Title
SPACE STRUCTURE OF A FIREBALL AND THE PION RADIUS

Authors
Hoang, T.F.
Cork, B.
Crawford, H.J.

Publication Date
1981-07-01

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Submitted to Zeitschrift für Physik C - Particles and Fields

SPACE STRUCTURE OF A FIREBALL AND THE PION RADIUS

T.F. Hoang, Bruce Cork, and H.J. Crawford

July 1981

NUCLEAR SCIENCE DIVISION

Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

LBL-12362

SPACE STRUCTURE OF A FIREBALL AND THE PION RADIUS*

T.F. Hoang
1749 Oxford Street, Berkeley, CA 94709

and

Bruce Cork and H.J. Crawford
Lawrence Berkeley Laboratory
University of California, Berkeley, CA 94720

Results of calculation of the fireball radius as measured by means of the Hanbury-Brown and Twiss (HBT) effect are interpreted in the context of Landau's hydrodynamical model. A method based on a simple geometric picture of fireball formation is used to deduce the pion radius. From available data of various $\bar{p}p$ annihilations and $\pi^{\pm}p$, $K^{+}p$, pp collisions, it is found $r_{\pi} = 0.76 \pm 0.20$ fm.

A derivation is given on the decoherence factor of the Kopylov formula. A discussion is presented on other applications of the HBT effect to particle physics.

*This work was supported in part by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

1. Introduction

One of the fundamental problems concerning Landau's hydrodynamical model of meson production [1] is the space-time structure of the fireball. That this property can actually be investigated by means of the Hanbury-Brown and Twiss (HBT) effect [2] has been suggested by several authors: Shuryak [3], Kopylov and Podgretskii [4], and Cocconi [5]. We recall that, as pointed out by Purcell [6], the HBT effect is essentially a quantum phenomenon dealing with interference of two partially incoherent waves from the same source. Its application was first made to investigate the diameter of a radiostar and now is used to study the π-π correlation of multiparticle production in high energy physics. This is remarkable in view of a tremendous difference in magnitudes as regards the characteristic dimensions of these two phenomena, astronomic on the one hand and subnuclear on the other.

There exists a wealth of data on the measurements of the fireball dimensions for a variety of reactions covering a wide range of energies, measurements made on the basis of the HBT effect [7]. In this paper, we interpret these measurements in terms of Landau's model and describe a method to estimate the pion radius from the fireball radius.

Our idea, non nova sed nove, is based on the assumption that the fireballs (albeit prematter) produced in, say, high energy collisions, undergo a stage of expansion and then break up and that this last stage takes place when the volume of a fireball becomes equal to that constituted by the mesons it emits. According to this view, the radius R of a fireball is related to the pion radius r_π by a simple geometrical relation, namely

$$R = n^{1/3} r_\pi \tag{1}$$

n being the number of the emitted mesons, charged as well as neutral.

Experimentally, R is estimated by means of the following correlation function for like-pions, due to Kopylov [4a]

$$N^{\pm\pm} \sim 1 + \left[\frac{J_1(Rq_\perp)}{Rq_\perp/2}\right]^2 \qquad (2)$$

where J_1 is the Bessel function of the first kind and $q_\perp$ denotes the component of the relative momenta $\vec{k}_1$ and $\vec{k}_2$ of the two pions in the direction normal to the resultant $\vec{K} = \vec{k}_1 + \vec{k}_2$. Note that R thus estimated may be either parallel or perpendicular to the collision axes, according to the convention that $|\cos\theta_{c.m.}|$ is either less or greater than 1/2, $\theta_{c.m.}$ being the angle of the pions in the center-of-mass system. However, in our analysis, unless otherwise stated (Sec. 5 and 6) we shall deal only with R normal to the colliding direction, which is free from the Lorentz contraction. We shall discuss this point in Sec. 2 and the energy dependence of R in Sec. 7.

It should be mentioned that the Kopylov formula assumes incoherence of interference to give rise to the HBT effect. Its modification to include the decoherence factor will be discussed in Sec. 9.

The results of our estimation of the pion radius using various reactions will be discussed in Sec. 3 through 6. A comparison of the values of r_π thus obtained with those derived by other methods based on the pion form factor for photo- or electro-production will be given in Sec. 8.

Although few data are now available regarding the space-time structure of the fireball using nuclear targets, it is interesting to compare these to the fireball radius obtained from simple hadron-hadron collisions. We devote Sec. 10 to this subject.

A discussion of the results of the present work will be given in Sec. 11, together with other applications, e.g. to estimate the radii of ρ and ω mesons and to investigate e^+e^- annihilations into hadrons.

2. Some Properties of the Fireball

Our interpretation of the fireball radius as measured by means of the HBT effect is based on the kinematical properties of the fireball under consideration. For this purpose, let us consider the general case of asymmetric fireballs from the following reaction:

$$a + p \rightarrow \pi + \pi + \ldots \tag{3}$$

where p denotes the proton target and a stands for an incident particle, v.g. π or K.

In the center-of-mass system, before collision, both particles move with the same momentum $|\vec{P}|$ and in the opposite directions as shown schematically in Fig. 1-(I). As their velocities are different because their masses m_a and m_p are different, so also are the corresponding Lorentz contraction factors.

During the collision, parts of the c.m. available energy are converted into hadronic matter around the colliding particles. As a consequence, they behave like two fireballs with masses $M_a^* > m_a$ and $M_p^* > m_p$.

After collision, the two fireballs fly away with velocities such that $\beta_a^*\gamma_a^* < P/m_a$ and $\beta_p^*\gamma_p^* < P/m_p$ as shown in Fig. 1 (III). Consequently, the Lorentz contraction factors for the fireballs (γ_a^* and γ_p^*) are less than those for the initial state. This is at variance with what is often found in the literature.

As regards the velocity β^* of the fireball (the subscript specifying the fireball is understood, for brevity), we recall that it is related to the parameter λ which has been introduced to modify the Bose-Einstein distribution in order to account for Feynman-Yang scaling [8], namely

$$\beta^{*} = 1 - \lambda \quad . \tag{4}$$

We refer to Appendix for the derivation of this relation and the following property required by the scaling, namely

$$\lambda = C/\gamma_{c.m.} \tag{5}$$

C being a constant, characteristic of the fireball. From previous investigations of meson production by pp, $\pi^{+}p$ and $K^{+}p$ reactions (cf. Appendix) we find

$$C_p = 2.0 \quad \text{and} \quad C_{\pi} = C_K = 4/3 \tag{6}$$

for the proton and π/K fireball, respectively.

As an illustration, we plot in Fig. 2 the Lorentz factors for the π and p fireballs of πp reactions vs. the lab momentum of the incident pion, together with the Lorentz factor $\gamma_{c.m.}$ for comparison.

Note the slow rise of the Lorentz factors of the fireballs in contrast with the increase of the Lorentz factor $\gamma_{c.m.}$ of the center-of-mass system with P_{Lab}. This behavior agrees with what has been observed with the experimental results of the fireball radii measured by means of the HBT effect (see, v.g., the compiled data of Fig. 7 of [7a]). We shall return to this point in Sec. 7.

3. $\bar{p}p$ Annihilation at Rest

We now proceed to estimate the pion radius r_{π} using the currently available data on the fireball radius R measured by $\pi\pi$ correlations of like and unlike charges for various reactions. We begin with $\bar{p}p$ annihilation at rest, which is the simplest case for our investigation, as we are dealing with a single and stationary fireball.

We use the data of an experiment by Aachen-Berlin-Bonn-CERN-Cracow-London-Vienna-Warsaw Collaboration [9]. They studied the $\pi\pi$ correlation using 4-prong events from pp annihilation at rest:

$$\bar{p} + p \to 2\pi^+ + 2\pi^- + x\pi^0$$

where x = 0, 1 or 2. We have reproduced in Fig. 3 their result of the ratio $N^{\pm\pm}/N^{\pm\mp}$ of numbers of pion pairs of like and unlike charges vs. $q_\perp^2$ [for definition, see (2) in Sec. 1] for $|E' - E| < 0.05$ GeV, $E = (\vec{k}^2 + m^2)^{1/2}$ being the total energy of a pion under consideration.

The solid curve in Fig. 3 is their fit to Kopylov's formula, modified by introducing a decoherence factor in front of the square of the bracket in (2). They treated this factor as a free parameter in order to overcome the difficult problem of the background; namely, according to Kopylov's formula, we must have $N^{\pm\pm}/N^{\pm\mp} \geqslant 1$, whereas there are data points lying below 1 for $q_\perp^2 > 0.15$ (GeV/c)^2. They found for the decoherence factor 1.22 ± 0.08 exceeding unity, which is unacceptable [see (20) in Sec. 9].

Therefore, we have tried instead another fit to Kopylov's original formula, Eq. (2), using a limited range $q_\perp^2 < 0.13$ (GeV/c)^2 in order to satisfy the condition $N^{\pm\pm}/N^{\pm\mp} \geqslant 1$. We find

$$R = 1.49 \pm 0.15 \text{ fm}.$$

Our fit is shown by the dashed curve, with χ^2/point = 12.5/15.

A comparison of our refit with the data points indicates that our curve fits better the part of the data near the origin, which, as is well known, is more important than the background at large $q_\perp^2$ as far as the HBT effect is concerned.

Next, we have to determine the average number of pions involved in the 4-prong events of $\bar{p}p$ annihilation at rest. For this purpose we make use of the data of an experiment by College de France and CERN [10] and find

$$n = 5.43 \pm 0.18.$$

With this information, we are able to estimate the pion radius according to (1). We thus obtain

$$r_\pi = 0.85 \pm 0.09 \text{ fm}.$$

Our results together with the characteristics of their experiment are listed in Table I.

4. $\bar{p}p$ and $\bar{p}n$ Annihilations in Flight

As for annihilations in flight, we first consider three cases of low energy, with $P_{Lab} \leqslant 1.6$ GeV/c, from experiments by Pisa-Paris Collaboration [11], Bombay-CERN-College de France-Madrid Collaboration [12], and Torino-Torino Collaboration [13]. The characteristics of these experiments are summarized in Table I. Note that these experiments deal with exclusive reactions, so that the number of mesons involved in the fireball is known exactly. As the values of $\gamma_{c.m.}$ differ little from unity (see Table I) we may neglect the fireball motion and treat these three cases as annihilations at rest. The results for r_π are listed in Table I.

Note that one of these experiments investigates correlation of like kaon pairs, $K^\circ_S K^\circ_S$, rather than $\pi^\pm\pi^\pm$ as in the other two cases. As the kaon radius is known to be approximately the same as the pion radius, within experimental errors [14], no attempt is made to separate the kaons from the pions; namely, we tentatively assume n = 4, as listed in Table I. It is interesting to note that r_π thus computed is consistent with other values involving only pions. This justifies our assumption $r_\pi \simeq r_K$. As the kaons under consideration are neutrals, this further indicates that the Coulomb effect is probably negligible in the case of $\pi^\pm\pi^\pm$ correlation from $\bar{p}p$ annihilation.

Consider next the data of $\bar{p}p$ annihilation at 5.7 GeV/c from an experiment by CERN-Prague Collaboration [15]. They investigated the $\pi\pi$ correlation using 4-prong events:

$$\bar{p} + p \to 2\pi^+ + 2\pi^- + x^0$$

where x^0 denotes missing neutrals. Their measurement of the fireball radius in the longitudinal direction is listed in Table I.

