Title
ALPHA RADIOACTIVITY

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DISCLAIMER

COMPLETE SET OF CORRECTIONS FOR UCRL-3424

p. 4 - Table of Contents

After Section 52 there should be a subheading:

II. Alpha Decay in the Region of Spheroidal Nuclei.

p. 14

Add missing Δ in two places.

p. 63, addition to footnote 2

Strictly speaking, this special definition would apply to summation over one or a limited number of nucleons with the center of mass of the residual Z protons and A-Z neutrons defining the origin of the coordinate system. If the summation were really carried over all nucleons with the nuclear center of mass as the origin, no special definition of $e_{\lambda p}$ need be made; it will be the unit charge e for a proton and zero for a neutron.

p. 67

$$\left[\frac{(2\ell + 1)\ (2\ell' + 1)}{2\lambda + 1}\right] \quad \text{should read} \quad \left[\frac{(2\ell + 1)\ (2\ell' + 1)}{2\lambda + 1}\right]^{1/2} \quad \text{in eq. (29.9).}$$

p. 68

$$\left[\frac{4\pi(2I_f' + 1)\ B_e(\lambda)}{2\lambda + 1}\right] \text{should read} \left[\frac{4\pi(2I_f' + 1)\ B_e(\lambda)}{2\lambda + 1}\right]^{1/2} \quad \text{in eq. (29.13).}$$

p. 71

$$\frac{\hbar^2}{Mr} \text{ should read } \frac{\hbar^2}{Mr^2} \qquad \left|\frac{u_1}{u_0}\right| \text{ should read } \left|\frac{u_1}{u_0}\right|^2$$

Section 51

In sentence beginning "Nielsen's results .... change "all" to "one". Delete last sentence of the paragraph. (Corrigendum to Pryce'article affects these changes).

Corrections for UCRL-3424, cont'd.

Section 56

Note added in proof: Recent work by Kalkstein, Stephens, Caretto, and Hollander has shown positively that Pu^{237} does decay to the extent of about 2% to the 60-kev state of Np^{237}. They can set an upper limit on electron capture to the 33-kev state of < 0.5%.

Ref. (9) Add the reference Proc. Phys. Soc. (London) A65, 962 (1952).

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Table I P. 2, Column 4 -- second group of Cf^{249} should read 6.14 not 6.04.

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P. 8, Column 4 -- in Th^{228} should read 5.338 not 5.348.

P. 15, Column 8 -- in Bi^{214}, 17 should be enclosed by parentheses. (17)

Table VIII There should be a minus (-) sign at the heading of Column 5.

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Table XIII Bk^{249} hindrance factor should read 0.70, not 19.

Figure 30 Energy of 4th level is 75.76, not 75.26.

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I. Perlman and J. O. Rasmussen

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Fig. 30 Energy of 4th level is 75.76, not 75.26.

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OUTLINE

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INTRODUCTION

The study of alpha-radioactivity has generated many of the fundamental concepts of atomic and nuclear structure. Important discoveries came from both the effort to understand the mechanism of the alpha-emission process and from the observation of events produced by the high velocity particles. In this latter category we should recall that it was from the analysis of large-angle scattering of alpha-particles that Rutherford (1) conceived of the atomic nucleus as the center of mass and positive charge. He also made the fundamental deduction that the subatomic particles bearing the charge within the nucleus must exhibit strong short range attractive force, otherwise the nucleus could not exist. As further examples, the discoveries of nuclear transmutation by Rutherford[1] and of artificial radioactivity by I. Curie and F. Joliot[2] came about from the irradiation of light elements with alpha particles.

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[1] E. Rutherford, Phil. Mag. 37, 581 (1919).

[2] I. Curie and F. Joliot, Compt. rend. 198, 254, 259 (1934).

In this review we shall deal principally with those phenomena concerned more directly with the mechanism of alpha-emission. Nevertheless, some derivative topics will be mentioned or partially developed because they currently receive much support from the study of alpha-radioactivity. Among these topics is the identification and classification of nuclear levels in the heavy element region. Much information on this subject is obtained from the study of alpha-spectra and furthermore the explanation of the degree of population of the various available levels is one of the central objectives in the current development of alpha emission theory.

It will become clear that current directions of interest in alpha-radioactivity had to await the events starting about 1945 in which much new experimental information became available. Up to that time fewer than thirty alpha-emitters had been reported and almost all of these lay in the natural radioactive series. At present there are about 160 species known and many of these have been studied intensively. Before pursuing the discussion of these a brief historical sketch will be presented in the following paragraphs.

As early as 1900 Mme. Curie[3] suggested that alpha-rays were heavy projected particles following an experiment in which she showed that they differed from Röntgen rays in that they became degraded in energy in traversing matter. Further work by Strutt (1901)[4] and Crookes (1902)[5] on the ionization produced in gases served to reinforce this conclusion. Measurements of the deflection of alpha-particles in magnetic and electrostatic fields by Rutherford (1903)[6] demonstrated clearly that these rays were indeed heavy charged particles and also afforded a measure of their velocities. Further deflection experiments (particularly those of MacKenzie[7]) showed that a mixture of alpha-emitters produced a mixture of alpha-groups of different velocity each of which was homogeneous. Almost three decades were to pass before Rosenblum[8] demonstrated that these homogeneous alpha groups often had "fine structure".

---

[3]M. Curie, Compt. rend. 130, 76 (1900).

[4]R. J. Strutt, Phil. Trans. Roy. Soc. 196A, 507 (1901).

[5]W. Crookes, Proc. Roy. Soc. 69A, 413 (1902).

[6]E. Rutherford, Phys. Zeit. 4, 235 (1903); Phil. Mag. 5, 177 (1903).

[7]D. R. MacKenzie, Phil. Mag. 10, 538 (1905).

[8]S. Rosenblum, J. Phys. 1, 438 (1930).

Along with the early deflection work another type of measurement came into use for characterizing alpha-groups. Mme. Curie[3] had demonstrated that alpha-particles from a thin polonium source had a definite range and an extension of this work by Bragg[9] firmly brought in the concept of range as a distinguishing feature of each alpha-emitter.

---

[9]W. H. Bragg, Phil. Mag. 8, 719 (1904); 10, 600 (1905); 11, 617 (1906); W. H. Bragg and R. Kleeman, Phil. Mag. 8, 726 (1904); 10, 318 (1905).

The relation between the range (or velocity) of the alpha-group and the half-life for emission was noted at an early date by Rutherford[10]. However, it remained for Geiger and Nuttall[11] to examine this relationship systematically and to show that the logarithm of the decay constant changed linearly with the logarithm of the range. It was even possible to predict from the range the half-life of ionium which had not been measured and to deduce that there was an extremely short-lived alpha activity associated with RaC. This was later shown to be RaC'(Po^{214}). Among the other significant deductions made was that alpha-groups of low energy would require a very long time for emission and it became clear in this way why all the energies observed did not vary continuously to include low values.

[10]E. Rutherford, Phil. Mag. 13, 110 (1907).

[11]H. Geiger and J. M. Nuttall, Phil. Mag. 22, 613 (1911); 23, 439 (1912).

In 1911, Rutherford [see (1)] produced the concept of the atomic nucleus as the center of mass and positive charge in the atom. Soon thereafter, considerable speculation arose on the structure of the nucleus and the relation of the alpha emission process to it. Among the early interesting theories of the alpha-emission process was that of Lindemann[12] who was able to obtain an expression which followed the Geiger-Nuttall relation. By 1919, Rutherford[13] had succeeded in transmuting light elements by irradiating with alpha-particles and the concepts of the potential barrier and short range attractive forces in the nucleus were brought into sharper focus. With regard to heavy atoms such as uranium the puzzle yet remained as to how the alpha-particle could leave the nucleus even though particles of even higher energy could not get into the nucleus. The development of wave mechanics was the necessary prelude to the explanation.

[12]F. A. Lindemann, Phil. Mag. 30, 560 (1915).

[13]E. Rutherford, Phil. Mag. 37, 581 (1919).

In 1928, Gamow (2) and Condon and Gurney (3) independently showed that the wave nature of matter permits the alpha-particle to penetratethe region of potential energy higher than its kinetic energy. Excellent quantitative agreement was obtained for the dependence of the decay constant upon the decay energy, and the principal feature of the Geiger-Nuttall relation was now understood as was the anomoly of how an alpha particle could leave a heavy nucleus but not be free to enter.

Despite the remarkable success of the theory it was soon obvious that important details were not yet explained. In the beginning it was noted that an alpha emitter such as AcX did not lie on the appropriate Geiger-

Nuttall line and the new theory did not help in this regard. The situation took on an added degree of interest and complexity when Rosenblum[8] showed by magnetic analysis that the alpha particles from a particular substance had "fine structure". The demands on alpha decay theory now took on the added dimension of explaining the relative competition between several alpha groups of a particular emitter. Ensuing work showed that the energy dependence was not the only factor involved, indeed cases began to appear in which the alpha-group of highest intensity did not have the highest energy. This problem takes us to the present day. Although a coherent quantitative theory to explain all alpha-spectra is still incomplete a number of the factors influencing alpha-emission rates are recognized. Much of the material in this article is concerned with just these problems.

Part A of this article has to do with the alpha-energies and nuclear levels and emphasizes separately the total decay energies of the alpha-emitters (Sections 1-7), the spectrum of alpha-groups for each species (Sections 8-11), and patterns of energy levels in the heavy element region (Sections 12-15). The total disintegration energy is now a property which can be predicted with fair accuracy. It is, of course, simply related to mass differences between parent and product, and it is now clear that these mass differences for the most part vary in regular fashion.

The next two major parts (B and C) are concerned with alpha-decay rates. A division is made between the even-even emitters and the other types in order to develop the theory first for the simplest cases. The first group in Sections 16-18 in Part B have to do with the semi-empirical correlations of decay rates. These have been of great practical value as a guide to experimental work and point out most clearly what is demanded of the theory.

Following this are sections devoted to the theory: Sections 19-27 are on the one-body classical theory, Sections 28-33 are on the effects of non-central fields and Sections 34-37 consider multi-body aspects.

Part C takes up those aspects of alpha-emission which are different for nuclei having unpaired nucleons. The first several sections (40-42) treat the semi-empirical correlations and parallel those for the even-even type (16-18). The second group (Sections 43-48) treat the directions which the theoretical interpretations are taking.

Finally, Part D discusses the energy level diagram of several cases selected to illustrate different types encountered. Information obtained from the alpha-emission is in this way correlated with that obtained from other sources.

Unfortunately, a number of interesting topics related to the alpha-emission process are omitted in this review. One of these, the experimental techniques employed in studying nuclear spectra, has become too elaborate to be dealt with in limited space. Others, such as the interaction of alpha particles with matter, were eliminated with fewer misgivings because detailed accounts can be found elsewhere.

A. ALPHA ENERGIES AND NUCLEAR STATES

I. ALPHA DISINTEGRATION ENERGIES

1. Conditions for alpha instability. We are concerned here with denoting the specific conditions which must apply in order that particular nuclear species can be alpha emitters, and further, to see in general how these conditions are met in different regions of the system of nuclei. From straightforward energy considerations it is seen that any nuclear species will be alpha-unstable if the sum of the separation energies of two neutrons and two protons is less than the binding energy of the alpha particle, 28.3 Mev.

As discussed by Kohman[1] and others (4), the slope of the packing fraction curve gives a rough indication of where alpha instability may be expected, but many of the important features are not revealed because the packing fraction curve only reflects the gross structure of nucleon-binding energies. The closed shells have a dramatic effect upon alpha energies and appear to be dividing lines between regions of alpha stability and alpha instability. Actually, the sudden appearance of alpha radioactivity in crossing a closed shell means only that the lifetimes have suddenly become short enough to permit detection of the instability.

[1] T. P. Kohman, Phys. Rev. 76, 448 (1949).

From existing atomic mass data it is possible to determine (with varying degrees of accuracy) alpha energies in regions where the actual decay process is too slow to permit observation and also where it is energetically impossible. Such regions can then be made continuous with those in which accurate direct measurements are possible and in this way to display a profile of alpha energies throughout the system of nuclei. Figure 1 contains such a plot of alpha energy as a function of mass number (heavy continuous line). This line attempts to show alpha energies for nuclei which lie along the "line of stability" of the energy surface; that is, it pertains to the most beta-stable isobar for each mass number.

Alpha energies are by no means simply a function of the mass number any more than a packing fraction curve reflects the masses of all isobars. It is more illuminating to consider an "alpha-energy surface" in analogy to a mass or energy surface. It has been found convenient to correlate alpha energies by relating the isotopes of each element separately[2-6] and on the alpha-energy surface this would correspond to contours of constant atomic number. Several such curves derived from experimental energy measurements have been entered in Fig. 1. That for uranium shows monotonic variation of alpha energy with mass number while the polonium curve exhibits a sharp inversion. These properties will be returned to presently.

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[2] G. Fournier, Compt. rend. 184, 878 (1927); K. Fajans Radioelements and Isotopes (McGraw-Hill Book Company, Inc., New York, 1931). Chap. I.

[3] J. Schintlmeister, Wien. Chem. Ztg., 46, 106 (1943).

[4] A. Berthelot, J. phys. radium VIII, 3, 17 (1942).

[5] B. Karlik, Acta Phys. Austriaca, 2, 182 (1948).

[6] I. Perlman, A. Ghiorso, and G. T. Seaborg, Phys. Rev. 77, 26 (1950); Phys. Rev. 74, 1730 (1948).

Returning to the main curve of Fig. 1 we note that at mass number 90 the alpha energy is a negative 3 Mev. Below this it drops rapidly to a negative 8 Mev at mass number 70. Somewhere between mass numbers 130 and 140 energies become positive and it is likely that all nuclei above this mass number (and within the band of beta-stability) are unstable toward alpha emission.

The prominent peaks in the curve above mass number 140 and again above 210 are consequences of major closed shells. Other irregularities, no doubt, occur, but mass data are not sufficiently refined to permit placement of any more detail in the curve. The effect of closed shells on alpha energies expressed in terms of the more familiar behavior of neutron and proton binding energies is illustrated by the sequence of energy cycles of Fig. 2. Selection was made of the alpha emitters Po^{212}, Ra^{222}, U^{234}, and Cm^{246} which represent points at the respective mass numbers on the curve of Fig. 1. It is seen that the alpha energy of Po^{212} is determined by the binding energies of two neutrons beyond Pb^{208} and two protons beyond Pb^{210}. These are the 127th and 128th neutrons and the 83rd and 84th protons all of which have low binding energies because they are just past closed shells. The sum of these four binding energies when subtracted from 28.3 Mev (the binding energy of the alpha particle) gives the large alpha energy, 8.9 Mev. As the closed shells are left behind, the nucleon-binding energies increase rather sharply as shown for the cycles pertinent to the decay of Ra^{222} and of U^{234}. The alpha energies consequently decrease. Superimposed upon the factors causing an increase in nucleon-binding energies beyond a closed shell are those which produce a gradual decrease as one progresses to higher and higher atomic numbers. The minimum in alpha energy which results from these opposite effects occurs at U^{234} in this illustration, and the energies then increase again. This increase in alpha energy may be seen by comparing the energy cycles for Cm^{246} and U^{234} in Fig. 2, and the increase is illustrated by the final upturn of the main curve in Fig. 1.

The guide lines in Fig. 1 labeled with half-lives of 1 hr and 10^8 yr show the approximate alpha energies necessary to produce the indicated half lives for alpha decay. The sudden appearance of natural alpha radioactivity above mass number 210 is readily seen, as is the fact that a beta-stable uranium isotope (U^{238}) has a half life longer than 10^9 yr, and therefore has persisted through geological time. Isotopes of other elements with sufficiently long alpha-decay lifetimes turn out to be beta unstable. (The exception is Th^{232}.) In the rare-earth region, on the contrary, almost all of the beta-stable species have alpha lifetimes too long for detection. The exception is Sm^{147} which is an alpha emitter found in nature and Sm^{146} which is beta-stable but has an alpha decay half life which is too short to have persisted to the present day. A number of neutron-deficient isotopes of this region have been prepared artificially and these have <u>both</u> electron capture and alpha-decay lifetimes too short to have persisted through geological time.

Further discussion of these and other alpha emitters will be found in Section 3-7.

2. The energy surface. We shall now make a more detailed examination of alpha energies in the heavy-element region. For this purpose it is convenient to refer to a "mass surface" or "energy surface" of the heavy-element region. One manner of exhibiting the array of relative masses is shown in Fig. 3a. Here the neutron number is plotted against the mass decrement (Δ) which is simply the difference between the mass (M) and the mass number (A).[1] Contour lines are shown connecting points of constant mass number (A)

and constant atomic number (Z). The "line of stability" which follows the bottom of the valley is also shown. The actual points in Fig. 3a would not lie on a smooth surface as shown because of the differences in mass depending upon whether the nucleons are paired or not. The data were normalized to make a smooth surface by subtracting a pairing energy term from the odd-odd species and adding a term to those with both even neutrons and protons. Since the alpha-decay process does not change the nuclear type, the same alpha-decay energy would be derived from mass differences whether the actual masses were used or normalized values used such as shown in Fig. 3a.

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[1]The data upon which this plot is based were taken from Ref. (5).

In order to obtain an alpha energy from Fig. 3a the mass decrement of the alpha particle (3.87 milli-mass units) is subtracted from the difference between the mass decrement of the alpha emitter (Δ_i) and its decay products (Δ_f).

$$E_\alpha = \Delta_i - (\Delta_f + \Delta_\alpha) \tag{2.1}$$

The curves of Fig. 3b are used to illustrate this derivation of alpha energy. Here the curve labeled "Pa" contains mass decrements of a series of protactinium isotopes, (the (Δ_i) of equation (2.1), while that labeled "Ac" consists of the mass decrements of actinium isotopes ($_f$) to which have been added the mass decrement of the alpha particle ($_\alpha$). The alpha energies, shown by the vertical arrows, are simply the differences between points on the two curves related by alpha emission. It is seen that the alpha energy increases with decreasing neutron number. This can be shown to be an expected consequence of a "regular" energy surface as defined by any of the semi-empirical statistical treatments of nuclear masses.

Figure 4 is an idealized sketch of such an energy surface and entered upon it are the uranium family and an artificially-produced chain collateral to the uranium series. Since this is a normalized surface the beta-decay steps (along constant mass number) are not portrayed accurately. For example, it would appear that Em^{222} should decay to Ra^{222} by sliding to the center of the trough through two successive β^- transitions. However, the intermediate nucleus Fr^{222} really lies on another surface above that of the even-even nuclei and the decay of Em^{222} to Fr^{222} is energetically impossible by a small margin.

It should be noted that alpha decay cuts across the valley in the progression downhill. If a chain of alpha emitters is sufficiently long it will eventually progress high enough up the side of the wall to produce a β^--unstable nucleus which will then decay toward the line of stability. In the series shown, this happens at $Th^{234}(UX_1)$, Pb^{214}(RaB) and Pb^{210}(RaD).

The marked change in the lower part of the energy surface displays the effects of the closed shells at 82 protons and 126 neutrons. Some of the consequences will be discussed presently.

3. Systematics of alpha energies. A convenient manner to display the trends in alpha energies is shown in Fig. 5 in which the decay energy is plotted against the mass number with points of constant atomic number joined.[1] Over a large area of this chart it is seen that for each element there is a nearly perfect monotonic increase in alpha energy with successive decreases in mass number. This regularity is the reflection of a regular trough-like energy surface as already mentioned. Differences in slope or spacing of the lines in Fig. 5 can be interpreted in terms of departure from extreme regularity of the surface such as small changes in the curvature of the trough or of its slope. Individual irregularities must occur wherever a particular nucleon-binding energy does not follow the smooth trends.

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[1] For further discussion and references see General Ref. (6).

It is often possible to predict rather accurately the alpha energies of unknown species simply by interpolation or extrapolation of the curves of Fig. 5. The ability to make such estimations has been an important aid in preparing new species.

The dramatic inversion in the alpha-energy trend around mass number 212 is a consequence of the major closed shells in this region[2] (6). We can see what happens more precisely by following the curve for the polonium isotopes. From Po^{218} down to Po^{212} the curve follows the normal trend; then Po^{211} is seen to have considerably lower alpha energy than Po^{212}. This can readily be shown to be a reflection of the fact that the binding energy of the 126th neutron in lead (Pb^{207}-Pb^{208}) is greater than that of the 128th neutron in polonium (Po^{211}-Po^{212}). Similarly, since the binding energy of the 125th neutron in lead (Pb^{206}-Pb^{207}) is greater than that of the 127th neutron in polonium (Po^{210}-Po^{211}), the alpha energy of Po^{210} is lower than that of Po^{211}. After the neutron shell of 126 is well past and the neutron-binding energies change monotonically for both parents and daughters, the regular trend of increasing alpha energy with decreasing neutron number is resumed (see region between Po^{208} and lower isotopes).

The curve for bismuth is seen to parallel the polonium curve with a wide energy spacing between the two. This energy spacing is presumably a consequence of the break in binding energies at the 82-proton shell. The reappearance of alpha radioactivity in highly neutron-deficient isotopes of bismuth was the clue needed to establish the generality of this effect of crossing the region of 126 neutrons[3].

---

[2] B. Karlik, Acta Phys. Austriaca 2, 182 (1948).

[3] D. H. Templeton and I. Perlman, Phys. Rev. 73, 1211 (1948).

It will be noted that the alpha energies for the isotopes with 128 neutrons would be expected to become progressively greater for each higher proton number. Accordingly, half lives will be very short in this region so that preparation and identification of such nuclides would be difficult. However, more neutron-deficient isotopes should be more stable just as Po^{210} is more stable than Po^{212}. Such a region has been found and includes isotopes of emanation, francium and even radium, showing that the effect of 126 neutrons extends at least this high[4-6]. Undoubtedly these points shown on Fig. 5 join with those of higher neutron number by going through sharp peaks higher than those shown for polonium and astatine.

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[4] E. K. Hyde, A. Ghiorso, and G. T. Seaborg, Phys. Rev. 77, 765 (1950).

[5] F. F. Momyer, Jr., and E. K. Hyde, J. Inorg. Nucl. Chem. 1, 274 (1955).

[6] W. E. Burcham, Proc. Phys. Soc. A67, 555 (1954).

The question arises as to whether or not there is evidence for other closed shells or subshells on the basis of alpha-decay data. A situation similar to that at 126 neutrons, but considerably more subdued, seems to occur at neutron number 152.[7] The evidence came initially from the alpha-energy trend of californium isotopes as seen in Fig. 5 showing that the one with 154 neutrons has a higher energy than that with 152 neutrons. The data on einsteinium and fermium isotopes are not out of line with this concept and the lines are drawn in Fig. 5 according to expectations.

It will be noted in Fig. 5 that the energy increments from isotope to isotope along many of the curves are not very uniform. Aside from the marked inversion in trend in the region of lead and the probable small inversion in conjunction with 152 neutrons, it is seen that in some places the isotopes seem bunched and in others relatively spread out. It has been suggested that there are subshells at 92 protons and at 88 protons in explanation of some of these irregularities. Other inferences concerning alpha energies as related to possible subshells have been discussed by Broniewsky[8].

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[7] Ghiorso, Thompson, Higgins, Harvey, and Seaborg, Phys. Rev. 95, 293 (1954).

[8] A. Broniewski, Can. J. Phys. 29, 193 (1951).

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4. Decay energies from energy-balance cycles. The ability to predict decay energies is of inestimable value in the preparation of new isotopes in the heavy-element region. Through judicious use of these decay energies it is possible to predict lifetimes and consequently to design the experiment accordingly. The method of prediction of decay energies of wide application consists of constructing a self-consistent system of energy-balance cycles from alpha- and beta-energies which are either measured or estimated by interpolation or extrapolation of curves such as those shown in Fig. 5.

To illustrate this method of correlating decay energies, a segment of the decay cycle representation of 4n+1 type[1] nuclei is shown in Fig. 6. A few examples of the uses of these cycles will be mentioned. It is noted that by making use of three measured-decay energies, the alpha energy of the 6.8 day beta-emitter U^{237} is calculated to be 4.25 Mev. This is almost identical with the decay energy of U^{238} which has a half life of 4.5×10^9 yr and one would expect the partial alpha half life of U^{237} to be at least that long.

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[1] The type "4n+1" means that all mass numbers are divisible by 4 with remainder 1. All nuclei connected by α- and β-decay processes are of the same type.

Consequently, the alpha-branching of U^{237} would be only of the order of 10^{-12} so that this mode of decay would be most difficult to observe. On the contrary, the alpha-branching of Pu^{241} was similarly estimated to be about 10^{-5} which was within reach for measurement. The alpha energy as subsequently measured is shown in Fig. 6.

Another use of these cycles has to do with predictions of beta-stability. If one considers the possibility that Cm^{245} is a β^--emitter and that its preparation would therefore also produce Bk^{245}, the idea should be rejected because it is seen that Bk^{245} is unstable with respect to Cm^{245} by about 0.7 Mev. The estimated alpha energy of Cm^{245} which went into this calculation could not possibly be in error by an amount to reverse this conclusion and it was subsequently measured and shown to be close to the value listed.

An extension of these cycles to still higher elements gives a means of making predictions into a region where measurements have not been made, and these predictions serve as an important guide in designing the experiments. Other cycles can be devised to join different nuclear types through measured neutron-binding energies. With a single neutron-binding energy measurement joining two series, other neutron-binding energies can be calculated.

Energy balance cycles showing alpha and beta energies in the transuranium region will be found in the article by Hyde and Seaborg in Volume 39 (7). In the same article are also presented neutron-and proton-bindinding energies and isotopic masses. Cycles covering the entire heavy-element region will be found in Ref. (5).

5. Table of alpha energies. Table I in Sec. 11 consists of a listing of the alpha groups found in the heavy-element region. The *Q*-values for alpha decay are shown in Column 9 and these are the numerical data appearing in Fig. 5. The *Q*-value is, of course, the total disintegration energy and in most cases is obtained by adding the recoil energy of the nucleus to the energy for the alpha group leading to the ground state. In some cases the highest energy alpha group detected leads to an excited state in which case the de-excitation energy of this state must also be included. The determination of the total decay energy is uncertain in some instances and when this is the case appropriate notation is made.

6. Alpha emitters just below lead. The removal of neutrons from any element increases the potential toward alpha decay and this is the basis for the main trend in Fig. 5. Alpha-active isotopes of gold and mercury have been prepared by removing many neutrons from the stable isotopes.[1] In the case of gold, the stable isotope Au^{197} is estimated to have an alpha energy of only 1 to 2 Mev, while the isotope observed with an alpha energy of 5.1 Mev is believed to lie in the mass number range 183-187. As neutrons are removed, successive isotopes become more unstable toward orbital electron capture also, but since alpha-decay lifetimes are extremely sensitive to energy, this mode of decay should at some point become discernible.

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[1] J. O. Rasmussen, S. G. Thompson, and A. Ghiorso, Phys. Rev. 89, 33 (1953).

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7. Rare earth alpha emitters. Among the rare-earth elements we pass through a region where stable or slightly neutron-deficient nuclides can decay by alpha emission to the closed shell of 82 neutrons. Such a nuclide with 84 neutrons is Sm^{146} which is beta-stable but missing in nature because of its relatively short alpha half life (~5 x 10^7 yr)[1]. The alpha energies are summarized in Fig. 7 and although the curves are fragmentary as compared with those in the heavy-element region, (Fig. 5) the basic structure as related to the 82-neutron is unmistakable. The point assigned to Nd^{144} is of special interest because Nd^{144} is a component of natural neodymium.[2]

The other alpha emitter existing in nature is the well-known samarium isotope, Sm^{147}, which has three neutrons beyond the closed shell and would correspond to Po^{213} in the heavy element region. The other species shown in Fig. 7 are all electron-capture unstable and their energies relative to each other conform well with their positions relative to the 82-neutron shell.[3,4]

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[1] D. C. Dunlavey and G. T. Seaborg, Phys. Rev. 92, 206 (1953).

[2] E. C. Waldron, V. A. Schultz, and T. P. Kohman, Phys. Rev. 93, 254 (1954).

[3] J. O. Rasmussen, Ph.D. Thesis, University of California (1952). (UCRL 1473 rev.)

[4] J. O. Rasmussen, S. G. Thompson, and A. Ghiorso, Phys. Rev. 89, 33 (1953)

II. COMPLEX ALPHA SPECTRA

8. Types of alpha spectra. Just as for other decay processes, the appearance of multiple groups in the alpha-emission process may be considered to be the result of competition in populating available energy levels. It will be seen that the transition probability or partial half life is influenced by a number of factors, and among these is the sharp dependence of lifetime with decay energy. Because of this dependence it is not to be expected that transitions to high-lying levels (say 1 Mev) would be readily observed. If all other factors are equal an alpha transition to the ground state would be about 10^6 times faster than one leading to an excited state at 1 Mev above ground.

The highest energy group is not always the most prominent despite the sensitive energy dependence mentioned above. In many cases there are selection processes operating which are strong enough to delay higher-energy groups and therefore make the lower-energy groups prominent. One of the most extreme cases noted is that of the decay of Cm^{243} to Pu^{239} in which two groups differing in energy by 230 kev show an 80-fold greater abundance of the lower-energy group[1]. (See Table I). From energy dependence considerations alone one would expect a ratio of 15 in the opposite order which means that by some selective mechanism this higher-energy group is "hindered" by a factor of 1200 relative to the lower-energy group.

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[1] F. Asaro, S. G. Thompson, and I. Perlman, Phys. Rev. 92, 694 (1953).

The observed alpha spectra do fall into certain patterns which can be correlated with nuclear type and systematic trends in available energy levels. The energy level patterns in the heavy-element region are discussed in Sec. 12-15 while the transition probabilities to these levels lie in the province of "kinetics of alpha decay" and are discussed in later sections. Here we shall examine some typical alpha spectra and tabulate information on alpha groups obtained from all heavy-element alpha emitters.

9. Spectra of even-even emitters. The portion of the alpha spectrum observable by alpha-particle spectroscopy[1] is fairly simple for even-even species. The four spectra shown in Fig. 8 illustrate this point. Starting

[1] It will be seen that gamma-ray measurements can indicate the positions of levels populated by alpha decay with a sensitivity which is often several orders of magnitude greater than the direct observation of alpha groups.

with a transuranium isotope, Cm^{242}, it is seen that the group of highest energy is most abundant and that there is a nearby group (~40 kev difference) also in good abundance. Somewhat farther away (~140 kev) is another group in extremely low intensity. As far as is known all even-even alpha emitters from plutonium to the highest elements exhibit a spectrum which is virtually identical with this in the energy spacing of these three groups. The symbols 0+, 2+, 4+ indicate the spins and partities of the first three levels of Pu^{238} which are reached by the respective alpha groups. These levels have been recognized as members of a rotational band (see Sec 13). The existence of other alpha groups in the decay of Cm^{242} can be inferred from gamma-ray data (see Table I),but more highly refined techniques would be required to observe them directly because of their low intensities. The alpha-and gamma-spectra associated with Cm^{242} decay are discussed in detail in Sec.54.

Turning to the decay of Th^{228}, the spectrum is seen to be quite similar to that of Cm^{242} except that a state of spin 1 and odd parity has appeared between the 4+ and 2+ states. Such odd-parity states appear as low-lying levels only in a limited region and are discussed in Sec.13 and 53. Another difference between this spectrum and that of Cm^{242} is the wider spacing between the 0+, 2+, 4+ sequence. The trend noted is that of increase in rotational level spacing in progressing toward a closed shell configuration, in this case the approach to the region of 126 neutrons and 82 protons. The spacing increases to such an extent that in Em^{218} (decay of Ra^{222}) the 2+ state is seen at 325 kev above the ground state and there are no intervening states. Consequently the spectrum is quite simple at least in terms of alpha groups which can readily be observed. Finally, we have in Fig. 8, the single-line spectrum for Po^{218}. The 2+ state for Pb^{214}, like that for other lead isotopes with even neutron number, probably lies well above the ground state and is hence so slightly populated by Po^{218} alpha decay that only the ground state transition can be observed.

10. Spectra of odd-nucleon types. All of the low-lying energy levels[1] in even-even nuclei appear to be due to collective aspects of internal motion and it is, of course, the low-lying states which are most prominent in alpha spectra. The spectra leading to these states are fairly regular with the ground states most heavily populated. In nuclei with unpaired nucleons the low-lying states are, in general, more dense and consequently the alpha

spectra are more complex. For these nuclei, states due to collective modes are superimposed upon each of a number of possible particle states and the selection rules which determine the relative alpha populations of the different configurations are not yet explicitly codified. Nevertheless certain types of spectra are now recognized and it is these which will be illustrated here. The meaning of some selected samples is discussed in detail in Sec. 55 and 56.

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[1] By "low-lying" we refer to energies up to several hundred kilovolts which is still well below the energy required to unpair nucleons.

