Title
Supporting Development of Mathematics Teaching Practices in the Context of the Common Core: An Action Research Model of Professional Development

Author
Michel, Lisa

Publication Date
2015

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA

Los Angeles

Supporting Development of Mathematics Teaching Practices in the Context of the Common Core: An Action Research Model of Professional Development for Upper Elementary Teachers

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Education

by

Lisa Michel

2015

ABSTRACT OF THE DISSERTATION

Supporting Development of Mathematics Teaching Practices
in the Context of the Common Core:
An Action Research Model of Professional Development
for Upper Elementary Teachers

by

Lisa Michel
Doctor of Education
University of California, Los Angeles, 2015
Professor Megan Loef Franke, Chair

Procedural learning has dominated the study of K-12 mathematics in the United States. A lack of emphasis on the development of mathematical thinking among educators has undermined American students' achievement in mathematics, thereby impacting students' readiness for college and careers in the 21st century. Elementary teachers, in particular, are often under-equipped to tackle teaching practices that promote mathematical thinking and the development of conceptual mathematical knowledge, both emphasized in the Common Core Standards for Mathematical Practice. They are frequently restricted by their own content knowledge and

confidence in understanding mathematics, and are themselves products of rote learning and an overemphasis on procedures.

This study was designed to support development of mathematical teaching practices among a group of upper elementary (third through fifth grade) teachers. The study utilized a qualitative action research design in order to investigate teachers' perceptions of the development of their practice over time. Ten upper elementary teachers participated in a bi-monthly mathematics study group over a period of 6 months. The study group initially focused on using warm-up tasks to promote mathematical discourse. Data was collected in the forms of transcriptions of the study group sessions, written and verbal reflections, and pre and post-questionnaires. Throughout the study, teachers regularly experimented with practices they had rarely or never tried before. Teachers reported that the warm-ups they learned in the professional development were the driving force for changes in their mathematics teaching practices. Additionally, teachers reported that there were substantial differences in the student discourse that occurred during math time between the previous school year and this year and that facilitation of student discourse during math time became a regular part of their practice.

The dissertation of Lisa Michel is approved.

Christina Christie

Jaana Juvonen

Jody Priselac

Megan Loef Franke, Committee Chair

University of California, Los Angeles

2015

DEDICATION

This dissertation is dedicated to the 10 elementary school teachers who participated in my study, and to teachers everywhere who, like them, give children plentiful opportunities to talk, who take an interest in what children say, and who work to ask the questions that will advance children's thinking in mathematics.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACKNOWLEDGEMENTS

I am honored to have had Dr. Megan Franke as the chair of my committee. I thank her for her generosity of time, her sharing of her expertise, and her ability to drive my thinking. She is the perfect model for asking purposeful questions. I also want to thank my other committee members, Dr. Tina Christie, Dr. Jody Priselac, and Dr. Jaana Juvonen, who have supported me throughout this process. I have learned so much from our conversations.

I thank Dr. Cindy Kratzer for her consistent willingness to give constructive and productive feedback. I thank Dr. Diane Durkin for her constant support and guidance, especially with regard to my writing. I thank Dr. Eugene Tucker, Dr. Barbara Jaffe, and Dr. Linda Rose who have served as wonderful supporters of my work throughout these last three years.

I would also like to thank my ELP Cohort 20 friends and study buddies. I don't know what I would have done without the support of my friends and colleagues in this program, especially since I had no time for others.

Lastly, I must thank my friends and family, who willingly allowed me to put them "on hold" while I have been in this program. I look forward to making up for lost time. And to my sister, Deborah Michel Rosch: you are my rock. I most literally could not have made it through without your support, your love, your edits, your cheerleading. I am the luckiest sister in the world.

VITA

1982-1986	B. A. in English Tufts University Medford, MA
1986-2000	Professional Jazz Vocalist
1992-1994	Coursework in Mathematics City University of New York: Brooklyn College Brooklyn, NY
1994-1995	M.S. in Teaching New School for Social Research New York, NY
1995-1997	Math Teacher Ballet Tech New York, NY
2001-2010	Math Teacher Culver City High School Culver City, CA
2010-2012	Research Associate WestEd, STEM Division Redwood City, CA
2012-present	Assistant Principal, Curriculum Culver City High School Culver City, CA

PUBLICATIONS

McCarthy, B., Michel, L., Tiu, M., Atienza, S., Rice, J., Nakamoto, J., & Tafoya, A. (October, 2011). Evaluation of *The Electric Company* summer learning program. WestEd: San Francisco, CA.

