

# Lawrence Berkeley National Laboratory

## Recent Work

### Title

PROBLEM: FOURIER SERIES FOR A COMBINATION OF JACOBIAN ELLIPTIC FUNCTIONS

### Permalink

<https://escholarship.org/uc/item/4tq2n994>

### Author

Pereira, N.R.

### Publication Date

1977-06-01

0 0 0 0 4 8 0 0 9 0 0

Submitted to SIAM, Review

UC-32  
LBL-6353  
Preprint c.1

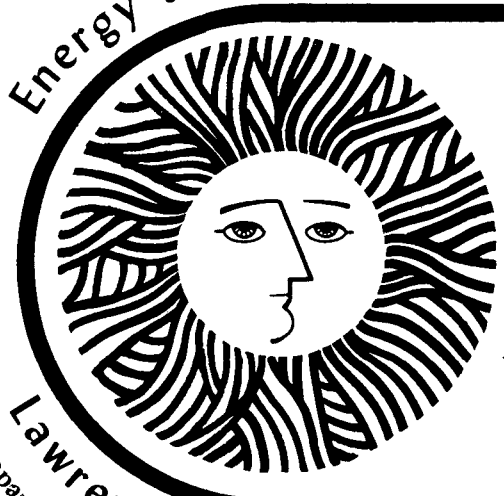
RECEIVED  
LAWRENCE  
BERKELEY LABORATORY

JUN 10 1977

LIBRARY AND  
DOCUMENTS SECTION

**For Reference**  
Not to be taken from this room

Energy and Environment Division



Problem: Fourier Series For A  
Combination of Jacobian  
Elliptic Functions

*N.R. Pereira*

June 1977

Lawrence Berkeley Laboratory University of California/Berkeley  
Prepared for the U.S. Energy Research and Development Administration under Contract No. W-7405-ENG-48

LBL-6353  
c.1

## **DISCLAIMER**

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

PROBLEM: FOURIER SERIES FOR A COMBINATION OF  
JACOBIAN ELLIPTIC FUNCTIONS \*

N. R. Pereira

LAWRENCE BERKELEY LABORATORY  
 Berkeley, California 94720

June 1977

Find a Fourier series for the function  $[\text{dn}(u) + ik \text{sn}(u)]^a$  for all real  $a$ . The problem appears in studies of perturbations on a particle oscillating in a sinusoidal potential well<sup>1</sup>  $V(x) = \cos x$ . The Fourier coefficient determines the influence of a perturbing wave with spatial dependence  $\cos ax/2$ . The presence of the Jacobian elliptic functions  $\text{dn}(u,k)$  and  $\text{sn}(u,k)$  shows that the particle is not deeply trapped, but can move near the separatrix.

The parameter  $k \leq 1$  then has a value of order unity. For integer powers  $a = m$  the Fourier coefficients are well-known.<sup>2</sup> They can be found from the integral

$$U_n(k,m) = (4K)^{-1} \int_0^{4K} [\text{dn}(u) + ik \text{sn}(u)]^m \exp(-in\pi u/2K) du$$

by extending the path of integration towards  $-i\infty$ , and adding the contributions from the two poles of  $\text{dn} + ik \text{sn}$  inside each fundamental

---

\*Work done under the auspices of the U. S. ERDA.

rectangle with sides  $4K$  and  $4iK'$ .  $K = K(k)$  is the first elliptic integral, and  $K' = K \left[ (1-k^2)^{\frac{1}{2}} \right]$ . Consequently the coefficients have the form

$$U_n(k,m) = (-1)^{m+n} (\pi/K)^m C_n(k,m) q^{n/2} / [1 + (-1)^{m+n} q^n] ,$$

where  $q$  is the nome  $q = \exp(-\pi K'/K)$  and  $C_n(k,m)$  reflects the contribution of each pole.  $C$  is unity for  $m = 1$  and  $m = 2$ , but different from unity for other  $m$ .

The corresponding problem for a particle that is not oscillating but instead traveling over the same potential gives rise to the Fourier transform of  $[\text{cn}(u) + i \text{sn}(u)]$  and its powers.

1. G. Smith, Phys. Rev. Lett. 38,970 (1977).
2. See e.g. E. T. Whittaker and G. N. Watson, Modern Analysis (Cambridge University Press, 1950), 4th ed., Chap. XXII.

This report was done with support from the United States Energy Research and Development Administration. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the United States Energy Research and Development Administration.

TECHNICAL INFORMATION DIVISION  
LAWRENCE BERKELEY LABORATORY  
UNIVERSITY OF CALIFORNIA  
BERKELEY, CALIFORNIA 94720

7.1