In Fig. 9 are shown four spectra of alpha emitters having an odd proton or an odd neutron. For both Cm^{243} and Am^{241} it is immediately seen that the alpha group in highest intensity is not the one of highest energy. In the case of Cm^{243} the most intense group populates an energy level of Pu^{239} which is 286 kev above the ground state. The transitions to the ground state and near-lying first excited state (indicated by vertical arrows in Fig. 9) have not yet been observed partly because of their low intensities and because they fall in the same position as the intense groups of Cm^{242} which were present with the Cm^{243}. The general observation to be made from this spectrum is that there is a "favored" alpha group analogous to the favored transition of an even-even nucleus but in the odd-nucleon case this group does not in general lead to the ground state. The low-intensity high-energy groups are interpreted as populating another rotational band based upon the ground-state configuration. Transitions to this band are highly hindered. The interpretation of these spectra will be found in Sec. 55 and 56.

The spectrum of Am^{241} also shows a favored transition to an excited state (60 kev above ground) and the rotational band based upon this state. The transitions to the ground state and its first rotational member are highly hindered as evidenced by the low intensities of the alpha groups.

The spectrum of U^{233} is shown to illustrate that sometimes the favored transition does lead to the ground state in which case the spectrum to the low-lying levels can look much like that of an even-even nucleus.

In the region just below thorium the density of low-lying levels appears to be very great because the alpha spectra become quite complex. The 12 alpha groups of Th^{227} are shown in Fig. 9 and thus far they have not been interpreted. The appearance of several groups in almost equal intensity probably means that there are at least that many particle states in this region. It is unlikely that the population of different members of the same rotational band can proceed with equal probability.

11. Table of alpha spectra. Table I presents a list of alpha groups associated with all heavy-element alpha emitters. Included are those groups which could not be seen directly but which are inferred from gamma-ray data. The gamma-ray intensities are used to deduce the relative abundances of the alpha groups. These data are of value in developing alpha-decay theory since it is primarily the relative intensities of competing groups which must be explained. It will also be noted that "long-range alpha groups" are listed. These are the high-energy groups which arise from the decay of excited states of alpha emitters and represent favorable competition between alpha emission and de-excitation of the levels by gamma emission. In all known cases, the levels are initially populated by β^--decay processes. The energies for long-range alpha groups are, of course, greater than the Q-values which represent transitions from ground state to ground state.

There are a considerable number of instances in which it is not certain that the highest energy group listed represents the ground-state transition, but the necessary information to determine total decay energy is lacking. Here, the Q-value is calculated from the highest energy group (usually the only group seen) and notation is made that there is insufficient evidence to consider this figure reliable. For the even-even alpha emitters in this category the assignment of a proper Q-value is a good deal more certain because it is known empirically that for these , the main alpha group invariably does lead to the ground state.

A statement should be made regarding the completeness of the information in Table I. To illustrate differences in available data for different species we may compare the spectra of Cm^{242} and Cm^{240}. For Cm^{242}, eight alpha groups are listed; for Cm^{240}, only a single group is listed. From what is known about the regularities of alpha spectra, it is fairly certain that the alpha spectra of these two substances are similar. In the case of

Cm^{242}, large amounts of the isotopically-pure substance can be made and consequently, the spectrum can be examined with high resolution and with great sensitivity. Close-lying groups may be resolved in instruments requiring intense sources and also very rare events may be sought, for example, in the range of one event in 10^5 to 10^7 disintegrations. As seen in Fig. 8 only three alpha groups have been observed directly; the other five groups of Table I are inferred from gamma-ray data which at present is obtained with greater sensitivity. In the case of Cm^{240}, it is much more difficult to prepare sizeable amounts and it is not possible to prepare it free of other curium isotopes. The only measurements made utilized an instrument of relatively low resolution and not even the expected intense group of energy some 45 kev lower than the ground-state transition has been resolved. As for gamma-ray analysis, the problem becomes quite difficult in view of the presence of other isotopes, particularly those which decay by orbital electron capture.

III. NUCLEAR STATES AND NUCLEAR MODELS

12. General remarks. Before one can hope to formulate any fundamental understanding of alpha decay kinetics, especially of odd nuclear types, it will be necessary to have considerable knowledge of the nuclear states involved. Thus, it is appropriate here to consider briefly the present state of this knowledge in the regions where alpha emission is prevalent.

One wishes first to know the energies of as many as possible of the low-lying nuclear states. Alpha-ray spectroscopy has played a key role in the establishment of the lowest levels (Sec 8-11). Next, one desires to classify states according to spin and parity. Such information is gained from experimental determination of ground-state spins by various methods and determination of the multipolarity of gamma transitions. Additional information on the details of nuclear states is given by nuclear magnetic and electric moments, by actual rates of nuclear transitions, and by energy level patterns.

For discussion of alpha-decay properties and for correlation of level energies and other nuclear properties with nuclear models, it is most logical to segregate heavy alpha emitters into three more-or-less distinct classes according to neutron number: a) a region comprising decays with parent and daughter having $N \leq 126$; b) a region with $126 < N \leq 138$; c) a region with

$N > 138$. Alpha decays (i.e., bismuth isotopes and 127-neutron isotones) crossing the closed shells of 82 protons or 126 neutrons constitute special cases.

For nuclei near the Pb^{208} doubly-closed configuration the nuclear models holding greatest promise are of the spherical well form deriving from the original Mayer-Jensen shell models[1,2](8). For nuclei more than one nucleon removed from Pb^{208} the modifications treating specific nucleon-nucleon interactions among the nucleons outside the closed shells are of greatest utility. (See the work of Pryce (9), Alburger and Pryce[3], and True[4]).

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1 M. Goeppert-Mayer, Phys. Rev. 75, 1969 (1949).

2 Haxel, Jensen, and Suess, Phys. Rev. 75, 1766 (1949).

3 D. E. Alburger and M. H. L. Pryce, Phys. Rev. 95, 1482 (1954).

4 W. W. True, Phys. Rev. 101, 1342 (1956).

For nuclei in the region with $N > 138$ it seems clear that a stabilized spheroidal deformation has set in, and the Bohr-Mottelson (10) "strong coupling" model has given many useful correlations of nuclear properties in this region.

On the low side of the border $N = 138$ the Goldhaber-Weneser[5] or Wilets-Jean[6] models offer hope, at least for even-even nuclei. The Goldhaber-Weneser model couples the motion of two or more individual nucleons with collective vibrational motion about a spherical equilibrium shape. The Wilets-Jean model treats the collective oscillations with stable total deformation but with little or no shape stability. For odd-nucleon types in the regions where the above two models apply the theory is less developed. Calculations along the lines of the intermediate coupling treatment of Bohr and Mottelson (10) and of Choudhury[7] might be of use in filling the gap.

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5 G. Goldhaber and J. Weneser, Phys. Rev. 98, 212 (1955).

6 L. Wilets and M. Jean, to be published. Preliminary report in Compt. rend 241, 1108 (1955).

7 D. C. Choudhury, Dan. Mat. Fys. Medd. 28, no. 4 (1954).

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13. Energies and other properties of nuclear states of even-even nuclei. The ground states of even-even nuclei are presumed all to have zero spin and even parity.

The first excited states of the heavy even-even nuclei (except Pb^{208})[1] have, wherever determined, spin two and even parity. Their energies have rather a regular dependence on mass number, being very high near Pb^{208} and descending to a broad flat region. Figs. 10 and 11 show plots of these energies *vs*. neutron number.

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[1] Pb^{208} has a first excited state of spin 3, odd parity.

The spectra of higher excited states of even-even nuclei become quite complicated in the regions near the closed shells. The discussion of these levels is not too essential in considering alpha decay since the higher excited levels of even-even nuclei in this region usually lie too high in energy to be detectably populated by alpha decay, although "long-range" alpha emission occurs from highly excited levels in Po^{212} and Po^{214} populated by beta decay of Bi^{212} and Bi^{214}. As the levels dip in energy further from the closed shell, a new regularity appears in the even spin-even parity excited states. That is, these energies approach a close agreement with a simple rotational energy formula.

$$E_I = \frac{\hbar^2}{2\mathfrak{J}} I(I+1) \qquad (13.1)$$

where I is the spin and $\mathfrak{J}$ is the moment of inertia.
Population by alpha decay of rotational band levels as high as the 8+ has been observed in Pu^{238} following alpha decay of Cm^{242}. (See Table I).

For a number of even-even nuclei in the heavy region, odd parity states of spin one have also been observed at moderate energies, lying lowest around 88 protons and 136 neutrons. The 1- states characteristically decay[2] by $E1$ gamma transitions to the 0+ and 2+ states, and the relative gamma intensities indicate that the component of total angular momentum along the nuclear symmetry axis (*K*-quantum number) is 0.

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[2] Stephens, Asaro, and Perlman, Phys. Rev. 100, 1543 (1955).

The 1- states are usually considered odd states of rotation belonging to the ground rotational band. In the usual form of the theory of spheroidally deformed nuclei, the nucleus is presumed to be symmetric with respect to rotation of π about a principal axis perpendicular to the cylindrical symmetry axis. Then for properly symmetrized strong coupling wave functions (cf. Ref. (10) Eq. II.15) the states of odd rotational angular momentum must vanish. If the symmetry of the nucleus with respect to the rotation of π is not perfect, states of odd rotational angular momentum may appear but displaced to higher energy with respect to the even members of their band. Christy[3] has thus proposed that a pear-shaped deformation must set in for nuclei in the region of low-lying 1- states.

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[3] R. F. Christy, unpublished data (1954).

It is beyond the scope of the present paper to speculate on details of the theory of the 1- states. We do wish to point out the significance of the fairly rapid E1 transitions (see Sec. 53) since available evidence indicates the lifetimes of the 1- states to be less than a millimicrosecond. If one grants the rotational nature of the 1- states, the occurrence of E1 transitions implies a separation between the center of mass and the center of charge in these deformed nuclei. It may well be that the two-fold rotational symmetry is broken down in these nuclei not so much by pear-shaped deformations as by opposite displacements of proton and neutron centers of mass along the cylindrical symmetry axis. The conditions for such displacements probably involve the availability of near-lying levels of opposite parity for both protons and neutrons near the top of the "Fermi sea." Such conditions are fulfilled in the region of Ra^{226} and Ra^{224}, since even-odd Ra^{225} and odd-even Ac^{227} exhibit low-energy E1 gamma transitions to ground.

14. Region of spheroidal nuclei. The appearance of rotational bands is associated by Bohr and Mottelson (10) (11) with onset of a stabilized spheroidal nuclear shape. An important quantity in the alpha decay treatments (12) in the rotational band region is the intrinsic quadrupole moment Q_o. The magnitude of Q_o may be determined either from the lifetime of the first excited state or from the coulomb excitation cross section for this state. The latter method has been applied by Divatia et al.[1] and from gamma-ray yield data with assumed total conversion coefficients of 340 for Th^{232} and 700 for U^{238} they calculate first excited state lifetimes of 7.8×10^{-10} and 4.4×10^{-10} sec, respectively. Their results may also be expressed in terms of intrinsic quadrupole moment values of $\pm 7.1 \times 10^{-24}$ cm^2 and $\pm 8.5 \times 10^{-24}$ cm^2, respectively. From other evidence we believe these intrinsic quadrupole moments to be positive. For both Am^{241} and Am^{243} ($I = 5/2$) Manning et al.[2] have determined spectroscopic quadrupole moments of +4.9 barns, corresponding to Q_o = +14 barns.

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[1] Divatia, Davis, Moffat, and Lind, Phys. Rev. 100, 1266A (1955); and verbal report quoted in National Research Council Nuclear Data Cards 56-1-118 and 56-1-120 (1956).

[2] Manning, Fred, and Tomkins, Phys. Rev. 102, 1108 (1956).

In the region of the heavy deformed nuclei many rotational bands have been identified, not only for even-even nuclei, but also for odd-mass nuclei. Most of these bands have been revealed by alpha spectroscopy (4) and some more recently by coulomb excitation.[3] Heavy nuclei exhibiting such bands with three or more member levels definitely identified are the following: Th^{229}, U^{235} (2 bands), Np^{237}, Np^{239}, Pu^{239}, and Bk^{249}.

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[3] J. O. Newton, Nature 175, 1028 (1955).

The occurrence of such bands implies an approximate conservation of angular momentum along the nuclear symmetry axis. The quantum number measuring this angular momentum component, usually designated as K, has proved to be of great usefulness in classifying nuclear states and correlating gamma, beta, and alpha transition rates.[5,6,7] There is a selection rule in the model for deformed nuclei that the multipolarity L of a radiation must equal or exceed ΔK, the change in K between states. (The intrinsic nucleonic angular momentum along the symmetry axis is designated Ω and will be equal to K for the low states, which do not involve shape vibrational excitation.)

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[5] Alaga, Alder, Bohr, and Mottelson, Dan. Mat. Fys. Medd. 29, no. 9 (1955).

[6] Rasmussen, Stephens, Strominger, and Åström, Phys. Rev. 99, 47 (1955).

[7] D. M. Chase and L. Wilets, Phys. Rev. 101, 1038 (1956).

In more detailed applications[8-10] of the model for deformed nuclei, attempts are made to understand transition rates and magnetic moments in terms of intrnsic nucleonic structure within the deformed well. Single particle wave functions[11,12] (13) in a spheroidal well are useful for such applications. An energy level diagram from Nilsson's work is given in Fig. 31 of Section 55. To further classify nucleonic states Nilsson (13) has suggested for use at large deformations the asymptotic quantum numbers, N (principal oscillator quantum number), n_z (or μ_z) (z-axis oscillator quantum number), and Λ (nucleon orbital angular momentum component along the symmetry axis) appropriate to a three-dimensional anisotropic harmonic oscillator with no spin-orbit interaction. The violation of selection rules in these quantum numbers for beta and gamma transitions appears generally to have a retarding effect on transitions though not so severe as violation of K-selection rules.[9,13]

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[8] B. R. Mottelson and S. G. Nilsson, Zeits. für Physik 144, 217 (1955).

[9] D. Strominger, Ph. D. Thesis, University of California (1956) (Radiation Laboratory Report UCRL 3374).

[10] J. M. Hollander, W. G. Smith and J. O. Rasmussen, University of California Radiation Laboratory Report, UCRL 3239 (1955).

[11] K. Gottfried, Ph. D. Thesis, Massachusetts Institute of Technology (1955) (unpublished).

[12] M. Rich, Bull. Am. Phys. Soc., Series II, 1, No. 5, F10 (1956).

[13] G. Alaga, Phys. Rev. 100, 432 (1955).

Concerning possible collective vibrational excitation of deformed nuclei less is presently known. Probably some recently discovered states populated weakly in alpha decay of radium isotopes to $Em^{218,220,222}$ are the second excited 2+ states predicted by Goldhaber-Weneser or Wilets-Jean models. Some states near 1 Mev are populated in Cm^{242} decay by mildly hindered groups; one may speculatively associate these states with beta and gamma vibrational states of spheroidal nuclei. (See Cm^{242} treatment in Section 54)

15. Region of spherical nuclei. In the region near Pb^{208} the equilibrium nuclear shape is probably spherical, and treatments involving the coupling of nucleons and/or holes outside the closed configuration seem most promising.[1] (9)

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[1] W. W. True, Phys. Rev. 101, 1342 (1956).

The simplest cases will be the nuclei immediately adjoining Pb^{208}. Their level systems with present probable shell model assignments are shown in Fig. 12, the assignments being somewhat different from Pryce's (9) by virtue of more recent work.

We will have occasion to use some of these shell model level assignments in Section 49-52 where alpha decay across closed shells (Po^{211}, Bi^{210} Bi^{211}, and Bi^{212}) is discussed in detail; these decays across the closed shell are of an especially highly hindered variety.

B. KINETICS OF ALPHA DECAY (EVEN-EVEN TYPE)

I. CORRELATIONS OF DECAY RATES OF EVEN-EVEN NUCLEI

16. Ground state transitions. As early as 1911, Geiger and Nutall found that a plot of the logarithms of decay constants for alpha emitters against the logarithms of the ranges resulted in a family of straight lines, one for each decay series. The aspect of these relations of greatest interest was that the slopes were about the same and demonstrated a sharp dependence of decay constant upon disintegration energy. After some two decades this relationship was interpreted by Gamow (2) and by Condon and Gurney (3) in terms of the potential barrier penetration problem.

The theory for alpha decay related the decay constant to three other parameters: the decay energy, the nuclear charge (atomic number), and the nuclear radius. Only the radius is not subject to direct measurement, but since no model of the nucleus permits widely varying nuclear radii in a limited region there was no difficulty in testing the major premise of the theory. In the main, the wide range of decay constants could be explained quantitatively in terms of the decay energy and the nuclear charge by letting the radius assume a simple $A^{1/3}$ dependence. As the theory is now developing, it is becoming difficult to ascribe fundamental significance to the nuclear radius as used in this way. Discussion of this problem will be found in Sections 19-27. Nevertheless, the concept is familiar and useful in explaining some of the observed trends and will be adhered to in this section.

The theory for the alpha-decay process has in recent years played an indirect but important practical role in the preparation of the many new alpha-emitting species. According to the regularities discussed in Section 3, it is possible to predict with fair accuracy the energy of any alpha-emitter. The theory then allows the calculation of the half-life in a manner summarized in Section 27. Such information quite obviously, constitutes a powerful guide for the preparation of new species. However, the calculations are too laborious to be made repeatedly, and it has become the practice to display them graphically. Several such sets of curves have been published[1,2] (6) and one is reproduced in Fig. 13. It should be stated at the start that there is excellent agreement between theory and experiment only for the ground state to ground state transitions in the even-even nuclei.

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[1] A. Berthelot, J. phys. radium VII 3, 52 (1942).

[2] S. Biswas, Ind. J. Phys. 23, 51 (1949).

It will be noted in Fig. 13 that nuclear radius does not appear. The manner by which it has been eliminated as an independent parameter is as follows. In essence, for each atomic number, the radius is not independent of the alpha-energy. This follows from the nearly monotonic change in alpha energy with mass number (see Section 3), and the radius in turn is taken to be a simple function of $A^{1/3}$. Therefore, to define a point on the curve for element Z, one simply takes the measured or estimated alpha energy for Z^A as the abscissa coordinate and calculates the half-life using A to define the radius. Since there will not generally be two values A for the particular decay energy the calculated half-life will be unique. The curves of Fig. 13 are calculated in the manner mentioned, and the points shown are experimental.

Aside from the utility of such curves for predicting the decay properties of unknown species, there has arisen another important function, namely, to serve as a baseline for discussing transitions which do not conform to the simple theory. Transitions to excited states of even-even nuclei and all transitions of odd nuclear types fall into this category.

For some purposes it is more convenient to present the information of Fig. 13 in another manner as illustrated by Fig. 14. Here the logarithm of the half-life is plotted against the reciprocal square root of the decay energy and the resulting family of straight lines is obtained.[3,4] The lines as shown were defined by least squares fitting using only those points for which accurate decay energies and half-lives are available. The relation of these lines to barrier penetration theory can be found from inspection of Equation (20.7) which predicts a nearly straight line dependence of the functions plotted.

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[3] A. Bohr, P. O. Fröman and B. R. Mottelson, Dan. Mat. Fys. Medd 29, No. 10 (1955).

[4] C. J. Gallagher and J. O. Rasmussen, University of California Radiation Laboratory Report, UCRL 3176 (1955).

Each line in Fig. 14 representing a single element can be expressed analytically as follows:

$$\text{Log } t_{1/2}(\text{sec}) = \frac{A}{\sqrt{Q_{eff}(\text{Mev})}} + B \qquad (16.1)$$

Here Q_{eff} is the effective decay energy which consists of the alpha-particle energy plus the recoil energy and plus an additive correction (~40 kev) for the orbital electron screening effect (see Section 25). Table II lists the values of A and B for each element.

A few alpha emitters for which reliable data are available were not used for determination of the lines of Fig. 14. These are the polonium alpha emitters Po^{210}, Po^{208}, and Po^{206}, which in common with all other even-even alpha emitters with neutron number equal to or less than 126 decay much slower than the simple theory would predict. Reasons for this behavior are advanced in Section 39. Po^{212} and Em^{218} have also been excluded from the determination of the lines since they appear to behave somewhat abnormally.

17. Hindrance factor in alpha-decay. It has already been mentioned that only the ground state transitions of the even-even alpha-emitters obey the simple barrier penetration theory. Other transitions are slower in varying degree than the demands of the theory and are consequently said to be hindered. It is convenient to express this effect quantitatively in terms of a hindrance factor (F) which is defined as the factor by which the observed alpha half-life is greater than that calculated. For some purposes rather subtle effects may be of importance, and we shall want to define this factor somewhat differently. The essence of the redefinition is that the hindrance factors for all even-even ground state transitions are taken to be unity whether or not the points fall precisely on the calculated curves of Fig. 13 or the best semi-empirical curves of Fig. 14. The reason for this small change will appear in the following discussion of transition to excited states.

18. Transitions to excited states. Corresponding to each ground state transition entered in Figs. 13 and 14, it would be possible to enter points for transitions leading to excited states. The alpha-particle energies can be obtained from Table I, and the partial alpha half-life for each transition may be calculated from its abundance and the total alpha half-life. It would be noted that such transitions are almost invariably hindered, some to a marked degree.

The transitions which have been well studied in the even-even nuclei are those to members of the rotational band based upon the ground state. The energy levels populated bear the designations 0+, 2+, 4+ .... where the 0+ state is the ground state. Fig. 15 shows the calculated hindrance factors for the various types of transitions. The hindrance factors for the ground state transitions are taken to be unity. The hindrance factor to an excited state is then obtained by normalizing the ground state transition to the proper curve of Fig. 14 and shifting the measured partial alpha half-life of the transition to the excited state accordingly. This normalization has the effect of simply referring hindrance factors of excited states to the ground state rather than to use the theory as an absolute guide.

Let us recapitulate the function of the hindrance factor. As well as can be told, the ground state transition of almost any even-even alpha-emitter can be described adequately in terms of simple barrier penetration. This transition is therefore taken to be "allowed" or unhindered. The hindrance factor of any other transition then expresses a retardation of decay rate which must be explained by some extension of the basic theory.

One of the factors long recognized as affecting the decay rate is the spin change. This aspect and others of the classical theory are discussed in Sections 19-27.

Other factors are now recognized which influence the alpha-emission lifetime and these are treated in succeeding sections. The curves of Fig. 14 will also be used (Section 40) to define hindrance factors for odd nucleon alpha-emitters.

II. DECAY RATE THEORY FOR PURELY CENTRAL FIELDS

19. Introduction. In the preceding chapter it was shown that the rates of ground state alpha transitions of even-even nuclei with few exceptions are amenable to simple exponential correlations with decay energy and atomic number. This sensitive exponential energy dependence of alpha decay rates is markedly different from the energy dependence of gamma and beta transition rates, where power laws of the energy are generally more applicable. An early triumph of quantum mechanics was the proposal by Gamow (2) and by Condon and Gurney (3) that alpha decay is essentially a coulombic barrier penetration process. With widely varying assumptions about the fundamental process of formation of alpha particles from nuclear matter the barrier rate formulations correlate the experimental rate data for even-even nuclei (rates varying over a factor of 10^{20}) quite well so long as one parameter, usually the coefficient of $A^{1/3}$ in an "effective nuclear radius" expression, is left free to be adjusted for the group of alpha emitters. The only significant breaks in the smooth correlations occur at 126 neutrons, those nuclei with 126 or less neutrons showing slower decay rates by factors of 5 to 20 than would be expected from correlations with nuclei of 128 or more neutrons.

In our first detailed applications of rate theory we will be concerned with the even-even nuclei, which possess zero angular momentum in their ground states.

After outlining some of the principal modern decay treatments and giving their formulas we will apply the experimental data in three ways, calculating (a) the effective nuclear radius for alpha decay as calculated by Kaplan's[1]

[1] I. Kaplan, Phys. Rev. 81, 962 (1951).

approximation to the Preston-Sexl (14, 15) decay formula, (Table VIII), (b) the normalized S-wave "surface probability" near the nucleus according to the treatment of G. H. Winslow. (16) (Table VII). (c) the reduced derivative width δ^2 according to the precise collision matrix formulation of R. G. Thomas (17) (Table VII).

20. A simple semi-classical treatment. Let us consider first a very simple approximate treatment of alpha decay along the lines first given by von Laue.[1] In common with other one-body treatments the alpha daughter nucleus is simply assumed to give rise to a potential function for the alpha particle. As the potential is usually idealized, it is taken as purely coulombic outside the "effective nuclear radius," R and is taken to be a constant value (less than the alpha decay energy) for distances less than R. An alpha particle is initially confined in a virtual state within the well, where it makes frequent collisions with the wall with a small probability of quantum mechanical penetration of the negative energy or barrier region. This three-dimensional problem with spherically symmetric potential can readily be reduced to a one-dimensional radial problem by familiar methods (Sec. 22). For S-wave alpha emission the effective potential in the one-dimensional problem is as shown in Fig. 16.

The decay constant λ will be a simple product of the frequency factor, f, giving the collisions per second with the wall, and the quantum mechanical penetration factor P.

$$\lambda = fP \qquad (20.1)$$

The collision rate will be equal classically to the alpha particle velocity in the well divided by the nuclear diameter, but variations in the model sometimes modify estimates of f.

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[1] M. von Laue, Zeits. f. Physik 52, 726 (1928).

From the WKB approximation we have the penetration factor below:

$$P \approx e^{-\frac{2}{\hbar}\int_R^{\frac{2Ze^2}{E}} \sqrt{2M\left(\frac{2Ze^2}{r} - E\right)}\, dr} \tag{20.2}$$

where $\underline{Z}$ is the charge of the recoil nucleus; $\underline{M}$, the reduced mass of the system (i.e., $M = \frac{M_\alpha M_r}{M_\alpha + M_r}$); $\underline{e}$, the elemental charge in electrostatic units. The integral is evaluated through the barrier.

The integral of (20.2) can be evaluated analytically and using Bethe's (18) notation we write,

$$P \approx e^{-2g(Z,R)\gamma(x)} \tag{20.3}$$

where $x = E/B$, $\underline{B}$, the barrier height being equal to $2Ze^2/R$. $g(Z,R)$ is a function independent of alpha decay energy and equal to

$$g = \frac{2e}{\hbar}\sqrt{MZR} \tag{20.4}$$

The function $\gamma(x)$ is given by

$$\gamma(x) = x^{-1/2} \arccos(x^{1/2}) - (1-x)^{1/2} \tag{20.5}$$

Bethe (18) has given a graph of γ vs. $\underline{x}$. For more accurate calculations we have included at the end of this section a five-place numerical tabulation (Table IX) of γ with argument $\underline{y}$, where $y = (1 - x)^{1/2}$. Winslow and Simpson have plotted (19) vs. $\underline{x}$ a function $f_o(x)$ which is $x^{1/2}$ times Bethe's $\gamma(x)$.

Formula (20.1) with (20.3) fairly satisfactorily correlates the variation of even-even ground state alpha decay rates with energy and atomic number.

One might in a semi-classical fashion include an extra factor in the penetrability expression to take into account special reflections due to the potential discontinuity at $\underline{R}$. That is,

$$P \approx \frac{(E - U)^{1/2}}{(B - E)^{1/2}}\, e^{-2g\gamma} \tag{20.6}$$

In optical analogy the new factor gives the transmission across a discontinuity in refractive index. The factor appears in more refined quantum mechanical treatments in the one-body model.

The nature of the function in (20.3) can be illustrated by a Taylor expansion about x = 0. Thus,

$$\log P = \frac{\sqrt{2MB}\,R}{\hbar}\left[\frac{\pi B^{1/2}}{E^{1/2}} - 4 + \frac{E}{B} - \frac{3E^2}{4B^2} + \ldots\right] \quad (20.7)$$

From the above we see that the main energy dependence of log P is an inverse square root dependence. Such is the mathematical plausibility argument why plots of log alpha half-life vs. $E^{-1/2}$ should be nearly linear for constant Z. (cf. Section 16 and Fig. 14).

There is an extensive literature on alpha decay theory, and a variety of special treatments in various degrees of approximation have been published. The better one-body treatments do not bring in any large corrections to the semi-classical formula (20.1), with (20.6).

The infinitely steep walls of the idealized potential are not realistic, and there have been treatments on sloping wall and rounded potentials by several authors.[2] (17), (20), ( ).

[2] M. L. Chaudhury, Phys. Rev. 88, 137 (1952).

A slight increase of the barrier has recently been suggested by Corben[3] as arising from vacuum polarization effects.

[3] H. C. Corben, Bull. Am. Phys. Soc. Ser. II. 1, 181 (1956).

21. Concerning applications to experimental data. It has been traditional in applying alpha decay formulas to experimental data to leave the channel radius or "effective nuclear radius for alpha decay" as a parameter to be calculated. These alpha decay radius values from one-body models are generally of the same order of magnitude but slightly smaller (see Table VIII) than radii from cross-section experiments involving alpha particles as projectiles (see Table IV). Some representative results for channel radii from various alpha decay theories are given in Table III. The values are strongly model-dependent, and there is still uncertainty on internal details giving rise to the frequency factor *f*. Until the fundamental process of alpha particle formation is better understood, such calculated "alpha decay radii" must be viewed with reservations.

With the present limited understanding of the intra-nuclear mechanics of the alpha decay process, it seems more logical, as Winslow (16) has argued, to summarize numerical calculations from experiment by some parameter not dependent on intra-nuclear assumptions. One such parameter is the squared normalized alpha wave amplitude near the nuclear surface.[1] For these calculations Winslow makes use of just that part of alpha decay theory which is undisputed and is common to all models. From a tabulation of these experimental wave amplitudes alone one may hope to draw certain conclusions, but the tabulation may also serve as a point of departure for any more detailed study of the internal mechanism of alpha decay. The squared alpha wave amplitudes when corrected for centrifugal barrier and non-central interaction effects we shall call reduced transition probabilities (RTP), and they bear a close analogy to the reduced transition probabilities for gamma transitions or *ft* values for beta transitions, where one makes

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[1] We shall call this quantity the "surface probability."

use of the general solutions to the problem without assumptions about the finer details of the intra-nuclear parts of the processes.

22. Coulomb wave functions. Let us now develop further the theory of alpha decay in central fields and in particular introduce the standard notation and mathematical relations involving the coulomb functions.

The general time-independent wave equation for an alpha particle in a repulsive nuclear coulomb potential may be written in spherical polar coordinates (center of mass system) as follows:

$$H\psi = E\psi \tag{22.1}$$

$$-\frac{\hbar^2}{2M}\nabla_{\underline{r}}^2\,\psi(\underline{r}) + \frac{2Ze^2}{r}\,\psi(\underline{r}) = E\,\psi(\underline{r}) \tag{22.2}$$

where M is the reduced mass $\frac{M_\alpha M_r}{M_\alpha + M_r}$ of the system, Ze is the charge of the recoil nucleus (of mass M_r), E is the total energy of the system (usually includes a correction to the experimental alpha plus recoil energy to get the decay energy that a bare nucleus with no orbital electrons would exhibit), and $\nabla_{\underline{r}}^2$ is the Laplacian operator. Since the potential is purely central, i.e., a function of the radial distance alone, the solutions may be conventiently expressed as

$$\psi_{nLm}(\underline{r}) = r^{-1} R_n(r)\, Y_L^m(\theta,\varphi) \tag{22.3}$$

where $Y_L^m(\theta,\varphi)$ are the normalized spherical harmonics.

With such a substitution the radial function must satisfy the equation

$$-\frac{\hbar^2}{2M}\frac{d^2R_n}{dr^2} + \left[\frac{2Ze^2}{r} - E + \frac{\hbar^2 L(L+1)}{2Mr^2}\right]R_n = 0 \quad (22.3)$$

This equation may be rearranged to a standard form involving the dimensionless parameters ρ and η, where $\rho = \underline{kr}$ and $\eta = \frac{Z_\alpha Ze^2}{\hbar v}$ with $\underline{k}$, the wave number, defined as $k = \frac{(2ME)^{1/2}}{\hbar}$

$$\frac{d^2u}{d\rho^2} + \left[1 - \frac{2\eta}{\rho} - \frac{L(L+1)}{\rho^2}\right]u = 0 \quad (22.4)$$

Two linearly independent solutions of this coulomb equation (the confluent hyper-geometric equation) are usually defined, a solution $F_L(\eta,\rho)$ regular at the origin and a solution $G_L(\eta, \rho)$ irregular at the origin. Many of the properties and approximations to these functions have been recently summarized by Fröberg.[1] Tables of the functions for small values of ρ and η have been published, the most extensive being those of the U.S. Bureau of Standards Group.[2] The tables do not extend nearly to the range of η values for the alpha-emitting nuclei ($\eta \sim 20$), but there are good approximation methods for calculating these functions. For alpha wave functions near the nuclear surface the most completely worked out approximation method appears to be the Riccati I method, as treated by Abramowitz.[3] The equations for determination of $\underline{F_o}$ and $\underline{G_o}$ are given by Fröberg in his equations. (9.1), (9.2), and (9.3).

Asymptotically, as $\rho \longrightarrow \infty$

$$F_L \longrightarrow \sin\left(\rho - \eta \log 2\rho - \frac{L\pi}{2} + \sigma_L\right) \qquad (22.5)$$
$$G_L \longrightarrow \cos\left(\rho - \eta \log 2\rho - \frac{L\pi}{2} + \sigma_L\right)$$

where $\sigma_L = \arg \Gamma(i\eta + L + 1)$

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[1] C. E. Fröberg, Revs. Modern Phys. 27, 399 (1955).

