) photoproduction, and (d) the nucleon anomalous magnetic moment. The ND^{-1} description of (a) allows a simple straightforward calculation of (b), (c), and (d). In principle, the calculational procedure would be as follows: (i) A multichannel ND^{-1} phenomenological fit is made to the $I=\frac{1}{2}$, $J=\frac{1}{2}$ πN partial-wave amplitude (a). (ii) A calculation is made of the photoproduction Born terms, which using fit (i) yields the photoproduction amplitudes (c). (iii) Fit (i) immediately gives form factor (b), and then finally (b) plus (c) yields $\mathrm{Im}F_2(W)$ (d).

We make use of our formalism together with previous ND^{-1} fits to the strong πN scattering[5] and photoproduction[6] to calculate the anomalous moments. We find that the contributions to the dispersion integrals up to $W \sim 1500$ MeV do not give good values for μ. [This agrees with previous calculations of μ (Refs. 2–4) which were done in a less systematic way.] We stress the importance of contributions from the inelastic channels and the need for ND^{-1} solutions to πN scattering and photoproduction at higher energies.

II. MULTICHANNEL ND^{-1} FORMALISM AND SIDEWISE VERTICES K

In general we treat two-body channels such as πN and ηN, but we also include the three-body channel $\pi\pi N$ by treating the two pions as a Breit-Wigner ϵ ($J^P=0^+$) resonance. (Other two-body or quasi-two-body channels can be included as desired.) The ND^{-1} formalism[5] treats the partial waves of given angular momentum and isospin by writing the multichannel symmetric scattering matrix

$$T=(1/2i)(S-1)=\rho^{1/2}t\rho^{1/2} , \tag{1}$$

where ρ is a diagonal matrix of kinematic factors which ensures the correct threshold behavior and acceptable asymptotic behavior. Unitarity is then [Fig. 1(a)]

$$\mathrm{Im}\, t = t^\dagger \rho t\,. \tag{2}$$

This may be solved by writing

$$t = ND^{-1}\,, \tag{3}$$

where D has only the unitarity cut $(m+m_\pi) < W < \infty$ and $-\infty < W < -(m+m_\pi)$, and N does not have this cut. Denoting the contribution of the other singularities to t by $B(W)$ (Born term), the solution is

$$N(W) = B(W) + \frac{1}{\pi}\left\{\int_{m+m_\pi}^{\infty} + \int_{-\infty}^{-m-m_\pi}\right\}\frac{dW'}{W'-W} \times \left[B(W') - \frac{W-W_0}{W'-W_0}\,B(W)\right] \times \rho(W')N(W')\,, \tag{4}$$

$$D(W) = 1 - \frac{W-W_0}{\pi}\left\{\int_{m+m_\pi}^{\infty} + \int_{-\infty}^{-m-m_\pi}\right\}\frac{dW'}{W'-W-i\epsilon} \times \frac{\rho(W')N(W')}{W'-W_0}\,, \tag{5}$$

where the solution is independent of the normalization point W_0. A symmetric input matrix B will ensure that t is symmetric as demanded by time-reversal invariance for the partial-wave amplitude. The P_{11} and S_{11} waves can be simultaneously determined by using the MacDowell relation[8]

$$f_{l+}(-W) = -f_{(l+1)-}(W). \tag{6}$$

In actual calculations, the contributions to B are approximated by a series of poles:

$$B = \sum_r \frac{1}{W-W_r}\, G_r\,, \tag{7}$$

where G_r are $n\times n$ real symmetric matrices for n channels. The subtraction point is taken at the nucleon mass $W_0 = m$ where the matrix G_0 is given by $(G_0)_{ij} = -g_{B_i NN}\, g_{B_j NN}/4\pi$ and the kinematic factors are chosen so that $\rho_i^{1/2}(G_0)_{ij}\rho_j^{1/2}/(W-m)$