For this case, $\gamma_{c.m.} = 1.88$, which is significantly different from 1; we therefore have to know the motion of the fireball or, what amounts to the same, the scaling parameter λ corresponding to the above inclusive reaction. In this regard we note that contrary to the p-p case, the scaling behavior, see (5), does not hold for $\bar{p}p$ annihilations as has been observed in a previous study [16]. We therefore have to determine λ directly for the above reaction with other available information.

For this purpose, we have at our disposal the angular distribution for π^- of the above annihilation observed in the c.m.s. system by the authors of the same experiment [17]. From their result, reproduced in Fig. 4, we find

$$\langle\mu\rangle = 0.700 \pm 0.028$$

where $\mu = \cos\theta_{c.m.}$, $\theta_{c.m.}$ being the meson emission angle in the c.m. system. Using the relation between λ and $\langle\mu\rangle$ (see Appendix) we find

$$\lambda = 1/\langle\mu\rangle - 1 = 0.43 \pm 0.06$$

The curve in Fig. 4 represents the angular distribution:

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{d\mu} = \frac{\lambda}{[1-(1-\lambda^2)\mu^2]^{3/2}} \tag{7}$$

expected from the modified Bose-Einstein distribution for $\lambda = 0.43$. We refer to Appendix for the derivation of this distribution and other related expressions used to estimate λ.

Knowing λ, we compute the Lorentz factor of the fireball using formulae (4) and (5) in Sec. 2. Its value is listed in Table I. It is now possible to estimate the order of magnitude of the fireball motion under investigation. Assuming a decay time $\tau \simeq 10^{-24}$ sec, characteristic of strong interaction [18], we find a displacement experienced by the fireball in the direction of the incident 5.7 GeV/c antiproton to be about $\beta^*\tau c = 0.17$ fm, amounting to $\sim$1/10 of the measured fireball radius ($R = 1.7$ fm, see Table I). Thus, here again we shall assume one stationary fireball as in the case of annihilation at rest.

There remains to estimate the average number of pions, charged and neutrals, for 4-prong events used in the experiment by CERN-Prague collaboration. In this regard, we use the results of a systematic study of particle production from $\bar{p}p$ annihilation at 5.7 GeV/c by Bonn-Hamburg-Milan Collaboration [19]. According to their investigation, the average number of neutral pions in the 4-prong events is $\sim$3.3. With this information, we obtain the pion radius as listed in Table I.

Referring to Table I, we are now in a position to compare the values of r_π estimated from various annihilation processes covering an energy range from $P_{Lab} = 0$ to 5.7 GeV/c. We notice that within experimental errors, they are consistent with each other, the average being

$$r_\pi = 0.81 \pm 0.14 \text{ fm}$$

in accord with the well-known value from the pion form factor (cf. infra).

It is worth noting that the values of R determined from annihilation are actually larger than those from hadron-hadron collisions [20]. (See Table II.) This difference is obvious from the point of view of fireball formation as mentioned in Sec. 1. It is because the meson multiplicity from $\bar{p}p$ annihilation is actually higher than that of, say, pp collision at the same energy [21]; nonetheless, this difference does not appear if we compare the values of r_π. We shall return to this point in Sec. 8.

5. $\pi^{\pm}p$ and $K^{+}p$ Collisions

Turning to the meson production by hadron collisions, we begin with $\pi^{+}p$ reactions at P_{Lab} = 5, 8, 16 and 23 GeV/c from a comprehensive work by Aachen-Berlin-Bonn-CERN-Cracow-Heidelberg-Warsaw (ABBCCHW) Collaboration [22]. They studied the $\pi\pi$ correlations using events with 5 πs in the final states for all four energies, as well as 7 πs for two reactions at 16 and 23 GeV/c. As we are dealing with exclusive reactions with only charged mesons $n_{\pm}$, involving no neutral ones $n_0 = 0$, it is a simple matter to determine the actual number of mesons of the reactions under investigation. The characteristics of these reactions and the values of R of their measurements are summarized in Table II.

As regards the fireball structure for $\pi^{+}p$ collisions, we note that here we are dealing with two asymmetric fireballs, one moving in the forward and the other in the backward direction of the c.m.s., and that the numbers of mesons emitted by the two fireballs are different, as has been investigated previously [23]. The ratios F/B of forward and backward mesons produced by $\pi^{+}p$ collisions have been estimated as in [23] and the results are summarized in Table II; the errors $\sim$6% are omitted for simplicity.

Note that in the measurement parallel to the collision axis, we need to consider only one fireball, which is facing the observer, whereas the one behind is in the shadow. As F/B > 1 (see Table II) and as we are looking for pion pairs of like charge, we tentatively assume the number of pions, n in Eq. (1), to be that of the forward π^{+} fireball, namely

$$n = (n_{\pm} + n_0)\,\frac{F}{F + B} \quad . \tag{8}$$

The values of n thus computed and those of r_π are listed in Table II.

For π^-p collisions, let us consider first the cases of exclusive reactions which can be analyzed in the same way as for the π^+p collisions discussed above. We have data at P_{Lab} = 4, 11, 16 and 25 GeV/c from the same group of ABBCCHW Collaboration [22], cf. supra, and P_{Lab} = 11.2 GeV/c by Pavia-Bologna-Firenze-Geneva-Milano-Oxford Collaboration [24]. The results are summarized in Table II.

In addition to these cases, we make use of two inclusive reactions at higher energies:

$$\pi^- + p \rightarrow \pi + \pi + \ldots$$

one at 40 GeV/c by the JINR group [25] and another at 200 GeV/c by Notre Dame-Duke-Toronto-Quebec (NDDTQ) Collaboration [26]. We have refitted the correlation data of these two experiments with Kopylov's formula, see (2). The values of R thus obtained are given in Table II.

As in the cases of exclusive reactions, here again, we have to consider only the forward fireball for the estimation of the average number of pions. We have used available information from other experiments at 40 and 200 GeV/c, namely [27,28]. We estimated the charged multiplicity for the proton fireball using the empirical formula (A-11) in Appendix with α_p = 0.26, then deduced the forward multiplicity from the observed value of $\langle n_{ch} \rangle$, account being taken of the charge of the target proton. The results together with the estimates of the pion radius are summarized in Table II.

Finally, we consider two exclusive K^+p reactions, one at 8.25 GeV/c by Mons-Brussel-CERN Collaboration [29] and another at 16 GeV/c by Brussel-Mons-Birmingham-CERN Collaboration [30]. Note that, as far as our purpose is concerned, these two cases are similar to the exclusive π^+p reactions discussed at the beginning of this section, except that here we have to exclude the incident K^+ meson which is the leading particle, since we are dealing with π-π correlations. With this remark, we proceed to calculate n and r_π according to (8) and (1). The values thus obtained together with the other quantities specific to the experiments are summarized in Table II.

In this section, we have estimated the values of r_π from π^+p, π^-p and K^+p reactions. If we take the averages separately, we find r_π = 0.65 ± 0.09, 0.72 ± 0.14, and 0.70 ± 0.07 fm, respectively. We notice that there is consistency among these averages, as well as with that of annihilation from the previous section.

6. p-p Collision

There remains the case of p-p collision. We have at our disposal the data of an <u>inclusive</u> reaction:

$$p + p \rightarrow \pi + \pi + \ldots$$

of a higher statistics counter experiment at P_{Lab} = 28.5 GeV/c by Purdue-Tufts-Strasbourg-FNAL Collaboration [31]. They have measured the fireball radius in both directions, parallel and perpendicular to the collision axis, and found

$$R_{\parallel} = 0.73 \pm {}^{0.10}_{0.11} \quad \text{and} \quad R_{\perp} = 1.65 \pm {}^{3.56}_{0.99} \text{ fm}$$

respectively. It is interesting to note that $R_\perp \simeq 2R_\parallel$ as expected since $\gamma_F \simeq 1.15$.

Indeed, according to our model, what is observed for $R_\perp$ is from either the foward or the backward fireball, or both, instead of one single fire-ball as in the case of measurement in the perpendicular direction, i.e. along the collision axis; then only the front fireball contributes, whereas the back one is in shadow. In this regard, we recall that the Lorentz contraction factor is not that of $\gamma_{c.m.}$ which is equal to 3.92 in the present case, but rather that of the fireball which is equal to 1.15 according to the recipe described in Sec. 2. This explains $R_{\perp} \simeq 2R_{\parallel}$; otherwise, it would be $R_{\perp} \simeq 2x(R_{\parallel}/3.92)$, namely 1/2 $R_{\parallel}$. This is ruled out by their data.

Therefore, for this experiment, we may take for the fireball radius the average of these two measurements, account being taken of the Lorentz contraction factor $\gamma_F = 1.15$; namely,

$$R = \frac{1}{2}\,(R_{\parallel} + \gamma_F R\ /2) \simeq 0.84\ \text{fm}.$$

To estimate the average number of mesons in each fireball, n, we find from available data [32] $\langle n_{\pm}\rangle = 2.54 \pm 0.06$ for charged pions and $\langle n_0\rangle =$ 1.75 ± 0.10 for neutral ones. Thus

$$n = (\langle n_{\pm}\rangle + \langle n_0\rangle)/2 = 2.15 \pm 0.06.$$

On the other hand, clearly $F/B \equiv 1$ as required by the symmetry of the pp system.

Knowing R and n and excluding the leading proton from the fireball under consideration, we obtain for the pion radius

$$r_{\pi} = 0.65 \pm 0.09\ \text{fm},$$

which is in good agreement with previous estimates from different reactions.

Finally, it should be mentioned that there exists another counter experiment investigating $\pi\pi$ correlations from inclusive pp reaction at ISR by CERN-College de France-Heidelberg-Karlsruhe Collaboration [33]. This experiment is particularly interesting, because of its energy, $\sqrt{s} = 52.5$ corresponding to a Lorentz contraction factor for the fireball $\gamma_F = 2.67$;

its effect could be detected. However, we are unable to include their result in the analysis of our present work, as we have tried in vain to refit the data using Kopylov's formula, see (2), with the decoherence factor equal to unity instead of being a free parameter as has been used in their paper. We shall discuss this point in Sec. 9.

7. Energy Dependence

We have presented in the preceeding sections results of analysis of the fireball radius R measured by means of the HBT effect for various reactions and at different energies. According to our model, the fireball radius R depends on the number of mesons n involved in the fireball under investigation, Eq. (1). Consequently, if measurements are made with fixed n along the colliding direction, we expect to find the same radius R regardless of the reaction and the incident energy. Indeed, an inspection of the data in Table II indicates that the values of R from exclusive π^+p, π^-p and K^+p reactions are actually consistent with $\sim$1 fm within experimental errors.

As for the inclusive reactions, we have to take for the number of mesons n the average multiplicity of mesons emitted by the fireball, which increases with energy. Therefore we expect R to increase with energy, as has been found with π^-p at 40 and 200 GeV/c (see Table II).