[2] Tables of Coulomb Wave Functions, Vol. I, NBS, Appl. Math. Series. 17 (Washington, D.C., 1952).

[3] M. Abramowitz, Quart. Appl. Math. 7, 75 (1949).

In any rigorous alpha decay treatment we will usually be interested in the linear combination $G_L + iF_L$, which represents a pure outgoing wave. The Wronskian relation is

$$G_L F_L' - F_L G_L' \equiv 1 \qquad (22.6)$$

where primes denote derivatives with respect to ρ.

More widely used approximations for the coulomb functions as applied to alpha decay theory are the JWKB approximation and the steepest descents approximation to the contour integral form for the coulomb functions.

The JWKB approximation yields the following expansion for the logarithmic derivatives

$$\left.\begin{matrix} G_L'/G_L \text{ upper sign} \\ F_L'/F_L \text{ lower sign} \end{matrix}\right\} = S_0 + S_1 + S_2 + \cdots\cdots\cdot$$

with $S_0 = \mp \kappa$

$$S_1 = -\frac{\kappa'}{2\kappa}$$

$$S_2 = \pm \frac{3(\kappa')^2}{8\kappa^3} \mp \frac{\kappa''}{4\kappa^2} \qquad (22.7)$$

where $\kappa = +\left[\frac{2\eta}{\rho} + \frac{L(L+1)}{\rho^2} - 1\right]^{\frac{1}{2}}.$

The second order and higher terms have generally been neglected in alpha decay treatments. Thus, to first order, after applying JWKB turning point formulas[4] at the transition point ($\kappa = 0$), we write in the barrier region,

$$G_L = \kappa^{-\frac{1}{2}} e^{\int_\rho^\tau \kappa \, d\xi}$$

$$F_L = \frac{1}{2}\kappa^{-\frac{1}{2}} e^{-\int_\rho^\tau \kappa \, d\xi} \qquad (22.8)$$

where τ is the value of ρ for which $\kappa = 0$.

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[4]L. I. Schiff, Quantum Mechanics, McGraw-Hill, New York: 1949. Equations (28.17) and (28.18).

As discussed earlier in connection with Equation (20.2), the L = 0 integral has a simple analytical expression (20.3).

More generally for all L, after performing the integration, the result can be expressed, in agreement with Thomas' (17) Equations (28) and Winslow's (29) Equation (11), (see also Bethe's (18) Equations (630) to (632)) as follows:

$$G_L(\eta,\rho) = \varkappa^{-\frac{1}{4}} (\eta^2 + \ell^2)^{-\frac{\ell}{2}} \left(\ell \varkappa + \frac{\ell^2}{\rho} + \eta\right)^{\ell} \cdot \exp\left[\pi\eta - \eta \arccos \frac{\eta - \rho}{\sqrt{\eta^2 + \ell^2}} - \rho\varkappa\right] \quad (22.9)$$

where $\varkappa$ is defined above (22.7) and the Langer modification[5] substituting for ℓ the value $L + 1/2$ (and substituting $(L + 1/2)^2$ for $L(L + 1)$ in Equation (22.7) for $\varkappa$). is recommended. Winslow and Simpson (21) have shown the accuracy of the above approximation for the values of η and ρ encountered in alpha decay theory to be within 0.1%. We shall not bother to reproduce the expression for F_L, since it is so much smaller than G_L that we generally only have need for G_L and the logarithmic derivative G_L'/G_L in practical alpha decay formulations.

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[5] R. E. Langer, Phys. Rev. 51, 669 (1937).

The ratio $(G_L/G_o)^{-2}$ is of some interest, since it enters as the centrifugal barrier reduction of the penetrability. Values calculated by Winslow and Simpson (21) and by Thomas (17) are given in Table V. The Thomas example corresponds to emission of a 5-Mev alpha particle by uranium from a radius R = 9.6 x 10^{-13} cm. Preston (14) and Thomas (17) derive frequency factors that increase with L for the literal one-body model. With such a model the decay rate is predicted to increase for small L values. Here in Table V we are concerned only with the barrier penetrability, a monotonically decreasing function of L.

It can be shown to a rough approximation that the logarithm of the reduction factor is inversely proportional to the square roots of the atomic number and the nuclear radius but is not very dependent on decay energy. A rough expression is

$$\left(\frac{G_L}{G_0}\right)^{-2} \approx e^{-\frac{2(L+\frac{1}{2})^2}{\sqrt{2\eta\rho}}} = e^{-\frac{\hbar(L+\frac{1}{2})^2}{\sqrt{MRZe^2}}} \qquad (22.10)$$

The values of Table V should be fairly representative for the heavy element alpha emitters. The corresponding values given by Devaney (22) and by Blatt and Weisskopf (23) in their text are in error, Winslow and Simpson (21) having pointed out that their factors are just the squares of what they should be.

23. Concerning time-dependent alpha decay treatments. While the majority of alpha decay theoretical treatments do not specifically introduce time-dependent wave functions, there are additional interesting insights to be gained by such considerations. Rasetti (24) has given an interesting one-body alpha decay treatment embodying time-dependent wave functions. Winslow and Simpson (25) have given a more detailed treatment along the same general lines as Rasetti. Kemble (26) has also treated alpha decay in this general fashion. At time zero an initial alpha particle wave function is constructed of finite spatial extension in a quasi-stationary state within the nuclear potential well.

This initial state is expressed as a wave packet formed by the infinitely extending regular solutions in the given potential, the energy distribution being about the virtual level within the potential well. A time-dependent wave function results, and this solution consists in the region outside the nucleus of purely outgoing coulomb waves decaying in time at any given spatial position according to the exponential decay law. The reader is referred to these original works for details.

24. Treatment in collision matrix formalism. A rigorous but simple formulation of alpha decay along the lines of the time-independent R-matrix theory of Wigner and collaborators[1] for nuclear reactions has been made by R. G. Thomas (17). The decay constants of radioactive states in this formulation are $2/\hbar$ times the imaginary parts of the energies of the poles of the collision matrix. Thomas first derives quite generally a formula giving the alpha decay constant in terms only of behavior of the internal alpha wave solution at the nuclear surface, as expressed by a parameter $\delta^2_{\lambda L}$ called the reduced width for the derivative of the radial alpha wave function (λth group with L angular momentum), is simply related to the reduced width $\gamma^2_{\lambda L}$ (commonly used in nuclear reaction applications) as follows:

$$\delta_{\lambda L} = \rho \frac{F_L F_L' + G_L G_L'}{F_L^2 + G_L^2} \gamma_{\lambda L} \approx \rho \frac{G_L'}{G_L} \gamma_{\lambda L} \qquad (24.1)$$

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[1] E. P. Wigner and L. Eisenbud, Phys. Rev. 72, 29 (1947). T. Teichman and E. P. Wigner, Phys. Rev. 87, 123 (1952).

The following equations express the decay rate in his formalism:

$$\lambda_L = \frac{2\rho\,\gamma_L^2}{\hbar(G_L^2+F_L^2)} \approx \frac{2\,\delta_L^2}{\rho\hbar(G_L')^2} = \frac{2\,\delta_L^2}{\rho\hbar\left(\frac{G_L'}{G_L}\right)^2 G_L^2} \quad (24.2)$$

We shall consider what various alpha decay models yield for the value of δ^2.

The formal definition of δ_λ, as given by Thomas, is

$$\delta_\lambda = \gamma_L + \sqrt{\frac{R\hbar^2}{2M}} \int_S \psi_c^* \,\mathrm{grad}_n X_\lambda \, dS \quad (24.3)$$

The integral is carried over the nuclear surface at radius $\underline{R}$; $X_\lambda = \frac{u(r)}{r} Y_L^m(\theta,\varphi)$ is the alpha wave function in the channel; ψ_c is a product wave function including internal coordinates of the alpha particle, the residual nucleus, and a spherical harmonic if there is angular momentum associated with that alpha channel. γ_λ, the reduced width, will be generally much smaller than δ_λ and Thomas neglects it for applications of the formula. Formally,

$$\gamma_\lambda = \sqrt{\frac{\hbar^2}{2MR}} \int_S \psi_c^* X_\lambda \, dS \quad (24.4)$$

Reducing (24.3) to a more directly applicable form we may write

$$\delta^2 \approx \frac{R\hbar^2}{2M} \left|\frac{du}{dr}\right|^2_{r=R} |\psi_c|^2 \quad (24.5)$$

The factor $|\psi_c|^2$ may be regarded as a formation probability factor and would be unity in a one-body treatment.

For the usual square well one body model Thomas' expression is

$$\delta_L^2 = \frac{\hbar^2}{MR^2} \frac{z_L^2 \rho^2 \left(\frac{G_L'}{G_L}\right)^2}{\left[\rho\left(\frac{G_L'}{G_L}\right)+L\right]\left[\rho\left(\frac{G_L'}{G_L}\right)-L-1\right] + z_L^2} \approx \frac{\hbar^2}{MR^2} z_L^2 \qquad (24.6)$$

where z_L is a zero (usually taken as the first) of the spherical Bessel function $j_{L+1/2}(z)$. The first z_0 is simply π, and substitution of this value in the above equations for $L = 0$ leads to results in agreement with other one-body treatments. That is $\delta_0^2 \approx 1.2$ Mev.

It may be of some interest to consider the reduced widths obtained from higher virtual levels than the lowest. (We discuss in Sec. 36 a recent estimate of 140 Mev for the kinetic energy of the alpha particle in the nucleus, whereas the lowest s-level is at about 0.5 Mev.) From Equation (24.6) for $L = 0$ we substitute a typical average value of -20 for $\rho\, G'/G$ and have

$$\delta^2 \approx \frac{\hbar^2}{MR^2} \frac{z^2}{\left(1+\frac{z^2}{400}\right)} \approx 0.12 \frac{z^2}{1+\frac{z^2}{400}} \text{Mev} \qquad (24.7)$$

For $L = 0$ the zeroes of the Bessel function are given by $z = n\pi$, with n a positive integer. For the lowest solution $\delta^2 \approx 1.2$ Mev, increasing for higher solutions and approaching the limit $\delta^2 \longrightarrow 48$ Mev.

The formula for the $L = 0$ reduced width is easily modified for the Winslow surface well model (21)(27) by substituting ΔR, the width of the surface well, for R, the nuclear radius. Evidently higher reduced widths than in the ordinary model will be obtained in the one body form of the surface well model, since $\Delta R < R$.

δ^2 is quite simply connected with the frequency factor f used in the early part of this chapter. Indeed, the reduced derivative width is approximately Planck's constant h times the semi-classical frequency factor.

$$\delta^2 \approx 2\pi\hbar f \tag{24.8}$$

δ^2 is 2π times Bethe's (18) G_α, the "α width in the absence of barrier" (cf. Bethe's Equation 593).

25. Influence of the atomic electron cloud. The alpha decay rate treatments are all formulated in terms of the decay of isolated nuclei, stripped of electrons. For many years the influence of the electron cloud was quite ignored. Credit for focussing attention on this question goes to Ambrosino and Piatier,[1] who in 1951 proposed that additive corrections to the observed alpha decay energy should be made before substitution into decay rate formulas derived for bare nuclei. Subsequent works of others[2] (21 p. 14) (17) generally concur in the necessity for energy correction but support corrections of about half the size proposed by Ambrosino and Piatier.

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[1] G. Ambrosino and H. Piatier, Compt. Rend., 232, 400 (1951).

(1953).

To understand the nature of these corrections let us imagine we have an assemblage of bare nuclei undergoing alpha decay with a measurable rate and alpha particle energy. What, if any, change will there be in these two observables as electrons are added to form normal atoms?

First, the decay rate will hardly change at all upon addition of orbital electrons. One might predict theoretically a negligibly slight increase in decay rate. To the extent that some electronic charge lies within the classical turning radius for alpha decay (4×10^{-12} cm) the potential barrier for alpha decay will be slightly lowered. A rough estimate with relativistic _K_-electron wave functions indicated the effect is negligible.

Second, the average observed alpha particle energy will decrease by an energy of the order of 40 kev in the heavy element region. In the main, this energy simply represents the work done against coulombic attraction of electrons during removal of the positively charged alpha particle from the center of the diffuse electron cloud to infinity. This is the potential energy of the alpha particle in the atomic electron field of the parent atom, and from Foldy's work[3] is well represented by the expression $65.3\ Z_i^{7/5}$ electron volts[4] (for Cm, 39.4 kev). If the alpha moved out with a velocity large compared to the average velocity of the orbital electrons, this potential would represent the difference between observed decay energy (alpha particle plus recoil nucleus kinetic energy) of the bare nucleus and that of the atom. However, the alpha actually moves at a lower velocity than the more tightly bound electrons, and some adiabatic adjustment of the electron wave functions is to be expected during alpha emission. With complete adiabatic readjustment, according to the results of Serber and Snyder[5] is associated an energy of 91.4 $Z_i^{2/5}$ (form Cm, -567 ev). By Thomas' estimate for a 4.5 Mev alpha particle from thorium there should be about 87 percent of the full adiabatic correction realized on the average. The best value for the alpha decay energy difference between bare nucleus and atom would then be

$$\Delta E = 65.3 Z_i^{2/5} - 80 Z_i^{2/5} \text{ ev.} \tag{25.1}$$

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[2]Rasmussen, Thompson, and Ghiorso, University of California Radiation Laboratory Report 1473 (1951).

[3]L. L. Foldy, Phys. Rev. 83, 397 (1951).

[4]Since Foldy's formula is based on non-relativistic Hartree calculations of total electron binding energy, his formula may give slightly small answers for heavy elements.

[5]R. Serber and H. S. Snyder, Phys. Rev. 87, 152 (1952).

The only important change necessary to use the conventional decay rate formulas is to add the above energy difference to the observed total alpha decay energy before substituting into the rate formulas.

The 13 percent of the adiabatic energy term (70 ev) not realized in the alpha decay energy correction represents the average electronic excitation of the recoil atom. In principle this average energy of excitation plus the 80 ev total energy of ionization of helium should be added to the experimental alpha decay energy for relative atomic mass calculations (5). The average energy of excitation is usually to be associated with ionization of outer-shell electrons.

Occasionally the alpha emission process will lead to ejection of a tightly bound electron. The ejection of K and L electrons has received attention both theoretically and experimentally, and some of the results are summarized in Table VI.

Migdal, (28) who first treated the problem of ionization induced by alpha decay, carried out a time-dependent perturbation treatment which effectively makes a semi-classical electric multipole expansion of the field from the moving alpha particle, similar to the semi-classical treatments of the coulomb excitation theory. Migdal used non-relativistic unscreened hydrogen-like electron wave functions, assumed uniform alpha particle velocity, and was able to derive analytical expressions for the ionization probability and the energy distribution of ejected electrons in the continuum.

Migdal considered only electric dipole terms and obtained a result of K ionizations per alpha disintegration of

$$W_K = \left(\frac{v}{v_K}\right)^2 \frac{2.2}{Z^2} \qquad (25.2)$$

where v is the velocity of the alpha particle, v_k the average velocity of a K-electron, and Z the atomic number of the alpha emitter.

Levinger[6] in addition treated electric quadrupole terms, which he found to be of significance for the p electrons of the L-shell. His treatment differs most sharply from Migdal's in his reduction of the dipole terms by a factor of about 25, due to special recoil considerations. Schaeffer[7] has treated the problem for K-electrons and estimates the dipole term as even slightly enhanced over Migdal's value. Recently Schwartz[8] has given theoretical arguments in support of Migdal's formulation of the dipole interaction over Levinger's.

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[6] J. S. Levinger, Phys. Rev. 90, 11 (1953); J. phys. radium 16, 556 (1955).

[7] G. W. Schaeffer, M. A. Thesis, University of Toronto, Canada, 1953. unpublished.

[8] H. M. Schwartz, Phys. Rev. 100, 135 (1955).

Schaeffer's treatment finds monopole interaction of some importance and for the constant alpha velocity treatment gets a total ionization probability about twice that of Migdal. He then corrects for the variation in alpha particle velocity, lowering the result by about a factor of two, numerically about equalling Migdal's value. Schaeffer feels that a relativistic treatment might further lower the probability in closer accord with the experimental value of Barber and Helm.[9]

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[9] W. C. Barber and R. H. Helm, Phys. Rev. 86, 275 (1952).

Levinger[6] has estimated that correction for electron screening effects should give an increase of about 40 percent in the K ionization.

We can generally conclude that the theory (except Levinger's) and experiment are in agreement within a factor of two for K-shell ionization.

The discrepancy (see Table VI) regarding L-shell ionization is still unresolved. The theoretical treatments of Migdal and of Levinger are an order of magnitude lower than the experimental value of Rubinson and Bernstein.[10] Levinger feels that relativistic and screening corrections will not be very large for the L-electron problem and cannot explain the discrepancy.

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[10] W. Rubinson and W. Bernstein, Phys. Rev. 82, 334 (1951); Phys. Rev. 86, 545 (1952).

The coincidence experiments of Lagasse and Doyen[11] are a neat confirmation of the Migdal theoretical energy distribution of ejected electrons.

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[11] A. Lagasse and J. Doyen, Compt. rend. 239, 670 (1954).

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26. Some numerical comparisons with decay rate data of even-even nuclei. In Table VII are presented data and calculated values of two parameters measuring the intrinsic decay rate calculated from experimental decay energy and rate data for even-even nuclei. An unusual procedure is used here. The ground state alpha groups are not treated separately but the total half lives for alpha decay were used in order to put all alpha emitters on an even basis for comparison. (Some cases probably have unresolved fine structure of appreciable intensity, Cf^{248}, Cm^{240}). The alpha particle energy for the ground-state transition, corrected for electron screening, appears in Col. 3. The logarithm to base 10 of the square of the irregular coulomb function G_0 evaluated at $R = 9.3 \times 10^{-13}$ cm

for heavy elements and at 8.0 x 10^{-13} cm for rare earths is given in Col. 4. In Col. 5 is the logarithmic derivative of G_o^2 (R in units of 10^{-13}) at R = 9.3. Finally, the parameters measuring intrinsic decay rate are given. In Col. 6 are Winslow's (16) surface probabilities; in Col. 7 are Thomas' (17) reduced derivative widths δ^2.

In Table VIII are summarized Asaro's[1] calculations of the effective nuclear radius using the alpha decay formulas of Preston (14) (Equations 4.3 and 4.4) or using a slight modification due to Kaplan[2] in which Preston's Equation (4.3) is supplanted by the equation

$$\mu^2 = \frac{0.52}{Q_\alpha(\text{Mev})},$$

μ^2 being the ratio of the alpha energy inside the well to that at infinity. Unlike the calculations of Table VII these calculations use the partial alpha half life for the ground state group alone, and in the cases of Cf^{248} and Cm^{240}, where ion chamber measurements have not resolved the expected alpha fine structure, reasonable branching ratios were assumed. In the last columns are given the r_o values from which the effective nuclear radius of the alpha decay daughter (mass number A) can be calculated by the formula

$$R = r_o A^{1/3} 10^{-13} \text{ cm}.$$

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[1] F. Asaro, unpublished calculations (1955).

[2] I. Kaplan, Phys. Rev. 81, 962 (1951).

A comparison between Tables VII and VIII reveals, of course, the same fluctuations in all three parameters, ("surface probability" δ^2 and r_o^2.

27. Summary of useful numerical formulas. Below are summarized a few formulas useful for numerical work on alpha decay rates. The nomenclature follows general practice. The numerical constants were calculated using 1955 revised values of Cohen, DuMond, Layton, and Rollett.[1] A value of 4.00278 atomic mass units was used for the mass of the alpha particle, based on the mass of 4.003873 ± 0.000015 for the helium atom given by Ajzenberg and Lauritsen.[2]

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[1]Cohen, DuMond, Layton, and Rollett, Revs. Modern Phys. 27, 363 (1955).

[2]F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. 24, 321 (1952).

Q_{eff}, the total alpha disintegration energy of a bare nucleus equals alpha particle energy (E) in the lab system plus recoil energy ($\frac{M_\alpha E}{M_r}$) plus ΔE_{sc}, the orbital electron "screening" correction. Q_{eff} will be expressed in Mev in the following formulas.

$$\Delta E_{sc} = 65.3\,(Z+2)^{7/5} - 80\,(Z+2)^{2/5}\ \text{ev}, \qquad (27.1)$$

where Z is the atomic number of the daughter nucleus.

η, an argument of the standard coulomb functions

$$\eta = \frac{2Ze^2}{\hbar v} = 0.63003_5 \frac{Z}{\sqrt{Q_{eff}}}\sqrt{\frac{A}{A+4}}, \qquad (27.2)$$

where v is the velocity of separation of alpha particle and recoil nucleus for large separation distance and A is the atomic weight of the daughter. More precisely $\frac{A}{A+4}$ should be replaced by $\frac{M_r}{M_r + M_\alpha}$.

ρ, an argument of the standard coulomb functions

$$\rho = kr$$

k, the wave number for large separation distance r

$$k = \frac{Mv}{\hbar} = 0.43756_6 \sqrt{\frac{A\ Q_{eff}}{A+4}} \cdot 10^{13}\ cm^{-1} \quad (27.3)$$

x, the dimensionless ratio of total energy Q_{eff} to the potential energy $\frac{2Ze^2}{r}$

$$x = \frac{Q_{eff}\ r}{2Ze^2} = \frac{\rho}{2\eta} = 0.34725_5 \frac{Q_{eff}\ r}{Z}, \quad (27.4)$$

where r is in units of 10^{-13} cm.

$$G_L(\rho,\eta) = G(\rho,\eta)\ \frac{G_L(\rho,\eta)}{G(\rho,\eta)}$$ the irregular coulomb function.[3]

$$\text{Log}\ G^2 = \frac{\sqrt{8\rho\eta}}{\log 10}\ \gamma(x) - \tfrac{1}{2}\text{Log}\left(\frac{1-x}{x}\right) \quad (27.5)^4$$

$$= 0.64496_1 \sqrt{\frac{ZrA}{A+4}}\ \gamma(x) - \tfrac{1}{2}\text{Log}\left(\frac{1-x}{x}\right)$$

$$\mathrm{Log}\left(\frac{G_L^2}{G^2}\right) = \frac{2}{\rho \log 10}\left[\sqrt{x(1-x)} - \frac{x}{4\rho(1-x)}\right]\left(L+\frac{1}{2}\right)^2 + \ldots \quad (27.6)^5$$

The dimensionless logarithmic derivative is given as follows:

$$\frac{d\,\mathrm{Log}\,G_L}{d\,\mathrm{Log}\,r} = -\rho\kappa - \frac{\rho}{2\kappa}\frac{d\kappa}{d\rho} \quad (27.7)$$

$$\kappa = \left[\frac{2\eta}{\rho} + \frac{(L+\frac{1}{2})^2}{\rho^2} - 1\right]^{\frac{1}{2}} \quad (27.8)$$

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[3] It is to be noted that in the Langer modification of the WKB approximation employed G is not quite equal to G_o. At the nuclear radius for most heavy alpha emitters G_o exceeds G by about 1%.

[4] The function γ and its first difference are tabulated to five decimal places in Table IX with argument y, where $y = (1-x)^{1/2}$.

[5] This is essentially the logarithm of the centrifugal barrier reduction factor. The $(L + 1/2)^4$ and $(L + 1/2)^6$ terms of this expansion are given in Winslow's (21) Equation (18). He has found for a representative case ($\eta = 20$, $\rho = 10$) that in the above expansion of the logarithm the fourth power term is equal to 0.002 for L = 3, and the sixth power term is 0.001 for L = 6.

"Surf.Prob." is our abbreviation for Winslow's (16) normalized squared alpha wave amplitude, evaluated arbitrarily at a radial distance of 9.3×10^{-13} cm for trans-lead alpha emitters and at 8.0×10^{-13} cm for rare earth nuclei. In Winslow's notation it is written $R\,|\phi_{J,I}(R)|^2$.

$$\text{surf. prob.} = \frac{M R \log 2}{\hbar K t_{1/2\alpha}} G_0^2(R)$$

$$= 0.998 \cdot 10^{-22} \sqrt{\frac{A}{A+4}}\, \frac{R\, G_0^2(R)}{Q_{eff}^{1/2}\, t_{1/2\alpha}(\text{sec.})} \tag{27.9}$$

δ_L^2 is Thomas' (17) reduced derivative width. (It is 2π times Bethe's (18) G_α, "alpha width in the absence of a potential barrier.") We have evaluated it at the same radii as for "surf. prob." above.

$$\delta_L^2 = \frac{\hbar \log 2}{2 K R t_{1/2\alpha}} \left(\frac{d \operatorname{Log} G_L}{d \operatorname{Log} r}\right)^2_{r=R} G_L^2(R)$$

(27.10)

$$\delta_L^2(\text{Mev}) = 5.2114 \cdot 10^{-22} \sqrt{\frac{A+4}{A\, Q_{eff}}}\, \frac{G_L^2(R)}{R\, t_{1/2\alpha}} \left(\frac{d \operatorname{Log} G_L}{d \operatorname{Log} r}\right)^2_{r=R}$$

III. DECAY RATE THEORY INCLUDING NON-CENTRAL FIELDS

28. Introduction. Whenever an alpha particle leaves the nucleus with the daughter nucleus in an excited state, we may consider semi-classically that the daughter nucleus has associated with it an electromagnetic radiation field or fields associated with the matrix elements of gamma transitions to other states. In calculation of internal conversion coefficients one calculates relative probabilities of the radiation field ejecting a bound electron into the continuum or of its creating a photon. In the same general manner, the radiation field (particularly the electric components) acts on the escaping alpha particle while it is in the vicinity of the nucleus, and there will be an opportunity for nuclear transitions to take place by inducing an energy increase and angular momentum change in the motion of the alpha particle. To be more general, we must consider that the escaping alpha particle may also lose energy and excite the daughter nucleus to higher excited states, again through the medium of the non-central electromagnetic transition fields. In a strict sense, then, it is not exact to consider only a single nuclear state of the daughter nucleus in solving the Schrödinger equation for alpha decay, as is done in Secs. 19-26.

These refinements to alpha decay theory have not received much attention until quite recently. M. A. Preston[1] (29) gave a general approximate treatment of the problem in 1949. Recent attention has been given to the special importance of coupling by large E2 matrix elements in even-even nuclei of the rotational band region.[2,3]

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[1] M. A. Preston, Phys. Rev. 82, 515 (1951).

[2] J. O. Rasmussen, University of California Radiation Laboratory Report 2431 (1953) (unpublished).

[3] R. F. Christy, Phys. Rev. 98, 1205A (1955).

29. Formulation relating gamma transition rates and non-central alpha decay interactions. One can make a general derivation of the alpha decay problem including non-central fields of electric multipole nature[1] and relate the interactions to reduced gamma transition probabilities.

[1]Magnetic multipole fields will probably be of little importance in alpha decay, since the alpha particle has no intrinsic spin.

Consider the total Hamiltonian for the alpha decay system where the alpha particle is at a distance beyond special short range nuclear forces. This may be written as

$$H = T + H_{nuc} + V, \tag{29.1}$$

where $\underline{T}$ is the kinetic energy operator for the alpha-recoil nucleus system excluding internal motion of the recoil nucleus, $\underline{H}_{nuc}$ is the Hamiltonian for internal structure of the recoil nucleus, and $\underline{V}$ represents the complete coulombic potential energy interaction.

$$T = -\frac{\hbar^2}{2M} \nabla_{\underline{\underline{r}}}^2$$

$$V = \sum_{p=1}^{A} \frac{2e \cdot e_p}{|\underline{\underline{r}} - \underline{\underline{r}}_p|}, \tag{29.2}$$

where the summation is carried out over all nucleons in the recoil nucleus, and the position vectors $\underline{\underline{r}}$ and $\underline{\underline{r}}_p$ are taken with the center of mass of the recoil nucleus as origin; $\underline{\underline{r}}$ gives the position of the alpha and $\underline{\underline{r}}_p$ of the proton.

Further, by an expansion of $|\underline{r} - \underline{r}_p|^{-1}$ valid for $r > r_p$,

$$V = 2e \sum_{p} \sum_{\lambda=0}^{\infty} \frac{e_{\lambda p}\, r_p^{\lambda}\, P_{\lambda}(\cos \gamma)}{\lambda!\; r^{\lambda+1}} \qquad (29.3)^2$$

where $P_{\lambda}(\cos \gamma)$ is a Legendre polynomial and γ is the angle between $\underline{r}_p$ and $\underline{r}$.

The λ=0 term of the summation is simply the ordinary central coulombic term

$$V_{cent} = \frac{2Ze^2}{r} ,$$

and it will be shown that the succeeding terms are quite generally related to electric transition probabilities and in special cases to such quantities as the intrinsic nuclear quadrupole moment.

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[2] We define an $e_{\lambda p}$ in a slightly different manner from Bohr and Mottelson's (10) Equation VII-3. We let $e_{\lambda p}$ be the effective charge of a nucleon which includes the recoil correction. $e_{\lambda p} = [1 - \frac{Z}{A^{\lambda}}]e$ for protons, and $e_{\lambda p} = - \frac{Ze}{A^{\lambda}}$ for neutrons except for $\lambda = 0$, where the recoil term vanishes.

The Schrödinger wave equation $H\psi = E\psi$ will in general not be separable when non-central interactions are included. It will be a partial differential equation of $3(\underline{A} + 4)$ independent variables. (A being the mass number of the recoil nucleus). To simplify the problem and reduce it to a set of ordinary differential equations in $\underline{r}$, one usually expands the total wave function in terms of an orthonormal set of functions in all variables except r, with the expansion coefficients functions of r. One can, for example, construct such a set using eigenstates of the recoil nucleus, thus utilizing the orthogonality properties of these eigenfunctions.[3] Different and more specialized expansions have been used where the spheroidal nuclear model is applicable[4] (12) and these will be taken up later in this paper.

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[3] M. A. Preston, Phys. Rev. 75, 90 (1949).

[4] J. O. Rasmussen, University of California Radiation Laboratory Report 2431 (1953) (unpublished).

In the present derivation we wish to deal only with total wave functions which conserve the angular momentum and parity of the parent alpha-emitting nucleus. Such functions can be constructed in familiar fashion by summations weighted by Clebsch-Gordan coefficients (cf. Blatt and Weisskopf's (23) Appendix A). That is, the orthonormal functions for the expansion can be represented by

$$\mathcal{Y}_{\ell I_f \tau}^{IM}(\theta,\varphi,x_i) = \sum_m (\ell\, I_f\, m\;\; M-m|\ell\;\, I_f\;\, I\, M) \cdot Y_{\ell m}(\theta,\varphi)\, \Phi_{I_f\, M-m\, \tau}(x_i) \quad (29.4)$$

where ℓ is the angular momentum associated with the alpha particle motion, m is its component on a space-fixed axis, I_i and I_f are initial and final nuclear angular momenta, M_i and M_f their components in space. τ represents all other quantum numbers specifying the nuclear state.

The orthogonality relation of these functions is as follows:

$$\int \mathcal{Y}^{IM\,*}_{\ell I_f \tau}\, \mathcal{Y}^{IM}_{\ell' I_f' \tau'}\, d\omega\, dx_i = \delta_{\ell\ell'}\,\delta_{I_f I_f'}\,\delta_{\tau\tau'} \tag{29.5}$$

Letting the total wave function be expressed as

$$\psi = \sum_{\ell' I_f' \tau'} r^{-1} u_{\ell' I_f' \tau'}(r)\, \mathcal{Y}^{IM}_{\ell' I_f' \tau'}(\theta, \varphi, x_i) \tag{29.6}$$

we substitute into the Schrödinger equation, $(H - E)\psi = 0$ with Hamiltonian of (29.1), multiply by $\mathcal{Y}^{IM\,*}_{\ell I_f \tau}$ and integrate over all space for all variables except r. Hereafter we write $\mathcal{Y}^{IM}_{\ell I_f \tau}$ as $|\tau \ell I_f; IM)$. There result the ordinary differential equations

$$-\frac{\hbar^2}{2M}\frac{d^2 u_{\ell I_f \tau}}{d r^2} + \left(\frac{\hbar^2}{2Mr^2}\ell(\ell+1) + E^{nuc}_{I_f \tau} - E\right) u_{\ell I_f \tau} + \frac{2Ze^2}{r} u_{\ell I_f \tau}$$

$$+ \sum_{\ell' I_f' \tau'} u_{\ell' I_f' \tau'} \left(\tau \ell I_f; IM \right| \sum_{\lambda=1}^{\infty} \frac{2e}{\lambda!\, r^{\lambda+1}} \sum_{p} r_p^{\lambda} e_{\lambda p} P_\lambda(\cos\gamma) \left| \tau' \ell' I_f'; IM \right) = 0. \tag{29.7}$$

From Racah's formula(30) (38) we can evaluate the desired matrix elements of the coulombic interaction between the alpha and the protons of the recoil nucleus as follows:

$$\frac{2e}{\lambda!\, r^{\lambda+1}} \left(\tau \ell I_f ; IM \middle| \sum_{p=1}^{A} e_{\lambda p}\, r_p^{\lambda}\, P_\lambda(\cos\gamma) \middle| \tau' \ell' I_f' ; IM\right)$$

$$= \frac{2e}{\lambda!\, r^{\lambda+1}} (-)^{\ell + I_f' - I} \left(\ell \,\middle\|\, \underline{\underline{C}}_\alpha^{(\lambda)} \,\middle\|\, \ell'\right) \cdot W(\ell I_f \ell' I_f' ; I \lambda) \tag{29.8}$$

$$\cdot \left(I_f \tau \,\middle\|\, \sum_p e_{\lambda p}\, r_p^{\lambda}\, \underline{\underline{C}}_p^{(\lambda)} \,\middle\|\, I_f' \tau'\right)$$

where we use Racah's notation throughout.