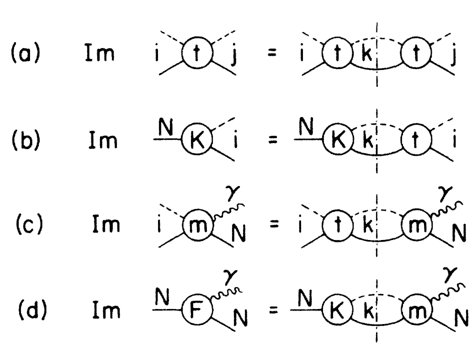

FIG. 1. Multichannel unitarity relations for (a) the strong scattering amplitude t, (b) the strong sidewise nucleon form factor $K(W)$, (c) the photoproduction multipoles m, and (d) the sidewise nucleon anomalous moment form factors $F_2(W)$.

equals the s-channel nucleon-pole terms. The subscript i denotes the ith channel which is also labeled by the boson $B_i = \pi, \eta$, or ϵ. Note that the calculation of the anomalous moment has assumed that the nucleon is an elementary particle and thus the nucleon must be included as a CDD (Castillejo-Dalitz-Dyson) pole term in the $B(W)$ and not as a dynamical zero in the determinant of D.

Denoting E_B as the energy of the nucleon in the c.m. system of the BN channel, and q_B the c.m. momentum, the kinematic factors are

$$\begin{aligned} \rho_1 &= \rho_\pi(\pm W) \\ &= 3q_\pi\,\frac{\pm E_\pi - m}{2W}\,, \\ \rho_2 &= \rho_\eta(\pm W) \\ &= q_\eta\,\frac{\pm E_\eta - m}{2W}\,, \\ \rho_3 &= \rho_\epsilon(\pm W) \\ &= \frac{1}{\pi}\int_{2m_\pi}^{W-m} dm_\epsilon\, f_{\mathrm{B.W.}}(m_\epsilon)\bar{\rho}_\epsilon(\pm W)\,, \end{aligned} \tag{8}$$

where m is the nucleon mass,

$$\begin{aligned} \bar{\rho}_\epsilon(\pm W) &= \bar{q}_\epsilon\,\frac{m \pm E_{m_\epsilon}}{2W}\,, \\ \bar{q}_\epsilon &= (m_\epsilon^2 - 4m_\pi^2)^{1/2}\,, \\ f_{\mathrm{B.W.}}(m_\epsilon) &= \frac{\Gamma/2}{(m_\epsilon - m_0)^2 + (\Gamma/2)^2}\,, \end{aligned} \tag{9}$$

and

$$\Gamma = \gamma \bar{q}_\epsilon\,.$$

In this formalism, the sidewise form factors for the strong vertices[1] $K_{B_i NN}(W)$ obey the unitarity relationship[9]

$$\mathrm{Im}K(W) = t^\dagger \rho K(W)\,, \tag{10}$$

where $K(W)$ is taken to be a column vector normalized to $K_{B_i NN}(m) = g_{B_i NN}$. Bincer[1] has shown that $K_{\pi NN}(W)$ has only unitarity cuts, or obeys the dispersion relation

$$K_{\pi NN}(W) = \frac{1}{\pi}\left\{\int_{m+m_\pi}^{\infty} + \int_{-\infty}^{-m-m_\pi}\right\}\frac{dW'}{W'-W-i\epsilon} \times \mathrm{Im}K_{\pi NN}(W')\,. \tag{11}$$

We assume this also for the other $K_{B_i NN}(W)$.

Equations (10) and (11) can be solved if we note that Eq. (2) with $t = ND^{-1}$ becomes, after using $t = \tilde{t}$,

$$D^{-1} - D^{-1*} = 2iD^{-1*}\rho N D^{-1}\,. \tag{12}$$

Taking the transpose,

$$\tilde{D}^{-1} - \tilde{D}^{-1*} = 2it\rho\tilde{D}^{-1*}\,. \tag{13}$$

Now by complex conjugation, we find that $\tilde{D}^{-1}$ satisfies the same form equation as K does:

$$\tilde{D}^{-1} - \tilde{D}^{-1*} = 2it^*\rho\tilde{D}^{-1} . \tag{14}$$

Therefore $K = \tilde{D}^{-1}\Psi$ satisfies Eqs. (10) and (11) for any column vector Ψ. Since $D(m) = I$, $\tilde{D}^{-1}(m) = I$, and we find the simple solution

$$K(W) = \tilde{D}^{-1}(W)K(m) , \tag{15}$$

where

$$K_{B_i NN}(m) = g_{B_i NN} . \tag{16}$$

We see that the form factors $K(W)$ in this formalism are obtained simultaneously with the partial-wave phase-shift fits. This is an improvement since unlike the Bincer solution, elastic unitarity does not have to be assumed for the highly inelastic P_{11} and S_{11} partial waves. To appreciate the simplicity of our result (15), compare it with the lengthy numerical calculations of coupled form factors in a two-channel problem done by Kreuzer and Kamal.[10]

