Title
Essays on Efficiency Measurement with Environmental Applications

Author
Levkoff, Steven Benjamin

Publication Date
2011

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA
RIVERSIDE

Essays on Efficiency Measurement with Environmental Applications

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Economics

by

Steven Benjamin Levkoff

August 2011

Dissertation Committee:
Dr. R. Robert Russell, Co-Chairperson
Dr. David Malueg, Co-Chairperson
Dr. Jang-Ting Guo

The Dissertation of Steven Benjamin Levkoff is approved:

Committee Co-Chairperson

Committee Co-Chairperson

University of California, Riverside

Acknowledgements

I would like to thank my *advisor and co-author, Bob Russell,*

for his unwavering guidance and support through my graduate school endeavor.

I would also like to thank my *two co-authors*, *Sushama Murty* at University of Exeter

Business School (UK) and *Bill Schworm* at the University of New South Wales (AUS),

for their collaborative efforts in Chapters 2 and 1 respectively,

and lastly,

I would like to thank *my students, past , present, and future*

for inspiring me to inspire them.

To *my parents*, *Jerry and Elaine*,

for teaching me the meaning of perseverance,

and

to *Katherine*,

for showing and reminding me of the meaning during times it was tempting to forget.

ABSTRACT OF THE DISSERTATION

Essays on Efficiency Measurement with Environmental Applications

by

Steven Benjamin Levkoff

Doctor of Philosophy, Graduate Program in Economics
University of California, Riverside, August 2011
Dr. R. Robert Russell, Co-Chairperson
Dr. David Malueg, Co-Chairperson

This study contributes to three branches of literature: index number theory, the modeling of environmental production technologies, and index decomposition analysis. The first part of the study begins by analyzing a previous adjustment made to a popular graph space index, which is used to compute input-output efficiency scores, and shows that this adjustment fails to correct two serious flaws it was intended to rectify – satisfaction of an indication and weak monotonicity condition. The failure arises at the boundary of output space when zeros arise frequently in the data. We propose an alternative formulation to correct for these deficiencies and implement the proposed modification in two different empirical applications.

Next, we identify a conceptual flaw regarding tradeoffs between inputs, outputs, and pollutants when one production relation is used to model the joint production of pollution-generating technologies. We propose a superior modeling tactic to correct for

the tradeoff implications when using only one implicit production relation. Using multiple production relations to capture intended and unintended production, and defining the reduced form technology as the intersection between these two production sets, we introduce the "by-production" technology. This novel formulation has direct implications on DEA estimation, which we derive and utilize in an application of efficiency measurement. Moreover, we show that constructing efficiency scores for "by-production" technologies isn't straight forward. We distinguish between efficiency in intended and unintended production, show that some traditional indexes are not appropriate for applications involving "by-production" technologies, and propose a modification of a previous index that has superior properties and decomposes into productive and environmental efficiency.

Lastly, this study applies the by-production modeling tactic developed earlier to reconsider index decomposition results previously derived under different assumptions regarding the nature of pollution-generating technologies. The application examines factors related to variations in electric power plant emissions through an index decomposition analysis and shows, in general, that previous results derived using the more primitive technological assumption of weak disposability and null-jointness are incongruent with the results derived under by-production. We conclude the final portion of the study by analyzing energy firm responses to the 1990 Amendment to the Clean Air Act.

Table of Contents

List of Tables

List of Figures

Preface

This dissertation was born from three branches of literature from the interdisciplinary fields of operations research, environmental economics, and productivity analysis. More specifically, this dissertation makes contributions utilizing results from index number theory, the modeling of environmental production technologies, and index decomposition analysis. Throughout the dissertation, the word "we" is used, not because some chapters are attributed to multiple authorship, but because the dissertation functions not only as an exercise in theory and application, but also as an exploratory discussion between the author(s) and the reader.

Chapter 1 begins with its foundations in the index number literature with applications in productivity analysis. Namely, we analyze Fare, Grosskopf, and Lovell's [1985] (FGL) adjustment to the coordinate-wise, graph space, "Russell" measure of technical efficiency, which is used to compute efficiency scores across both input and output space. We show first, that this adjustment fails to correct two serious flaws it was intended to rectify at the boundary of output space – satisfaction of indication and weak monotonicity conditions. While FGL's modification works for input oriented models, the failure of the proposed modification arises at the boundary of output space when zeros arise frequently in the data.

We propose three methods for altering FGL's modification to satisfy the desirable indication and monotonicity properties, and implement one of the proposed modifications, an epsilon perturbation of the technology, in two different empirical applications: the first, an output efficiency exercise using a carefully constructed, synthetic data set; the second, an application of the modification to measure a high frequency data set where zeros occur frequently in the defined output space – measuring Babe Ruth's 1923 season relative batting performance.

Chapter 2 begins with a discussion of past tactics used to model the joint production of output when some of these outputs are undesirable to the public. We then discuss how the previous studies, which typically treated pollution as an input or as a weakly disposable, null-joint output, impose on themselves a conceptual flaw regarding tradeoffs between inputs, outputs, and pollutants when only a single implicit production relation is used to model the joint production of pollution-generating technologies. A superior modeling tactic is proposed to correct for the tradeoff implications when using only one implicit production relation.

Using multiple production relations to capture intended and unintended production, the reduced form technology is defined as the intersection between the intended production set and nature's residual generation mechanism, which treats pollution as an output satisfying "costly disposability." We introduce the term "by-production" technology to refer to such technologies that are modeled accordingly with both multiple production relations, and satisfaction of the costly disposability condition. This novel formulation has direct implications on DEA estimation of by-production technologies, which we derive and utilize in an application of efficiency measurement. Using a data set of polluting firms, we show that constructing efficiency scores for "by-production" technologies isn't straight forward, and that in general, it is important to distinguish between efficiency in intended production and efficiency in unintended production. The importance is further elicited since in our sample since there is a very strong negative correlation between efficiency scores of intended production and unintended production. This implies that firms in general, face a tradeoff between operating efficiently in terms of intended output expansions and unintended pollutant reductions: firms that tend to be efficient in intended production tend not to be efficient by environmental criteria and vice versa. Moreover, we show that some popular indexes, the directional distance function (DDF) and hyperbolic indexes , which are frequently applied in the productivity analysis literature, are not appropriate for applications involving "by-production" technologies. We propose a modification of the FGL index from Chapter 1 that has superior properties relative to its somewhat more traditional predecessors, and tractably decomposes into productive and environmental efficiency as to allow policy makers and researchers alike to distinguish between these two notions of efficiency.

Finally, Chapter 3 applies the by-production modeling tactic developed in the previous chapter in reconsideration of an index decomposition analysis conducted by Pasurka [2006], who analyzed factors associated with emissions reductions of sulfur dioxide and nitrogen oxides from coal-fired, electric power plants by treating pollutants as weakly disposable, null-joint outputs. Sulfur dioxide and nitrogen oxides are primarily responsible for causing acid rain deposition in the environment. The reconsideration, assumes that the technology satisfies by-production, as introduced in Chapter 2, and shows, in general, that previous results derived using the standard technological assumption of weak disposability and null-jointness are significantly different than when by-production is assumed. Results are computed using output oriented hyperbolic and coordinate-wise distance functions and we report the results of the index decomposition using notions of both efficiency in intended production and efficiency in unintended production. Chapter 3 concludes the final portion of the study by analyzing energy firm responses during 1985-1995 to the 1990 Amendment to the Clean Air Act Amendment. Evidence is suggestive of firms responding almost immediately in preparation to binding emissions reductions, despite the fact that active compliance isn't required until 1995.

Chapter 4 contains some concluding remarks to summarize the results of these studies and discusses future prospective research applications spawned from this dissertation.

Boundary Problems with the "Russell" Graph Measure of Technical Efficiency: A Refinement

Steven B. Levkoff

University of California, Riverside

R. Robert Russell

University of California, Riverside, and

University of New South Wales

and

William Schworm

University of New South Wales

Abstract: In an influential paper, Fare and Lovell [1978] proposed an (input based) technical efficiency index designed to correct two fundamental inadequacies of the Debreu-Farrell index: its failure to satisfy (1) indication (the index is equal to 1 if and only if the input bundle is technically efficient) and (2) weak monotonicity (an increase in any one input quantity cannot increase the value of the index). Fare, Lovell, and Grosskopf [1985] extended the index to measure efficiency measurement in the full space of input and output quantities. Unfortunately, this index fails to satisfy not only indication and monotonicity at the boundary (of output space), but also weak monotonicity. We show, however, that a simple modification of the index corrects these flaws. To demonstrate the tractability of our proposal, we apply it to baseball batting performance, in which zero outputs occur frequently.

I. Introduction.

In an influential paper, Färe and Lovell [1978] proposed an (input based) technical efficiency index designed to correct two fundamental inadequacies of the Farrell [1956] index: its failure to satisfy (1) *indication* (the index is equal to 1 if and only if the input bundle is technically efficient in the sense of Koopmans [1951]) and (2) *monotonicity* (an increase in any one input quantity lowers the value of the index). In sharp contrast to the (maximal) radial-contraction construction of Farrell, the Färe-Lovell

(FL) index is essentially a (maximal) average of coordinate-wise contractions of input quantities, with an adjustment to correct for potential violations of both indication and monotonicity at the boundary of input space. As it turns out the FL index also fails to satisfy monotonicity, but it does satisfy indication and *weak monotonicity* (an increase in any one input or a decrease in any one output can not increase the value of the index).[1]

In recent years, much more emphasis has been placed on technical efficiency measurement in the full space of input and output quantities—often referred to as "graph space"—as opposed to efficiency measurement in input (or output) space alone. A prominent index in graph space, proposed by Färe, Grosskopf, and Lovell [1985], is a straightforward extension of the FL index. It is a (maximal) average of coordinate-wise expansions of outputs and contractions of inputs, with an adjustment to correct for potential violations of indication and monotonicity at the boundary of graph space. Variations of this Färe-Grosskopf-Lovell (FGL) index have played a prominent role in the operations-research literature in recent years (see, *e.g.*, Cooper, Seiford, Tone, and Zhu [2007] and the references therein). These indexes are commonly referred to as "slacks-based measures" (and, for obscure historical reasons, as "Russell measures").

In this paper, we show that the the FGL adjustment to correct for problems at

[1] See Russell [1985].

the boundary of graph space fails to do the job. In particular, the FGL index does not distinguish between efficient and inefficient points on the boundary of output space. Furthermore, an increase in an output quantity starting at an inefficient boundary output vector can *lower* the value of the FGL index. The FGL index, therefore, satisfies neither indication nor weak monotonicity.

In our view, this is a serious flaw, leaving the FGL index with no attractive properties whenever zero values of output quantities may occur. Of course, a zero output quantity would be highly improbable if there were a single output, but the FGL measure is designed to accommodate multiple outputs. Since a multiple-output firm might not produce some types of outputs, zeros in production vectors can be sensible choices and may arise naturally in empirical work for several reasons. A panel data application of agricultural firms may contain zeros in output space owing to seasonal crop rotation: outputs are only produced during specific growing seasons and thus appear as zeros during off seasons. Another situation where problems may arise because of zero output values is in the comparison of relative efficiencies of firms that produce separate, but non-disjoint sets of outputs. For example one firm may produce goods A, B and C, while the other produces only goods B, C, and D. A third example where zero outputs may appear is in the use of panel data with a rolling-window DEA analysis, as in the examination of the efficiency of maintenance units in the U.S. Air Force by Charnes, Clark, Cooper, and Golany [1984]. While their data base contains no zero outputs, it is likely that zeros would occur if the frequency of data acquisition were to increase in a rolling-window or panel application.

The flaw in the FGL index is attributable to the boundary adjustment that cannot distinguish between efficient and inefficient production vectors when some output quantities are zero. We show, however, that a simple modification of the FGL index—in particular, a modification of the correction factor at the boundary—corrects these flaws, restoring the indication and weak-monotonicity properties.[2]

To demonstrate the tractability of our proposal, we study an example in which zero outputs occur frequently. We measure a baseball player's batting performance by counting his singles, doubles, triples, and home runs during each game. If we measure these outputs for an entire season, zeros are likely to be rare, since most players produce all four types of hits over the course of an entire season. By increasing the frequency of observations to a per-game basis, we ensure that zero outputs are common. Thus, while our empirical application is intended primarily to illustrate the practicality of our approach, it also contributes modestly to the DEA literature on efficiency of baseball players (*e.g.*, Anderson and Sharp [1997] and Mazur [1995]).

The paper unfolds as follows: Section II lays out the framework of our analysis

[2] Our modification is not needed to maintain indication and weak monotonicity of the input-based FL index at the boundary of input space.

and defines efficiency indexes and the indication and monotonicity axioms. Section III shows that the FGL index violates indication and weak monotonicity at the boundary. Section IV introduces our modified FGL index and proves that it satisfies indication and weak monotonicity. Section V discusses the implementation of the modified FGL, and Section VI illustrates the practicality of the concept by applying it to two data sets, one synthetic and the other an actual data set on baseball performance. Section VII concludes.

II. Efficiency Indexes and Axioms.

The ⟨input, output⟩ production vector $\langle x, y\rangle \in \mathbf{R}_+^{n+m}$ is constrained to lie in a technology set $T \subset \mathbf{R}_+^{n+m}$. Denote the origin of this space by $\langle 0^{[n]}, 0^{[m]}\rangle$ and the unit vector by $\langle 1^{[n]}, 1^{[m]}\rangle$. The input requirement set for output y is $L(y) = \{x \in \mathbf{R}_+^n \mid \langle x, y\rangle \in T\}$, and the output possibility set for input x is $P(x) = \{y \in \mathbf{R}_+^m \mid \langle x, y\rangle \in T\}$.

We consider the collection of non-empty, closed technology sets that satisfy the following conditions:[3,4]

[3] All but free disposability of these conditions are necessary to guarantee that our efficiency indexes are well defined. Free disposability could be dispensed with (theoretically); the only change that would be needed in what follows would be to redefine the inefficiency indexes on the free-disposal hull of T, $T + (\mathbf{R}_+^n \times -\mathbf{R}_+^m)$, rather than on T itself (as in Russell [1987] for input-based efficiency indexes).

[4] Vector notation: $\bar{x} \geq x$ if $\bar{x}_i \geq x_i$ for all i; $\bar{x} > x$ if $\bar{x}_i \geq x_i$ for all i and $\bar{x} \neq x$; and $\bar{x} \gg x$ if $\bar{x}_i > x_i$ for all i.

(i) $\langle x, y\rangle \in T$ and $\langle \bar{x}, -\bar{y}\rangle > \langle x, -y\rangle$ implies $\langle \bar{x}, \bar{y}\rangle \in T$ (free disposability of inputs and outputs),

(ii) $y > 0^{[m]} \implies \langle 0^{[n]}, y\rangle \notin T$ (no free lunch), and

(iii) $P(x)$ is non-empty and bounded for all $x \in \mathbf{R}^n_+$.

Denote by $\mathcal{T}$ the set of non-empty, closed technologies satisfying these conditions.

A production vector $\langle x, y\rangle \in T$ is technologically efficient if $\langle x, -y\rangle > \langle \bar{x}, -\bar{y}\rangle$ implies $\langle \bar{x}, \bar{y}\rangle \notin T$; denote the efficient subset of T by $\mathrm{Eff}(T)$. An efficiency index is a mapping, $E : \Xi \to (0, 1]$, with image $E(x, y, T)$, where

$$\Xi = \left\{ \langle x, y, T\rangle \in T \times \mathcal{T} \mid \langle x, y\rangle \in T \ \wedge \ x \neq 0^{[n]} \right\}. \tag{2.1}$$

Färe, Grosskopf, and Lovell [1985] proposed a "graph" efficiency index on the full space of inputs and outputs. This index is an extension of the Färe-Lovell index defined in the input space. We formulate their index as follows:[5]

$$E_{FGL}(x,y,T) = \min_{\alpha,\beta} \left\{ \frac{\sum_i \delta(x_i)\alpha_i + \sum_j \delta(y_j)\beta_j}{\sum_i \delta(x_i) + \sum_j \delta(y_j)} \;\Big|\; \langle \alpha,\beta\rangle \in \Omega(x,y,T) \right\}, \tag{2.2}$$

where

$$\Omega(x,y,T) = \left\{ \langle \alpha,\beta\rangle \mid \langle\, \alpha \otimes x, y \oslash \beta\rangle \in T \;\wedge\; 0^{[n]} \leq \alpha \leq 1^{[n]} \;\wedge\; 0^{[m]} \ll \beta \leq 1^{[m]} \right\}, \tag{2.3}$$

$\alpha \otimes x = \langle \alpha_1 x_1, \ldots, \alpha_n x_n\rangle$, $y \oslash \beta = \langle y_1/\beta_1, \ldots, y_m/\beta_m\rangle$, and

$$\delta(z) = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z = 0 \end{cases} \tag{2.4}$$

for $\alpha = \langle \alpha_1, \ldots, \alpha_n\rangle$ and $\beta = \langle \beta_1, \ldots, \beta_m\rangle$.

[5] FGL order coordinates so that the first k input quantities and first l output quantities are positive and then minimize only over the sum of these $k+l$ coordinates, all of which have positive values. Our characterization does not require a permutation of the coordinates whenever a production vector (or a technology) is changed.

For the sake of symmetry, we weight each input-contraction factor by $\delta(x_i)$, $i = 1, \ldots n$, but the index would be unaffected by omitting these weights, since the value of α_i at the minimum is zero if $x_i = 0$.

In the initial specification of the FGL index (page 154), α is restricted to be strictly positive, in which case their minimization problem has no solution if, for some i, $x_i > 0$ and the minimum value of α_i is zero; this case is illustrated in Figure 2, where x' would be contracted to $\hat{x}$. In their characterization of the domain of the index (page 153), however, they restrict α to be non-negative. We choose the latter formulation to ensure that the index is well-defined on the domain, which includes input and output quantities with zero values.

Note that we are able to use the min operator instead of inf even though the constraint set $\Omega(x,y,T)$ is not closed because, using inf, $\overset{*}{\beta}_j = 0$ only if $\delta(y_j) = 0$, in which case β_j does not appear in the objective function in (2.2).

The use of the indicator function δ is a fundamentally important aspect of their formulation, aimed at correcting serious problems at the boundary. If $\langle x, y \rangle \gg 0^{[n+m]}$ so that $\sum_i \delta(x_i) + \sum_j \delta(y_j) = n + m$, the objective function in (2.2) is a simple average of the proportional, coordinate-wise, input-contraction and output-expansion factors. If $\sum_i \delta(x_i) + \sum_j \delta(y_j) < n + m$, the objective function is a simple average of proportional, coordinate-wise, input-contraction and output-expansion factors for *positive* input and output quantities, in which case inputs and outputs with zero quantity values are simply ignored in the efficiency calculation.

The axioms relevant to our analysis are as follows:[6]

Indication of Efficiency (I): For all $\langle x, y, T \rangle \in \Xi$, $E(x, y, T) = 1$ if and only if $\langle x, y \rangle \in \text{Eff}(T)$.

Weak Monotonicity (WM): For all pairs $\langle x, y, T \rangle \in \Xi$ and $\langle \bar{x}, \bar{y}, T \rangle \in \Xi$ satisfying $\langle \bar{x}, -\bar{y} \rangle > \langle x, -y \rangle$, $E(\bar{x}, \bar{y}, T) \leq E(x, y, T)$.

III. The Failure of the Färe-Grosskopf-Lovell Index on the Boundary.

The inability of the FGL Index to satisfy the indication and weak monotonicity axioms is demonstrated by the simple example with $m = 2$ displayed in Figure 1. Assume that $\langle x, y'' \rangle$ is efficient with $x \gg 0$ and note that $\langle x, y^o \rangle$ is inefficient. Calculate

[6] Neither of the indexes we consider satisfies the stronger property of (strict) monotonicity. Nor does either satisfy continuity. See Russell and Schworm [2011] for details.

the FGL Index at $\langle x, y^o \rangle$ as follows:

$$E_{FGL}(x, y^o, T) = \min_{\alpha,\beta} \left\{ \frac{\sum_{i=1}^{n} \alpha_i + \beta_1}{n+1} \;\middle|\; \langle \alpha, \beta \rangle \in \Omega(x, y^o, T) \right\} \tag{3.1}$$

where

$$\Omega(x, y^o, T) = \left\{ \langle 1^{[n]}, 1, \beta_2^o \rangle \mid 0 < \beta_2^o \leq 1 \right\}. \tag{3.2}$$

The objective function is simplified since $\delta(x_i) = 1$ for all $i = 1, \ldots, n$, $\delta(y_1^o) = 1$, and $\delta(y_2^o) = 0$. The constraint set is reduced to the line between $\langle 1^{[n]}, 1, 0 \rangle$ and $\langle 1^{[n]}, 1, 1 \rangle$. Since β_2^o does not affect the objective function, all points in $\Omega(x, y^o, T)$ are minimizing vectors and the efficiency index is

$$E_{FGL}(x, y^o, T) = \frac{n+1}{n+1} = 1. \tag{3.3}$$

Therefore, indication (I) is violated.

Next consider a point like y' in Figure 1 and note that

$$E_{FGL}(x, y', T) = \frac{n+1+\overset{*}{\beta}_2}{n+2} < 1, \tag{3.4}$$

where $\overset{*}{\beta}_2 = y_2'/y_2'' < 1$. Therefore, $y^0 < y'$ and $E_{FGL}(x, y^o, T) = 1$, so that (WM) is violated.

We summarize with the following theorem:

Theorem 1: E_{FGL} violates (I) and (WM).

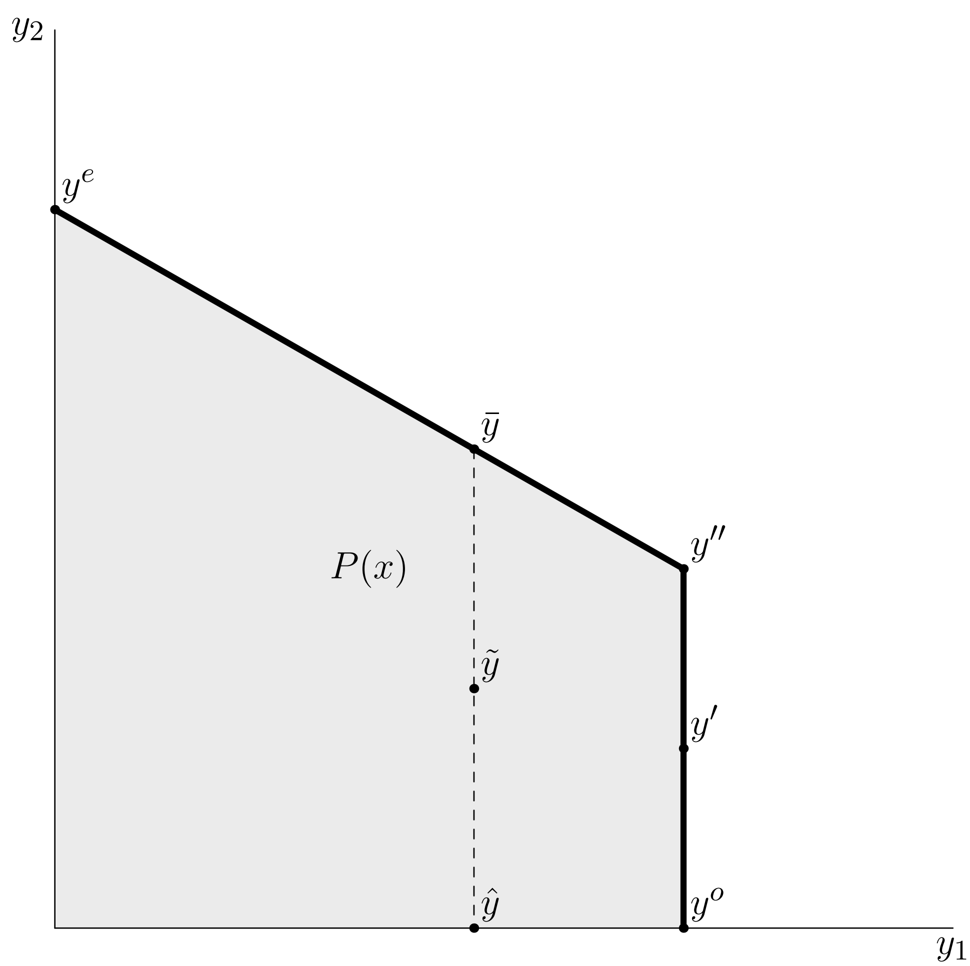

Figure 1: P(x) with Efficient and Inefficient Boundary Point

While the above example places the output vector at a "corner" of the production possibility set, the violation of weak monotonicity occurs for any feasible production vector in this diagram with a zero value of y_2. To see this, consider the points $\hat{y}$ and $\tilde{y}$ in Figure 1. So long as the slope of the frontier segment $[y^e y'']$ is low enough (so that y'' is the reference point for $\tilde{y}$), the FGL efficiency index values (again assuming efficiency in input space) is given by

$$E_{FGL}(x, \tilde{y}, T) = \frac{n + \hat{y}_1/y_1^o + \tilde{y}_2/y_2''}{n+2}. \tag{3.5}$$

The last term in the numerator of (3.5) can be made arbitrarily close to zero by shifting the cusp y'' vertically until the following inequality is established

$$E_{FGL}(x, \tilde{y}, T) = \frac{n + \hat{y}_1/y_1^o + \tilde{y}_2/y_2''}{n+2} < \frac{n + \hat{y}_1/y_1^o}{n+1} = E_{FGL}(x, \hat{y}, T). \tag{3.6}$$

This inequality shows that weak monotonicity is violated.

The example suggests that the problem is caused by the boundary adjustment when some element of the output vector is zero. Eliminating this adjustment for output and defining the objective function for a modified FGL Index by[7]

$$\frac{\sum_i \delta(x_i)\alpha_i + \sum_j \beta_j}{\sum_i \delta(x_i) + m} \tag{3.7}$$

would ensure that the index is less than one at $\langle x, y^o \rangle$. This alteration, however, would fail at the point y^e in Figure 1 where $\langle x, y^e \rangle$ an efficient point. In this case, the boundary adjustment is needed to ensure that $E_{FGL}(x, y^e, T) = 1$.

The difficulty with the FGL Index is that it is unable to distinguish between inefficient boundary points like y^0 and efficient boundary points like y^e in Figure 1. In the next section, we propose a modification that allows this distinction.

[7] And replacing min with inf.

A natural question is whether an analogous problem arises for inputs on the boundary that would also affect the (input oriented) Färe-Lovell Index, defined as follows:

$$E_{FL}(x,y,T) = \min_{\alpha} \left\{ \frac{\sum_i \delta(x_i)\alpha_i}{\sum_i \delta(x_i)} \;\middle|\; \langle \alpha \otimes x, y \rangle \in T \;\wedge\; 0^{[n]} \leq \alpha \leq 1^{[n]} \right\}. \qquad (3.8)$$

Figure 2 displays an example in which $\hat{x}$ is efficient but the simple average of minimal contraction factors is 1/2; on the other hand, $E_{FL}(\hat{x}, y, T) = 1$. Therefore, the boundary adjustment works correctly for input vectors with zero components.

These examples show that boundary points for inputs and outputs require different treatments if an index is to satisfy the indication and weak monotonicity axioms. Any input vector can be contracted by each component (with zero elements remaining unchanged) until an efficient point is reached. An output vector cannot always be expanded until an efficient point is reached, because the zero elements remain unchanged.[8]

IV. The Modified Färe-Grosskopf-Lovell Index.

To eliminate these boundary problems for the FGL index, we need to enable the index to distinguish between the two boundary points, y^0 and y^e, in Figure 1. First

[8] These output boundary problems also arise for the FGL output-oriented index formulated by Färe, Grosskopf, and Lovell [1985, pp. 148–149] (and further analyzed by Färe, Grosskopf, and Lovell [1994, pp. 115–118]).

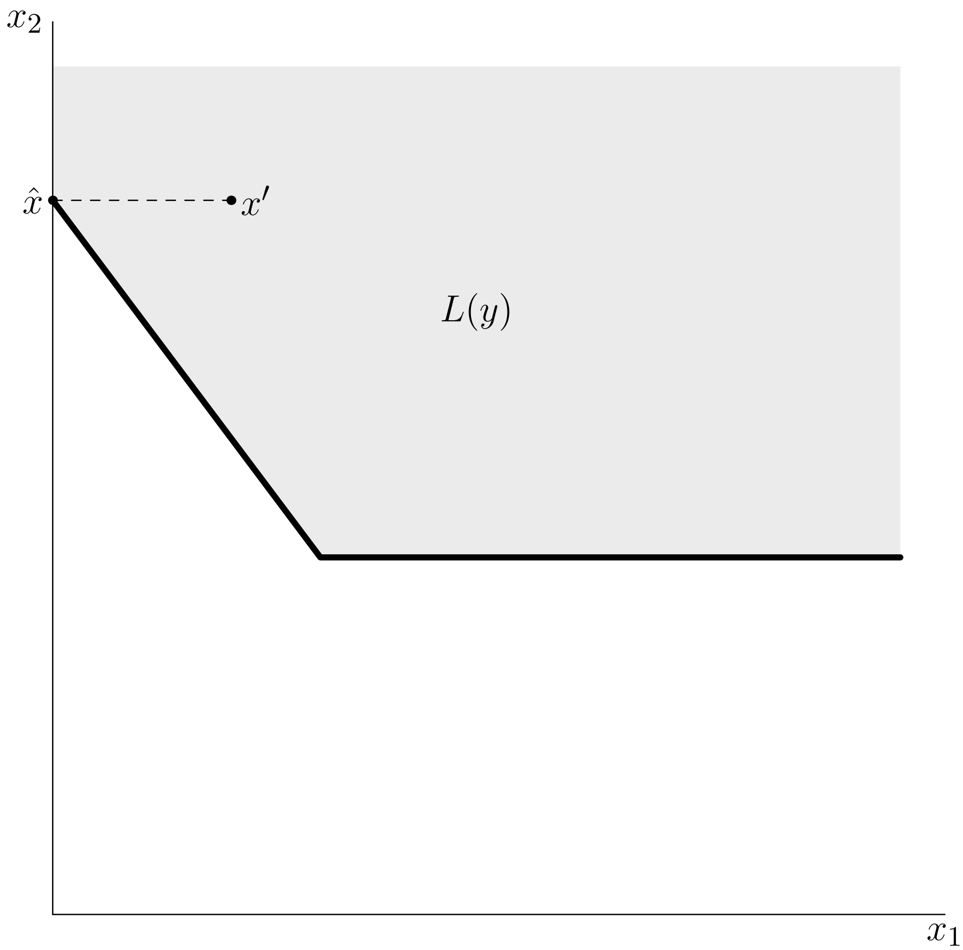

Figure 2: L(y) with Efficient Boundary Point

define $\epsilon^j \in \mathbf{R}^m_+$ as the vector with the j^{th} coordinate equal to $\epsilon \in \mathbf{R}_{++}$ and all other coordinates equal to zero. For inputs, we use the indicator for interior points $\delta(x_i)$ as defined in (2.4). For outputs, we use indicators for interior points or inefficient boundary points:

$$\psi_j(x, y, T) = \begin{cases} 1 & \text{if } y_j > 0 \text{ or } \left[y_j = 0 \ \wedge \ \langle x, y + \epsilon^j \rangle \in T \text{ for some } \epsilon > 0 \right] \\ 0 & \text{if } y_j = 0 \ \wedge \ \langle x, y + \epsilon^j \rangle \notin T \text{ for all } \epsilon > 0 \end{cases} \tag{4.1}$$

for $j = 1, \ldots, m$.

The indicator functions for output variables are output specific and are enhanced to distinguish between efficient and inefficient zero values for output variables. This modification requires that the output indicator functions, $\psi_j,\ j = 1, \ldots m$, depend on technologies and all input and output quantities rather than on y_j alone. Define the modified Färe-Grosskopf-Lovell index as follows:[9]

$$\bar{E}_{FGL}(x,y,T) = \inf_{\alpha,\beta} \left\{ \frac{\sum_i \delta(x_i)\alpha_i + \sum_j \psi_j(x,y,T)\beta_j}{\sum_i \delta(x_i) + \sum_j \psi_j(x,y,T)} \,\middle|\, \langle \alpha, \beta \rangle \in \Omega(x,y,T) \right\}, \quad (4.2)$$

where Ω is defined in (2.3). Let $\langle \overset{*}{\alpha}, \overset{*}{\beta} \rangle$ be values of $\langle \alpha, \beta \rangle$ that yield the infimum of the objective function in (4.2) and note that $\overset{*}{\beta}_j$ is an arbitrary selection from $(0,1]$ if $\psi_j(x,y,T) = 0$.[10]

Let us begin by seeing how the modified index works for the example in Figure 1. For the inefficient point $\langle x, y^o \rangle$ in Figure 1, $\psi_2(x, y^o, T) = 1$ and $\overset{*}{\beta}_2 = 0$, so that the infimum is

$$\bar{E}_{FGL}(x, y^0, T) = \frac{n+1+0}{n+1+1} < 1, \quad (4.3)$$

and indication is no longer violated. Also, note that $\psi_2(x, y', T) = 1$ and $\beta_2 = y_2'/y_2'' <$

[9] Note that we must use the infimum here, because $\psi_j(x,y,T)$ can be non-zero when $y_j = 0$.

[10] And, of course, in this formulation $\overset{*}{\alpha}_i$ is an arbitrary selection from [0,1] if $\delta(x_i) = 0$. Note that, in addition to this arbitrariness, the "solution" values for α_i or β_j not associated with zero values of x_i or y_j need not be unique, since there can be ties in the optimization problem.

1, so that

$$\bar{E}_{FGL}(x, y', T) = \frac{n+1+\beta_2}{n+1+1} > \frac{n+1}{n+2} = \bar{E}_{FGL}(x, y^0, T) \tag{4.4}$$

and weak monotonicity is satisfied for the change from y^o to y'.

For the efficient point $\langle x, y^e \rangle$ in Figure 1, $\psi_1(x, y^e, T) = 0$ and $\langle \overset{*}{\alpha}, \overset{*}{\beta} \rangle = \langle 1^{[n]}, \beta_1, 1 \rangle$ for any $\beta_1 \in (0, 1]$, so that

$$\bar{E}_{FGL}(x, y^e, T) = \frac{n+0+1}{n+0+1} = 1, \tag{4.5}$$

and indication is satisfied. As there is no feasible variation in y_1^e alone, there is no violation of weak monotonicity.

Now, we turn to the general result.

Theorem 2: *$\bar{E}_{FGL}$ satisfies (I) and (WM).*

We begin with a lemma regarding the indicator functions.

Lemma 1: *If $\langle x, y \rangle \in T$, $\langle \bar{x}, \bar{y} \rangle \in T$, and $\langle x, -y \rangle < \langle \bar{x}, -\bar{y} \rangle$, then (i) $\delta(x_i) \leq \delta(\bar{x}_i)$ for all $i = 1, \ldots, n$ and (ii) $\psi_j(x, y, T) \leq \psi_j(\bar{x}, \bar{y}, T)$ for all $j = 1, \ldots, m$.*

Proof: As (i) is obvious, we need only prove (ii). It suffices to consider two cases: (a) $x < \bar{x}$ (and $y = \bar{y}$) and (b) $y > \bar{y}$ (and $x = \bar{x}$).

In case (a), $x < \bar{x}$ and free disposability imply $P(x) \subseteq P(\bar{x})$. Since $y = \bar{y}$ by assumption, $\psi_j(x, y, T) \neq \psi_j(\bar{x}, \bar{y}, T)$ for some j only if $y_j = \bar{y}_j = 0$, $y + \epsilon^j \notin P(x)$

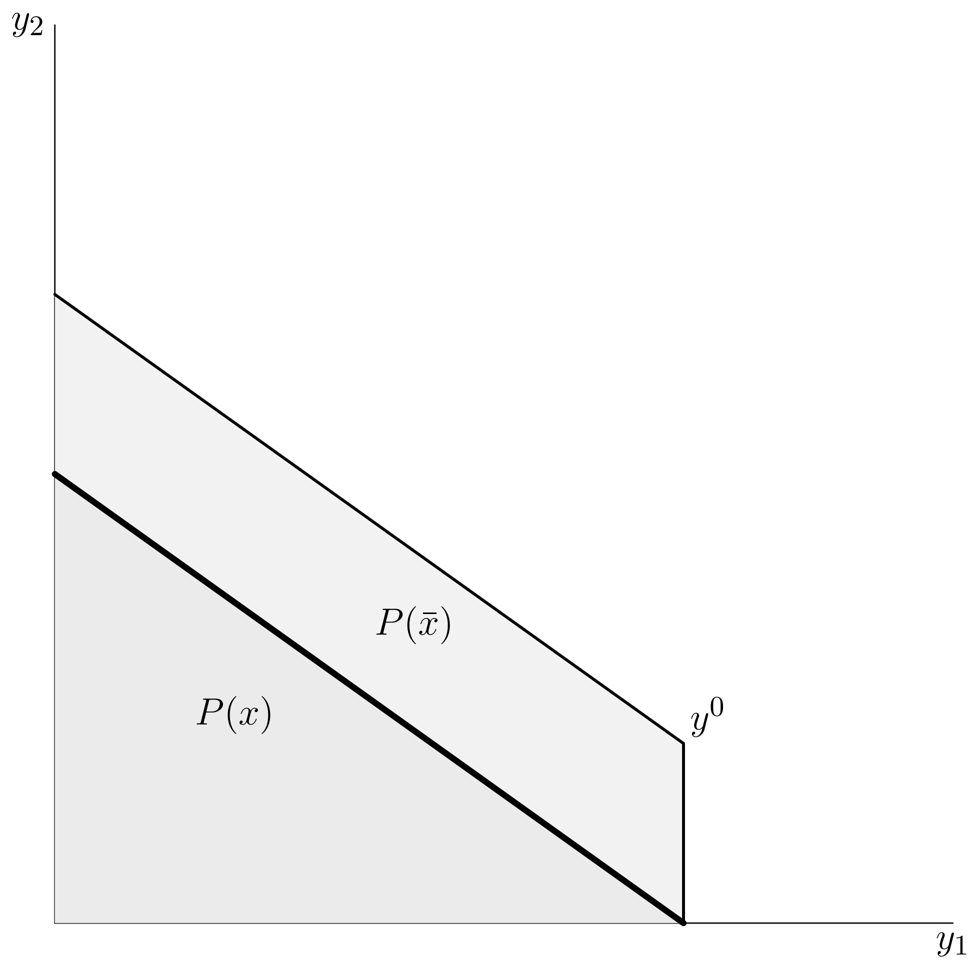

Figure 3: $\psi_2(x, y) < \psi_2(\bar{x}, y)$

for all $\epsilon > 0$, and $y + \epsilon^j \in P(\bar{x})$ for some $\epsilon > 0$ (refer to definition (4.1)). Figure 3 displays this situation. In this case, $\psi_j(x, y, T) = 0$ and $\psi_j(\bar{x}, \bar{y}, T) = 1$ so that the inequality in (ii) is satisfied.