CHAPTER 1: STATEMENT OF THE PROBLEM

Rote learning and attention to procedure at the expense of propelling mathematical thinking have dominated mathematics teaching and learning in the United States (Conference Board of the Mathematical Sciences [CBMS], 2012; Ma, 1999; National Research Council [NRC], 2001). Such teaching and learning has led to too many American students unable to show understanding of basic concepts in algebra by the time they leave high school, thereby restricting both their access to and persistence in college (Adelman, 2006). In addition, career opportunities brought on by the current economy in science, technology, engineering, and mathematics (STEM) fields are severely restricted for the many students who leave high school without understanding basic algebra concepts (EdSource, 2011b).

American teachers are products of an overemphasis on procedural learning. Though it has been more than a decade since mathematics teaching experts have documented the necessary processes and practices for teaching and learning mathematics (National Council of Teachers of Mathematics [NCTM], 2000; NRC, 2001), teachers' practices have not changed, and American students continue to underperform in mathematics (National Center for Education Statistics [NCES], 2013; Organisation for Economic Co-operation and Development [OECD], 2012; EdSource, 2011b). The Common Core Standards (National Governors Association Center for Best Practices [NGACBP] & Council of Chief State School Officers [CCSSO], 2010) were collaboratively developed by experts and educators in order to improve American students' college and career readiness. The mathematical content standards include eight standards for mathematical practice that demand that teachers have a strong conceptual foundation for the mathematics they are teaching. Unless teachers have opportunities for professional development, they will be ill equipped to address the learning needs of their students.

Background Information

A recent longitudinal study (Finkelstein et al., 2012) examined mathematics course-taking patterns for 24,000 California students who were continually enrolled in districts from grade 7 through grade 12. Researchers found that mathematics performance in grade 7 is predictive of high-school mathematics course taking, with those performing well in grade 7 more likely to take more advanced courses in high school. Grade 7 mathematics standards in California are generally considered to be pre-algebra standards. A little less than 50% of students in California do not score at the proficiency level in mathematics in grade 7 (EdSource, 2011a), making it more difficult to successfully complete algebra and advanced algebra (what they need to be considered *college-ready*) by the time they leave high school. If performance in seventh grade mathematics is the strongest predictor of success in college preparatory mathematics, it is essential to examine mathematics preparation prior to the seventh grade.

American students spend most of their time learning mathematics through extensive teacher-directed explanation, followed by seatwork (CBMS, 2012; NRC, 2001). Despite the prevalence of student underachievement in mathematics, mathematical teaching practices continue to be dominated by demonstrations of computational procedures and repeated practice (CBMS, 2012). Two broad features of instruction have been shown to promote students' conceptual knowledge of mathematics: explicit attention to concepts through mathematical discourse and giving students the space to struggle (Richland, Stigler, & Holyoak, 2012). Neither of these features of instruction is often seen in U.S. classrooms (Stigler & Hiebert, 1999).

Among school-related factors, teachers "matter more to student achievement than any other aspect of schooling" (RAND Corporation, 2009, p. 1). Yet many elementary teachers do not have requisite conceptual understanding of important mathematical concepts so that they can

help students' mathematical understanding (Hill, Rowan, & Ball, 2005; Ma, 1999). Hill et al. (2005) further distinguish mathematical content knowledge from "mathematical knowledge for teaching" (p. 373), finding that elementary teachers' knowledge used to carry out the work or practice of teaching mathematics positively predicts student gains in mathematics achievement.

While no single model of professional development emerges as the most successful in supporting school mathematics reform, professional development focused on teacher practice has the potential to impact both teacher learning and student achievement in mathematics (Borasi & Fonzi, 2002). Additionally, best practices in professional development invite teachers to take an inquiry approach to learning and teaching: to be active participants in constructing their professional development experience and to have the ability to share and analyze their own classroom experiences (Sowder, 2007; Walker, 2007). This requires a sustained and ongoing model of professional development within a school or classroom context so that teachers have the time to develop their own deep understanding of the mathematics that they are required to teach (Ma, 1999; Walker, 2007) directly tied to their practice of teaching it (Sowder, 2007).

Research Questions

As districts moved towards implementing the Common Core standards, I conducted a study that engaged elementary school teachers to improve their practice for teaching mathematics. I established a cross-school, cross-grade level mathematics study group setting for upper elementary teachers (grades 3, 4, and 5). The mathematics study group used particular classroom activity structures to engage teachers in developing their practice in relation to each other and in line with the Common Core Standards for Mathematical Practice. In particular, the study group initially focused on using warm-up tasks to promote student discourse. The following research questions guided the study:

1. In what ways does a mathematics study group support upper elementary teachers' self-reported development of classroom practice?
2. In particular, what aspects of the professional development support the development of their practice?

