Title
PROBLEM: FOURIER SERIES FOR A COMBINATION OF JACOBIAN ELLIPTIC FUNCTIONS

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PROBLEM: FOURIER SERIES FOR A COMBINATION OF JACOBIAN ELLIPTIC FUNCTIONS *

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Find a Fourier series for the function $[\mathrm{dn}(u) + ik\ \mathrm{sn}(u)]^a$ for all real a. The problem appears in studies of perturbations on a particle oscillating in a sinusodial potential well[1] $V(x) = \cos x$. The Fourier coefficient determines the influence of a perturbing wave with spatial dependence $\cos ax/2$. The presence of the Jacobian elliptic functions $\mathrm{dn}(u,k)$ and $\mathrm{sn}(u,k)$ shows that the particle is not deeply trapped, but can move near the separatrix.

The parameter $k \leq 1$ then has a value of order unity. For integer powers $a = m$ the Fourier coefficients are well-known.[2] They can be found from the integral

$$U_n(k,m) = (4K)^{-1} \int_0^{4K} [\mathrm{dn}(u) + ik\ \mathrm{sn}(u)]^m \exp(-in\pi u/2K)\, du$$

by extending the path of integration towards $-i\infty$, and adding the contributions from the two poles of dn + ik sn inside each fundamental

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rectangle with sides 4K and 4iK'. K = K(k) is the first elliptic integral, and $K' = K\left[(1-k^2)^{\frac{1}{2}}\right]$. Consequently the coefficients have the form

$$U_n(k,m) = (-1)^{m+n}\,(\pi/K)^m\,n\,C_n(k,m)\,q^{n/2}/\left[1+(-1)^{m+n}\,q^n\right] \quad ,$$

where q is the nome $q = \exp(-\pi K'/K)$ and $C_n(k,m)$ reflects the contribution of each pole. C is unity for m = 1 and m = 2, but different from unity for other m.

The corresponding problem for a particle that is not oscillating but instead traveling over the same potential gives rise to the Fourier transform of [cn(u) + i sn(u)] and its powers.

1. G. Smith, Phys. Rev. Lett. 38,970 (1977).

2. See e.g. E. T. Whittaker and G. N. Watson, Modern Analysis (Cambridge University Press, 1950), 4th ed., Chap. XXII.

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