In case (b), it suffices to establish the result for an arbitrary coordinate; suppose, therefore, that $y_{j'} > \bar{y}_{j'}$ for some j' and that $y_j = \bar{y}_j$ for all $j \neq j'$. Suppose that

$\psi_{j'}(x, y, T) \neq \psi_{j'}(\bar{x}, \bar{y}, T)$. Then $y_{j'} > 0$ and $\bar{y}_{j'} = 0$ (refer again to definition (4.1)). It follows from $\langle x, y \rangle \in T$ that $\langle \bar{x}, \bar{y} + \epsilon^{j'} \rangle \in T$, where $\epsilon = y_{j'} - \bar{y}_{j'}$. Thus, in fact, $\psi_{j'}(x, y, T) = \psi_{j'}(\bar{x}, \bar{y}, T) = 1$.

Consider now $\psi_j(x, y, T)$ where $j \neq j'$. Since $y_j = \bar{y}_j$ by assumption, either $y_j > 0$ and $\bar{y}_j > 0$, in which case $\psi_j(x, y, T) = \psi_j(x, \bar{y}, T) = 1$ or $y_j = \bar{y}_j = 0$. In the latter case, $\psi_j(x, y, T) > \psi_j(\bar{x}, \bar{y}, T)$ only if $\langle x, y + \epsilon^j \rangle \in T$ for some $\epsilon > 0$ and $\langle x, \bar{y} + \epsilon^j \rangle \notin T$ for all $\epsilon > 0$, in which case $\psi_j(x, y, T) = 1$ and $\psi_j(x, \bar{y}, T) = 0$. Writing out these vectors more explicitly (assuming, without loss of generality, that $1 < j < j' < m$), we have, for some $\epsilon > 0$,

$$\langle y_1, \ldots, 0_j + \epsilon, \ldots, y_{j'}, \ldots, y_m \rangle \in P(x) \tag{4.6}$$

and

$$\langle y_1, \ldots, 0_j + \epsilon, \ldots, \bar{y}_{j'}, \ldots, y_m \rangle \notin P(x) \tag{4.7}$$

where 0_j is the placeholder for the zero value of $y_j = \bar{y}_j$. But since $y_{j'} > \bar{y}_{j'}$, this violates free disposability. ■

We now prove the theorem.

Proof of Theorem:

(i) Indication.

To show that $\bar{E}_{FGL}$ satisfies (I), suppose first that $\bar{E}_{FGL}(x, y, T) < 1$. There are three (nonexclusive) possibilities: (a) there exists at least one i' with $\delta(x_i) = 1$ and $0 \leq \overset{*}{\alpha}_{i'} < 1$; (b) there exists at least one j' with $\psi_{j'}(x, y, T) = 1$ and $0 < \overset{*}{\beta}_{j'} < 1$; and (c) there exists at least one j' with $\psi_{j'}(x, y, T) = 1$ and $\overset{*}{\beta}_{j'} = 0$.

Case (a) implies that there is a vector $\bar{x} = \overset{*}{\alpha} \otimes x$ with $\bar{x} < x$ and $\langle \bar{x}, y \rangle \in T$. Case (b) implies that either there is a vector $\bar{y} = y \oslash \overset{*}{\beta}$ with $\bar{y} > y$ and $\langle x, \bar{y} \rangle \in T$. In both cases, $\langle x, y \rangle$ is inefficient. Case (c) implies that there exists a vector $\bar{y}$, with $\bar{y}_{j'} > y_{j'} = 0$ for some j' and $\bar{y}_j = y_j$ for all $j \neq j'$, satisfying $\langle x, \bar{y} \rangle \in T$, so that $\langle x, y \rangle$ is inefficient.

Now suppose that $\langle x, y \rangle$ is inefficient in T, implying the existence of a vector $\langle \bar{x}, \bar{y} \rangle \in T$ satisfying $\langle \bar{x}, -\bar{y} \rangle < \langle x, -y \rangle$. There are again three (nonexclusive) possibilities: (a) there exists at least one i' such that $0 \leq \bar{x}_{i'} < x_{i'}$; (b) there exists at least one j' such that $0 < y_{j'} < \bar{y}_{j'}$; or (c) there exists at least one j' such that $0 = y_{j'} < \bar{y}_{j'}$.

In cases (a) and (b), there exists a vector $\langle \alpha, \beta \rangle \in \Omega(x, y, T)$ with $\alpha_{i'} < \overset{*}{\alpha}_{i'} \leq 1$ in case (a) and $\beta_{j'} < \overset{*}{\beta}_{j'} \leq 1$ in case (b); moreover, $\delta(\bar{x}_{i'}) = \delta(x_{i'}) = 1$ in case (a) and $\psi_{j'}(\bar{x}, \bar{y}, T) = \psi_{j'}(x, y, T) = 1$ in case (b), so that, *ceterus paribus*, $\sum_i \delta(x_i) + \sum_j \psi_j(x, y, T) = \sum_i \delta(\bar{x}_i) + \sum_j \psi_j(\bar{x}, \bar{y}, T)$ in either case. In case (c), we have $\overset{*}{\beta}_{j'} = 0$ and $\psi_{j'}(x, y, T) = 1$. In each case, we obtain $\bar{E}_{FGL}(x, y, T) < 1$.

(ii) Weak Monotonicity.

Consider two production vectors satisfying $\langle \bar{x}, -\bar{y} \rangle > \langle x, -y \rangle$, which implies

$$\Omega(x, y, T) \subset \Omega(\bar{x}, \bar{y}, T). \tag{4.8}$$

This implies that

$$\begin{aligned} \bar{E}_{FGL}(x, y, T) &= \inf_{\alpha,\beta} \left\{ \frac{\sum_i \delta(x_i)\alpha_i + \sum_j \psi_j(x, y, T)\beta_j}{\sum_i \delta(x_i) + \sum_j \psi_j(x, y, T)} \;\middle|\; \langle \alpha, \beta \rangle \in \Omega(x, y, T) \right\} \\ &\geq \inf_{\alpha,\beta} \left\{ \frac{\sum_i \delta(x_i)\alpha_i + \sum_j \psi_j(x, y, T)\beta_j}{\sum_i \delta(x_i) + \sum_j \psi_j(x, y, T)} \;\middle|\; \langle \alpha, \beta \rangle \in \Omega(\bar{x}, \bar{y}, T) \right\}. \end{aligned} \tag{4.9}$$

(Note that, while the constraints in these two optimization problem differ, the objective functions are identical.) Our task now is to show that replacing the weights in the right-hand-side (RHS) optimization problem, $\delta(x_i),\ i = 1, \ldots n$, and $\psi_j(x, y, T),\ j = 1, \ldots, m$, with $\delta(\bar{x}_i),\ i = 1, \ldots n$, and $\psi_j(\bar{x}, \bar{y}, T),\ j = 1, \ldots, m$, does not affect the inequality. To this end, denote the solution to the RHS optimization problem by $\langle \bar{\alpha}, \bar{\beta} \rangle$.

By Lemma 1, $x_{i'} < \bar{x}_{i'}$ and $\delta(x_{i'}) \neq \delta(\bar{x}_{i'})$ imply $\delta(x_{i'}) = 0$ and $\delta(\bar{x}_{i'}) = 1$, hence $x_{i'} = 0$ and $\bar{x}_{i'} > 0$, in which case replacing $\delta(x_{i'})$ with $\delta(\bar{x}_{i'})$ replaces zeros in both the numerator and denominator with $\alpha_{i'}$ in the numerator and 1 in the denominator. Clearly, $\langle \bar{\alpha}, \bar{\beta} \rangle$ remains feasible in the new optimization problem, so that the infimum is at least as small as before the substitution.[11]

[11] In fact, it will be lower, since the zeros in the optimal values of the numerator and denominator will be replaced by a zero in the numerator and a 1 in the denominator.

By Lemma 1, $\psi_{j'}(x,y,T) \neq \psi_{j'}(\bar{x},\bar{y},T)$ implies $\psi_{j'}(x,y,T) = 0$ and $\psi_{j'}(\bar{x},\bar{y},T) = 1$. From the definition of $\psi_{j'}$, this implies that $y_{j'} = 0$ and either (i) $\bar{y}_{j'} > 0$ or (ii) $\bar{y}_{j'} = 0$ and there exists an ϵ such that $\langle \bar{x}, \bar{y} + \epsilon^{j'} \rangle \in T$. Alternative (i) is ruled out, since $y > \bar{y}$ by assumption. Under alternative (ii), replacing $\psi_{j'}(x,y,T)$ with $\psi_{j'}(\bar{x},\bar{y},T)$ replace zeros in the numerator and denominator of the RHS objective function with $\beta_{j'}$ and 1 in the numerator and denominator, respectively. Again, $\langle \bar{\alpha}, \bar{\beta} \rangle$ remains feasible in the new optimization problem, so that the infimum is at least as small as before the substitution.

This completes the proof. ■

V. Empirical Implementation.

Implementation of the modified FGL index is not straightforward, since the functions ψ_j, $j = 1, \ldots, m$, depend on infinitesimal comparisons, the constraint set is not closed, and the use of the infimum is salient (as the minimum does not always exist). In this section, we discuss possible solutions to these problems and suggest methods for calculating $\bar{E}_{FGL}(x,y,T)$.

If the technology is known, then a frequently employed method of dealing with zero inputs or outputs can be used: simply replace the zeros with small positive numbers. Since the FGL index handles zero elements of the input vector correctly, we need only perturb the output vector.

Consider $\langle x, y\rangle \in T$ with some elements of the input and output vectors possibly zero. Define y^ϵ by replacing all zero components of y with a small number $\epsilon > 0$ and consider the perturbed production vector $\langle x, y^\epsilon\rangle$, which may or may not be feasible.

Define the functions ψ_j^ϵ for $j = 1, \ldots, m$ as follows:

$$\psi_j^\epsilon(x, y, T) = \begin{cases} 1 & \text{if } y_j > 0 \text{ or } \left[\, y_j = 0 \ \wedge \ \langle x, y + \epsilon^j\rangle \in T \,\right] \\ 0 & \text{if } y_j = 0 \ \wedge \ \langle x, y + \epsilon^j\rangle \notin T \end{cases} \tag{5.1}$$

Then replace $\bar{E}_{FGL}$ with a modification $\bar{E}^\epsilon_{FGL}$ defined by

$$\bar{E}^\epsilon_{FGL}(x, y, T) = \min_{\alpha,\beta} \left\{ \frac{\sum_i \delta(x_i)\alpha_i + \sum_j \psi_j^\epsilon(x, y, T)\beta_j}{\sum_i \delta(x_i) + \sum_j \psi_j^\epsilon(x, y, T)} \;\middle|\; \langle \alpha, \beta\rangle \in \Omega(x, y^\epsilon, T) \right\} \tag{5.2}$$

which is well defined, since the minimum is attained at $\overset{*}{\beta} \gg 0^{[m]}$.

The formulation in (5.2) provides a good approximation for small ϵ only if $\bar{E}^\epsilon_{FGL}$ is continuous at the boundary of output space for sequences approaching the boundary from the interior. Although Russell and Schworm [2011] have shown that the FGL index itself is not in general continuous in input or output quantities, we can show that $\bar{E}^\epsilon_{FGL}$ converges to $\bar{E}_{FGL}$ at the boundary for the restricted paths $y^\epsilon \to y$ as $\epsilon \to 0$.

For any $\langle x, y, T\rangle$ define the reference point $\langle \bar{x}, \bar{y}$ by $\bar{x} = \overset{*}{\alpha} \otimes x$ and $\bar{y} = y \oslash \overset{*}{\beta}$.

Then we can reformulate (5.2) as

$$\bar{E}^{\epsilon}_{FGL}(x,y,T) = \min_{\alpha,\bar{y}} \left\{ \frac{\sum_i \delta(x_i)\alpha_i + \sum_j \psi^{\epsilon}_j(x,y,T) y^{\epsilon}_j / \bar{y}_j}{\sum_i \delta(x_i) + \sum_j \psi^{\epsilon}_j(x,y,T)} \;\Big|\; \langle \alpha, \beta \rangle \in \Omega(x, y^{\epsilon}, T) \right\}. \tag{5.3}$$

From this formulation, it is clear that $\bar{E}^{\epsilon}_{FGL}(x,y,T) \to \bar{E}_{FGL}(x,y,T)$ as $\epsilon \to 0$.

If the technology is known and convex, we can sketch a alternative method for calculating $\psi_j(x,y,T)$. If $y_j > 0$, set $\psi_j(x,y,T) = 1$. If $y_j = 0$, calculate the shadow prices for $\langle x,y \rangle$. If the shadow price of the j^{th} output is positive for any shadow price vector supporting $\langle x,y \rangle$, set $\psi_j(x,y,T) = 0$. Otherwise, set $\psi_j(x,y,T) = 1$. If the technology is not convex, shadow prices may not exist and it is necessary to compute $\psi_j(x,y,T)$ directly by checking feasibility of small changes in output as above.

If the technology is not known, it is necessary to use the data to estimate the technology. We sketch here a method of calculating—or at least approximating arbitrarily closely—the modified FLG index using DEA methods.

Assume we have data on inputs and outputs for D decision making units (DMUs): $\langle x^d, y^d \rangle,\ d = 1, \ldots, D$. Define $I^d_+ = \{i \mid x^d_i > 0\}$ and $J^d_+ = \{j \mid y^d_j > 0\}$.

The original FGL index for a specific DMU, d', is calculated by first running the following program:[12]

$$
\begin{aligned}
\min_{\alpha,\beta,\lambda} \quad & \sum_{i \in I_+^{d'}} \alpha_i + \sum_{j \in J_+^{d'}} \beta_j \quad \text{s.t.} \\
& \alpha_i x_i^{d'} \geq \sum_{d=1}^{D} \lambda_d \; x_i^d \quad \wedge \quad 0 \leq \alpha_i \leq 1 \quad \forall \; i \in I_+^{d'}, \\
& y_j^{d'}/\beta_j \leq \sum_{d=1}^{D} \lambda_d \; y_j^d \quad \wedge \quad 0 < \beta_j \leq 1 \quad \forall \; j \in J_+^{d'}, \\
& \lambda_d \geq 0, \quad d = 1, \ldots, D.
\end{aligned} \tag{5.4}
$$

Let A be the solution to this program. Then

$$
E_{FGL} = \frac{A}{S\left(x^{d'}, y^{d'}\right)}, \tag{5.5}
$$

where

$$
S(x^{d'}, y^{d'}) = \sum_i \delta \left(x_i^{d'} \right) + \sum_j \delta_j \left(y_j^{d'} \right) = |I_+^{d'}| + |J_+^{d'}|. \tag{5.6}
$$

The proposed method of calculating the data-based, modified FGL index has three steps:

Step 1 distinguishes between zero values of outputs with $\overset{*}{\beta}_j = 1$ and those with $\overset{*}{\beta}_j < 1$ (*i.e.*, between efficient points with zero outputs and inefficient points with

[12] As noted above, the minimum is well defined (no attempt to divide by zero): since $y_j^{d'}$ is non-zero and the output constraint set is bounded, the solution value for each β_j will be non-zero.

zero outputs). For a selected DMU, d', solve

$$
\begin{aligned}
&\min_{\alpha,\beta,\lambda} \quad \sum_{i\in I_+^{d'}} \alpha_i + \sum_{j=1}^{m} \beta_j \quad \text{s.t.} \\
&\qquad \alpha_i x_i^{d'} \geq \sum_{d=1}^{D} \lambda_d \, x_i^d \quad \wedge \quad 0 \leq \alpha_i \leq 1 \quad \forall\, i \in I_+^{d'}, \\
&\qquad \left(y_j^{\epsilon}\right)^{d'} / \beta_j \leq \sum_{d=1}^{D} \lambda_d \, \left(y_j^{\epsilon}\right)^{d} \quad \wedge, \quad 0 < \beta_j \leq 1, \quad j = 1, \ldots, m, \\
&\qquad \lambda_d \geq 0, \quad d = 1, \ldots, D.
\end{aligned}
\tag{5.7}
$$

Denote the minimizing values of β in (5.7) by $\overset{*}{\beta}$ and set $J_\epsilon^{d'} = \left\{ j \mid y_j = 0 \wedge \overset{*}{\beta}_j < 1 \right\}$. $J_\epsilon^{d'}$ is the set of zero output coordinates belonging to inefficient production vectors (since $j \in J_\epsilon^{d'}$ implies that $y_j^\epsilon / \overset{*}{\beta}_j \in T$ for $\overset{*}{\beta}_j < 1$).

Step 2 calculates the minimal value of the numerator of the objective function under the constraint that $\beta_j = 0$ for all zero outputs that are coordinate-wise inefficient. Solve

$$
\begin{aligned}
&\min_{\alpha,\beta,\lambda} \quad \sum_{i\in I_+^{d'}} \alpha_i + \sum_{j\notin J_\epsilon^{d'}} \beta_j \quad \text{s.t.} \\
&\qquad \alpha_i x_i^{d'} \geq \sum_{d=1}^{D} \lambda_d \, x_i^d \quad \wedge \quad 0 \leq \alpha_i \leq 1 \quad \forall\, i \in I_+^{d'}, \\
&\qquad (y_j^{\epsilon})^{d'} / \beta_j \leq \sum_{d=1}^{D} \lambda_d \, \left(y_j^{\epsilon}\right)^{d}, \quad \wedge \quad 0 < \beta_j \leq 1 \qquad \forall j \notin J_\epsilon^{d'}, \\
&\qquad \lambda_d \geq 0, \quad d = 1, \ldots, D.
\end{aligned}
\tag{5.8}
$$

Step 3 generates the value of the index by dividing the the value function in (5.8) by the sum of the coordinates with positive values of input or output quantities plus the number of outputs with zero values that are coordinate-wise inefficient. Denote the solution to (5.8) by A. The (approximate) modified FGL index is then given by

$$E^{\epsilon}_{FGL} = \frac{A}{|I^{d'}_{+}| + |J^{d'}_{+}| + |J^{d'}_{\epsilon}|}. \tag{5.9}$$

This algorithm could yield some values that are only approximately correct (if zero output values exist), owing largely to the use of the ϵ perturbation of the data. In fact, the method could treat a point with zero output values as efficient if y were, in some sense, less than ϵ below the frontier. But such a point would be approximately efficient. Moreover the chance of such incorrect identification of efficient points is extremely remote if ϵ is chosen small enough.

VI. Empirical Example.

In this section, we employ two data sets to illustrate the practicality of our proposed approach to implementing the modified FGL. The first, a synthetic data set with two inputs and two outputs, illustrates the practical restoration of the indication and monotonicity properties at the boundary of output space. The second is an actual data base on an athlete's relative (self) performance over the course of a season.

The synthetic data base contains 10 observations on the two outputs and two inputs. Because the problems with the FGL index arise only at the boundary of output space, we set input quantities to unity, allowing us to represent the technology in output space. The output data and efficiency scores of the two indexes are displayed in Table 1, and the production possibility set (the DEA hull in output space) is depicted in Figure 4.

Table 1: $\bar{E}_{FLG}$ versus E_{FGL} for the Synthetic Data

Observation	y_1	y_2	$E_{FGL}(x,y,T)$	$\bar{E}_{FGL}(x,y,T)$
1	0	6	1	.81
2	3	6	1	1
3	4	5	1	1
4	5	4	1	1
5	9	0	1	1
6	0	3	.67	.50
7	1	5	.77	.77
8	3	4	.88	.88
9	5	3	.94	.94
10	5	0	.70	.54

As required by theory, the two indexes agree at all points except inefficient points containing a zero value (observations 1, 6, and 10). Observation 1 is identified as output efficient by the original FGL index, whereas the modified index corrects for inefficiency at this boundary point. Observation 5 is properly identified as an output-efficient point despite the zero value of output 2.

As discussed in the Introduction, zero outputs may arise naturally in empirical

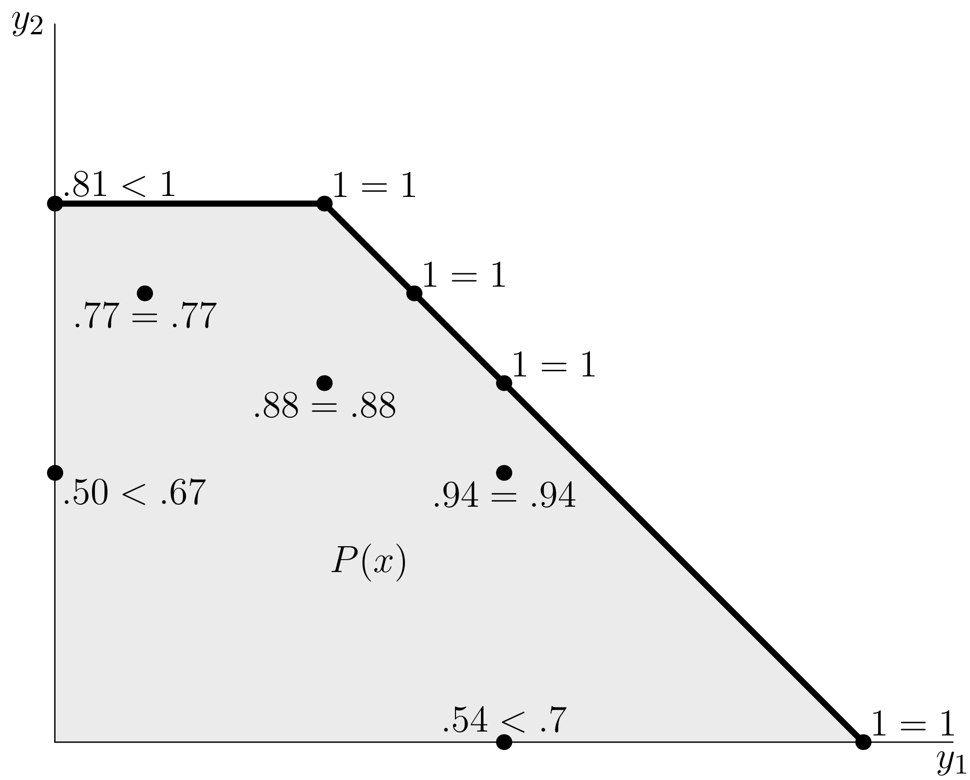

Figure 4: The Technology for the Synthetic Data

work for several reasons. To impose a demanding test on our proposed modification of the FGL index, we select a problem with frequent observations with zeros in the outputs. In particular, we measure a baseball player's batting performance in each game by counting the singles, doubles, triples, and home runs during the game.

Mazur [1995] evaluates players' relative efficiencies by utilizing a zero-dimensional input space and a three-dimensional output space composed of a player's batting average, runs batted in (RBIs), and home runs. Many baseball statisticians, however, have rejected this formulation, because RBI's are a function of the number of players on base during an at-bat, which may be unrelated to a particular batter's performance.

Anderson and Sharp [1997] utilize the composite batter index (CBI) as a performance measure of a baseball players batting prowess. They adopt plate appearances—the number of at bats plus the number of walks—as the one-dimensional input. The five outputs are walks, singles, doubles, triples, and home runs. Their analysis, however, uses inputs and outputs aggregated over an entire season, in which case zero values in output space are unlikely.

We proceed in a fashion similar to Anderson and Sharp [1997], adopting a modification of their CBI. We drop from the output space the number of walks a player accumulates and look only at active hit performance, singles, doubles, triples, and home runs, since walks are mainly a function of the pitcher's performance, not the batter's. Specifically, we track the batting performance of Babe Ruth, one of history's most renowned baseball figures, on a per-game basis, using both the FGL and the modified FGL for the 1923 season with the New York Yankees.

We are unaware of any studies that use such a high-frequency statistic as batting performance on a per game basis. The flexibility of the modified FGL index in handling zeros in the output data allows us to increase the frequency at which we are able to calculate relative efficiencies. The data used can be found at http://baseball-reference.com. We track the performance of Babe Ruth over the duration of the 1923 regular season, in which he won the MVP award. The season consisted of 152 games total, and we treat Babe Ruth's performance in each game as a separate DMU to assess performance relative to his performance in other games played during that season.[13]

We compute, for each of the 152 games, the original FGL index and the modified FGL index numerically, by solving the non-linear programs laid out in the previous section using a combination of active-set, trust-region, and line-search algorithms.[14] The results are depicted in Figure 5, where solid dots reflect the standard FGL scores and circles reflect the modified FGL scores. The lines connecting these scores for each game illustrate the gaps between the two scores. As can be seen from this figure, the correction for inefficient boundary points with zero outputs by the modified FGL index is large. Moreover, the Spearman rank and Pearson product moments between the FGL and the modified FGL indexes for this exercise are 0.7073 and 0.7102 respectively. Thus, the modified FGL index, by adjusting for inefficient boundary points, also significantly alters the relative rankings of performance across games.

VII. Conclusion.

[13] As pointed out by a referee, a more ambitious analysis would take account of the fact that the production set T would vary across baseball parks, time of day, etc. As our main purpose here is to illustrate the practicability of our approach, we leave such refinements to sabremetric specialists.

[14] The programming codes are available at http://economics.ucr.edu/people/russell/index.html.

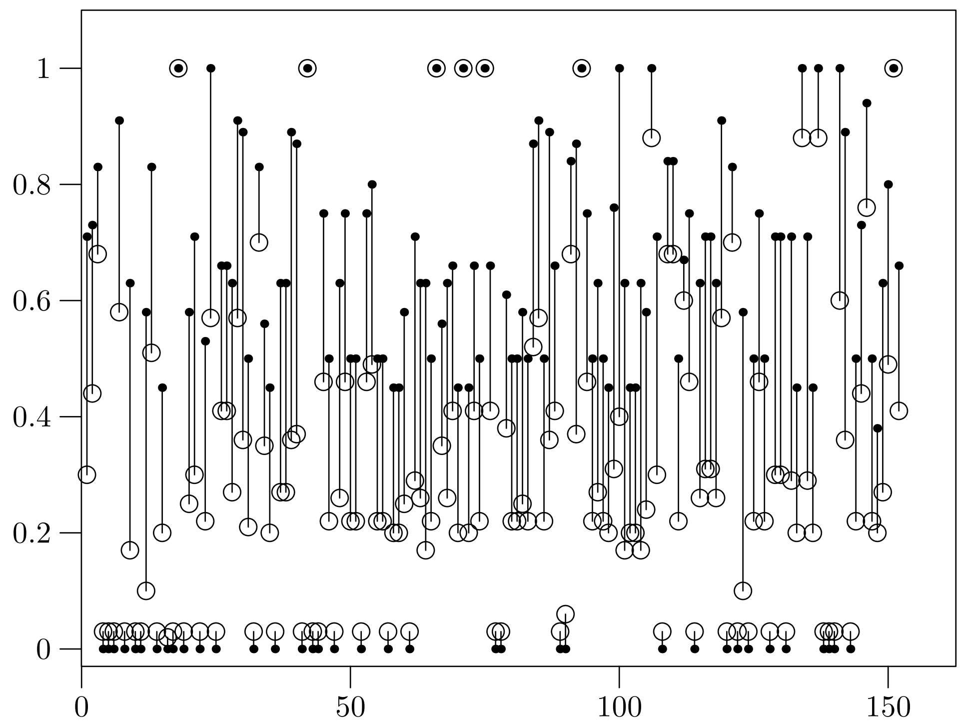

Figure 5: $\bar{E}_{FLG}$ *versus* E_{FGL}

Satisfaction of the indication condition is the principal (putative) advantage of the FGL index (as well as related "slacks based" indexes that contract inputs and expand outputs in coordinate-wise directions) over indexes that contract inputs and expand outputs in a radial or arbitrary direction—*e.g.*, the hyperbolic index (Färe, Grosskopf, and Lovell [1985]), the Briec [1997] index, and the directional-distance index (Luenberger [1992], Chung, Färe, and Grosskopf [2000], and Färe and Grosskopf [2000]). Failure to satisfy indication at the boundary is therefore a serious inadequacy of the FGL index, as is the failure to satisfy weak monotonicity, which *is* satisfied by the radial and directional indexes. We therefore believe that the modification of the FGL index we propose to restore indication and weak monotonicity is essential. In addition, we provide some illustrative empirical evidence, using baseball performance data, that the modified index can be practicably implemented and can yield results that are markedly different from those generated by the original FGL index.

References

Anderson, T., and G P. Sharp [1997], "A New Measure of Baseball Batters Using DEA," *Annals of Operations Research*, **73**: 141-155.

Briec, W. [1997], "A Graph Type Extension of Farrell Technical Efficiency," *Journal of Productivity Analysis*, **8**: 95–110.

Charnes, A., C. T. Clark, W. W. Cooper, and B. Golany [1984], "A Developmental Study of Data Envelopment Analysis in Measuring the Efficiency of Maintenance Units in the U.S. Air Force," *Annals of Operations Research*, **2**: 95-112.

Chung, Y, R. Färe, and S. Grosskopf [1997], "Productivity and Undesirable Outputs: A Directional Distance Function Approach," *Journal of Environmental Management*, **51**: 229–240.

Cooper, W. W., L. M. Seiford, K. Tone, and J. Zhu, "Some Models and Measures for Evaluating Performances with DEA: Past Accomplishments and Future Prospects," *Journal of Productivity Analysis*, **28**: 151–164.

Färe, R., and S. Grosskopf [2000], "Theory and Application of Directional Distance Functions," *Journal of Productivity Analysis*, **13**: 93–103.

Färe, R., S. Grosskopf, and C. A. K. Lovell [1985], *The Measurement of Efficiency of Production*, Boston: Kluwer-Nijhoff.

Färe, R., S. Grosskopf, and C. A. K. Lovell [1994], *Production Frontiers*, Cambridge: Cambridge University Press.

Färe, R., and C. A. K. Lovell [1978], "Measuring the Technical Efficiency of Production," *Journal of Economic Theory*, **19**: 150–162.

Farrell, M. J. [1956], "The Measurement of Productive Efficiency," *Journal of the Royal Statistical Society, Series A, General*, **120**: 253–282.

Koopmans, T. C. [1951], "Analysis of Production as an Efficient Combination of Activities," in *Activity Analysis of Production and Allocation* (T. C. Koopmans, Ed.), Heidelberg: Physica-Verlag.

Luenberger, D. G. [1992], "New Optimality Principles for Economic Efficiency and Equilibrium," *Journal of Optimization and Applications*, **75**: 221–264.

Mazur, M.J. [1995], "Evaluating the Relative Efficiency of Baseball Players?," in *Data Envelopment Analysis: Theory, Methodology and Applications* (A. Charnes, W. W. Cooper, A. Y. Lewin and L. M. Seiford, eds.), Boston: Kluwer Academic Press.

Russell, R. R. [1985], "Measures of Technical Efficiency," *Journal of Economic Theory*, **35**: 109–126.

Russell, R. R. [1987], "On the Axiomatic Approach to the Measurement of Technical Efficiency," in *Measurement in Economics: Theory and Application of Economic*

Indices (W. Eichhorn, Ed.), Heidelberg: Physica-Verlag.

Russell, R. R., and W. Schworm [2011], "Properties of Inefficiency Indexes on ⟨Input, Output⟩ Space," *Journal of Productivity Analysis*, forthcoming.

On Modeling Pollution-Generating Technologies.

Sushama Murty
University of Exeter

R. Robert Russell
University of California, Riverside

Steven B. Levkoff
University of California, Riverside

Abstract

We argue analytically that many commonly used models of pollution-generating technologies, which treat pollution as a freely disposable input or as a weakly disposable and null-joint output, may generate unacceptable implications for the trade-offs among inputs, outputs, and pollution. We show that the correct trade-offs in production are best captured if a pollution generating technology is modeled as an intersection of an intended-production technology of the firm and nature's residual-generation set. The former satisfies standard disposability properties, while the latter violates free-disposability of pollution and pollution-causing goods. As a result, the intersection—which we call a by-production technology—violates standard free-disposability of pollution and pollution-causing goods. Employing data envelopment analysis on an electric-power-plant database, we illustrate shortcomings, under by-production, of two popular efficiency indexes: the hyperbolic and directional-distance-function indexes. We propose and implement an alternative index with superior properties. Under by-production, most efficiency indexes are decomposable into intended-production and environmental efficiency indexes.

Journal of Economic Literature Classification Number: D20, D24, D62, Q50

Keywords: pollution-generating technologies, free disposability, weak disposability, data envelopment analysis, environmental and technical efficiency measurement.

1. Introduction.

Our reading of the environmental economics literature reveals three broad features of pollution that economists aim to capture. First, the generation of pollution/residuals seems to proceed hand-in-hand with the processes of consumption and production. Second, the residuals so generated require the use of the assimilative capacity of the environment for their disposal. Third, the generation of the residuals and the consequent use of environmental resources for their disposal generate external effects on both consumers and producers and hence the need for policies to regulate the generation of pollution.

In this paper, we confine ourselves to addressing the first feature alone.[1] In particular, we focus on pollution generated by firms. We distinguish between outputs that firms intend to produce and outputs that unintentionally (incidentally) get generated by firms when they engage in the production of intended outputs. Pollution is such an unintended output. We are mainly concerned with studying the specification of technology sets that best captures the link between production of outputs intended by firms and the generation of pollution.

It is reasonable to say that, in the case of pollution generated by firms, there are some specific aspects about the process of transformation of inputs into intended outputs (*e.g.,* the use of certain inputs such as coal or the production of certain

[1] See Murty [2010a] for a general equilibrium study of the second feature in the light of the first feature.

outputs such as varieties of cheese that release a strong odor) that trigger additional reactions in nature and (abstracting from abatement activities) *inevitably* result in the generation of pollution as a by-product. In this paper, we refer to these natural reactions, which occur alongside intended production by firms, as *by-production* of pollution.

In the case of technologies exhibiting by-production, we observe an inevitability of a certain *minimal* amount of the incidental output (the by-product), given the quantities of certain inputs and/or certain intended outputs. Inefficiencies in production could generate more than this minimal amount of the unintended output. At the same time, in such technologies, we also observe the usual menu of *maximal* possible vectors of intended outputs, given an input vector. Such a menu generally reflects the negative tradeoffs in the production of intended outputs when inputs are held fixed, as production of each of these commodities is costly in terms of the inputs used. Inefficiencies in intended production may imply that less than this maximal amount may get produced. An increase in the amounts of the inputs used increases the menu of intended output vectors that are technologically feasible. At the same time, it increases the minimal amount of the unintended output that can be generated.[2]

The above underscores two crucial points to note about pollution-generating technologies:

[2] *E.g.*, a greater amount of usage of coal increases the quantity of both smoke and electricity generated.

(i) technologies of pollution-generating firms do not satisfy free disposability of by-products such as pollution (pollution cannot be disposed of below the minimal level described above if inputs and intended outputs are held fixed) and

(ii) in such technologies there is a mutual interdependence between changes in inputs, intended outputs, and pollution—an interdependence that we will argue is more a correlation than a causation.

In most of the existing literature, the standard building block employed in constructing pollution-generating technologies is the positive correlation between intended and unintended outputs that is usually observed in such technologies. This literature attributes this observed positive correlation to abatement activities by firms rather than directly to the phenomenon of by-production. Abatement activities of firms involve a diversion of resources (inputs) to mitigate or clean up the pollution they produce. In this paper, we model abatement activities as outputs of the firm. Examples are end-of-pipe treatment plants (that treat and clean water to remove the pollutant) and production of outputs like scrubbers (which reduce sulphur emissions).[3] The production of these abatement activities is hence costly, given fixed amounts of resources: the more resources are diverted to abatement activities, the less they are available for producing intended outputs. Hence, an increase in the level

[3] We abstract from long-run abatement options of development, purchase, and installation of new technologies that generate less pollution. See *e.g.,* Barbera and McConnell [1998], where abatement activities include both a purchase of abatement capital and a diversion of some amounts of the usual inputs of a firm towards running of the abatement capital.

of abatement activities leads concomitantly to both lower residual generation and lower production of intended output.