By Racah's equations (30) (50) and (51)

$$(\ell \| \underset{=}{C}_{\alpha}^{(\lambda)} \| \ell') = (-)^{\ell} \left[\frac{(2\ell+1)(2\ell'+1)}{2\lambda+1}\right] (\ell\, \ell' 0 0 | \ell\, \ell' \lambda 0) \tag{29.9}$$

we may establish the connection between $\sum_{p} (I_f \tau \| r_p^{\lambda} \underset{=}{C}_p^{(\lambda)} \| I_f' \tau')$ and the electric 2^{λ}-pole reduced transition probability connecting the states.

The electric 2^{λ}-pole transition probability from Bohr and Mottelson's (10) equation (VII.I) is as follows:

$$T(\lambda) = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{1}{\hbar} \left(\frac{\omega}{c}\right)^{2\lambda+1} B_e(\lambda) \tag{29.10}$$

where $B_e(\lambda)$, the reduced transition probability is given by

$$B_e(\lambda) = \sum_{M_f \mu} \left| (I_f M_f \tau | \sum_{p=1}^{A} e_{\lambda p} r_p^{\lambda} Y_{\lambda\mu}(\theta_p, \varphi_p) | I_f' M_f' \tau') \right|^2 \tag{29.11}$$

The transition operator transforms with rotation of coordinates like a single spherical harmonic of order λ, so we may use Racah's equations to evaluate matrix elements of it. By Racah's equation (29) the above is equal to

$$B = \sum_{M_f \mu} \Big| (-)^{I_f - M_f} (\tau_f I_f \| \sum_p e_{\lambda p} r_p^{\lambda} \left(\frac{2\lambda+1}{4\pi}\right)^{1/2} C^{(\lambda)} \| \tau_f' I_f') \cdot V(I_f I_f' \lambda; -M_f M_f' \mu) \Big|^2 = \left(\frac{2\lambda+1}{4\pi}\right) (\tau_f I_f \| \sum_p e_{\lambda p} r_p^{\lambda} C^{(\lambda)} \| \tau_f' I_f')^2 \cdot \sum_{M_f \mu} V(I_f I_f' \lambda; -M_f M_f' \mu)^2 \tag{29.12}$$

and by Racah's equation (20a) the summation reduces to $(2I_f' + 1)^{-1}$, so

$$\underset{I_f' \to I_f}{B}(\lambda) = \left(\frac{2\lambda+1}{4\pi}\right) \frac{(\tau_f I_f \| \sum_p e_{\lambda p} r_p^\lambda C^{(\lambda)} \| \tau_f' I_f')^2}{2I_f' + 1}$$

Therefore

$$(\tau_f I_f \| \sum_p e_{\lambda p} r_p^\lambda C^{(\lambda)} \| \tau_f' I_f') = \pm \left[\frac{4\pi(2I_f'+1)\, B_e(\lambda)}{2\lambda+1}\right] \tag{29.13}$$

Thus, combining (29.8), (29.9), and (29.13) we may write for the coefficients of the coupling terms in (29.7)

$$\begin{aligned}(\tau \ell I_f ; IM \Big| \frac{2e}{\lambda!\, r^{\lambda+1}} \sum_p e_{\lambda p} r_p^\lambda P_\lambda(\cos \gamma_p) \Big| \tau' \ell' I_f' ; IM) \\ = \pm (-)^{I_f' - I} \frac{2e\left[(2\ell+1)(2\ell'+1)\, 4\pi (2I_f'+1)\, B_{I_f' \to I_f}(\lambda)\right]^{1/2}}{\lambda!\ r^{\lambda+1}\ (2\lambda+1)} \\ \cdot (\ell \ell' 00 | \ell \ell' \lambda 0)\ W(\ell I_f \ell' I_f' ; I \lambda)\end{aligned} \tag{29.14}$$

With this relationship the magnitude of non-central coupling terms in alpha decay may be calculated from a knowledge of the gamma transition probability or coulomb excitation cross section. There is an uncertainty in sign associated with this procedure, corresponding to the two phase possibilities for a gamma transition. In some cases information on relative phases of gamma radiation may be gained from gamma angular distribution experiments with mixed multipolarities. In other cases, the sign may be determined from a theoretical model.

30. Numerical calculation of some E1 and E2 interaction matrix elements.

In order to get an idea of the importance in actual alpha emitters of including coupling induced by electric multipole fields, let us consider numerical examples.

Throughout the strong deformation region of heavy isotopes ($A > 224$) many low energy E1 transitions have been identified. About half of these decay from metastable states of measurable half life, meaning a large retardation of ($\sim 10^3 - 10^5$) from the single proton transition lifetime formula. Throughout the whole region of alpha emitters many E2 transitions have been identified, and in the region $A > 224$ these transitions are usually of the collective rotational type, exhibiting transition probabilities much larger than single proton formulas estimate.

We shall consider briefly first E1 coupling, where the coupling effects beyond the range of specifically nuclear forces are expected to be relatively unimportant. A special section will be devoted to E2 coupling, the inclusion of which is of essential importance in any fundamental treatment of alpha decay in the region of the enhanced E2 rotational transitions.

In Fig. 17 are shown the decay schemes of alpha emitters to be considered in numerical calculation of coupling terms here. The gamma transitions and levels taken into account in the examples are shown as solid lines and others as dashed lines.

In Table X are calculated according to Equation (29.14) values of the matrix elements coupling the alpha decay Equation (29.7) for actual alpha emitters. By far the largest coupling terms are found for the enhanced E2 transitions in the rotational region.

31. Approximate treatment of E1 interaction in alpha decay of Am^{241}. The E1 coupling terms for Am^{241} seem very small, but we should like to carry through an approximate treatment to determine whether the E1 coupling can

provide a simple explanation for the intensity of the highly hindered alpha group to the ground state. Preston[1] has suggested that some rare alpha groups might be explained by assuming the wave of the rare group to have zero amplitude at the nuclear surface. The wave would attain finite amplitude at larger distances by virtue of coupling terms to more abundant alpha groups. Preston's method[1] for solving the coupled equations for small energy difference should be suited to Am^{241}, since the energy difference of the coupled alpha groups is quite small. Another method due to Dancoff[2] we believe to be incorrect, in that the total rate of electric transitions from one alpha group to another is interpeted as the rate of emergence of the latter alpha group from the barrier. This method gives answers vastly over-estimating the importance of coupling.

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[1] M. A. Preston, Phys. Rev. 82, 515 (1951).

[2] S. M. Dancoff, U.S. Atomic Energy Commission Unclassified Document 2853 (1950) (unpublished).

Let us consider the S-wave to the 60-kev state as having unit amplitude at the nuclear surface and any waves to the ground state as having zero amplitude. Considering just these two states and only S- and P-waves, the wave equation of (29.7) reduces to two equations:

$$\frac{d^2u_0}{dr^2} - \frac{2M}{\hbar^2}\left(\frac{2Ze^2}{r} + E_0 - E\right)u_0 \pm \frac{2M}{\hbar^2}\frac{\epsilon}{r^2}u_1 = 0$$

$$\frac{d^2u_1}{dr^2} - \left[\frac{2M}{\hbar^2}\left(\frac{2Ze^2}{r} + E_1 - E\right) + \frac{2}{r^2}\right]u_1 \pm \frac{2M}{\hbar^2}\frac{\epsilon}{r^2}u_0 = 0 \qquad (30.1)$$

with $E_0 = 60$ kev and $E_1 = 0$.

The diagonal energy difference $(H_{11} - H_{00})$ is $\frac{\hbar^2}{Mr^2} + E_1 - E_0$, which shows a different radial dependence from that of the off-diagonal energies. $\frac{\hbar^2}{Mr^2}$ has the value of about 0.11 Mev near the nuclear surface ($r = 1 \times 10^{-12}$ cm). Thus, the diagonal energy difference will be positive out to about 1.4×10^{-12} cm and negative thereafter. Rather than attempting an exact solution of Equation (30.1) we shall solve two simpler approximate problems for order of magnitude estimates of the coupling effects.

In the first we let the diagonal energy difference be zero for all r. Then by addition of Equations (30.1) we obtain an uncoupled equation in $u_0 + u_1$, and by subtraction, an equation in $u_1 - u_0$. These equations differ only in the sign of an ϵ/r^2 diagonal energy term. The effect of such a term on the penetration factor for alpha decay has been derived in connection with centrifugal barrier effects. That is, the addition of an ϵ/r^2 term to the potential barrier causes an extra attenuation of an outgoing wave approximately given by the factor $\exp\left[\frac{-\epsilon}{\hbar e}\sqrt{\frac{M}{ZR}}\right]$. By this method if we take the boundary conditions at the nuclear surface of $\left|\frac{u_1}{u_0}\right|^2_{r=R} = 0$, then we find $\left|\frac{u_1}{u_0}\right|^2_{r\to\infty} = 2.5 \times 10^{-8}$, representing a build-up many orders of magnitude less than the experimental relative intensity ratio $\left|\frac{u_1}{u_0}\right| = 3.5 \times 10^{-3}$.

In the second approximate method we set $E_1 - E_0$ equal to zero and retain $\frac{\hbar^2}{Mr}$ as the diagonal energy difference. In a similar manner we find the linear combinations which decouple the equations. With the same boundary conditions as before we get $\left|\frac{u_1}{u_0}\right|^2_{r\to\infty} = 1.0 \cdot 10^{-7}$, a factor of four larger than by the first treatment, but still far less than the experimental intensity ratio.

We conclude that alpha coupling by the E1 radiation field is quite negligible in the Am^{241} case. It is possible that there could be larger E1 coupling effects in the diffuse surface region or nuclear interior due to short range nuclear forces. However, this phase of the problem cannot be as precisely formulated as the coulomb interaction problem.

It might be argued that external coupling would be larger in cases where E1 transitions are not so retarded as in the Am^{241} case. These retarded E1 transitions are probably fairly typical, at least of odd-mass nuclei, in the heavy region. It seems likely that all the known low-energy E1 transitions of odd A nuclei in the heavy region are much slower than given by the single-particle lifetime formula, by virtue principally of selection rules[3] in the asymptotic quantum numbers appropriate to large spheroidal deformation (See Sec. 56). We have no measured E1 lifetimes in the even-even nuclei, as only limits have been set at present. If E1 transition matrix elements of single-proton magnitude were taken in the Am^{241} problem above, the coupling could build up the relative wave amplitude $\left|\frac{u_1}{u_0}\right|^2$ from zero at the nuclear surface to order 10^{-2} at large distance.

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[3] G. Alaga, Phys. Rev. 100, 432 (1955).

32. Alpha decay of spheroidal nuclei; the E2 interaction. An important special case where the non-central coupling is especially large occurs for nuclei with large spheroidal deformation. These nuclei, odd mass as well as even-even types, generally exhibit rotational band structure in their level schemes. Many of these rotational band levels are interconnected by especially large E2 matrix elements (cf. Table X for E2 interactions for Cm^{242} and Am^{241}). The formulation of the alpha decay equations may be made in the same general manner as in the preceding section. The necessary equations giving the reduced transition probabilities within a given rotational band are given by Bohr and Mottelson (10) Equations (VII.18) and (VII.19).

These expressions together with Equation (29.14) allow the calculation of the magnitude of rotational quadrupole coupling terms in odd or even nuclear types. A general expression[1] giving the sign of the term as well may be derived by using Bohr-Mottelson strong coupling wave functions (10) (Equation II.15) to evaluate the invariant $\left(I_f \tau \middle\| \sum_p e r_p^2 \underline{\underline{C}}_p^{(2)} \middle\| I_f' \tau'\right)$. One makes use of the transformation (10) (Equation VII.13) of the electric multipole operator from the space-fixed to the body-fixed coordinate system.

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[1] J. O. Rasmussen and B. Segall, University of California Chemistry Division Quarterly Report (UCRL 2531) (April, 1954) (unpublished).

The general result for the quadrupole matrix element connecting states of a given rotational band (i.e., K and Ω unchanged) is

$$\left(K \Omega \ell I_f ; I M \middle| \frac{2e}{r^3} \frac{Q_0 e P_2(\cos\gamma)}{2} \middle| K \Omega \ell' I_f' ; I M\right)$$
$$= (-)^{I_f + I_f' - K - I} \frac{Q_0 e^2}{r^3} \frac{[(2I_f+1)(2I_f'+1)(2\ell+1)(2\ell'+1)]^{1/2}}{5} (I_f I_f' K -K | I_f I_f' 2 0) \tag{32.1}$$
$$\cdot (\ell \ell' 0 0 | \ell \ell' 2 0)\, W(\ell I_f \ell' I_f' ; I 2) .$$

For decay of even-even nuclei (I = 0) to the ground rotational band ($K = \Omega = 0$) the above expression reduces[2] simply to

$$\frac{[(2\ell+1)(2\ell'+1)]^{1/2}}{5}(\ell\ell'00|\ell\ell'20)^2,$$

which for $\ell = \ell'$ is equal to $\dfrac{\ell(\ell+1)}{(2\ell+3)(2\ell-1)}$ and for $\ell' = \ell + 2$ is equal to $\dfrac{3(\ell+1)(\ell+2)}{2(2\ell+3)\sqrt{(2\ell+1)(2\ell+5)}}$, and which vanishes whenever ℓ and ℓ' differ by other than 0 or ± 2.

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[2] J. O. Rasmussen, University of California Radiation Laboratory Report 2431 (1953) (unpublished).

For treating alpha emission of odd mass nuclei in the region of validity of the Bohr-Mottelson spheroidal deformation model it is for some purposes better to use a different representation for the functions in terms of which expansion (29.6) is made. Rasmussen and Segall (12) have used an expansion in which not only the nucleon coordinates (of recoil nucleus) are referred to a rotating body-fixed coordinate system (as in Bohr-Mottelson strong deformation model wave functions), but also the coordinates of the alpha particle are referred to this body-fixed system. The cylindrical symmetry axis of the nuclear spheroid is taken as the polar axis of the body-fixed system. We write the new functions

$$\Upsilon^{IM}_{K_f \ell m;\tau} = |\tau K_f \ell m; IM) = \chi_\tau(x_p') \sqrt{\frac{2I+1}{8\pi^2}}\, \mathcal{D}^{I}_{M\,K_f+m}(\Theta_i)\, Y_{\ell m}(\theta',\varphi') \tag{32.2}$$

where Θ_i are the three Eulerian angles specifying the position of the body-fixed nuclear coordinate system in space, the primed coordinates are with reference to the body-fixed system (the x_p' being coordinates of the nucleons in the recoil nucleus, and the θ', φ' being the alpha angular coordinates). $\sqrt{\frac{2I+1}{8\pi^2}} \; \mathcal{D}^{I}_{M \; K_f+m}(\Theta_i)$ is the normalized symmetric top rotational wave function, for angular momentum of I, component on the space-fixed z-axis of M, and component on the body-fixed z'-axis of $K_f + m$. The Υ representation contrasts with the $\mathcal{Y}$ representation used in (29.6) mainly in that m supplants I_f for the specification of the state of alpha particle motion. In general, a state in the Υ representation is a linear combination of several different states in the $\mathcal{Y}$ representation. The transformation (Equation 11) of Reference (12) is

$$\Upsilon^{IM}_{K_f \ell m ; \tau} = \sum_{I_f} (-)^{m+I_f-I} (I \ell \; K_f+m \; -m | I \ell \; I_f K_f) \; \mathcal{Y}^{IM}_{I_f K_f \ell ; \tau} \qquad (32.3)$$

When the expansion

$$\Psi = \sum_{\ell' m'} r^{-1} w_{\ell' m'}(r) \; \Upsilon^{IM}_{K_f \ell' m' ; \tau} \qquad (32.4)$$

is substituted into the Schrödinger equation with Hamiltonian (29.1), multiplied by $\Upsilon^{IM\,*}_{K_f \ell m ; \tau}$ and integrated over all coordinates except r, one gets a set of coupled ordinary differential equations similar to (29.7).

$$\left[\frac{d^2}{dr^2} + \frac{2M}{\hbar^2}(E - E_{\tau K_f}) - \frac{\ell(\ell+1)}{r^2} - \frac{4MZe^2}{\hbar^2 r}\right] w_{\ell m}(r)$$

$$- \frac{2M}{\hbar^2}\sum_{\ell' m'} w_{\ell' m'}(\ell m | H_{rot} | \ell' m') - \frac{2M}{\hbar^2}\sum_{\ell' m'} w_{\ell' m'}\left(\ell m \middle| V - \frac{2Ze^2}{r} \middle| \ell' m'\right) \tag{32.5}$$

$$= 0.$$

H_{rot} is the rotational energy Hamiltonian for the recoil nucleus, and it gives rise to the characteristic rotational band structure of energy levels. From the eigenvalue equation

$$H_{rot}\, \mathcal{Y}^{IM}_{I_f K_f \ell;\tau} = \frac{\hbar^2}{2\mathfrak{J}} I_f(I_f+1)\, \mathcal{Y}^{IM}_{I_f K_f \ell;\tau} \tag{32.6}$$

and the expansion (32.3) the required matrix elements of H_{rot} may be evaluated.

$$(\ell m | H_{rot} | \ell' m') = \frac{\hbar^2}{2\mathfrak{J}}\, \delta_{\ell\ell'} \sum_{I_f} (-)^{m-m'} I_f(I_f+1) \cdot$$

$$\cdot (I\, \ell\; K_f{+}m\; {-m} | I\, \ell'\, I_f\, K_f)(I\, \ell\; K_f{+}m'\; {-m'} | I\, \ell\; I_f\, K_f) \tag{32.7}$$

The last term in (32.5) involves matrix elements of the quadrupole interaction, and in this representation has a simpler form than in (32.1)

$$\left(\tau K_f \ell m; IM \middle| \frac{2e}{r^3}\frac{Q_0 e P_2(\cos\gamma)}{2} \middle| \tau K_f \ell' m'; IM\right)$$

$$= \frac{Q_0 e^2}{r^3}\int Y^*_{\ell m}(\theta,\varphi)\, P_2(\cos\theta)\, Y_{\ell' m'}(\theta,\varphi)\, d\omega \tag{32.8}$$

The integral is easily evaluated and contains no off-diagonal elements in m. The integral is equal to the $c^2(\ell' m', \ell m)$ tabulated by Condon and Shortley[3] but can be simply given in terms of Clebsch-Gordan coefficients as shown by Racah(30).

$$c^2(\ell' m', \ell m)$$

$$= \delta_{mm'} \frac{[(2\ell+1)(2\ell'+1)]^{1/2}}{5} (\ell \ell' 00 | \ell \ell' 20)(\ell \ell' {-m}\, m | \ell \ell' 20). \tag{32.9}$$

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[3] E. U. Condon and G. H. Shortley, (Theory of Atomic Spectra, Cambridge University Press, London, 1935,) p. 175.

An illustration of the usefulness of this representation is provided in the consideration of the favored alpha decay cases[4] (31) in odd mass number deformed nuclei. Favored alpha decay transitions are those in which the intrinsic wave function of the odd nucleon remains essentially unchanged. In such cases $\Delta K = \Delta \Omega = 0$. When the alpha particle is in the surface region of the nucleus we would expect nearly to conserve the component of total angular momentum of the system along the symmetry axis; hence, that near the nucleus all alpha components except those with $m = 0$ will be negligibly small. The matrix elements of the electric potential have no off-diagonal elements in m. Hence, the only build-up of alpha components with $m \neq 0$ will result from the matrix elements of H_{rot}, the nuclear rotational energy. In the limiting case of $H_{rot} = 0$ (infinite nuclear moment of inertia) the $m \neq 0$ components in favored alpha decay would be identically zero. We could set down exactly the branching of a given ℓ value between various nuclear rotational states I_f. The expansion (31.3) gives the desired branching. That is, a given ℓ-group branches as the squares of Clebsch-Gordan coefficients, $(I \ell K_f 0 | I \ell I_f K_f)^2$. The intensity relationship is quite similar to relationships for branching of gamma and beta transitions to various members of a rotational band. It is not obvious how good the relationship in alpha decay will be, since the matrix elements in H_{rot} are not zero. Bohr, Fröman, and Mottelson (31) take H_{rot} into account approximately by reducing the alpha intensity to higher rotational members by the reduction in barrier penetrability appropriate to the energy differences of the final states. They have used their approximation to calculate expected intensities to various final states of odd nuclei based on the empirical relative alpha intensities for neighboring even-even nuclei. The comparison of their approximation with experimental intensities is given in Table XVI.

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[4] J. O. Rasmussen, Arkiv f. Fysik 7, 185 (1953).

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33. Numerical results for alpha decay of spheroidal nuclei. Some numerical work on the alpha decay of deformed nuclei has been carried out.

R. F. Christy[1] has developed a useful analytical approximation with a new form of the WKB method. This approximation, which is most applicable in the limit of very large nuclear moments of inertia, treats the wave function from each point on the nuclear surface as penetrating outward radially while diffusing transversely in angle. A total solution corresponding to an arbitrary wave function at the nuclear surface can be constructed by superposition of the solutions from points on the surface.

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[1] R. F. Christy, Phys. Rev. 98, 1205A (1955).

L. Dresner[2] also has made use of the WKB method to derive a formula for the alpha penetration problem. He, like Christy, considers the limiting case of infinitely large nuclear moment of inertia.

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[2] L. Dresner, Ph.D. Thesis, Princeton University (1955) (unpublished).

Since Christy's equations have not been published, we are unable to make a detailed comparison between his and Dresner's WKB treatments. One significant difference seems to be that Dresner has dropped from his Equation (III.2) the term that would give rise to the centrifugal barrier reduction (or diffusion in angle), while Christy has retained this feature. Rasmussen and Segall (12) have derived the alpha decay equations in prolate spheroidal coordinates and have done both outward numerical integrations for Cm^{242} with simple assumptions on nuclear surface boundary conditions and inward integrations for Cm^{242} and Th^{228} with experimental alpha intensities setting the boundary conditions.

The work of Christy[1] and of Rasmussen and Segall (12) has shown that the boundary conditions making the alpha wave function constant over the spheroidal nuclear surface does not give for Cm^{242} the correct $\ell = 4$ alpha group intensity, the experimental value being smaller than that calculated. Work of Nosov[3] indicates that the assumption of constant wave function over the surface may be more consistent with experimental Cm^{242} alpha intensities if the nucleus is not prolate but oblate spheroidal. Other evidence weighs strongly for prolate deformation in the heavy region, so we are inclined to discount this interpretation. Furthermore the work of Rasmussen and Segall and of Dresner shows that the one-body model, with the alpha in its lowest virtual level within the well, does not give a low enough $\ell = 4$ group intensity, although the values are somewhat lower than for the uniform boundary condition.

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[3]V. G. Nosov, Doklady Akad. Nauk (SSSR) 103, 65 (1955). English translation, University of California Radiation Laboratory Translation 258.

Christy has found that by decreasing the alpha wave on the nuclear surface near the poles or by introducing some fourth order deformation in the nuclear surface the relative alpha group intensities observed in Cm^{242} can be theoretically obtained.

Rasmussen and Segall have for Cm^{242} and Th^{228} with $\ell = 0$, 2, and 4 groups taken the experimental alpha intensities to set the boundary conditions outside the barrier and have carried solutions of the wave equation into the spheroidal nuclear surface. (Recourse is made to a WKB approximation method.) There is an ambiguity in the choice of relative phases, the experimental intensities giving only a measure of wave amplitudes. For the treatment including three groups there are in general four choices of phase leading to different physically acceptable solutions. Most choices lead to a narrow wave distribution at the equator on the nuclear surface, and we are inclined to believe one of the choices of phase other than these to be the true one. The most direct evidence in support of our choice of relative $\ell = 0$, $\ell = 2$ phase comes from the interpretation of the α-γ angular correlation in Am^{241}, in which the phase difference determines the sign of the interference term in the correlation function. (See Section 56.) Fig. 18 shows the alpha probability ($|\psi|^2$) as a function of position on the nuclear surface from these inward integrations for the most likely choice of relative phases.

These results are of special interest in that they demonstrate most clearly the inadequacy of a one-body model, and they are suggestive of a model in which the alpha wave function at the surface is a measure of the angular overlap of the wave function of the most lightly bound nucleons. The distribution seems more dependent on proton than on neutron number.

The results of Fig. 18 are subject to some modification, since the deformations and intrinsic quadrupole moments assumed for the study appear to be about 50 percent too large compared to recent quadrupole moment determinations (Th^{232}, U^{238}, Am^{241}, Am^{243}) (See Sec. 14).

The coupled equations of alpha decay in spherical coordinates lend themselves to solution by high-speed digital computers, and computations in progress[8] may shed more light on the whole interesting problem of deducing alpha angular distributions near the nuclear surface.

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[8] J. O. Rasmussen and E. Hansen (unpublished).

On the basis of numerical work to date it is felt that the observed regular variations of $\ell = 2$ and $\ell = 4$ group hindrance (cf. Fig. 15) represent a regular shift of the area of highest alpha formation probability in the nuclear surface from near the poles (thorium alpha emitters) to zones midway between poles and equator (curium), and, perhaps, somewhat further toward the equator for higher elements. The development and shift of higher order deformations in the nuclear surface would provide an alternative interpretation, though.

IV. ALPHA DECAY AS A MULTIBODY PROCESS - (THE EVEN-EVEN NUCLEI)

34. Introduction. There have been numerous approaches suggested for treating the internal mechanics of the alpha decay process in a more realistic manner than in the one-body model. There is a general feeling that for a given radius of cut-off of the coulombic potential barrier a realistic model would give smaller decay rates for alpha decay than the one-body model. Bethe (18) early advanced arguments that the alpha particle should probably not be thought of as existing within the nucleus a large fraction of time.

The attempts at many body modifications of alpha theory may be classified into three categories: (1) statistical model ("level spacing") approaches analogous to the statistical theory of nuclear reactions, (2) "shell model" (overlap integral) approaches based on the individual particle (nucleon) nuclear models, and (3) the "resonanting group" approaches treating configuration mixing involving alpha particles explicitly in some configurations.

35. Statistical model. Bethe (18) suggested that the formation and emission of an alpha particle (~ 6 Mev) in the absence of barrier should occur about as rapidly as the emission of a neutron from a nucleus with slightly more excitation energy (~ 6 Mev) than needed to emit a neutron. Namely, he suggested a "decay constant for alpha emission in the absence of a barrier" of $\sim 10^{15}$ sec^{-1}, (i.e. $\delta^2 \approx 6$ ev), whereas Preston's lowest virtual level one-body treatment yields $\sim 2 \times 10^{20}$ sec^{-1} ($\delta^2 \approx 1.2$ Mev). The alpha decay channel radii calculated on this theory seem a little too large to accord with our present ideas of nuclear sizes.

Cohen[1] suggested that Bethe's approach should be modified, increasing Bethe's fundamental "reduced decay constant' of 10^{15} sec^{-1} by the ratio of the "average level spacing" near ground to the average level spacing at an excitation of ~ 6 Mev, thus giving a reduced decay constant ~ 10^{19} sec^{-1} ($\delta^2 \approx 50$ kev) which leads to more reasonable channel radii. The proportionality of neutron level widths to average level spacings had been noted experimentally and derived theoretically in the statistical theory of nuclear reactions of Feshbach, Peaslee, and Weisskopf.[2] The concept of an average level spacing at the ground state is a vague one. Cohen suggested taking ~ 100 kev for this average from the energies of first excited states in even-even nuclei. On this point Rasmussen[3] (32) asserted that the spacings to first excited states were not relevant, since they are generally of a different spin from ground. He suggested that the spacing to the next 0+ level would be more appropriate and proposed in the absence of experimental information that this energy might be of the order of the lowest order surface vibrational quantum energy, ~ 1 Mev.

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[1]B. L. Cohen, Phys. Rev. 80, 105 (1950).

[2]Feshbach, Peaslee, and Weisskopf, Phys. Rev. 71, 150 (1947).

[3]J. O. Rasmussen, Ph.D. Thesis, University of California (1952). University of California Radiation Laboratory Report UCRL-1473 rev.

Devaney (22) suggested taking first excited state energies for the average level spacing in absence of better information.

Various authors[3] (22) (23) have given theoretical derivations basing alpha decay rate on an average level spacing. In terms of the one-body model the level spacing treatment reduces the decay rate by the ratio of the real nuclear level spacing to that in the one-body model (~ 1 Mev). One assumes the reciprocal of the logarithmic derivative of the alpha wave function at the nuclear surface $S_i(W)$ to be a function of the form $1/S_i(W) = \tan z(W)$ with virtual levels at $z(W) = n\pi$, where $z(W)$ is some monotonic function of the total energy of the system. It can be shown (17) that δ^2, the reduced derivative width, is related to S_i and its derivative with respect to energy as follows:

$$\delta^2 = -\frac{S_i^2}{\frac{\partial S_i}{\partial E}} = \left[\frac{\partial\left(\frac{1}{S_i}\right)}{\partial E}\right]^{-1} \qquad (34.1)$$

The assumption is made[4] that $dz/dE = \pi/D^*$, where D^* is an average level spacing. Then at a resonance

$$\delta^2 = \frac{D^*}{\pi} \qquad (34.2)$$

Thomas (17) has briefly discussed a modification of the level spacing approach also. He gives as a formula

$$\left\langle \delta_\lambda^2 \right\rangle_{ave} \approx q_\alpha \, T_\alpha \left(\frac{D}{W}\right) \qquad (34.3)$$

with D the actual level spacing, T_α the kinetic energy of the alpha particle within the nucleus, w the characteristic single-particle level spacing within the nucleus ($W = \pi\hbar^2 K/MR$ where $K \sim 10^{13}$ cm^{-1}, characteristic of a nucleon of mass M within the nucleus). $q_\alpha \lesssim 1$ is a probability factor for the existence of the alpha particle on the nuclear surface. Thomas mentions nuclear reaction work as supporting values near unity for q_α.

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[4] This assumption is not correct in the square well one-body case, where $z(W)$ varies as $W^{1/2}$.

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36. Surface well model. Another modification of the level spacing approach was made by Winslow (21,27) who proposed a "surface well model." The attractive nuclear potential for the alpha particle is presumed to exceed the coulombic repulsive potential when the alpha particle is very near the nuclear surface but not yet "within" the nuclear matter. The boundary conditions from the level spacing estimate are applied at the inner surface, and the alpha will move in a surface region where its kinetic energy is positive before it enters the coulombic barrier. Winslow derives the decay rate theory for the one-body model of a single alpha particle trapped in the surface well and obtains a higher decay probability for given barrier radius than the traditional model yields. This result is to be expected, since the confinement of the alpha particle within the small volume of the surface means a greater kinetic energy for the lowest state and classically a higher collision rate with the barrier. Winslow then introduces the many-body concept of an equilibrium between the nuclear configuration with the alpha particle in the surface well and the configuration with none in the well but with constituent nucleons moving individually within the nucleus. The ratio of the probability of the former configuration to the total of configurations is designated as the preformation factor, P_L, which should reduce the one-body decay rate.

$$\delta^2 \approx P_L \cdot T_\alpha \qquad (35.1)$$

The level spacing model of Devaney (22) is used by Winslow to estimate P_L as

$$P_L \sim \frac{D}{2\pi\hbar}\left[\frac{v}{2\Delta R} + \frac{D}{2\pi\hbar}\right]^{-1} \approx \frac{D\cdot\Delta R}{\pi\hbar v}$$

where ΔR is the width of the surface well. Taking from Blatt and Weisskopf's Table 3.2 (23) a D for Po^{212} decay of 0.7 Mev, Winslow finds that an inner (nuclear matter) radius of $1.31 \times 10^{-13} A^{1/3}$ cm and $\Delta R = 1.2 \times 10^{-13}$ cm give agreement with the experimental rate. He points out that the slower decay rate of Po^{210} could be accounted for by a decrease of ΔR to 0.6×10^{-13} cm, leaving the inner radius unchanged.

The surface well picture may prove valuable in the future development of a detailed model of the alpha formation process. The square surface well model, as Winslow points out, represents an extreme idealization. The necessity of adjustment of the potential in the surface well to give the experimentally observed alpha decay energy to the lowest virtual state seems artifical, though, and the procedure of making the inside potential infinite needs more theoretical study.

Winslow's surface well model shows a regular decrease in decay rate with increasing angular momentum instead of the increase to a maximum at L = 2 as found in the traditional model (14).

37. Concerning the intra-nuclear alpha potential energy. The recent paper of Tolhoek and Brussard (33) offers fresh and promising lines of attack on the question of the relation between the nuclear density distribution and the attractive potential for alpha particles and also on the fundamental mechanism of formation of the alpha particle from constituent nucleons.