Research Population

The study took place in a small, southern Californian, urban unified school district that represents a diverse student population. The district consists of five elementary schools, one middle school, and one high school serving approximately 6,500 students. It also includes a pre-school program, an adult school, a continuation high school, and an independent study school. In 2013 the district's California Standards Test (CST) results showed that 57% of seventh grade students were proficient in mathematics, and less than 50% of students were proficient in Algebra 1, Geometry, and Algebra 2 across all grade levels.

A preliminary analysis of fifth and sixth grade CST results from a cohort of students ($N = 269$)[1] who graduated from the district's high school in 2013 revealed that in this district, mathematics performance in grade 6 may be predictive of success in college-ready mathematics courses (Algebra 2 and above) in high school. Sixty-three percent of the students scored Proficient or Advanced on the fifth grade test, while 52% scored Proficient or Advanced on the sixth grade test. Further, 92% of the students that scored Proficient or Advanced on the sixth grade test reached Algebra 2 or higher level mathematics by their junior year. Of these students, 71% reach proficiency on their Algebra 2 or higher level mathematics CST. In contrast, only 39% of students who did not score Proficient on their sixth grade CST go on to Algebra 2 or

[1] These students were enrolled in the district from at least fourth grade on, and had CST results for fourth, fifth, and sixth grade.

higher level mathematics by their junior year and only 8% of them reach proficiency on their Algebra 2 or higher level mathematics CSTs.

Although this school district outperforms the average California urban district, it is still representative of the problem of students unable to successfully master college preparatory mathematics. By recruiting upper elementary (grades 3 through 5) teachers in the district to participate in a mathematics study group, the study addressed the problem through focusing on these teachers' perceived changes in practice and built upon not only their content and pedagogical knowledge, but specifically their pedagogical knowledge for teaching mathematical content.

Overview of the Research Design

This study utilized a qualitative action research design in order to investigate teachers' perceptions of the development of their practice over time. The collaborative, participatory nature of the study demanded an action research design, as teachers worked together towards understanding and changing their classroom practice over time. Ten upper elementary teachers participated in a mathematics professional development opportunity that I facilitated. The format of the professional development was a mathematics study group initially focused on using warm-up tasks to promote mathematical discourse. The study group met approximately twice a month outside of school hours. Teachers engaged in tasks that I modeled and then developed their own tasks that they tried out in their classrooms. The study group was sustained over a period of months, according to best practices in professional development for teachers, so that teachers had the opportunity to not only apply their learning to their own classroom, but also to debrief, analyze, and reflect upon their practice. After the first five study group sessions, teachers had the opportunity to participate in the planning of the rest of the study group sessions.

Significance of the Research

Results from the study have the potential to guide how districts plan to develop their elementary teachers' practices to teach mathematics. Professional developers can also learn from what teachers say about what helps them to develop their practice. The mathematics study group model has the potential to positively impact teacher practice, thereby impacting student success in mathematics in urban public school districts.

CHAPTER 2: LITERATURE REVIEW

Rote learning and attention to procedure rather than mathematical thinking have dominated mathematics teaching and learning in the United States (CBMS, 2012; Lambdin & Walcott, 2007; Ma, 1999; NRC, 2001). Such teaching and learning undermines American students' mathematical achievement, thereby restricting both their access to and persistence in college, as well as career opportunities in our STEM-focused economy (Adelman, 2006; EdSource, 2011b). Additionally, teachers are themselves products of rote learning and an overemphasis on procedures. Elementary teachers, in particular, often lack both content knowledge and confidence to combat their own beliefs and attitudes about mathematics, making it difficult for them to positively impact students' mathematical proficiency (Briley, 2012; Gresham, 2008; Harbin & Newton, 2013; Mewborn, 2001; Stipek, Givvin, Salmon, & MacGyvers, 2001; Tatto & Senk, 2011; Walker, 2007). Though it has been more than a decade since mathematics teaching experts have documented the necessary processes and practices for teaching and learning mathematics (NCTM, 2000; NRC, 2001), teachers' practices have not fundamentally changed, and American students continue to underperform in mathematics (EdSource, 2011b; NCES, 2013; OECD, 2012). In an effort to improve American students' college and career readiness, experts and educators collaborated to develop the Common Core State Standards (NGACBP & CCSSO, 2010).

Currently, 42 out of 50 of states are in the process of implementing the Common Core Content Standards for Mathematics (NGACBP & CCSSO, 2010). These grade level content standards include eight Standards for Mathematical Practice, all of which are to be incorporated at each grade level. The Standards for Mathematical Practice emphasize mathematical proficiency through the development of problem solving processes and a conceptual

understanding of mathematics. These standards require teachers to engage in practices with their students that have not been commonly utilized in the United States, and therefore teachers need opportunities to develop these practices. This literature review makes the case for addressing the problem of U.S. mathematics underachievement by focusing on professional development that supports teachers' practice.