In this literature, however, abatement activities are not usually explicitly modeled as another set of outputs produced by firms.[4] Rather, what is proposed is a "reduced form" of the technology in the space of inputs, by-products, and intended outputs. Special assumptions are made to allow the technology to exhibit a positive correlation between by-products and intended outputs, which is implicitly explained by abatement options open to firms. At the same time, it is also assumed that the technology satisfies the standard disposability assumptions with respect to *all* inputs and intended outputs. The approaches taken in the literature to model the positive correlation include: (a) treating pollution as a standard input (technology satisfies input free disposability with respect to pollution),[5] or (b) treating pollution as an output but with the technology satisfying the assumptions of weak disposability and null-jointness with respect to intended and unintended outputs.[6] In empirical works, both parametric and non-parametric specifications of such technologies are often employed for measuring technical efficiency, marginal abatement cost, productivity, and growth when economic units also produce incidental outputs like pollution. Both

[4] For an exception, see Barbera and McConnell [1998].

[5] See, *e.g.,* Baumol and Oates [1988], Cropper and Oates [1992], Reinhard, Lovell, and Thijhssen [1999], and Reinhard, Lovell, and Geert [2000].

[6] See Section 4 for formal definitions of these concepts.

Data Envelopment Analysis (DEA)[7] and econometric approaches are employed in this literature.[8]

We propose a model of pollution-generating technologies that captures the salient features (i) and (ii) of the phenomenon of by-production identified above. Our model of technology, also called a by-production technology, is obtained as a composition of two technologies: an intended-production technology and a residual-generation technology. The former is a standard technology that describes how inputs are transformed into intended outputs in production. The latter reflects nature's residual generation mechanism, which is a relationship between pollution (an output) and commodities that cause pollution. Thus, if we assume that it is some inputs (*e.g.,* coal) that cause pollution, then an increase in the use of these inputs results (under standard assumptions) in an increase in intended outputs (say electricity). At the same time, such an increase in the use of these inputs causes also an increase in pollution via nature's residual generating technology. Thus, even without any reference to explicit abatement efforts by firms, the model generates a positive correlation between pollution generation and intended outputs.

---

[7] See Färe, Grosskopf, and Lovell [1994] for a basic description of DEA and Fried, Lovell, and Schmidt [2008] for surveys of more recent developments.

[8] For measurement issues based on parametric specifications of a technology that treats by-products as outputs and employs weak disposability and null jointness see, *e.g.,* Pittman [1983], Färe, Grosskopf, Noh, and Yaisawarng [1993], Coggins and Swinton [1994], Hailu and Veeman [1999], Murty and Kumar [2002, 2003], and Murty, Kumar, and Paul [2006]. For non-parametric set-theoretic approaches under similar assumptions on the technology see, *e.g.,* Färe, Grosskopf, and Pasurka [1986], Färe, Grosskopf, Lovell, and Pasurka [1989], Färe, Grosskopf, Noh, and Weber [2005], and Boyd and McClelland [1999]. See Zhou and Poh [2008] for a comprehensive survey of over a hundred papers employing this approach to the modeling of pollution-generating technologies.

We show that abatement options available to firms can also be explicitly factored into our model. When they are available, they form a part of both the intended production technology (as their production is also costly in terms of resources/inputs of the firm) and the residual generation mechanism (as they mitigate residual generation). Moreover, we show that the presence of abatement options implies that data generated by pollution-generating technologies can violate the null-jointness assumption that is often made in the literature, *i.e.,* positive levels of intended output may be consistent with zero levels of pollution. The weak-disposability restriction on pollution-generating technologies does not preclude regions of *negative* correlation between intended and unintended outputs.[9] On the other hand, in the by-production technology we formulate, no such regions of negative correlations will be observed.

The intended production technology satisfies standard free-disposability properties with respect to inputs and intended outputs and is assumed to be independent of the level of pollution. As in Murty [2010a], the nature's residual generating technology treats pollution as an output that satisfies a new assumption of "costly disposability" and violates standard disposability properties with respect to goods that result in (affect) pollution generation. As a result, the by-production technology, which is an intersection of the intended production technology and nature's residual generating technology, violates standard disposability with respect to goods that cause (affect) pollution generation and exhibits costly disposability with respect to pollution. In

[9] A fact already noted in the literature cited in Footnote 8 above.

these ways, our proposed by-production approach, is different from the standard input and output approaches to modeling pollution-generating technologies.

We show how our by-production technology can be constructed using DEA methods as the intersection of two DEA technologies, one for intended production and one for residual generation, and discuss the calculation of efficiency of individual firms using these methods. With the help of a simple example we show that the sets of (weakly) efficient points obtained from the weak-disposability approach usually employed in the DEA literature and the new by-production approach are generally different (the former will be a larger set of points than the latter). In the context of by-production, the conventional (in)efficiency indexes decompose nicely into an intended-output efficiency index and an environmental efficiency index. We use our example to show that the common indexes employed in this literature, the hyperbolic index and the directional-distance-function index, are seriously flawed when the technology satisfies by-production. In particular, standard indexes tend to overstate efficiency. We then propose an alternative index, a modification of an index proposed by Färe, Grosskopf, and Lovell [1985], for measurement of efficiency for by-production technologies. This index corrects for the flaws in the hyperbolic and directional-distance-function indexes. A comparison of the values of this index with those of the hyperbolic and directional-distance-function indexes, using a data base for electric power firms, confirms our arguments about the inadequacies of the latter.

In Section 2, we show that a single implicit relation between outputs and inputs is not rich enough to capture, simultaneously, all the trade-offs between commodities that are implied by the phenomenon of by-production. In Section 3, we propose a model of a pollution-generating technology based on multiple production relations in which these inconsistencies in trade-offs are resolved. This is true regardless of whether or not abatement options are open to firms. Multiple production relations are required to distinguish between intended production by firms and nature's residual generation mechanism. In Section 4, we use a numerical example to show how by-production technologies can be constructed by DEA methods. Section 5 discusses issues related to efficiency measurement under by-production. In Section 6, we carry out an empirical analysis of efficiency measurement using an empirical data base. In Section 7, we extend our DEA formulation of a by-production technology to incorporate abatement efforts of firms. We conclude with Section 8.

2. Single-equation representation of pollution-generating technologies.

We show that a single implicit relation between outputs and inputs is not rich enough to capture, simultaneously, all the trade-offs between commodities that are implied by the phenomenon of by-production.

2.1. The case without abatement output.

The vectors of input quantities (indexed by $i = 1, \ldots, n$), intended-output quantities (indexed by $j = 1, \ldots, m$), and incidental-output quantities (indexed by $k = 1, \ldots, m'$), are given, respectively, by $y \in \mathbf{R}^m_+$, $z \in \mathbf{R}^{m'}_+$, and $x \in \mathbf{R}^n_+$.

Suppose pollution is caused by the use of certain inputs like coal or because of the production of certain intended outputs like cheese. Suppose also that the firm does not participate in any abatement activity to reduce the pollution that it generates. A single-equation formulation of such a pollution-generating technology, an extension of the standard functional representation of a multiple-output technology, is as follows:

$$T = \left\{ \langle x, y, z \rangle \in \mathbf{R}^{n+m+m'}_+ \middle| \; f(x, y, z) \leq 0 \right\},$$

where f is differentiable, with derivatives with respect to inputs and intended outputs given by[10]

$$\begin{aligned} &\text{(a)} \quad f_i(x, y, z) \leq 0, \;\; i = 1, \ldots, n, \\ &\text{(b)} \quad f_j(x, y, z) \geq 0, \;\; j = 1, \ldots, m. \end{aligned} \tag{2.1}$$

The constraints (a) and (b) are standard differential restrictions to impose "free disposability" of, respectively, inputs and intended outputs:[11]

$$\langle x, y, z \rangle \in T \;\wedge\; \bar{x} \geq x \Longrightarrow \langle \bar{x}, y, z \rangle \in T \tag{2.2}$$

and

$$\langle x, y, z \rangle \in T \;\wedge\; \bar{y} \leq y \Longrightarrow \langle x, \bar{y}, z \rangle \in T. \tag{2.3}$$

[10] Subscripts on f indicate partial differentiation with respect to the indicated variable.
[11] The symbol $\wedge$ stands for "and".

To capture the fact that pollution is an output of the production process for which disposal is not free, Murty [2010a] introduces and formalizes an assumption that is the polar opposite of free output disposability with respect to the unintended outputs:

$$\langle x, y, z \rangle \in T \ \wedge \ \bar{z} \geq z \Longrightarrow \langle x, y, \bar{z} \rangle \in T. \tag{2.4}$$

Following Murty [2010a], we refer to this property as "costly disposability" of residuals.[12] Costly disposability implies the possibility of inefficiencies in the generation of pollution (*e.g.,* if a given level of coal generates some level of smoke, then inefficiency in the use of coal may imply that this level of coal can also generate a greater amount of pollution. The differential restrictions required to impose costly disposability on T are

$$f_k(x, y, z) \leq 0, \ \ k = 1, \ldots, m'. \tag{2.5}$$

Quantity vectors satisfying $f(x, y, z) = 0$ are points on the frontier of the technology.[13] Those satisfying $f(x, y, z) < 0$ are inefficient: more intended output could be produced with given quantities of inputs and pollution; less pollution could be generated with given intended output and input quantities; and smaller input quantities could be used to produce the given output quantities, given the pollution level.

[12] At this stage, though the assumption that the technology satisfies costly disposability of pollution seems similar to the assumption that it also satisfies input free disposability with respect to pollution, two differences between these assumptions and their implications will become clear later: (1) in our by-production approach this assumption is satisfied by nature's residual generation mechanism and not by the intended production technology and (2) the nature's residual generation mechanism treats pollution as an output of production and not as an input.

[13] We adopt the following convention in this paper: A point $\langle x, y, z \rangle \in T$ lies on the frontier of T (or is a weakly efficient point of T) if there exists no other point $\langle \bar{x}, \bar{y}, \bar{z} \rangle \in T$ with $\bar{x}_i < x_i$ for all i, $\bar{y}_j > y_j$ for all j, and $\bar{z}_k < z_k$ for all k. A point $\langle x, y, z \rangle \in T$ lies on the efficient frontier of T (or is an efficient point of T) if there exists no other point $\langle \bar{x}, \bar{y}, \bar{z} \rangle \in T$ with $\bar{x} \leq x$, $\bar{y} \geq y$, and $\bar{z} \leq z$.

Assume, in this section and without loss of generality, that $m' = 1$. Suppose $f_k(\hat{x}, \hat{y}, \hat{z}) < 0$ for some $\langle \hat{x}, \hat{y}, \hat{z} \rangle$ satisfying $f(\hat{x}, \hat{y}, \hat{z}) = 0$. Then, from the implicit function theorem, there exist neighborhoods $U \subseteq \mathbf{R}_+^{m+n}$ and $V \subseteq \mathbf{R}_+$ around $\langle \hat{x}, \hat{y} \rangle \in \mathbf{R}_+^{n+m}$ and $\hat{z} \in \mathbf{R}_+$ and a function[14] $\zeta : U \rightarrow V$ such that

$$\hat{z} = \zeta(\hat{x}, \hat{y}) \tag{2.6}$$

and

$$f\big(x, y, \zeta(x, y)\big) = 0. \tag{2.7}$$

The trade-off between each intended output j and unintended output k (with inputs and all other outputs held fixed) implied by the implicit function theorem is

$$\frac{\partial \zeta(x, y)}{\partial y_j} = -\frac{f_j(x, y, z)}{f_k(x, y, z)} \geq 0, \quad j = 1, \ldots, m. \tag{2.8}$$

The trade-off between each input i and unintended output k (with intended outputs and all other inputs held fixed) is

$$\frac{\partial \zeta(x, y)}{\partial x_i} = -\frac{f_i(x, y, z)}{f_k(x, y, z)} \leq 0, \quad i = 1, \ldots, n. \tag{2.9}$$

Noting that all these trade-offs are evaluated at points in the technology set that are weakly technically efficient (that is, $f(x, y, z) = 0$), the foregoing formulation of a pollution-generating technology seems to be inconsistent with the phenomenon of by-production for the following reasons:

[14] See the appendix for a statement of the implicit function theorem.

(a) The existence of the function ζ satisfying (2.8) as a strict inequality implies that there exists a rich menu (a manifold) of (weakly) technically efficient $\langle y, z \rangle$ combinations, with varying levels of z, that are possible with *holding all inputs fixed*. If pollution is generated by input usage, this menu is contrary to phenomenon of by-production, since by-production implies that at fixed levels of inputs (*e.g.,* coal), there is only *one* (weakly) technically efficient (minimal) level of pollution.[15]

(b) Furthermore, if pollution is generated by inputs such as coal, as is very often the case, the non-positive trade-offs between pollution generation and these inputs (derived by holding the levels of intended outputs fixed), apparent in (2.9), are inconsistent with by-production, as by-production implies that this trade-off should be non-negative.

How should one interpret the trade-offs observed under single equation modeling of pollution-generating technologies when one abstracts from abatement options? As discussed above, these trade-offs are not reflective of the phenomenon of by-production. Rather, the non-negative trade-offs observed in (2.8) between each intended output and pollution and the non-positive trade-offs observed in (2.9) between each input and pollution suggest that this approach treats pollution like any other input in production: first, increases in its level, holding all other inputs fixed, increases

[15] If pollution is caused by some intended outputs (*e.g.,* strong odor from some varieties of cheese produced by a dairy) and (2.9) holds as a strict inequality, then it implies that there exists a rich menu of (weakly) technically efficient $\langle x, z \rangle$ combinations, with varying levels of z, that are possible with *given levels of all intended outputs*. Such a menu is inconsistent with by-production.

intended outputs and, second, pollution is a substitute for all other inputs in intended production—the same level of intended outputs can be produced by decreasing other inputs and increasing pollution. This also does not seem to be intuitively correct: it is not a correct description of the role pollution plays in intended production.

2.2. The case with abatement output.

Consider the case where the technology of a pollution-generating firm is defined by a single restriction on all inputs and outputs, including the abatement output:

$$T = \big\{ \langle x, y, z, y^a \rangle \in \mathbf{R}_+^{n+m+m'+1} \mid f(x, y, z, y^a) \leq 0 \big\}. \tag{2.10}$$

We assume that

$$f_a(x, y, z, y^a) \geq 0. \tag{2.11}$$

This restriction captures the fact that the abatement output is also freely disposable:

$$\langle x, y, z, y^a \rangle \in T \ \wedge \ \bar{y}^a \leq y^a \Longrightarrow \langle x, y, z, \bar{y}^a \rangle \in T, \tag{2.12}$$

so that producing it is costly in terms of input usage, implying a non-positive trade-off between it and the other intended outputs. In that case, the implicit function theorem can again be invoked to show that the trade-off between the abatement output and pollution, evaluated in a neighborhood of a (weakly) technically efficient point $\langle \hat{x}, \hat{y}, \hat{z}, \hat{y}_a \rangle \in \mathbf{R}_+^{n+m+m'+1}$ such that $f(\hat{x}, \hat{y}, \hat{z}, \hat{y}_a) = 0$ and $f_k(\hat{x}, \hat{y}, \hat{z}, \hat{y}_a) < 0$, is

$$\frac{\partial \zeta(x, y, y_a)}{\partial y^a} = -\frac{f_a(x, y, z, y_a)}{f_k(x, y, z, y_a)} \geq 0 \tag{2.13}$$

whenever $f(x, y, z, y_a) = 0$, contradicting the fact that abatement output is produced by firms to mitigate, and not to enhance, pollution.

3. A by-production approach to modeling pollution.

Given the above analysis, a sound foundation must be identified for introducing multiple production relations to adequately capture the features of by-production. We feel that the resolution to the problem lies in early work of Frisch [1965] on production theory, in which he envisaged situations where the correct functional representation of a production technology may require more than one implicit functional relation between inputs and outputs. More recently, Førsund [2009] explores these ideas of Frisch. We build on the works of Frisch and Førsund and show that the phenomenon of by-production requires distinguishing explicitly the by-product-generating mechanism from the production relation that describes the production of intended commodities. We show that when this is done the inconsistencies among trade-offs elucidated in Section 2 get resolved.

3.1. A by-production approach: the case without abatement.

In this sub-section, we abstract from explicit abatement efforts. The production of the intended output sets a residual-generation mechanism in motion, leading to the generation of the by-product. To fix our ideas on the salient aspects of by-production and to simplify notation, we continue to assume, without loss of generality, that

$m' = 1$ and that the pollution is generated by usage of a single input (such an input could be coal), say input $\imath$.[16] Denote the input quantity vector purged of the quantity of input $\imath$ by x^1. Specify the technology as

$$T = T_1 \cap T_2, \tag{3.1}$$

where

$$T_1 = \left\{ \langle x^1, x_\imath, y, z \rangle \in \mathbf{R}_+^{n+m+1} \mid f(x^1, x_\imath, y) \leq 0 \right\}, \tag{3.2}$$

$$T_2 = \left\{ \langle x^1, x_\imath, y, z \rangle \in \mathbf{R}_+^{n+m+1} \mid z \geq g(x_\imath) \right\}, \tag{3.3}$$

and f and g are continuously differentiable functions. The set T_1 is a standard technology set, reflecting the ways in which the inputs can be transformed into intended outputs. The standard free disposability properties (2.3) and (2.4) can be imposed on this set by assuming that f satisfies

$$\begin{aligned} &f_i(x, y) \leq 0, \;\; i = 1, \ldots, n, \;\text{ and} \\ &f_j(x, y) \geq 0, \;\; j = 1, \ldots, m. \end{aligned} \tag{3.4}$$

Note that (3.2) imposes no constraint on z, that is, it is implicitly assumed that the by-product does not affect the production of intended outputs.[17]

The set T_2 reflects nature's residual-generation mechanism. T_2 treats pollution as an output and satisfies costly disposability with respect to pollution as defined in

[16] The analysis can easily be extended to the case where pollution (such as a strong odour) is also caused by the production of an intended output (such as cheese). See Murty [2010b].

[17] This could be generalized, of course, allowing pollution to have an effect on intended production as well; *e.g.,* smoke could adversely affect the productivity of labor engaged in producing intended outputs. See Murty [2010b] for a generalization.

(2.4), with the function g defining the minimal level of pollution that gets generated for given level of $x_\imath$.[18] The derivative of g satisfies

$$g'(x_\imath) \geq 0. \tag{3.5}$$

The condition in (3.5) capture the fact that the efficient (minimal) level of pollution rises with the increase in the usage of input $\imath$. This means, however, that T_2 violates standard free disposability of input $\imath$. In fact it satisfies the polar opposite condition in this good:

$$\langle x^1, x_\imath, y, z\rangle \in T_2 \;\wedge\; \bar{z} \geq z \;\wedge\; \bar{x}_\imath \leq x_\imath \Longrightarrow \langle x^1, \bar{x}_\imath, y, \bar{z}\rangle \in T_2. \tag{3.6}$$

This implies that if a given level of coal generates some amount of pollution, then inefficiencies in residual generation may imply that lower amounts of the coal input can also generate the same level of pollution if the firm operates more efficiently.

It is easy to infer the disposability properties of T from the disposability properties of the intended production technology T_1 and the residual generation mechanism T_2

Theorem 1: *T satisfies free disposability with respect to all intended outputs and non-pollution-causing inputs. It, however, violates free disposability with respect to the pollution-causing input $\imath$. It satisfies costly disposability with respect to the quantity of pollution z.*

[18] Costly disposability, as defined in (2.4), could be considered to be too extreme. It implies that an infinite amount of pollution can be generated by given amount of input $\imath$. In general, there may also be an upper bound for the generation of the unintended output. See Murty [2010b] for a generalization.

The technology violates standard disposability conditions with respect to the quantity of the pollution-causing input $x_\imath$ because, while T_1 satisfies standard free-disposability conditions in $x_\imath$, T_2 satisfies the polar opposite conditions with respect to this input.

Quantity vectors $\langle x, y, z\rangle \in T$ that satisfy $f(x, y) = 0$ and $z = g(x_\imath)$ are the weakly efficient points of T. If a quantity vector in $\langle x, y, z\rangle \in T$ is such that $f(x, y) < 0$, then it is technologically possible to decrease the levels of the non-pollution-causing inputs without changing the production levels of the remaining goods. If a quantity vector in $\langle x, y, z\rangle \in T$ is such that $z > g(x_\imath)$, then it is technologically possible to decrease the level of pollution without changing the production levels of all other goods.

To sign the trade-offs between pollution and a (non-pollution-causing) intended output j at a weakly efficient point of T, we invoke the implicit function theorem. Let $\langle \hat{x}, \hat{y}, \hat{z}\rangle$ be a weakly efficient point of T. Then

$$\begin{aligned} f(\hat{x}, \hat{y}) &= 0 \\ \hat{z} - g(\hat{x}_\imath) &= 0. \end{aligned} \tag{3.7}$$

Denote y^{-j} to be the vector obtained by purging the j^{th} element from the vector y. Suppose that $f_j(\hat{x}, \hat{y}) \neq 0$ and $g_\imath(\hat{x}_\imath) \neq 0$. Then the matrix

$$\begin{bmatrix} f_j(\hat{x}, \hat{y}) & f_\imath(\hat{x}, \hat{y}) \\ 0 & -g_\imath(\hat{x}_\imath) \end{bmatrix} \tag{3.8}$$

has full row rank. By the implicit function theorem, there exists a neighborhood U around $\langle \hat{x}^1, \hat{y}^{-j}, \hat{z} \rangle$ in $\mathbf{R}_+^{n+m-1}$, a neighborhood V around $\langle \hat{x}_\imath, \hat{y}_j \rangle$ in $\mathbf{R}_+^2$, and continuously differentiable mappings $\psi^j : U \to \psi^j(U)$ and $h : U \to h(U)$ with images

$$\begin{aligned} y_j &= \psi^j(x^1, y^{-j}, z) \\ x_\imath &= h(x^1, y^{-j}, z) \end{aligned} \tag{3.9}$$

such that $\big\langle h(x^1, y^{-j}, z),\ \psi^j(x^1, y^{-j}, z) \big\rangle \in V$ and

$$\begin{aligned} f(x^1, h(x^1, y^{-j}, z), \psi^j(x^1, y^{-j}, z), y^{-j}) = 0 \\ z - g(h(x^1, y^{-j}, z)) = 0. \end{aligned} \tag{3.10}$$

In that case, assuming that $g'(x_\imath) > 0$, the trade-off between y_j and z is[19]

$$\frac{\partial \psi^j(x^1, y^{-j}, z)}{\partial z} = -\frac{f_\imath\big(x, y\big)\ h_k(x^1, y^{-j}, z)}{f_j(x, y)} \geq 0. \tag{3.11}$$

How should one interpret this non-negative "trade-off" between y_j? Starting at a weakly efficient point in a local neighborhood of $\langle \hat{x}, \hat{y}, \hat{z} \rangle \in T$, an increase in z is attributable, because of the by-production phenomenon inherent in T_2, to an increase in $x_\imath$ (as $h_k(x^1, y^{-j}, z) > 0$). Under the conventional assumptions on intended production in (3.4), the trade-off between the pollution-generating input $\imath$ and intended output j is

$$-\frac{f_\imath(x, y)}{f_j(x, y)} \geq 0, \tag{3.12}$$

hence, the increase in $x_\imath$ implies an increase in y^j. The "trade-off" in (3.11), thus, reflects a non-negative *correlation* between the residual and an intended output via

[19] Note, as we have assumed a single unintended output, $h_k(x^1, y^{-j}, z)$ is the derivative of the function h with respect to z. Note also that h is the inverse of g, *i.e.*, $h(x^1, y^{-j}, z) = g^{-1}(z)$, so that, if $z = g(x_\imath)$ and $g'(x_\imath) > 0$, then $h_k(x^1, y^{-j}, z) = 1/g'(x_\imath) > 0$.

$x_\imath$, because a change in $x_\imath$ affects both y^j (non-negatively in intended production) and z (positively with respect to residual generation).

To summarize, the non-negative "trade-off" between an intended and an unintended output in the reduced form model is explained by (a) the phenomenon of by-production, which relates the use of inputs such as $\imath$ to the by-product, and (b) the non-negative marginal product of input $\imath$ in producing intended outputs like j.

3.2. A by-production approach: incorporating abatement activities.

We again keep the analysis simple by sticking to a single abatement output (as well as a single unintended output). On the other hand, we make the model more general to allow the possibility of input substitutability in the generation of the by-product.[20] We do so by partitioning the vector of all n inputs into n_1 non-residual-generating inputs and n_2 residual-generating inputs. Denote the respective input quantity vectors by x^1 and x^2. Let y^a denote the level of the firm's abatement activities, which are also costly in terms of the input resources of the firm. Without loss of generality, we assume that the intended outputs do not cause pollution. Similarly to the previous section, we specify the technology as $T = T_1 \cap T_2$, where

$$\begin{aligned} T_1 &= \left\{ \langle x^1, x^2, y, z, y^a \rangle \in \mathbf{R}^{n+m+2} \;\middle|\; f(x^1, x^2, y, y^a) \leq 0 \right\} \\ T_2 &= \left\{ \langle x^1, x^2, y, z, y^a \rangle \in \mathbf{R}^{n+m+2} \;\middle|\; z \geq g(x^2, y^a) \right\}. \end{aligned} \tag{3.13}$$

T reflects both the transformation of inputs into intended outputs and abatement output (as indicated by the definition of T_1) and the use of the abatement output by

[20] For example, substituting a cleaner variety of coal for a less pure variety or vice-versa.

the firm to control the by-production of the residual that results from use of pollution-generating inputs in producing intended outputs (as indicated by the definition of T_2 in (3.13)). We confine ourselves again to a local analysis and posit the following signs of the partial derivatives at a weakly efficient point $\langle \hat{y}, \hat{y}_a, \hat{x}^1, \hat{x}^2, \hat{z} \rangle$ of T:

$$
\begin{aligned}
f_j(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) &\geq 0, \;\; j = 1, \ldots m, \\
f_a(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) &> 0, \\
f_i(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) &\leq 0, \;\; i = 1, \ldots n, \\
g_a(\hat{x}^2, \hat{y}_a) &< 0, \\
g_\imath(\hat{x}^2, \hat{y}_a) &\geq 0 \quad \text{for all } \imath = n_1 + 1, \ldots, n, \\
g_\imath(\hat{x}^2, \hat{y}_a) &> 0 \quad \text{for some } \imath = n_1 + 1, \ldots, n.
\end{aligned}
\tag{3.14}
$$

It is easy to see that (3.13) and (3.14) imply that T_1 satisfies standard free disposability conditions for inputs, abatement output, and intended outputs. In addition, there is a negative (or at least non-positive) trade-off between standard outputs and the abatement output and a positive (or a non-negative) trade-off between each intended output and the inputs in intended production.

With respect to residual generation, (3.13) and (3.14) imply that T_2 satisfies costly disposability for the unintended output and a condition that is the polar opposite of standard input and output free disposability for the abatement output and non-pollution-generating inputs:

$$\langle x^1, x^2, y, z, y^a \rangle \in T_2 \wedge \bar{z} \geq z \wedge \bar{x}^2 \leq x^2 \wedge \bar{y}^a \geq y^a \Longrightarrow \langle x^1, \bar{x}^2, y, \bar{z}, \bar{y}^a \rangle \in T_2. \tag{3.15}$$

We call (3.15) "costly disposability of pollution, abatement output, and inputs that generate pollution."[21] The trade-offs between z and each of the pollution-generating input quantities $x_\imath^2$ implied by (3.14) are non-negative and that between z and abatement output y^a is negative. Thus, the sign of g_a captures the mitigating effect abatement has on residual generation and the sign of $g_\imath$ captures the increase in pollution attributable to the increase in inputs causing pollution.

It is easy to infer the disposability properties of T from the above characteristics of T_1 and T_2:

Theorem 2: *T satisfies free disposability with respect to all intended outputs and non-pollution-causing inputs. It, however, violates free disposability with respect to each of the pollution-causing inputs and the abatement output. It satisfies costly disposability with respect to pollution.*

Let the inequalities in (3.14) hold. We now sign the trade-off between z and an intended output y_j at a weakly efficient point of T. As in the previous section, we do so by employing the implicit function theorem. Let $\langle \hat{x}^1, \hat{x}^2, \hat{y}, \hat{z}, \hat{y}_a \rangle$ be a weakly efficient point of T. Then

$$\begin{aligned} f(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) &= 0 \\ \hat{z} - g(\hat{x}^2, \hat{y}^a) &= 0. \end{aligned} \tag{3.16}$$

[21] This assumption reflects the inefficiencies in the production of pollution: if given levels of coal and abatement activities generate some amount of pollution, then inefficiencies in the use of coal or abatement activities imply that a lower amount of the coal input or a higher level of abatement activities could generate the same level of pollution if the firm were to operate more efficiently.

Let $f_j(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) \neq 0$ and $g_a(\hat{x}^2, y^a) \neq 0$. Then the matrix

$$\begin{bmatrix} f_j(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) & f_a(\hat{x}^1, \hat{x}^2, \hat{y}, \hat{y}_a) \\ 0 & -g_a(\hat{x}^2, \hat{y}_a) \end{bmatrix} \tag{3.17}$$

is full-row ranked. The implicit function theorem implies that there exists a neighborhood U around $\langle \hat{x}, \hat{y}^{-j}, \hat{z} \rangle$ in $\mathbf{R}_+^{n+m}$, a neighborhood V around $\langle \hat{y}_j, \hat{y}^a \rangle$ in $\mathbf{R}_+^2$, and continuously differentiable mappings $\psi^j : U \to \psi^j(U)$ and $h : U \to h(U)$ with images

$$\begin{aligned} y_j &= \psi^j(x, y^{-j}, z) \\ y^a &= h(x, y^{-j}, z) = g^{-1}(z, x^2) \end{aligned} \tag{3.18}$$

such that $\langle \psi^j(x, y^{-j}, z),\ h(x, y^{-j}, z) \rangle \in V$ and

$$\begin{aligned} f(x, \psi^j(y^{-j}, z), y^{-j}, h(x, y^{-j}, z)) = 0 \\ z - g(x^2, h(x, y^{-j}, z)) = 0. \end{aligned} \tag{3.19}$$

In that case, the trade-off between y_j and z is

$$\frac{\partial \psi^j(x, y^{-j}, z)}{\partial z} = -\frac{f_a\big(x, y, y^a\big)\ h_k(x, y^{-j}, z)}{f_j(x, y, y^a)} \geq 0. \tag{3.20}$$

As in the previous section, this non-negative trade-off between an intended output and pollution at a weakly efficient point of T reflects a correlation between these commodities; in this case, this correlation is effected by abatement effort of the firm to mitigate by-production of pollution.[22] Precisely, holding the levels of all inputs (including pollution-causing inputs) fixed, an increase in z must have come about because of reductions in abatement efforts y^a by firms, and hence there is an increase

[22] Note that, as in the previous section, a (generally different) non-negative correlation between the intended and unintended outputs effected by an input that causes pollution could also be derived.

in resources diverted towards production of other intended outputs y (assuming, of course, that firms are operating in a weakly efficient way).

From our analysis above, we can derive the reduced-form functional representation of the technology T. By substituting out abatement efforts from the function f in (3.13), we can rewrite T equivalently as

$$T = \big\{\langle x^1, x^2, y, z, y^a\rangle \in \mathbf{R}^{n+m+2} \mid \tilde{f}\big(x, y, z\big) \leq 0 \ \wedge \ \ y^a \geq h(x, y^{-j}, z)\big\}, \quad (3.21)$$

where

$$\tilde{f}\big(x, y, z\big) := f\big(x, y, h(x, y^{-j}, z)\big). \quad (3.22)$$

Using (3.21), we can define a reduced-form technology in the space of intended and unintended outputs and inputs as

$$\tilde{T} := \{\langle x^1, x^2, y, z\rangle \in \mathbf{R}_+^{n+m+1} \mid \tilde{f}(x, y, z) \leq 0\}. \quad (3.23)$$

The input and output approaches in the conventional literature model a reduced-form technology—quite in the spirit of $\tilde{T}$—in the space of intended and unintended outputs and inputs that exhibits a positive correlation between intended and unintended outputs but satisfies *all* of the standard free disposability assumptions with respect to intended outputs and inputs. The technology is modeled only in reduced form because, although this literature attributes the positive correlation to abatement options available to firms, abatement activities are not explicitly modeled.

In the case of the by-production, it is easy to check that, in the neighborhood of a point $\langle x, y, z\rangle$ that satisfies $\tilde{f}(x, y, z) = 0$, the trade-off between an intended and an unintended output, $-\tilde{f}_j(x, y, z)/\tilde{f}_k(x, y, z)$, is given by (3.20) and hence is non-negative. This is consistent with conventional modeling of the reduced form of a pollution-generating technology. However, the derivative of the function $\tilde{f}$ with respect to a pollution-causing input $i = n_1 + 1, \ldots, n$ is

$$\tilde{f}_i(x^1, x^2, y, z) = f_a(x^1, x^2, y, y^a)h_i(x, y^{-j}, z) + f_i(x^1, x^2, y, y^a). \tag{3.24}$$

Given (3.18) and the sign conventions in (3.14), the sign of $\tilde{f}_i$ is ambiguous, contrary to the conventional literature, where it is signed as per a normal input. As seen in Theorems 1 and 2 in Section 3, this follows from the fact that the residual generating technology T_2 (and hence the by-production technology $T = T_1 \cap T_2$) violates standard free disposability in such inputs.

4. Data-based pollution-generating technologies.

The foregoing analysis reveals that modeling the phenomenon of by-production requires more than one implicit production relation among inputs and outputs. One of these relations captures intended production activities of firms (that is, describes the set T_1), while the other captures the inevitability of residual generation when firms engage in intended production (that is, describes the set T_2). The former identifies an upper bound for the intended outputs of firms for every given level of inputs, while the latter identifies a lower bound for pollution generation given every level of intended outputs and inputs that are responsible for causing pollution.

In this paper, we adopt a data envelopment analysis (DEA) approach to constructing pollution-generating technologies.[23] These methods have become increasingly common in recent years.[24] Assuming (as we do in this paper) constant returns to scale, this approach essentially envelops the data in the "smallest" or "tightest fitting" convex cone. Additionally, in the case of conventional inputs and outputs, the technology is the free disposal hull of this convex cone,[25] but the problem is more complicated in the case of pollution-generating technologies, where some goods violate the free disposability assumption.

To lay out these concepts formally, we consider a more general model than the one presented above, incorporating multiple pollution-generating inputs and multiple pollutants. We restrict ourselves to the case where pollution is caused by the use of certain inputs by firms.[26]

First augment the notation in Section 2 as follows:

(i) p decision making units (DMUs),[27] indexed by d.

[23] See attached working paper version of this paper for comments on the econometric approach to modeling by-production.

[24] The survey of these methods for modeling pollution generating technologies by Zhou, Ang, and Poh [2008] contains 150 references.

[25] See, *e.g.*, Färe, Grosskopf, and Lovell [1994] for details and generalizations to other alternative returns to scale assumptions.

[26] The data set used below for our empirical application does not contain information on abatement. Extension to the case where some intended outputs also cause pollution is straightforward. In Section 7 we consider a numerical example to illustrate the extension to the case with abatement efforts of firms.

[27] Here we follow the standard nomenclature in the literature on technical efficiency measurement. The generic DMU could be a firm, a plant belonging to a specific firm, or any of a number of types of units of study.

(ii) m intended outputs, indexed by j, with quantity vector $y \in \mathbf{R}^m_+$. The $p \times m$ matrix of observations on intended output quantities is denoted by Y.

(iii) n inputs, indexed by i. The first n_1 are non-pollution-generating, while the remaining $n_2 = n - n_1$ are pollution generating. The quantity vector is $x = \langle x^1, x^2 \rangle \in \mathbf{R}^n_+$. The $p \times n$ matrix of observations on the input quantities is denoted by $X = \langle X^1, X^2 \rangle$.

(iv) m' pollutants, indexed by k, with quantity vector $z \in \mathbf{R}^{m'}_+$. The $p \times m'$ matrix of observations on pollutants is denoted by Z.