We now proceed to investigate this behavior of R as a function of P_{Lab} for inclusive πp reactions. We assume the pions emitted by the π-fireball and estimate the average multiplicity according to our fireball model [16,21]. Referring to Appendix for the recipe of our model, we write

$$n = 3\alpha_\pi(M_\pi^* - m_p)/m_p + 1 \qquad (9)$$

where m_π, m_p and M_π^* are the masses of pion, proton and π-fireball, respectively, whereas $3\alpha_\pi = 3 \times 0.36$ is a constant (cf. [16]), account being taken of the charge symmetrty for charged and neutral pions, as well as the incident pion, i.e. second term of the right-hand side of (9). The fireball mass M_π^* has been computed using the following formulae: (6) and (A-3), (A-9) and (A-10) in Appendix.

Knowing n and assuming r_π = 0.76 fm, as discussed in the next section, we deduce R using (1) and find the behavior of R shown by the curve in Fig. 5, which agrees well with the experimental data at 40 and 200 GeV/c shown by the full circles. For comparison, we have also plotted the data from exclusive $\pi^- p$ reactions in open circles.

Finally, we note that in the high energy limit, we expect $M^* \sim \gamma_{c.m.}^{1/2}$ [21]; therefore, $R \sim \gamma_{c.m.}^{1/6}$ or $P_{Lab}^{1/12}$, indicating a very slow increase with energy. Furthermore, at a given energy, we expect the fireball radii from inclusive πp and Kp to be the same, in view of $\alpha_\pi = \alpha_K$, the similarity property of meson production by hadron-hadron collisions, as discussed in [21].

8. The Pion Radius

We have estimated the pion radius using available data on the fireball radius, summarized in Tables I and II. We now compare our results with those obtained by other methods.

We recall that the conventional method consists of determining the pion electromagnetic form factor F_π as a function of q^2, the invariant momentum transfer squared. As a direct measurement, one may use the pion-electron scattering

$$\pi + e^- \to \pi + e^-$$

and investigate F_π in the space-like region of $q^2 > 0$. An alternative method is to use its inverse reaction, namely e^+e^- annihilation into 2π via one-photon exchange

$$e^+ + e^- \to \pi^+ + \pi^-$$

and to measure F_π in the time-like region, i.e. $q^2 < 0$. In passing, we recall that the electromagnetic form factor for π^0 is zero because of the CPT invariance. But this does not imply its radius to be zero as assumed in the present work (see Sec. 11).

As for the indirect measurement of F_π, one may use the electro-production of pion, v.g.,

$$e + p \to e + \pi^+ + n$$

or

$$e + n \to e + \pi^- + p.$$

However, in this case, the determination of F_π is less straightforward because of the nuclear effects of the proton and neutron.

Many efforts have been made to estimate the pion radius using the conventional methods. The latest results compiled by the Harvard-Cornell group [34] are shown in Fig. 6. The average of these measurements yield[35]

$$r_\pi = 0.70 \pm 0.06 \text{ fm}.$$

Another method based on the Chou-Yang model [36] of hadron-hadron collisions is due to Chou [14] using high energy πp and Kp elastic scattering. His analysis gives for the π and K radius

$$r_\pi = 0.61 \pm 0.03 \text{ fm}$$

$$r_K = 0.54 \pm 0.14 \text{ fm},$$

somewhat smaller than the result from the conventional methods.

As our method of estimating r_π is different, it is interesting to compare those measurements with our results. For this purpose, we have plotted in Fig. 7 our values of r_π. The overall average

$$r_\pi = 0.76 \pm 0.20 \text{ fm}$$

is consistent with previous results, within large statistical errors of our estimates.

In this regard, it should be mentioned that all the data used in the present analysis, except the case of pp at 28.5 GeV/c (cf supra), came from bubble chamber experiments. It would be interesting to have in the future high statistics data so that more accurate estimates of r_π can be attempted to compare with values determined by other methods and to investigate if there is actually any disparity of results from different methods as pointed out by Chou [14].

9. The Decoherence Factor

As mentioned earlier, Sec. 1, the Hanbury-Brown and Twiss effect is based on the well-known property of cross-correlation between two partially incoherent radio waves. In this regard, it should be mentioned that the situation of partial incoherence as required by the HBT effect is entirely different for meson emission by the fireball we are concerned with. In particular, we note that the momenta k of the mesons under investigation are rather distributed over a wide range and that their intensity depends on k. Clearly, this difference from a radiostar has to be taken into account for an accurate parametrization of the fireball radius R for various experiments that we have used in the present work.

For this purpose, we derive in this section an approximate formula on more general grounds than were used in the literature to obtain Kopylov's formula (2). We then discuss the use of the decoherence factor as a free parameter.

Consider two mesons of momenta k and $k' \neq k$ emitted from two points S and S' of a fireball and two detectors D_1 and D_2 at a distance D from SS' as shown in Fig. 8. We propose to investigate the HBT effect under the general condition that the intensities of the two sources S and S' are not equal.

For simplicity, we shall assume plane waves associated with the two mesons under consideration and use the optical analogy as in [5]. Now, the interference observed by D_1 is given by the following amplitude (we set $\hbar = c = 1$)

$$A_1 = \alpha e^{ika_1} + \beta e^{ik'a_2} . \tag{10}$$

The meaning of the symbols is self-explanatory: $|\alpha|^2$ and $|\beta|^2$ are intensities of the two sources S and S', respectively; whereas a_1 and a_2 are distances from D_1 to the sources S and S' (see Fig. 8). The rate observed by D_1 is

$$I_1 = |A_1|^2 = |\alpha|^2 + |\beta|^2 + 2\mathrm{Re}\,\alpha^* \beta e^{i(k'a_2-ka_1)} . \tag{11}$$

Likewise, the rate observed by D_2 is obtained from the above expressions by interchanging

$$a_1 \leftrightarrow b_2, \qquad a_2 \leftrightarrow b_1 \tag{12}$$

namely

$$I_2 = |A_2|^2 = |\alpha|^2 + |\beta|^2 + 2\mathrm{Re}\,\alpha^* \beta e^{i(kb_2-k'b_1)} . \tag{13}$$

Assuming random phase between the two sources of emission, namely incoherent superposition of two interfering waves in (11), as required by the HBT effect, we have to average I_1 and I_2 over the relative phase of α and β. This leads to

$$\langle I_1 \rangle = \langle I_2 \rangle = |\alpha|^2 + |\beta|^2 . \tag{14}$$

Consider next the coincidence between the two detectors D_1 and D_2. Here again, for simplicity, we assume the mesons to be emitted simultaneously. The rate of coincidence is then

$$I_{12} = \left|A_1 A_2\right|^2$$

$$= |\alpha|^4 + |\beta|^4 + |\alpha|^2|\beta|^2\left|e^{i(ka_1+k'b_1)} + e^{i(kb_2+k'a_2)}\right|^2 . \quad (15)$$

Once more we have to average this expression over the random phase between α and β as before, except that here account is to be taken of the indistinguishability of the two mesons. This amounts to exchanging $k \leftrightarrow k'$ in (15); in other words, we have to add a complex conjugate term. Thus the third term in (15) actually yields

$$|\ldots|^2 = 2 + 2\mathrm{Re}\; e^{i[k(a_1-b_2)-k'(a_2-b_1)]} + \text{c.c.} \quad (16)$$

Consequently, assuming that $|\alpha|^2 \neq |\beta|^2$, we find the general expression for the coincidence rate, i.e. the cross correlation as follows:

$$\langle I_{12}\rangle = \langle I_1\rangle\langle I_2\rangle[1 + a(k,k')\mathrm{Re}\; e^{i\psi}] \quad (17)$$

where

$$\psi = k(a_1 - b_2) - k'(a_2 - b_1) \quad (18)$$

is the phase and

$$a(k,k') = \frac{4|\alpha|^2|\beta|^2}{(|\alpha|^2+|\beta|^2)^2} \quad (19)$$

is, by definition, the <u>decoherence factor</u> which has the following properties:

$$\begin{aligned} a(k,k') &\leqslant 1, \text{ whatever } k \text{ and } k' \\ &= 1, \text{ if } k-k' \to 0 \\ &= 0, \text{ if } k-k' \to \infty. \end{aligned} \quad (20)$$

Note that the coherence condition requires $a(k,k') = 1$, and that $a(k,k')$ depends only on the ratio, say, $z = |\alpha|^2/|\beta|^2$. A plot of $a(z)$ vs. z is shown in Fig. 9.

There remains the integration of the above expression (17) over the surface of the fireball, which is rather intricate because of the decoherence factor. However, for practical purposes, we may assume $a(k,k')$ to be constant and follow Kopylov and Podgretskii [4], to approximate the ratio of two particle correlation including the decay time τ as follows:

$$\frac{\sigma^{\pm\pm}}{\sigma^{+-}} = 1 + a(k,k') \, \frac{[2J_1(Rq_\perp)/Rq_\perp]^2}{1+\tau^2 q_0^2} \tag{21}$$

where

$$\vec{q}_\perp = \vec{q} - n(\vec{q}\cdot\vec{n}), \quad q_0 = |E - E'| \tag{22}$$

with $\vec{q} = \vec{k} - \vec{k}'$ and $\vec{n} = (\vec{k} + \vec{k}')/|\vec{k} + \vec{k}'|$, where $E = (k^2 + m_\pi^2)^{1/2}$. J_1 is the Bessel function of the first kind.

Note that in case $|k - k'| \ll k$ or k', $|\alpha| \simeq |\beta|$; then $a(k,k') \simeq 1$. This leads to the well-known Kopylov formula, Eq. (2) for $\tau q_0 \ll 1$, i.e. $|E - E'|$ small.

Consider now the decoherence factor a [37], which has often been treated as a free parameter (see, v.g., [7,40,41]. The value of the radius R thus estimated turns out to be larger than without it, i.e. assuming $a = 1$, as has been noted in the case of $\bar{p}p$ annihilation at rest discussed in Sec. 3 (1.89 ± 0.06 to compare with 1.49 ± 0.15 fm, see [9]). If we take $a \simeq 0.4$ as mentioned (see, v.g., [7]) we find it to correspond to a ratio $z = |\alpha|^2/|\beta|^2$ about 1/7. This implies rather a large difference between the momenta k and k' of the two mesons under investigation; thus $q_\perp$ is large, indicating that we are rather far away from the correlation peak which occurs near $q_\perp = 0$ (see Fig. 3). We therefore believe that by treating the decoherence factor as a free parameter, one has emphasized the distribution due to background [38], but this procedure compromises the information in the peak near the origin.

10. Fireballs from Nuclear Reactions

Finally, we note that at present only few data are available on the fireball radius measured with nuclear targets. In spite of scarcity, it is not without interest to analyze the results in the same way as has been done with hadron-hadron collisions and to compare with what is expected from the fireball model.

We find two propane bubble chamber experiments with $\pi^- C$ at JINR, one at 5 GeV/c by Bairamov, et al. [39], and another at 40 GeV/c by Angelov, et al. [40]. To estimate the fireball radius from the HBT effect, both groups have used the decoherence factor as a free parameter and approximated the Bessel function by a series expansion. They arrive at the following expression:

$$\frac{N^{\pm\pm}}{N^{+-}} = A_1 + A_2 e^{-R'^2 q_\perp^2/4} \tag{23}$$

where we have used R' to denote the radius thus estimated, whereas R is kept for the value obtained using (2), i.e. the decoherence factor set to unity.