Suura and Simmons[7] have shown that the ratio g_A/g_V is obtainable from this form factor as

$$g_A/g_V = K_{\pi NN}(m)/K_{\pi NN}(-m) . \tag{17}$$

III. PHOTOPRODUCTION AMPLITUDES AND THE ANOMALOUS MAGNETIC MOMENTS

We summarize here the application of the ND^{-1} formalism to photoproduction as presented by Ball, Campbell, and Shaw.[6]

The amplitudes with correct threshold and asymptotic behavior are defined in terms of the CGLN[11] amplitudes M_i by

$$m_i = q_i^{\,1/2}\rho_i^{\,-1/2}M_i\, q_\gamma^{\,-1} , \tag{18}$$

where the subscripts i on the m and M denote the amplitude for $\gamma N \to B_i N$. For the photoproduction Born terms B_i^γ we similarly have

$$b_i^\gamma = q_i^{\,1/2}\rho_i^{\,-1/2}B_i^\gamma q_\gamma^{\,-1} . \tag{19}$$

The unitarity relation for photoproduction is [Fig. 1(c)]

$$\mathrm{Im}m = t^\dagger\rho m = t^*\rho m \tag{20}$$

for the column vector m of the m_i. In particular the m_{1-} (M_{1-}) transition leads to the P_{11} partial wave, and the e_{0+} (E_{0+}) to the S_{11} wave.

The solution is given in terms of the ND^{-1} solution to the hadron scattering[6]:

$$m(W) = b^\gamma(W) + \frac{1}{\pi}\tilde{D}^{-1}\left\{\int_{m+m_\pi}^{\infty} + \int_{-\infty}^{-m-m_\pi}\right\} dW' \times \frac{N\rho\, b^\gamma(W')}{W' - W - i\epsilon} . \tag{21}$$

In practice, the Born terms for the πN channel are calculated from a few simple diagrams.[6] The Born terms for the other channels are parametrized by using the poles of Eq. (7) with real residues.

The isoscalar and isovector photon amplitudes are related to those of CGLN[11] by

$$\begin{aligned} M_\pi^S &= 3M_\pi^{(0)} , \\ M_\pi^V &= M_\pi^{(+)} + 2M_\pi^{(-)} = M_\pi^{(1)} , \\ M_0^S &= \sqrt{3}\, M_0^{(0)} , \\ M_0^V &= \frac{1}{\sqrt{3}} M_0^{(1)} , \end{aligned} \tag{22}$$

where the π, 0 subscripts indicate the photoproduction of pions and isoscalar bosons (η or ϵ), respectively. The unitarity relations Eq. (20) are obeyed by $m^{(0)}$ and $m^{(1)}$.

The calculation of absorptive contributions to the sidewise $F_2(W)$ form factors, Fig. 1(d), proceeds as in Ref. 1 (see footnote 9), and the result may be written in the matrix form

$$\begin{aligned} \frac{e}{2m}\mathrm{Im}F_2^V(+W) &= \frac{1}{\sqrt{3}} K^+(W)\rho(W)m_{1-}^{(1)}(W) , \\ \frac{e}{2m}\mathrm{Im}F_2^V(-W) &= \frac{1}{\sqrt{3}} K^+(-W)\rho(-W)e_{0+}^{(1)}(W) , \\ \frac{e}{2m}\mathrm{Im}F_2^S(+W) &= \sqrt{3}\, K^+(W)\rho(W)m_{1-}^{(0)}(W) , \\ \frac{e}{2m}\mathrm{Im}F_2^S(-W) &= \sqrt{3}\, K^+(-W)\rho(-W)e_{0+}^{(0)}(W) . \end{aligned} \tag{23}$$

The multipoles in Eq. (23) are related by the MacDowell symmetry

$$M_{1-}(W) = E_{0+}(-W) .$$

With the solution for $K(W)$, Eq. (15), Eqs. (23) are unitary; that is, $\mathrm{Im}F_2(W)$ is real, although it is a sum over complex numbers from each channel. This is the final consequence of having hadron scattering, hadron sidewise vertices, photoproduction, and electromagnetic sidewise vertices all tied together consistently by the unitary multichannel formalism.