For illustrative purposes, we posit an example for a very simple special case with five decision making units, one intended output, one unintended output, and one input:

Example 1: $p = 5$, $m = 1$, $n = n_1 = 1$, and $m' = 1$. The (artificial) data are as follows:

DMU	x	y	z
1	1	2	4
2	1	3/2	1
3	1	2/3	2
4	2	3	5
5	2	2	3

(4.1)

In the conventional output approach to modeling pollution-generating technologies, all intended outputs and inputs are assumed to satisfy standard disposability conditions, but two key assumptions are made regarding the unintended outputs. The first,

$$\langle x, y, z \rangle \in \tilde{T} \ \wedge \ \lambda \in [0, 1] \implies \langle x, \lambda y, \lambda z \rangle \in \tilde{T}, \tag{4.2}$$

is called "weak disposability", a concept originally attributable to Shephard [1953, 1974]. The second,

$$\langle x, y, z\rangle \in \tilde{T} \ \wedge \ z = 0 \Longrightarrow y = 0, \tag{4.3}$$

is called "null jointness". Weak disposability and null-jointness imply that (a) while pollution is not freely disposable, it is possible to jointly and proportionately decrease pollution and the intended outputs and (b) production of *any* positive level of intended output always results in positive amounts of the residual being generated. This literature is predicated on the belief that these two assumptions can capture the fact that, starting at any efficient point of the technology, it is not possible to decrease pollution without decreasing the production of the intended outputs, and hence that, together, they model the positive reduced-form correlation between pollution and other intended outputs. The standard DEA construction of a pollution-generating technology (based on the assumptions of weak disposability and null-jointness) first formulated by Färe, Grosskopf, and Pasurka [1989], is given by

$$\tilde{T}_{WD} = \Big\{\langle x, y, z\rangle \in \mathbf{R}_+^{n+m+m'} \mid \lambda X \leq x \ \wedge \ \lambda Y \geq y \ \wedge \ \lambda Z = z \text{ for some } \lambda \in \mathbf{R}_+^p\Big\}. \tag{4.4}$$

The production possibility set satisfying weak disposability for Example 1, with $x = 1$ is shown in Panel 4 of Figure 7 (where points A and B are the $\langle z, y\rangle$ combinations for DMUs 2 and 1, respectively, and the other DMU vectors fall below the frontier.

Denote the overall technology $T_1 \cap T_2$ that satisfies by-production by T_{BP}. We assume that T_1 satisfies free disposability of inputs and intended outputs (as defined in (2.2) and (2.3)) and that it is closed, convex, and satisfies constant returns to scale. In addition, T_1 satisfies the following assumption, which we call "independence of T_1 from z" and which states that pollution does not directly affect production of intended outputs:[28]

$$\langle x, y, z \rangle \in T_1 \Longrightarrow \langle x, y, \bar{z} \rangle \in T_1 \ \forall \ \bar{z} \in \mathbf{R}_+^{m'}. \tag{4.5}$$

The intended-output technology T_1 that satisfies these assumptions is obtained in a standard way using DEA techniques as follows:

$$T_1 = \Big\{ \langle x, y, z \rangle \in \mathbf{R}_+^{n+m+m'} \mid \lambda X \leq x \ \wedge \ \lambda Y \geq y \text{ for some } \lambda \in \ \mathbf{R}_+^{p} \Big\}. \tag{4.6}$$

We assume T_2 satisfies costly disposability of pollution and inputs that cause pollution (as defined in (3.6)) and constant returns to scale. Also note that, since we have assumed that only x^2 affects residual generation, T_2 also satisfies "independence of T_2 from x^1 and y":

$$\langle x, y, z \rangle \in T_2 \Longrightarrow \langle \bar{x}^1, x^2, \bar{y}, z \rangle \in T_2 \ \forall \ \langle \bar{x}^1, \bar{y} \rangle \in \mathbf{R}_+^{n_1+m}. \tag{4.7}$$

The DEA version of T_2, which satisfies these assumptions, is obtained as

$$T_2 = \Big\{ \langle x^1, x^2, y, z \rangle \in \mathbf{R}_+^{n_1+n_2+m+m'} \mid \mu X^2 \geq x^2 \wedge \mu Z \leq z \ \text{ for some } \mu \in \mathbf{R}_+^{p} \Big\}. \tag{4.8}$$

[28] This assumption would have to be relaxed if, *e.g.*, the presence of pollution could adversely affect labor productivity in producing intended outputs. See Murty [2010b].

The first inequality in (4.8) reflects costly disposability of inputs that cause pollution and the second reflects costly disposability of pollution. Since T_2 is independent of x^1 and y, no inequalities need to be specified for x^1 and y.

A data set coming from pollution-generating units must simultaneously belong to both T_1 and T_2. The overall technology that exhibits by-production is the intersection of T_1 and T_2:

$$T_{BP} = \Big\{\langle x^1, x^2, y, z\rangle \in \mathbf{R}^{n_1+n_2+m+m'} \mid \lambda[X^1\ X^2] \leq \langle x^1,\ x^2\rangle,\ \lambda Y \geq y, \\ \mu X^2 \geq x^2,\ \mu Z \leq z, \\ \text{for some } \langle \lambda, \mu\rangle \in \mathbf{R}^{2p}_{+}\Big\}. \tag{4.9}$$

The above construction of T_{BP} using activity analysis involves two sets of production relations. These are reflected in the two different intensity vectors λ and μ, each of which is applied to the same data set.

These sets under the assumptions of Example 1 are depicted in the first three panels of Figure 6. Noting that T_1 is independent of z and T_2 is independent of y, Panels 1 and 3 of Figure 6 show the DEA constructions of projections of T_1 (in the space of the input and the intended output) and T_2 (in the space of the input and the unintended output), respectively.[29]

Panels 2 and 4 of the same figure show the combinations of intended and unintended outputs that are feasible with $x = 1$, under the by-production (BP) and the

[29] With an abuse of notation, but with no confusion, we also call these projections T_1 and T_2 in Figure 6. Panels 1 and 3 of this figure are drawn under the maintained assumption of constant returns to scale.

weak disposability (WD) approaches, respectively. It is clear from Panel 2 that, in the case of BP, the output possibility set has only one efficient point, $e = \langle 1, 2 \rangle$ (the efficient frontier of the output possibility set is a singleton). This gives the minimum level of the unintended output and the maximum level of the intended output that can be produced when $x = 1$ and corresponds to efficient points of T_1 and T_2 as seen in Panels 1 and 3.[30] On the other hand, Panel 4 shows that the efficient frontier of the output possibility set satisfying weak disposability OAB has a far greater number of points. This illustrates that the efficient frontier of the output possibility set under the BP approach is smaller than under the WD approach.

5. Measuring technical efficiency.

Two conventional efficiency indexes have been extensively employed in the DEA pollution literature: the output-oriented hyperbolic (HYP) index employed in the original DEA pollution study of Färe, Grosskopf, and Pasurka [1986] and the output-oriented directional-distance-function (DDF) index employed in more recent studies (*e.g.*, Färe, Grosskopf, Noh, and Weber [2005]).[31] These indexes are "output-

[30] Note that, while e is not a point in our artificial data set, the data are used to find e. The rest of the frontier of the output possibility set in Panel 2 reflects the fact that T_{BP} satisfies standard output free disposability in the direction of the intended output and costly disposability in the direction of pollution.

[31] The HYP efficiency index was formulated for standard technologies by Färe, Grosskopf, and Lovell [1985, pp. 110–111]. The DDF index was adapted from the shortage function of Luenberger [1992] to the measurement of efficiency by Chambers, Chung, and Färe [1996] and Chung, Färe, and Grosskopf [1997]. For a comparison of the properties of these two efficiency indexes, among others, see Russell and Schworm [2010].

oriented" because they measure efficiency in (intended and unintended) output space (*i.e.*, in the output direction).

For each technology $T = \tilde{T}_{WD}, T_{BP}$ and for each decision making unit ($d = 1, \ldots, p$), the output-oriented HYP efficiency index is defined by

$$E_H\left(x^d, y^d, z^d, T\right) = \min_{\beta>0}\left\{\beta \mid \left\langle x^d, y^d/\beta, \beta z^d\right\rangle \in T\right\}, \tag{5.1}$$

and the output-oriented DDF index of *inefficiency* is defined by

$$I_{DD}\left(x^d, y^d, z^d, T\right) = \max\left\{\beta \mid \left\langle x^d,\ y^d + \beta g_y,\ z^d - \beta g_z\right\rangle \in T\right\}, \tag{5.2}$$

where $g = \langle g_y, g_z\rangle \in \mathbf{R}_+^{m+m'}$ is the arbitrary (output) "direction vector." E_H maps into the (0,1] interval, while E_{DD} maps into $\mathbf{R}_+$. For points on the frontier of T, $E_H(x, y, z, T) = 1$ and $I_{DD}(x, y, z, T) = 0$.[32] The vectors $\left\langle x^d, y^d/\overset{*}{\beta}, \overset{*}{\beta} z^d\right\rangle$ and $\left\langle x^d,\ y^d + \overset{*}{\beta} g_y,\ z^d - \overset{*}{\beta} g_z\right\rangle$, where $\overset{*}{\beta}$ is the solution value in each case, are referred to as "reference points"; they are comparison vectors for assessing the efficiency of a particular production vector.

5.1. Inadequacies of conventional efficiency indexes for the by-production approach: the hyperbolic and directional-distance-function indexes.

Using our proposed BP approach under the assumptions that T_1 is independent of z and T_2 is independent of y, the HYP and DDF efficiency indexes in (5.1) and (5.2)

[32] Note that an HYP output-oriented index of *inefficiency* can be defined by $1/E_H(x, y, z, T)$, which lies in the interval $[1, \infty)$.

implicitly decompose total (in)efficiency into (in)efficiency in intended production and environmental (in)efficiency:

$$\begin{aligned}
E_H(x, y, z, T_{BP}) &= \min_{\beta>0} \{\beta | \ \langle x, y/\beta, \beta z\rangle \in T_{BP}\} \\
&= \min_{\beta>0} \{\beta | \langle x, y/\beta, \beta z\rangle \in T_1 \text{ and } \langle x, y/\beta, \beta z\rangle \in T_2\} \\
&= \max\{\beta_1, \beta_2\}, \ \text{where} \\
\beta_1 &= \min_{\beta>0} \{\beta | \langle x, y/\beta, z\rangle \in T_1\} =: E_H^1(x, y, z, T_{BP}) \text{ and} \\
\beta_2 &= \min_{\beta>0} \{\beta | \langle x, y, \beta z\rangle \in T_2\} =: E_H^2(x, y, z, T_{BP})
\end{aligned} \tag{5.3}$$

and

$$\begin{aligned}
I_{DD}(x, y, z, T_{BP}) &= \max_{\beta} \{\beta | \ \langle x, y + g_y\beta, z - g_z z\rangle \in T_{BP}\} \\
&= \max_{\beta} \{\beta | \langle x, y + g_y\beta, z - g_z\beta\rangle \in T_1 \text{ and } \langle x, y + g_y\beta, z - g_z\beta\rangle \in T_2\} \\
&= \min\{\beta_1, \beta_2\}, \ \text{where} \\
\beta_1 &= \max_{\beta} \{\beta | \langle x, y + g_y\beta, z\rangle \in T_1\} =: I_{DD}^1(x, y, z, T_{BP}) \text{ and} \\
\beta_2 &= \max_{\beta} \{\beta | \langle x, y, z - g_z\beta\rangle \in T_2\} := I_{DD}^2(x, y, z, T_{BP}).
\end{aligned} \tag{5.4}$$

If max $\{\beta_1, \beta_2\} = \beta_1 \neq \beta_2$ for the HYP output-oriented measure of efficiency, the data point is compared to a reference point that is weakly efficient in intended production but is not weakly environmentally efficient. If max $\{\beta_1, \beta_2\} = \beta_2 \neq \beta_1$, the reference point is weakly environmentally efficient but not weakly efficient in intended production. A similar logic applies in an obvious way for the DDF measure of inefficiency. Thus, the reference points with which different data points are compared to measure (in)efficiency may not be fully efficient when the BP approach is used, and we argue below that they typically are *not* fully efficient.

Consider the quantity vector of DMU 3 in Example 1, represented by point $a = \langle a_z, a_y \rangle = \langle 2, 2/3 \rangle$ in the output possibility set corresponding to $x = 1$ in Panel 2. If the BP approach is used to measure HYP efficiency, (5.3) and Panels 1 to 3 show that $\beta_1 = 1/3$ and $\beta_2 = 1/2$ so that $\max\{\beta_1, \beta_2\} = \beta_2$.[33] This implies that the reference point that is being used to measure efficiency of $\langle 2, 2/3 \rangle$ is $e' = \langle 1, 4/3 \rangle$. In contrast to the fully efficient point e, e' is environmentally efficient but not efficient in intended production. On the other hand, the HYP efficiency of a using the WD approach in Panel 4 is .47, and the reference point is e'', which is technologically efficient with respect to the WD technology.[34]

Suppose that, as is common in the literature, we adopt a direction vector $g = \langle g_z, g_y \rangle = \langle 1, 1 \rangle =: \mathbf{1}$ to compute the DDF index of inefficiency for DMU 3. If the BP

---

33 The intuition as to why the efficiency measure chooses $\beta = \beta_2 = \max\{\beta_1, \beta_2\}$ as a full measure of output efficiency is that, while $\langle 2\beta_2, \frac{2}{3\beta_2} \rangle$ is feasible both with respect to T_1 and T_2 with $x = 1$, $\langle 2\beta_1, \frac{2}{3\beta_1} \rangle$ is feasible only with respect to T_1 and not T_2, as it implies a reduction in the level of the unintended output z below the minimum that $x = 1$ can produce.

34 The frontier of the output possibility set in Panel 4 can be represented functionally by

$$\begin{aligned} y &\leq \frac{3}{2}z \qquad 0 \leq z \leq 1 \\ y &\leq \frac{z}{6} + \frac{4}{3} \quad 1 \leq z \leq 4. \end{aligned} \tag{5.5}$$

The HYP efficiency index in this case will choose a reference point that either lies on line-segment OA (for $z \in [0, 1]$) or on line-segment AB (for $z \in [1, 4]$). The reference point will be of the form $\langle 2\beta, \frac{2}{3\beta} \rangle$. Suppose, the reference point is on OA, then (5.5) implies that it should solve $\frac{2}{3\beta} = \frac{3}{2}2\beta$ and this yields $\beta = \frac{\sqrt{2}}{3} = 0.471$. If it is on AB then (5.5) implies that it should solve $\frac{2}{3\beta} = \frac{2\beta}{6} + \frac{4}{3}$ and this yields $\beta = \sqrt{6} - 2 = 0.449$. However, for this case, (5.5) implies that the underlying reference point, $\langle 0.899, 1.483 \rangle$, is not feasible. Hence, HYP efficiency associated with a is 0.471, which takes us to the reference point $e'' = \langle 0.943, 1.414 \rangle$ lying on OA.

approach is employed, then β_1 is implicitly defined by $\frac{2}{3} + \beta_1 = 2$, so that $\beta_1 = 4/3$. Similary, β_2 is implicitly defined by $2 - \beta_2 = 1$ so that $\beta_2 = 1$. Thus, the DDF inefficiency score of DMU 3 is max $\{\beta_1, \beta_2\} = \beta_2 = 1$, and this leads to a reference point $\langle 1, 5/3 \rangle$ that is environmentally efficient but not efficient in intended production.

Now consider the quantity vector of DMU 2 represented by point $b = \langle 1, 3/2 \rangle$ in the output possibility set corresponding to $x = 1$ in Panel 2. For the HYP measure, program (5.3) and Panels 1 to 3 of Figure 6 imply that $\beta_2 = 1$ while $\beta_1 = 3/4 < 1$. Thus, the conventional HYP measure computed using the BP approach gives DMU 2 an efficiency score $\beta = 1$ even though DMU 2 is not efficient in both the environmental and the intended output dimensions: it is only environmentally efficient.[35]

These examples illustrate a fundamental problem with the conventional measures of efficiency when using the BP approach for constructing the technology: the efficiency score for a firm may take the value 1 for HYP measures or 0 for the DDF measure even though the firm is not weakly efficient in both environmental and intended output directions. In addition, the reference point with which the firm is compared may not be weakly efficient in both these dimensions, resulting in an understatement (overstatement) of overall inefficiency (efficiency).

It is well known that the HYP and DDF indexes do not satisfy the indication condition: score equal to 1 or 0, respectively, if and only if the point is (fully) effi-

[35] Similarly, it is easy to verify that the conventional DDF measure of inefficiency also gives DMU 2 an inefficiency score of 0.

cient.[36] But, because of another problem, the DDF is particularly unsuitable for use as an inefficiency index for a BP technology. The inefficiency scores obtained from the DDF measure are very sensitive to the choice of the direction vector $g = \langle g_z, g_y \rangle$. While computing the DDF index of inefficiency, the direction and size of vector g are held fixed across all data points.[37] For this choice of g, the DDF inefficiency indexes for DMUs, 1, 2, and 3 in Example 1, are obtained as below:

$$\begin{array}{cccc} \text{DMU} & \beta_1 & \beta_2 & \beta = \min\{\beta_1, \beta_2\} \\ 1 & 0 & \frac{3}{g_z} & \beta_1 = 0 \\ 2 & \frac{1/2}{g_y} & 0 & \beta_2 = 0 \\ 3 & \frac{4/3}{g_y} & \frac{1}{g_z} & \begin{array}{l} \beta_2 \quad \text{if } g_y < 4g_z/3 \\ \beta_1 \quad \text{if } g_y > 4g_z/3 \end{array} \end{array} \tag{5.6}$$

Thus, except when a DMU is environmentally efficient or efficient in intended production, the DDF measure chooses β_1 or β_2 as the overall measure of inefficiency depending on the choice of the direction vector g. It is a common practice in the literature to choose $g = \mathbf{1}$. In this example with $g = \mathbf{1}$, the DDF measure selects the environmental inefficiency component for DMU 3 as the overall measure of inefficiency. It is, of course, obvious that the DDF inefficiency score is sensitive, in general, to the choice of the direction vector. This sensitivity seems to be more salient in the BP approach, however, since the choice of g is typically tantamount to predetermining a choice between the selection of the environmental or the intended production

[36] See Russell and Schworm [2010].

[37] The components g_y and g_z of g are interpreted to be measured in the units in which intended output and pollution are measured, respectively, so that the inefficiency scores can be interpreted to be independent of units of measurement.

inefficiency components as the measure of overall inefficiency.[38]

Many (in)efficiency indexes have been proposed in the literature.[39] In empirical work on pollution-generating technologies, however, HYP and DDF are among the more widely used of these conventional indexes. Given the above problems with these two indexes under the BP approach, we propose, in the next subsection, a modification of another conventional efficiency index that is better behaved for use in measuring efficiency on BP production technologies.

5.2. A proposed efficiency index for by-production technologies: modification of the Färe-Grosskopf-Lovell index.

The previous subsection shows that the principal problem with the widely used hyperbolic and directional-distance-function efficiency indexes applied to BP technologies is the endemic understatement of the degree of inefficiency.

The index we propose for measuring efficiency on by-production technologies is motivated by the input-oriented index proposed by Färe and Lovell [1978] and extended to the full ⟨input, output⟩ space for standard technologies (with no unintended outputs) by Färe, Grosskopf, and Lovell [1985, pp. 153–154]. The key feature of this index is that the reference points it uses to assign efficiency scores to the DMUs are

[38] It is well known that the HYP inefficiency index can be interpreted as an alternative kind of DDF inefficiency index in which the direction vector varies across DMUs and, in particular, is equated to the quantity vector $\langle z, y \rangle$. This alteration alleviates the above problem with the conventional DDF index (where g is held fixed across all DMUs). (See Chambers, Chung, and Färe [1996].)

[39] See Russell and Schworm [2010] for an analysis of these indexes and their properties.

all efficient, in contrast to the the HPY and DDF indexes, for which the reference points are all weakly efficient.[40] Define

$$y \oslash \theta = \langle y_1/\theta_1, \ldots, y_m/\theta_m \rangle \tag{5.7}$$

and

$$\gamma \otimes z = \langle \gamma_1 z_1, \ldots, \gamma_{m'} z_{m'} \rangle. \tag{5.8}$$

As our modification is minor, we continue to refer to it as the (output oriented) Färe-Grosskopf-Lovell (FGL) index and define it as follows:

$$E_{FGL}(x, y, z, T) := \frac{1}{2} \min_{\theta,\gamma} \left\{ \frac{\sum_j \theta_j}{m} + \frac{\sum_k \gamma_k}{m'} \Big| \ \langle x, y \oslash \theta, \gamma \otimes z \rangle \in T \right\}. \tag{5.9}$$

This index maps into the (0,1] interval and is equal to 1 if and only if the output vectors are technically efficient.

In the case of BP technologies, and under the assumption that T_1 is independent of z and T_2 is independent of y, the index decomposes as follows:

$$\begin{aligned} E_{FGL}(x, y, z, T_{BP}) &:= \frac{1}{2} \min_{\theta,\gamma} \left\{ \frac{\sum_j \theta_j}{m} + \frac{\sum_k \gamma_k}{m'} \Big| \ \langle x, y \oslash \theta, \gamma \otimes z \rangle \in T_{BP} \right\} \\ &= \frac{1}{2} \min_{\theta,\gamma} \left\{ \frac{\sum_j \theta_j}{m} + \frac{\sum_k \gamma_k}{m'} \Big| \ \langle x, y \oslash \theta, \gamma \otimes z \rangle \in T_1 \ \wedge \ \langle x, y \oslash \theta, \gamma \otimes z \rangle \in T_2 \right\} \\ &= \frac{1}{2} \min_{\theta} \left\{ \frac{\sum_j \theta_j}{m} \ \Big| \ \langle x, y \oslash \theta, z \rangle \in T_1 \right\} + \frac{1}{2} \min_{\gamma} \left\{ \frac{\sum_k \gamma_k}{m'} \ \Big| \ \langle x, y, \gamma \otimes z \rangle \in T_2 \right\} \\ &=: \frac{1}{2} \left[E^1_{FGL}(x, y, z, T_1) + E^2_{FGL}(x, y, z, T_2) \right] =: \frac{1}{2}[\beta_1 + \beta_2] = \beta, \end{aligned} \tag{5.10}$$

[40] This feature is attributable to the fact that The Färe-Grosskopf-Lovell index involves a maximal contraction/expansion of inputs/outputs in coordinate-wise directions (rather than in a maximal radial or hyperbolic direction). Hence, all the slack in inputs and outputs is removed. (Of course, the input-oriented or output-oriented version of this index takes up all slack only in the input or output space, leaving the possibility of residual slack in outputs or inputs.)

where the third identity follows from independence of T_1 from z and independence of T_2 from y. This index is one-half of the sum of the average maximal coordinate-wise expansions of intended-output quantities and the average maximal coordinate wise contractions of unintended-output quantities subject to the constraint that the expanded/contracted output-quantity vector remain in the production possibility set for a given input vector.[41] Under our independence assumptions, the index decomposes into the sum of a standard intended-output-oriented index defined on T_1 (β_1) and an environmental index defined on T_2 (β_2).

The properties of this proposed index can be illustrated using the artificial data in Example 1 above. Consider first the case of DMU 3, represented by point a in Panel 3 of Figure 6. It is clear that $E^1_{FGL}(1, 2/3, 2, T_1) = 1/3$ and $E^2_{FGL}(1, 2/3, 2, T_2) = 1/2$, so that $E_{FGL}(1, 2/3, 2, T_{BP}) = 5/12 < E_H(1, 2/3, 2, T_{BP}) = 1/2$. Moreover, the reference point for a is the fully efficient point e in Panel 3; thus, unlike the HYP and DDF indexes, this proposed index takes up all the slack in the measurement of efficiency. Consider now the quantity vector of DMU 2 represented by point $b = \langle 1, 3/2 \rangle$ in Panel 3. Although this point is not fully efficient, the values of both HYP and DDF are equal to 1. On the other hand, for this DMU, $E^2_{FGL}(1, 3/2, 1, T_2) = 1$ but $E^1_{FGL}(1, 3/2, 1, T_1) = (3/2)/2 = 3/4$, so that $E_{FGL}(1, 3/2, 1, T_{BP}) = 7/8$. These examples illustrate the fact that the proposed index corrects the principal problem

[41] Note that, instead of weighting each index equally, one could adopt different weights (summing to 1) if there were a reason to give more importance to one type of efficiency than the other.

with the HYP and DDF indexes in the measurement of efficiency on BP technologies. In particular, the FGL efficiency scores will typically be lower than the HYP efficiency scores.[42]

It can also be verified that, for DMU 3, $E_{FGL}(1, 2/3, 2, T_{WD}) = .47$, and the associated reference point is e'' in Panel 4 of Figure 6. Hence, the FGL efficiency score for DMU 3 under the WD approach is higher than under the BP approach. Further, e'' is technologically infeasible under the BP approach, while the analogous reference point e for DMU 3 under the BP approach is technologically infeasible under the WD approach. The output quantity vector associated with DMU 2 is efficient under the WD approach ($E_{FGL}(1, 2/3, 2, T_{WD}) = 1$ and it involves no slack viz-a-viz the WD technology). But, this vector is only weakly efficient under the BP approach and hence FGL gives DMU 2 a lower efficiency score. Thus, the efficiency scores for DMUs and the associated reference points for FGL efficiency index are typically quite different across the BP and WD approaches. In particular, if a DMU is judged efficient by the FGL index under the BP approach, this index will also judge it efficient under the WD approach. But the converse is not true. This implies that the FGL efficiency scores under the WD approach will typically be at least as high as those under the BP approach.

[42] This is true for both WD and BP technologies.

6. Empirical application.

We now illustrate the implementation of the modified FGL index on a BP technology constructed with an actual data base. We use annual data for 92 coal-fired electric power plants from 1985 to 1995.[43] This data base includes observations for one intended output: net electricity generation (in kWh); two unintended outputs: sulfur dioxide (SO_2) and nitrogen oxide (NO_x) (in short-tons); two non-polluting inputs: the capital stock and the number of employees; and three pollution-generating inputs: the heat content (in Btu) of coal, oil, and natural gas consumed at each power plant. Thus $p = 92$, $m = 1$, $m' = 2$, $n_1 = 2$, and $n_2 = 3$.

The various efficiency indexes are calculating by executing mathematical programming problems. In particular, the appropriate objective function in (5.1), (5.2), or (5.9) is optimized subject to the constraints in (4.4), (4.6), or (4.8), respectively.[44]

The results depicted in Table 2 underscore the sensitivity of the the DDF measure to the choice of the direction vector (illustrated above using Example 1). In our data set, the consequence of choosing $g = \mathbf{1}$ is that the DDF measure of inefficiency picks up the environmental inefficiency component as the overall measure for most DMUs.

---

43 A detailed description of the data can be found in Pasurka [2006].

44 Recall that the BP approach involves decompositions of (in)efficiency indexes (see (5.3), (5.4), and (5.10)). As an example, the linear programs for calculating the FGL index under by-production can be found in the attached working-paper version at page 27. We should note that calculation of the HYP and FGL indexes on WD technologies employ the linear approximation used by Färe, Grosskopf, and Pasurka [1986] and much of the subsequent literature. Owing to the relatively large dimensionality of our data set, calculation of solutions to the nonlinear programs needed to calculate these indexes explicitly on WD technologies is impractical. In the case of the BP approach, however, the programs required to computations all three (in)efficiency indexes—HYP, DDF, and FGL—are linear and hence pose no such calculation problems.

Table 2 reports the (in)efficiency scores of a sample of ten DMUs for the year 1985 under the BP approach. The magnitudes of the HYP efficiency figures for β_1 and β_2 for these firms are reasonably comparable (ranging from 0.7416 to 1.000 for β_1 and from 0.3052 to 1.000 for β_2), so that the operation $\beta = \max\{\beta_1, \beta_2\}$ is, in some sense, non-discriminatory in choosing between β_1 and β_2. The magnitudes of β_1 and β_2 for the DDF measure, however, are in orders ranging from 10^8 to 10^{10} and from 10^3 to 10^5, respectively, so that, except when $\beta_1 = 0$, the operation $\beta = \min\{\beta_1, \beta_2\}$ predominantly favors β_2 over β_1. Primarily for this reason we do not present further results for the DDF measure of inefficiency.

Table 3 contains the mean values of the HYP and FGL efficiency indexes for each year in our sample. Columns (1) and (2) pertain to the WD technology and Columns (3)–(8) pertain to the BP technology underlying our data set. The BP approach is our proposed method of constructing pollution-generating technologies and the FGL index is our proposed method of calculating efficiency on BP technologies.

Columns (1) and (2) and Columns (5) and (8) of Table 3 show that, under both the WD and BP approaches, the HYP index runs higher than the FGL index. As in Example 1, this comparison reflects the fact that the expansion/contraction to the frontier of the latter takes up all the slack in outputs, thus comparing the output quantity vector to a reference vector on the efficient frontier, whereas the expansion/contraction of the former leaves some slack, comparing the output quantity vector to a point on the frontier but not necessarily in its efficient subset.

Table 3 also indicates that, for our data set, both the HYP and FGL efficiency estimates are consistently higher for the WD technology than for the BP technology, a phenomenon that we explained above using Example 1. These differences in the efficiency scores across the BP and WD technologies suggest that, for both HYP and FGL measures, the reference points with respect to which efficiency is measured are different under the two approaches. In particular, in the FGL case, all the reference points are efficient, whereas for the HYP case, all are only weakly efficient. Thus, our results show that the sets of efficient and the sets of weakly efficient points differ across WD and BP technologies.

In the case of our particular data set, regardless of the index used, Table 3 also shows that the degree of inefficiency in the pollution technology T_2 is much larger than that in the intended-production technology T_1: apparently, the DMUs in our data set are less concerned about the environmental dimension of their production activities or environmental efficiency is more difficult to achieve.

The FGL index records greater pollution-generation inefficiency than does the HYP index. An obvious expanation could again be the differences in the way in which the two indexes treat slacks in outputs.[45]

Table 4 provides counts of weakly efficient and efficient firms using the HYP and FGL indexes, respectively, for the two technologies. Columns (1) and (6) and Columns

[45] Note that the (output oriented) HYP and FGL indexes take the same values for the intended production technology T_1 because, with only a single intended output, they collapse to the same index.

(2) and (10) provide a comparison across WD and BP technological specifications of numbers of firms that receive an efficiency score of 1 under the HYP and FGL measures, respectively. The table shows that, for both the HYP and FGL indexes, the WD technological specification results in a larger number of firms receiving an efficiency score 1 than does the BP technological specification. This seems consistent with the findings from Example 1: the frontier of the output possibility set is larger under the WD specification than under the BP specification. Hence, the probability of a DMU being assigned an efficiency value of 1 is greater under the WD approach than under the BP approach.

Columns (3)–(10) of Table 4 also help to compare the performance of FGL and HYP indexes under the BP approach. First, it is not surprising that the HYP index, which allows slack to remain in reference output vectors, judges at least as many DMUs to be efficient (environmentally, in intended production, and overall) as does the FGL measure. This comparison is indicated by comparing Column (3) with Column (7), Column (4) with Column (8), and Column (6) with Column (10).[46] Second, it follows that all the DMUs that are judged environmentally efficient by FGL are a subset of the DMUs judged environmentally efficient by HYP. Finally, as demonstrated by Example 1, the HYP index gives efficiency score 1 to DMUs that are efficient in intended outputs *or* are environmentally efficient *or* are both. Hence,

[46] In particular, with respect to the intended production technology T_1, since there is only one intended output, there is no slack remaining in the reference vector when the HYP index gives a DMU an efficiency score of 1. Hence, Columns (3) and (7), are identical.

Column (6) is obtained by adding Columns (3) and (4) and subtracting Column (5) from this sum. On the other hand, as also demonstrated by Example 1, FGL is more demanding in judging a DMU efficient: it gives efficiency score 1 to a DMU if and only if it is efficient both environmentally and in intended production. Thus, Column (10) is equal to Column (9).

Table 5 shows how the rankings of firms on the basis of their efficiency scores compare across the two efficiency indexes HYP and FGL, across the two technological specifications, and across the environmental and intended-production efficiency scores. Columns (1) and (2) of Table 5 show that, for both HYP and FGL, the Spearman correlation coefficients between the efficiency scores under the WD and BP approaches are moderately high and positive: the rank correlation coefficients lie in the range .5 to .71 and .66 to .89 for the HYP and FGL measures, respectively. In the light of the significant conceptual differences between the two approaches (in particular, the differences in the frontiers of the BP and WD technologies), which are reinforced strongly by our empirical findings above, the BP approach seems to make a larger difference in the levels than in the ranking of the efficiency scores of the DMUs.

Table 5 also allows comparison of rankings under the the HYP and FGL indexes applied to BP technologies. Given that in our data set there is only a single intended output, there are no differences in the efficiency scores for intended production obtained from the HYP and FGL measures. Hence, the Spearman correlation coefficients in Column (4) are all equal to 1. Our data set also exhibits high rank correlations between environmental efficiency scores obtained from the FGL and HYP measures: as seen in Column (5), the rank correlation coefficients lie in the range .87 to .99. Nevertheless, the rank correlation coefficients between overall efficiency scores obtained under FGL and HYP are on the lower side: as seen in Column 3, these lie in the range .42 to .72. This could be explained by the differences in the way HYP an FGL indexes aggregate over environmental and intended output efficiency scores. In Example 1, we saw that the HYP gives an efficiency score of 1 to a DMU that is environmentally efficient but not efficient in intended production or vice-versa. The FLG index, however, penalizes such DMUs for the slack in production of the intended or the unintended output and gives them a lower score. Thus, the strength of the association between the rankings of DMUs on the basis of their overall efficiency under the HYP and FGL measures is not clear: in our particular data set, the association is low.

Columns (6) and (7) of Table 5 show the rank correlation coefficients between efficiency scores in intended and unintended productions for the HYP and FGL indexes under the BP approach. These values are all negative and low; *e.g.,* the Spearman correlation coefficients range between -.08 to -.28 and -.01 to -.27 for the HYP and FGL indexes, respectively. Negative correlation values indicate that DMUs that are more efficient in intended production are also likely to be more environmentally inefficient, and vice-versa. This may suggest that the DMUs face some trade-offs between efficiency in intended production and in pollution generation. In our data set, however, these trade-offs are weak, as the correlation values are very low. Thus, one may conclude that most DMUs in our data set do not face significant trade-offs between intended production and residual generation and can improve simultaneously on both environmental and intended output efficiencies.

7. By-production versus weak disposability: Comparisons of DEA formulations in the presence of abatement efforts.

The WD approach explains the positive correlation between intended outputs and pollution through abatement efforts of firms that are not modeled. Hence, it considers only a reduced form of the overall technology in the space of inputs and all unintended and intended outputs other than the abatement output. In this section, we extend the DEA formulation of a BP technology to include abatement efforts made by firms and derive the DEA analogue of its reduced form defined in (3.23). With the help of an example, we then compare the reduced forms of the two technologies.

A DEA version of the BP technology in the presence of an abatement output is derived as follows: With respect to the intended technology T_1, abatement is a standard output that satisfies standard output free disposability. The residual-generating mechanism T_2, on the other hand, satisfies costly disposability of abatement output.

Thus,

$T_{BP} = T_1 \cap T_2,$ where

$$T_1 = \Big\{\langle x^1, x^2, y, y^a, z\rangle \in \mathbf{R}_+^{n_1+n_2+m+1+m'} \mid \lambda[X^1 \ X^2] \leq \langle x^1,\ x^2\rangle,\ \lambda Y \geq y,\ \lambda A \geq y^a,$$
$$\text{for some } \lambda \in \mathbf{R}_+^p\Big\},\ \text{and}$$
$$T_2 = \Big\{\langle x^1, x^2, y, y^a, z\rangle \in \mathbf{R}_+^{n_1+n_2+m+1+m'} \mid \mu X^2 \geq x^2,\ \mu A \leq y^a,\ \mu Z \leq z,$$
$$\text{for some } \mu \in \mathbf{R}_+^p\Big\}, \tag{7.1}$$

where A is the vector of abatement outputs for the p firms.

Holding all input quantities fixed at x, we next derive a DEA version of the reduced form of T_{BP}. Precisely, this is the projection of the output possibility set of T_{BP} (corresponding to input-quantity level x) defined in the $\langle z, y, y^a\rangle$ space into the $\langle z, y\rangle$ space.

Noting that technology T_1 is independent of z, the DEA construction of the projection of the output-possibility set for technology T_1 (corresponding to input level x) into the $\langle y^a, y\rangle$ space is denoted by $\hat{P}_1(x)$.[47] In a similar manner, noting that technology T_2 is independent of y, we define the DEA construction of the projection $\hat{P}_2(x)$ of T_2 into the $\langle y^a, z\rangle$ space.[48]

[47] This is the set of all combinations $\langle y^a, y\rangle$ that are possible with input level x for technology T_1.
[48] This is the set of all combinations $\langle y^a, z\rangle$ that are possible with input level x for technology T_2.

The DEA versions of the WD technology (see (4.4)) and the reduced form of T_{BP} in the $\langle z, y\rangle$ space, for a fixed level x of input quantities, are defined as follows:

$$\hat{P}_{BP}(x) = \Big\{\langle z, y\rangle \in \mathbf{R}_+^{m+m'} \mid \exists\, y^a \in \mathbf{R}_+ \text{ such that } \langle y^a, y\rangle \in \hat{P}_1(x) \ \wedge\ \langle y^a, z\rangle \in \hat{P}_2(x)\Big\}$$

$$\tilde{P}_{WD}(x) = \Big\{\langle z, y\rangle \in \mathbf{R}_+^{m+m'} \mid \langle x, y, z\rangle \in \tilde{T}_{WD}\Big\}. \tag{7.2}$$

In Example 2 below, we compare $\hat{P}_{BP}(x)$ and $\tilde{P}_{WD}(x)$. It is assumed that $n_2 = 1,\ n_1 = 0,\ m = m' = 1$, and $x = 1$.