We have to convert their values R' in order to compare with results we have obtained in Sec. 5 for $\pi^- p$ at the same energy. In this regard, we note that if we make the same approximation with (2) assuming an adjustable background, we then obtain a radius R related to R' by

$$R = R'[A_2/2(A_1 + A_2)]^{1/2} \quad . \tag{24}$$

We shall use these refitted values R (see Table III) in our discussion.

We note that R increases from P_{Lab} = 5 to 40 GeV/c as in the $\pi^- p$ case shown in Fig. 5. A comparison with this figure indicates that these values for $\pi^- C$ are significantly larger than those for $\pi^- p$ at the same energy. We have to interpret this difference.

As for meson multiplicities for these $\pi^- C$ reactions, we use the information from other experiments by other JINR groups [42]. Using their data on average charged multiplicity $<n_\pm>$ we deduce the average number of mesons emitted by $\pi^- C$ collision assuming charge symmetry, i.e. $<n_{\pm 0}> = 3 <n_\pm>/2$. The results thus obtained are listed in Table III.

No information is available on the properties of fireballs from π-nucleus reactions. Nonetheless, it seems reasonable to assume F/B = 1, i.e. $n = <n_{\pm 0}>/2$ in view of the fact that at a given P_{Lab}, pions from $\pi^- C$ reactions are more isotropic than those of $\pi^- p$ collisions (cf. [41b]). The values of r_π thus obtained are listed in Table III.

We note that in spite of large errors, the values of r_π here found are significantly larger than the average 0.72 ± 0.14 fm from $\pi^- p$ collisions (cf. Sec. 5). This indicates that the fireballs formed in $\pi^- C$ collisions appears more diffused than those from $\pi^- p$ collisions. This is probably due to such effects as nuclear scattering, secondary production, Coulomb interaction, etc. As is usually the case, the structure of the fireball from nuclear reactions is undoubtedly more complicated and more complex [42]; no attempt will be made to apply further our simple model to these nuclear reactions.

11. Conclusion

We have analyzed the currently available data on the fireball radius measured by means of the Hanbury-Brown and Twiss effect for various reactions in terms of the fireball model formulated by Landau. We have used a simple geometric picture to estimate the pion radius, see (1), and found $r_\pi = 0.76 \quad 0.20$ fm in accord with the values determined by direct measurements, Sec. 8.

Bearing in mind that we have applied the method in its simplest form, we mention further refinements to be made, for instance, by taking properly into account the decoherence factor as discussed in Sec. 9. Furthermore, if events from different channels are used for measuring R, account should be taken of appropriate weights of these channels in order to estimate the correct number of s involved in the fireball.

As for other applications of the method, we mention first the radius of π^0. Since its electromagnetic form factor is identically zero (cf. Sec. 8), the conventional method is no more applicable to this case [43]. Yet we may estimate its value using the method described in this paper. Consider for instance the folowing two $\overline{p}n$ annihilations at rest into five pions, namely

$$\begin{aligned} \overline{p} + n &\rightarrow \pi^- + \pi^- + \pi^+ + \pi^0 + \pi^0 \\ &\rightarrow \pi^- + \pi^- + \pi^- + \pi^+ + \pi^+ \quad . \end{aligned}$$

It is possible to estimate the π^0 radius by comparing the fireball radii of these two exclusive reactions by means of the HBT effect, the second reaction used merely for comparison. Note that for the first reaction we have to know only the number of mesons, whereas the fireball radius can be estimated with negative pions so that the measurements of π^0 momenta are not indispensable for this purpose.

Next, we mention that the method can also be applied to estimate the radius of a resonance such as ρ or ω, which is rather difficult to measure by the conventional method. Yet the knowledge of the ρ radius is interesting as far as its nature of <u>elementary</u> particle is concerned. The question arises: What is it like in comparison with the pion radius? In this regard, it should be mentioned that the nuclear mean free path of ρ is found to be the same as that of π from photo-production experiments by the groups of DESY-MIT [44], Cornell [45], and SLAC [46]. It is another challenge to determine the ρ radius.

One may use the $\rho\pi$ correlation measured by means of the HBT effect. Indeed, this seems feasible in principle, as the lifetime of ρ deduced from its decay width is $\sim 4 \times 10^{-24}$ sec., which is much longer than the decay time of a fireball (see, v.g., [31]). It is therefore possible to observe the ρ particle before its decay into two pions, provided that the fireball radius is not too large.

As an illustration, we may consider, v.g., the $\bar{p}p$ annihilation at rest, which, as is well known, has a copious ρ production. We know that the 5π final state amounts to $18.7 \pm 0.9\%$, from the Yale experiment [47], that the fireball radius in this case is ~ 1.5 fm (see Table I) and that the rates for

$$
\begin{aligned}
\bar{p}p &\to \pi^0 + \pi^+ + \pi^- + \pi^0 \\
&\to \pi^\pm + \pi^\mp + \pi^+ + \pi^-
\end{aligned}
$$

are 7.3 ± 1.7 and $6.4 \pm 1.8\%$, respectively. An accurate estimate of the fireball radius of these exclusive reactions might shed light on the ρ radius.

From the general point of view, needless to say that the Hanbury-Brown and Twiss effect, which is a well-founded quantum mechanical phenomenon, provides a unique tool to investigate the $\pi\pi$ correlation which has attracted great interest since its first observation by Goldhaber, et al. [48]. A wealth of data on the subject has already been accumulated, and e^+e^- annihilation measurements are still needed. Of particular interest is this unique feature that there is no volume defined in the initial state due to the fact that the electron and the positron are structureless, a situation quite different from that of $\bar{p}p$ annihilation. It is particularly interesting to know what the space structure looks like for meson fireballs produced by either the quark or the gluon fragmentation as described by the QCD. In this regard, an investigation at the $\Upsilon(9.5)$ production and its vicinity will be very interesting, since Υ decay gives rise to three gluon jets; whereas off the $\Upsilon(9.5)$ resonance, $e^+e^- \to \bar{q}q \to$ hadrons leads to two jets in opposite directions, as the gluon bremstrahlung is negligible at this energy.

Finally, it should be mentioned that clearly the HBT effect may also be used to investigate the correlation of fermions such as μ's observed in high energy hadron collisions. The correlation function in this case is similar to that for bosons, except the sign and an extra factor 1/2; this amounts to replacing the decoherence factor a of (21) by -1/2 to account for the Fermi-Dirac instead of the Bose-Einstein statistics. This case has special interest, partly because there is no problem caused by the resonance background as in $\pi\pi$ correlation and partly because of lepton number conservation; the $\mu^+\mu^-$ pair is produced at the same point. The fireball thus explored is expected to be a point source, because the muons are structureless [49]. Thus the information on the fireball radius measured by means of the HBT effect using $\mu^+\mu^-$ pairs may provide a method to test this fundamental law of the conservation of the muon number.

Acknowledgement

The authors wish to thank Drs. M. Gyulassy and J. Lapore for reading and commenting on the manuscript. One of the authors (TFH) acknowledges the support of the Tsi Jung Fund. This work was supported in part by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

Appendix

Here we explain some basic formulae used in the analysis presented in this paper. To describe the single particle distribution, we have used the Bose-Einstein distribution modified for the Feynman-Yang Scaling:

$$\frac{d\sigma}{dP_T^2 dP_L} \sim \frac{1}{e^{\varepsilon(\lambda)/T}-1} \qquad \text{(A-1)}$$

where P_T and P_L are the transverse and the longitudinal momentum of a meson of mass m, in the c.m. system and

$$\varepsilon(\lambda) = (P_T^2 + \lambda^2 P_L^2 + m^2)^{1/2} \ . \qquad \text{(A-2)}$$

The two parameters are the temperature T and the scaling parameter λ; the Boltzmann constant and the velocity of light are set to unity.

Note that $\langle P_T \rangle$ depends only on T, whereas $\langle P_L \rangle \sim 1/\lambda$, a property used for the scaling, according to which $\langle P_L \rangle \sim E_{c.m.}$; therefore

$$\lambda = C/\gamma_{c.m.} < 1 \ . \qquad \text{(A-3)}$$

The constant C has been determined experimentally from pp, π^+p, K^+p, μ^+p, ep, γp, and νp reactions (cf. [8(b),16,21]).

In the case of scaling, the P_L distribution can be approximated with an exponential law, see [8(a)] and may be written as follows:

$$\frac{d\sigma}{dx} \sim e^{-x/\langle x \rangle} \qquad \text{(A-4)}$$

where $x = 2P_L/E_{c.m.}$ is the Feynman variable.

According to (A-1), the averages of P_T and P_L are related by (see [8(a)]):

$$\lambda = \frac{2}{\pi} \cdot \frac{\langle P_T \rangle}{\langle P_L \rangle} \qquad \text{(A-5)}$$

which may be used to estimate λ. Another method is to use the angular distribution of mesons in the c.m. system (cf. [8(e)]). From the modified Bose-Einstein distribution, it is found

$$\frac{d\sigma}{d\mu} \sim \frac{1}{[1-(1-\lambda^2)\mu^2]^{3/2}} \qquad \text{(A-6)}$$

where $\mu = \cos\theta^*$, θ^* being the c.m. angle of the meson. The first and the third moments in the interval $\mu = 0$ to 1 are as follows

$$\langle\mu\rangle = \frac{1}{1+\lambda} \quad \text{and} \quad \langle\mu\rangle^3 = \frac{1}{(1+\lambda)^2} \qquad \text{(A-7)}$$

so that $\langle\mu\rangle^2 = \langle\mu^3\rangle$ may be used as a validity test for (A-1). Note that for $\lambda = 1$, i.e. isotropic distribution, $d\sigma/d\mu$ = const. as is well known for black body radiation, and that in general $\lambda < 1$.

From λ we may deduce the velocity β^* of the fireball with respect to the c.m. system (see [8(d)]). Indeed, the introduction of λ in (A-1) amounts to changing the frame of reference, characterized by β^*. The general expression for the Bose-Einstein distribution, i.e. in its covariant form, is as follows:

$$\frac{d\sigma}{dP_T^2 dP_L} \sim \frac{1}{e^{(E-\vec{\beta}^*\cdot\vec{P})/T} - 1} \qquad \text{(A-8)}$$

where $(E,\vec{P})$ is the 4-momentum vector of the meson. As the transverse motion of the fireball is in general negligible, $\vec{\beta}^*\cdot\vec{P} \simeq \beta^* P_L$ and in the relativistic approximation, we find by comparing the exponentials in (A-1) and (A-8)

$$\beta^* = 1 - \lambda \quad . \qquad \text{(A-9)}$$

Note that $\lambda \leqslant 1$ by definition and that, at a given energy, β^* is not the same for the inclusive and the exclusive reactions.

Knowing β^* we deduce the Lorentz factor of the fireball $\gamma_F = 1/(1 - \beta^{*2})^{1/2}$ with respect to the c.m.s. and its mass M_F^* by energy conservation:

$$\gamma_F M_F^* = E_{c.m.}, \tag{A-10}$$

$E_{c.m.}$ being the c.m. energy of the colliding particles under consideration (see [8(c)]).

Finally, we mention that the average multiplicity of pions produced by the collision $a + p \to \pi^- + \ldots$ is found to be proportional to the fireball mass (cf. [8(c)]). For instance, we may write for mesons from the proton fireball

$$\langle n \rangle_p = \alpha_p (M_p^* - m_p)/m_p \quad , \tag{A-11}$$

m_p being the proton mass and α_p a coefficient, characteristic of the meson production, and a similar expression for the other fireball. Experimentally, it has been found that only three coefficients are needed to describe meson production by various high energy collisions, related to the proton, the meson and the lepton fireballs. We refer to [21] for a detailed discussion on this remarkable property of similarity.