I begin the review contextualizing the need for mathematics professional development opportunities with a discussion of American K-12 students' lack of proficiency in mathematics and related outcomes, particularly in the state of California. The review then focuses on longstanding American teaching practices and beliefs about mathematics that may thwart students' mathematical proficiency. I then present the research behind mathematical teaching practices and beliefs in countries that fare better than the United States on student achievement tests. Following this discussion, I give an overview of some of the possible areas for focus in the planning of how to support mathematics teacher learning. Finally, and most importantly, I review effective professional development models for teaching mathematics and changing teacher practice.

U. S. Students' Lack of Proficiency in K-12 Mathematics and Related Outcomes

State, national, and international mathematics achievement exams demonstrate that the United States has done at best a mediocre job of developing students' mathematical proficiency (EdSource, 2011b; NCES, 2013; OECD, 2012). The 2013 results of the NAEP mathematics assessment (NCES, 2013), while better than previous years, still had 58% of fourth graders and 64% of eighth graders performing below proficient. In California the results were worse: 68% of fourth graders and 73% of eighth graders performed below proficient. The NAEP assessment measures not only students' mathematical knowledge and skills, but also their ability to apply

their knowledge in problem-solving situations across content areas such as number properties and operations, measurement, data analysis, and algebra.

Internationally, the United States does not fare well. The results of the 2012 Program for International Student Assessment (PISA; OECD, 2013), which tests 15-year-olds in mathematics, science, and reading, found that the United States ranked below the average in all categories. American students placed 31st in mathematics, slipping from 29th place in 2009. While the results of such assessments keep public attention on the need to reform mathematics teaching, a more compelling argument for reform is how a lack of mathematics achievement impacts other student outcomes.

A recent longitudinal study (Finkelstein, Fong, Tiffany-Morales, Shields, & Huang, 2012) examined mathematics course-taking patterns for 24,000 California students who were continually enrolled in districts from grade 7 through grade 12. Researchers found that mathematics performance in grade 7 is predictive of high-school mathematics course taking, with those performing well in grade 7 more likely to take more advanced courses in high school. Grade 7 mathematics standards in California are generally considered to be pre-algebra standards. A little less than 50% of students in California do not score at the proficiency level in mathematics in grade 7 (EdSource, 2011a), making it more difficult to successfully complete algebra and advanced algebra (what they need to be considered 'college-ready') by the time they leave high school. In fact, a key marker toward completion of a bachelor's degree is getting beyond the Algebra 2 level in high school (Adelman, 2006). If performance in seventh grade mathematics is the strongest predictor of success in college preparatory mathematics, it is essential to examine mathematics preparation prior to the seventh grade. The prevalent picture of

what a mathematics lesson looks like, however, is consistent across grade levels in the United States.

Mathematical Teaching and Learning Practices and Beliefs

American students spend most of their time learning mathematics through extensive teacher-directed explanation, followed by seatwork (CBMS, 2012; NRC, 2001). Observations of mathematics classrooms over the past few decades (NRC, 2001; Stigler & Hiebert, 1999) found that the typical lesson begins with a homework check or warm-up activity. The teacher then presents several problems and demonstrates the process for solving them. Students either observe passively, or respond to close-ended questions in several words or less. Students then spend class time practicing exercises similar to those demonstrated by the teacher. Despite the prevalence of student underachievement in mathematics, mathematical teaching practices continue to be dominated by demonstrations of computational procedures and repeated practice (CBMS, 2012).

Demonstrating this attention to procedure, Richland et al. (2012) presented results from two studies on the mathematical knowledge of students in a community college. They found that students tend to believe that mathematics is a collection of rules, procedures, and facts that should be memorized. In addition, students often inappropriately apply procedures that have been memorized to problems that don't warrant them. Students also fail to make comparisons across problems that might help them to identify structural or conceptual similarities that, in turn, would help them solve the problems.

Teaching methods correlate with such findings. A recent study found that when elementary teachers emphasized memorizing facts over solving novel problems, student achievement grew more slowly (Desimone, Smith, & Phillips, 2013). Richland et al. (2012) cite Hiebert and Grouws' 2007 review of the research in which measures of student learning were

empirically related to features of teaching. Two broad features of instruction were shown to promote students' conceptual knowledge of mathematics: explicit attention to concepts through mathematical discourse and giving students the space to struggle. Discourse provides the opportunity to understand the meaning behind the procedures, and struggling refers to the process of making sense of mathematics. Students need to spend time and effort to figure out what is not immediately apparent to them. Neither of these features of instruction is often seen in U.S. classrooms. Americans traditionally view good teaching as minimizing confusion and frustration during the learning process. Frustration and confusion are assumed to be signs that previous material has not been mastered (Stigler & Hiebert, 1999). The American practice of teaching mathematics by incrementally introducing procedures and having students practice them over and over correlates to the general American view of mathematics as a set of procedures to be learned. Elementary mathematics teaching is particularly susceptible to this view.