Example 2: $p = 8$. The (artificial) data are as follows:

DMU	x	y^a	y	z
1	1	0	8	9
2	1	1	7	6
3	1	2	6	8
4	1	3	6	3
5	1	4	1	2
6	1	5	4	0
7	1	6	2	0
8	1	7	1	11

(7.3)

After plotting the data, we find that $\hat{P}_1(1)$ and $\hat{P}_2(1)$ can be represented functionally by piece-wise linear functions:

$$\begin{aligned} \psi^1(y^a) &= 8 - \frac{2}{3}y^a \text{ if } y^a \in [0,3] & \psi^2(y^a) &= 9 - 3y^a \text{ if } y^a \in [0,1]. \\ &= 9 - y^a \text{ if } y^a \in [3,5] & &= \frac{15}{2} - \frac{3}{2}y^a \text{ if } y^a \in [1,5] \\ &= \frac{23}{2} - \frac{3}{2}y^a \text{ if } y^a \in [5,7] & &= 0 \text{ if } y^a \geq 5. \end{aligned} \tag{7.4}$$

The sets $\hat{P}_1(1)$ and $\hat{P}_2(1)$ are shown in Panels 1 and 2 of Figure 7. (7.2) implies that $\hat{P}_{BP}(1)$ (shown in Panel 3 of Figure 7) is constructed as follows:

$$\hat{P}_{BP}(1) = \big\{\langle z, y\rangle \in \mathbf{R}_+^2 \mid z \geq \psi^2(y^a) \ \wedge\ y \leq \psi^1(y^a) \ \wedge\ y^a \in [0,7]\big\}. \tag{7.5}$$

Note that the construction of $\hat{P}_{BP}(1)$ involves explicit reference to the abatement output.[49] No reference was made, however, to data on y^a in the DEA construction of $\tilde{P}_{WD}(1)$ in Panel 4 of Figure 7.

Moreover, while weak disposability holds for $\tilde{P}_{WD}(1)$, the data are such that null jointness is violated. This can be rationalized by the fact that the abatement output of a firm can completely mitigate pollution even when it is producing positive amounts of the intended outputs.[50] Further, the boundary of $\tilde{P}_{WD}(1)$ has a negatively sloped region, indicating a negative correlation between intended and unintended outputs in that region. The frontier of $\hat{P}_{BP}(1)$, on the other hand, is everywhere non-negatively sloped.

8. Conclusions.

Pollution is an unintended output that cannot be freely disposed of. Underlying its production are a set of chemical and physical reactions that take place in nature when firms engage in the production of intended outputs. These natural reactions define nature's residual generation mechanism, which is a relation between the residuals generated and some inputs that are used or some intended outputs that are produced by the firm: hence, the inevitability of a certain minimal amount of pollution being

[49] In particular, we have been able to express the frontier of $\hat{P}_{BP}(1)$ as a vector-valued function of y^a.

[50] This could be true, *e.g.,* in the presence of abatement activities such as recycling of wastes or if all wastes are biodegradable and can hence be completely eliminated.

generated when firms engage in intended production. We call this phenomenon by-production of pollution. The larger is the scale of intended production, the greater are the pollution-causing inputs being used or the greater are the pollution-causing intended outputs being produced, and hence, the more is the pollution generated. This provides the fundamental explanation for the positive correlation that is observed between intended production and residual generation.[51]

Standard approaches in the existing literature, on the other hand, usually attribute the observed positive correlation between pollution generation and intended production to resource-costly abatement options of firms. Such options, however, are not explicitly modeled, and only a reduced form of the technology is considered. Pollution is either treated as an input satisfying standard input free disposability or is considered as an output that is weakly disposable.

To capture the phenomenon of by-production, we model pollution-generating technologies as a composition of two technologies: an intended-production technology and a residual-generation technology. The former describes how inputs are transformed into intended outputs, is assumed to be independent of the level of pollution, and satisfies standard free-disposability properties. The latter reflects nature's residual generation, violates standard disposability properties with respect to goods that result in (affect) pollution generation, and exhibits costly disposability with respect

[51] Some of the literature has adopted physical science terminology to describe these relationships in terms of the "material balance" condition (see Ayres and Kneese [1969] and, more recently, Coelli, Lauwers, and van Huylenbroeck [2007].

to pollution. As a result, the overall technology violates standard disposability with respect to inputs that cause (affect) pollution generation and exhibits costly disposability with respect to pollution. In these ways, a by-production technology, which is based on multiple production relations, is different and better able to capture the observed trade-offs in production than the usual input and output approaches to modeling pollution-generating technologies based on just a single production relation.

We formulate DEA specifications of technologies that satisfy by-production, with or without pollution-abatement activities, and employ them to measure technical efficiency of firms. In the context of by-production, standard measures of efficiency decompose very naturally into environmental and intended output efficiencies. However, we find that, in the context of by-production, the commonly used indexes of (in)efficiency, the hyperbolic and the directional-distance-function index, overstate efficiency. In the existing set of (in)efficiency indexes proposed in the literature, we find that a modification of an index proposed by Färe, Grosskopf, and Lovell [1985] corrects the flaws in the hyperbolic and directional-distance-function indexes for measurement of efficiency for by-production technologies. A comparison of the values of this index with those of the hyperbolic and directional-distance-function indexes, using a database for electric power firms, supports our arguments about the inadequacies of the latter.

Appendix.

Implicit function theorem: Let $f : \mathbf{R}^n_+ \times \mathbf{R}^m_+ \to \mathbf{R}^m$ be a continuously differentiable vector valued function with image $f(x, y) = z$, where $x \in \mathbf{R}^n_+$ and $y \in \mathbf{R}^m_+$. Let $\langle \bar{x}, \bar{y} \rangle \in \mathbf{R}^{n+m}_+$ be such that $f(\bar{x}, \bar{y}) = 0$ and the $m \times m$ matrix of first derivatives, $\nabla_y f(\bar{x}, \bar{y})$, is full ranked (has a non-zero determinant). Then there exist neighborhoods U and V around $\bar{x}$ and $\bar{y}$ in $\mathbf{R}^n_+$ and $\mathbf{R}^m_+$, respectively, and a continuously differentiable function $\Phi : U \to V$ with image $\Phi(x) = y$ such that, for all $x \in U$, we have $f(x, \Phi(x)) = 0$ and the $m \times n$ matrix of first derivatives, $\nabla_x \Phi(x)$, is obtained as

$$\nabla_x \Phi(x) = - \left[\nabla_y f(x, \Phi(x))\right]^{-1} \nabla_x f(x, \Phi(x)).$$

References.

Ayres, Robert U., and Allen V. Kneese.1969. "Production, Consumption and Externalities." *American Economic Review*, 59: 282–297.

Barbera, Anthony J., and Virginia D. McConnell. 1990. "The Impact of Environmental Regulation on Industry Productivity: Direct and Indirect Effects." *Journal of Environmental Economics and Management*, 18: 50–65.

Baumol, William. J., and Wallace E. Oates. 1988. *The Theory of Environmental Policy.* Second Edition, Cambridge: Cambridge University Press.

Boyd, Gale. A., and John D. McClelland.1999. "The Impact of Environmental Constraints on Productivity Improvement in Integrated Paper Plants." *Journal of Environmental Economics and Management*, 38: 121–142.

Chambers, Robert G., Y. Chung, and Rolf Färe.1996. "Benefit and Distance Functions." *Journal of Economic Theory* **70**: 407–419.

Chung, Y. H., Rolf Färe, and Shawna Grosskopf. 1997. "Productivity and Undesirable Outputs: A Directional Distance Function Approach." *Journal of Environmental Management*, **51**: 229–240.

Coelli, Tim, Ludwig Lauwers, and Guido Van Huylenbroeck. 2007. "Environmental Efficiency Measurement and the Materials Balance Condition." *Journal of Productivity Analysis*, 28: 3–12.

Coggins, Jay S., and John R. Swinton. 1996. "The Price of Pollution: A Dual Approach to Valuing SO_2 Allowances." *Journal of Environmental Economics and Management*, 30: 58–72.

Cropper, Maureen L., and Wallace E. Oates. 1992. "Environmental Economics: A Survey." *Journal of Economic Literature*, 30: 675–740.

Färe, Rolf, Shawna Grosskopf, and C. A. Knox Lovell. 1985. *The Measurement of Efficiency of Production*. Boston: Kluwer-Nijhoff.

Färe, Rolf, Shawna Grosskopf, and C. A. Knox Lovel. 1994. *Production Frontiers*. Cambridge: Cambridge University Press.

Färe, Rolf, Shawna Grosskopf, C. A. Knox Lovell, and Carl Pasurka. 1989. "Multilateral Productivity Comparisons When Some Outputs are Undesirable: A Nonparametric Approach." *The Review of Economics and Statistics*, 71: 90–98.

Färe, Rolf, Shawna Grosskopf, C. A. Knox Lovell, and Suthathip Yaisawarng. 1993. "Derivation of Shadow Prices for Undesirable Outputs: A Distance Function Approach." *The Review of Economics and Statistics*, 75: 374–380.

Färe, Rolf, Shawna Grosskopf, Dong-Woon Noh, and William Weber. 2005. "Characteristics of a Polluting Technology: Theory and Practice." *Journal of Econometrics*, 126: 469–492.

Färe, Rolf, Shawna Grosskopf, and Carl Pasurka. 1986. "Effects of Relative Efficiency in Electric Power Generation Due to Environmental Controls." *Resources and Energy*, 8: 167–184.

Färe, Rolf, Shawna Grosskopf, and Osman Zaim. 2002. "Hyperbolic Efficiency and Return to the Dollar." *European Journal of Operational Research*, 136: 671–679.

Førsund, Finn. 2009. "Good Modelling of Bad Outputs: Pollution and Multiple-Output Production." *International Review of Environmental and Resource Economics*, 3: 1–38.

Fried, Harold, C. A. Knox Lovell, and Shelton S. Schmidt. 2008. *The Measurement of Productive Efficiency and Growth*. Cambridge: Cambridge University Press.

Frisch, Ragnar. 1965. *Theory of Production*. Dordrecht: D. Reidel Publishing Company.

Hailu, Atakelty, and Terrance S. Veeman. 2000. "Environmentally Sensitive Productive Analysis of the Canadian Pulp and Paper Industry, 1959–1994: An Input

Distance Function Approach." *Journal of Environmental Economics and Management*, 40: 251–274.

Luenberger, David G. 1992. "New Optimality Principles for Economic Efficiency and Equilibrium." *Journal of Optimization and Applications*, **75**: 221–264.

Murty, M. N., and Surender Kumar. 2002. "Measuring Cost of Environmentally Sustainable Industrial Development in India: A Distance Function Approach." *Environment and Development Economics*, 7: 467–86.

Murty, M. N., and Surender Kumar. 2003. "Win-Win Opportunities and Environmental Regulation: Testing of Porter Hypothesis for Indian Manufacturing Industries." *Journal of Environmental Management*, 67: 139–44.

Murty, M. N., Surender Kumar, and Mahua Paul. 2006. "Environmental Regulation, Productive Efficiency, and Cost of Pollution Abatement: A Case Study of Sugar Industry in India." *Journal of Environmental Management*, 79: 1–9.

Murty, Sushama. 2010a. "Externalities and Fundamental Nonconvexities: A Reconciliation of Approaches to General Equilibrium Externality Modeling and Implications for Decentralization." *Journal of Economic Theory*, 145: 331-53.

Murty, Sushama. 2010b. "On the Theory of a Firm: The Case of By-Production of Emissions." The Warwick Economics Research Papers Series (TWERPS), No. 934.

Pasurka, Carl A. 2006. "Decomposing Electric Power Plant Emissions Within a Joint Production Framework." *Energy Economics*, 28: 26–43.

Pittman, Russell W. 1983. "Multilateral Productivity Comparisons with Undesirable Outputs." *The Economic Journal*, 93: 883–391.

Reinhard, Stijn, C. A. Knox Lovell, and Geert J. Thijssen. 1999. "Econometric Estimation of Technical and Environmental Efficiency: An Application to Dutch Dairy Farms." *American Journal of Agricultural Economics*, 81: 44–60.

Reinhard, Stijn, C. A. Knox Lovell, and Geert J. Thijssen. 2000. "Environmental Efficiency with Multiple Environmentally Detrimental Variables; Estimated with SFA and DEA." *European Journal of Operational Research*, 121: 287–303.

Russell, R. Robert, and William Schworm. 2010. "Properties of Inefficiency Indexes on ⟨Input, Output⟩ Space." Working Paper, University of California, Riverside, and University of New South Wales.

Shephard, Ronald W. 1953. *Cost and Production Functions*. Princeton: Princeton University Press.

Shephard, Ronald W. 1974. *Indirect Production Functions*. Meisenheim Am Glan: Verlag Anton Hain.

Zhou Peng, Beng Wah Ang, and Kim-Leng Poh. 2008. "A Survey of Data Envelopment Analysis in Energy and Environmental Studies." *European Journal of Operational Research*, 189: 1–18.

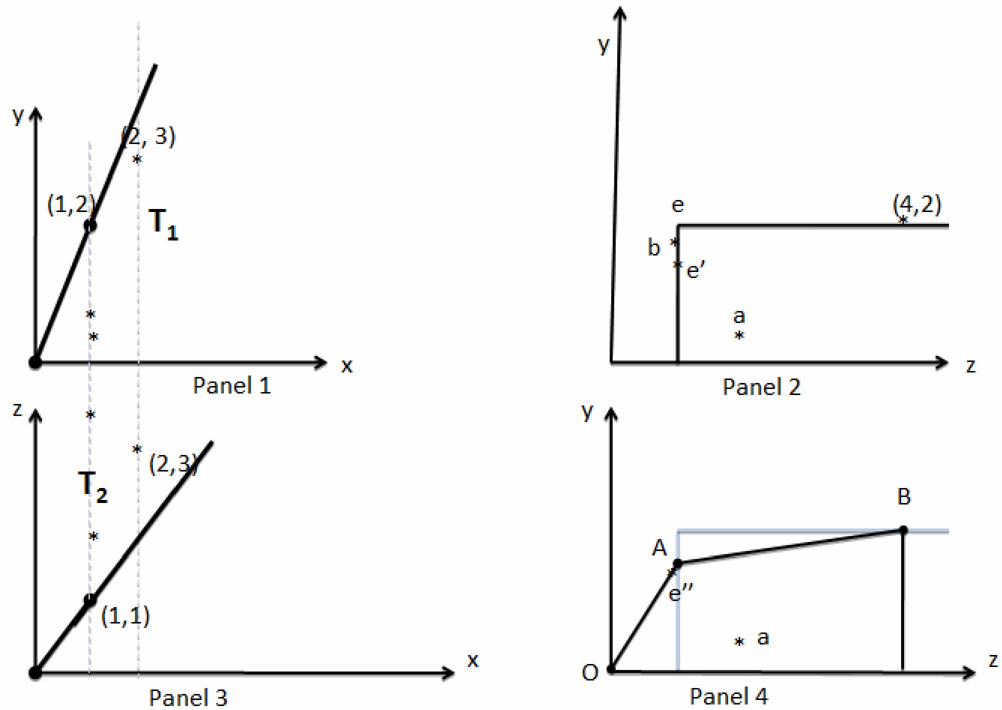

Figure 6

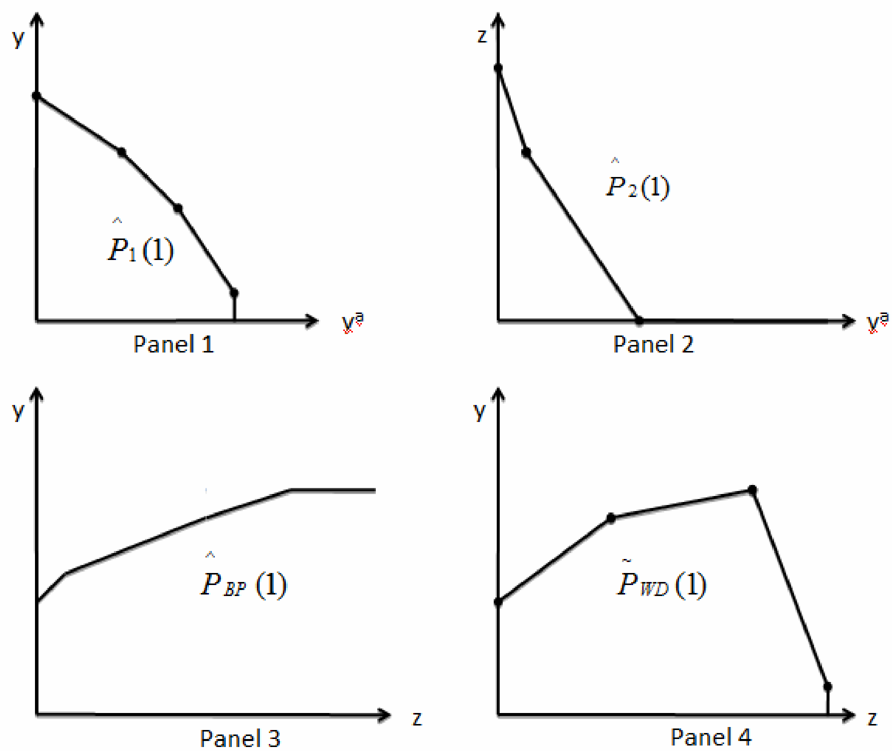

Figure 7

Table 2 : HYP and DDF (in)efficiency indexes for BP technology.

HYP β_1	HYP β_2	HYP $\max\{\beta_1, \beta_2\}$	DDF β_1	DDF β_2	DDF $\min\{\beta_1, \beta_2\}$
1	0.3425	β_1	0	43996.691	β_1
1	0.3052	β_1	0	46160.821	β_1
0.9024	0.4192	β_1	377628130	12283.274	β_2
0.8440	1	β_2	256197050	0	β_2
0.8412	0.6914	β_1	1226089000	7287.4051	β_2
0.7734	0.4540	β_1	304909890	3174.1978	β_2
0.8191	0.8304	β_2	89913470	252.83995	β_2
0.8909	0.5577	β_1	127072340	1888.807	β_2
0.8754	0.9262	β_2	511323320	814.31455	β_2
0.7416	0.5194	β_1	125872500	933.20682	β_2

Notes: Results in this table pertain to a sample of 10 DMUs for the year 1985. The direction vector employed for computing DDF is $g = \mathbf{1}$.

Table 3 : Mean efficiency values.

	WD technology		BP technology					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	HYP	FGL	HYP1	HYP2	HYP	FGL1	FGL2	FGL
1985	.94	.78	.89	.64	.90	.89	.52	.70
1986	.94	.78	.87	.62	.88	.87	.49	.68
1987	.95	.79	.90	.65	.92	.90	.54	.72
1988	.95	.81	.88	.63	.90	.88	.60	.74
1989	.95	.82	.90	.63	.92	.90	.60	.75
1990	.94	.82	.88	.62	.91	.88	.59	.74
1991	.95	.80	.89	.59	91	.89	.54	.71
1992	.95	.79	.89	.58	.91	.89	.53	.71
1993	.95	.79	.89	.60	.91	.89	.54	.72
1994	.94	.77	.88	.60	.90	.88	.56	.72
1995	.91	.74	.80	.61	.84	.80	.55	.68

Table 4: Counts of efficient DMUs.

	WD technology		BP technology							
	HYP	FGL	HYP				FGL			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Year	$\beta = 1$	$\beta = 1$	$\beta_1 = 1$	$\beta_2 = 1$	$\beta_1 = 1$ $\beta_2 = 1$	$\beta = 1$	$\beta_1 = 1$	$\beta_2 = 1$	$\beta_1 = 1$ $\beta_2 = 1$	$\beta = 1$
1985	35	3	9	9	1	17	9	4	0	0
1986	36	3	5	6	1	10	5	4	0	0
1987	43	3	12	10	1	11	12	6	0	0
1988	41	7	8	8	0	16	8	5	0	0
1989	41	6	9	11	1	19	9	9	0	0
1990	36	6	7	11	0	18	7	8	0	0
1991	39	4	8	10	2	16	8	7	1	1
1992	38	5	10	8	1	17	10	7	1	1
1993	44	5	7	7	0	14	7	5	0	0
1994	43	3	6	6	0	12	6	5	0	0
1995	34	3	9	9	0	18	9	5	0	0

Table 5: Spearman rank correlation coefficients among efficiency indexes.

	Across BP and WD technologies		Within BP technology				
			ρ(HYP,FGL)			$\rho(\beta_1, \beta_2)$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Year	HYP	FGL	β	β_1	β_2	HYP	FGL
1985	.71	.82	.60	1.00	.89	-.08	-.01
1986	.70	.89	.53	1.00	.87	-.12	-.09
1987	.60	.78	.54	1.00	.91	-.13	-.12
1988	.60	.77	.42	1.00	.97	-.23	-.23
1989	.63	.66	.45	1.00	.99	-.28	-.27
1990	.58	.71	.50	1.00	.98	-.24	-.24
1991	.52	.79	.46	1.00	.96	-.20	-.17
1992	.57	.87	.43	1.00	.94	-.21	-.13
1993	.50	.82	.42	1.00	.94	-.18	-.18
1994	.54	.76	.47	1.00	.96	-.13	-.16
1995	.59	.78	.72	1.00	.96	-.18	-.14

Decomposing NO_x and SO_2 Electric Power Plant Emissions in a "By-production" Framework: A Nonparametric DEA Study

Steven Benjamin Levkoff

University of California, Riverside

Abstract

In a recent paper, Murty, Russell, and Levkoff [2011] introduce the concept of "by-production." The by-production approach differs significantly in two respects from previous studies, which treat unintended output either as a freely disposable input or as an weakly disposable, null-joint output. This study reconsiders a quadripartite decomposition of emissions fluctuations using a mixed-period distance function approach following in the spirit of Pasurka [2006], which assumed weak disposability and null-jointness of unintended production. Using an input-output panel data set of coal-fired electric power plants, this study implements a quadripartite decomposition utilizing two common efficiency indexes: the hyperbolic index and a modification of the Fare, Grosskopf, Lovell [1985] coordinate-wise graph space index. The indexes are computed using data envelopment (DEA) methods. Lastly, we distinguish between efficiency in intended production and environmental efficiency (efficiency in unintended production). This distinction is important in implementation of the factor decomposition since, in general, the two notions provide differing results - firms that tend to be environmentally efficient are typically inefficient in intended production and vice versa.

1 Introduction

In a recent paper, Murty, Russell, and Levkoff [2011] (MRL) introduce the concept of "by-production." Technologies that exhibit the property of by-production satisfy a “costly disposability” condition, and violate standard free disposability with respect to goods that cause pollution. Moreover, MRL shows that in general, more than one implicit production relation is needed in order to properly identify all of the technological trade-offs associated with by-production of ”bad” outputs. The necessity of more than one implicit production relation to model by-production of unintended outputs has direct implications on specifications of the DEA technology and efficiency measurement. This study models by-production technologies in an application of decomposing pollution emissions by coal-fired electric power plants. Following Pasurka [2006], this study takes a distance function approach in modeling the joint by-production of both good and bad outputs. This study decomposes changes in bad output production resulting from changes in technical efficiency, technical progress, and the input-output mix in a manner similar to growth accounting studies of total factor productivity. This study also tries to help resolve some debate on the measurement of technical efficiency in the presence of unintended outputs.

The remainder of the study is organized as follows: The rest of the first section provides background on disposability and conducts a survey of previous decomposition studies related to firm emissions. The following section explains the derivation of the joint by-production model used in decomposing emissions via Data Envelopment Analysis (DEA) under the assumption of residual by-production. Section 3 outlines the efficiency indexes used to calculate the decomposition, the decomposition components, and discusses issues related to using a mixed-period distance function. Section

4 presents the results of the index decomposition for the various technological assumptions and concludes with a discussion related to firm response to the 1990 Clean Air Act Amendment. The study concludes with Section 5.

1.1 Previous Studies

The literature on decomposing factors associated with changes in pollutant emissions has been extensive. Many other studies have carried out analyses related to carbon emissions using either index decomposition (ID) models[1] similar to the one herein or structural decomposition analysis (SDA) models[2], which utilize input-output tables[3]. While carbon emissions are of great importance in the environmental literature, this study focuses on NO_x and SO_2 emissions. NO_x and SO_2 emissions are now subject to federal cap and trade policy under the Acid Rain Ruling, which was established under Title IV of the 1990 Clean Air Act Amendment to reduce acid deposition. We discuss the implications of this policy in the last section of results. Unllike studies analyzing only CO_2 emissions, where the production frontier consists of a single combination of good and bad output production for a given technology and input-intended output combination, when NO_x and SO_2 emissions are generated, abatement activities allow for multiple combinations of good and bad outputs for a given technology and input-

[1]See Lin and Chang [1996], Selden et al. [1999], Viguier [1999], Hammer and Lofgren [2001], Bruvoll and Medin [2003], and Cherp et al. [2003].

[2]See Leontief and Ford [1972], Meyer and Stahmer [1989], Wier [1998], Wier and Hasler [1999].

[3]For a comparison of these models, see Hoekstra and ven der Bergh [2003]

intended output vector. Carbon emission reductions, on the other hand, require either substitution between types of fuels or substitution of non-fuel inputs for fuel inputs. Aiken and Pasurka [2002] specify a joint production model and attempt to quantify variations in SO_2 emissions associated with changes in technical efficiency, the output mix, and production levels in the United States manufacturing sector during the 80's and 90's. Pasurka [2003] extends this analysis by calculating the change in SO_2 emissions associated with the lack of free disposability of pollutants. Throughout the remainder of the paper, the standard technology set will be denoted by T, input vectors are denoted by $x \in R^I_+$, intended output vectors by $y \in R^J_+$, and unintended output vectors by $z \in R^K_+$.

1.2 Disposability

Past studies focused on capturing the positive relationship between intended production and residual generation of by-products typically treated pollution in one of two fashions: either as a standard input as in Baumol and Oates [1975], Cropper and Oates [1992], Pittman [1981], and Barbera and McConnell [1990], or as a weakly disposable, null- joint output as in Pittman [1983], Fare, Grosskopf, Lovell, and Pasurka[1989], Pasurka[2006], and Fare, Grosskopf, and Pasurka [1986].

A technology satisfies weak disposability of outputs if[4]

$$\langle x, y, z\rangle \in T \implies \langle x, \lambda y, \lambda z\rangle \in T \quad \forall \lambda \in [0, 1].$$

[4]This was first formalized by Shephard [1953]

This implies that while pollution is not freely disposable, it is possible to reduce, in tandem, pollution and intended outputs.

Null jointness is satisfied if

$$\langle x, y, z \rangle \in T \wedge z = 0 \implies y = 0.$$

This condition implies that any positive level of intended production always generates some residual by-product.

This study follows in the spirit of Pasurka [2006], in which NO_x and SO_2 emissions are decomposed. Pasurka [2006] operates under the assumption of weak disposability and null-jointness of unintended output production. We alter these assumptions herein.

In addition to much of the previous work done on emissions decomposition, there have been several studies within the DEA literature focused on calculating relative efficiencies in production when "bad" outputs exist. Fare, Grosskopf, Lovell, and Pasurka [1989] modify the Farell [1956] approach to handle asymmetric treatment of good and bad outputs by also treating unintended production as weakly disposable.

However, MRL shows that weak disposability of unintended outputs is inconsistent with the trade offs implied by the by-production process. It is reasonable to purport that, in the case of pollution generating firms, there are specific characteristics about stages in the production process of applying a technology to a set of inputs to produce some desired output that can set reactions in motion in nature and inevitably result in the generation of pollution as a by-product (e.g., the use of gas, fuel, or coal generates NO_x and SO_2 emissions which in turn react to the atmosphere causing acid rain). MRL define these natural reactions that occur contemporaneously with intended production as by-production. In the case of technologies exhibiting this property, it is clear that one can observe a certain minimal amount of the by-product for a particular level of input and/or output utilization. Inefficiencies in the production process can lead to excess generation past this minimal amount of by-product. This means that the technology should also satisfy what is known as a costly disposability condition[5]

$$\langle x, y, z\rangle \in T \wedge \bar{z} \geq z \implies \langle x, y, \bar{z}\rangle \in T.$$

That is, inefficiencies in unintended production lead to levels of residual generation that are larger than the lower bound for a given input-intended output combination and technology. This condition highlights the possibility of inefficiencies arising within the process of residual generation itself. As a preliminary discussion, lets consider first, the flaw associated with utilizing only one implicit production relation when the technology satisfies costly disposability. The technology set is given by[6]

$$T = \{\langle x, y, z\rangle \in R_+^{I+J+K} | f(x, y, z) \leq 0\},$$

where f is differentiable and vectors satisfying $f(x, y, z) = 0$ are points on the boundary of the technology set. Any vectors satisfying $f(x, y, z) < 0$ are inefficient production vectors. The standard differential restrictions imposed on the technology

[5] first formalized by Murty [2010]

[6] We abstract from including abatement output as it is not in our data set. For a theoretical discussion including abatement output, see Murty, Russell, and Levkoff [2011]

are

$$f_x(x, y, z) \leq 0$$

$$f_y(x, y, z) \geq 0$$

$$f_z(x, y, z) \leq 0.$$

The first two differential restrictions are the standard free disposability of inputs and intended outputs respectively

$$\langle x, y, z\rangle \in T \wedge \bar{x} \geq x \implies \langle \bar{x}, y, z\rangle \in T$$

$$\langle x, y, z\rangle \in T \wedge \bar{y} \leq y \implies \langle x, \bar{y}, z\rangle \in T$$

The third differential restriction captures the costly disposability condition.

Next, consider an efficient vector $\langle \hat{x}, \hat{y}, \hat{z}\rangle$ such that $f(\hat{x}, \hat{y}, \hat{z}) = 0$ and $f_z(\hat{x}, \hat{y}, \hat{z}) < 0$. Then by the implicit function theorem, there exist neighborhoods $U \subseteq R_+^{I+J+K-1}$ and $V \subseteq R_+$ around $\langle \hat{x}, \hat{y}, \hat{z_{-k}}\rangle \in R_+^{I+J+K-1}$ and $\hat{z_k} \in R_+$ and a function $\varphi : U \to V$ such that

$$\hat{z_k} = \varphi(\hat{x}, \hat{y}, \hat{z_{-k}})$$

and

$$f(x, y, \varphi(x, y, z_{-k}), z_{-k}) = 0.$$

where z_{-k} is the vector z with the kth element purged.

Then the ceteris paribus trade off between any input and unintended by-product z_k implied by the implicit function theorem is given by

$$\frac{\partial \varphi(x, y, z_{-k})}{\partial x_i} = -\frac{f_x(x, y, z)}{f_z(x, y, z)} \leq 0 \quad \forall i, k$$

Clearly, this non-positive trade off between inputs and residual pollution is inconsistent with the phenomena of by-production, especially the case in which the usage of input i results in residual generation. That is, if pollution is generated as a by-product of input usage, then the usage of inputs should correspond in a positive fashion to the residual generation of pollution.

Likewise, the ceteris paribus trade off between any intended output and unintended by-product z_k implied by the implicit function theorem is given by:

$$\frac{\partial \varphi(x, y, z_{-k})}{\partial y_j} = -\frac{f_y(x, y, z)}{f_z(x, y, z)} \geq 0 \quad \forall j, k$$

If the trade off between intended production and pollution is strictly positive, this implies the existence of a wide variety of technically efficient $\langle y, z \rangle$ combinations that are possible with fixed levels of all inputs. This is also incongruent with the by-production phenomena since by-production itself implies that only one technically efficient, minimal level of pollution should exist given a fixed level of inputs.

Finally, the implied trade off between one pollutant and another along the efficient frontier is given by:

$$\frac{\partial \varphi(x, y, z_{-k})}{\partial z_{-k}} = -\frac{f_z(x, y, z)}{f_z(x, y, z)} \leq 0 \quad \forall k$$

This implies that pollutants are substitutable for one another and that there exists a rich menu of $\langle z_k, z_{-k} \rangle$ for a given input and intended production vector. Again, this trade off is not consistent with the fact that the residual generation mechanism may be specific to the utilization of particular inputs (ie: coal) and one combination of inputs may not be able to produce such a rich menu of unintended by-product.

Clearly, these trade offs are not consistent with the phenomena of by-production. Moreover, it seems that the single equation implicit specification of a pollution generating technology seems to treat residual generation like any other input: increases in levels of the by-product holding all other inputs fixed increases output; secondly, pollution is a substitute for all other inputs in the production process so that decreases in non-polluting inputs can be easily substituted with pollution to generate the same level of output.

To reconcile this seeming paradox between by-production and implicit trade offs, MRL utilize multiple production relations to model the residual generation mechanism as an independent and separate process from intended production and resolve the paradoxical trade offs highlighted above. That is, MRL suggest specifying two production relations - one for intended production and another for residual generation of the by-product. The by-production technology is then the set of vectors in the intersection of these two technology sets. Under the by-production specification, the reduced form technology satisfies free disposability with respect to intended outputs and non-pollution causing inputs. It violates free disposability with respect to pollution causing inputs and satisfies the costly disposability condition with respect to residually generated, unintended outputs. A few studies have already explored some of these ideas.[7] Forsund [2009] utilizes a welfare maximization problem to show that optimal policy is ambiguous when only a single production relation is used to model

[7] Frisch [1965] provides some foundation on using multiple production relations.

a pollution generating technology.

The next section describes the technology under by-production and the implications for data envelopment. The following sections discuss the choice of distance functions to be used in the quadripartite decomposition.

2 By-production Technology

This section explains the construction of the by-production technology and the derivation of the DEA technology to be utilized in calculating relative efficiencies. In order to correct for the aforementioned flaws using the single equation, implicit production relation when costly disposability is invoked, MRL specify a by-production technology as the intersection of two technological mechanisms: one governing the intended production process and another governing the residual generation mechanism. MRL show that the reduced form by-production technology corrects the trade off distortions present when only one implicit production relation is used to model residual generation of pollutants. First, we specify a partition among the inputs in the following fashion: $x = \langle x_{-\imath}, x_\imath \rangle$ where $x_{-\imath}$ is the input vector purged of the first $I' \leq I$ inputs that are associated with the pollutants z and let $x_\imath$ denote the subset of the input vector that are associated with the residual generation of pollutants[8]. Then we

[8] In general, we need not restrict ourselves to the case where only input usage causes pollutants. For example, the production of the output of say, cheese, could in some sense cause an unintended odor from the presence of the output, not necessarily the input usage. Murty, Russell, and Levkoff

specify the technology as

$$T_{BP} = T_1 \cap T_2,$$

where

$$T_1 = \left\{ \langle x_{-\imath}, x_\imath, y, z \rangle \in \mathbf{R}_+^{I+J+K} \;\middle|\; f(x_{-\imath}, x_\imath, y) \leq 0 \right\},$$

$$T_2 = \left\{ \langle x_{-\imath}, x_\imath, y, z \rangle \in \mathbf{R}_+^{I+J+K} \;\middle|\; g(z, x_\imath) \geq 0 \right\},$$

where f and g are continuously differentiable functions.[9] T_1 is the standard technology set specifying the ways in which inputs are transformed into intended outputs. The standard free disposal properties can be imposed on this set by assuming that

$$f_x(x, y) \leq 0 \;\; \forall \;\; i = 1 \ldots I$$

$$f_y(x, y) \geq 0 \;\; \forall \;\; j = 1 \ldots J$$

T_2 is nature's residual-generation set reflecting the physical and chemical mechanism underlying the production of pollutants. T_2 satisfies costly disposability with respect to pollution as the function $g(z, x_\imath)$ defines the minimum level of bad outputs generated by a given level of input usage and satisfies

$$g_{x_\imath}(z, x_\imath) \leq 0 \;\; \forall \;\; \imath = 1 \ldots I'$$

$$g_z(z, x_\imath) \geq 0 \;\; \forall \;\; k = 1 \ldots K$$

[2011] address this issue as well and show that is trivial to adjust the model to capture this behavior

[9] We assume that T_{BP} is non-empty. In fact, as long as a production vector in T_1 is feasible given the same component of pollution generating input causes some amount of pollution through T_2, then the intersection will not be null. If the no free lunch assumption holds, then the zero vector lies in both T_1 and T_2, so that T_{BP} is non-empty.

to reflect the fact that increases in input usage associate with bad output production will increase this minimal amount. However, notice that T_2 violates standard free disposability of inputs that are associated with pollution and satisfies a completely different condition with respect to these inputs:

$$\langle x_{-\imath}, x_\imath, y, z\rangle \in T_2 \wedge \bar{z} \geq z \wedge \bar{x_\imath} \leq x_\imath \implies \langle x_{-\imath}, \bar{x_\imath}, y, \bar{z}\rangle \in T_2$$

This condition reflects that fact that inefficiencies arising in the residual generation process imply that it may be possible to either reduce input usage to generate the same level of pollutant or reduce pollution generation for a given level of input usage.

Thus, we can infer that T_{BP} satisfies free disposability with respect to all intended outputs and inputs not associated with residual generation. However, the reduced form by-production technology violates standard free disposability with respect to inputs associated with residual generation and satisfies costly disposability with respect to residual generation of pollutants.