References and Footnotes

1. L.D. Landau, Izv. Akad. Nauk. SSSR, ser. fiz. 17, 51 (1953).
2. R. Hanbury-Brown and R.O. Twiss, Phil. Mag. 45, 663 (1954).
3. E.V. Shuryak, Phys. Lett. 44B, 387 (1973).
4. G.I. Kopylov, Phys. Lett. 50B, 472 (1974), and G.I. Kopylov and M.I. Podgoretskii, Sov. J. Nucl. Phys. 18, 336 (1974).
5. G. Cocconi, Phys. Lett. 49B, 459 (1974).
6. E.M. Purcell, Nature, 177, 1449 (1956).
7. For a recent survey, see (a) R. Diebolt's Rapporteur Talk, Proc. of the 19th International Conference on High Energy Physics, Tokyo, 1978, Editor S. Homa (Physics, Japan, 1979), p. 668, and (b) A. Giovanni, et al., Riv. Nuovo Cimen. 2 Nr 10 (1979).
8. T.F. Hoang, Nucl. Phys. 38B, 333 (1972); Phys. Rev. D6, 1328 (1972); D8, 2315 (1973); D12, 296 (1975); and D13, 1881 (1976); referred to as a, b, c, d, and e, respectively.
9. Aachen-Berlin-Bonn-CERN-Cracow-London-Vienna-Warsaw Collaboration, M. Deutschmann, et al., CERN/EP/Phys. 78-1 (1978). The data here used can also be found in Fig. 7 of Diebold's Rapporteur Talk [7].
10. College de France-CERN Collaboration, C. Chesquiere, in Symposium on Antinucleon-Nucleon Interactions, Liblice-Prague, 1974, edited by L. Montanet (CERN, Geneva, 1974), p. 436.
11. Pisa-Paris Collaboration, C. Angelini, et al., Nuovo Cim. 19, 279 (1977).
12. Bombay-CERN-College de France-Madrid Collaboration, A.M. Cooper, et al., Nucl. Phys. 139B, 45 (1978).
13. Torino-Torino Collaboration, Nuovo Cim. 36A, 245 (1976).

14. See, v.g., T.T. Chou, Phys. Rev. D19, 3327 (1979). $r_K = 0.54 \pm 0.14$ fm and $r_\pi = 0.61 \pm 0.03$ fm.

15. V. Šimák, in Proc. Europ. Symp. on NN Interactions, Stockholm, 1976, edited by G. Ekspong and S. Nielsen (Pergamon, NY 1977), p. 319.

16. T.F. Hoang, et al., Phys. Rev. D17, 927 (1978).

17. CERN-Prague Collaboration, J. Žáček, in Proc. of Europ. Symp. on $N\bar{N}$ Interactions, Liblice-Prague, 1974, edited by L. Montanet (CERN-74-18, 1974), p. 179.

18. We mention that the decay time of a fireball is found to be $\sim 10^{-24}$ sec, see, v.g., [11,12].

19. Bonn-Hamburg-Milan Collaboration, K. Böckmann, et al., Nuovo Cim. 42A, 954 (1966).

20. This has already been noted by A.W. Cooper, et al. See Sec. 5 of [12].

21. For further discussion, see T.F. Hoang, "Similarity Property of Meson Production," Phys. Rev. D, in press.

22. Aachen-Berlin-Bonn-CERN-Cracow-Heidelberg-Warsaw Collaboration, M. Deutchmann, et al., Nucl. Phys. 103B, 198 (1976).

23. T.F. Hoang, et al., Phys. Rev. D20, 692 (1979).

24. Pavia-Bologna-Firenze-Genova-Milano-Oxford Collaboration, E. Calligarich, et al., Nuovo Cim. 16, 129 (1976).

25. JINR group, N. Angelov, et al., Soviet J. Nucl. Phys. 26, 419 (1977).

26. NDDTQ Collaboration, N.N. Biswas, et al., Phys. Rev. Lett. 37, 175 (1976).

27. For the multiplicity of $\pi^- p$ at 40 GeV/c, see Bucharest-Budapest-Cracow-Dubna Collaboration, O. Balea, et al., Nucl. Phys. 52B, 414 (1973), and E.N. Kladnitskaya et al. Soviet J. Nucl. Phys. 24, 307 (1976).

28. For the multiplicity of $\pi^- p$ at 200 GeV/c, see NAL-LBL Collaboration, D. Bogert, et al., Phys. Rev. Lett. 31, 1271 (1973) and Preprint NAL Conf. 74/55 (1974).

29. Mons-Brussel-CERN Collaboration, F. Grand, et al., Nucl. Phys. 102B, 221 (1976).

30. Brussel-Mons-Birmingham-CERN Collaboration, E. de Wolf, et al., Nucl. Phys. 132B, 383 (1977).

31. Purdue-Tufts-Strasbourg-FNAL Collaboration, C. Ezell, et al., Phys. Rev. Lett. 38, 873 (1977).

32. We have used the pp data at 29 GeV/c of D. Smith, et al., Phys. Rev Lett. 23, 1964 (1969) for charged multiplicity and those at 24 GeV/c of Bonn-Hamburg-München Collaboration, V. Blobel, et al., Nucl. Phys. 69B, 454 (1974) for neutral multiplicity.

33. CERN-College de France-Heidelberg-Karlsruhe Collaboration, D. Drijard, et al., Nucl. Phys. 155B, 269 (1979).

34. Harvard-Cornell Collaboration, C.J. Bebeck, et al., Phys. Rev. D17, 1693 (1978).

35. The measurements are taken from S.F. Berezhnev, et al., Soviet J. Nucl. Phys. 16, 99 (1971); G.T. Adylov, Phys. Lett. 51B, 402 (1974); S. Dubnicka and O.V. Dumbrajas, Phys. Lett. 53B, 285 (1974); C.J. Bebeck, et al., Phys. Rev. D13, 25 (1976); G. Bardin, et al., Nucl. Phys. 120B, 45 (1977); A. Quenzer, et al., contributed paper, Int. Symp. on Lepton and Photon Interactions at High Energy, Hamburg (unpublished), and E.B. Dalley, et al., Phys. Rev. Lett. 39, 1176 (1978).

36. T.T. Chou and C.N. Yang, Phys. Rev. 170, 1591 (1968).

37. For an interesting discussion from another point of view on this subject, see G.N. Fowler and R.M. Weiner, Phys. Lett. 70B, 201 (1977).

38. See, v.g., P. Grossberger, talk at VIII Int. Symp. on Multiparticle Dynamics, Kayserberg (France), 1977. Preprint NTH 77/7, Universite de Nice.

39. JINR Group, A.A. Bairamov, et al., Soviet J. Nucl. Phys. 28, 59 (1978).

40. JINR Group, N. Angelov, et al., Soviet J. Nucl. Phys. 27, 361 (1977).

41. JINR Group, S.A. Azimov, et al., Nucl. Phys. 107B, 45 (1976) and Soviet J. Nucl. Phys. 28, 55 (1978); referred to as a and b.

42. For a survey on this situation, see, v.g., M. Gyulassy, "Pion Interferometry of Nuclear Collisions," LBL-8655, to appear in Nature.

43. Angular correlations for $\pi^0\pi^0$ have been observed with a Xe bubble chamber by K. Eskreys, Acta Phys. Pologna, 36, 237 (1969).

44. DESY-MIT Group, J.G. Asbury, et al., Phys. Rev. Lett. 19, 865 (1967).

45. Cornell Group, G. McCellan, et al., Phys. Rev. Lett. 22, 373 (1969).

46. SLAC Group, F. Bulos, et al., Phys. Rev. Lett. 22, 490 (1969).

47. Yale Group, C. Baltay, et al., Phys. Rev. 145, 1103 (1966).

48. G. Goldhaber, et al., Phys. Rev. Lett. 3, 181 (1959).

49. A recent measurement of the muon radius gives $r_\mu < 10^{-16}$ cm. See Mark-J Collaboration, D.P. Barber, et al., Phys. Rev. Lett. 45, 1904 (1980).

Table I - Estimates of the pion radius from $\overline{N}N$ annihilations

Reaction	P_{Lab}(GeV/c)	$\gamma_{c.m.}$	γ_F	n	R(fm)	r_π(fm)	Ref.
$\bar{p}p \rightarrow 2\pi^+ 2\pi^- x\pi^0$	0	1	1	5.43 0.18	<u>1.49±0.15</u>	0.85±0.09	9,10
$2\pi^+ 2\pi^- \pi^0$	0-0.76	1-1.07	1	5	1.8 ±0.1	1.05±0.06	11
$K_s^0 K_s^0 \pi^+ \pi^-$	0.76	1.07	1	4	0.9 ±0.2	0.57±0.13	12
$\bar{p}n \rightarrow 3\pi^- 2\pi^+$	0.1-1.6	1.15	1	5	1.23±0.09	0.72±0.05	13
$\bar{p}p \rightarrow 2\pi^+ 2\pi^- x\pi^0$	5.7	1.88	1.22	7.3	1.7 ±0.3	0.88±0.16	15,17,19

$\gamma_{c.m.}$ and γ_F are Lorentz factors of the c.m.s. and the fireball.

Underlined value of R is a refit with decoherence factor set to 1, see text in Sec. 3.

Table II - Estimates of the pion radius from $\pi^{\pm}p$, $K^{+}p$ and pp reactions

Reaction	P_{Lab}(GeV/c)	$\gamma_{c.m.}$	γ_F	F/B	n	R(fm)	r_{π}(fm)	Ref.
$\pi^{+}p \rightarrow p5\pi$	5	1.86	1.05	2.13	3.40	1.13±0.23	0.75±0.16	22
$p5\pi$	8	2.24	1.09	1.54	3.03	2.03±0.50	1.40±0.71	22
$p5\pi$ $p7\pi$	16	3.06	1.23	1.50	3.50	0.87±0.28	0.57±0.18	22
$p5\pi$ $p7\pi$	23	3.50	1.30	1.48	3.52	0.87±0.18	0.57±0.12	22
$\pi^{-}p \rightarrow p5\pi$	4	1.70	1.03	4.30	4.06	0.86±0.14	0.54±0.09	22
$p5\pi$	11	2.58	1.17	1.71	3.14	0.90±0.30	0.61±0.20	22
$p5\pi$ $n6\pi$	11.2	2.60	1.17	1.71	3.53	1.04±0.28	0.68±0.19	24
$p5\pi$ $p7\pi$	16	3.06	1.27	1.50	3.53	1.07±0.78	0.71±0.19	22
$p5\pi$	25	3.76	1.31	1.40	2.92	1.41±0.39	0.98±0.27	22
$\pi^{-}p \rightarrow \pi\pi + \cdots$	40	4.70	1.44	1.21	4.02	1.20±0.11	0.75±0.14	25,27
	200	10.32	2.04	1.11	5.84	1.48±0.11	0.82±0.14	26,28
$K^{+}p \rightarrow K^{+}p2\pi^{+}2\pi^{-}$	8.25	2.27	1.12	1.25	2.20	0.80±0.08	0.62±0.06	29
	16	3.01	1.21	1.21	2.20	1.00±0.09	0.77±0.06	30
$pp \rightarrow \pi\pi + \cdots$	28.5	3.95	1.15	1.0	2.15	0.84±0.10	0.65±0.09	31,32

γ_F refers to Lorentz factor for the forward fireball.