By a four-step energy cycle argument, assembling and breaking up an alpha particle inside and outside the nucleus, they estimate an intra-nuclear potential energy of ~ -134 Mev for alpha particles. This large attractive potential "tails off" beyond the nuclear matter radius to counteract the coulombic potential for some distance and gives a large effective channel radius for alpha decay. They assume an attractive potential between alpha and an element of nuclear matter of the form $-V_o \exp(-r^2/\beta^2)$ with $\beta = 1.6 \times 10^{-13}$ cm, thus yielding a potential from nuclear forces of

$$V(\underline{\underline{x}}) = \frac{-V_o}{\rho_o \pi^{3/2} \beta^3} \int d\underline{\underline{\xi}}\, \rho(\underline{\underline{\xi}})\, e^{-\frac{(\underline{\underline{x}} - \underline{\underline{\xi}})^2}{\beta^2}} \qquad (36.1)$$

where $\underline{V_o}$ = 167 Mev, $\rho(\underline{\underline{\xi}})$ is the density function for nuclear matter (for which they substitute a specific form), and $\underline{\underline{x}}$ is the position vector of the alpha particle. The results of the high energy electron scattering work are used to estimate the density function, and a function with the surface fall-off thickness parameter $\underline{s} = 1.26 \times 10^{-13}$ cm (as calculated by Ravenhall and Yennie[1] for gold) was used.

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[1]D. G. Ravenhall and D. R. Yennie, Phys. Rev. 96, 239 (1954).

It has been conventional to argue (18) that the alpha particle in the nuclear potential well, being a Bose-Einstein particle, must be in its lowest level. For a well of nuclear volume this lowest level involves about 0.5 Mev of kinetic energy, in strong contrast to the ~ 140 Mev of the above-discussed estimate. Devaney (22) has also discussed the question of the intra-nuclear alpha particle kinetic energy. He points out that the alpha particle, as a conglomerate of four Fermi-Dirac particles, can only be considered to obey Bose-Einstein statistics when the perturbations on its constituent particles are small compared to the binding of the particles in the conglomerate. This condition is not fulfilled in nuclear matter, and the four-particle must obey the exclusion principle to some extent because of its Fermi-Dirac constituents. Devaney estimates a kinetic energy of 5 Mev if the Fermi-Dirac statistics were obeyed by the constituents, and he then proposes an average figure of 3 Mev to be most nearly correct.

We cannot resolve the vast discrepancy between these estimates and feel the matter is still open to question. The Tolhoek-Brussard well depth of -134 Mev represents a large departure from the customary one-body model estimates. The question of the potential (and consequently the average alpha particle kinetic energy) seems a difficult one, and there are unanswered questions attached to the energy cycle estimate. First, the energy cycle argument (33) has as its weakest link the energy associated with the assembly of nucleons into an alpha particle within nuclear matter; in view of the saturation nature of nuclear forces and the extra kinetic energy of alpha particle internal motion the formation of the alpha cluster in nuclear matter is in all probability a rather endothermic process. The problem is related to the problem of treating residual nucleon-nucleon interactions in the individual particle model. These interactions, are responsible for configuration mixing in the shell model, and perhaps eventually the studies of specific nucleon-nucleon interactions within the nuclear well can shed light on questions concerning alpha particle clusters in nuclear matter.

38. Nucleon overlap model. Tolhoek and Brussard (33) propose a "shell model" picture for the probability of formation of alpha particles. The probability of finding an alpha particle in a particular position is equated approximately to the probability of finding the four constituent nucleons within the distance of the alpha particle radius (taken as $\beta_\alpha \approx 1.6 \times 10^{-13}$ cm). Then the alpha-clustering probability P_α of four particular nucleons is just proportional to the square of their overlap integral. Making the rough assumption of nucleon wave functions constant throughout the nuclear volume they obtain

$$P_\alpha = 64 \left(\frac{\beta_\alpha}{R}\right)^9 \tag{37.1}$$

The frequency factor for an alpha hitting the potential barrier is taken as

$$f = \frac{v_i}{2R} P_\alpha n_\alpha \tag{37.2}$$

where n_α is the number of ways the energetic alpha particle can be formed from nucleons in outer orbits. n_α is taken by them to be about 3. Applying the above frequency factor with a potential falling off as determined by Equation (36.1) to alpha decay of Po^{214} they calculate a nuclear matter radius of 6.8×10^{-13} cm, which corresponds to a constant of r_o in the $R = r_o A^{1/3}$ formula of 1.13×10^{-13} cm; consistent with the charge radii from electron scattering work. For this Po^{214} solution the preformation factor P_α is equal to 1.4×10^{-4}. This preformation factor of 1.4×10^{-4} will reduce the conventional one-body reduced width

($\delta^2 \approx 1$ Mev), but the 300-fold increase in kinetic energy acts in the opposite direction with a factor of about 40 (see Sec. 24 on one-body δ^2 estimates). The decreased reduced width is consistent with experimental data for the larger effective channel width derived by Tolhoek and Brussard from the exponential tailing off of the strong nuclear attractive force.

Tolhoek and Brussard also present a so-called dynamical estimate of the alpha formation probability, based on collision cross-section and mean free path arguments. The estimate yielded similar numerical results to the overlap integral estimate and was cited as adding confidence to the overlap integral results.

The nucleon overlap approach to the problem of alpha particle formation, while needing much further study, seems realistic and offers hope for future theoretical work. The great successes of the indiviudal nucleon models, contrasted with the limited utility of the alpha particle nuclear model, supports the idea that within the body of nuclear matter the state of motion is more easily described by representations involving independent nucleon wave functions than by representations involving clustered groups of nucleons. The tendency toward some clustering is expressible in the individual nucleon representation by configuration mixing. As we move from the center of a heavy nucleus into the surface region and beyond, we must expect an increasing tendency toward association or clustering in the alpha particle grouping. That is, the total probability of finding the energetic nucleons at a given distance falls off with distance, but the relative probability of their being associated in an alpha particle must increase with distance. This conclusion follows from the fact that for alpha emitters there is some small probability of finding alpha particles at any distance, though unassociated nucleons cannot be found at large distances. It would seem logical to set the channel radius near the transition between the inner region of independent nucleons and the outer region of associated nucleons. The matching of boundary conditions on the alpha wave function could be made at this radius. The channel radius is obviously arbitrary, but if set too far out, the individual nucleon representation will encounter difficulty in adequately representing the alpha cluster probability at the channel radius. On the other hand, if the channel radius is set too small, the treatment of the alpha particle as experiencing simply a real potential may fail close to the channel radius. The optimum channel radius $\underline{R_c}$ probably lies near the outside of the nuclear density distribution. If $\underline{R_c}$ lies in a region of positive kinetic energy of the alpha, then a form of Winslow's surface well picture may be applicable. If $\underline{R_c}$ lies in the negative kinetic energy or barrier region, more conventional treatments with coulomb functions determining the boundary conditions would be applicable.

The theoretical analysis of relative intensities to rotational band members for alpha decay of spheroidal nuclei promises to add much to our understanding of the process of alpha formation. These studies appear to indicate that the mean free path of alpha particles in nuclear matter is quite small. The explanation in terms of the individual nucleon wave function overlap seems promising. For a more detailed discussion refer to Sec. 32. and 33.

Wheeler[1] has set up a formalism incorporating mixing of configurations specifically composed of groups or clusters, such as the alpha particle. Although the use of a one-body mo el frequency factor reduced by a "preformation" factor lies essentially in the resonating group category, no rigorous attempt has been made to formulate such an approach for alpha emitters with due attention to redundancy of coordinates and the supplementary conditions on them, and the partial observance of the exclusion principle (discussed by Devaney (22)).

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[1] J. A. Wheeler, Phys. Rev. 52, 1083, 1107 (1937).

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39. The abnormal decay rates of alpha emitters with neutron number 126 or less. It is easy to recognize from the individual nucleon approach that the alpha formation probability would be lower for the even-even polonium isotopes with 126 or less neutrons. The 6*h* proton radial function is concentrated at the outermost part of the nucleus and overlaps well with the 7*i* or 6*g* neutron functions beyond 126 neutrons, but the 6*h* protons would overlap poorly with the 4*p* neutrons (most lightly bound) for neutron numbers of 126 and just below.

It also would seem reasonable that the cut-off radius for the barrier might be somewhat less for daughter nuclei with no nucleons outside the closed shells than for those possessing a few. The extra slowness of polonium alpha emitters has been discussed in terms of a shrinkage of the nuclear radius at the closed shell.(6).

It may be possible to decide whether formation probability (overlap) or radius shrinkage is the more important effect by examining in detail the reduced transition probabilities (RTP) of some polonium and emanation alpha emitters.

Of even-even alpha emitters with daughters from two to six nucleons beyond the closed shells, three (Em^{218}, Em^{220}, Ra^{222}) can be taken as a basis of the normal alpha decay. Their mean RTP[1] is 0.086. Po^{212} decays to Pb^{208} and should reflect any shrinkage of the effective channel radius at the closed shell but should not suffer any reduction in alpha formation probability from the overlap integral arguments. Its RTP is 0.025, a factor of 3.4 smaller than the base comparison. (The heavier poloniums have a mean RTP a factor of 2.3 smaller than the base comparison). The six even-even alpha emitters of polonium and emanation with 126, 124, and 122 neutrons exhibit a mean RTP of 0.0027, a factor of 32 smaller than the base comparison. Semi-quantitatively we may divide the retardation factor for these alpha emitters of $N \leq 126$ into a factor of three due to effective radius decrease and a factor of 10 due to lowered formation probability.

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[1] Em^{218} seems somewhat anomalously to decay too fast compared to its neighbors. Exclusion of Em^{218} from the comparison base would give an average RTP (between Em^{220} and Ra^{222}) of 0.073.

C. KINETICS OF ALPHA DECAY OF ODD NUCLEON TYPES

I. CORRELATIONS OF DECAY RATES OF ODD NUCLEI

40. Hindrance factors for odd nuclei. In contrast to the regular behavior of decay rates of even-even alpha emitters the odd nucleon types show great variation in their rates. Sometimes transition probabilities of alpha groups are comparable to those of neighboring even-even nuclei; in other cases the rates are much slower.

It is most useful in the region of regular decay rate behavior of even-even nuclei to try to compare the reduced decay rates of alpha groups of an odd nucleus to the ground state rates of its nearest even-even neighbors. The factor by which the alpha group in question decays slower than this "normal" rate is usually called the hindrance factor (which we designate by F) (See Section 17 and 42.)

One might calculate effective nuclear radii for the even-even neighbors using some form of one-body rate theory and apply an average radius value to calculation of the "normal" decay rate for the odd emitter. A simpler and nearly equivalent method is to construct a family of normal half-life vs. energy curves (Fig. 14, Sec. 16) that closely represent the behavior of the even-even ground decay groups and then to read normal alpha half-lives for odd groups from these curves. For the odd-Z isotopes it is necessary to define lines between the neighboring even-Z lines, and these lines are usually chosen to give a mean decay rate between even-even neighbors of the same decay energy.

As discussed in Section 16, in the region above the closed shells the partial alpha half-lives of even-even alpha emitters of a given element may be well represented by expression (16.1).

$$\text{Log } t_{\frac{1}{2}\alpha}(\text{sec.}) = A\, Q_{eff}^{-\frac{1}{2}}(\text{Mev}) + B \qquad (16.1)$$

The justification of this particular form by decay rate theory was discussed in Section 20. The constants A and B obtained from a least squares analysis were tabulated in Table II.

In Table XI we supplement Table II with the interpolated constants for the odd-Z lines used as a basis for hindrance factor calculations.

Fig. 19 shows a plot of the "normal" decay rate lines with points for all the known ground state groups of even-odd nuclei. The distance of a point above its line is proportional to the logarithm of its hindrance factor, F. That is,

$$\text{Log } F = \text{Log } t_{1/2\alpha} - \left(A\, Q_{eff}^{-\frac{1}{2}} + B\right) \qquad (40.1)$$

Tables XII, XIII, and XIV list Log F for all alpha groups of even-odd, odd-even, and odd-odd nuclei with 128 or more neutrons. (This excludes the region around 126 neutrons where decay rate properties of even-even nuclei change sharply and the hindrance factor concept is not well defined.)

The distributions of logarithms of hindrance factors for all the established alpha groups of odd nuclei beyond the closed shells ($Z \geq 84$, $N \geq 128$) are given as histograms in Fig. 20 (odd-evens), Fig. 21 (even-odds), and Fig. 22 (odd-odds). The bold line in each case divides the alpha emitters with $N \leq 138$ (above) from those with $N > 138$ (below), roughly the dividing line between the region of spherical nuclear shape and that of stable spheroidal shape.

In the $N \leq 138$ region one notes in Figs. 20, 21, and 22 generally many alpha groups with low hindrance factors for all odd types including odd-odd. Exceptional are the alpha emitters Ra^{223}, Ac^{225}, and Th^{227}, which exhibit great complexity with many highly hindered groups.

In the N>138 spheroidal nuclear region the odd mass nuclei fall roughly into two broad groupings, the majority of cases in a broad group of most probable hindrance factor about five and a few cases in a group around hindrance factor 500.

Odd-odd nuclei of the N>138 region are probably generally very highly hindered. E^{252} and E^{254} constitute two of the three known cases in the whole region. It may be significant that these alpha emitters lie beyond 152 neutrons, where a small discontinuity ("subshell effect") in neutron binding energies is observed. Also, it is probably significant that E^{253} exhibits essentially unhindered ("favored") decay to the ground state of Bk^{249}, a unique case for odd-even nuclei of the N>138 region. One might be justified in treating the region of N>152 as distinct from the other regions, although we have not done so.

It is evident that the hindrance factor as defined and calculated above is not necessarily a very fundamental measure of the alpha group probability at the nuclear surface. The centrifugal barrier effects associated with angular momentum of alpha decay are not taken into account, and neither are the effects of the large non-central electric quadrupole interactions in the spheroidal region.

Centrifugal barrier effects on the wave function in the external region are well worked out and should be incorporated in more detailed comparison of alpha decay rates, where spin changes are known. The large electric quadrupole interactions in the region of deformed nuclei should be taken into account, but this is a problem of greater difficulty.

For the odd nuclei not included in the hindrance factor tables (XII, XIII, and XIV) we have calculated by the prescription of Winslow (16) (our Equation 27.9) the alpha surface probabilities, assuming L = 0 and, where the true angular momentum values may be inferred (see Section 49-52), the reduced transition probabilities (RTP) which are surface probabilities correctly including the centrifugal barrier factor, (Equation 27.6) (See the list of definitions in Section 42). These results are presented in Table XV.

41. Decays across the major closed shells. The order of magnitude of reduced hindrance factor (cf. Section 42) for decay across closed shells, compared to the even-even alpha emitters above the closed shells (Em^{218}, Em^{220}, Ra^{222}) (mean RTP = .086) may be summarized as follows:

(α) Decays crossing one closed configuration.

The odd and even mass bismuth isotopes with $N \geq 126$ have reduced hindrance factors of order 200. The Po^{211} (9/2+) groups to p configurations experience a similar reduced hindrance of about 100 and to $f_{5/2}$ about 1000. We tentatively associate this retardation mainly with the reduced overlap integral where one constituent nucleon forming the alpha particle is concentrated well within the nuclear volume and its radial wave function overlaps poorly with the three high angular momentum nucleons beyond the shell. The smaller RTP to $f_{5/2}$ than to the p configurations in Po^{211} decay is the reverse of the simple expectation, though. The high spin Po^{211} isomer has reduced hindrance factors (assuming 19/2-) of 5.7×10^5 to $p_{1/2}$, 1.5×10^5 to $p_{3/2}$, and 2.5×10^3 to $i_{13/2}$.

(β) Decays crossing both closed configurations, Bi^{210}.

The low spin isomer (RaE) has a reduced hindrance of 5.4×10^4 for L = 0, while the high spin isomer has reduced hindrance of 2.0×10^6 with L = 4 assumed.

(γ) Decays for $Z \geq 84$ and $N \leq 126$.

As discussed in Section 39 most even-even nuclei of the region below 126 neutrons have a reduced hindrance factor of about 30. Several of the odd alpha emitters exhibit more than one group. They range in hindrance from about that of their immediate even-even neighbors to as much as an additional factor of 50.

42. Summary of proposed terminology.

Hindrance factor (F): Ratio of the experimental partial alpha half life to a "normal" value based on ground-state transitions of even-even neighbors. For excited state transitions in an even-even nucleus the basis of comparison is to be taken as its own ground-state transition rate. For all odd nuclei we have defined the "normal" rate from the semi-empirical correlation of even-even decay rates in Section 16. Formally, $\text{Log } F = \text{Log } t_{1/2\,\alpha} - (A_z Q_{eff}^{-1/2} + B_z)$. Centrifugal barrier and non-central interaction effects are ignored in calculating hindrance factors.

Reduced hindrance factor: The same as hindrance factor except that centrifugal barrier effects (and non-central interaction effects where important) have been taken into account. The correction of F for centrifugal barrier effects consists simply in multiplying F by $(G_L/G_o)^{-2}$. See Table V and Equation 27.6.

Surface probability: A quantum mechanical probability expression for a given alpha decay group, proposed by Winslow (16) and denoted as $R\,|\phi_{J,I}(R)|^2$. The outgoing coulomb wave $\phi_{J,I}(r)$ is normalized such that the total outgoing flux is proper for the given partial alpha half-life if there is a unit probability of finding the alpha in the nuclear region. The probability function $|\phi_{J,I}(R)|^2$ is made dimensionless by multiplying by R.

For the calculations of the present paper the function is evaluated at $R = 9.3 \times 10^{-13}$ cm. for heavy elements and at $R = 8.0 \times 10^{-13}$ cm. for rare earths. For the present paper the "surface probability" will be calculated without considering centrifugal barrier or non-central interaction effects, just as is the hindrance factor.

Reduced transition probability (RTP): This is the real "surface probability", correctly taking into account centrifugal barrier and non-central interaction effects.

Reduced derivative width (δ_L^2): A fundamental measure of the hypothetical "decay rate in the absence of a barrier." The decay rate constant without barrier, λ_0, is equal to δ_L^2 divided by Planck's constant, h. (See Equation 27.1).

II. ALPHA DECAY AS A MULTIBODY PROCESS
(THE ODD NUCLEI)

43. General observations. It was noted early that odd nuclei tend to decay at a generally slower rate than neighboring even-even nuclei. From the known data on alpha groups of odd nuclei we see by referring to the histogram distributions (Figs. 20, 21, and 22 of Section 40) of hindrance factors that the most probable value of the hindrance factor is not much greater than unity, with the probability gradually falling off toward the higher hindrance factors. The only possibly significant separate groups in the distributions lie near hindrance factor 500.

The attempts to explain hindrance factors solely on the basis of the centrifugal barrier for higher angular momentum waves cannot possibly account for the hindrance factors near 1000. An angular momentum of 5, the maximum allowable for the ground state alpha group of Am^{241} (Initial, 5/2-; Final, 5/2+) has an extra centrifugal barrier reduction factor of only

12, compared to the actual hindrance factor of 520. Following such arguments as this, Perlman et al. (6) suggested that the large hindrance factors are to be associated mainly with abnormally small preformation factors for the assembly of an alpha particle from unpaired nucleons.

44. Favored alpha decay intensities. The development of the Bohr-Mottelson model for deformed nuclei and the accumulation of more precise data on alpha fine structure and nuclear excited states has made possible some really quantitative correlations of decay rates of a few selected alpha groups in odd nuclei. It was suggested[1] that the main three alpha groups

[1] J. O. Rasmussen, Arkiv. f. Fysik 7, 185 (1953).

of Am^{241} populate excited levels of a single rotational band, with the spacings suggesting the assignment of I = 5/2 to the base state of the band. The spin of the Am^{241} ground state was known to be 5/2, and the main alpha decay group to the 60-kev state exhibited no hindrance from normal even-even alpha decay rate behavior (hindrance factor about unity). Thus, it seemed logical to theorize that the odd proton wave functions in Am^{241} and the 60-kev state of Np^{237} were essentially the same. Perlman and Asaro (4) were soon to point out cases in other odd-mass nuclei where essentially unhindered alpha decay groups populated three members of a rotational band (Am^{243}, U^{233}, Cm^{243}). Bohr, Fröman, and Mottelson (31) (referred to hereafter as BFM) directed further attention to these special decay groups and proposed the term "favored" for those alpha transitions in which the intrinsic wave function of the odd nucleon remains essentially unchanged. They, furthermore, derived a simple approximate relationship between the intensities of favored alpha decay to various rotational band members and the intensities of alpha decay groups in neighboring even-even nuclei. (See Section 32). Their formula is exact only in the limit of infinite nuclear moment of inertia. For favored alpha decay from a nucleus with spin $I_o = K_o$ to a band in the daughter of like K, the L = 0 alpha group can only go to the base level by conservation of angular momentum. The L = 2 group can generally divide between more than one band member. The relative intensities of the L = 2 branches by the BFM theory are proportional to $P(Z,E) \cdot (I_o 2 K_o 0 | I_o 2 I_f K_o)^2$, where the P's are barrier penetration factors and the other factor is the square of a Clebsch-Gordan coefficient. The L = 4 wave generally populates more levels, and the relative branching is determined in a similar way. The relative intensities of L = 0, L = 2, and L = 4 waves are estimated from the relative alpha intensities to rotational band members in neighboring even-even nuclei. For much of the heavy region the L = 4 group is almost negligible. The relative transition probabilities are given by

$$\lambda \propto P_0(Z, E_f) \sum_L C_L \left(I_i L K 0 | I_i L I_f K\right)^2 \qquad (44.1)$$

where C_L is the reciprocal of the hindrance factor for the alpha group of angular momentum L in neighboring even-even nuclei.

As a demonstration of the kind of agreement between this theory and experiment we reproduce in Table XVI the Table IV of BFM, brought up to date with the most recent data and using revised estimates for the C_L's.

There is good semi-quantitative agreement, and most discrepancies may possibly be explainable when numerical work is performed without the assumption of infinite nuclear moment of inertia.

Two cases were calculated for E^{253}. The level spacings support the K = 11/2 assignment, but in view of possible level perturbations the K = 7/2 possibility has also been calculated. (These two spins are the only high spins that appear reasonable for Z = 99 by Nilsson's (13) spheroidal well calculations.) The intensity comparison rather supports the lower K.

The theory of intensities of favored alpha decay really avoids the basic problem of alpha formation, since it makes use of experimental intensity data from even-even nuclei. This theory neatly exploits the similarity between favored alpha decay and decay of even-even nuclei.

45. Hindered decay; selection rules; and approaches to the problem. Going beyond the favored decay groups to the hindered groups of odd nuclei we find a territory still uncharted by theory. Here we only hope to define the problem and indicate some of the possible paths that may bear future exploration. The empirical hindrance factor distributions of Section 40 offer few clues, as they show only slight tendency toward any grouping. There is a group of hindrance factors in the neighborhood of 500, separated slightly from the main body of lower hindrance factors.

At the outset we may set down the strict selection rules for alpha decay, based on conservation of total angular momentum and parity.

$$|I_i - I_f| \leq L \leq I_i + I_f \qquad (45.1)$$

Only even values of L are allowed if the parity of initial and final nuclear states is the same and only odd values of L allowed if there is a parity change. If either I_i or I_f is zero, it is possible to have cases where alpha decay is strictly forbidden. For example, decay of an even-even nucleus (0+) cannot populate odd spin states of even parity or even spin states of odd parity.

Alpha decay rate behavior stands in marked contrast to that of beta and gamma decay in that angular momentum change per se plays a relatively minor role. Alpha decay involving an angular momentum of eight units (Cm^{242}) has been detected in competition with L = 0 emission. In contrast, the associated angular momentum is the dominant factor separating beta and gamma emission rates into groups. The relative insensitivity of alpha transition rates to angular momentum means that mixtures of L values may frequently be encountered in alpha decay of odd nuclei, a matter of special importance to angular correlation studies involving alpha radiation.

For theoretical interpretation of the alpha emission process for groups other than "favored" we can recognize three areas that need consideration and exploration: First, the influence of non-central interactions, both electromagnetic and short-range nuclear; second, configuration mixing in parent or daughter that may allow alpha emission from a paired nucleon structure; and third, the direct formation of alphas from constituent nucleons in unpaired configurations.

46. Influence of non-central interactions. The first area, non-central interactions, has been dealt with in Secs. 28-33, and we only take time briefly to recapitulate here.

The coupled radial wave equations in the region beyond the nucleus have been precisely formulated to include the effects of electric multipole radiation fields. Numerical application of the equations has been quite limited to date, but it seems clear that the fast rotational electric quadrupole fields play an important role in the relative alpha group intensities to states connected by these E2 matrix elements. As yet, only the E2 coupling within a given rotational band has been treated numerically, but relatively large

E2 matrix elements may also connect states of different bands where there has been configuration mixing.

E1 matrix elements of the order of magnitude of the single proton estimate[1] or larger could play a significant role. However, many of the low-energy E1 transitions in the alpha emitter region are of measured half-life and retarded by more than 10^4 from the single particle lifetime. The numerical example of Am^{241} in Section 30 showed that the intensity of the hindered ground state group could not be accounted for by assuming the group to be of zero amplitude at the nuclear surface and considering the build-up due to E1 coupling to the favored group to the 60-kev state. There is theoretical reason to believe that the odd mass nuclei with E1 lifetimes too short to be measured may still be considerably retarded from the single proton formula[1] expectation. Hence, electric dipole coupling probably has negligible influence on alpha group intensities of odd-mass nuclei. (The lifetimes of E1 transitions from 1- states of even-even nuclei are more uncertain, only upper limits being known.)

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[1] S. A. Moszkowski, Chap. XIII in Beta and Gamma Ray Spectroscopy; Kai Siegbahn, Editor; North-Holland Publishing Co., Amsterdam, 1955. p. 391.

The possibility exists that short-range nuclear forces or surface polarizations may exert important coupling on the alpha particle in the nuclear surface region. The formulas of Secs. 28-33 could be adapted to study this coupling if some form and strength of the interaction were to be assumed.

47. Influence of configuration mixing. In the second area, we must consider configuration mixing in parent or daughter that may allow alpha emission from the even-even paired structure of the nucleus.

In the region of a strongly deformed nuclei the ordinary Bohr-Mottelson wave function (Equation II.15 of Ref. 10) for an odd-mass nucleus will consider all nucleons paired except the odd one, and the odd nucleon will move within the deformed well such that the projection (Ω) of its angular momentum about the cylindrical symmetry axis is conserved. Certain perturbations, as the rotation-particle-coupling (RPC)[1] and residual nucleon-nucleon interactions will mix the zero order wave functions somewhat. Kerman's analysis[1] of the levels of W^{183} shows the effect of the rotation particle coupling in mixing of bands of the same parity but differing in Ω (and K) by one. Other interactions may connect still other pairs of intrinsic states.

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[1]A. K. Kerman, Dan. Mat. Fys. Medd 30, No. 15 (1956).

Obviously, we can again, as for favored alpha decay, exploit empirical knowledge of the alpha decay rates of neighboring even-even nuclei to interpret decay of odd-mass alpha emitters wherever configuration mixing of states gives in parent and daughter some configurations with identical odd nucleon wave functions. Expression of these concepts might be treated in the formalism of the fractional parentage coefficients.

Knowing the general importance of the rotation-particle-coupling, we may predict that high hindrance factors in deformed nuclei should not usually occur where initial and final states in the alpha transition differ by one unit in Ω (and K) and S- or D-wave alpha emission is allowed by spin and parity selection rules. No systematic test of this hypothesis will be attempted here, but we know of no violations in the highly hindered (~500) groups, most of which involve a parity change and some of which (i.e. Cm^{243}) involve an Ω change of two units with no parity change.

Alpha decay groups involving a parity change may also be partly or wholly accountable by configuration mixing, especially in the region (near $\underline{A} = 224$) where the $\underline{I} = 1-$, $\underline{K} = 0$ states in even-even nuclei lie lowest and where the hindrance factors for decay to them are relatively low. That is, one would consider the mixing in of configurations in which the odd nucleon is promoted to an orbital of opposite parity (from the principal configuration) and is coupled with an even-even core in an odd parity state of collective motion.

48. Direct formation from unpaired nucleons; the overlap model. The third area to consider for non-favored alpha decay groups includes the process of direct formation of alpha particles from constituent nucleons, at least two of which are not paired to each other in the individual nucleon configuration. The elucidation of the participation of this third mechanism would be of great fundamental interest.

The direct formation mechanism may be amenable to theoretical treatment by a Tolhoek-Brussard-type individual nucleon approach, (33) in which the alpha formation probability inside the nucleus is proportional to the overlap of the four constituent nucleon wave functions. It seems obvious that the overlap will be a good deal less for orbitals not in a pair than for paired orbitals, since the configuration interaction whereby paired nucleons may increase their overlap would generally be absent for the nucleons not paired together. For alpha emitters decaying across a closed shell (the Bi isotopes and 127-neutron isotones) the radial overlap integral should be extremely poor, since it involves nucleons in different shells with greatly differing angular momentum. (See Section 41).

D. ILLUSTRATIVE INDIVIDUAL DECAY SCHEMES

I. ALPHA DECAY ACROSS A CLOSED SHELL

49. Bismuth-211 and Bismuth-213. Bi^{211} (AcC) decays across the 82-proton shell according to the decay scheme of Fig. 23. With the spherical well shell-model assignments of Pryce (9) (refer to Section 15, Fig. 12) the ground-state alpha group must be pure *L* = 5 and the lower-energy group may have *L* = 3 or 5. When Winslow's (16) (see our Table XV) calculated surface probabilities at 9.3×10^{-13} cm are corrected for angular momenta 5 and 3, respectively, the reduced-transition probabilities are 6.1×10^{-4} for the ground-state transition and 5.8×10^{-4} for the other transition. It is interesting to note how nearly equal these values are though both are highly hindered in comparison with nearby even-even nuclei (Em^{218}, Em^{220}, Ra^{222}: RTP ≈ 0.1). The extraordinary slowness may be associated qualitatively with a lowered formation probability (poor overlap of constituent protons in parent nucleus).

The 353-kev gamma transition in Tl^{207} has been shown[1] to have a *K*-conversion coefficient $\alpha_k = 0.18 \pm 0.03$ and a K/L conversion ratio of 5.5. With the screened relativistic theoretical conversion coefficients of Rose[2] the transition appeared to be an *M1*-*E2* mixture, but with the recent refined theoretical coefficients of Sliv[3] the best interpretation would be pure *M1*. (Theoretical α_k (*M1*) = 0.20.)

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[1]Falk-Vairant, Teillac, and Victor, J. phys. radium 13, 313 (1952).

[2]M. E. Rose, Privately circulated tables and Appendix IV of Beta and Gamma Ray Spectroscopy; Kai Siegbahn, editor (North-Holland Publishing Co., Amsterdam, 1955).

[3]L. A. Sliv, Unpublished tables (1956).

Gorodetzky et al.[4] have performed alpha-gamma angular correlation measurements with the 353-kev gamma transition and find an essentially isotropic distribution. The lifetime of the state was measured[1] as less than, or equal to, 1.2×10^{-9} sec.; with such a short lifetime attenuation effects due to extranuclear fields would not be expected to be too serious. From the lack of angular correlation it was suggested[4] that the intermediate state be assigned $s_{1/2}$ and the ground state of Tl^{207}, $d_{3/2}$, the reverse of Pryce's (9) proposals and of our Fig. 23. We still prefer the $s_{1/2}$ ground-state assignment, mainly on grounds of the beta decay of Tl^{207} being "favored" first forbidden (log ft = 5.2) by the analysis of King and Peaslee.[5] With the Fig. 23 spin assignments isotropy in the alpha-gamma correlation should occur with 87% L = 3 and 13% L = 5 alpha radiation, a reasonable admixture (or with 13% L = 3 and 87% L = 5).

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[4] Gorodetzky, Gallmann, Knipper, and Armbruster, Compt. rend. 237, 245 (1953).
[5] R. W. King and D. C. Peaslee, Phys. Rev. 94, 1285 (1954).

Bi^{213} exhibits a single alpha group to the ground state in Tl^{209}. We assume angular momentum 5 for the alpha-decay group and calculate a reduced transition probability of 9.0×10^{-4}, quite comparable to those in Bi^{211}.

50. The Polonium-211 isomers. Po^{211} (AcC') decays with a 0.52-sec. half-life across the 126-neutron shell to the well-studied Pb^{207}. A 25-sec. alpha-emitting isomer was discovered by Spiess.[1] Further work of Jentschke et al.[2] led to the decay scheme of Fig. 24. The spin of the ground state, AcC', is expected to be 9/2, involving mainly a configuration with the two protons beyond Pb^{208} as $(h_{9/2})^2$ $J = 0$ and the odd-neutron as $g_{9/2}$. Arguments[2] have been given for the spin of the isomer being at least 19/2. The most

reasonable configuration might involve $[(j_{15/2})\ (h_{9/2})^2_{J=2}]_{19/2}$ the two $h_{9/2}$ protons coupled to a $J = 2$ state, with the odd neutron in $j_{15/2}$.

[1]F. N. Spiess, Phys. Rev. 94, 1292 (1954).