2.1 Constructing the DEA Technologies

In this section, we describe the fundamental differences between the weak disposability/null jointness and by-production approaches with respect to the DEA technologies. We consider the case where only input usage causes pollution since this is the case relevant to our data set. Construction of the DEA technologies requires the following elements:

(i) D decision making units (DMUs), indexed by d.

(ii) J intended outputs, indexed by j, with quantity vector $y \in R_+^J$. Let Y be the

$D \times K$ matrix of intended output observations.

(iii) I inputs, indexed by i. The first I' are polluting inputs, indexed by $\imath$. The remaining $I - I'$ inputs are non pollution-generating. The quantity vector is $x = \langle x^\imath, x^{-\imath} \rangle$. The $D \times I$ matrix of input observations is then partitioned into $X = \langle X^\imath, X^{-\imath} \rangle \in R_+^I$.

(iv) K pollutants indexed by k, with quantity vector $z \in R_+^K$. Let Z be the $D \times K$

matrix of pollution observations.

The standard DEA construction of a pollution-generating technology satisfying weak disposability and null-jointness, as formulated by Fare, Grosskopf, and Pasurka [1989] is given by

$$T_{WD} = \left\{ \langle x, y, z \rangle \in R_+^{I+J+K} | \lambda X \leq x \wedge \lambda Y \geq y \wedge \lambda Z = z \;\; \text{for some} \;\; \lambda \in R_+^D \right\}$$

The by-production technology is constructed in two stages. First, T_1 is constructed as follows:

$$T_1 = \left\{ \langle x, y, z \rangle \in R_+^{I+J+K} | \lambda X \leq x \wedge \lambda Y \geq y \;\; \text{for some} \;\; \lambda \in R_+^D \right\}$$

Next, construct T_2 by:

$$T_2 = \left\{ \langle x^\imath, x^{-\imath} y, z \rangle \in R_+^{I+J+K} | \mu X^\imath \geq x^\imath \wedge \mu Z \leq z \;\; \text{for some} \;\; \mu \in R_+^D \right\}$$

The by-production technology is defined as the set of vectors lying within the intersection of T_1 and T_2 and can be written as:

$$T_{BP} = \left\{ \langle x^{\imath}, x^{-\imath} y, z \rangle \in R_+^{I+J+K} | \lambda [X^{\imath} \;\; X^{-\imath}] \leq \langle x^{\imath}, x^{-\imath} \rangle \;\wedge\; \lambda Y \geq y \;\wedge\; \mu X^{\imath} \geq x^{\imath} \;\wedge\; \mu Z \leq z \right.$$

$$\left. \text{for some} \;\; \langle \lambda, \mu \rangle \in R_+^{2D} \right\}$$

3 Efficiency Measurement and Decomposition

This study will utilize two different distance functions in implementing the quadripartite decomposition [10]. The popular hyperbolic index will first be implemented as in Pasurka [2006]. The second index used only for the by-production technology, is a modification of the Fare, Grosskopf, and Lovell [1985] coordinate-wise efficiency index as utilized by MRL to characterize efficiencies in by-production technologies. The advantage of this index is that it satisfies an indication axiom that the hyperbolic does not, and that it is easily decomposable into measures of efficiency in intended production and inefficiency in unintended residual generation of by-products.

3.1 Distance Functions: Weak Disposability vs. By-production

The most popular efficiency index used in decomposition studies is the hyperbolic efficiency index. Pasurka [2006] measures producer efficiency under weak disposability

[10] Although, within these two categories of distance functions, we will actually compute seven different indexes, five of which utilize mixed-period distance functions to run the decomposition.

and null jointness by employing the hyperbolic index:

$$D_{HYP}^{-1}(x, y, z, T_{WD}) = \max_{\beta > 0} \{\beta | \langle x, y\beta, \beta z \rangle \in T_{WD}\}$$

That is, efficiency is measured by crediting the producer for expanding both intended and unintended outputs to the boundary of the technology set. However, this implies, that a producer will be ranked more efficiently if they are able to produce more (not less) pollution for a given input and intended output combination. we argue, this is not conceptually consistent with the idea that pollution has negative societal implications[11]. Before considering by-production technologies, we correct the direction of measurement under weak disposability by employing the subsequent decomposition utilizing the hyperbolic efficiency index whereby the producer is credited for expansions of intended output and retractions of unintended output:

$$D_{HYP}^{-1}(x, y, z, T_{WD}) = \min_{\beta > 0} \{\beta | \langle x, y/\beta, \beta z \rangle \in T_{WD}\}$$

This hyperbolic efficiency index is a distance function calculated by measuring the radial expansion of intended outputs and radial contraction of unintended outputs, in a fashion we argue is similarly applicable to by-production technologies.

Under the by-production technological specification, one can think of measuring the distance from two different frontiers. Thus, two notions of efficiency arise under by-production: efficiency in intended production, as measured by the expansion of an intended output vector toward the production possibilities frontier, and environmental

[11] Although there are other reasons why this approach has been taken in the past. See Pasurka [2006]

efficiency, as measured by retracting unintended pollution to its minimally possible level given an input-intended output vector. We calculate the hyperbolic index by solving the following optimization problem:

$$
\begin{aligned}
D_{HYP}(x, y, z, T_{BP}) &= \min_{\beta>0} \{\beta |\ \langle x, y/\beta, \beta z\rangle \in T_{BP}\} \\
&= \min_{\beta>0}\{\beta|\langle x, y/\beta, \beta z\rangle \in T_1 \text{ and } \langle x, y/\beta, \beta z\rangle \in T_2\} \\
&= \max\{\beta_1, \beta_2\} \quad \text{where} \\
\beta_1 &= \min_{\beta>0}\{\beta|\langle x, y/\beta, z\rangle \in T_1\} =: D_H^1(x, y, z, T_{BP}) \\
\beta_2 &= \min_{\beta>0}\{\beta|\langle x, y, \beta z\rangle \in T_2\} =: D_H^2(x, y, z, T_{BP})
\end{aligned}
$$

The equalities after the first equation follow from the independence of T_1 and T_2. Note that β_1 measures efficiency in intended production while β_2 measures environmental efficiency. That is, if $D_{HYP} = \beta_1$, then the reference point in the by-production technology set is efficient (weakly) in intended production, but not environmentally efficient. If $D_{HYP} = \beta_2$, then the reference point in the by-production technology set is environmentally efficient (weakly), but not output efficient. Therefore, it is important for policy makers and researchers to distinguish between these two notions of efficiency as the objectives of producers and policy makers may not coincide.

The second distance function that will be used in computing efficiency is a modification of a graph space [12] index first introduced by Fare, Grosskopf, and Lovell[1985]. Define $y \oslash \beta = \langle y_1/\beta_1, \ldots, y_J/\beta_J\rangle$ and $\gamma \otimes z = \langle \gamma_1 z_1, \ldots, \gamma_K z_K\rangle$. In the case of by-

[12] Graph space refers to the full space of inputs and outputs

production and assuming independence of T_1 and T_2 the by-production index proposed by MRL decomposes as follows:

$$\begin{aligned}
D_{FGL}(x, y, z, T_{BP}) &:= \frac{1}{2} \min_{\beta,\gamma} \left\{ \frac{\sum_j \beta_j}{J} + \frac{\sum_k \gamma_k}{K} \middle| \ \langle x, y \oslash \beta, \gamma \otimes z \rangle \in T_{BP} \right\} \\
&= \frac{1}{2} \min_{\beta,\gamma} \left\{ \frac{\sum_j \beta_j}{J} + \frac{\sum_k \gamma_k}{K} \middle| \ \langle x, y \oslash \beta, \gamma \otimes z \rangle \in T_1 \ \wedge \ \langle x, y \oslash \beta, \gamma \otimes z \rangle \in T_2 \right\} \\
&= \frac{1}{2} \min_{\beta} \left\{ \frac{\sum_j \beta_j}{J} \ \middle| \ \langle x, y \oslash \beta, z \rangle \in T_1 \right\} + \frac{1}{2} \min_{\gamma} \left\{ \frac{\sum_k \gamma_k}{K} \ \middle| \ \langle x, y, \gamma \otimes z \rangle \in T_2 \right\} \\
&=: \frac{1}{2} \left[D^1_{FGL}(x, y, z, T_1) + D^2_{FGL}(x, y, z, T_2) \right]
\end{aligned}$$

This index differs significantly from the hyperbolic in that no "slack" is left in the technology[13]. In the hyperbolic index, only one constraint involving β binds with equality. However, D_{FGL} is measured through coordinate-wise expansions and contractions of intended output and pollution respectively. That is, D_{FGL} takes up all of the slack in the technology, and additionally, satisfies weak indication.[14], which the hyperbolic does not.

[13]The weight of 1/2 is used here. Any weights on (0,1) would suffice to preserve the indication property for by-production technologies. An index satisfies weak indication if it is equal to unity if and only if the vector lies on the efficient frontier of the technology set.

[14]An index satisfies indication if it equals unity if and only if the observation is on the efficient frontier of the technology set. See Levkoff, Russell, and Schworm[2010] for a discussion of this property and its relationship to the FGL index.

3.2 Quadripartite and Pentipartite Emissions Decomposition

In this section, the quadripartite decomposition of emissions is described and extended to a pentipartite decomposition by considering fuel and non-fuel input growth. Let z_k^{t+1}/z_k^t represent the gross rate of change in pollutant k between periods t and $t+1$. Then, using distance functions, we can rewrite the gross rate of emissions change as follows:

$$\frac{z_k^{t+1}}{z_k^t} = \frac{D(x^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^{t+1})(z_k^{t+1}/D(x^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^{t+1}))}{D(x^t, y^t, z^t, T_{BP}^t)(z_k^t/D(x^t, y^t, z^t, T_{BP}^t))}$$

which can be rewritten as

$$\begin{aligned}
\frac{z_k^{t+1}}{z_k^t} = & \left[\left(\frac{D(x^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^{t+1})}{D(x^t, y^t, z^t, T_{BP}^t)}\right)\right] \times \left[\left(\frac{D(x^t, y^t, z^t, T_{BP}^t)}{D(x^t, y^t, z^t, T_{BP}^{t+1})}\right)\left(\frac{D(x^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^t)}{D(x^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^{t+1})}\right)\right]^{1/2} \\
& \times \left[\left(\frac{D(x^t, y^t, z^t, T_{BP}^{t+1})}{D(x^{t+1}, y^t, z^t, T_{BP}^{t+1})}\right)\left(\frac{D(x^t, y^{t+1}, z^{t+1}, T_{BP}^t)}{D(x^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^t)}\right)\right]^{1/2} \\
& \times \left[\left(\frac{z_k^{t+1}/D(x^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^{t+1})}{z_k^t/D(x^{t+1}, y^t, z^t, T_{BP}^{t+1})}\right)\left(\frac{z_k^{t+1}/D(x^t, y^{t+1}, z^{t+1}, T_{BP}^t)}{z_k^t/D(x^t, y^t, z^t, T_{BP}^t)}\right)\right]^{1/2} \\
& = TECHEFF_t^{t+1} \times TECHCHANGE_t^{t+1} \times INPUTGROWTH_t^{t+1} \times OUTPUTMIX_{t,k}^{t+1}
\end{aligned}$$

where $TECHEFF_t^{t+1}$ is the index component associated with variations in technical efficiency (movements toward or away from the frontier), $TECHCHANGE_t^{t+1}$ is the index component associated with technical change (shifting of the frontier), $INPUTGROWTH_t^{t+1}$ is the index component associated with variations in input usage, and $OUTPUTMIX_t^{t+1}$ is the index component associated with changes in the output mixture (movements along the frontier). It is important to note that $TECHEFF_t^{t+1}$, $TECHCHANGE_t^{t+1}$, and $INPUTGROWTH_t^{t+1}$ have equal effects on all pollutants being emitted by a given producer and only the k subscript appears on $OUTPUTMIX_{t,k}^{t+1}$). Thus, the $OUTPUTMIX_{t,k}^{t+1}$ component accounts for all variation in emissions across bad outputs for a producer. If any of the components have a value larger than unity, then increased emissions of the pollutant are associated with that component. If the component's value is less than unity, this indicates decreased bad output production associated with the component. $INPUTGROWTH_t^{t+1}$ can be further decomposed to analyze factors related to growth of fuel and non-fuel inputs:

$$INPUTGROWTH_t^{t+1} = \left[\left(\frac{D(x_F^t, x_{NF}^t, y^t, z^t, T_{BP}^{t+1})}{D(x_F^t, x_{NF}^{t+1}, y^t, z^t, T_{BP}^{t+1})}\right)\left(\frac{D(x_F^t, x_{NF}^{t+1}, y^t, z^t, T_{BP}^{t+1})}{D(x_F^{t+1}, x_{NF}^{t+1}, y^t, z^t, T_{BP}^{t+1})}\right)\right]^{1/2} \times \left[\left(\frac{D(x_F^t, x_{NF}^t, y^{t+1}, z^{t+1}, T_{BP}^t)}{D(x_F^t, x_{NF}^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^t)}\right)\left(\frac{D(x_F^t, x_{NF}^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^t)}{D(x_F^{t+1}, x_{NF}^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^t)}\right)\right]^{1/2}$$

where the set of inputs $x = \langle x_F, x_{NF}\rangle$ is partitioned into the set of fuel inputs x_F and non-fuel inputs x_{NF}. We can also rewrite this expression as:

$$\begin{aligned} INPUTGROWTH_t^{t+1} &= \left[\left(\frac{D(x_F^t, x_{NF}^t, y^t, z^t, T_{BP}^{t+1})}{D(x_F^t, x_{NF}^{t+1}, y^t, z^t, T_{BP}^{t+1})}\right)\left(\frac{D(x_F^t, x_{NF}^t, y^{t+1}, z^{t+1}, T_{BP}^t)}{D(x_F^t, x_{NF}^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^t)}\right)\right]^{1/2} \\ &\times \left[\left(\frac{D(x_F^t, x_{NF}^{t+1}, y^t, z^t, T_{BP}^{t+1})}{D(x_F^{t+1}, x_{NF}^{t+1}, y^t, z^t, T_{BP}^{t+1})}\right)\left(\frac{D(x_F^t, x_{NF}^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^t)}{D(x_F^{t+1}, x_{NF}^{t+1}, y^{t+1}, z^{t+1}, T_{BP}^t)}\right)\right]^{1/2} \\ &= NONFUEGROWTH_t^{t+1} \times FUELGROWTH_t^{t+1} \end{aligned}$$

to observe how changes in fuel usage are associated with emissions reductions. Note that both of the above decompositions utilize the mixed-period distance function: using production vectors from period t and the technology form period $t+1$ or vice versa. Using mixed period distance functions presents complications with solution feasibility when calculating indexes under the assumption of weak disposability, and are addressed in the next section.

3.3 Modeling the Best Practice Frontier Using Panel Data

As was discussed in the previous section, the pentipartite decomposition employs seven different distance functions for a given technological specification, five of which involve mixed periods. As is true for Malmquist-Luenberger productivity indexes, some of the solutions to the mixed period distance functions may be infeasible.[15] To mitigate this problem, Pasurka[2006] "pools" the technology by using a three period rolling window, so that the technology in period t is derived from observations from periods t, $t-1$, and $t-2$. While this does lessen the frequency of the infeasibility problem [16], the problem still exists and is further exacerbated by the large number of different mixed period distance functions utilized for each year. If any one out of the five has an infeasible solution for any given year, then the firm must be removed from the sample as the decomposition between those two years cannot be calculated. Pasurka[2006] removed more than 20% of firms in the sample due to this problem, dramatically limiting the analytical power of the results. Another down side to using the rolling window technology is that the size of the window restricts the use of all

[15] There may exist vectors such that $\langle y_{t+1}, z_{t+1} \rangle \notin T_t(x_{t+1})$. If β is bounded above by unity, then solutions to these programs will not be feasible.

[16] Relative to using only a one period window.

years of data.[17]

There are three alternatives to remedy this problem. The first, is allowing the efficiency scores to exceed unity, and characterizing hyper-efficient[18] points in the technology set. However, interpretation of this notion is not clearly understood in applications to the decomposition literature.

Another solution, is to allow observations from the entire panel horizon to define the best practice technology. This guarantees never running into an infeasibility problem. However, it removes all dynamics from the frontier - if the frontier is generated from all observations, it will not change from period to period and will not allow us to assess differences between technical efficiency and technical progress. In fact, there is vacuously no technological progress if all observations are used in constructing the technology.

The third solution, and the one employed herein, is to use the sequential technology so that the technology in period t includes observations from the current and all previous periods. Unlike the pooled window specification, which allows for the possibility of technological regress or implosion, the sequential technology assumes no possibility of technological implosion - if a vector was feasible in the past, then it is feasible in the present. There may be some applications where allowing for implo-

[17]A rolling window using W periods of observations in constructing the technology means that the first period that can be fully analyzed in implementing the decomposition is period W and information is lost from periods before W.

[18]Not technologically feasible in a given period.

sion is warranted. However, for our purposes herein, it is not likely that technological regress has occurred in the electricity generation industry. Some may argue that government regulation may prevent past observations from being feasible. However, this argument is no longer discussing technological feasibility, but rather policy feasibility.[19] Unlike the polled rolling window production set, the other advantage of using the sequential production set is that it no longer limits the time period in which we can begin conducting the analysis. We immediately extend Pasurka[2006] to the sequential technology and show that his results are robust to the choice of technological dynamics. For the remainder of the study, the sequential frontier is employed.[20]

4 Data Analysis and Results

4.1 Description of the Data

This section describes the data utilized in the analysis of the by-production model. Observations from 92 coal-fired power plants from the years 1985 through 1995 are used to construct the efficiency indexes and to estimate the by-production technology. Each observation (electrical plant) produces one intended output, net electrical

[19] We can reconcile this argument by defining another set of policy feasible vectors and define the technology over the intersection of the production set and the feasible policy vector set, the latter of which may implode.

[20] We include the results for the three-period rolling window for each index as well in the appendix to show that the results, in general, are robust to the assumption regarding the use of the sequential frontier.

generation, measured in kWh, and two unintended outputs, sulfur dioxide (SO_2) and nitrogen oxides (NO_x), measured in short tons. The inputs used by each plant consist of the capital stock, the number of employees, and the heat content of coal, oil, and natural gas, measured in Btus.[21] In order to model homogeneous production technologies via data envelopment, coal must provide a minimum of 95 percent of the Btu of fuels consumed by each plant[22].

The number of employees is calculated as an average taken from data in the U.S. Federal Energy Regulatory Commission Form 1 survey. Additionally, the FERC 1 survey also collects information on the historical cost of plants and equipment and does not consider investment expenditures. Thus, variation in the value of plants and equipment reflect the value of additional plant and equipment less the value of depreciated plant and equipment. In constructing the capital stock in each period for each plant, this study assumes that changes in the costs of plants and equipment reflect net investment[23]. Historical costs are converted to constant dollar values via the HWI[24]. The net constant dollar capital stock is then the sum of the ratios of net investment to HWI over all previous years. Thus, in the first year of operation,

[21] This study ignores the consumption of fuel inputs other than coal, oil, and natural gas if the consumption of these fuel inputs constitutes less than .0001 percent of a plant's total fuel consumption.

[22] Otherwise, the firm is not considered to be a coal-fired electric plant. DEA assumes that technologies are homogeneous across decision making units

[23] See Yaisawarng and Klein [1994] and Carlson et al. [2000] for studies that operates under the same assumption

[24] See Whitman, Requardt, and Associates, LLP [2002]

the net investment of a power plant is equal to the aggregate value of its plant and equipment.

The U.S. Department of Energy's Form EIA-767 survey provides the information on fuel consumption and net electrical output, which is utilized to derive estimates of SO_2 and NO_x emissions[25].

4.2 Hyperbolic Index Under Weak Disposability and Null Jointness

The hyperbolic index program under weak disposability and null jointness, as implemented by Pasurka [2006], is calculated by solving the following optimization problem

[25] A common criticism of DEA in this type of environment is that it does not consider measurement error, of which there most likely is in deriving emissions estimates based on observables in the production process.

using the sequential frontier:[26]

For each plant d' and for each year τ, solve:

$$D^{d',\tau}_{HYP}(x,y,z,T_{WD}) = \max_{\lambda,\beta} \beta \qquad s.t.$$

$$\sum_{t=1}^{\tau}\sum_{d=1}^{D} \lambda_{d,t} y^{j}_{d,t} \geq \beta y^{j}_{d',t} \quad \forall j = 1 \ldots J$$

$$\sum_{t=1}^{\tau}\sum_{d=1}^{D} \lambda_{d,t} x^{i}_{d,t} \leq x^{i}_{d',t} \quad \forall i = 1 \ldots I$$

$$\sum_{t=1}^{\tau}\sum_{d=1}^{D} \lambda_{d,t} z^{k}_{d,t} = \beta z^{k}_{d',t} \quad \forall k = 1 \ldots K$$

$$\lambda \geq 0.$$

Results of computing the decomposition factors for this index are listed in the appendix in Table 6[27]. Table 6 contains geometric means of each decomposition component across all years for each firm. Table 7 shows geometric means across all firms in the sample for each factor in the decomposition by two-year pairs. Note that the relative magnitudes and impacts of the factors using the sequential frontier are the same using the pooled frontier.[28] Tables 6 and 7 confirm that technical change, on average, is associated with increased emissions. Moreover, these results are consistent with Pasurka [2006] in that changes in the output mix are most associated with

[26] Note that for each of the following programs, changing the index range on the first summation in any constraint from $t = 1$ to $t = \tau - W + 1$ where W is the rolling window size, will correspond to the same index calculated under the rolling window, pooled technology.

[27] We maintain the assumption of constant returns to scale throughout this study.

[28] In fact, this is the case with every index calculated in considering the decomposition. We report only the sequential frontier results as there is minimal infeasibility problem to consider.

emissions reductions. Changes in bad outputs per unit of intended output can be the result of a regulatory induced change, requiring the producer to mitigate pollutants relative to intended production. Again, the results suggest that changes in the output mix, or movements along the production frontier, account for greater reductions in SO_2 than for NO_x emissions.

Note that the above program[29] credits producers for expanding both intended and unintended output production as increases in efficiency. However, under by-production, being able to produce more pollutant for a given input vector is not more efficient, but less. Thus, we first correct for the direction maintaining weak disposability by considering crediting a producer for expanding intended outputs and contracting unintended outputs by solving:

For each plant d' and for each year τ, solve:

$$D_{HYP}^{d',\tau}(x, y, z, T_{WD}) = \min_{\lambda,\beta} \beta \qquad s.t.$$

$$\sum_{t=1}^{\tau} \sum_{d=1}^{D} \lambda_{d,t} y_{d,t}^{j} \geq y_{d',t}^{j} / \beta \quad \forall j = 1 \dots J$$

$$\sum_{t=1}^{\tau} \sum_{d=1}^{D} \lambda_{d,t} x_{d,t}^{i} \leq x_{d',t}^{i} \quad \forall i = 1 \dots I$$

$$\sum_{t=1}^{\tau} \sum_{d=1}^{D} \lambda_{d,t} z_{d,t}^{k} = \beta z_{d',t}^{k} \quad \forall k = 1 \dots K$$

$$\lambda \geq 0.$$

Results for this program are reported in Table 8.[30] The one clear difference once we

[29] as in Pasurka [2006]

[30] Since this problem is non-linear in the constraint set, and due to the size of the choice variable

switch to crediting producers for retracting unintended outputs, is that input growth of fuel inputs is now associated with increased emissions. Again, the results from Table 8 suggest that changes in the output mix account for greater reductions in SO_2 than for NO_x emissions, but the effect of the output mix for SO_2 is slightly lower under retractions of pollutants and the effect of the output mix for NO_x is slightly higher. Thus, by not retracting output, the weakly disposable, null-joint index decomposition tends to understate the effect of fuel growth on emissions increases over the panel horizon, understate the effects of output mix changes on reductions of NO_x, and overstate the effects of the output mix on reductions of SO_2 emissions, relative to the case where producers are credited for retracting unintended outputs.

set, we use a first order Taylor series approximation to the output constraint around $\beta = 1$ as is standard practice and utilized in Fare, Grosskopf, Lovell, and Pasurka [1989]. Blank cells indicate firms with some component of the decomposition yielding an infeasible LP problem.

4.3 Hyperbolic Index Under By-production

Under the assumption of by-production, and crediting producers for expanding outputs and retracting unintended outputs, the hyperbolic index is calculated as follows:

For each plant d' and for each year τ, solve:

$$D^{d',\tau}_{HYP}(x, y, z, T_{BP}) = \max\{\beta_1, \beta_2\} \quad \text{where}$$

$$\beta_1 = \min_{\lambda,\beta} \beta \qquad s.t.$$

$$\sum_{t=1}^{\tau} \sum_{d=1}^{D} \lambda_{d,t} y^j_{d,t} \geq y^j_{d',t}/\beta \quad \forall j = 1 \ldots J$$

$$\sum_{t=1}^{\tau} \sum_{d=1}^{D} \lambda_{d,t} x^i_{d,t} \leq x^i_{d',t} \quad \forall i = 1 \ldots I$$

$$\lambda \geq 0.$$

and

$$\beta_2 = \min_{\mu,\beta} \beta \qquad s.t.$$

$$\sum_{t=1}^{\tau} \sum_{d=1}^{D} \mu_{d,t} z_{d,t} \leq \beta z^k_{d',t} \quad \forall k = 1 \ldots K$$

$$\sum_{t=1}^{\tau} \sum_{d=1}^{D} \mu_{d,t} x_{d,t} \geq x^i_{d',t} \quad \forall i = 1 \ldots I'$$

$$\mu \geq 0.$$

If the $\max\{\beta_1, \beta_2\} = \beta_1$, then the counter factual reference point on the frontier is efficient in intended output, but inefficient in unintended output. That is, in minimizing β, the constraint on intended output binds before the constraint on unintended output. If $\max\{\beta_1, \beta_2\} = \beta_2$, then the counter factual reference point on the frontier is efficient in retracting unintended output to its minimally necessary level, but it is inefficient in expansions of intended output. It is important to note that in this study, we do not take the max when running the decomposition. This is because mixing notions of intended and unintended efficiency does not make sense when evaluating relative distance functions. However, we report the decomposition results for β_1 and β_2 separately as it is crucial to differentiate between these notions of efficiency. The results for these two hyperbolic indexes are reported in Table 9 by two year pairs, and in Tables 10 and 11 by firms averaged across years.

4.4 The By-production Coordinate-wise Index: A Modification of the FGL Index

Lastly, we calculate, for by-production technologies, a proposed modification of the Fare, Grosskopf, and Lovell[1985] graph space index that measures a coordinate-wise average of intended output expansions and unintended output retractions. This index measures joint efficiency by taking a weighted average of efficiency scores for intended and unintended production. However, this index is easily decomposable into intended output efficiency (D^1_{FGL}) and unintended output efficiency (D^2_{FGL}). The algorithm for computing this index is given by:

For each plant d' and for each year τ, solve:

$$D^{d',\tau}_{FGL}(x,y,z,T_{BP}) = \min_{\beta,\gamma,\lambda,\mu} \frac{1}{2}\left\{\frac{\sum_{j=1}^{J}\beta_j}{J} + \frac{\sum_{k=1}^{K}\gamma_k}{K}\right\} \qquad \text{s.t.}$$

$$\sum_{t=1}^{\tau}\sum_{d=1}^{D}\lambda_{d,t}y^j_{d,t} \geq y^j_{d',t}/\beta_j \quad \forall j = 1\dots J$$

$$\sum_{t=1}^{\tau}\sum_{d=1}^{D}\lambda_{d,t}x^i_{d,t} \leq x^i_{d',t} \quad \forall i = 1\dots I$$

$$\sum_{t=1}^{\tau}\sum_{d=1}^{D}\mu_{d,t}z^k_{d,t} \leq \gamma_k z^k_{d',t} \quad \forall k = 1\dots K$$

$$\sum_{t=1}^{\tau}\sum_{d=1}^{D}\mu_{d,t}x^i_{d,t} \geq x^i_{d',t} \quad \forall i = 1\dots I'$$

$$\lambda,\mu \geq 0.$$

In general, this problem is non-linear. However, since we have only one intended output in our data set, and by independence of T_1 from T_2 , D^1_{FGL} and D^2_{FGL} can be calculated separately by running the following linear programs:

$$[D^{1,d',\tau}_{FGL}(x,y,z,T_{BP})]^{-1} = \max_{\beta,\lambda}\beta \qquad \text{s.t.}$$

$$\sum_{t=1}^{\tau}\sum_{d=1}^{D}\lambda_{d,t}y^j_{d,t} \geq \beta y^j_{d',t}$$

$$\sum_{t=1}^{\tau}\sum_{d=1}^{D}\lambda_{d,t}x^i_{d,t} \leq x^i_{d',t} \quad \forall i = 1\dots 5$$

$$\lambda \geq 0$$

$$D_{FGL}^{2,d',\tau}(x,y,z,T_{BP}) = \min_{\gamma,\mu} \frac{\gamma_1+\gamma_2}{2} \qquad \text{s.t.}$$

$$\sum_{t=1}^{\tau}\sum_{d=1}^{D}\mu_{d,t}z_{d,t}^{k} \leq \gamma_k z_{d',t}^{k} \quad \forall k=1,2$$

$$\sum_{t=1}^{\tau}\sum_{d=1}^{D}\mu_{d,t}x_{d,t}^{i} \geq x_{d',t}^{i} \quad \forall i=3,4,5$$

$$\mu \geq 0$$

Then $D_{FGL} = \frac{1}{2}D_{FGL}^{1} + \frac{1}{2}D_{FGL}^{2}$. Again, we give equal weights to intended output efficiency and unintended output efficiency, but the policy maker is free to choose weights more appropriate for any particular use.[31] Results for the coordinate-wise modification of the FGL index are reported in Table 9 by two-year pairs and in Table 12 by firms.

4.5 Efficiencies of By-production Technologies

Perhaps the most striking difference between the treatment of technologies as weakly disposable and null-joint verses by-production, is the size of the efficient frontier. Since the by-production technology lies at the intersection of two manifolds implied by T_1 and T_2, the efficient frontier is smaller in dimension[32]. This results in far fewer firms operating efficiently. In fact, there is no single firm in the sample that received an efficiency score of unity for both β_1 and β_2 using the hyperbolic and no firm with an efficiency score of unity using the coordinate-wise index. Moreover, Table

[31] ie: A regular interested in environmental efficiency may choose to put greater weight on D_{FGL}^{2}

[32] see MRL for a discussion related to this issue

13 includes both rank and product moments between intended output efficiency and unintended output efficiency for both the hyperbolic and coordinate-wise indexes. The large negative correlations provides evidence consistent with the idea that firms in general, face a trade off between efficiency in intended and unintended output. That is, firms that tend to operate efficiently in terms of intended output expansions tend to operate relatively inefficiently in terms of unintended output reductions and vice versa.

4.6 Evidence of Firm Response to Title IV of the 1990 Clean Air Act Amendment

The use of fossil fuels in energy generation is one of the primary sources of NO_x and SO_2 atmospheric by-products that lead to the creation of acid rain when they react with water in the air. While Title IV of the 1990 Clean Air Act Amendment was passed in 1990, the first phases of action required by firms related to emissions reductions was not mandated until January of 1995. The Acid Rain Ruling specifically mandated emissions reductions in two phases, requiring aggregate reductions of 10 million tons per year of SO_2 and 2 million tons per year of NO_x relative to the levels prevailing in the 1980's. In the 10 years following the implementation of Phase I in 1995, annual emissions of SO_2 fell by almost 23% while annual emissions of NO_x fell by more than 33%. Currently, both NO_x and SO_2 allowances are subject to the federal cap and trade system under the 2005 Clean Air Interstate Rule (CAIR)[33]. End-of-pipe abatement efforts and fuel switching are important methods utilized by electricity generating firms to remain in compliance with the Acid Rain Ruling. Phase I required very specific reductions in SO_2 across 110 power plants in the United States listed in Table A of section 404 of U.S.7651c 1990 Clean Air Act. Fortunately, 34 of the 92 plants in our sample are present on this list and are highlighted in the rows of the appendix tables. Also at the bottom of Tables 2-5, we have considered the geometric averages across firms and decomposition components before and after the initial 1990 ruling. However, while the first phase of mandated reductions did not go into effect until 1995, the firms that would be forced to comply were made aware of the necessity when the ruling was made in 1990. Thus, the timing of the data set at hand provides us with an interesting perspective to identify whether or not firms would anticipate and adjust their emissions earlier to be ready for the 1995 implementation of Phase I. Unfortunately, Phase I also mandated specific NO_x reductions for specific firms, but there is not information available on the levels of reduction specific to a given firm as the mandated reduced levels depend on the various types of boilers used.

Observing tables 7 and 9, it is clear that regardless of how the technology is specified, the variations in the output mix play a much more significant role in explaining reductions after the 1990 Amendment was announced. For the weak disposability technology as used in Pasurka[2006], the OM factors drop from .9920 and .9973 be-

[33]despite the fact that the program was suspended and then re-implemented in 2008

tween 1985-1990 to .9452 and .9727 between 1990-1995 for SO_2 and NO_x, respectively. The decomposition under the three by-production indexes shares this trend as well. In Table 9, when we consider only efficiency in intended output, we observe decreases in the OM components from .9965 and 1.0018 between 1985-1990 to .9573 and .9851 from 1990-1995 for SO_2 and NO_x, respectively. Measuring only environmental efficiency in Table 9, we can also see the effects of the output mix for both pollutants contributes dramatically to the reduction in emissions during the 1990-1995 period from .9982 and 1.0035 in 1985-1990 to .9015 and .9277 during 1990-1995 for SO_2 and NO_x, respectively. In fact, all by-production decompositions in Table 9 share this trend, even when joint efficiency is considered. However, the decomposition factors when only considering environmental efficiency contradict one another for the by-production technologies with respect to the effects of input growth before and after the policy change. From 1985-1990, the IG component of Table 9 for enviromental efficiency switches from contributing to reductions in emissions to contributing to increases during the 1990-1995 period.

However, there is one major respect that the weakly disposable, null-joint technological specification and decomposition differs from its by-production counterparts. Observing Table 7, note that the OM effect contributes to reductions of both pollutants over both the 1985-1990 and 1990-1995 periods. However, Table 9 illustrates that the OM effect for NO_x was actually contributing to increased emissions during the 1985-1990 period, and the direction of this effect changed after the 1990 Clean

Air Act Amendment was passed.

5 Conclusion

This study not only implements a previous decomposition with a novel modeling philosophy for pollution-generating technologies, but it also contributes to the discussion of efficiency measurement when some outputs are not necessarily socially desirable. We have extended Pasurka[2006] to the sequential frontier, and show that his results are, for the most part, robust to the sequential technological specification over the entire sample duration relative to when the three period, pooled, rolling window was used. Moreover, we have extended the analysis by implementing the same decomposition under by-production, and show that efficiency measurement for by-production technologies is not straightforward, and the criteria used to measure producer efficiency has direct implications on how producers seem to respond to abatement mandates. The final section of the results provided evidence of firm responses to the 1990 Clean Air Act Amendment related to Title IV's Acid Rain Rule, despite the fact that Phase I of the program was not implemented until 1995.

References

Aiken, D. and C. Pasurka. 2002. "Sources of emission changes: a joint production perspective of existing decomposition models." Presented at the Fourteenth International Conference on Input-Output Techniques, Universite du Qubec a Montreal, Canada.

Barbera, A. and V. McConnell. 1990. "The Impact of Environmental Regulation on Industry Productivity: Direct and indirect Effects." *Journal of Environmental Economics and Management*, **18**: 50-65.

Baumol, W. and W. E. Oates. 1988. *The Theory of Environmental Policy*. Second Edition, Cambridge: Cambridge university Press.

Bruvoll, A. and H, Medin. 2003. "Factors behind the environmental Kuznets curve: a decomposition of the changes in air pollution." *Environmental and Resource Economics*, **24**: 27-48.

Carlson, C., D. Burtraw, M. Cropper, and K. Palmer. 2000. "Sulfur dioxide control by electric utilities: what are the gains from trade?" *Journal of Political Economy*, **108**: 1292-1326.

Cherp, A. I. Kopteva, and R. Mnatsakanian. 2003. "Economic transition and environmental sustainability: effects of economic restructuring on air pollution in the Russian Federation." *Journal of Environmental Management*, **68**: 141-151.

Cropper, M. and W. E. Oates. 1992. "Environmental Economics: A Survey." *Journal of Economic Literature*, **30**: 675-740.

Fare, R., S. Grosskopf, and C. A. K. Lovell. 1985. *The Measurement of Efficiency of Production*, Boston: Kluwer-Nijhoff.

Fare, R., S. Grosskopf, C. A. K. Lovell, and C. Pasurka. 1989. "Multilateral Productivity Comparisons When Some Outputs are Undesirable: A Nonparametric Approach." *The Review of Economics and Statistics*, **71**: 90-98.

Fare, R., S. Grosskopf, and C. Pasurka. 1986. "Effects of Relative Efficiency in Electric Power Generation Due to Environmental Controls." *Resources and Energy*, **8**: 167-184.

Farrell, M. J. 1956. "The Measurement of Productive Efficiency," *Journal of the Royal Statistical Society, Series A, General*, **120**: 253-282.

Forsund, Finn. 2009. “Good Modeling of Bad Outputs: Pollution and Multiple-Output Production.” *International Review of Environmental and Resource Economics*, **3**: 1-38.