Underlined R values are refits with decoherence factor equal to 1 (see text in Sec. 5).

Table III - Estimates of the pion radius from π^-C reactions

Reaction	P_{Lab}(GeV/c)	$n_{\pm 0}$	R(fm)	r_π(fm)	Ref.
$\pi^-C \to \pi\pi...$	5	4.5 ± 0.1	1.09 ± 0.03	0.83 ± 0.19	40,42
	40	9.6 ± 0.1	1.70 ± 0.05	1.00 ± 0.11	41,42

Values of R are reduced for decoherence factor = 1 with Eq. (24).

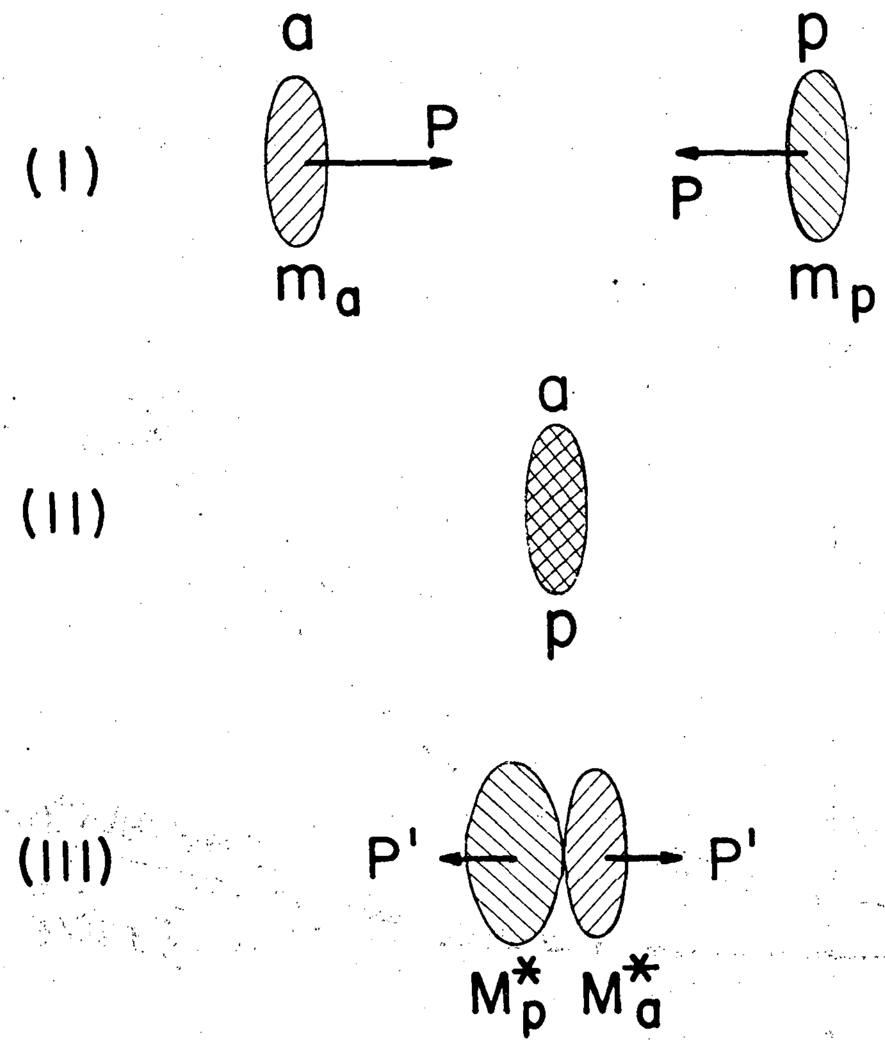
$n = \langle n_{\pm 0} \rangle /2$ assuming F/B = 1 (see text in Sec. 10).

Figure Captions

1. Schemas of fireball formation resulting from collision between two particles a and p. (I) The two particles approach each other in c.m.s. with momentum P. (II) During collision, parts of their kinetic energies are converted into prematter to form fireballs surrounding the colliding particles. (III) After collision, the two excited particles go away with momentum $P' < P$.
2. Plots of Lorentz factors for πp collisions as a function of P_{Lab}. The Lorentz factors for the π and p fireballs are calculated with Eqs. (4)-(6) (see also Appendix).
3. $\pi\pi$ correlation observed with 4-prong events from $\bar{p}p$ annihilation at rest. Data from [9]. The solid curve is the original fit by the authors of the experiment, assuming the decoherence factor a as a free parameter. The dashed curve is a refit with a = 1 (see text in Sec. 3).
4. Angular distribution of forward π's in c.m.s. from $\bar{p}p$ annihilation at 5.7 GeV/c. Data from [17]. The curve is a one-parameter fit with the modified Bose-Einstein distribution, Eq. (7), to estimate the scaling parameter $\lambda = 1/\langle\mu\rangle - 1$. See Appendix.
5. Energy dependence of the fireball radius from the inclusive $\pi^- p$ reactions. The curve is the prediction by the fireball model and experimental points are shown by full circles. For comparison, data from exclusive reactions are shown by open circles. See text in Sec. 7.
6. Values of the pion radius measured by the conventional methods based on the electromagnetic form factors. The average is $r_\pi = 0.70 \pm 0.06$ fm.
7. Values of the pion radius estimated from the fireball radius, using Eq. (1). The average is $r_\pi = 0.76 \pm 0.20$ fm.

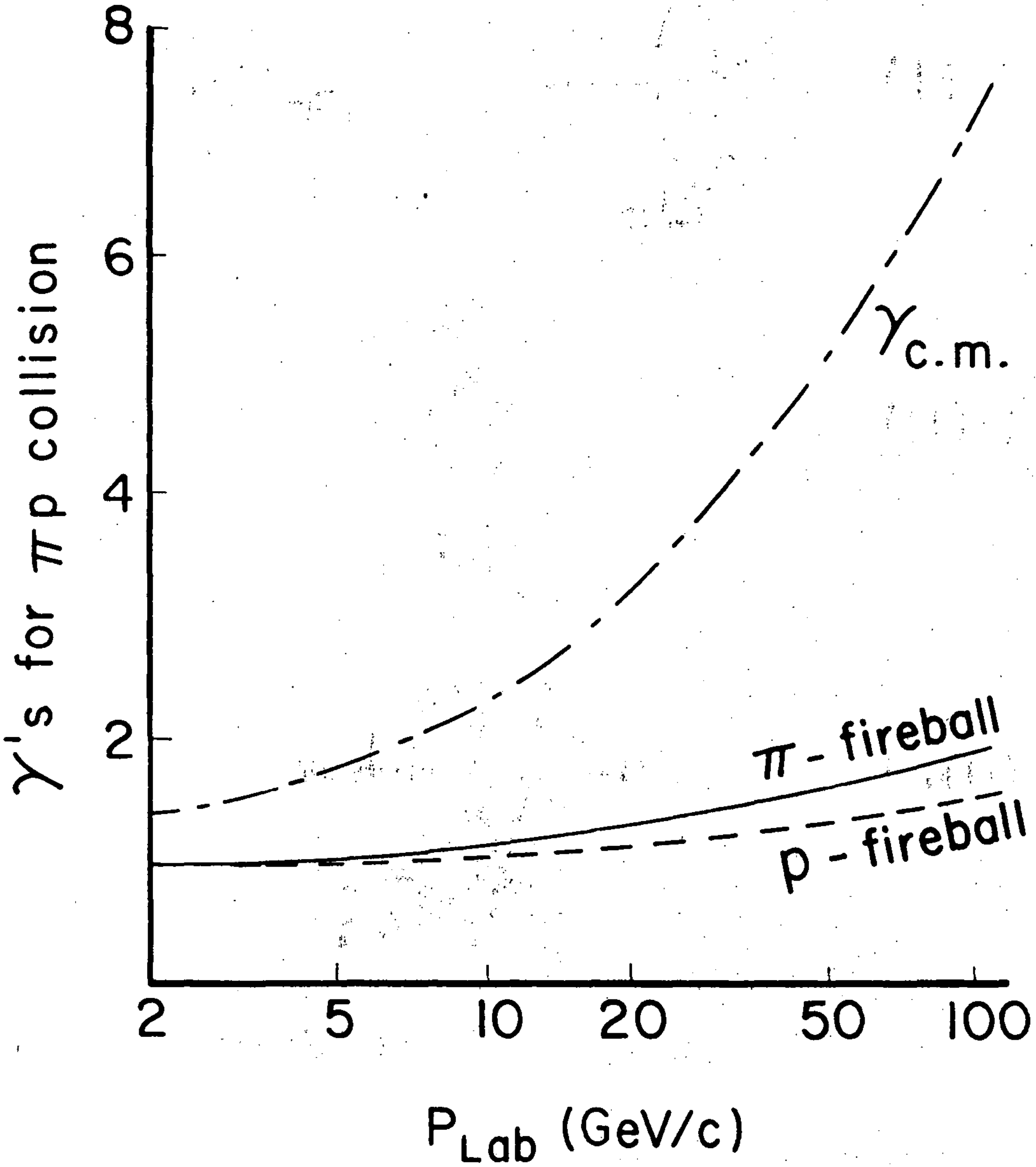
8. Geometrical rays of two π's issued from sources S and S'. D and D' are two detectors for observing the Hanbury-Brown and Twiss effect.

9. Plot of the decoherence factor a as a function of z, the ratio of the intensities of the two π's under consideration for the Hanbury-Brown and Twiss effect. Coherence requires a = 1.



