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# EFFECTS OF THE PION-PION RESONANCE AND THE THREE-PION RESONANCE OR BOUND STATE ON NEUTRAL PION DECAY*

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Recently Frazer and Fulco[1] and Chew[2] have proposed that a two-pion P-wave resonance and a three-pion resonance or bound state may account respectively for the isotopic vector and scalar components of nucleon electromagnetic structure. The purpose of this note is to investigate the effects of such resonances on neutral-pion decay.

The dispersion analysis of neutral-pion decay was first considered by Goldberger and Treiman,[3] but they assumed nucleon-antinucleon pairs to be the most important intermediate states and neglected multipion states. Here we adopt a different approach and consider the contributions of the least massive states. This can be done if we extend a photon variable $q^2$ into the complex plane instead of the meson variable $p^2$ used by Goldberger and Treiman.

Following the standard method, one has[4] (see Fig. 1)

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† A preliminary account of this work was given at the 1959 Thanksgiving meeting of the American Physical Society, November 27-28, 1959, Cleveland, Ohio, [How-sen Wong, Bull. Am. Phys. Soc. 4, 407 (1959)].

$$(q(\mu), k(\nu)\ |T|\ p(5)) = \frac{i(2\pi)^4\ \delta^4(p-q-k)\ F_\nu(-q^2; -k^2; -p^2) e_\nu'}{(8\ q_0\ k_0\ p_0)^{1/2}}, \tag{1}$$

where we have

$$F_\nu = (4\ p_0\ q_0)^{1/2}\ \langle\ q(\mu)\ |\ J_\nu(0)\ |\ p(5)\ \rangle\ ,$$

and $p$ is the pion four-momentum. The indices $\mu$ and $\nu$ refer to the polarization of the photons of momenta $q$ and $k$, respectively. The number "5" inside the matrix element represents a neutral pion in the initial state.

From general invariance arguments, the $F$ function can be written in the form

$$F(-q^2; -k^2; -p^2) = \epsilon_{\alpha\beta\mu\nu}\ q_\alpha\ k_\beta\ e_\mu\ e_\nu'\ f(-q^2; -k^2; -p^2)\ . \tag{2}$$

We assume that, with both $p^2$ and $k^2$ on the mass shell, the scalar function $f(-q^2)$ satisfies the following dispersion relation without subtraction:

$$f(-q^2) = \frac{1}{\pi} \int_4^\infty \frac{\mathrm{Im}\ f(t)\ dt}{t + q^2}\ . \tag{3}$$

The lifetime of $\pi^0$ is then given by

$$\tau = 64\ \pi\ [\ f(0)\ ]^{-2}\ . \tag{4}$$

Using the unitary condition, we can express the absorptive part of $F$ as

$$\mathrm{Im}\, F = \pi\, \epsilon_\mu\, \epsilon_\nu' \sqrt{2p_0} \sum_n \delta^4(q - p_n) \langle 0|J_\mu(0)|n\rangle\langle n|J_\nu(0)|\,p(3)\rangle \,.$$

Since our approach is to assume that the function is determined by near-by singularities, no intermediate state except the $2\pi$ and $3\pi$ states are considered here. Let us first investigate the effect from the two-pion P-wave resonance alone and leave the $3\pi$ contribution to be discussed later; then we can show

$$\mathrm{Im}\, F(-q^2) = \frac{e_r}{48\pi}\, \epsilon_{\alpha\beta\mu\nu}\, q_\alpha k_\beta \epsilon_\mu \epsilon_\nu' \times \frac{(-q^2-4)^{3/2}}{(-q^2)^{1/2}}\, F_\pi^\dagger(-q^2)\, M_1(-q^2) \,, \tag{5}$$

where $F_\pi^\dagger$ is the Hermitian conjugate of $F_\pi$, the pion form factor, and $M_1$ is the P-wave amplitude for photopion production from pions. It has been shown by the author that the $M_1$ function is linearly related to a real constant $\Lambda$ and the $F_\pi$ function.[5] Comparing Eqs. (2) and (5), we obtain

$$f_{2\pi}(-q^2) = \frac{e_r}{48\pi^2} \int_4^\infty \frac{(t-4)^{3/2}}{\sqrt{t}\;(t+q^2)}\, F_\pi^\dagger(t)\, M_1(t)\, dt \,. \tag{6}$$

If the Frazer-Fulco form-factor function is used for a resonance at $t_{2\pi} = 10$ we find $2 \times 10^{-16}$ sec $< \tau <$ $4 \times 10^{-16}$ sec for $1.8\, e \geq |\Lambda| \geq 1.3\, e$.[6] Thus we have shown that the contribution of two-pion resonance is capable of producing a large effect in the neutral pion decay.