The conventional American belief about elementary mathematics is that it is *simple*, and one does not need to know more than prescribed facts and computational algorithms in order to teach it (CBMS, 2001; Walker, 2007). This belief that teaching elementary mathematics does not require a deep understanding of mathematics is supported by findings from a comparative study of U.S. and Chinese elementary teachers' mathematical knowledge (Ma, 1999). Ma (1999) found that in contrast to the U.S. teachers' view of elementary mathematics as basic, Chinese teachers' attitudes mirrored the attitudes of practicing mathematicians, in that they were concerned with justifying explanations, giving multiple solutions for a problem, and discussing relationships among arithmetic operations. Chinese teachers worked at making sense of procedures and understanding the rationale behind the algorithms they were teaching.

Like the Chinese, the German and Japanese have differently structured classrooms where more advanced reasoning is expected, and students fare better than Americans on mathematics achievement tests. The Third International Mathematics and Science Study (TIMSS) included a video study component that compared the teaching of eighth-grade mathematics in the United States, Germany, and Japan. While it was found that in German classrooms, like in classrooms in the United States, teachers focused on procedure, the level of procedure development was much more advanced. In contrast, Japanese teachers take a less active role in presenting procedures, and expect students to struggle to invent their own procedures for solving problems. Japanese students are generally presented with a problem, and then work individually or in groups to work out solution methods. Then class time is taken to discuss different solution methods. Teachers often strategically select students, rather than take volunteers, to share their methods. The practice of Japanese teachers is generated by the belief that mathematics involves relationships between concepts, facts, and procedures. These relationships are revealed through talking explicitly about the process of developing, studying, and improving upon solution methods to problems. Japanese teachers' teaching practices honor confusion and frustration as a natural part of the learning process, seeing it as an important step in making sense of the content. Japanese teachers create opportunities for their students to struggle, and then participate in discussions about the pros and cons of solving problems through different methods, as well as the relationships between these methods (Stigler & Hiebert, 1999).

Attempts over the last century to reform the teaching and learning of mathematics in the United States have been thwarted by an entrenched culture of what teaching looks like. Stigler and Hiebert (1999) frame teaching as a cultural activity, where people within a culture share a mental picture, or *script* of what teaching is like. A cultural script for teaching begins even

before a child enters school. Most Americans could enter a classroom and *act like a teacher* because this script is so ingrained. The New Math of the 1960s and '70s exemplifies the failure of a particular reform to change the culture of teaching. While this reform changed textbooks to shift the focus from the drill and practice phase of the early part of the twentieth century to an understanding of the structure of mathematics (Lambdin & Walcott, 2007), teaching practices remained unchanged (Stigler & Hiebert, 1999). The culture of teaching includes both the roles that teachers play and the roles that students learn to play. Changing one feature, such as a textbook, or a particular teaching strategy, will generally backfire as the basic system of teaching remains intact.

In fact, in a recent study of elementary mathematics teachers, Harbin and Newton (2013) found that the most powerful influence on teaching practice is the teachers' experience as a student. Teachers' practices were most influenced by their own teachers, whether or not those practices aligned with their beliefs about mathematics. Even a desire to be innovative is often thwarted by the power of former teacher models (Harbin & Newton, 2013; Mewborn & Cross, 2007). Many teachers come to the classroom with their own mathematical anxiety and lack of confidence with respect to teaching mathematics (Brady & Bowd, 2005; Bursal & Paznokas, 2006; Cornell, 1999; Gresham, 2008), and research shows a clear negative correlation between math anxiety and confidence to teach elementary mathematics. Research also shows a significant association between teachers' self-confidence as mathematics teachers and their students' self-confidence as mathematics learners (Stipek et al., 2001). The American culture for teaching and learning mathematics should be addressed in order to make meaningful reforms that will better serve students. The United States has traditionally done little more than prepare elementary

teachers to teach a set of discrete mathematical procedures (Greenberg & Walsh 2008; Tatto & Senk, 2011).

What do Elementary Teachers Need to Know in Order to Teach Mathematics?

Because elementary teachers may not be well prepared to teach mathematics, especially in the context of the newer Common Core standards, on-the-job professional development plays an important role in elementary teachers' learning (Borko, 2004; Sowder, 2007). Designing professional development that supports their learning is complicated by the fact that there are many possible lenses for professional development experiences. What follows is a discussion of several types of knowledge and understanding necessary for effective mathematics teaching that should be considered when planning professional development.