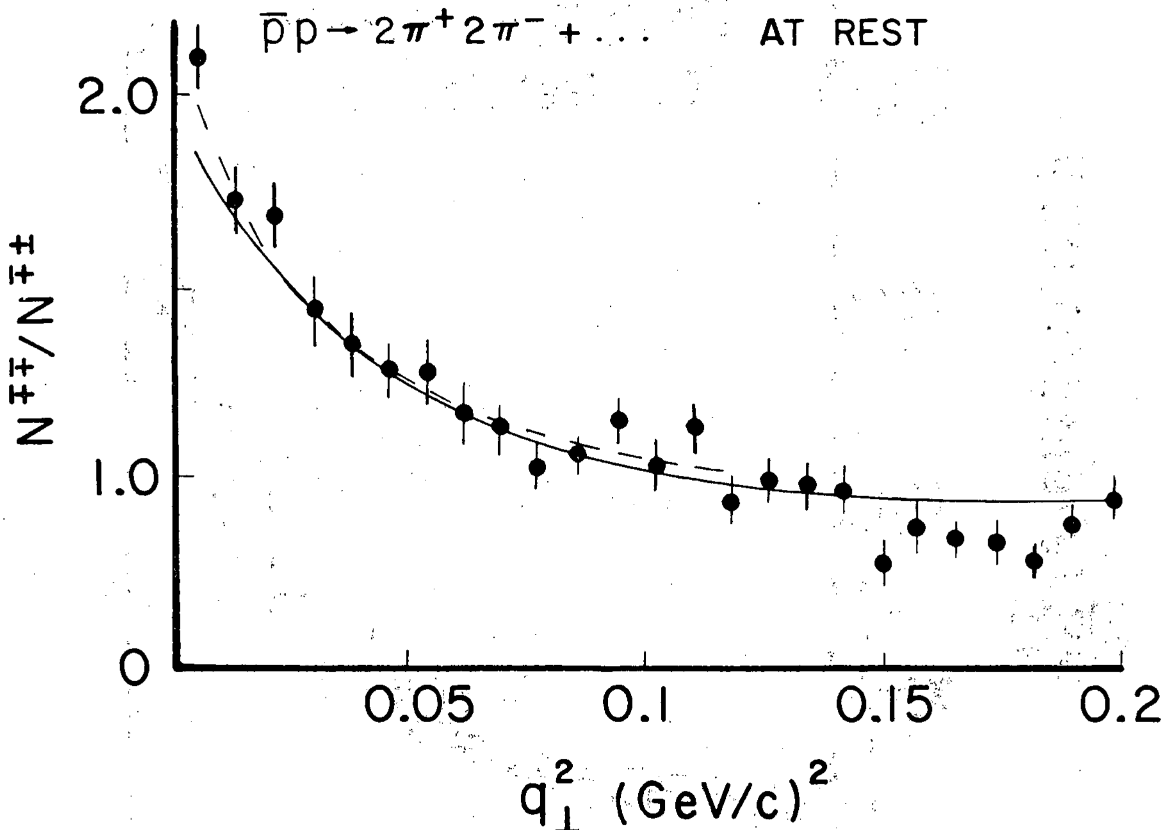
XBL 817-10566

Fig. 1



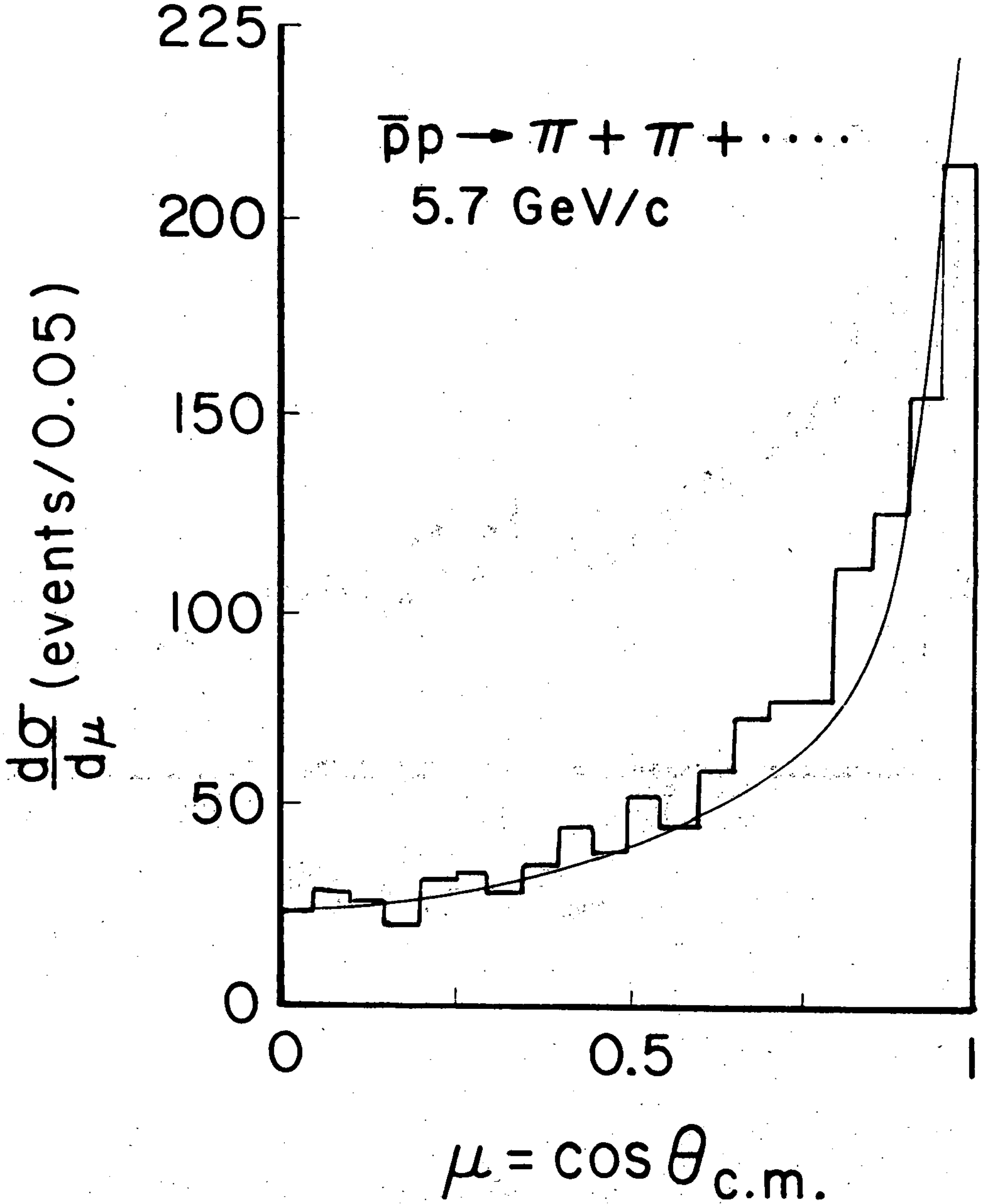
XBL 817-10567

Fig. 2



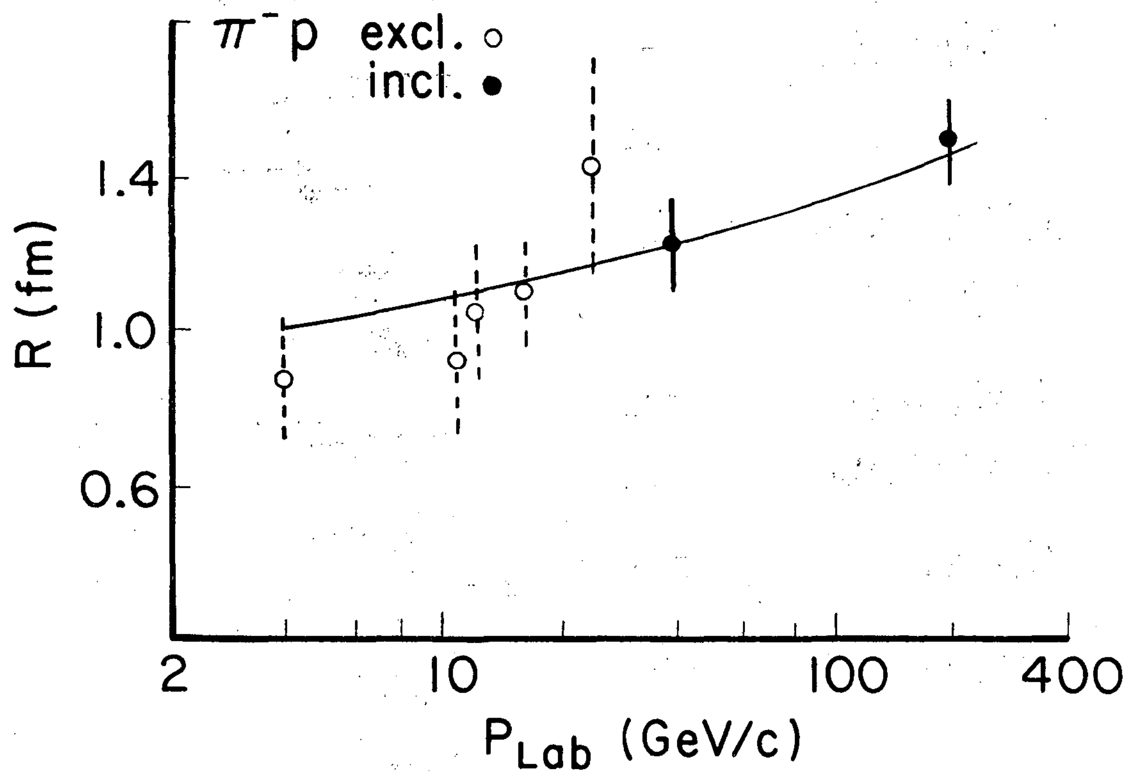
XBL 817-10568

Fig. 3



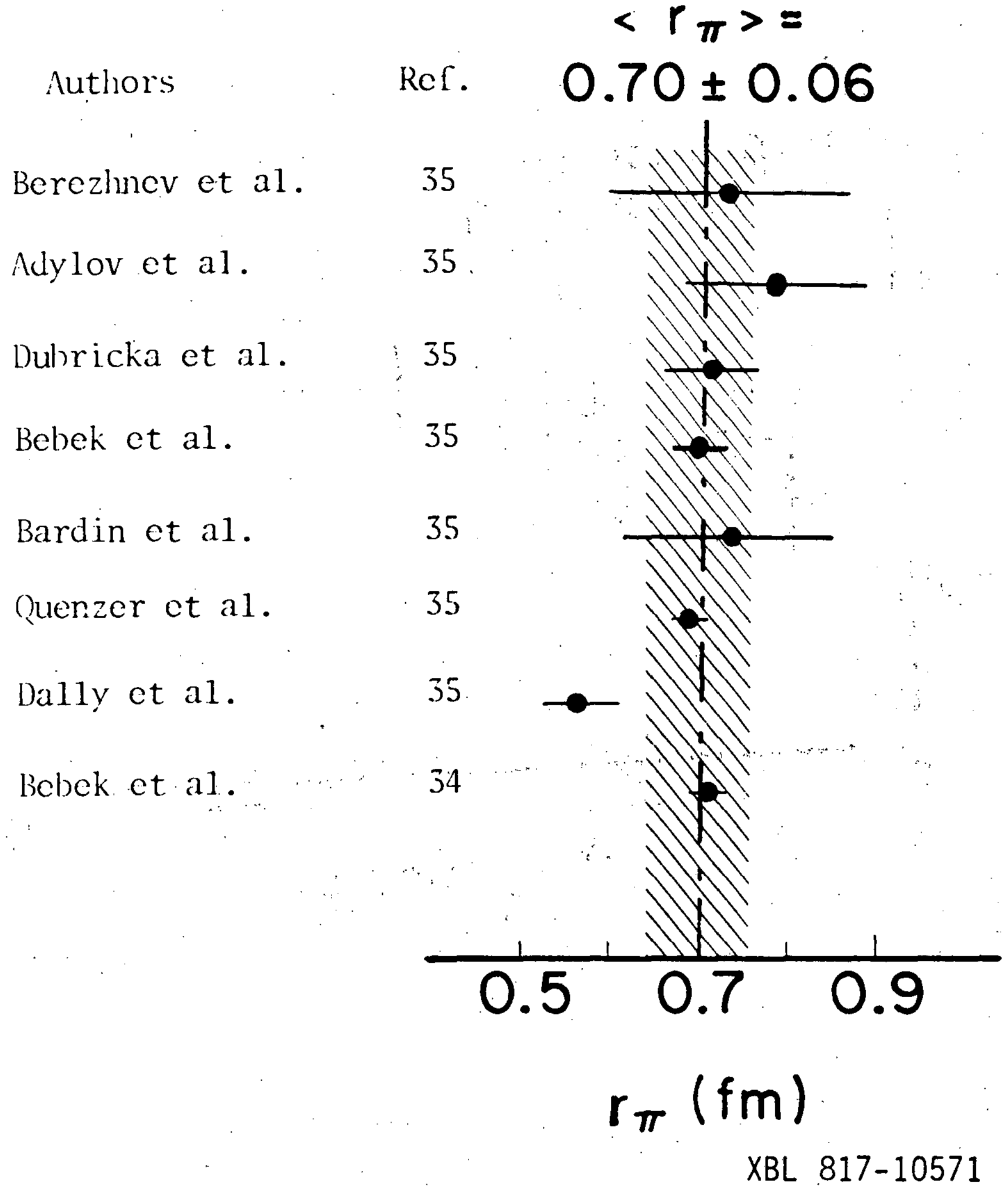
XBL 817-10569

Fig. 4



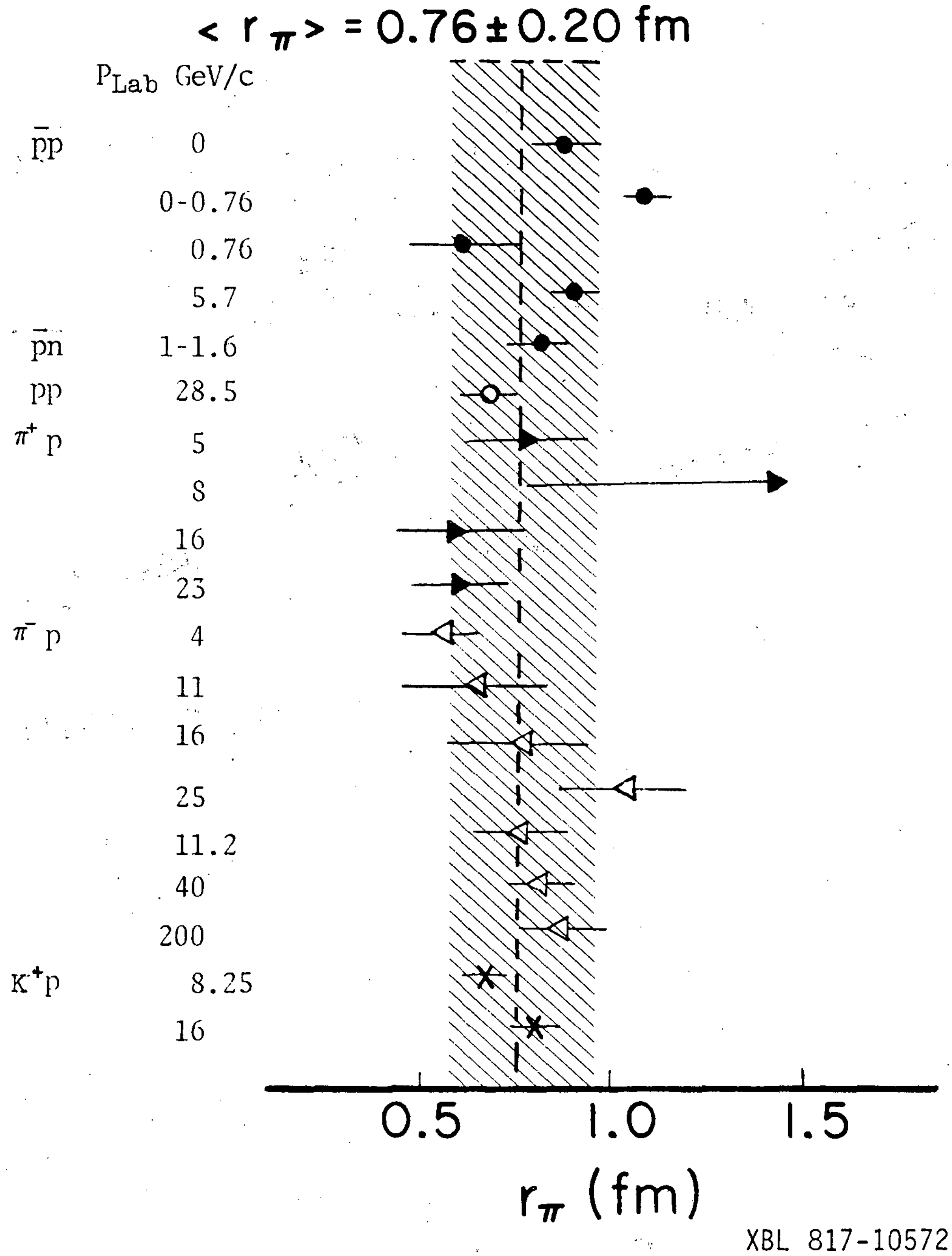
XBL 817-10570

Fig. 5



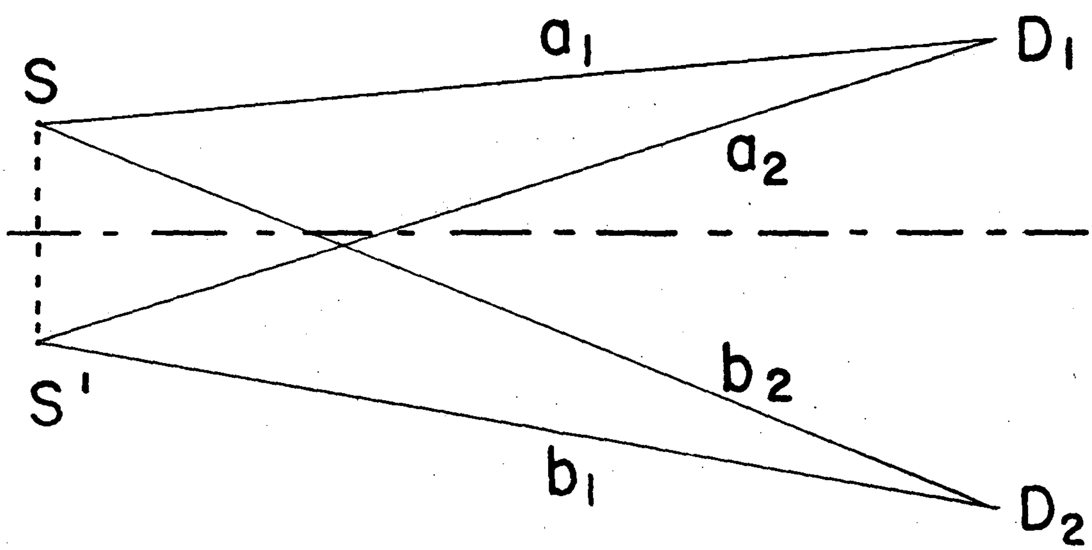
< r_π > =
0.70 ± 0.06
Authors
Ref.
Berezhnev et al.
35
Adylov et al.
35
Dubricka et al.