[2]W. Jentschke, A. C. Juveland, and G. H. Kinsey, Phys. Rev. 96, 231 (1954).

The possible alpha angular momenta for AcC' are as shown in Fig. 24. We calculate, taking the lowest values of L in all cases, reduced transition probabilities of 7.2×10^{-4} to $p_{1/2}$ ground state, 5.9×10^{-5} to $f_{5/2}$, and 9.5×10^{-4} to $p_{3/2}$. The similarity in the values to the p states is interesting, as is their similarity to the Bi^{211} values. The greater retardation of the transition to $f_{5/2}$ is the reverse of expectation in terms of the naive radial wave-function overlap considerations. (See Sec. 48).

If we take, speculatively, the assignment 19/2- for the 1.30-Mev isomer, the allowed angular momenta are 10 to the $p_{1/2}$; 8 or 10 to the $p_{3/2}$; and 3, 5, 7, 9, 11, 13, or 15 to the $i_{13/2}$. The calculated reduced transition probabilities (lowest L) are as follows:

to $p_{1/2}$, 1.5×10^{-7},
to $p_{3/2}$, 5.9×10^{-7},
to $i_{13/2}$, 2.9×10^{-5}.

The extra amount by which these transitions are slower than from the AcC' isomer might be attributed to the unpaired proton configuration.

51. Bismuth-212. Bi^{212} (ThC) decays 33.7 percent by alpha emission and the remainder by beta minus decay. Six alpha groups are known, the two main groups populating ground state and a 39.85-kev excited state which decays to ground state by an M1 transition. The weaker alpha groups populate a group of four upper excited states.

The spin assignments 5+ and 4+ for ground state and first excited state were proposed theoretically by Pryce (9) and are supported by angular-correlation studies and analysis by J. W. Horton.[1] The spins of the upper four levels are somewhat uncertain and are omitted from our Fig. 25. Pryce has suggested that they constitute a quartet of levels with a $(d_{3/2})_P^{-1}$ $(g_{9/2})_N$ configuration and spins of 3, 4, 5, and 6. All the gamma transitions drawn on Fig. 25 are of M1 multipolarity, the assignment for the energetic gamma rays being due to work by O. B. Nielsen.[2] Nielsen's results appear to contradict all of Pryce's spin assignments for the upper quartet of levels. (Nielsen's Fig. 2 is supposed to present Pryce's spin assignments, but these were apparently incorrectly copied from Pryce's (9) Table 3.)

[1] J. W. Horton, Phys. Rev. 101, 717 (1956).

[2] O. B. Nielsen, Dan. Mat. Fys. Medd. 30, No. 11 (1955).

The alpha group to the ground state exhibits a reduced transition probability of 3.2×10^{-4}, and that to the upper state, 2.7×10^{-4}. It is interesting that the values are quite comparable to one another and to those in Bi^{211} and Bi^{213}. Alpha-gamma angular correlation offers a test for the alpha angular momenta of the group to the 40-kev state. Several workers have performed alpha-gamma angular correlation experiments, and we quote the value obtained by Horton[1], for the anisotropy $W(\pi)/W(\pi/2) - 1 = -0.229$. Horton[1] has discussed many possible spin assignments. Here we consider only those of Fig. 25. Figure 26 plots the theoretical anisotropy as a function of the mixing percentage between $L = 3$ and 5. The calculation assumes pure M1 gamma radiation. The experimental anisotropy agrees with pure $L = 3$ or nearly pure $L = 5$. $L = 3$ seems the better choice on decay-rate grounds. Actually it seems rather surprising to find such pure radiation in this case, since the centrifugal barrier factor for L = 5 is only 4.6 times the factor for L = 3. Angular correlation involving alpha radiation is especially susceptible to attenuation; therefore, it is conceivable that the unperturbed anisotropy slightly exceeds that measured, though the lifetime[3] is extremely short ($t_{1/2} < 7 \cdot 10^{-11}$ sec.)

[3] R. L. Graham and R. E. Bell, Can. J. Phys. 31, 377 (1953).

52. The bismuth-210 isomers. The long-lived isomer of Bi^{210} decays by alpha emission across both closed shells to Tl^{206} with an alpha-particle energy of 4.93 Mev and a half-life of $2.6 \cdot 10^{6}$ yr.[1] Its decay scheme is shown in Fig. 27. From Levy and Perlman's[2] determination of β^- branching (log ft ~18.7) and lack of isomeric transition the most likely assignment is (4-). Tl^{206} is (1-) $(s_{1/2})_P^{-1}\ (p_{1/2})_N^{-1}$. An L-value of 4 is permitted, and we calculate a reduced-transition probability of $4.2 \cdot 10^{-8}$. This result must be taken as somewhat uncertain, with the possibility that the spin may be in error or that the decay may not proceed to the ground state of Tl^{206}.

The five-day beta-emitter Bi^{210m} (RaE) is probably (1-) with mainly a mixture of configurations $(i_{13/2})_P(j_{15/2})_N$ and $(f_{7/2})_P(g_{9/2})_N$, and it has been observed to undergo alpha branching to the extent of $5 \cdot 10^{-5}$%(3), or $1.7 \cdot 10^{-4}$%(4). L values of 0 and 2 should be allowed. Assuming L = 0 and taking the latter branching ratio value we calculate the RTP as $1.6 \cdot 10^{-6}$.

[1] D. J. Hughes and H. Palevsky, Phys. Rev. 92, 1206 (1953).

[2] H. B. Levy and I. Perlman, Phys. Rev. 94, 152 (1954).

[3] E. Broda and N. Feather, Proc. Roy. Soc. (London) A190, 20 (1947).

[4] Fink, Warren, Robinson, and Edwards, Bull. Am. Phys. Soc. 1, Ser. II, 171, (1956).

II. ALPHA DECAY IN THE REGION OF SPHEROIDAL NUCLEI

53. Thorium-230 and the levels of radium-226. The spectrum of levels exhibited by Ra^{226} (Fig. 28) is an interesting one, for in addition to the normal even-parity rotational sequence A0, A2, A4, and perhaps A6 at 416 kev, there appear at least one (B1) and probably two other (B3, B5) odd-parity states at surprisingly low energy for even-even nuclei. Th^{230} has been the subject of many careful α-γ and γ-γ angular correlation measurements, and this work constitutes an outstanding example of the wealth of information obtainable by the angular correlation method.

Alpha spectroscopy[1] has directly established the two lowest levels. The next three levels are firmly established by α-γ and γ-γ coincidence studies.[2] The evidence for the 416- and 445-kev levels is not entirely conclusive, consisting of the observation[3] of two weak gamma rays of 206 ± 5 kev and 237 ± 5 kev in coincidence with the 124-kev gamma (A4-A2).

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[1]Rosenblum, Valadares, Blandin-Vial, Bernas, Compt. rend. 238, 1496 (1954).
Hummel, Asaro and Perlman, Unpublished data (1955).

[2]Booth, Madansky, and Rasetti, Phys. Rev. 102, 800 (1956).
G. Valladas and R. Bernas, Compt. rend. 91, 2230 (1953).
Perlman, Asaro, Stephens, Hummel, and Pilger, University of California Radiation Laboratory Unclassified Report 2932 (Chemistry Division Quarterly Report, Dec.,1954 to Feb.,1955) p. 59.

[3]Stephens, Asaro and Perlman, (to be published) (1956).

The spin and parity assignments of the levels A2, A4, and B1 rest firmly on alpha-gamma angular correlation work (cf. especially Valladas, *et al.*[4], Falk-Vairant and Petit[5], and Stephens[6] with additional references therein), internal conversion coefficient determinations, and gamma-gamma angular correlations.

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[4]Valladas, Teillac, Falk-Vairant, and Benoist, J. phys. radium 16, 123 (1955).
[5]P. Falk-Vairant and G. Y. Petit, Compt. rend. 240, 296 (1955).
[6]F. S. Stephens, Ph. D. Thesis, University of California Radiation Laboratory Unclassified Report 2970 (1955), unpublished.

Since a good rotational level sequence is exhibited by Ra^{226}, we may reason that there is a stable spheroidal nuclear shape and that the *K*-quantum number may be fairly good. Band *A* necessarily has *K* = 0, the spin of its lowest member. The *K*-quantum number applying to state B1 can be deduced from the relative reduced gamma transition probabilities for the two *E1* gamma rays depopulating it. Study of the electron-capture decay of Ac^{226}, which mainly populates B1, has given a good relative intensity figure.

Experimentally, the relative reduced transition probabilities are

$$\frac{B_{B1 \longrightarrow A0}}{B_{B1 \longrightarrow A2}} = 1.32 \left(\frac{185}{253}\right)^3 = 0.50.$$

Theoretically,

$$\frac{B_{1 \rightarrow 0}}{B_{1 \rightarrow 2}} = \frac{(1\ 1\ K_i\ {-K_i} \mid 1\ 1\ 00)^2}{(1\ 1\ K_i\ {-K_i} \mid 1\ 1\ 20)^2} = \begin{cases} 2.00 \text{ for } K_i = 1 \\ 0.50 \text{ for } K_i = 0 \end{cases}$$

The assignment $\underline{K_i} = 0$ is clearly demanded, and the similar comparison for 1- levels in Em^{218}, Ra^{222}, Ra^{224}, Ra^{226}, Th^{226}, Th^{228}, and Pu^{238} also favors *K* = 0. (cf. Section 13). If the 1- level were an ordinary nucleonic excited state involving two unpaired nucleons, the Bohr-Mottelson theory would require K = 1. Hence, the state B1 must be regarded as a state of collective excitation; more specifically, B1 is to be regarded as a member of the ground rotational band. (See Section 13 for discussion.)

The state at 320 kev is thought to be 3- and K = 0, on the basis of its decay properties. It is probably to be considered as another odd member of the ground rotational band. Curiously, the effective energy constant $\hbar^2/2\mathcal{J}$ as calculated from the spacing between B1 and B3 is only 60 percent of that from the spacing between A0 and A2.

The very tentative spin and parity assignments of Fig. 28 for the 416- and 445-kev states are based principally on expectations from the rotational energy formula, but their observed decay only to A4 is consistent with the assignments.

It is interesting to compare the levels with a rotational energy formula. In order to secure agreement with the even parity energy levels it is necessary to employ a three-parameter equation

$$E_I = AI(I + 1) - BI^2(I + 1)^2 + CI^3(I + 1)^3 \qquad (53.1)$$

in which case, with the energies 67.62, 210, and 416 kev for A2, A4, and A6 one obtains

$$A = 11.69 \text{ kev}, \; B = 0.075 \text{ kev}, \text{ and } C = 0.0008 \text{ kev}.$$

For the odd-parity levels

$$E_I = 240 + 6.75\, I\, (I + 1) \qquad (53.2)$$

gives a satisfactory fit within experimental uncertainty. The perturbation from the simple formula appears to be of the oppsite sense from that of the even-parity levels (see discussion in work of A. K. Kerman[7]), but the level energies are not known precisely enough to establish the point.

[7] A. K. Kerman, Dan. Mat. Fys. Medd. 30, No. 15 (1956).

The alpha-gamma angular correlation involving the 68-kev A2-A0 gamma has been the object of careful study.[8,9] Theoretically, for the sequence $0 \xrightarrow[2]{\alpha} 2 \xrightarrow[2]{\gamma} 0$ one should obtain an angular correlation function of form $(\sin 2\,\theta)^2$. That is, the coincidence rate should be zero at $\theta = \pi$ or $\pi/2$. The experimental function has maxima and minima at the proper angles, but they are less pronounced, and the function does not get very near zero at the minima. Such attenuation of the correlation function is attributed to the interaction of external magnetic fields on the nuclear magnetic moment of the intermediate state or of electric field gradients on the nuclear electric quadrupole moment. One conventionally defines attenuation factors

[8] G. M. Temmer and J. M. Wyckoff, Phys. Rev. 92, 913 (1953).

[9] Valladas, Teillac, Falk-Vairant, and Benoist, J. phys. radium 16, 123 (1955).

G_2 and G_4 for the separate terms in the expansion of the correlation function in Legendre functions. That is, for the above alpha-gamma sequence we have for no attenuation

$$W(\theta) = 1 + 0.714\, P_2(\cos\theta) - 1.714\, P_4(\cos\theta) \qquad (53.3)$$

while in the actual case with attenuation

$$W(\theta) = 1 + 0.714\, G_2\, P_2(\cos\theta) - 1.714\, G_4\, P_4(\cos\theta) \qquad (53.4)$$

Valladas *et al.*[9] find $G_2 = 0.47_3$ and $G_4 = 0.56_5$.

As discussed in the papers of Temmer and Wyckoff[8] and Valladas *et al.*[9], the attenuation here is to be attributed predominantly to electric rather than magnetic interactions, since magnetic interaction should give $G_2 > G_4$

in disagreement with experiment. By theories of Alder[10] and Abragam and Pound[11] the attenuation coefficients are given as functions of $\omega\tau$, where τ is the mean life of the intermediate state and $2\pi\omega$ is the smallest non-vanishing Larmor precession frequency for the quadrupole interaction. The above attenuations are $(\omega\tau)_1 \sim 1$.

[10] K. Alder, Helv. Phys. Acta. 25, 234 (1952).

[11] Abragam and Pound, Phys. Rev. 92, 943 (1953).

The attenuation coefficients for the $0 \xrightarrow[4]{\alpha} 4 \xrightarrow[2]{\gamma} 2$ sequence have been determined[13] as $G_2 = 0.74$ and $G_4 = 0.61$, consistent with electric interaction with $(\omega\tau)_2 \sim 0.13$.

Using Abragam and Pound's relationship between ω and the quadrupole moment and Bohr and Mottelson's (10) relationships between lifetime, quadrupole moment, and intrinsic quadrupole moment, Valladas et al.[12] calculate an expected theoretical ratio

$$\frac{(\omega\tau)_2}{(\omega\tau)_1} = 0.11 \pm 0.04$$

in agreement with their experimental ratio of 0.13.

Alpha-gamma angular correlation has been done[13] with state B1 as intermediate, confirming the spin assignment of unity. The attenuation is small: $0.8 < G_2 < 1$. That is, assuming electric quadrupole interactions, $\omega\tau < 1$.

[12] Valladas, Teillac, Falk-Vairant, and Benoist, J. phys. radium 16, 123 (1955).

[13] P. Falk-Vairant and G. Y. Petit, Compt. rend. 240, 296 (1955).

54. Curium-242 and the levels of plutonium-238. The energy levels of Pu^{238} have been investigated thoroughly both from the alpha decay[1] of Cm^{242} and the β^- decay[2] of Np^{238}. Some work on electron capture[3] of Am^{238} has also been done. The level pattern and alpha spectrum (Fig. 29) are expected to be representative for even-even nuclei in the region of pronounced spheroidal distortion.

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[1] Asaro, Perlman, and Thompson, Phys. Rev. 92, 694 (1953), F. Asaro, unpublished results (1956).

[2] Freedman, Jaffey, and Wagner, Phys. Rev. 79, 410 (1950), Rasmussen, Slätis, and Passell, Phys. Rev. 99, 42 (1955), Rasmussen, Stephens, Strominger, and Åström, Phys. Rev. 99, 47 (1955), S. A. Baranov and K. N. Shlyagin, Atomnia Energia 1, 52 (1956).

[3] R. J. Carr, Ph. D. Thesis, University of California (1956) (Radiation Laboratory Report UCRL 3395).

The energy levels A0, 2, 4, 6, and 8 are believed to comprise a rotational band based upon the ground state. A part of this sequence of states is well established. The alpha groups to states A0, A2, and A4 have been observed directly, and the energy level spacing agrees with the I(I+1) dependence expected of a rotational band. Conversion coefficients for the transitions A2 $\longrightarrow$ A0 and A4 $\longrightarrow$ A2 have shown them to be E2 and no crossover (A4 $\longrightarrow$ A0) could be found. From this information the state A4 is assigned uniquely 4+ only on the basis of the I(I+1) level spacing although the other data support this assignment. However, the same sequence has been investigated in other even-even nuclei and alpha-gamma and gamma-gamma angular correlations show quite definitely that the third member is a 4+ state. (See Section 53).

The 44-kev E2 transition (A2 $\longrightarrow$ A0) has an L-shell conversion coefficient of several hundred, and since this state is by far the most heavily populated of the excited states, the most prominent electromagnetic radiation consists of L x-rays. The 102-kev transition (A4 $\longrightarrow$ A2) is found to be in coincidence with L x-rays as expected. The remainder of the cascading sequence A8 $\longrightarrow$ A6 $\longrightarrow$ A4 was characterized by showing that each gamma ray was in coincidence with the one below. The spins of these states (A8 and A6) were assigned on the basis of the cascading transitions and because the energy level spacings agree with the expectations for higher members of the rotational band. The low intensity of population of these states makes it difficult to obtain more detailed information, such as conversion coefficients, for these transitions. It will be noted later that the 6+ and 8+ states are not seen in the β^- decay of Np^{238}, and this fact conforms with a consistent set of level assignments for Np^{238} and the higher energy levels of Pu^{238}.

The energy level B1, assigned 1-, was deduced from a pair of gamma-rays of 605 kev and about 560 kev. The higher-energy component was not in coincidence with any photons, and the lower-energy component was in coincidence with L x-rays proving that it leads to level A2. Such pairs of gamma rays have been seen in other heavy even-even nuclei[4] and have been proved to be E1 transitions arising from 1- states. In all cases examined in which the 1- assignment was established, the reduced transition probabilities of the competing E1 transitions were found in a ratio characteristic for the K-quantum number equal to zero for the 1- state as well as for the rotational band based upon the ground state. This same relationship of the reduced transition probabilities was found in the present case. Beyond this reasoning by analogy there is no other supporting evidence for the (0,1-) assignment of state B1 and it must, therefore, be considered provisional.

[4]Stephens, Asaro, and Perlman, Phys. Rev. 96, 1568 (1954).
F. S. Stephens, Jr. and I. Perlman, to be published.

The best information on the higher levels comes from the study of Np^{238} beta decay. The best high-energy conversion electron data are those of Baranov and Shlyagin,[5] and they show five gamma transitions near 1 Mev: 1032, 1030, 988, 942 and 927 kev.[6] The K-lines of the two highest-energy

[5]S. A. Baranov and K. N. Shlyagin, Atomnia Energia 1, 52 (1956).

[6]Baranov and Shlyagin (Ref. 5) reported the absence of the 927-kev transition seen by Rasmussen, Slätis, and Passell (Ref. 2), but in private communication to J. M. Hollander, Baranov reported later observing it. The K-line is certainly much weaker than reported by Rasmussen, Slätis, and Passell.

transitions are barely identifiable as two separate lines,[5] but coincidence measurements showed earlier[7] that there are indeed two gamma rays of about 1030 kev, the more intense component being not in coincidence with L x-rays. Hence this gamma leads to the ground state, definitely establishing a level at 1030 kev designated D2 in Fig. 29. The 988-kev gamma-ray is found in about equal intensity to the 1030-kev gamma-rays, and its energy agrees with the D2-A2 difference, but also with the C2-A0 difference, and Baranov and Shlyagin[5] divide the line intensity between these transitions. The 942-kev gamma-ray is probably in coincidence with L x-rays, and hence does not go to the ground state. Whether this gamma goes to level A4 or to A2 was regarded as an unsettled question by Rasmussen et al.[7] Baranov and Shlyagin's[5] work favors the latter alternative of directing γ_{942} to level A2, thus defining a level at 986 kev.

[7]Rasmussen, Stephens, Strominger, and Åström, Phys. Rev. **99**, 47 (1955).

Level A4 is populated in about 3 percent of the beta transitions. Gamma-gamma coincidence work[7] suggested population of A4 by a hard gamma-ray of energy consistent with either the 942 or 927 kev gamma rays. Electron-electron coincidence work[5] suggested population of A4 directly by a hard beta group (1133 kev), roughly to the extent of 2.8 percent of total disintegrations. There is evidently population of A4 by both beta- and gamma-rays, although the relative amounts must surely be regarded as somewhat uncertain.

Most of the gamma transitions must be of electric quadrupole nature, and the data are consistent with all being E2. Both of the recent studies[5,7] proposed sets of relative intensities for all transitions, but with the unresolved beta and gamma transitions these relative intensities must be based on balancing transition intensities to and from the lower levels. The proposed sets of intensities are presently to be taken with some reservation.

The levels D2 and D3 have been designated[7,8] as (2,2+) and (2,3+) states and could possibly have the predominant character of a so-called gamma vibration (shape vibration) band predicted theoretically by Bohr and Mottelson (10). The spacing between D2 and D3 is roughly 46 kev, and this corresponds to the same or slightly lower rotational moment of inertia as the ground state. If the levels are of vibrational nature, there must, however, also be a sizeable admixture of an excited nucleonic structure, since the beta decay would necessarily be much slower if the levels were purely vibrational.

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[8]The usual convention of A. Bohr and co-workers of listing quantum numbers in the order (K, I, π) is used here.

The assignments of level D2 to be a (2,2+) state receives its strongest support from the relative gamma-ray intensities. For large spheroidal distortion, where K may be a fairly good quantum number, one can calculate relative transition rates to different members of a rotational band. (The reason why all transitions from state D2 are expected to be pure E2 will be dealt with presently.) The reduced transition probabilities should go simply as the squares of appropriate Clebsch-Gordan coefficients involving I- and K-quantum numbers. In particular, this analysis often permits an unambiguous assignment of the K-quantum number. When applied to the present case, it turns out that only if K = 2 for state D2 would one expect to find transitions D2 $\longrightarrow$ A0 and D2 $\longrightarrow$ A2 in the relative intensities observed and have the D2 $\longrightarrow$ A4 transition of unobservably low intensity.

One of the demands of these assignments is that the transition D2 $\longrightarrow$ A2, be E2. The reason why it does not exhibit measurable M1 admixture is because of the selection rule that the multipolarity must exceed or be equal to ΔK.

The most reasonable spin assignment for Np^{238} with the decay scheme of Fig. 29, would seem to be 3 with even or odd parity admissible.

The alpha group shown to populate the state D2 is assigned from rather fragmentary evidence. A definite, but very weak, gamma-ray of ~1010 kev was observed in the Cm^{242} spectrum, and this could be a mixture of the 988- and 1030-kev transitions from level D2 seen in the beta decay of Np^{238}.

The level designated C0 is based on a weak ~890-kev gamma transition observed in alpha decay of Cm^{242}. It was found to be in coincidence with L x-rays. Our tentative interpretation of this level, placed at 935 kev, is as a 0+ level, perhaps to be associated with a Bohr-Mottelson (10) beta vibrational excitation.

It is very interesting to note that the 986-kev level from beta decay of Np^{238} is spaced above the level C0 by about 50 kev which is of the right magnitude to be a rotational excitation. In Fig. 29 we have associated these levels with a single rotational band, calling the 986-kev level C2. It should be emphasized that the interpretation of the states C0 and C2 is highly speculative and is advanced here mainly as a guide to further measurements.

Attention will be called here to the obvious need of relating the observed intensities of alpha population of the several levels with alpha-decay theory. The hindrance factor which expresses the ratio of the partial alpha half-life of each transition to that of the ground state transition after removing the energy dependence is given in the caption to Fig. 29. Also given is the factor taken from the treatment of Winslow (21) which expresses the retardation due to the centrifugal barrier resulting from alpha-particle angular momentum.

The assignment of the ground-state rotational band follows partly from the identification of cascading E2 transitions and partly from the level spacing. The Bohr-Mottelson rotational formula is of the form

$$E = AI(I + 1) - BI^2(I + 1)^2 \qquad (54.1)$$

where $B \ll A$ in the region of applicability of the model. The agreement between experiment and theory may be judged from Table XVII.

55. Curium-243 and the levels of plutonium-239. Our knowledge of this alpha emitter and the levels of its daugher nucleus Pu^{239} is considerable as a result of study of the three isotopes decaying to Pu^{239}; i.e., Np^{239}, Am^{239}, and Cm^{243}, as well as coulombic excitation of Pu^{239}. The essential results of these studies[1-4] are embodied in the decay scheme of Fig. 30.

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[1](Np^{239}) J. M. Hollander, W. G. Smith and J. W. Mihelich, Phys. Rev. 102, 740 (1956), S. A. Baranov and K. N. Shlyagin, Atomnia Energia 1, 52 (1956), D. Engelkemeir and L. B. Magnusson, Phys. Rev. 99, 135 (1955), H. W. Lefevre, E. M. Kinderman and H. H. Van Tuyl, Phys. Rev. 100, 1374 (1955), Other references in "Table of Isotopes" by Hollander, Perlman and Seaborg Revs. Mod. Phys. 25, 469 (1953).

[2](Am^{239}) W. G. Smith, W. M. Gibson, and J. M. Hollander, University of California Radiation Laboratory Report 3356 (1956) (to be published).

[3](Cm^{243}) F. Asaro, S. G. Thompson, and I. Perlman, Phys. Rev. 92, 694 (1953).

[4](coul. exc.) J. O. Newton, Nature 175, 1028 (1955) and private communication (Oct. 1955).

The levels may be grouped into rotational bands, levels A-1/2, A-3/2, A-5/2, and A-7/2 comprising one band, and the measured[5] ground-state spin of 1/2 identifies this band, with its irregular spacings, as an "anomalous" $K = \Omega = 1/2$ band with energy level spacings given by

$$E_I = \frac{\hbar^2}{2\mathfrak{J}}\left[I(I+1) + (-)^{I+1/2}\,a\left(I+\tfrac{1}{2}\right)\right] \qquad (55.1)$$

with a, the decoupling parameter, depending on details of the intrinsic nucleonic structure. Levels B-5/2 and B-7/2 evidently belong to a common rotational band, and the occurrence of three M1 gamma transitions to levels of spins 3/2, 5/2, and 7/2 fixes the spin and relative parity of level B-5/2.

Level C-5/2 is the opposite parity from the lower levels and is presumed of spin 5/2 on the basis of the transitions depopulating it. All the above levels presumably receive some alpha population, although experimental difficulties (interference from Cm^{242}) has hitherto prevented the actual observance of any alpha decay to the lowest two levels.

[5] M. Van den Berg and P. F. A. Klinkenberg, Physica 20, 37, 461 (1954).

Coulomb excitation gives rise to observation of gamma transitions A-5/2 $\rightarrow$ A-3/2 and A-5/2 $\rightarrow$ A-1/2. Level D-7/2 is populated by decay of Np^{239} or Am^{239}, but its excitation has not been observed in alpha decay, perhaps on account of its high energy. Spins of 5/2 or 7/2 are consistent with experiment, but 7/2 seems more likely from inspection of the Nilsson diagram (13), Fig. 31.

The rotational spacing factor $\hbar^2/2\mathfrak{J}$ is 6.25 kev for band A and 6.39 kev for band B. This is to be compared with 4.74 kev and 6.20 kev for bands A and B, respectively, in nearby Np^{237} and to 7.35 kev for Pu^{238}.

Level B-5/2 has a measured lifetime[6] of 1.1×10^{-9} sec. representing a slow-down from the single-particle formula of about a factor of 10^4 for the main M1 transition B-5/2 $\rightarrow$ A-3/2 depopulating it. This slowness is attributed to the violation of the K selection rule, since $\Delta K = 2$ exceeds the gamma multipolarity.

[6] R. L. Graham and R. E. Bell, Phys. Rev. 83, 222A (1951).

Level C-5/2 has a measured lifetime[7] of $1.9 \cdot 10^{-7}$ sec, representing a retardation of $\sim 2 \times 10^6$ on the single-proton lifetime estimate for E1 transition C-5/2 $\rightarrow$ B-5/2. The slowness is rather commonly observed for E1 transitions in the heavy region and may be associated[8,9] with a violation of selection rules in the asymptotic quantum numbers n_z, or Λ (defined in Section 14). With the assignments in Fig. 30 there is $\Delta n_z = 3$, exceeding the multipolarity.

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[7] D. Engelkemeir and L. B. Magnusson, Phys. Rev. 99, 135 (1955).

[8] J. O. Rasmussen and D. Strominger, Bull. Am. Phys. Soc. 1, Ser. II, No. 4 Paper R4 (1956).

[9] D. Strominger, Ph. D. Thesis, University of California (1956) (Radiation Laboratory Report UCRL 3374).

The relative reduced-transition probabilities of the E2 transition A-5/2 → A-1/2 and the E2 component of mixed M1-E2 transition A-5/2 → A-3/2 have been shown[10] to be proportional to the squares of the appropriate Clebsch-Gordan coefficients $(I_i\ 2\ K\ 0\ |\ I_i\ 2\ I_f\ K)^2$, as they should be for K a fairly good quantum number.

The ground state magnetic moment ($|\mu| = 0.4$) and decoupling parameter ($a = 1.418$) for the band have been compared with theoretical calculations[9] using Nilsson's spheroidal well nucleonic wave functions (13), and the assignment to the particular Nilsson state giving agreement was made.

Likewise, a consistent interpretation of the beta- and electron-capture ft values in terms of Alaga's selection rules[11] in N, n_z, and Λ can be obtained with the assignments of Fig. 30. It is necessary to assume assignment of 5/2+ to Np^{239}, the same as Np^{237}, but in contradiction to the spectroscopically measured spin[12] of 1/2. The violation of selection rules in n_z and Λ for the allowed beta transitions from Np^{239} to bands B and D slows them to a comparable rate (log ft ~7) with the unhindered first-forbidden transition to C (log ft ~6.5). The most dramatic effect of the asymptotic quantum number selection rules results from the violation $\Delta N = 2$ for the allowed transition from Am^{239} to C-5/2; the transition is too weak to be observed and must have a log ft > 8.

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[10] Hollander, Smith and Mihelich, Phys. Rev. 102, 740 (1956).

[11] G. Alaga, Phys. Rev. 100, 432 (1955).

[12] J. G. Conway and R. D. McLaughlin, Phys. Rev. 96, 541 (1954).

The main alpha group to level B-5/2 has a hindrance factor of only 1.4 and is presumed to be of the "favored" type in which the odd-neutron in state B-5/2 is in nearly the same state as in Cm^{243}. See Table XVI for comparison of favored alpha groups with the Bohr-Fröman-Mottelson (31) theory.

The alpha transitions to the ground band fall in the category of highly-hindered transitions. It is significant that the E2 radiations between the ground band and the favored band are weak, indicating that the electric coupling of the hindered group with the favored is not large. Beyond this we are unable to say why the ground transitions are so hindered. The alpha transition to state C-5/2 is hindered by a factor of 16. The parity change restricts the transition to L = 1, 3, or 5.

56. Americium-241 and the levels of neptunium-237. Here again (Fig. 32) the three radioactivities decaying to Np^{237} have been carefully studied,[1-3] and coulomb excitation experiments[4] have been carried out.

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[1] (Am^{241}) Jaffe, Passell, Browne, and Perlman, Phys. Rev. 97, 142 (1955).

F. Asaro and I. Perlman, Phys. Rev. 93, 1423 (1954).

P. P. Day, Phys. Rev. 97, 689 (1955).

Milsted, Rosenblum, and Valadares, Compt. rend. 239, 259, 700 (1954).

J. L. Wolfson, private communication (1954).

J. M. Hollander, W. G. Smith, and J. O. Rasmussen, Phys. Rev. 102, 1372 (1956).

Gol'din, Tret'yakov, and Novitsova, Conf. Acad. Sci. USSR on Peaceful Use of Atomic Energy, Phys. Math. Sci. p. 226 (July 1955).

J. F. Turner, Phil. Mag. 46, 687 (1955).

S. A. Baranov and K. N. Shlyagin, Conf. Acad. Sci. USSR on Peaceful Use of Atomic Energy, Phys. Math. Sci. p. 251 (July 1955).

[2](U^{237}) Wagner, Freedman, Engelkemeir, and Huizenga, Phys. Rev. 89, 502 (1953).

L. Melander and H. Slätis, Phys. Rev. 74, 709 (1948); Arkiv Mat. Astron. Fysik 36A, No. 15 (1949).

S. A. Baranov and K. N. Shlyagin, Conf. Acad. Sci. USSR on Peaceful Use of Atomic Energy, Phys. Math. Sci. p. 251 (July 1955).

Zhur. Exp. Theor. Fys. 30, 225 (1956).

Hollander, Canavan, and Rasmussen, to be published (1956).

[3](Pu^{237}) R. W. Hoff, Ph. D. Thesis, University of California (1954) (Radiation Laboratory Report UCRL 2325); R. W. Hoff and J. L. Olsen, private communication (1956).

[4]J. O. Newton, Nature 175, 1028 (1955) and private communication (Oct. 1955).

Levels A-5/2, A-7/2, and A-9/2 form a regular rotational band as shown by coulomb excitation. Levels B-5/2, B-7/2, and B-9/2 form a second rotational band of opposite parity to the first. The base spins of these bands and their K-values are 5/2. These levels constitute the main levels receiving alpha decay, although alpha decay to level A-9/2 has not been seen and is probably almost unobservable.

The rotational spacing factor $\hbar^2/2\mathcal{J}$ for band A is 4.74 kev, the smallest known for any nucleus, while $\hbar^2/2\mathcal{J}$ for band B is 6.20 kev.