Many policy reports over the last number of years (National Academy of Education, 2009; NCTM, 2000; NRC, 2001) have made recommendations to improve the preparation and professional development of mathematics teachers. Educators and researchers agree that both teachers' content knowledge and pedagogical content knowledge are necessary to improve student achievement in mathematics (CBMS, 2012; NRC, 2001; Walker, 2007). The Conference Board of Mathematical Sciences (CBMS, 2012) issued a report containing recommendations for the mathematics that teachers should know in the context of teaching the Common Core State Standards (NGACBP & CCSSO, 2010). The report stemmed from themes espoused in the first CBMS (2001) report such as the intellectual substance of school mathematics, and the expectation that teachers have more mathematical knowledge than just proficiency with school mathematics (CBMS, 2012). The report outlined the widespread agreement among mathematicians and mathematics education researchers that teachers cannot rely on their past experiences as mathematics learners. It is not enough for teachers to study more advanced

mathematics than they will teach. Prior to teaching, elementary teachers, defined as K-5 in the report, should study the vast majority of K-5 mathematics, its connections to pre-kindergarten mathematics, and its connections to grades 6–8 mathematics. As teachers are products of the K-12 instruction they received, and undergraduate education does little to correct how mathematics is taught, many who opt to teach elementary school avoid all but required mathematics courses (CBMS, 2001). Compared to other countries with high-achieving mathematics students, elementary teachers in the United States do not have the same opportunities to learn many areas of school mathematics (Tatto & Senk, 2011).

A large quantitative comparative study that investigated the mathematics preparation of elementary and secondary teachers in 17 countries found that not only was there tremendous variation in the structure of teacher education systems, but also in mathematical content knowledge for teaching. Whereas future elementary teachers in the United States did well in applying whole number arithmetic in simple problem-solving situations, fewer than 50% of these teachers demonstrated ability to use fractions to solve story problems and familiarity with linear expressions and functions (Tatto & Senk, 2011). The U.S. future teacher performance aligns with teachers' overall learning opportunities at both the university and K-12 level.

Conceptual understanding is one of the five interdependent components that define mathematical proficiency[2] (NRC, 2001); however, many American teachers do not demonstrate this level of understanding themselves (CBMS, 2012; Mewborn, 2001; NRC, 2001; Walker, 2007). People with conceptual understanding are able to make connections both to prior knowledge and among mathematical facts, procedures, and ideas. They are able to contextualize why and when a mathematical idea is useful. Elementary teachers in China, where students

[2] The other components are: procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

outperform Americans on every international measure (Ma, 1999; OECD, 2013), demonstrate a conceptual structure for mathematics that has eluded many U.S. teachers. In a groundbreaking comparative study of U.S. and Chinese elementary teachers' knowledge of mathematics (Ma, 1999), Chinese teachers were found to be not only significantly more proficient in the procedural aspect of several elementary mathematics topics, but also demonstrated a deep conceptual understanding. Though the Chinese teachers had significantly fewer years of formal education than the U.S. teachers, they had clearly been educated differently. The Chinese teachers' knowledge of mathematics was based in understanding the rationale behind the algorithms they taught, enabling them to better direct their students' learning.

In addition to developing content knowledge and conceptual understanding, elementary teachers must develop their mathematical pedagogy (CBMS, 2012). In other words, pedagogy for teaching the content of mathematics is distinguished from mathematical content knowledge and pedagogy in general, and is necessary for effective teaching. The concept of pedagogical content knowledge, or knowledge that goes beyond subject matter and into the dimension of subject matter knowledge for teaching, stems from the work of Shulman (1986, 1987) and his associates. This type of knowledge includes "the ways of representing and formulating the subject that makes it comprehensible to others" (Shulman, 1986, p. 9) as well as an understanding of the preconceptions and misconceptions that students often bring to the subject, and what makes the learning of specific topics easy or hard. Research shows that teachers' mathematical knowledge for teaching relates significantly to student achievement (Hill et al., 2005; Ma, 1999).

Ideally, professional development experiences will support not only teachers' mathematical and pedagogical content knowledge and conceptual understanding, but will also

support their confidence and productive disposition towards mathematics and the teaching of mathematics. Trying to explicitly address all of these things at once, however, is a daunting prospect. By focusing on teachers' practice—the things teachers do on a daily or weekly basis with their students when teaching mathematics—professional developers can potentially support the necessary teacher learning for improving student achievement in mathematics. But how do we best frame professional learning experiences to support teachers' practice?

How Do We Develop Elementary Teachers' Practice for Teaching Mathematics?