We note that the $f$ function in Eq. (6) is function in the virtual-photon mass variable. Thus $-x \equiv q^2 = (P_+ + P_-)^2$ represents the square of total four momentum of the electron-positron pair in the process $\pi^0 \to \gamma + e^+ + e^-$. Since $x$ is less than 1, we can write

$$f(x) = f_{2\pi}(0)(1 + \alpha x) \cong f_{2\pi}(0)(1 + \frac{x}{t_{2\pi}}) \tag{7}$$

for small $x$.[7] Thus, we see that $\alpha$ is always positive in this approximation.

The calculation so far is based on the assumption that only the $2\pi$ state contributes to the dispersion integral, but there is no good reason to expect the $3\pi$ contribution to be negligible. Since no one has succeeded in treating the matrix element $(\gamma\pi \mid 3\pi)$, we are not able to handle this part. However, we may observe that if the three-pion $I = 0$ and $J = 1$ state is resonant or bound at energy $\sqrt{t_{3\pi}}$, as suggested by Chew, then

$$f_{3\pi}(-q^2) = \frac{\text{constant}}{q^2 + t_{3\pi}} ,$$

so that

$$f(-q^2) = f_{2\pi}(-q^2) + f_{3\pi}(-q^2) \approx \frac{f_{2\pi}(0)}{1 + \frac{q^2}{t_{2\pi}}} + \frac{f_{3\pi}(0)}{1 + \frac{q^2}{t_{3\pi}}} .$$

Thus, if we define

$$\beta = \frac{f_{3\pi}(0)}{f_{2\pi}(0)} ,$$

then for small $x (\equiv -q^2)$,

$$f(x) \cong f(0)\left[1 + \left(\frac{1}{1+\beta}\right)\left(\frac{1}{t_{2\pi}} + \frac{\beta}{t_{3\pi}}\right)x\right]. \qquad (8)$$

Thus, if the $2\pi$ and $3\pi$ states make comparable contributions (as they do in the nucleon charge structure) and $\beta$ has a negative sign, it is possible that the parameter $\alpha$ defined in Eq. (7) to be negative.[8] An attempt is in progress to determine $\beta$ from other experimental data involving the $3\pi$ state.

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## REFERENCES

1. W. R. Frazer and J. R. Fulco, Phys. Rev. Letters **2**, 365 (1959); Phys. Rev. **117**, 1609 (1960).

2. G. F. Chew, Phys. Rev. Letter **4**, 142 (1960).

3. M. L. Goldberger and S. Treiman, Nuovo cimento **9**, 451 (1958).

4. We use the fundamental metric tensor such that $-p^2 = M^2$. Units are used in which $\hbar = c = \mu = 1$, where $\mu$ is the mass of the pion.

5. How-sen Wong, Phys. Rev. Letters (to be published in July 15, 1960 issue).

6. James S. Ball, "The Application of the Mandelstam Representation to Photoproduction of Pions from Nucleons," UCRL-9172, April 11, 1960 (unpublished). Dr. Ball has applied Mandelstam representation to the $\gamma + N \rightarrow \pi + N$ problem, and finds that $|\lambda|$ is less than 1.8 in order to make his calculated cross section compatible with experimental data.

7. S. M. Berman and D. A. Geffen, "Electromagnetic Form Factor of the $\pi^0$ and the Decay Mode $\pi^0 \rightarrow \gamma + e^+ + e^-$," preprint (no date). These authors have also studied the $f$ function from a different approach and obtained $\alpha \approx 0.06$.

8. The author is indebted to Dr. N. Samios of Brookhaven National Laboratory for advance communication of his preliminary experimental result that $\alpha = -0.24 \pm 0.16$.

## FIGURE CAPTION

Fig. 1. Neutral pion decay. Wavy lines are photons; broken line, pion.

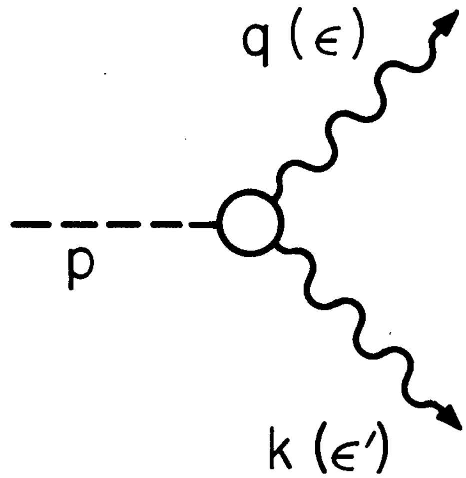


Fig 1
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