Level B-5/2 is a metastable state[5] of half-life 6.3 x 10^{-8} sec, decaying by E1 transitions B-1/2 → A-5/2 and B-5/2 → A-7/2. Like other E1 transitions of this region these violate a selection rule in n_z (Δn_z = 2). The gyromagnetic ratio for state B-5/2 has been measured[6] as +0.8 ± 0.2 by the attenuation of the alpha-gamma angular correlation in an applied magnetic field.

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[5]Beling, Newton, and Rose, Phys. Rev. 87, 670 (1952).

[6]Krohn, Novey, and Raboy, Phys. Rev. 98, 1187 (1955).

The alpha decay to band A is of the highly-hindered type. Alpha decay to band B appears to be of the favored type. Here, it would seem, is an interesting opportunity to test the favored alpha-decay hypothesis that the odd-nucleon wave function remains essentially unchanged. The measured magnetic moment[7] of Am^{241} is +1.4 nuclear magnetons while that of state B-5/2 of Np^{237} by the attenuation measurement[6] is +2.0 ± 0.5 nuclear magnetons. The moments are not the same but the check is probably close enough to be consistent with the favored decay hypothesis, particularly when it is remembered that magnetic moments are often extremely sensitive to details of the wave function.

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[7]T. E. Manning, M. Fred, and F. S. Tomkins, Phys. Rev. 102, 1108 (1956).

The M1-E2 mixing ratios for cascade radiation in band B have been measured by conversion electron intensity work[8] and, coupled with the experimental magnetic moment for level B-5/2, provide a test in satisfactory agreement with the model of a single free proton in a spheroidal well. This agreement is in contrast to similar tests in the ground band which do not agree with the extreme single-particle picture. The unusually large magnetic moment[9] of

+6.0 ± 2.5 nuclear magnetons for Np^{237} is not calculable on the extreme single-particle picture, though the even-parity assignment of Fig. 32 gives a larger (+3 nm) moment than other possible states.

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[8]Hollander, Smith, and Rasmussen, Phys. Rev. 102, 1372 (1956).

[9]Bleaney, Llewellyn, Pryce, and Hall, Phil. Mag. 45, 992 (1954).

The U^{237} main beta transition to state C-3/2 has log *ft* ~6.1 and is classified with the state assignments of the figure as first forbidden ($\Delta I = 1$, yes), unhindered. Beta decay to A-5/2 should be second forbidden, and is not observed. Beta decay to B-5/2 would by spin and parity change be unique first forbidden ($\Delta I = 2$, yes), but it is hindered by the $\Delta \Lambda$ selection rule. This beta group has not been observed, and experimentally a limit has been set of log *ft* >9.

State D-7/2 cannot be significantly populated directly by beta decay as it has ($\Delta I = 3$) and would be second forbidden. The spin 5/2 is also consistent with experiment, but 7/2 seems more likely by inspection of the Nilsson diagram. (Fig. 31). Experimentally somewhat less certain is the level designated tentatively E-3/2. For most of the gamma-rays depopulating this level only one conversion line has been observed. The beta branch to this level is quite weak. There are also a few weak conversion lines not accounted for by the present decay scheme.

Pu^{237} decays predominantly to the ground state (or possibly some to other members of the ground band). An upper limit on decay giving the 60-kev gamma (B-5/2 → A-5/2) has been set as less than 2 percent of total *K*-capture. The *L*/*K*-capture ratio is 0.88, about normal for allowed or $\Delta I = 0, 1$, yes, first forbidden for the decay energy ~180 kev, estimated[10] from closed decay energy cycles.

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[10] R. A. Glass, private communication (1956).

The alpha decay hindrance factors are given in Table XIII.

The decay to band B is of favored type, and one is referred to Table XVI for comparison of intensities with the Bohr-Fröman-Mottelson (31) theory.

The BFM formulation can be used to predict that the main alpha group to level B-5/2 is about 80% L = 0 and 20% L = 2. The alpha-gamma angular correlation with gamma B-5/2 ⟶ A-5/2 will be sensitive to this admixture. Fig. 33 plots the anisotropy for various mixtures of L = 0 and 2, with the gamma radiation assumed pure dipole.

The experimental anisotropy, $\frac{W(\pi)}{W(\frac{\pi}{2})} - 1 = -0.41$, obtained by Novey[11] with solution sources, is slightly lower than the theoretical.

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[11] T. B. Novey, private communication (1956).

The angular correlation is especially important on another score, as it determines the L = 0 and L = 2 waves to be nearly in phase instead of 180° out of phase. This allows us to select the more likely cases for the inward integration of coupled alpha-decay equations for nearby even-even isotopes. (See Section 32).

For Fig. 33, we have used for the two possible phase differences a formula given by Frauenfelder.[12] The formula is subject to modification where strong non-central electric interactions exist, as they do here. The greater the inequality in the mixing ratio, the greater the modification. Evaluation of the phase shift modification must await numerical studies of the appropriate alpha decay differential equations, but any modification would be expected to bring both branches of the loop of Fig. 33 closer to a straight line connecting the points for pure L = 0 or L = 2.

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[12]H. Frauenfelder, Chap. XIX(I), Beta and Gamma Ray Spectroscopy; Kai Siegbahn, Editor, North Holland Publishing Co., Amsterdam, 1955. Eq.(48).

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We wish to express our thanks to all who have helped by corresponding with us on various matters pertaining to this work. We also wish to thank many of our colleagues at the University of California Radiation Laboratory for supplying us with data and discussing topics with us. Especially are we indebted to Dr. F. Asaro for his indispensable assistance in assembling and assessing the alpha decay data of Table I and to Mr. C. J. Gallagher for calculation of alpha decay hindrance factors.

Table I

ALPHA SPECTRA OF HEAVY ELEMENTS

Explanation of Columns

COLUMNS 1 AND 2

Column 1 indicates the alpha emitters with measured spectra. Those emitters in which alpha decay was deduced only by the chemical separation of the daughter are not included.

Column 2 shows the partial alpha half-life. If the partial alpha half-life is unknown or is measurably different from the overall half life, the latter is shown in column 1.

COLUMN 3

Column 3 indicates the degree of certainty that the half life and the most prominent alpha group belong to the emitter listed in column 1. The meaning of the symbols is as follows:

A Element certain, mass number certain;

B Element certain, mass number probable;

C Element probable, mass number probable;

D Element certain, mass number uncertain or not well established;

E Element probable and mass number not well-established or known;

F Insufficient evidence.

COLUMN 4

Column 4 shows the measured alpha particle energies. Where there is some uncertainty about the existence of an alpha group, a question mark appears after the energy. Where there is considerable uncertainty about the existence of alpha groups, they are omitted from this table.

Explanation of Columns (cont'd) COLUMN 5

Column 5 indicates the method of measurement which best defines. the energy and existence of the various alpha groups.

spect	magnetic or electrostatic spectrograph.
ion ch	ionization chamber coupled with some form of pulse-height analyzer
range air (or mica)	range determination in air (or mica).
range emuls	range of alpha tracks in a photographic emulsion.
γ - α	pulse height analyzed alpha spectrum observed in coincidence with gamma rays or electrons. The listed energy is the measured alpha energy or that deduced from the energy of the gamma ray (or electron), whichever is known better.
γ - γ	The alpha group was deduced from gamma ray-gamma ray coincidences and a knowledge of the decay scheme.
conv emuls	Conversion electrons were observed in coincidence with alpha particles in a photographic emulsion. The listed energy is equal to the energy of the ground state alpha group minus the corrected gamma energy corresponding to the

Explanation of Columns (cont'd)

	conversion electrons. A small correction in the gamma energy is necessary to compensate for the difference in recoil of the alpha groups populating the states spanned by the gamma ray.
α - γ	A gamma ray was observed in coincidence with alpha particles and the energy of the alpha group was deduced from a knowledge of the decay scheme.
γ or c	A gamma ray or conversion line was observed and the energy of the alpha group was deduced from a knowledge of the decay scheme.

COLUMN 6

Column 6 gives the reference for the energy determinations shown in columns 4 and 5. Where no references are given they may be found in the Table of Isotopes, Revs. of Modern Phys. 25, 469 (1953) by Hollander, Perlman, and Seaborg.

COLUMN 7

Column 7 indicates the energies of the excited states corresponding to the various alpha groups. Where the excited state energy was deduced from measurements other than the type shown in column 4, the following symbols are used following the energy value in column 7:

γ	deduced from a gamma ray measurement
c	deduced from a conversion line measurement

Explanation of Columns (cont'd)

γ' (or c') deduced from a gamma ray (or conversion line) measurement of a nuclide other than the alpha emitter which decays to the same residual nuclide as the alpha emitter.

The absence of a symbol in column 7 does not mean necessarily that the value was taken from the measurement referred to in column 5 and 6 but only that the measurement was the same type.

COLUMN 8

Column 8 shows the relative abundances of the various alpha groups. These values were determined from the same type of measurement as shown in columns 5 and 6 but are not necessarily the same measurement. One exception is a low energy group of Po^{210} where the abundance was deduced from a gamma abundance. The designation " ~ 100" signifies low-energy alpha groups have been looked for without success with a high resolution instrument, and very low limits may be set on their abundances.

COLUMN 9

In a large fraction of cases the "highest-energy group" of column 4 is either known to be the ground state transition or is assumed to be so in the absence of information regarding the complexity of the alpha spectrum. The Q-values, unless otherwise stated under "comments," were calculated by adding the recoil energy of the residual nucleus to the alpha particle energy listed in column 4. The recoil energy is $4 \cdot E/(A-4)$, where E is the alpha particle energy and A is the mass number of the emitter.

Explanation of Columns (cont'd) COLUMN 10

The comments in this column for the most part reinforce the decision on the decay energy.

ins.evid	Insufficient evidence to know whether or not the alpha energy measured is that of the ground state.
e - e	No direct evidence, but since the nucleus is of the even-even type, it can be assumed that the measured energy is that of the ground state transition.
γ, c, c', α - γ	These designations indicate, respectively, that gamma rays, conversion electrons from alpha emitters, conversion electrons from beta emitters, or coincidences between alpha particles and gamma rays have been observed which show some doubt that the highest-energy alpha group is the ground-state transition or the most accurate measurement of it. Where the evidence is not sufficiently definite to deduce an alpha decay energy based on anything other than the highest-energy alpha group, this is reflected by the value in columns 4 and 9 differing only by the recoil energy. Where the evidence is sufficiently

Explanation of Columns

definite to deduce the decay energy, it will be found that columns 4 and 9 differ by more than the recoil energy. In Ra^{223} and Po^{211m} decay a more accurate disintegration energy was obtained by adding to the particle energy of the most prominent alpha group its nuclear recoil energy and the energy of the gamma ray thought to span the corresponding energy level and the ground state.

REFERENCES FOR TABLE I

1. Jones, Schuman, Butler, Cowper, Eastwood, and Jackson, Phys. Rev. 102, 203 (1956).
2. Asaro, Stephens, Thompson, and Perlman, Phys. Rev. 98, 19 (1956).
3. Friedman, Gindler, Barnes, Sjoblom, and Fields, Phys. Rev. 102, 585 (1956).
4. Harvey, Chetham-Strode, Ghiorso, Thompson, and Choppin, unpublished data (1955).
5. Hummel, Asaro, Stephens, Thompson, and Perlman, unpublished data (1955).
6. A. Ghiorso and B. G. Harvey, unpublished data (1954) mentioned in Ref. 7.
7. F. S. Stephens, Jr., Ph.D. Thesis, University of California, (U.C. Radiation Laboratory Unclassified Report UCRL-2970), June 1955.
7a. A. Chetham-Strode and L. W. Holm, to be published (1956).
8. Ghiorso, Rossi, Harvey, and Thompson, Phys. Rev. 93, 257 (1956).
9. Asaro, Stephens, Harvey, and Perlman, Phys. Rev. 100, 137 (1955).
10. L. B. Magnusson, unpublished data mentioned in Ref. 11.
11. Magnusson, Friedman, Engelkemeir, Fields, and Wagner, Phys. Rev. to be published (1956).
12. Hulet, Thompson, and Ghiorso, Phys. Rev. 95, 1703 (1954).
13. Hummel, Stephens, Asaro, Chetham-Strode, and Perlman, Phys. Rev. 98, 22 (1955).
14. Asaro, Stephens, Chetham-Strode and Perlman, unpublished data (1955).
15. A. Chetham-Strode, Jr., G. R. Choppin, and B. G. Harvey, Phys. Rev. 102, 747 (1956).
16. A. Chetham-Strode, Jr., Ph. D. Thesis, University of California (U.C. Radiation Laboratory Unclassified Report UCRL-3322), June 1956.
17. A. Chetham-Strode, Jr., F. S. Stephens, Jr., and F. Asaro, unpublished data (1955).
18. Fields, Studier, Diamond, Mech, Inghram, Pyle, Stevens, Fried, Manning, Ghiorso, Thompson, Higgins, and Seaborg, Phys. Rev. 102, 180 (1956).
19. Private communication from E. K. Hulet, 1956.
20. E. K. Hulet, S. G. Thompson, and A. Ghiorso, Phys. Rev. 95, 1703 (1954).
21. Asaro, Thompson, and Perlman, unpublished data (1954).
22. Hummel, Asaro, Thompson, and Perlman, unpublished data (1955).

Ref. for Table I (cont'd)

23. F. Asaro, S. G. Thompson, and I. Perlman, Phys. Rev. 92, 694 (1953).
24. F. Asaro, B. G. Harvey, and I. Perlman, unpublished data (1955-1956).
25. R. A. Glass, Ph.D. Thesis, University of California, (U.C. Radiation Laboratory Unclassified Report UCRL-2560), April 1954.
26. G. H. Higgins, Ph.D. Thesis, University of California, (U.C. Radiation Laboratory Unclassified Report UCRL-1796), June 1952.
27. Stephens, Hummel, Asaro, and Perlman, Phys. Rev. 98, 261 (1955) and unpublished data (1955).
28. F. Asaro and I. Perlman, Phys. Rev. 93, 1423 (1954).
29. L. L. Gol'din, E. F. Tret'yakov, and G. I. Novikova, Conf. Acad. Sci. USSR on Peaceful Uses of Atomic Energy, Phys. Math. Sci., p. 226 (July 1955) UCRL translation 242.
30. Jaffe, Passell, Browne, and Perlman, Phys. Rev. 97, 142 (1955).
31. F. Asaro and I. Perlman, unpublished data mentioned in Ref. 32; also F. Asaro, Ph.D. Thesis, University of California (U.C. Radiation Laboratory Unclassified Report UCRL-2180), June 1952.
32. I. Perlman and F. Asaro, Annual Review of Nuclear Science 4, 157 (1954).
33. F. Asaro and I. Perlman, unpublished data (1956).
34. F. Asaro and I. Perlman, Phys. Rev. 94, 381 (1954).
35. F. Asaro, F. S. Stephens, Jr., and I. Perlman, unpublished data (1955).
36. J. P. Hummel, F. Asaro, G. H. Higgins, and I. Perlman, unpublished data (1956).
37. D. C. Dunlavey and G. T. Seaborg, unpublished data listed in Ref. 32.
38. T. D. Thomas and R. A. Glass, unpublished data (1956).
39. Magnusson, Engelkemeir, Freedman, Porter, and Wagner, Phys. Rev. 100, 1237A (1955).
40. H. G. Jackson and B. G. Harvey, private communication mentioned in Ref. 32.
41. B. Zajac, Phil. Mag. 43, 264 (1952).
42. G. Valladas, Dr. Thesis, Commissariat a L'Energie Atomique Rapport C.E.A. No. 483 (1955).
43. J. R. Grover and G. T. Seaborg, private communication (1952), reported in University of California Radiation Laboratory Unclassified Report UCRL-2455, p. 22 (Jan. 1954).

Ref. for Table I (cont'd)

44. G. Valladas, Compt. rend. 237, 1673 (1953); see also Ref. 42.

45. F. Asaro and I. Perlman, Phys. Rev. 99, 37 (1955).

46. F. Asaro and I. Perlman, Phys. Rev. in press (1956); see also Ref. 48.

47. W. G. Smith, F. Asaro, and J. M. Hollander, Phys. Rev. in press (1956).

48. F. S. Stephens, Jr., F. Asaro, and I. Perlman, Phys. Rev. 96, 1568 (1954).

49. J. P. Hummel, F. Asaro, and I. Perlman, unpublished data (1955); see also Ref. 50 and 29.

50. J. P. Hummel, F. Asaro, and I. Perlman, Phys. Rev. 98, 261A (1955).

51. S. W. Peat and M.A.S.Ross, Proc. Phys. Soc. (London) A68, 923 (1955).

52. G. Valladas and R. Bernas, Compt. rend. 91, 2230 (1953).

53. F. S. Stephens, Jr., F. Asaro, and I. Perlman, unpublished data (1955).

54. F. Asaro, F. S. Stephens, Jr., and I. Perlman, Phys. Rev. 92, 1495 (1953).

55. Pilger, Hummel, Asaro, and Perlman, unpublished data (1955). See University of California Radiation Laboratory Unclassified Report UCRL-3240, p.72 (Dec.1955).

56. Stephens, Hummel, Asaro, and Perlman, unpublished data (1956); see also Ref. 7.

57. F. S. Stephens, Jr., F. Asaro, and I. Perlman, unpublished data (1956).

58. F. F. Momyer, Jr., F. Asaro, and E. K. Hyde, J. Inorg. Nucl. Chem. 1, 267 (1955).

59. L. Madansky and F. Rasetti, Phys. Rev. 102, 464 (1956).

60. F. F. Momyer, Jr. and E. K. Hyde, Phys. Rev. 101, 136 (1956).

61. F. Asaro and I. Perlman, unpublished data (1953) listed in Ref. 32.

62. G. H. Briggs (Ref.63) recalculated the energy of the main Em^{219} alpha group from the original data (Ref. 64) using later values for the reference energies.

63. G. H. Briggs, Revs. Modern Phys. 26, 1 (1954). This is a compilation of the best values for the energies of natural alpha particles.

64. M. Curie and S. Rosenblum, Compt. rend. 196, 1598 (1933).

65. The alpha particle energy of Em^{207} was increased 20 kev above the value given in Ref. 66 in order to correspond to the best available value for Em^{208}.

66. F. F. Momyer, Jr. and E. K. Hyde, J. Inorg. Nucl. Chem. 1, 274 (1955).

67. R. W. Hoff, J. P. Hummel, and F. Asaro, unpublished data (1954); see also University of California Radiation Laboratory Unclassified Report UCRL-3157 p. 67 (Sept.1955).

Ref. for Table I (cont'd)

68. Stoner, Asaro, Stephens, and Hyde, University of California Radiation Laboratory Unclassified Report UCRL 2455, p. 14 (Jan. 1954).

69. Perlman, Stephens, Hummel, Pilger, and Asaro, University of California Radiation Laboratory Unclassified Report UCRL 3068, p. 37 (July 1955).

70. W. Jentschke, A. C. Juveland, and G. H. Kinsey, Phys. Rev. 96, 231 (1954).

71. F. Asaro and I. Perlman, Revs. Modern Phys. 26, 237 (1954). This is a compilation of alpha disintegration energies.

72. S. Rosenblum and H. Tyren, Compt. rend. 239, 1205 (1954).

73. H. B. Levy and I. Perlman, Phys. Rev. 94, 152 (1954).

Table II. Semi-empirical Constants[1] from Correlation of Ground State Decay Rates of Even-Even Nuclei

Element	A	B	Remarks
100 Fm	156.38	-53.3742	Extrapolated
98 Cf	152.86	-52.9506	
96 Cm	152.44	-53.6825	
94 Pu	146.23	-52.0899	
92 U	147.49	-53.6565	
90 Th	144.19	-53.2644	
88 Ra	139.17	-52.1476	
86 Em	137.46	-52.4597	
84 Po	129.35	-49.9229	

[1]Constants to use with Equation (16.1) or (40.1), where $t_{1/2\alpha}$ is in seconds and Q_{eff} in Mev.

Table III. Effective Nuclear Radii from Alpha Decay Rate Theories

Model	Apparent radius at A=232 (10^{-13} cm)	Ref.
One body model (Preston) (lowest virtual level, $f \sim 3 \cdot 10^{20}$ sec^{-1})	9.4	1(14)
One body model (Biswas-Patro) ($f = v/R \sim 2 \cdot 10^{21}$ sec^{-1})	9.1	2, 3
Extreme many body model (Bethe) ($f \sim 10^{15}$ sec^{-1})	12.6	(18)

1 I. Kaplan, Phys. Rev. 81, 962 (1951).

2 S. Biswas and J. Patro, Indian J. Phys. 22, 540 (1948).

3 I. Perlman and T. J. Ypsilantis, Phys. Rev. 79, 30 (1950).

Table IV. Effective Nuclear Radii from Alpha Particle Cross Section Studies

Type of determination	Radius expression (in 10^{-13}cm)	Corresponding radius at A=232 (10^{-13} cm)	Ref.
Alpha scattering (20-40 Mev)	$\sim(1.4 \text{ to } 1.5)A^{1/3}+2.5$	11.1 to 11.7	1
Total inelastic alpha cross sections (240 Mev)	$1.84\, A^{1/3} + 0.35$	11.7	2
Alpha-fission cross sections (15-40 Mev)	---	10.3	3

1 D. L. Hill, Vol. --, Sec. 33, in Handbuch der Physik, S. Fluegge, editor; (Springer-Verlag, Berlin, 1956).

2 Millburn, Birnbaum, Crandall, and Schechter, Phys. Rev. 95, 1268 (1954).

3 J. Jungermann, Phys. Rev. 79, 640 (1950).

Table V. Values of centrifugal reduction factor $(G_L/G_o)^{-2}$

η	ρ	L = 0	1	2	3	4	5	6	Ref.
20.0	10.0	1.000	0.844	0.601	0.363	0.1854	0.0806	0.0299	(21)
25.4	9.42	1.000	0.854	0.588	0.354	0.181	0.082	---	(17)

Table VI. Some Theoretical and Experimental K- and L-shell Ionization Probabilities for Alpha Decay of Po^{210}

	Theoretical		Experimental		
Shell	Migdal[a]	Levinger[b]	Barber and Helm[c]	Rubinson and Bernstein[d]	Roy and Goes[e]
K	$2.6 \cdot 10^{-6}$	$1.0 \cdot 10^{-7}$	$1.4 \cdot 10^{-6}$	--	--
Total L	$0.24 \cdot 10^{-4}$	$0.50 \cdot 10^{-4}$	--	$8.8 \cdot 10^{-4}$	--
Total all shells	--	--	--	--	$2.7 \cdot 10^{-3}$

[a] A. Migdal, J. Phys. (U.S.S.R.) 4, 449 (1941).

[b] J. S. Levinger, Phys. Rev. 90, 11 (1953); J. phys. radium 16, 556 (1955).

[c] W. C. Barber and R. H. Helm, Phys. Rev. 86, 275 (1952).

[d] W. Rubinson and W. Bernstein, Phys. Rev. 82, 334 (1951); Phys. Rev. 86, 545 (1952).

[e] R. Roy and M. L. Goes, Compt rend. 237, 1515 (1953).

Table VII. Alpha Surface Probabilities and Reduced Derivative Widths for Even-Even Nuclei

(evaluated at R=9.3 for heavy region and R=8.0 for rare earths R in units of 10^{-13} cm.)

Alpha Emitter	Log $t_{1/2\alpha}$ (sec)	E_α^1	Log $G_o^2(R)$	$-\frac{d}{dR}(\text{Log } G_o^2)$	Surface probability	δ^2 (Mev)
Fm^{254}	4.061	7.242	24.49	1.778	.088	1.96
Cf^{252}	7.821	6.154	28.32	1.797	.120	2.74
Cf^{250}	8.472	6.066	28.76	1.801	.072	1.65
Cf^{248}	7.298	6.302	27.60	1.791	.072	1.64
Cf^{246}	5.109	6.795	25.34	1.771	.059	1.31
Cm^{244}	8.782	5.839	29.00	1.785	.062	1.39
Cm^{242}	7.147	6.151	27.39	1.772	.064	1.42
Cm^{240}	6.365	6.291	26.70	1.767	.080	1.76
Pu^{242}	13.198	4.932	33.42	1.798	.069	1.57
Pu^{240}	11.317	5.202	31.66	1.787	.088	1.98
Pu^{238}	9.451	5.535	29.66	1.773	.063	1.39
Pu^{236}	7.930	5.790	28.26	1.762	.080	1.82
Pu^{234}	5.908	6.230	26.05	1.744	.051	1.08
U^{238}	17.151	4.219	37.67	1.802	.146	3.35
U^{236}	14.877	4.538	35.11	1.789	.073	1.65
U^{234}	12.894	4.802	33.18	1.778	.081	1.81
U^{232}	9.366	5.357	29.65	1.756	.076	1.65
U^{230}	6.255	5.928	26.56	1.733	.076	1.60
U^{228}	2.842	6.709	23.04	1.697	.055	1.12
Th^{232}	17.642	4.031	38.14	1.786	.143	3.20
Th^{230}	12.402	4.719	32.66	1.757	.075	1.64
Th^{228}	7.778	5.458	28.06	1.725	.074	1.55
Th^{226}	3.268	6.373	23.55	1.687	.070	1.40
Ra^{226}	16.709	4.813	30.88	1.728	.061	1.29
Ra^{224}	5.498	5.717	25.67	1.689	.057	1.14
Ra^{222}	1.580	6.590	21.94	1.652	.082	1.58

Table VII. (cont.)

Alpha Emitter	Log $t_{1/2\alpha}$ (sec)	E_α [1]	Log $G_o^2(R)$	$-\frac{d}{dR}(\text{Log } G_o^2)$	Surface probability	δ^2 (Mev)
Em^{222}	5.520	5.521	25.69	1.671	.057	1.12
Em^{220}	1.736	6.317	21.99	1.635	.065	1.22
Em^{218}	-1.721	7.162	18.80	1.597	.120	2.06
Em^{216}	--	8.045	16.07	1.555	--	--
Em^{214}	--	--	--	--	--	--
Em^{212}	3.140	6.297	22.05	1.636	.0030	0.056
Em^{210}	3.987	6.071	23.02	1.646	.0039	0.075
Em^{208}	3.799	6.173	22.56	1.641	.0019	0.0366
Po^{218}	2.262	6.032	22.29	1.621	.040	0.741
Po^{216}	-0.801	6.808	19.21	1.586	.036	0.640
Po^{214}	-3.786	7.714	16.27	1.543	.038	0.63
Po^{212}	-6.517	8.810	13.40	1.489	.025	0.395
Po^{210}	7.078	5.332	25.64	1.652	.0014	0.0273
Po^{208}	7.966	5.142	26.67	1.660	.0020	0.0388
Po^{206}	6.891	5.252	26.05	1.655	.0056	0.109
Po^{204}	6.136	5.404	25.23	1.648	.0049	0.0939
Po^{202}	5.193	5.624	24.12	1.638	.0032	0.0614
Po^{200}	--	5.874	22.93	1.627	--	--
Gd^{148}	9.65	3.18	30.13	1.636	.153	2.78
Sm^{146}	15.2	2.57	34.88	1.633	.027	0.49
Nd^{144}	22.7	1.92	42.49	1.631	.040	0.72

[1] E_α is particle energy plus screening correction used for these calculations. (Mev)

Table VIII. Calculated Alpha Decay "Nuclear Radii" from One-Body Theory

Alpha emitter	Partial alpha half-life	Abundance gd. state group(α_o)	Partial α_o half-life	Alpha particle energy (Gd. state)	Effective decay energy (Q_{eff})	Radius formula coefficient (r_o) Preston	Kaplan
Fm^{254}	3.3 h	0.83	4.0 h	7.22	7.378	1.48_0	--
Cf^{252}	2.2 y	0.845	2.6 y	6.112	6.252	1.512	--
Cf^{250}	10.0 y	0.83	12.1 y	6.024	6.163	1.495	--
Cf^{248}	250 d	0.80*	313 d	6.26	6.404	--	1.497
Cf^{246}	35.7 h	0.78	45.8 h	6.753	6.906	1.492	1.495
Cm^{244}	19 y	0.767	25 y	5.798	5.935	1.498	--
Cm^{242}	162.5 d	0.737	220.5 d	6.110	6.253	1.502	1.504
Cm^{240}	26.8 d	0.70*	38.3 d	6.27	6.416	1.503	--
Pu^{242}	$3.76x10^5$y	0.74	$5.08x10^5$y	4.898	5.019	1.516	--
Pu^{240}	6580 y	0.755	$8.71x10^3$y	5.162	5.289	1.520	1.522
Pu^{238}	89.6 y	0.72	$1.24x10^2$y	5.495	5.628	1.509	--
Pu^{236}	2.7 y	0.689	3.9 y	5.763	5.901	--	1.51_3
U^{238}	$4.49x10^9$y	0.77	$5.84x10^9$y	4.182	4.290	--	1.548
U^{236}	$2.39x10^7$y	0.73	$3.28x10^7$y	4.499	4.613	--	1.523
U^{234}	$2.48x10^5$y	0.74	$3.35x10^5$y	4.763	4.883	--	1.532
U^{232}	73.6 y	0.68	108 y	5.318	5.448	1.527	--
U^{230}	20.8 d	0.679	30.6 d	5.884	6.026	1.532	--
Th^{232}	$1.39x10^{10}$y	0.76	$1.83x10^{10}$y	3.994	4.110	--	1.551
Th^{230}	$8.0x10^4$y	0.763	$1.05x10^5$y	4.682	4.801	1.534	--
Th^{228}	1.90 y	0.71	2.68 y	5.421	5.554	1.534	1.536
Th^{226}	30.9 m	0.79	39.1 m	6.330	6.480	1.542	--
Ra^{226}	1622 y	0.943	1720 y	4.777	4.898	--	1.545
Ra^{224}	3.64 d	0.948	3.84 d	5.681	5.819	--	1.546
Ra^{222}	38 s	0.95	40 s	6.551	6.706	1.545	--

Table VIII (cont.)

Alpha emitter	Partial alpha half-life	Abundance gd. state group (α_o)	Partial α_o half-life	Alpha particle energy (Gd. state)	Effective decay energy (Q_{eff})	Radius formula coefficient (r_o)	
						Preston	Kaplan
Em^{222}	3.825 d	--	3.825 d	5.486	5.621	--	1.552
Em^{220}	54.5 s	0.997	54.7 s	6.282	6.432	--	1.560
Em^{218}	0.019 s	0.998	0.019 s	7.127	7.294	--	1.585
Em^{212}	23 m	--	23 m	6.262	6.417	--	1.445
Em^{210}	2.8 h	--	2.8 h	6.037	6.188	--	1.461
Po^{218}	3.05 m	--	3.05 m	5.998	6.143	--	1.543
Po^{216}	0.158 s	--	0.158 s	6.774	6.935	--	1.541
Po^{214}	1.637×10^{-4} s	--	1.637×10^{-4} s	7.680	7.859	1.537	1.545
Po^{212}	3.04×10^{-7} s	--	3.04×10^{-7} s	8.776	8.978	--	1.527
Po^{210}	138.4 d	--	138.4 d	5.299	5.435	--	1.422
Po^{208}	2.93 y	0.997	2.94 y	5.108	5.241	--	1.442
Po^{206}	180 d	--	180 d	5.218	5.354	--	1.460

*Value assumed.