While there are specific considerations for professional development around the teaching of elementary mathematics, Darling-Hammond, Wei, Andree, Richardson, and Orphanos (2009) of the School Redesign Network (SRN) at Stanford University found that, in general, effective professional development: is sustained, intensive, and connected to practice; focuses on student learning and addresses the teaching of specific curriculum content; and helps to develop strong working relationships among teachers. Effective professional development is defined as that which results in improvements both in teachers' knowledge and instructional practice and in student learning outcomes. The SRN report was based on a survey of the existing research, both qualitative and quantitative, on effective professional learning as well as data from a variety of national surveys[3] in order to examine available opportunities and supports for teachers' learning.

The amount of contact hours for professional development and how long it lasts impacts the effect on student achievement gains (Yoon, Duncan, Lee, Scarloss, & Shapley, 2007). Professional development that has a significant effect on teachers and student achievement gains offers substantial contact hours, ranging from 30-100 hours spread over 6-12 months. Limited professional development involving one-time workshops or hours ranging from 5 to 14 in total

[3] National Center for Education Statistics 2003-04 Schools and Staffing Survey, MetLife Survey of the American Teacher, The National Education Association's Survey of America's Teachers and Support Professionals on Technology, and the National Staff Development Council's Standards Assessment Inventory

does not have an impact on either teacher or student learning (Darling-Hammond et al., 2009; Yoon et al., 2007). Intensive professional development best influences teachers' practices when it includes applications of knowledge to teachers' planning and instruction (Garet, Porter, Desimone, Birman, & Yoon, 2001). This kind of professional development not only leads to gains in student learning, but also is most valued by teachers (Darling-Hammond et al., 2009; Garet et al., 2001).

Franke, Carpenter, Levi, and Fennema (2001) found that teachers who participated in a three-year professional development program on understanding the development of students' mathematical thinking continued to maintain some level of implementation of the program four years after it ended. Moreover, 10 out of 22 of the study participants expanded upon their learning and went on to create sustained learning communities at their sites. The professional development program did not provide instructional materials or prescribe practices; rather, teachers developed their own materials and practices through listening to and struggling to understand their students, and building on their students' mathematical thinking.

Teachers also value professional development that takes into account the resources and curriculum guidelines by which they must abide and models classroom practices they can immediately try out in their classrooms (Darling-Hammond et al., 2009; Garet et al. 2001). Kabasakalian (2007) facilitated professional development sessions with a group of mathematics teacher leaders that introduced a problem-solving protocol that teachers could utilize with their students. The purpose of her study was to utilize a study group to address the dynamic relationship between mathematics discourse and mathematics problem exploration. The teacher discourse that followed the presentation of a written problem revealed gaps in content knowledge and mathematical misconceptions, some of which were clarified through group discussion.

Kabasakalian found that the problem-solving protocol provided a "safe venue for teachers to talk about the underlying mathematics in a problem, the language in which it can be illuminated, and the pedagogical strategies for conveying it effectively" (p. 839). Teachers who had been with the professional development project for more than two years reported that not only was the protocol a part of their regular teaching practice, but that they engaged in much more problem solving with their students since working with the protocol. Effective professional development addresses the daily realities of learning specific academic subject matter (Darling-Hammond et al., 2009). Additionally, effective professional development creates and encourages the conditions for professional collaboration, thereby breaking the cultural norm of teachers working alone behind closed doors (Crespo, 2006; Darling-Hammond et al., 2009).

The SRN's findings are echoed in the research specific to professional development for teaching mathematics (Hill & Ball, 2004; NRC, 2001; Sowder, 2007; Walker, 2007). Sowder (2007) outlines research-based goals for professional development for teaching mathematics, which include: developing mathematical content knowledge; developing an understanding of how students think about and learn mathematics; developing pedagogical content knowledge; and developing a sense of self as a teacher of mathematics. This last goal involves developing the ability to explore mathematics intellectually and regularly engage in reflection of his/her own learning and practice within a community of teachers. These goals won't be achieved by telling teachers what to do or how to teach differently. Rather, professional development programs must give teachers the opportunities to develop new ways of thinking about learning, learners, and mathematics so that they can make the desired changes themselves. In other words, effective professional development allows teachers to construct their own professional knowledge base so that they can impact student learning.

Additionally, teachers are unlikely to change instructional practices if they do not see how change will enhance their students' learning (Crespo, 2006; Guskey, 2002; Franke et al., 2001; Sowder, 2007). Therefore, effective professional development includes opportunities for teachers to reflect both on their practice and on student responses to the practices they try out in the classroom. Sakshaug and Wohlhuter (2010) sought to not only improve elementary teachers' problem solving abilities, but also their abilities to teach mathematics through problem solving. Their study involved having teachers conduct action research in their own classrooms around the problems they were working on in a graduate course entitled Teaching Elementary School Mathematics. All 41 teachers participating in the study "demonstrated at least some belief in the power of the problem-solving approach for teaching mathematics" (p. 406).