Frisch, Ragnar. 1965. *Theory of Production*. Dordrecht: D. Reidel Publishing Company.

Hammer, H. and A. Lofgren. 2001. “The determinants of sulfur emissions from oil consumption in Swedish manufacturing industries.” *The Energy Journal*, **22**: 107 -126.

Hoekstra, R. and J. van der Bergh. 2003. “Comparing structural and index decomposition analysis.” *Energy Economics*, **23**: 39-64.

Leontief, W. and D. Ford. 1972. “Air pollution and the economic structure: empirical results of input-output computations.” In: Brody, A. and A.P. Carter (Eds.), *Input -Output Techniques*. North-Holland Publishing Co., Amsterdam, pp. 9-29.

Levkoff, S. B., R. R. Russell, and W. Schworm. 2010. “Boundary Problems with the Russell Graph Measure of Technical Efficiency: A Refinement," Working Paper. University of California, Riverside and University of New South Wales.

Li, S. and H. L. Chan. 1998. "Decomposing output growth in the presence of multiple outputs." Presented at the University of Georgia Productivity Workshop.

Lin, S. and T. C. Chang. 1996. "Decomposition of SO2, NOx, and CO2 emissions from energy use of major economic sectors in Taiwan." *The Energy Journal*, **17**: 1-17.

Meyer, H. and S. Stahmer. 1989. "Energy consumption and sulfur dioxide emissions and the Federal Republic of Germany in 1980 and 1986," Ninth International Conference on Input-Output Techniques, Keszthely, Hungary, Sept. 4th-Sept 9th, mimeo.

Murty, Sushama. 2010. "On the Theory of a Firm: The Case of By-Production of Emissions." The Warwick Economics Research Paper Series (TWERPS), No. 934.

Murty, S. R. R. Russell, and S. B. Levkoff. 2011. "On modeling pollution-generating technologies." Working Paper, UC Riverside and University of Exeter Business School.

Pasurka, Carl. A. 2006. "Decomposing Electric Power Plant Emissions Within a Joint Production Framework." *Energy Economics*, **28**:26-43.

Pasurka, Carl. 2003. "Changes in emissions from U.S. manufacturing: a joint production perspective, SSRN: http://papers.ssrn.com/abstract=418720.

Pittman, R. W.. 1981. "Issues in Pollution Control: Interplant Cost Differences and Economies of Scale." *Land Economics*, **57**: 1-17.

Pittman , R. W.. 1983. "Multilateral Productivity Comparisons with Undesirable outputs. *The Economic Journal*, **93**: 383-391.

Shephard, Ronald. 1953. *Cost and Production Functions.* Princeton: Princeton University Press.

Selden, T., A. Forrest, and J. Lockhart. 1999. "Analyzing the reductions in U.S. air pollution: 1970 to 1990." *Land Economics* **75**: 1-21.

Viguier, Laurent. 1999. "Emissions of SO2, NOx, and CO2 in transition economies: emission inventories and divisia index analysis." *The Energy Journal*, **20**: 59-87.

Wier, Mette. 1998. "Sources of changes in emissions from energy: a structural decomposition analysis." *Economic Systems Research*, **10**: 99-112.

Wier, M. and B. Hasler. 1999. "Accounting for nitrogen in Denmark a structural decomposition analysis." *Ecological Economics* **30**: 217-331.

Whitman, Requardt, & Associates, LLP,. 2002. "The Handy-Whitman index of public utility construction costs: trends of construction costs," Bulletin No. 156 (1912 to July 1, 2002), Baltimore, MD.

Yaisawarng, S. and J. D. Klein. 1994. "The effects of sulfur dioxide controls on productivity change in the U.S. electric power industry." *Review of Economics and Statistics* **76**: 447-460.

Plant Name	DELTA_SO2	DELTA_NOx	TE	TC	IG	IG_NF	IG_F	OM_SO2	OM_NOx
Barry	1.0336	1.0120	0.9930	1.0080	1.0130	1.0000	1.0130	1.0193	0.9980
Gorgas	1.0573	1.0163	0.9918	1.0153	1.0160	1.0002	1.0158	1.0336	0.9935
Comanche	0.9886	1.0127	1.0013	1.0007	1.0116	1.0000	1.0116	0.9753	0.9991
Brandon Shores	1.1312	1.1323	0.9999	1.0024	1.1307	0.9997	1.1310	0.9981	0.9990
Crist	0.8818	0.9539	0.9764	1.0200	0.9839	0.9998	0.9840	0.8999	0.9735
Hammond	0.8934	0.9157	0.9703	1.0317	0.9353	1.0000	0.9353	0.9542	0.9780
Harllee Branch	0.9737	0.9831	0.9905	1.0044	0.9790	0.9999	0.9791	0.9998	1.0095
Yates	0.8227	0.8911	0.9598	1.0327	0.9213	1.0000	0.9213	0.9009	0.9757
E.D. Edwards	1.0641	0.9933	0.9840	1.0153	1.0160	1.0000	1.0160	1.0483	0.9785
Coffeen	0.8525	0.9791	0.9805	1.0199	0.9792	0.9999	0.9792	0.8707	0.9999
Grand Tower	0.9711	0.9674	0.9673	1.0324	0.9664	1.0000	0.9664	1.0062	1.0024
Hustonville	0.9048	0.8770	0.9608	1.0325	0.9092	1.0000	0.9092	1.0031	0.9723
Meredosia	0.9643	0.9781	0.9674	1.0321	0.9773	1.0000	0.9773	0.9881	1.0023
Kincaid	0.7620	0.9199	0.9879	1.0102	0.9307	0.9998	0.9309	0.8204	0.9903
Powerton	0.9327	0.9858	0.9856	1.0107	0.9787	0.9999	0.9787	0.9567	1.0111
Joppa Steam	0.8710	1.0160	0.9917	1.0107	1.0197	0.9997	1.0200	0.8522	0.9940
Baldwin	0.9952	0.9872	0.9841	1.0134	0.9932	0.9998	0.9934	1.0047	0.9966
Clifty Creek	0.9012	1.0090	0.9909	1.0131	0.9917	0.9979	0.9937	0.9054	1.0136
Tanners Creek	0.9435	1.0054	0.9839	1.0180	1.0144	1.0011	1.0133	0.9287	0.9896
H.T. Pritchard	0.9462	0.9805	0.9682	1.0321	0.9992	1.0000	0.9992	0.9477	0.9820
Petersburgh	1.0298	1.0309	0.9920	1.0080	1.0459	0.9999	1.0460	0.9845	0.9856
Edwardsport	0.9525	0.9744	0.9633	1.0321	0.9745	1.0000	0.9745	0.9831	1.0057
R. Gallagher	0.9535	0.9790	0.9827	1.0136	1.0213	0.9997	1.0216	0.9373	0.9624
F.B. Culley	0.8068	0.9639	0.9949	1.0051	0.9992	0.9968	1.0024	0.8075	0.9647
Lansing	0.9854	0.9444	0.9698	1.0181	0.9760	1.0000	0.9760	1.0225	0.9800
Lawrence	0.9735	0.9916	0.9963	1.0096	1.0220	1.0000	1.0220	0.9470	0.9646
E.W. Brown	0.9278	0.9369	0.9835	1.0168	0.9718	0.9998	0.9720	0.9548	0.9641
Ghent	0.9461	1.0314	1.0011	1.0042	1.0558	1.0002	1.0556	0.8913	0.9717
Green River	0.9865	0.9527	0.9676	1.0262	0.9732	1.0000	0.9732	1.0209	0.9860
Mill Creek	1.1458	0.9992	0.9878	1.0133	1.0166	0.9999	1.0166	1.1260	0.9820
R.P. Smith	0.9285	0.8934	0.9836	1.0321	0.9162	1.0000	0.9162	0.9984	0.9605
Mount Tom	0.9680	0.9609	1.0086	1.0273	0.9565	1.0000	0.9565	0.9768	0.9697
B.C. Cobb	1.0312	1.0636	0.9814	1.0199	1.0515	1.0000	1.0515	0.9797	1.0105
Trenton Channel	0.9914	1.0258	0.9727	1.0277	1.0040	0.9990	1.0050	0.9877	1.0220
Hoot Lake	0.8964	1.0111	0.9756	1.0316	1.0025	1.0000	1.0025	0.8885	1.0021
Montrose	0.7142	0.9554	1.0177	1.0221	0.9767	1.0000	0.9767	0.7029	0.9403
Labadie	0.9141	0.9869	0.9915	1.0096	1.0057	0.9997	1.0060	0.9081	0.9803
Sioux	0.9770	0.9831	0.9877	1.0182	1.0315	1.0000	1.0315	0.9419	0.9477
Goudey	0.9483	0.9251	0.9993	1.0146	0.9429	0.9990	0.9438	0.9919	0.9677
Greenidge	0.9310	0.9216	0.9872	1.0252	0.9396	0.9963	0.9431	0.9791	0.9692
Milliken	0.9047	1.0045	0.9865	1.0150	1.0123	1.0001	1.0122	0.8926	0.9910
C.R. Huntley	0.9758	0.9773	0.9966	1.0085	0.9755	0.9848	0.9905	0.9952	0.9968
Dunkirk	0.9960	0.9628	0.9772	1.0369	0.9919	0.9969	0.9950	0.9910	0.9579
Rochester	0.9838	0.9843	0.9647	1.0321	0.9870	1.0000	0.9870	1.0011	1.0016
Asheville	1.0368	1.0232	0.9938	1.0084	1.0187	1.0000	1.0187	1.0156	1.0023
G.G. Allen	1.0220	1.0061	0.9849	1.0115	1.0432	0.9993	1.0440	0.9833	0.9681
Cliffside	1.0124	0.9817	0.9843	1.0188	1.0108	0.9988	1.0119	0.9988	0.9685
Marshall	1.0347	0.9959	1.0000	1.0015	1.0304	0.9997	1.0307	1.0026	0.9650
R.M. Heskett	0.8701	0.8981	0.9864	1.0068	0.9400	1.0000	0.9400	0.9320	0.9621
J.M. Stuart	0.9487	0.9859	0.9895	1.0098	0.9973	1.0001	0.9972	0.9520	0.9894
R.E. Burger	0.9375	0.9488	0.9941	1.0048	0.9466	0.9987	0.9478	0.9916	1.0035

Table 6

Plant Name	DELTA_SO2	DELTA_NOx	TE	TC	IG	IG_NF	IG_F	OM_SO2	OM_NOx
Muskingum River	0.9255	0.9574	0.9744	1.0226	0.9770	1.0000	0.9771	0.9507	0.9835
Kyger Creek	0.9128	0.9920	0.9974	1.0020	1.0004	1.0000	1.0004	0.9130	0.9923
Elrama	1.0683	1.0068	0.9798	1.0243	1.0498	0.9999	1.0500	1.0139	0.9556
Seward	0.9927	0.9731	0.9821	1.0192	0.9950	0.9991	0.9959	0.9968	0.9770
Shawville	0.9879	0.9617	0.9926	1.0079	0.9956	0.9990	0.9966	0.9918	0.9655
New Castle	0.9815	0.9750	0.9871	1.0238	0.9732	0.9973	0.9758	0.9979	0.9912
Brunner Island	0.9681	0.9463	0.9974	1.0062	0.9825	0.9988	0.9837	0.9818	0.9597
Montour	1.0231	0.9596	0.9961	1.0036	0.9917	0.9991	0.9926	1.0320	0.9680
Armstrong	0.9537	0.9241	0.9996	1.0248	0.9378	1.0000	0.9378	0.9926	0.9619
Watertree	1.0237	1.0250	0.9979	1.0046	1.0254	1.0001	1.0253	0.9959	0.9971
Big Brown	1.0315	0.9973	0.9845	1.0160	0.9907	1.0066	0.9842	1.0409	1.0064
Carbon	1.0758	1.1008	1.0003	1.0040	1.1020	1.0000	1.1021	0.9721	0.9946
Clinch River	1.0160	1.0150	0.9939	1.0077	1.0107	0.9999	1.0107	1.0037	1.0027
Glen Lyn	1.0116	0.9776	0.9687	1.0272	1.0181	0.9997	1.0184	0.9985	0.9650
Potamac River	0.9721	0.9792	0.9715	1.0290	0.9812	1.0001	0.9811	0.9910	0.9983
Bremo	1.0388	1.0260	0.9785	1.0274	1.0174	1.0000	1.0174	1.0156	1.0030
Kanawha River	0.9796	0.9816	0.9696	1.0263	0.9914	1.0000	0.9914	0.9930	0.9950
Rivesville	0.9178	0.9195	0.9634	1.0322	0.9218	1.0000	0.9218	1.0013	1.0031
Willow Island	1.0007	0.9913	1.0187	1.0127	0.9641	1.0000	0.9641	1.0061	0.9967
Kammer	0.9774	0.9978	0.9959	1.0020	1.0019	1.0000	1.0019	0.9775	0.9980
Mitchell	0.9507	0.9608	0.9941	1.0078	0.9979	0.9999	0.9980	0.9508	0.9610
Nelson Dewey	0.8331	0.9272	0.9884	1.0111	1.0021	0.9990	1.0030	0.8320	0.9259
Pulliam	0.8519	1.0576	1.0041	1.0032	1.0472	1.0000	1.0472	0.8077	1.0027
Dave Johnston	0.9793	1.0291	0.9960	1.0077	1.0164	1.0000	1.0164	0.9600	1.0088
Naughton	1.0052	1.0189	0.9954	1.0057	1.0167	1.0000	1.0166	0.9877	1.0011
J.H. Miller Jr.	1.1362	1.1543	0.9986	1.0066	1.1368	1.0000	1.1367	0.9944	1.0102
Pleasants	1.2178	0.9974	1.0106	1.0086	0.9937	0.9999	0.9938	1.2024	0.9847
Duck Creek	0.9956	1.0076	0.9888	1.0142	1.0081	1.0000	1.0081	0.9848	0.9967
Newton	1.0660	1.0316	0.9904	1.0130	1.0274	0.9998	1.0276	1.0342	1.0008
Sooner	1.0493	1.0218	0.9986	1.0046	1.0407	0.9980	1.0428	1.0050	0.9787
Welsh	0.9780	0.9593	0.9912	1.0055	0.9980	0.9994	0.9985	0.9832	0.9644
Martin Lake	1.0840	1.0064	0.9996	1.0064	1.0048	1.0001	1.0047	1.0723	0.9956
Monticello	0.9540	0.9758	0.9776	1.0289	0.9631	1.0013	0.9619	0.9848	1.0073
Rush Island	0.9425	0.9830	0.9880	1.0047	1.0107	1.0000	1.0108	0.9394	0.9798
Coleto Creek	1.0170	0.9797	0.9950	1.0133	0.9916	0.9854	1.0063	1.0174	0.9800
Harrington	1.0193	0.9881	1.0000	1.0504	0.9608	0.9879	0.9725	1.0099	0.9790
Pawnee	1.0399	0.9917	0.9946	1.0081	1.0120	1.0000	1.0120	1.0248	0.9773
Mountaineer	0.9787	0.9748	0.9868	1.0125	0.9744	0.9999	0.9745	1.0053	1.0012
Belews Creek	0.9785	1.0058	0.9992	1.0287	0.9777	0.9918	0.9858	0.9736	1.0007
Gen. J.M. Gavin	0.8000	1.0107	0.9937	1.0114	1.0094	1.0002	1.0092	0.7885	0.9962
Cheswick	1.0046	0.9671	1.0030	1.0073	0.9964	1.0000	0.9964	0.9979	0.9606
AVG	**0.9680**	**0.9846**	**0.9878**	**1.0159**	**0.9962**	**0.9992**	**0.9969**	**0.9683**	**0.9849**

Table 6 (cont)

WD/NJ	HYP								
Year	**DELTA_SO2**	**DELTA_NOx**	**TE**	**TC**	**IG**	**IG_NF**	**IG_F**	**OM_SO2**	**OM_NOx**
1985-1986	0.9538	0.9587	0.9792	1.0196	0.9570	0.9991	0.9579	0.9983	1.0034
1986-1987	0.9878	0.9908	1.0025	1.0018	0.9971	0.9999	0.9972	0.9865	0.9894
1987-1988	1.0284	1.0586	1.0041	1.0018	1.0548	0.9996	1.0553	0.9692	0.9977
1988-1989	1.0573	1.0493	1.0010	1.0014	1.0384	1.0003	1.0380	1.0159	1.0082
1989-1990	0.9592	0.9564	0.9964	1.0063	0.9654	1.0001	0.9653	0.9909	0.9880
1990-1991	0.9331	0.9444	0.9965	1.0020	0.9516	1.0001	0.9515	0.9821	0.9939
1991-1992	0.9924	0.9741	1.0049	1.0001	0.9960	0.9998	0.9962	0.9914	0.9731
1992-1993	1.0141	1.0294	1.0002	1.0011	1.0458	0.9999	1.0459	0.9684	0.9830
1993-1994	0.9663	0.9185	0.9988	1.0003	0.9795	0.9979	0.9816	0.9874	0.9385
1994-1995	0.8099	0.9752	0.8990	1.1315	0.9824	0.9955	0.9868	0.8104	0.9759
AVG(pre-1990)	**0.9965**	**1.0018**	**0.9966**	**1.0062**	**1.0018**	**0.9998**	**1.0020**	**0.9920**	**0.9973**
AVG(post-1990)	**0.9403**	**0.9676**	**0.9790**	**1.0257**	**0.9906**	**0.9986**	**0.9919**	**0.9452**	**0.9727**
AVG	**0.9680**	**0.9846**	**0.9878**	**1.0159**	**0.9962**	**0.9992**	**0.9969**	**0.9683**	**0.9849**

Table 7

Plant Name	DELTA_SO2	DELTA_NOx	TE	TC	IG	IG_NF	IG_F	OM_SO2	OM_NOx
Barry	1.0336	1.0120	0.9911	1.0097	1.0064	0.9998	1.0066	1.0263	1.0048
Gorgas	1.0573	1.0163	0.9986	1.0111	1.0077	1.0001	1.0076	1.0393	0.9990
Comanche	0.9886	1.0127	1.0078	0.9382	1.0668	1.0048	1.0617	0.9801	1.0041
Brandon Shores	1.1312	1.1323	1.0003	1.0084	1.0465	0.9980	1.0485	1.0716	1.0726
Crist	0.8818	0.9539	0.9789	1.0151	0.9574	1.0001	0.9573	0.9268	1.0026
Hammond	0.8934	0.9157	0.9813	1.0204	0.9569	1.0003	0.9566	0.9324	0.9557
Harllee Branch	0.9737	0.9831	0.9898	1.0059	0.9899	1.0002	0.9897	0.9879	0.9975
Yates	0.8227	0.8911	0.9866	1.0113	0.9659	1.0001	0.9658	0.8537	0.9246
E.D. Edwards	1.0641	0.9933	0.9825	1.0158	1.0190	1.0000	1.0190	1.0464	0.9767
Coffeen	0.8525	0.9791	0.9589	1.0354	0.9750	1.0002	0.9748	0.8807	1.0115
Grand Tower	0.9711	0.9674	0.9747	1.0247	1.0124	1.0000	1.0124	0.9604	0.9567
Hustonville	0.9048	0.8770	0.9825	1.0182	0.9722	1.0000	0.9722	0.9302	0.9016
Meredosia	0.9643	0.9781	0.9813	1.0137	1.0000	1.0000	1.0000	0.9693	0.9832
Kincaid	0.7620	0.9199	1.0000	0.9028	1.0583	1.0143	1.0434	0.7976	0.9628
Powerton	0.9327	0.9858	0.9889	0.9768	1.0098	1.0014	1.0085	0.9562	1.0106
Joppa Steam	0.8710	1.0160	0.9972	1.0113	1.0100	0.9999	1.0101	0.8551	0.9974
Baldwin	0.9952	0.9872	1.0000	0.9952	1.0073	1.0037	1.0037	0.9927	0.9847
Clifty Creek	0.9012	1.0090	1.0000	0.9721	1.0135	1.0010	1.0125	0.9148	1.0241
Tanners Creek	0.9435	1.0054	0.9736	1.0260	1.0111	1.0023	1.0088	0.9342	0.9955
H.T. Pritchard	0.9462	0.9805	0.9847	1.0131	0.9979	1.0000	0.9979	0.9505	0.9849
Petersburgh	1.0298	1.0309	0.9876	1.0201	1.0065	1.0001	1.0065	1.0155	1.0166
Edwardsport	0.9525	0.9744	0.9652	1.0274	1.0168	1.0000	1.0168	0.9445	0.9662
R. Gallagher	0.9535	0.9790	0.9823	1.0120	1.0379	1.0000	1.0379	0.9241	0.9489
F.B. Culley	0.8068	0.9639	0.9976	0.9827	1.0393	1.0110	1.0280	0.7918	0.9460
Lansing	0.9854	0.9444	0.9534	1.0131	1.0157	0.9860	1.0301	1.0043	0.9626
Lawrence									
E.W. Brown	0.9278	0.9369	0.9886	1.0191	0.9929	1.0015	0.9914	0.9275	0.9366
Ghent	0.9461	1.0314	1.0076	1.0172	1.0145	1.0011	1.0133	0.9099	0.9919
Green River	0.9865	0.9527	0.9854	1.0173	0.9700	1.0000	0.9700	1.0145	0.9798
Mill Creek	1.1458	0.9992	0.9973	1.0247	1.0129	0.9943	1.0187	1.1070	0.9654
R.P. Smith	0.9285	0.8934	1.0075	1.0095	0.9676	1.0000	0.9676	0.9435	0.9078
Mount Tom									
B.C. Cobb	1.0312	1.0636	0.9812	1.0157	1.0264	0.9998	1.0266	1.0081	1.0398
Trenton Channel	0.9914	1.0258	0.9769	1.0141	1.0078	1.0000	1.0078	0.9930	1.0275
Hoot Lake	0.8964	1.0111	0.9785	1.0433	0.9287	1.0000	0.9287	0.9454	1.0664
Montrose									
Labadie	0.9141	0.9869	0.9818	1.0166	1.0004	0.9981	1.0023	0.9156	0.9884
Sioux	0.9770	0.9831	0.9679	1.0311	0.9667	1.0001	0.9666	1.0127	1.0190
Goudey	0.9483	0.9251	1.0085	1.0119	0.9673	1.0000	0.9673	0.9606	0.9371
Greenidge	0.9310	0.9216	1.0022	1.0127	0.9651	0.9999	0.9652	0.9505	0.9409
Milliken	0.9047	1.0045	0.9901	1.0174	0.9975	0.9998	0.9978	0.9004	0.9997
C.R. Huntley									
Dunkirk									
Rochester	0.9838	0.9843	0.9794	1.0151	0.9928	1.0000	0.9928	0.9967	0.9972
Asheville	1.0368	1.0232	0.9931	1.0092	1.0140	1.0004	1.0136	1.0202	1.0068
G.G. Allen	1.0220	1.0061	1.0018	1.0111	1.0255	0.9998	1.0257	0.9838	0.9685
Cliffside	1.0124	0.9817	1.0000	1.0116	1.0010	1.0000	1.0010	0.9997	0.9694
Marshall	1.0347	0.9959	1.0000	1.0260	0.9959	0.9963	0.9997	1.0125	0.9746
R.M. Heskett	0.8701	0.8981	0.9999	1.0360	0.9792	0.9978	0.9814	0.8578	0.8854
J.M. Stuart	0.9487	0.9859	0.9844	1.0154	0.9961	1.0004	0.9956	0.9529	0.9903
R.E. Burger	0.9375	0.9488	0.9833	1.0076	0.9935	0.9995	0.9940	0.9525	0.9640

Table 8 (*blank cells indicate infeasible programming problems)

Plant Name	DELTA_SO2	DELTA_NOx	TE	TC	IG	IG_NF	IG_F	OM_SO2	OM_NOx
Muskingum River	0.9255	0.9574	0.9702	1.0051	0.9923	1.0010	0.9913	0.9564	0.9894
Kyger Creek	0.9128	0.9920	0.9963	0.9926	1.0062	1.0005	1.0057	0.9173	0.9969
Elrama									
Seward	0.9927	0.9731	0.9931	1.0108	1.0003	1.0000	1.0003	0.9886	0.9691
Shawville	0.9879	0.9617	0.9964	1.0050	0.9914	1.0000	0.9914	0.9951	0.9687
New Castle	0.9815	0.9750	0.9851	1.0262	0.9738	1.0000	0.9738	0.9970	0.9903
Brunner Island	0.9681	0.9463	0.9988	1.0205	0.9857	1.0005	0.9852	0.9636	0.9419
Montour	1.0231	0.9596	1.0000	1.0302	0.9869	0.9989	0.9880	1.0064	0.9439
Armstrong	0.9537	0.9241	1.0084	1.0256	0.9476	1.0002	0.9475	0.9730	0.9429
Watertree	1.0237	1.0250	0.9982	1.0044	1.0131	0.9998	1.0133	1.0079	1.0091
Big Brown	1.0315	0.9973	0.9941	1.0467	0.9650	0.9779	0.9868	1.0274	0.9933
Carbon	1.0758	1.1008	1.0100	1.0080	1.0075	1.0000	1.0075	1.0489	1.0733
Clinch River	1.0160	1.0150	0.9916	1.0084	1.0196	0.9998	1.0197	0.9966	0.9956
Glen Lyn	1.0116	0.9776	0.9872	1.0089	1.0194	1.0000	1.0194	0.9963	0.9629
Potamac River	0.9721	0.9792	0.9898	1.0068	0.9965	1.0002	0.9964	0.9788	0.9860
Bremo	1.0388	1.0260	0.9679	1.0280	1.0455	1.0004	1.0450	0.9987	0.9863
Kanawha River	0.9796	0.9816	0.9635	1.0292	1.0018	1.0000	1.0018	0.9861	0.9881
Rivesville	0.9178	0.9195	0.9806	1.0146	1.0148	1.0000	1.0148	0.9091	0.9108
Willow Island	1.0007	0.9913	1.0175	1.0322	0.9974	1.0000	0.9974	0.9553	0.9463
Kammer	0.9774	0.9978	0.9929	1.0101	1.0103	1.0072	1.0031	0.9646	0.9848
Mitchell	0.9507	0.9608	0.9917	1.0102	0.9862	0.9999	0.9863	0.9623	0.9726
Nelson Dewey									
Pulliam									
Dave Johnston	0.9793	1.0291	1.0239	0.9485	1.0261	0.9998	1.0263	0.9827	1.0328
Naughton	1.0052	1.0189	1.0020	1.0043	1.0035	1.0002	1.0033	0.9955	1.0090
J.H. Miller Jr.	1.1362	1.1543	0.9988	1.0104	1.0528	0.9997	1.0531	1.0694	1.0864
Pleasants	1.2178	0.9974	1.0086	0.7392	1.2904	1.0009	1.2892	1.2658	1.0367
Duck Creek	0.9956	1.0076	0.9867	1.0094	1.0086	1.0000	1.0086	0.9911	1.0031
Newton	1.0660	1.0316	0.9939	1.0085	1.0065	1.0000	1.0065	1.0566	1.0225
Sooner	1.0493	1.0218	1.0000	1.0455	0.9759	1.0006	0.9753	1.0284	1.0015
Welsh	0.9780	0.9593	0.9991	1.0121	0.9948	0.9999	0.9949	0.9723	0.9538
Martin Lake	1.0840	1.0064	0.9909	1.0130	0.9950	0.9999	0.9951	1.0853	1.0076
Monticello	0.9540	0.9758	0.9816	1.0139	0.9902	1.0010	0.9891	0.9681	0.9902
Rush Island	0.9425	0.9830	0.9931	1.0076	1.0072	1.0000	1.0072	0.9351	0.9754
Coleto Creek	1.0170	0.9797	1.0000	1.0254	0.9738	0.9858	0.9878	1.0185	0.9811
Harrington									
Pawnee	1.0399	0.9917	0.9799	0.9284	1.1108	1.0086	1.1014	1.0290	0.9813
Mountaineer	0.9787	0.9748	0.9848	1.0132	0.9816	1.0006	0.9810	0.9992	0.9952
Belews Creek	0.9785	1.0058	1.0000	0.9900	1.0236	1.0055	1.0180	0.9656	0.9925
Gen. J.M. Gavin	0.8000	1.0107	1.0000	0.9909	1.0203	1.0044	1.0159	0.7913	0.9997
Cheswick									
AVG	**0.9721**	**0.9853**	**0.9906**	**1.0059**	**1.0046**	**1.0001**	**1.0045**	**0.9712**	**0.9843**

Table 8 (cont; *blank cells indicate infeasible programming problems)

BP TECH	HYP	OUTPUT EFF							
Year	**DELTA_SO2**	**DELTA_NOx**	**TE**	**TC**	**IG**	**IG_NF**	**IG_F**	**OM_SO2**	**OM_NOx**
1985-1986	0.9538	0.9587	0.9792	1.0196	0.9435	1.0048	0.9391	1.0125	1.0177
1986-1987	0.9878	0.9908	1.0025	1.0018	0.9955	1.0001	0.9954	0.9881	0.9910
1987-1988	1.0284	1.0586	1.0041	1.0018	1.0537	1.0003	1.0534	0.9703	0.9988
1988-1989	1.0573	1.0493	1.0010	1.0014	1.0371	1.0004	1.0367	1.0171	1.0094
1989-1990	0.9592	0.9564	0.9964	1.0063	0.9610	1.0018	0.9592	0.9954	0.9925
1990-1991	0.9331	0.9444	0.9965	1.0020	0.9497	1.0001	0.9496	0.9840	0.9959
1991-1992	0.9924	0.9741	1.0049	1.0001	0.9959	0.9998	0.9961	0.9915	0.9731
1992-1993	1.0141	1.0294	1.0002	1.0011	1.0447	1.0000	1.0448	0.9694	0.9840
1993-1994	0.9663	0.9185	0.9988	1.0003	0.9792	0.9978	0.9814	0.9877	0.9388
1994-1995	0.8099	0.9752	0.8990	1.1315	0.9252	1.0585	0.8741	0.8605	1.0362
AVG(pre-1990)	**0.9965**	**1.0018**	**0.9966**	**1.0062**	**0.9973**	**1.0015**	**0.9958**	**0.9965**	**1.0018**
AVG(post-1990)	**0.9403**	**0.9676**	**0.9790**	**1.0257**	**0.9781**	**1.0110**	**0.9675**	**0.9573**	**0.9851**
AVG	**0.9680**	**0.9846**	**0.9878**	**1.0159**	**0.9876**	**1.0062**	**0.9816**	**0.9767**	**0.9934**
BP TECH	**HYP**	**ENV EFF**							
Year	**DELTA_SO2**	**DELTA_NOx**	**TE**	**TC**	**IG**	**IG_NF**	**IG_F**	**OM_SO2**	**OM_NOx**
1985-1986	0.9538	0.9587	0.9758	1.0267	1.0391	1.0128	1.0260	0.9163	0.9210
1986-1987	0.9878	0.9908	1.0038	1.0090	0.9966	1.0026	0.9940	0.9786	0.9815
1987-1988	1.0284	1.0586	0.8770	1.1460	0.9418	1.0695	0.8806	1.0864	1.1184
1988-1989	1.0573	1.0493	0.9701	1.0075	0.9753	1.0027	0.9727	1.1092	1.1008
1989-1990	0.9592	0.9564	0.9915	1.0288	1.0254	1.0143	1.0110	0.9171	0.9144
1990-1991	0.9331	0.9444	1.0379	1.0004	1.0264	0.9998	1.0265	0.8756	0.8861
1991-1992	0.9924	0.9741	0.9756	1.0057	1.0358	1.0025	1.0332	0.9765	0.9584
1992-1993	1.0141	1.0294	1.0177	1.0148	0.9456	1.0075	0.9386	1.0384	1.0540
1993-1994	0.9663	0.9185	1.0175	1.0629	0.9972	1.0285	0.9696	0.8959	0.8516
1994-1995	0.8099	0.9752	1.0180	1.0333	1.0284	1.0164	1.0117	0.7487	0.9016
AVG(pre-1990)	**0.9965**	**1.0018**	**0.9626**	**1.0423**	**0.9950**	**1.0201**	**0.9754**	**0.9982**	**1.0035**
AVG(post-1990)	**0.9403**	**0.9676**	**1.0132**	**1.0232**	**1.0061**	**1.0109**	**0.9953**	**0.9015**	**0.9277**
AVG	**0.9680**	**0.9846**	**0.9875**	**1.0327**	**1.0005**	**1.0155**	**0.9853**	**0.9486**	**0.9649**
BP TECH	**CW**	**JOINT EFF**							
Year	**DELTA_SO2**	**DELTA_NOx**	**TE**	**TC**	**IG**	**IG_NF**	**IG_F**	**OM_SO2**	**OM_NOx**
1985-1986	0.9538	0.9587	0.9613	1.0416	0.9765	1.0169	0.9602	0.9754	0.9805
1986-1987	0.9878	0.9908	1.0064	1.0012	0.9942	0.9995	0.9947	0.9861	0.9890
1987-1988	1.0284	1.0586	1.0021	1.0046	1.0114	1.0016	1.0098	1.0101	1.0398
1988-1989	1.0573	1.0493	0.9903	1.0006	1.0142	0.9999	1.0142	1.0521	1.0442
1989-1990	0.9592	0.9564	0.9976	1.0103	0.9825	1.0042	0.9784	0.9687	0.9658
1990-1991	0.9331	0.9444	1.0121	1.0012	0.9745	0.9999	0.9746	0.9450	0.9564
1991-1992	0.9924	0.9741	0.9956	1.0011	1.0127	1.0002	1.0124	0.9833	0.9651
1992-1993	1.0141	1.0294	1.0047	1.0076	0.9989	1.0033	0.9956	1.0029	1.0180
1993-1994	0.9663	0.9185	1.0149	1.0127	0.9800	1.0039	0.9762	0.9595	0.9120
1994-1995	0.8099	0.9752	0.9485	1.0876	0.9676	1.0388	0.9315	0.8113	0.9770
AVG(pre-1990)	**0.9965**	**1.0018**	**0.9914**	**1.0115**	**0.9956**	**1.0044**	**0.9913**	**0.9980**	**1.0033**
AVG(post-1990)	**0.9403**	**0.9676**	**0.9948**	**1.0215**	**0.9866**	**1.0091**	**0.9777**	**0.9378**	**0.9651**
AVG	**0.9680**	**0.9846**	**0.9931**	**1.0165**	**0.9911**	**1.0068**	**0.9844**	**0.9675**	**0.9840**