To show that $\mathrm{Im}F_2$ is real we use (15) to write (23) as

$$\mathrm{Im}F_2 \propto K^+\rho m = \tilde{K}(m)D^{-1*}\rho m . \tag{24}$$

Now from the unitarity relation for m, Eq. (20),

$$\mathrm{Im}m = ND^{-1*}\rho m . \tag{25}$$

Multiplying Eq. (25) by $\tilde{K}(m)N^{-1}$ we see that $\mathrm{Im}F_2$ is real since $\mathrm{Im}m$ is real.

Bincer[1] proved that the $F_2(W)$ form factors obey a dispersion relation with only the unitarity cut. The isoscalar and isovector anomalous magnetic moments

$$F_2^{S,V}(m)=\tfrac{1}{2}(\mu^p \pm \mu^n) \tag{26}$$

can then be calculated from

$$F_2^{S,V}(m)=\frac{1}{\pi}\left\{\int_{m+m_\pi}^{\infty}+\int_{-\infty}^{-m-m_\pi}\right\}dW' \times \frac{\mathrm{Im}F_2^{S,V}(W')}{W'-m}. \tag{27}$$

IV. DISCUSSION

We have seen how unitarity ties together the four processes shown in Fig. 1. A phenomenological ND^{-1} description for the strong $J=\frac{1}{2}$, $I=\frac{1}{2}$ (S_{11} and P_{11}) πN partial-wave amplitudes t then allows, in principle, a parameter-free determination of the other three processes: strong form factor, photoproduction, and anomalous magnetic moment.

In Sec. II we explicitly considered three channels: πN, ηN, and ϵN. We would then simultaneously fit the phase-shift data for the S_{11} and P_{11} waves in a three-channel ND^{-1} form. This would, from (15) and (16), yield the strong vertex $K(W)$. Calculating the photoproduction Born terms b^γ we then find the multipoles m^γ using (21). Finally we use (23) to find the contribution to the anomalous magnetic moments integral (27) up to the energy of the ND^{-1} fit.

We have evaluated the anomalous moments in this formalism, using previously obtained[6] strong partial waves and photoproduction multipoles. In these calculations, it was assumed that the πN and ϵN channels saturate unitarity for the P_{11},[12] and that the πN and ηN do likewise for the S_{11}.[6] Each of these waves was fitted separately, since we neglected the $-W$ contribution in each case. $K(+W)$ is obtained from the P_{11} fit and is normalized in the usual way. $K(-W)$, however, is obtained from the S_{11} fit and cannot be normalized at the nucleon pole, which is too far away to be accurate. To calculate the anomalous moments, we have normalized $K(-W)$ and, at the same time, joined the two solutions by imposing, first, that $K(+0)=K(-0)$, and second, that[7] $K_{\pi NN}(m)/K_{\pi NN}(-m)=g_A/g_V=1.23$. Equations (23) are calculated, except that each of the matrices is now 2×2 instead. The P_{11} partial-wave[12] fit was good to W past 1600 MeV, but the fit for the S_{11} was good only up to the opening of the ηN channel at approximately 1500 MeV. The photoproduction multipoles were taken from the work of Ref. 6. There the Born terms b^γ for the inelastic channels were treated as parameters which were determined from fits to the photoproduction cross sections. We find that the contribution (27) to μ up to $W\sim 1500$ MeV is too small for the isovector moment and still increasing at a substantial rate. The isoscalar contribution was much smaller and less rapidly varying. This agrees with previous calculations of μ (Refs. 2–4), which were done in a less systematic way, however.

In conclusion, our calculations have demonstrated the importance of the inelastic channels to the anomalous moments dispersion relations. They also point out the need for accurate fits to the scattering and photoproduction data up to much higher energies in order to determine μ.

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