Table IX. TABLE OF γ(y)

$$y = \sqrt{1 - x}$$

where $x = E/B$

$$\gamma = x^{-1/2} \text{ arc cos } x^{1/2} - (1-x)^{1/2}$$

y	γ(y)	δ_1	y	γ(y)	δ_1
0.750	0.53215	0.00350	0.790	0.69556	0.00480
1	0.53565	352	1	0.70036	485
2	0.53917	355	2	0.70521	488
3	0.54272	357	3	0.71009	493
4	0.54629	361	4	0.71502	497
5	0.54990	363	5	0.71999	501
6	0.55353	366	6	0.72500	506
7	0.55719	368	7	0.73006	510
8	0.56087	372	8	0.73516	514
9	0.56459	374	9	0.74030	519
0.760	0.56833	377	0.800	0.74549	524
1	0.57210	380	1	0.75073	528
2	0.57590	383	2	0.75601	533
3	0.57973	386	3	0.76134	537
4	0.58359	389	4	0.76671	543
5	0.58748	392	5	0.77214	547
6	0.59140	396	6	0.77761	552
7	0.59536	398	7	0.78313	557
8	0.59934	401	8	0.78870	563
9	0.60335	405	9	0.79433	567
0.770	0.60740	408	0.810	0.80000	572
1	0.61148	411	1	0.80572	578
2	0.61559	414	2	0.81150	584
3	0.61973	418	3	0.81734	588
4	0.62391	421	4	0.82322	594
5	0.62812	424	5	0.82916	600
6	0.63236	428	6	0.83516	605
7	0.63664	431	7	0.84121	611
8	0.64095	435	8	0.84732	617
9	0.64530	438	9	0.85349	623
0.780	0.64968	442	0.820	0.85972	629
1	0.65410	446	1	0.86601	635
2	0.65856	449	2	0.87236	640
3	0.66305	453	3	0.87876	647
4	0.66758	457	4	0.88523	654
5	0.67215	460	5	0.89177	660
6	0.67675	465	6	0.89837	666
7	0.68140	468	7	0.90503	673
8	0.68608	472	8	0.91176	679
9	0.69080	476	9	0.91855	687

Table IX. (cont'd)

y	$\Upsilon(y)$	δ_1	y	$\Upsilon(y)$	δ_1
0.830	0.92542	0.00693	0.870	1.27014	0.01084
1	0.93235	700	1	1.28098	098
2	0.93935	707	2	1.29196	112
3	0.94642	715	3	1.30308	127
4	0.95357	722	4	1.31435	140
5	0.96079	729	5	1.32575	156
6	0.96808	737	6	1.33731	171
7	0.97545	744	7	1.34902	187
8	0.98289	753	8	1.36089	202
9	0.99042	760	9	1.37291	219
0.840	0.99802	768	0.880	1.38510	235
1	1.00570	776	1	1.39745	252
2	1.01346	785	2	1.40997	269
3	1.02131	793	3	1.42266	287
4	1.02924	801	4	1.43553	305
5	1.03725	810	5	1.44858	323
6	1.04535	819	6	1.46181	342
7	1.05354	828	7	1.47523	362
8	1.06182	838	8	1.48885	381
9	1.07020	846	9	1.50266	401
0.850	1.07866	856	0.890	1.51667	421
1	1.08722	865	1	1.53088	443
2	1.09587	876	2	1.54531	464
3	1.10463	885	3	1.55995	487
4	1.11348	895	4	1.57482	508
5	1.12243	905	5	1.58990	532
6	1.13148	916	6	1.60522	556
7	1.14064	927	7	1.62078	580
8	1.14991	938	8	1.63658	604
9	1.15929	948	9	1.65262	631
0.860	1.16877	959	0.900	1.66893	656
1	1.17837	971	1	1.68549	683
2	1.18808	983	2	1.70232	711
3	1.19791	994	3	1.71943	739
4	1.20785	0.01007	4	1.73682	767
5	1.21792	019	5	1.75449	798
6	1.22811	031	6	1.77247	828
7	1.23842	045	7	1.79075	859
8	1.24887	057	8	1.80934	892
9	1.25944	070	9	1.82826	924
0.870	1.27014	0.01084	0.910	1.84750	959

Table IX. (cont'd)

y	Υ(y)	δ_1
0.910	1.84750	0.01959
1	1.86709	994
2	1.88703	0.02029
3	1.90732	067
4	1.92799	104
5	1.94903	145
6	1.97048	185
7	1.99233	226
8	2.01459	269
9	2.03728	314
0.920	2.06042	359
1	2.08401	407
2	2.10808	455
3	2.13263	505
4	2.15768	558
5	2.18326	611
6	2.20937	667
7	2.23604	724
8	2.26328	784
9	2.29112	846
0.930	2.31958	911
1	2.34867	976
2	2.37843	0.03044
3	2.40887	116
4	2.44003	190
5	2.47193	266
6	2.50459	347
7	2.53806	430
8	2.57236	517
9	2.60753	606
0.940	2.64359	701
1	2.68060	799
2	2.71859	901
3	2.75760	0.04008
4	2.79768	120
5	2.83888	237
6	2.88125	359

Table X. Representative Non-Central Coupling Term Magnitudes in Alpha Emission

Multi-polarity	Alpha Emitter	$\underline{I}$	$\underline{I_f}$	ℓ	$\underline{E}$ (kev)	$\underline{I_f'}$	ℓ	$\underline{E'}$ (kev)	$\underline{B}$[3] $\underline{I_f'} \rightarrow \underline{I_f}$	Coupling Energy at $r=1\cdot10^{-12}$ cm (Mev)	Determined from
E1	Am^{241}	5/2	5/2	1	0	5/2	0	60	$1.7\cdot10^{-38}$ ev·cm^3	$\pm1.2\cdot10^{-4}$	Delay coincidence[1] lifetime of 60-kev state ($6.3\text{x}10^{-8}$ sec).
		5/2	5/2	1	0	5/2	2	60	"	$\pm0.98\cdot10^{-4}$	
		5/2	5/2	3	0	5/2	2	60	"	$\pm0.90\cdot10^{-4}$	
		5/2	7/2	1	33	5/2	0	60	$1.4\cdot10^{-38}$	$\pm1.1\cdot10^{-4}$	
		5/2	7/2	1	33	5/2	2	60	"	$\pm0.28\cdot10^{-4}$	
E2	Cm^{242}	0	0	0	0	2	2	44	$2.3\cdot10^{-55}$ ev·cm^5	+0.58	$\underline{Qo}$ value of $9\text{x}10^{-24}$ cm^2 estimated from coulomb excitation work on Th^{232} and U^{238}. (see Sec. 14)
	Am^{241}	5/2	5/2	0	60	5/2	2	60	--	+0.54	$\underline{Qo} \approx +14\text{x}10^{-24}$ cm^2 from Am^{241} spectroscopic determination[2] used.
		5/2	5/2	0	60	7/2	2	103	$1.0\cdot10^{-54}$ (both rows)	-0.62	
		5/2	5/2	2	60	7/2	2	103		-0.40	
	Em^{218}	0	0	0	0	2	2	609	$1.1\cdot10^{-56}$	±0.13	Lifetime ($6\cdot10^{-12}$ sec) estimation from long range alpha particle data (see Table XXVII of Ref. (10)).

[1] Beling, Newton and Rose, Phys. Rev. 87, 670 (1952).

[2] Manning, Fred and Tomkins, Phys. Rev. 102, 1108 (1956).

[3] $\underline{B}$ is the reduced gamma transition probability in the notation of Bohr and Mottelson (10).

Table XI. Interpolated Semi-Empirical Constants[1] for Hindrance Factor Calculations

Element	A	B	Remarks
99 E	155.04	-53.3141	Extrapolated
97 Bk	152.65	-53.3166	
95 Am	149.33	-52.8862	
93 Np	146.86	-52.8732	
91 Pa	145.84	-53.4604	
89 Ac	141.68	-52.7060	
87 Fr	138.31	-52.3037	
85 At	133.40	-51.1913	

[1]Constants for the even-*Z* elements are listed in Table II. The constants are for use in Equation (40.1), where $t_{1/2\alpha}$ is in seconds and Q_{eff} is in Mev.

Table XII. Hindrance Factors for Even-Odd Nuclei

Alpha Emitter	Alpha Particle Energy (Mev)	Hindrance Factor F
Fm^{255}	7.08	2.6
	7.04	1.8
Fm^{251}	6.9	5.2
Cf^{249}	6.19, 6.04	N.R.[1]
	5.94, 5.90	N.R.[1]
	5.80	2.9
Cf^{245}	7.11	1.4
Cm^{245}	5.45	42.
	5.36	2.5
	5.31	5.5
Cm^{243}	6.003	1700.
	5.985	280.
	5.777	1.4
	5.732	4.8
	5.679	16.
Cm^{241}	5.95	12.
Pu^{241}	4.893	3.2
	4.848	5.1
Pu^{239}	5.150	2.9
	5.137	10.
	5.099	9.3
	4.98	900.
	4.78	140.
	4.73	62.
Pu^{235}	5.85	1.7
U^{235}	4.58	950.
	4.47?	450.
	4.40	4.6
	4.20	2.3
U^{233}	4.816	1.2
	4.773	3.3
	4.717	12.
	4.489	14.
U^{231}	5.45	1.1
U^{229}	6.42	2.3
Th^{229}	5.02	110.
	4.94	18.
	4.85	1.3
Th^{227}	6.030	120.
	6.001	500.
	5.970	58.
	5.952	380.
	5.907	700.
	5.859	140.
	5.800	200.
	5.792	1850.
	5.749	4.9
	5.706	7.6
	5.699	12.
	5.661	19.
Th^{225}	6.57	2.5
Ra^{223}	5.860	large
	5.735	59.
	5.704	6.8
	5.592	4.2
	5.525	4.9
	5.487	14.
	5.418	3.9
Ra^{221}	6.71	3.4
Em^{221}	6.0	7.7
Em^{219}	6.807	14.
	6.542	5.8
	6.417	2.2
	6.197	0.76
Em^{217}	7.74	4.5
Po^{215}	7.38	1.2
Po^{213}	8.35	2.0

[1] Alpha groups not resolved.

Table XIII. Hindrance Factors for Odd-Even Nuclei

Alpha Emitter	Alpha Particle Energy (Mev)	Hindrance Factor F
E^{253}	6.633	1.2
	6.592	8.8
	6.545	24.
	6.493	80.
	6.25	17.
E^{251}	6.48	3.0
E^{249}	6.76	5.8
Bk^{249}	5.40	6.4
	5.06	19.
Bk^{247}	5.67	70.
	5.51	5.9
	5.30	3.9
Bk^{245}	6.37	450.
	6.16	34.
	5.90	2.5
Bk^{243}	6.72	670.
	6.55	68.
	6.20	4.9
Am^{243}	5.339	1500.
	5.308	1000.
	5.266	1.1
	5.224	4.5
	5.169	18.
Am^{241}	5.535	520.
	5.503	600.
	5.476	1.2
	5.433	4.2
	5.379	20.
	5.314	750.
	5.27	1500.
Am^{239}	5.75	2.3
Am^{237}	6.01	4.6
Np^{237}	4.872	230.
	4.816	140.
	4.787	3.5
	4.767	4.8
	4.713	34.
	4.674	9.2
	4.644	3.1
	4.589	15.
	4.52	120.
Np^{235}	5.06	1.7
Np^{233}	5.53	0.32
Pa^{231}	5.046	230.
	5.017	68.
	5.001	51.
	4.971	330.
	4.938	21.
	4.921	130.
	4.839	73.
	4.722	1.5
	4.696	7.6
	4.666	3.2
Pa^{229}	5.69	5.7
Pa^{227}	6.46	1.5
Ac^{227}	4.942	3.3
Ac^{225}	5.818	7.1
	5.781	4.6
	5.719	10.5
	5.627	11.
Ac^{223}	6.64	3.1
Fr^{221}	6.332	3.3
	6.116	1.9
Fr^{219}	7.30	1.2
At^{219}	6.27	1.9
At^{217}	7.05	.60
At^{215}	8.00	3.7

Table XIV. Hindrance Factors for Odd-Odd Nuclei

Alpha Emitter	Alpha Particle Energy (Mev)	Hindrance Factor (F)
E^{254}	6.42	1.7
E^{252}	6.64	16.
Bk^{244}	6.67	1900.
Pa^{228}	6.09 5.85	63. 13.
Ac^{224}	6.17	22.
Fr^{220}	6.69	7.8
At^{218}	6.63	2.0
At^{216}	7.79	2.6

Table XV. Surface Probabilities and Reduced Transition Probabilities for Odd Nuclei with Z < 84 or N < 128.

Alpha emitter	Measured alpha particle energy (Mev)	Log $t_{1/2\alpha}$ (sec)	Neg. logarithm of surface probability (base 10)	Lowest alpha angular momentum[1]	Reduced transition probability[2] (RTP)
Fr^{212}	6.411	3.85	3.43	--	--
	6.387	3.83	3.31	--	--
	6.342	4.04	3.33	--	--
Em^{211}	5.847	5.82	3.38	(2)	$(7.1 \cdot 10^{-4})$
	5.779	5.54	2.77	--	--
	5.613	7.05	3.45	--	--
Em^{209}	6.037	4.03	2.39	--	--
Em^{207}	6.14	4.21	3.08	--	--
At^{211}	5.862	4.819	2.92	(0)	$(1.2 \cdot 10^{-3})$
At^{210}	5.519	7.73	4.17	--	--
	5.437	7.75	3.77	--	--
	5.355	7.67	3.26	--	--
At^{209}	5.42	5.64	2.73	(0)	$(1.9 \cdot 10^{-3})$
At^{208}	5.65	6.10	3.20	--	--
	5.52	7.56	4.00	--	--
At^{207}	5.75	4.86	2.45	(0)	$(3.5 \cdot 10^{-3})$
Po^{211}	7.44	-0.28	4.20	5	$7.2 \cdot 10^{-4}$
	6.88	1.99	4.63	3	$5.9 \cdot 10^{-5}$
	6.56	2.02	3.49	3	$9.5 \cdot 10^{-4}$
Po^{211m}	8.70	2.55	10.55	(10)	$(1.5 \cdot 10^{-7})$
	7.85	3.00	8.73	(8)	$(5.9 \cdot 10^{-7})$
	7.14	1.83	5.36	(3)	$(2.9 \cdot 10^{-5})$
Po^{209}	4.877	9.50	2.88	2	$2.3 \cdot 10^{-3}$
	4.62	11.90	3.64	--	--
Po^{207}	5.10	8.28	2.98	--	--
Po^{205}	5.2	6.86	2.13	--	--
Bi^{214}	5.505	6.82	4.14	--	--
	5.444	6.73	3.75	--	--
Bi^{213}	5.86	5.14	4.15	(5)	$(9.0 \cdot 10^{-4})$
Bi^{212}	6.086	4.60	4.60	5	$3.2 \cdot 10^{-4}$
	6.047	4.19	4.02	3	$2.7 \cdot 10^{-4}$
	5.765	5.80	4.38	--	--
	5.622	6.86	4.76	--	--
	5.603	5.99	3.80	--	--
	5.481 ?	7.83	5.04	--	--

Table XV (cont.)

Alpha emitter	Measured alpha particle energy (Mev)	Log $t_{1/2\alpha}$ (sec)	Neg. logarithm of surface probability (base 10)	Lowest alpha angular momentum[1]	Reduced transition probability[2] (RTP)
Bi^{211}	6.620	2.198	4.31	5	$6.1 \cdot 10^{-4}$
	6.273	2.875	3.65	3	$5.8 \cdot 10^{-4}$
Bi^{210}	4.94	13.92	8.14	4	$4.2 \cdot 10^{-8}$
$Bi^{210m}_{(RaE)}$	(4.96) ?	11.41	5.80	(0)	$(1.6 \cdot 10^{-6})$
Bi^{203}	4.85	11.80	5.57	--	--
Bi^{201}	5.15	8.10	3.61	--	--
Bi^{199}	5.47	7.20	4.40	--	--
Bi^{198}	5.83	5.96	4.88	--	--

1. Parentheses indicate uncertainty in assignment of angular momenta.

2. Reduced transition probability is calculated for the lowest permitted alpha angular momentum, but this does not imply that mixed angular momenta will not be encountered in reality.

Table XVI. Alpha Group Intensities in Favored Alpha Decay

Parent Nucleus	I_i	Observed Relative Intensities				Calculated Relative Intensities (BFM)[31]				Assumed Constants	
		$I=K_f$	K_f+1	K_f+2	K_f+3	$I=K_f$	K_f+1	K_f+2	K_f+3	C_2	C_4
E^{253}	(11/2) or	100	8.5	2.0	0.3	100	5.3	0.5	0	.27	0*
	(7/2)					100	7.0	1.2	0.1	.27	.013
Cm^{243}	(5/2)	100	17	<2	--	100	12	1.9	--	.56	--
Am^{243}	5/2	100	13	1.5	--	100	12	1.9	--	.56	.002
Am^{241}	5/2	100	16	1.2	0.02	100	13	2.2	0.02	.59	.005
Pu^{241}	5/2	100	33	--	--	100	13	--	--	.59	--
Pu^{239}	1/2	100	23	15	--	100	20	9.5	--	.63	--
U^{233}	5/2	100	18	2.4	--	100	16	2.9	--	.87	.067
Pa^{231}	3/2	100	19	---	--	100	15	--	--	1.0	--

*Assumed zero because necessary Clebsch-Gordan coefficients not readily calculable.

Table XVII. Energies of Ground Rotational Band Members in Pu^{238}

Member of band (I, Π)	0+	2+	4+	6+	8+
Measured energy	0	44.11	146.0	303.7	514*
Calculated energies based upon $E=A\ I(I+1)$; A from 44.11-kev 2+ state		(44.11)	147.0	308.8	529.3
Calculated energies including $I^2(I+1)^2$ term; $A = 7.37$, $B = 0.0034$ from measured energies of 2+ and 4+ states		(44.11)	(146.0)	303.4	513.0

*This energy is based upon a very weak gamma ray whose energy is known only to ±10 kev. The close agreement with the calculated energy is therefore fortuitous.

Fig. 3 Energy surface expressed in terms of mass decrements.

Fig. 3a shows mass decrement (M-A) in millimass units plotted against neutron number. Data were obtained from Ref. (5) and these include nucleon pairing energy terms used to normalize the different nuclear types. Contours are shown at constant A (for odd values) and at constant Z (all values). The heavy line running along the bottom of the valley is the "line of stability" and goes through points of greatest beta stability for each mass number.

Fig. 3b illustrates alpha energy variation with neutron number for a series of protactinium isotopes. The mass decrements for protac-isotopes (Δ_{Pa}) are taken from Fig. 3a and make up the curve labelled Pa. The curve labelled Ac consists of $\Delta_{Ac} + \Delta_{\alpha}$. The energy differences between points on the two curves which are related through alpha decay are simply the alpha-energies, indicated by lengths of arrows. These data illustrate the increase in alpha energy with decrease in neutron number.

Fig. 14 Plot of logarithms of partial alpha half lives for ground-state transitions *versus* the inverse square root of the effective total alpha-decay energy (Q_{eff} = alpha-particle energy + recoil energy + electron screening correction). The points are experimental, and the straight lines are based on a least squares analysis of the points where energies have been determined by magnetic spectrographs (excluding Em^{218} which exhibits an apparently anomalous decay rate). There is only one point for element fermium (Fm); hence, the slope of the fermium line was arrived at by extrapolation from the slopes of lower elements. Points used in the analysis are indicated by triangles, and points not used, by circles. Nuclei with 126 or fewer neutrons are not shown on this plot. Constants in the equations for these lines are given in Table II. The last digit in the mass number of the alpha emitter is given beside each point.

Fig. 15 Plots of hindrance factors for alpha decay of even-even nuclei to excited states. a. Even parity, even spin final states. b. Odd parity, spin one final states.
Numerical values of the factors to these states are tabulated below:

Alpha Emitter	Spin of State				
	2	4	6	8	1
Fm^{254}	3.5	57			
Cf^{252}	3.2	82			
Cf^{250}	2.9				
Cf^{246}	2.3	120	280		
Cm^{244}	1.9	830	480		
Cm^{242}	1.7	390	350	5100	480
Pu^{242}	1.4				
Pu^{240}	1.6	89			
Pu^{238}	1.5	116	360	15000	
Pu^{236}	1.2	50	640		
Pu^{234}	1.7				
U^{238}	1.3	30			
U^{236}	1.2	8.8			
U^{234}	1.1	14			
U^{232}	1.0	16			71
U^{230}	0.91	11			15
Th^{232}	0.96				
Th^{230}	1.1	12	(8200)		38
Th^{228}	0.85	13			11
Th^{226}	1.3	5.3			2.4
Ra^{226}	0.99				
Ra^{224}	1.0				2.3
Ra^{222}	1.1	8.7			.97
Em^{222}	2.2				
Em^{220}	1.1				
Em^{218}	2.7				

Fig. 28 Alpha-decay scheme of Th^{230} and electron-capture decay scheme of Ac^{226} to Ra^{226}. The vertical arrows representing the experimentally observed gamma transitions indicate qualitatively by their width the relative transition intensities. Spin and parity assignments are given on the left hand side of a level, those in parentheses being somewhat uncertain. The K-quantum number for a rotational band is shown in parentheses near the center of the base level of the band. The energies in kev are given at the right of the levels. Each known level is drawn in both decay schemes but is dashed in the decay scheme where it is not detectably populated.

Hindrance Factors for Th^{230}

Final state		Alpha		Centrifugal
Energy (kev)	Spin and parity	intensity (%)	Hindrance factor	barrier factor
0	0+	74	(1)	1
67.62	2+	26	1.1	1.7
210	4+	0.2	12	5.4
253	1-	0.03	38	1.2
320	((3-)	0.001	370	(2.8)
416	(6+)	$\sim 8 \cdot 10^{-6}$	8200	(40)
445	(5-)	$\sim 8 \cdot 10^{-6}$	4900	(14)

Fig. 29 Alpha-decay scheme of Cm^{242}, beta-decay scheme of Np^{238}, and preliminary electron-capture decay scheme of Am^{238} to the common daughter nucleus Pu^{238}. For comments on the conventions followed in drawing these decay schemes see the caption to Fig. 28. The placement of the level C2 and its association with the level C0 in a common rotational band is quite tentative.

Hindrance Factors for Cm^{242}

Final state: Energy (kev)	Final state: Spin and parity	Alpha intensity (%)	Hindrance factor	Centrifugal barrier factor
0	0+	73.7	(1.0)	1.0
44	2+	26.3	1.7	1.6
146	4+	0.035	390	4.9
304	(6+)	0.006	350	(29)
514	(8+)	3×10^{-5}	5000	(340)
605	(1-)	1×10^{-4}	500	(1.2)
935	(0+)	3×10^{-5}	20	(1)
~1030	(2+)	4×10^{-6}	45	(1.6)

Fig. 31 Plot of eigenvalues for a nucleon in a prolate spheroidal three-dimensional harmonic oscillator potential with strong spin-orbit coupling according to the calculations of S. G. Nilsson (13). As calculations of the 7th oscillator shell were not available, the $j_{15/2}$ levels (dashed lines) were estimated using the correct limiting slope at zero deformation. The vertical positioning of the $j_{15/2}$ levels is rather arbitrary, in this diagram being somewhat lower than proposed by Nilsson (13). The lower position gives a natural gap in levels at large deformation for 152 particles thus offering some rationale for the observed neutron "subshell" at 152.

The levels are labelled in the ordinary shell model convention according to ℓ and j at the extreme left of Fig. 31, corresponding to a spherical well. At the right hand side, corresponding to large prolate deformation, the levels are labelled according to the Ω- quantum number (the spin of the base state of a nuclear rotational band except in some cases for $\Omega = 1/2$) and the parity of the level. In parentheses are the asymptotic quantum numbers N and n_z most appropriate to the eigenfunction at the largest deformation ($\eta = 6$) here plotted. The asymptotic quantum number Λ can be determined as follows: Λ can differ from Ω only by 1/2 and takes the even or odd value according to whether $N-n_z$ is even or odd. (N is the principal oscillator quantum number, n_z is the symmetry-axis oscillator quantum number, Λ is the symmetry-axis component of orbital angular momentum, and Ω is the symmetry-axis component of total nucleonic angular momentum. For details see Nilsson (13)).

GENERAL REFERENCES

(1) E. Rutherford, Radioactive Substances and their Radiations (Cambridge University Press, London, 1913).

A work such as this, written during the early formative period of radioactivity, provides insight into the development rather than the results of one of the most brilliant chapters in the history of science.

(2) G. Gamow, Zeits. f. Physik 51, 204 (1928).

The original paper propounding the wave mechanical explanation of the alpha decay process. The explanation was independently advanced by Condon and Gurney (3).

(3) E. U. Condon and R. W. Gurney, Nature 122, 439 (1928); Phys. Rev. 33, 127 (1929).

The original papers proposing, independently of Gamow (2), the wave mechanical explanation of the alpha decay process.

(4) I. Perlman and F. Asaro, Ann. Revs. Nucl. Sci. 4, 157 (1954).

This review article on alpha radioactivity presents for the first time numerous results from high resolution alpha spectroscopic work. The paper touches on a large number of topics related to alpha emission, including the then new spectroscopic investigations revealing population of numerous nuclear rotational bands.

(5) R. A. Glass, S. G. Thompson, and G. T. Seaborg, J. Nucl. Inorg. Chem. 1, 3 (1955).

This paper presents a comprehensive set of nuclear masses for the heavy region, calculated from alpha- and beta-decay energy balance cycles supplemented by a few experimental neutron binding energies. By estimating alpha decay energies the table is extrapolated to include masses of as-yet-undiscovered nuclei.

(6) I. Perlman, A. Ghiorso, and G. T. Seaborg, Phys. Rev. 77, 26 (1950).

This paper systematizes the trends of alpha decay energies and alpha decay rates, drawing on the large number of new artificial alpha emitters produced and studied for the first time in the preceding decade. Many of the principles developed in the present article were laid down in this 1950 paper.

(7) E. K. Hyde and G. T. Seaborg, Handbuch der Physik, S. Fluegge, editor; Vol. 39 (Springer-Verlag, Berlin, 1956).

This article has a thorough modern review of the transuranium isotopes. Its presentation of chemical and nuclear properties makes it an indispensable reference for those interested in the transuranium region.

(8) E. Feenberg, Shell Theory of the Nucleus (Princeton University Press, Princeton, 1955).

M. G. Mayer and J. H. D. Jensen, Elementary Theory of Nuclear Shell Structure (John Wiley and Sons, New York; Chapman and Hall, London, 1955).

M. H. L. Pryce, "Nuclear Shell Structure", Repts. Progr. in Phys. 17, 1 (1954).

These works thoroughly cover the theory and applications of the spin-orbit coupling spherical well nuclear shell models, which have affected nuclear science so profoundly since their inception in 1949.

(9) M. H. L. Pryce, Proc. Phys. Soc. (London) A65, 773 (1952).

In this paper the spherical well shell model is applied to the problem of the level systems of nuclei near Pb^{208}, which has closed neutron and proton configurations. This work is an indispensable reference for theoretical studies or correlations involving nuclei of this region.

(10) A. Bohr and B. R. Mottelson, Dan. Mat. Fys. Medd. 27, No. 16 (1953).

This work gives the original comprehensive application of the unified nuclear model, which introduces features of collective motion into the individual particle model. Most of the data constituting the strongest support of the model have been accumulated subsequent to this 1953 paper. The paper contains a wealth of useful formulas for application of the model.

(11) A. Bohr and B. R. Mottelson, Dan Mat. Fys. Medd. 30, No. 1 (1955).
This paper is concerned largely with the rotational properties of nuclei in the regions of stabilized spheroidal deformation. A large number of data on nuclear rotational bands are presented, but the main emphasis is on the theoretical problem of explaining the empirical moments of inertia.

(12) J. O. Rasmussen and B. Segall, Phys. Rev., in press (1956), (University of California Radiation Laboratory Report, UCRL 3040, 1955).
This paper develops the theory of alpha decay of nuclei with stable spheroidal deformation. The derived wave equations are applied numerically to the cases Th^{228} and Cm^{242} to give information regarding the most probable surface zones for formation of energetic alpha particles.

(13) S. G. Nilsson, Dan. Mat. Fys. Medd. 29, No. 16 (1955).
This paper presents tables and graphs of eigenfunctions and eigenvalues for a nucleon in a three-dimensional harmonic oscillator potential with varying degrees of spheroidal distortion. Semi-empirical terms for spin-orbit interaction and an $\boldsymbol{\ell} \cdot \boldsymbol{\ell}$ interaction were included. These tables have many applications in correlating nuclear properties in the regions of spheroidal nuclear shapes.

(14) M. A. Preston, Phys. Rev. 71, 865 (1947).
A careful derivation (by Gamow's complex eigenvalue method) of the decay rate probability of an alpha particle trapped in the lowest virtual level (for given angular momentum) of the one-body square potential well is given. The rather involved derivation of the coulomb function from the contour integral form leads to essentially the same result as does the ordinary WKB method.

(15) T. Sexl, Zeits. f. Physik 81, 163 (1933).

This paper contains an interesting review of several early treatments of alpha decay rate theory in the five years following the original papers (2) and (3). Sexl's own thorough treatment of the problem is along the lines described in the preceding reference (14) and seems indeed to form the basis for Preston's treatment.

(16) G. H. Winslow, "On the One-Body Model of Alpha Radioactivity, IV. Calculated Wave Amplitudes near the Nucleus," Argonne National Laboratory Report ANL-5381, Jan., 1955 (unpublished).

This work contains an extensive set of tables calculating squares of the radial alpha wave function near the nuclear surface from data for known alpha emitters. Trends of these calculated values are discussed. Tables and graphs for the calculation of centrifugal barrier effects are given.

(17) R. G. Thomas, "A Formulation of the Theory of Alpha-particle Decay from Time-independent Equations," Prog. Theo. Physics 12, 253 (1954).

This work concisely presents alpha decay rate theory in a formalism closely related to the collision matrix theory often applied to nuclear reactions. Results are derived for the one-body model. Many-body modifications are also discussed. The paper contains a lucid discussion of electron screening effects and contains several references to modern literature on alpha decay rate theory.

(18) H. A. Bethe, Revs. Modern Phys. 9, 69 (1937).

For alpha decay and barrier penetration see pp. 161-171 and pp. 177-178.

This work on alpha decay is still a valuable reference work nearly two decades after its appearance. The penetration factor formulas and symbols are still widely used. Although the particular form of the many-body model Bethe proposed is not generally favored today, this treatment of the one-body model and discussion of the fundamental problems are lucid and timely even today.

(19) G. H. Winslow and O. C. Simpson, "On the One-body Model of Alpha Radioactivity, I. Survey of the Existing Results," Argonne National Laboratory Report ANL-4841, June, 1952 (unpublished).

This paper gives a good review and numerical comparisons of several earlier one-body alpha decay rate formulations. Graphs and tables serving as aids to computation of barrier penetrabilities are also presented.

(20) G. H. Winslow and O. C. Simpson, "On the One-body Model of Alpha Radioactivity, IIb. S-state Decay with the Limiting Square Well Model," Argonne National Laboratory Report ANL-4910, Jan., 1953 (unpublished).

A one-body treatment of alpha decay with sloping potential walls at the nuclear surface is given.

(21) G. H. Winslow and O. C. Simpson, "On the One-body Model of Alpha Radioactivity, III. Non-S-state Decay, Preformation Factors, Model Variations, General Discussion," Argonne National Laboratory Report ANL-5277, April, 1954 (unpublished).

This paper tests WKB approximations of the coulomb functions. The authors also discuss the atomic electron screening correction. They present a "surface well" one-body model and propose many-body modifications. Some of the material of this paper is published in Reference (27).

(22) J. J. Devaney, Phys. Rev. 91, 587 (1953).

This paper develops an alpha decay rate theory of many-body type based on statistical considerations of average nuclear level spacings near ground.

(23) J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley and Sons, Inc., New York, 1952). For alpha decay see pp. 568-578.

In the space devoted to alpha decay the statistical many-body model based on average level spacings is developed and applied to nuclear data for calculation of effective nuclear radii. Selection rules for alpha decay are presented.

(24) F. Rasetti, Elements of Nuclear Physics (Prentice-Hall, Inc., New York, 1936) p. 100.

The one-body model of alpha decay is treated by construction of a wave packet of time-dependent wave functions, and the behavior with time is derived. This is a more general and fundamental approach than the usual treatments by the complex eigenvalue or other methods using time-independent wave functions, but the numerical results are essentially the same.

(25) G. H. Winslow and O. C. Simpson, "On the One-body Model of Alpha Radioactivity, IIa. S-state Decay with the Consistent Square Well Model," Argonne National Laboratory Report ANL-4901, Dec., 1952 (unpublished).

The method of Rasetti (24) is employed and extended to derive a one-body decay rate expression, and they demonstrate that the methods of Bethe (18) and Preston (14) lead to the same results as theirs when sufficiently accurate approximations are employed.

(26) E. C. Kemble, The Fundamental Principles of Quantum Mechanics (McGraw-Hill Co., New York, 1937) p. 192.

Kemble also gives an interesting treatment of the one-body alpha decay model using wave packets of time-dependent wave functions.

(27) G. H. Winslow, Phys. Rev. 96, 1032 (1954).

A "surface well" one-body model and its many-body modification are treated and discussed. Peripheral subjects such as the atomic eledtron screening correction are also discussed.

(28) A. Migdal, J. Phys. (USSR) 4, 449 (1941).

Formulas for the probability of orbital electron ejection during alpha emission are derived. Expressions for the energy distribution of ejected electrons are also given.

(29) M. A. Preston, Phys. Rev. 75, 90 (1949).

The effects of non-central interactions between alpha particle and recoil nucleus are explored, and the general methods underlying later developments (e.g. Ref. (12)) are set forth.

(30) G. Racah, Phys. Rev. 62, 438 (1942).

This paper contains a wealth of formulas useful for quantum mechanical treatment of systems of interacting particles. The very general tensor algebra developed here offers powerful methods for attacking problems of non-central interactions in alpha decay, as well as a host of other problems including many-electron or many-nucleon systems.

(31) A. Bohr, P. O. Fröman, and B. R. Mottelson, Dan. Mat. Fys. Medd. 29, No. 10 (1955).

This paper is notable for its applications of the spheroidal well nuclear model to alpha decay and for its extension of the concept of "favored" alpha decay groups.

(32) J. O. Rasmussen, S. G. Thompson, and A. Ghiorso, Phys. Rev. 89, 33 (1953).

Data on a number of artificial alpha emitters in the rare earth region are presented, and correlations of decay rates with alpha decay theories are attempted.

(33) H. A. Tolhoek and P. J. Brussard, Physica 21, 449 (1955).

This paper presents fresh approaches to alpha decay rate theory. A very strong attractive nuclear potential for alpha particles is proposed. The diffuseness of the nuclear surface and the finite range of nuclear forces are included in the derivation of the alpha particle potential energy function. An overlap integral expression for the probability of formation of an alpha particle from constituent nucleons in nuclear matter is derived.