Table 9

Plant Name	DELTA_SO2	DELTA_NOx	TE	TC	IG	IG_NF	IG_F	OM_SO2	OM_NOx
Barry	1.0336	1.0120	0.9930	1.0080	1.0081	1.0028	1.0053	1.0243	1.0029
Gorgas	1.0573	1.0163	0.9918	1.0153	1.0096	1.0092	1.0004	1.0400	0.9997
Comanche	0.9886	1.0127	1.0013	1.0007	1.0113	1.0004	1.0109	0.9755	0.9994
Brandon Shores	1.1312	1.1323	0.9999	1.0024	1.1295	1.0008	1.1286	0.9991	1.0001
Crist	0.8818	0.9539	0.9764	1.0200	0.9738	1.0093	0.9649	0.9092	0.9835
Hammond	0.8934	0.9157	0.9703	1.0317	0.9212	1.0162	0.9065	0.9688	0.9930
Harllee Branch	0.9737	0.9831	0.9905	1.0044	0.9770	1.0015	0.9755	1.0018	1.0115
Yates	0.8227	0.8911	0.9598	1.0327	0.9066	1.0162	0.8921	0.9155	0.9916
E.D. Edwards	1.0641	0.9933	0.9840	1.0153	1.0082	1.0075	1.0007	1.0564	0.9861
Coffeen	0.8525	0.9791	0.9805	1.0199	0.9676	1.0077	0.9602	0.8811	1.0119
Grand Tower	0.9711	0.9674	0.9673	1.0324	0.9510	1.0159	0.9361	1.0225	1.0187
Hustonville	0.9048	0.8770	0.9608	1.0325	0.8946	1.0160	0.8806	1.0194	0.9881
Meredosia	0.9643	0.9781	0.9674	1.0321	0.9620	1.0159	0.9470	1.0038	1.0183
Kincaid	0.7620	0.9199	0.9879	1.0102	0.9263	1.0051	0.9216	0.8243	0.9951
Powerton	0.9327	0.9858	0.9856	1.0107	0.9735	1.0053	0.9684	0.9617	1.0165
Joppa Steam	0.8710	1.0160	0.9917	1.0107	1.0157	1.0055	1.0101	0.8556	0.9979
Baldwin	0.9952	0.9872	0.9841	1.0134	0.9880	1.0074	0.9808	1.0100	1.0019
Clifty Creek	0.9012	1.0090	0.9909	1.0131	0.9799	0.9993	0.9806	0.9162	1.0258
Tanners Creek	0.9435	1.0054	0.9839	1.0180	1.0044	1.0103	0.9942	0.9379	0.9994
H.T. Pritchard	0.9462	0.9805	0.9682	1.0321	0.9835	1.0159	0.9681	0.9628	0.9977
Petersburgh	1.0298	1.0309	0.9920	1.0080	1.0419	1.0041	1.0377	0.9883	0.9893
Edwardsport	0.9525	0.9744	0.9633	1.0321	0.9592	1.0159	0.9442	0.9987	1.0217
R. Gallagher	0.9535	0.9790	0.9827	1.0136	1.0140	1.0058	1.0082	0.9440	0.9693
F.B. Culley	0.8068	0.9639	0.9949	1.0051	0.9976	1.0023	0.9953	0.8088	0.9663
Lansing	0.9854	0.9444	0.9698	1.0181	0.9670	1.0087	0.9586	1.0321	0.9891
Lawrence	0.9735	0.9916	0.9963	1.0096	1.0171	1.0048	1.0123	0.9515	0.9692
E.W. Brown	0.9278	0.9369	0.9835	1.0168	0.9620	1.0061	0.9562	0.9645	0.9740
Ghent	0.9461	1.0314	1.0011	1.0042	1.0537	1.0027	1.0509	0.8931	0.9736
Green River	0.9865	0.9527	0.9676	1.0262	0.9578	1.0100	0.9483	1.0372	1.0018
Mill Creek	1.1458	0.9992	0.9878	1.0133	1.0099	1.0066	1.0033	1.1335	0.9885
R.P. Smith	0.9285	0.8934	0.9836	1.0321	0.9018	1.0159	0.8877	1.0143	0.9758
Mount Tom	0.9680	0.9609	1.0086	1.0273	0.9315	1.0002	0.9313	1.0030	0.9957
B.C. Cobb	1.0312	1.0636	0.9814	1.0199	1.0388	1.0076	1.0310	0.9917	1.0229
Trenton Channel	0.9914	1.0258	0.9727	1.0277	0.9917	1.0131	0.9788	1.0000	1.0347
Hoot Lake	0.8964	1.0111	0.9756	1.0316	0.9868	1.0153	0.9720	0.9026	1.0181
Montrose	0.7142	0.9554	1.0177	1.0221	0.9614	1.0061	0.9556	0.7141	0.9553
Labadie	0.9141	0.9869	0.9915	1.0096	1.0014	1.0048	0.9967	0.9120	0.9845
Sioux	0.9770	0.9831	0.9877	1.0182	1.0254	1.0119	1.0133	0.9475	0.9534
Goudey	0.9483	0.9251	0.9993	1.0146	0.9384	1.0078	0.9311	0.9967	0.9724
Greenidge	0.9310	0.9216	0.9872	1.0252	0.9295	1.0067	0.9233	0.9896	0.9797
Milliken	0.9047	1.0045	0.9865	1.0150	1.0072	1.0095	0.9977	0.8972	0.9961
C.R. Huntley	0.9758	0.9773	0.9966	1.0085	0.9704	0.9881	0.9821	1.0004	1.0021
Dunkirk	0.9960	0.9628	0.9772	1.0369	0.9713	1.0108	0.9609	1.0120	0.9782
Rochester	0.9838	0.9843	0.9647	1.0321	0.9715	1.0159	0.9563	1.0170	1.0176
Asheville	1.0368	1.0232	0.9938	1.0084	1.0153	1.0050	1.0102	1.0190	1.0056
G.G. Allen	1.0220	1.0061	0.9849	1.0115	1.0375	1.0045	1.0329	0.9887	0.9734
Cliffside	1.0124	0.9817	0.9843	1.0188	1.0023	1.0080	0.9944	1.0072	0.9767
Marshall	1.0347	0.9959	1.0000	1.0015	1.0297	1.0003	1.0294	1.0033	0.9657
R.M. Heskett	0.8701	0.8981	0.9864	1.0068	0.9370	1.0035	0.9337	0.9350	0.9652
J.M. Stuart	0.9487	0.9859	0.9895	1.0098	0.9928	1.0049	0.9880	0.9563	0.9939
R.E. Burger	0.9375	0.9488	0.9941	1.0048	0.9450	0.9996	0.9454	0.9933	1.0052

Table 10

Plant Name	DELTA_SO2	DELTA_NOx	TE	TC	IG	IG_NF	IG_F	OM_SO2	OM_NOx
Muskingum River	0.9255	0.9574	0.9744	1.0226	0.9660	1.0109	0.9556	0.9616	0.9948
Kyger Creek	0.9128	0.9920	0.9974	1.0020	0.9997	1.0007	0.9990	0.9137	0.9930
Elrama	1.0683	1.0068	0.9798	1.0243	1.0377	1.0122	1.0252	1.0258	0.9668
Seward	0.9927	0.9731	0.9821	1.0192	0.9855	1.0077	0.9780	1.0064	0.9865
Shawville	0.9879	0.9617	0.9926	1.0079	0.9926	1.0028	0.9898	0.9948	0.9684
New Castle	0.9815	0.9750	0.9871	1.0238	0.9641	1.0088	0.9557	1.0074	1.0006
Brunner Island	0.9681	0.9463	0.9974	1.0062	0.9804	1.0016	0.9788	0.9840	0.9618
Montour	1.0231	0.9596	0.9961	1.0036	0.9905	1.0006	0.9900	1.0332	0.9691
Armstrong	0.9537	0.9241	0.9996	1.0248	0.9242	1.0098	0.9152	1.0072	0.9760
Watertree	1.0237	1.0250	0.9979	1.0046	1.0229	1.0024	1.0204	0.9984	0.9996
Big Brown	1.0315	0.9973	0.9845	1.0160	0.9789	1.0114	0.9678	1.0536	1.0186
Carbon	1.0758	1.1008	1.0003	1.0040	1.1001	1.0018	1.0982	0.9737	0.9963
Clinch River	1.0160	1.0150	0.9939	1.0077	1.0069	1.0038	1.0031	1.0075	1.0065
Glen Lyn	1.0116	0.9776	0.9687	1.0272	1.0039	1.0123	0.9917	1.0126	0.9786
Potamac River	0.9721	0.9792	0.9715	1.0290	0.9668	1.0141	0.9534	1.0058	1.0132
Bremo	1.0388	1.0260	0.9785	1.0274	1.0060	1.0159	0.9902	1.0271	1.0144
Kanawha River	0.9796	0.9816	0.9696	1.0263	0.9783	1.0126	0.9661	1.0064	1.0084
Rivesville	0.9178	0.9195	0.9634	1.0322	0.9073	1.0159	0.8931	1.0173	1.0191
Willow Island	1.0007	0.9913	1.0187	1.0127	0.9637	1.0124	0.9520	1.0065	0.9970
Kammer	0.9774	0.9978	0.9959	1.0020	1.0020	1.0005	1.0015	0.9775	0.9979
Mitchell	0.9507	0.9608	0.9941	1.0078	0.9939	1.0037	0.9903	0.9546	0.9648
Nelson Dewey	0.8331	0.9272	0.9884	1.0111	0.9947	1.0015	0.9932	0.8381	0.9327
Pulliam	0.8519	1.0576	1.0041	1.0032	1.0441	1.0003	1.0438	0.8100	1.0056
Dave Johnston	0.9793	1.0291	0.9960	1.0077	1.0127	1.0040	1.0087	0.9634	1.0125
Naughton	1.0052	1.0189	0.9954	1.0057	1.0144	1.0034	1.0110	0.9899	1.0033
J.H. Miller Jr.	1.1362	1.1543	0.9986	1.0066	1.1344	1.0043	1.1295	0.9965	1.0123
Pleasants	1.2178	0.9974	1.0106	1.0086	0.9895	1.0041	0.9855	1.2075	0.9889
Duck Creek	0.9956	1.0076	0.9888	1.0142	1.0012	1.0073	0.9940	0.9915	1.0036
Newton	1.0660	1.0316	0.9904	1.0130	1.0211	1.0064	1.0147	1.0405	1.0070
Sooner	1.0493	1.0218	0.9986	1.0046	1.0399	0.9996	1.0403	1.0058	0.9794
Welsh	0.9780	0.9593	0.9912	1.0055	0.9970	1.0042	0.9928	0.9841	0.9654
Martin Lake	1.0840	1.0064	0.9996	1.0064	1.0002	1.0020	0.9982	1.0772	1.0001
Monticello	0.9540	0.9758	0.9776	1.0289	0.9499	1.0174	0.9337	0.9986	1.0213
Rush Island	0.9425	0.9830	0.9880	1.0047	1.0077	1.0014	1.0063	0.9422	0.9827
Coleto Creek	1.0170	0.9797	0.9950	1.0133	0.9794	0.9863	0.9930	1.0300	0.9922
Harrington	1.0193	0.9881	1.0000	1.0504	0.9073	0.9838	0.9222	1.0695	1.0367
Pawnee	1.0399	0.9917	0.9946	1.0081	1.0082	1.0043	1.0039	1.0287	0.9810
Mountaineer	0.9787	0.9748	0.9868	1.0125	0.9670	1.0047	0.9625	1.0130	1.0089
Belews Creek	0.9785	1.0058	0.9992	1.0287	0.9508	0.9922	0.9583	1.0012	1.0291
Gen. J.M. Gavin	0.8000	1.0107	0.9937	1.0114	1.0037	1.0061	0.9976	0.7929	1.0018
Cheswick	1.0046	0.9671	1.0030	1.0073	0.9922	1.0029	0.9893	1.0021	0.9646
AVG	**0.9680**	**0.9846**	**0.9878**	**1.0159**	**0.9876**	**1.0062**	**0.9816**	**0.9767**	**0.9934**

Table 10 (cont.)

Plant Name	DELTA_SO2	DELTA_NOx	TE	TC	IG	IG_NF	IG_F	OM_SO2	OM_NOx
Barry	1.0336	1.0120	0.9496	1.0281	1.0088	1.0131	0.9957	1.0494	1.0275
Gorgas	1.0573	1.0163	0.9554	1.0399	0.9871	1.0200	0.9677	1.0783	1.0365
Comanche	0.9886	1.0127	0.9893	1.0233	0.9878	1.0097	0.9783	0.9885	1.0127
Brandon Shores	1.1312	1.1323	0.9495	1.0173	0.9140	1.0086	0.9062	1.2813	1.2825
Crist	0.8818	0.9539	1.0299	1.0329	0.9878	1.0172	0.9711	0.8392	0.9078
Hammond	0.8934	0.9157	1.0065	1.0504	1.0468	1.0222	1.0240	0.8073	0.8274
Harllee Branch	0.9737	0.9831	0.9710	1.0334	1.0209	1.0164	1.0044	0.9504	0.9596
Yates	0.8227	0.8911	1.0102	1.0484	1.0751	1.0215	1.0524	0.7226	0.7827
E.D. Edwards	1.0641	0.9933	0.9803	1.0200	0.9843	1.0116	0.9730	1.0812	1.0092
Coffeen	0.8525	0.9791	1.0000	1.0366	1.0275	1.0182	1.0091	0.8005	0.9193
Grand Tower	0.9711	0.9674	0.9648	1.0482	1.0231	1.0241	0.9991	0.9385	0.9350
Hustonville	0.9048	0.8770	1.0123	1.0496	1.0733	1.0230	1.0492	0.7933	0.7690
Meredosia	0.9643	0.9781	0.9763	1.0863	0.9739	1.0215	0.9534	0.9335	0.9469
Kincaid	0.7620	0.9199	1.0339	1.0493	1.0644	1.0268	1.0366	0.6600	0.7967
Powerton	0.9327	0.9858	0.9942	1.0362	1.0127	1.0174	0.9954	0.8941	0.9451
Joppa Steam	0.8710	1.0160	1.0086	1.0457	0.9604	1.0216	0.9401	0.8599	1.0030
Baldwin	0.9952	0.9872	0.9611	1.0456	1.0080	1.0223	0.9860	0.9825	0.9746
Clifty Creek	0.9012	1.0090	0.9874	1.0454	1.0002	1.0218	0.9789	0.8728	0.9772
Tanners Creek	0.9435	1.0054	0.9796	1.0238	1.0047	1.0093	0.9955	0.9364	0.9978
H.T. Pritchard	0.9462	0.9805	0.9561	1.0390	1.0294	1.0197	1.0095	0.9253	0.9588
Petersburgh	1.0298	1.0309	0.9682	1.0417	0.9569	1.0190	0.9391	1.0669	1.0681
Edwardsport	0.9525	0.9744	1.0003	1.0653	0.9657	1.0353	0.9328	0.9256	0.9469
R. Gallagher	0.9535	0.9790	1.0078	1.0508	0.9641	1.0279	0.9379	0.9339	0.9589
F.B. Culley	0.8068	0.9639	1.0477	1.0445	0.9975	1.0217	0.9764	0.7391	0.8830
Lansing	0.9854	0.9444	1.0006	1.0000	1.0245	0.9993	1.0252	0.9612	0.9212
Lawrence	0.9735	0.9916	1.0000	1.0417	0.9709	1.0122	0.9592	0.9625	0.9804
E.W. Brown	0.9278	0.9369	0.9931	1.0475	1.0261	1.0245	1.0015	0.8693	0.8779
Ghent	0.9461	1.0314	1.0008	1.0385	0.9558	1.0186	0.9383	0.9524	1.0383
Green River	0.9865	0.9527	0.9748	1.0463	1.0291	1.0226	1.0064	0.9399	0.9077
Mill Creek	1.1458	0.9992	0.9709	1.0172	0.9858	1.0050	0.9810	1.1769	1.0264
R.P. Smith	0.9285	0.8934	1.0053	1.0354	1.0698	1.0184	1.0504	0.8338	0.8022
Mount Tom	0.9680	0.9609	0.9886	1.0333	1.0200	1.0153	1.0046	0.9291	0.9223
B.C. Cobb	1.0312	1.0636	0.9661	1.0279	0.9546	1.0142	0.9413	1.0878	1.1220
Trenton Channel	0.9914	1.0258	0.9570	1.0233	1.0030	1.0143	0.9889	1.0092	1.0443
Hoot Lake	0.8964	1.0111	1.0133	1.0088	0.9958	0.9911	1.0048	0.8805	0.9932
Montrose	0.7142	0.9554	1.0406	1.0310	1.0347	1.0068	1.0278	0.6434	0.8606
Labadie	0.9141	0.9869	0.9771	1.0504	0.9928	1.0266	0.9671	0.8971	0.9685
Sioux	0.9770	0.9831	1.0141	1.0353	0.9644	1.0152	0.9499	0.9649	0.9710
Goudey	0.9483	0.9251	0.9728	1.0323	1.0767	1.0160	1.0597	0.8771	0.8556
Greenidge	0.9310	0.9216	0.9701	1.0371	1.0777	1.0155	1.0612	0.8587	0.8501
Milliken	0.9047	1.0045	0.9977	1.0468	0.9802	1.0224	0.9588	0.8837	0.9811
C.R. Huntley	0.9758	0.9773	0.9649	1.0456	1.0192	1.0190	1.0002	0.9490	0.9506
Dunkirk	0.9960	0.9628	1.0014	1.0544	0.9837	1.0255	0.9593	0.9590	0.9270
Rochester	0.9838	0.9843	0.9593	1.0499	1.0087	1.0262	0.9829	0.9684	0.9689
Asheville	1.0368	1.0232	0.9728	1.0231	0.9829	1.0116	0.9716	1.0599	1.0460
G.G. Allen	1.0220	1.0061	1.0060	1.0254	0.9635	1.0133	0.9508	1.0283	1.0123
Cliffside	1.0124	0.9817	0.9888	1.0326	0.9862	1.0174	0.9694	1.0053	0.9748
Marshall	1.0347	0.9959	0.9858	1.0320	0.9778	1.0182	0.9603	1.0401	1.0011
R.M. Heskett	0.8701	0.8981	1.0000	1.0540	1.0644	1.0303	1.0331	0.7755	0.8005
J.M. Stuart	0.9487	0.9859	0.9936	1.0402	1.0028	1.0207	0.9824	0.9152	0.9512
R.E. Burger	0.9375	0.9488	0.9556	1.0452	1.0552	1.0226	1.0319	0.8895	0.9002

Table 11

Plant Name	DELTA_SO2	DELTA_NOx	TE	TC	IG	IG_NF	IG_F	OM_SO2	OM_NOx
Muskingum River	0.9255	0.9574	0.9789	1.0484	1.0176	1.0228	0.9949	0.8861	0.9167
Kyger Creek	0.9128	0.9920	0.9876	1.0440	0.9994	1.0206	0.9792	0.8859	0.9627
Elrama	1.0683	1.0068	1.0000	0.9770	0.9631	0.9759	0.9869	1.1353	1.0700
Seward	0.9927	0.9731	0.9535	1.0420	1.0297	1.0190	1.0104	0.9704	0.9512
Shawville	0.9879	0.9617	0.9886	1.0498	0.9974	1.0243	0.9737	0.9544	0.9291
New Castle	0.9815	0.9750	0.9660	1.0296	1.0311	1.0177	1.0132	0.9571	0.9507
Brunner Island	0.9681	0.9463	0.9904	1.0558	1.0105	1.0253	0.9856	0.9162	0.8956
Montour	1.0231	0.9596	0.9911	1.0513	0.9989	1.0232	0.9762	0.9830	0.9220
Armstrong	0.9537	0.9241	0.9757	1.0488	1.0560	1.0240	1.0313	0.8825	0.8551
Watertree	1.0237	1.0250	0.9542	1.0227	0.9987	1.0106	0.9882	1.0505	1.0518
Big Brown	1.0315	0.9973	0.9858	1.0370	0.9794	1.0137	0.9662	1.0303	0.9961
Carbon	1.0758	1.1008	0.9890	1.0132	0.9114	1.0080	0.9042	1.1779	1.2052
Clinch River	1.0160	1.0150	0.9824	1.0137	0.9894	1.0059	0.9835	1.0313	1.0302
Glen Lyn	1.0116	0.9776	0.9631	1.0234	1.0327	1.0145	1.0179	0.9938	0.9605
Potamac River	0.9721	0.9792	0.9761	1.0234	1.0236	1.0108	1.0126	0.9507	0.9577
Bremo	1.0388	1.0260	0.9629	1.0123	0.9977	1.0097	0.9881	1.0683	1.0551
Kanawha River	0.9796	0.9816	0.9981	1.0144	1.0099	1.0078	1.0020	0.9582	0.9601
Rivesville	0.9178	0.9195	0.9955	1.0145	1.0716	1.0079	1.0632	0.8480	0.8496
Willow Island	1.0007	0.9913	0.9640	1.0197	1.0252	1.0098	1.0153	0.9931	0.9837
Kammer	0.9774	0.9978	0.9599	1.0463	0.9978	1.0228	0.9755	0.9752	0.9956
Mitchell	0.9507	0.9608	1.0021	1.0251	1.0158	1.0157	1.0001	0.9110	0.9208
Nelson Dewey	0.8331	0.9272	1.0881	1.0214	0.9968	1.0088	0.9881	0.7521	0.8370
Pulliam	0.8519	1.0576	1.0188	1.0458	0.9477	1.0275	0.9224	0.8437	1.0474
Dave Johnston	0.9793	1.0291	0.9917	1.0289	0.9835	1.0124	0.9714	0.9759	1.0255
Naughton	1.0052	1.0189	0.9900	1.0157	0.9808	1.0078	0.9732	1.0192	1.0330
J.H. Miller Jr.	1.1362	1.1543	0.9559	1.0263	0.8876	1.0077	0.8808	1.3050	1.3257
Pleasants	1.2178	0.9974	0.9533	1.0292	1.0044	1.0151	0.9894	1.2359	1.0121
Duck Creek	0.9956	1.0076	0.9916	1.0138	0.9923	1.0067	0.9857	0.9981	1.0102
Newton	1.0660	1.0316	0.9697	1.0264	0.9772	1.0187	0.9592	1.0960	1.0607
Sooner	1.0493	1.0218	0.9867	1.0095	0.9603	1.0045	0.9561	1.0969	1.0682
Welsh	0.9780	0.9593	1.0051	1.0165	1.0035	1.0062	0.9973	0.9539	0.9357
Martin Lake	1.0840	1.0064	0.9590	1.0059	0.9925	1.0034	0.9892	1.1322	1.0512
Monticello	0.9540	0.9758	0.9794	1.0157	1.0318	1.0090	1.0226	0.9295	0.9507
Rush Island	0.9425	0.9830	1.0044	1.0409	0.9902	1.0196	0.9711	0.9105	0.9496
Coleto Creek	1.0170	0.9797	0.9938	1.0058	0.9991	1.0024	0.9967	1.0184	0.9810
Harrington	1.0193	0.9881	0.9831	1.0117	0.9931	1.0035	0.9896	1.0319	1.0003
Pawnee	1.0399	0.9917	1.0323	1.0149	0.9246	1.0053	0.9197	1.0734	1.0237
Mountaineer	0.9787	0.9748	0.9834	1.0169	1.0284	1.0096	1.0187	0.9517	0.9479
Belews Creek	0.9785	1.0058	0.9922	1.0202	0.9951	1.0108	0.9845	0.9713	0.9984
Gen. J.M. Gavin	0.8000	1.0107	1.0268	1.0493	0.9879	1.0243	0.9645	0.7516	0.9496
Cheswick	1.0046	0.9671	0.9988	1.0380	0.9975	1.0212	0.9767	0.9715	0.9352
AVG	**0.9680**	**0.9846**	**0.9875**	**1.0327**	**1.0005**	**1.0155**	**0.9853**	**0.9486**	**0.9649**

Table 11 (cont.)

Plant Name	DELTA_SO2	DELTA_NOx	TE	TC	IG	IG_NF	IG_F	OM_SO2	OM_NOx
Barry	1.0336	1.0120	0.9832	1.0095	1.0044	1.0030	1.0014	1.0368	1.0151
Gorgas	1.0573	1.0163	0.9845	1.0156	1.0033	1.0092	0.9941	1.0540	1.0131
Comanche	0.9886	1.0127	0.9964	1.0110	1.0016	1.0051	0.9965	0.9797	1.0037
Brandon Shores	1.1312	1.1323	0.9797	1.0081	1.0251	1.0028	1.0223	1.1173	1.1183
Crist	0.8818	0.9539	1.0055	1.0177	0.9808	1.0095	0.9716	0.8786	0.9504
Hammond	0.8934	0.9157	0.9892	1.0229	0.9606	1.0153	0.9461	0.9192	0.9421
Harllee Branch	0.9737	0.9831	0.9896	1.0074	0.9896	1.0035	0.9862	0.9869	0.9964
Yates	0.8227	0.8911	0.9956	1.0240	0.9626	1.0127	0.9505	0.8383	0.9079
E.D. Edwards	1.0641	0.9933	0.9835	1.0138	1.0002	1.0070	0.9933	1.0670	0.9960
Coffeen	0.8525	0.9791	0.9920	1.0191	0.9810	1.0079	0.9733	0.8596	0.9872
Grand Tower	0.9711	0.9674	0.9780	1.0245	0.9736	1.0095	0.9644	0.9954	0.9917
Hustonville	0.9048	0.8770	0.9870	1.0233	0.9568	1.0103	0.9470	0.9363	0.9076
Meredosia	0.9643	0.9781	0.9909	1.0443	0.9610	1.0100	0.9515	0.9696	0.9835
Kincaid	0.7620	0.9199	1.0060	1.0144	0.9593	1.0085	0.9512	0.7784	0.9397
Powerton	0.9327	0.9858	0.9891	1.0197	0.9890	1.0087	0.9805	0.9351	0.9883
Joppa Steam	0.8710	1.0160	1.0105	1.0166	1.0022	1.0078	0.9944	0.8460	0.9868
Baldwin	0.9952	0.9872	0.9834	1.0149	0.9924	1.0082	0.9844	1.0047	0.9966
Clifty Creek	0.9012	1.0090	0.9951	1.0160	0.9829	1.0023	0.9806	0.9069	1.0153
Tanners Creek	0.9435	1.0054	0.9898	1.0172	1.0030	1.0090	0.9940	0.9343	0.9956
H.T. Pritchard	0.9462	0.9805	0.9712	1.0249	1.0104	1.0154	0.9951	0.9407	0.9748
Petersburgh	1.0298	1.0309	0.9929	1.0112	1.0103	1.0054	1.0049	1.0152	1.0163
Edwardsport	0.9525	0.9744	0.9958	1.0356	0.9582	1.0145	0.9445	0.9639	0.9860
R. Gallagher	0.9535	0.9790	1.0062	1.0120	0.9970	1.0053	0.9918	0.9392	0.9644
F.B. Culley	0.8068	0.9639	1.0221	1.0122	0.9955	1.0058	0.9897	0.7834	0.9359
Lansing	0.9854	0.9444	0.9825	1.0126	0.9909	1.0059	0.9851	0.9995	0.9579
Lawrence	0.9735	0.9916	0.9983	1.0162	1.0039	1.0013	1.0026	0.9557	0.9736
E.W. Brown	0.9278	0.9369	0.9995	1.0168	0.9864	1.0062	0.9803	0.9255	0.9346
Ghent	0.9461	1.0314	1.0150	1.0098	1.0163	1.0058	1.0105	0.9083	0.9902
Green River	0.9865	0.9527	0.9743	1.0246	0.9774	1.0099	0.9679	1.0110	0.9765
Mill Creek	1.1458	0.9992	0.9776	1.0149	0.9985	1.0067	0.9919	1.1567	1.0087
R.P. Smith	0.9285	0.8934	1.0003	1.0209	0.9804	1.0067	0.9738	0.9275	0.8924
Mount Tom	0.9680	0.9609	1.0084	1.0246	0.9532	1.0022	0.9511	0.9830	0.9758
B.C. Cobb	1.0312	1.0636	0.9802	1.0204	1.0031	1.0077	0.9955	1.0278	1.0601
Trenton Channel	0.9914	1.0258	0.9696	1.0223	1.0000	1.0121	0.9881	1.0001	1.0349
Hoot Lake	0.8964	1.0111	0.9998	1.0212	0.9895	1.0089	0.9808	0.8873	1.0008
Montrose	0.7142	0.9554	1.0471	1.0216	0.9910	1.0070	0.9841	0.6737	0.9012
Labadie	0.9141	0.9869	1.0023	1.0105	0.9984	1.0058	0.9926	0.9040	0.9759
Sioux	0.9770	0.9831	1.0009	1.0161	1.0103	1.0102	1.0001	0.9509	0.9568
Goudey	0.9483	0.9251	0.9963	1.0148	0.9780	1.0073	0.9709	0.9591	0.9357
Greenidge	0.9310	0.9216	0.9897	1.0215	0.9723	1.0083	0.9643	0.9471	0.9375
Milliken	0.9047	1.0045	1.0034	1.0179	0.9970	1.0109	0.9863	0.8885	0.9864
C.R. Huntley	0.9758	0.9773	0.9937	1.0118	0.9828	0.9949	0.9879	0.9875	0.9891
Dunkirk	0.9960	0.9628	0.9892	1.0344	0.9764	1.0130	0.9639	0.9969	0.9636
Rochester	0.9838	0.9843	0.9732	1.0262	0.9863	1.0150	0.9717	0.9987	0.9993
Asheville	1.0368	1.0232	0.9889	1.0103	1.0036	1.0056	0.9980	1.0340	1.0204
G.G. Allen	1.0220	1.0061	0.9997	1.0117	1.0074	1.0049	1.0025	1.0030	0.9874
Cliffside	1.0124	0.9817	0.9934	1.0158	0.9988	1.0072	0.9916	1.0044	0.9740
Marshall	1.0347	0.9959	1.0011	1.0063	1.0087	1.0027	1.0059	1.0182	0.9801
R.M. Heskett	0.8701	0.8981	1.0111	1.0175	1.0011	1.0074	0.9937	0.8448	0.8720
J.M. Stuart	0.9487	0.9859	0.9960	1.0122	0.9960	1.0064	0.9897	0.9448	0.9819
R.E. Burger	0.9375	0.9488	0.9896	1.0103	0.9697	1.0030	0.9668	0.9671	0.9787

Table 12

Plant Name	DELTA_SO2	DELTA_NOx	TE	TC	IG	IG_NF	IG_F	OM_SO2	OM_NOx
Muskingum River	0.9255	0.9574	0.9858	1.0211	0.9752	1.0101	0.9655	0.9428	0.9753
Kyger Creek	0.9128	0.9920	1.0025	1.0062	1.0001	1.0029	0.9972	0.9049	0.9834
Elrama	1.0683	1.0068	0.9917	1.0056	0.9935	0.9986	0.9948	1.0782	1.0162
Seward	0.9927	0.9731	0.9797	1.0188	1.0023	1.0063	0.9960	0.9923	0.9727
Shawville	0.9879	0.9617	1.0000	1.0098	0.9967	1.0049	0.9918	0.9817	0.9556
New Castle	0.9815	0.9750	0.9868	1.0201	0.9828	1.0085	0.9746	0.9921	0.9855
Brunner Island	0.9681	0.9463	1.0036	1.0122	0.9933	1.0059	0.9875	0.9593	0.9377
Montour	1.0231	0.9596	0.9992	1.0104	0.9922	1.0046	0.9877	1.0214	0.9580
Armstrong	0.9537	0.9241	0.9984	1.0231	0.9581	1.0090	0.9496	0.9744	0.9442
Watertree	1.0237	1.0250	0.9868	1.0080	1.0172	1.0042	1.0129	1.0119	1.0131
Big Brown	1.0315	0.9973	0.9802	1.0172	0.9881	1.0102	0.9781	1.0472	1.0124
Carbon	1.0758	1.1008	0.9931	1.0122	1.0069	1.0048	1.0020	1.0629	1.0876
Clinch River	1.0160	1.0150	0.9890	1.0110	1.0015	1.0059	0.9957	1.0146	1.0136
Glen Lyn	1.0116	0.9776	0.9627	1.0243	1.0223	1.0123	1.0098	1.0035	0.9698
Potamac River	0.9721	0.9792	0.9772	1.0204	0.9972	1.0097	0.9876	0.9776	0.9848
Bremo	1.0388	1.0260	0.9730	1.0225	1.0028	1.0144	0.9886	1.0412	1.0283
Kanawha River	0.9796	0.9816	0.9795	1.0211	0.9917	1.0110	0.9809	0.9877	0.9897
Rivesville	0.9178	0.9195	0.9794	1.0213	0.9774	1.0077	0.9699	0.9389	0.9406
Willow Island	1.0007	0.9913	1.0061	1.0132	0.9810	1.0116	0.9697	1.0008	0.9914
Kammer	0.9774	0.9978	0.9940	1.0060	1.0014	1.0025	0.9989	0.9760	0.9964
Mitchell	0.9507	0.9608	1.0041	1.0090	0.9977	1.0046	0.9932	0.9404	0.9505
Nelson Dewey	0.8331	0.9272	1.0200	1.0162	0.9921	1.0048	0.9874	0.8102	0.9016
Pulliam	0.8519	1.0576	1.0183	1.0177	1.0096	1.0071	1.0025	0.8142	1.0107
Dave Johnston	0.9793	1.0291	0.9950	1.0143	0.9996	1.0074	0.9923	0.9707	1.0202
Naughton	1.0052	1.0189	0.9914	1.0107	0.9995	1.0055	0.9941	1.0037	1.0173
J.H. Miller Jr.	1.1362	1.1543	0.9733	1.0187	1.0158	1.0065	1.0093	1.1281	1.1460
Pleasants	1.2178	0.9974	0.9815	1.0136	0.9934	1.0063	0.9873	1.2322	1.0091
Duck Creek	0.9956	1.0076	0.9890	1.0150	0.9976	1.0078	0.9898	0.9942	1.0063
Newton	1.0660	1.0316	0.9812	1.0127	0.9978	1.0060	0.9918	1.0751	1.0404
Sooner	1.0493	1.0218	0.9936	1.0099	1.0069	1.0039	1.0030	1.0385	1.0113
Welsh	0.9780	0.9593	0.9979	1.0094	1.0001	1.0058	0.9943	0.9708	0.9523
Martin Lake	1.0840	1.0064	0.9785	1.0080	0.9953	1.0026	0.9927	1.1041	1.0251
Monticello	0.9540	0.9758	0.9819	1.0209	0.9844	1.0136	0.9712	0.9669	0.9889
Rush Island	0.9425	0.9830	1.0034	1.0089	1.0008	1.0036	0.9972	0.9302	0.9702
Coleto Creek	1.0170	0.9797	0.9921	1.0137	0.9881	0.9955	0.9926	1.0234	0.9859
Harrington	1.0193	0.9881	0.9918	1.0403	0.9364	0.9935	0.9426	1.0550	1.0227
Pawnee	1.0399	0.9917	1.0105	1.0119	0.9749	1.0070	0.9682	1.0431	0.9947
Mountaineer	0.9787	0.9748	0.9858	1.0119	0.9900	1.0039	0.9862	0.9911	0.9871
Belews Creek	0.9785	1.0058	0.9974	1.0259	0.9641	0.9971	0.9669	0.9919	1.0195
Gen. J.M. Gavin	0.8000	1.0107	1.0183	1.0150	0.9986	1.0081	0.9906	0.7751	0.9792
Cheswick	1.0046	0.9671	1.0031	1.0144	0.9911	1.0080	0.9832	0.9962	0.9589
AVG	**0.9680**	**0.9846**	**0.9931**	**1.0165**	**0.9911**	**1.0068**	**0.9844**	**0.9675**	**0.9840**

Table 12 (cont.)

	Rolling Window			
	Pearson		Spearman	
Year	HYP	CW	HYP	CW
1987	-0.1151	-0.1107	-0.0624	-0.0437
1988	-0.207	-0.1879	-0.1721	-0.1318
1989	-0.2204	-0.2252	-0.2154	-0.2031
1990	-0.2353	-0.2402	-0.253	-0.2537
1991	-0.2027	-0.1895	-0.2003	-0.1639
1992	-0.1877	-0.1463	-0.1842	-0.1332
1993	-0.2067	-0.21	-0.1662	-0.1541
1994	-0.141	-0.1836	-0.1574	-0.1693
1995	-0.0625	-0.1035	-0.0523	-0.0851
AVG	**-0.175377778**	**-0.17743**	**-0.162589**	**-0.14866**
	Sequential Technology			
	Pearson		Spearman	
Year	HYP	CW	HYP	CW
1985	-0.1057	-0.0688	-0.0824	-0.0124
1986	-0.1446	-0.1429	-0.12	-0.1003
1987	-0.1151	-0.1107	-0.0624	-0.0437
1988	-0.2034	-0.1869	-0.1691	-0.133
1989	-0.2195	-0.1781	-0.1991	-0.1335
1990	-0.2605	-0.2239	-0.2832	-0.208
1991	-0.1814	-0.1833	-0.192	-0.1551
1992	-0.184	-0.1597	-0.1913	-0.1415
1993	-0.1948	-0.195	-0.1705	-0.165
1994	-0.1157	-0.1617	-0.1219	-0.1464
1995	-0.0615	-0.0854	-0.0398	-0.0621
AVG	**-0.162381818**	**-0.15422**	**-0.148336**	**-0.11827**

Table 13

Concluding Remarks

This dissertation has made three major contributions to the literature. The first contribution, introduced in Chapter 1, was correcting a vital flaw of the coordinate-wise "Russell" measure of technical efficiency that occurred at the boundary of output space. Without the proposed adjustment herein, this measure is left with no desirable properties when zero output values are frequent in the data. We have implemented the theoretical contribution empirically by analyzing a high frequency data set with zero output values, Babe Ruth's 1923 season relative batting performance. We observe that in general, our modification tends to lower relative efficiency scores and is indicative of an upward bias in the index values when the proposed modification is not implemented. This contribution was presented first as an introduction to applications of index number theory as the coordinate-wise index is applied in the following studies.

The second contribution the dissertation makes is in the realm of technological modeling. Chapter 2 introduces the concept of by-production, which we argue more adequately captures the tradeoffs implied by the natural physical and chemical processes implied by utilization of pollution-generating inputs (or outputs). The key was utilizing multiple production relations to identify separately, the intended production set and nature's residual generation set, and to define the reduced form by-production technology as the feasible production vectors in the intersection of these two sets. Accordingly, we also show that this novel specification of pollution-generating technology has direct implications on DEA estimation, which we derive herein. More importantly, the by-production specification requires us to distinguish between technical efficiency in intended production, and technical efficiency in unintended production. The identification of this dichotomy is important, as many of the more popular indexes utilized in growth decomposition analysis and efficiency measurement are not appropriate for applications over by-production technologies. Moreover, we show empirically, that firms tending to be efficient in intended output production tend to be inefficient in terms of pollution reduction, and vice versa.

The final contribution this dissertation has made is in the realm of index decomposition. In Chapter 3, we apply the by-production specification of a pollution-generating technology derived in Chapter 2 to analyze factors related to emissions reductions of sulfur dioxide and nitrogen oxides by coal-fired, electric power plants using a panel data set. First, we extend previous results derived under more the more primitive assumptions of weak disposability and null-jointness to the sequential frontier and show that prior results are not sensitive to the specification of the dynamics in the technology. Secondly, maintaining weak disposability and null-jointness, we juxtapose the results of models that credit producers for expanding unintended outputs to those that credit producers from retracting unintended outputs. Lastly, we apply the index decomposition under the assumption of by-production, and show that the index decomposition results are extremely sensitive to the notion of efficiency (intended production or unintended production) used to capture distances from the frontier. The final study concludes by validating empirically, that energy firms required to comply with the 1990 Clean Air Act

Amendment typically began readjusting input-output allocations long before the binding requirement became active in 1995.