35
Bebek et al.
35
Bardin et al.
35
Quenzer et al.
35
Dally et al.
35
Bebek et al.
34
0.5
0.7
0.9
r_π (fm)
XBL 817-10571

Fig. 6



< r_π > = 0.76 ± 0.20 fm
P_{Lab} GeV/c
$\bar{p}p$
0
0-0.76
0.76
5.7
$\bar{p}n$
1-1.6
pp
28.5
π^+p
5
8
16
23
π^-p
4
11
16
25
11.2
40
200
K^+p
8.25
16
0.5
1.0
1.5
r_π (fm)
XBL 817-10572

Fig. 7



XBL 817-10573

Fig. 8

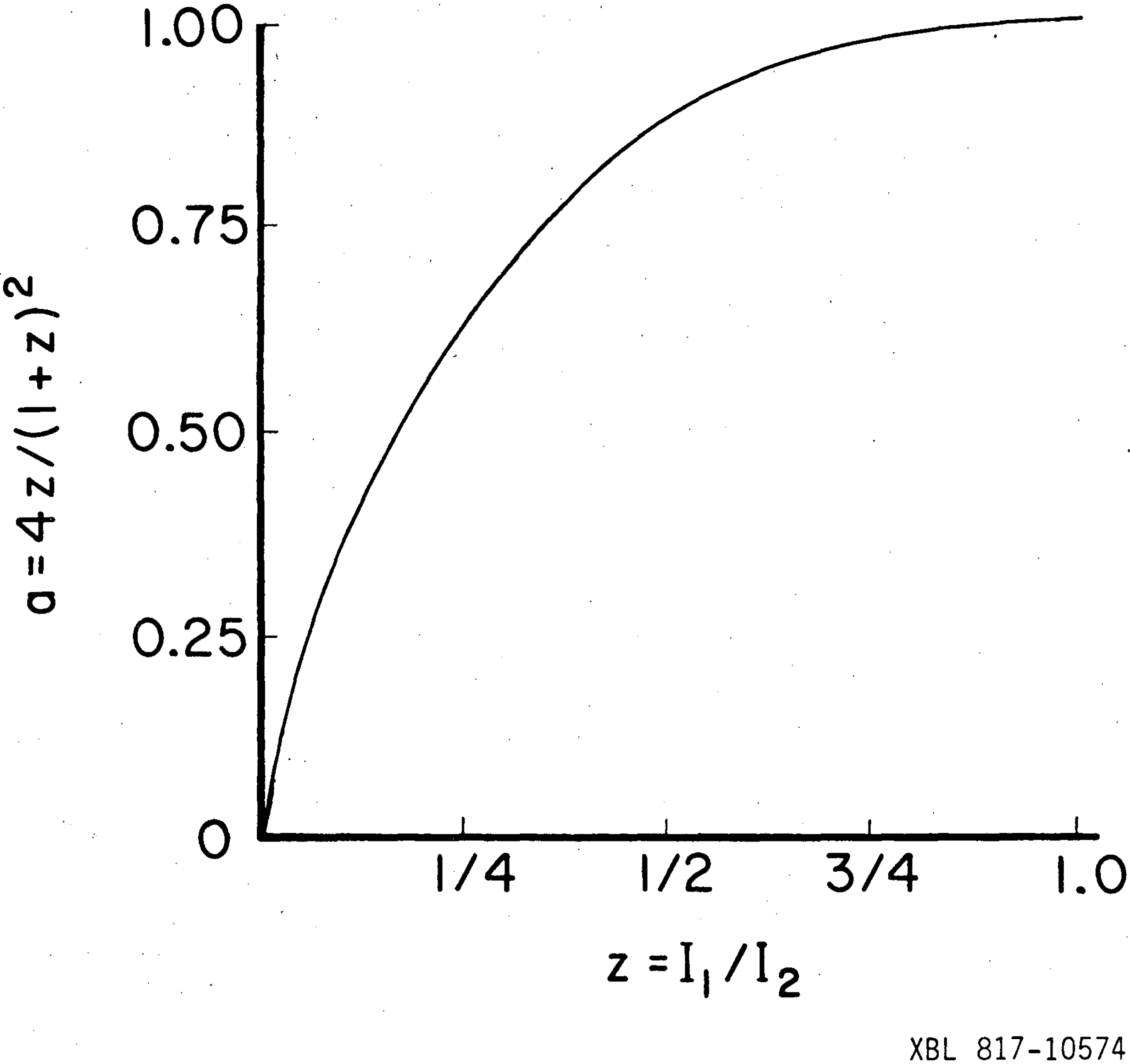


Fig. 9

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720