Mathematics professional development in the form of facilitated teacher study groups have shown promise in expanding teachers' knowledge of students' thinking, and filling gaps in teachers' content knowledge and knowledge for teaching mathematics (Crespo, 2006; Franke et al., 2001; Kabasakalian, 2007; Zevenbergen, 2004). Few studies have followed teachers more than a year after professional development to track sustained teacher change, but the current movement towards in-school professional learning communities provide the structure for continued learning and improvement of practices among teachers (Schmoker, 2006).

As teacher change is a process, rather than an event, professional development that utilizes an action research design engages teachers in identifying their own needs for reforming their practice. The collaborative, problem-solving nature of action research (Coghlan & Brannick, 2007) provides the frame for accomplishing the goals of generating new knowledge while working towards solving the problem of students' underachievement in mathematics.

Summary

This review of the literature substantiates the need for my study, which took place at the moment when a district expected teachers to fully implement new standards. While content knowledge, and beliefs and attitudes about mathematics were indirectly addressed as a result of my study, the mathematics study group that I facilitated with upper elementary teachers directly addressed teacher practices for teaching mathematics. Because of the urgency of the change necessary in order to meet the standards, my study provides a lens for rich understanding around the complexities of developing practice in a new era for teaching and learning mathematics. I provided a sustained model of professional development where teachers developed their own practices and worked collaboratively to construct understanding of both how they learn and how their students learn through the practices of teaching mathematics. My study answers the research questions: In what ways does a mathematics study group support upper elementary teachers' self-reported development of classroom practice? In particular, what aspects of the professional development support the development of their practice?

CHAPTER 3: METHODOLOGY

The goal of my action research study was to support upper elementary teachers' practice of teaching mathematics by providing professional development in the format of a mathematics study group. My study addressed the following research questions:

1. In what ways does a mathematics study group support upper elementary teachers' self-reported development of classroom practice?
2. In particular, what aspects of the professional development support the development of their practice?

Ten upper elementary teachers (grades 3-5) participated in a mathematics professional development opportunity that I facilitated. My qualifications include 12 years as secondary mathematics teacher and department chair, as well as two years of professional development experience with mathematics teachers as a research associate in the STEM division of WestEd—a nonprofit research and development agency. The format of the professional development was a mathematics study group initially focused on using warm-up tasks to promote mathematical discourse. The study group met approximately twice a month outside of school hours. Teachers engaged in tasks that I modeled and then developed their own tasks that they tried out in their classrooms. The study group was sustained over a period of 6 months, so that teachers had the opportunity to not only apply their learning to their own classroom, but also to debrief, analyze, and reflect upon their practice. After the first five study group sessions, teachers had the opportunity to participate in the planning of the rest of the study group sessions.

Research Design

This study utilized a qualitative action research design in order to investigate teachers' perceptions of the development of their practice over time. The collaborative, participatory

nature of the study demanded an action research design, as teachers worked together towards understanding and changing their classroom practice over time. Action research requires more than observation of teachers' learning, but active engagement of teachers to take an inquiry approach to their practice as well as active work on my part as a researcher to bring them appropriate learning opportunities. As change is often a messy process, an action research design most appropriately captures such *mess* (Cook, 1998). Additionally, developing strong working relationships among teachers and participating in sustained professional development that is connected to practice are both features of effective models for teacher learning (Darling-Hammond et al., 2009).

I, as the researcher and the facilitator of the mathematics study group, took on many cycles of action research based on Coghlan and Brannick's (2007) four-step model: diagnosing, planning action, taking action, and evaluating action. The first action research cycle involved the planning of the first two study group sessions, as well as a mid-study session that would engage the teachers in the action research process. The planning of each successive study group session demanded that I evaluate the impact of the previous session based on the input and reflections from participating teachers, and then diagnose and plan for what needed to happen next in order to keep participants engaged and involved in the learning process.

The data collected for the study was qualitative in nature using an inductive process. These data included quick-write reflections during study group sessions, researcher field notes, session audio recordings, verbal reflections at two key points during the span of the mathematics study group, and embedded teacher assessments. There was also data collected in the form of a pre-post questionnaire focused on classroom practices. A qualitative approach enabled me to

understand and explore different aspects of the action research process and intricacies involved in studying and developing practice (Creswell, 2014).

My study was guided by a social constructivist worldview with an element of advocacy. Vygotsky (1978) theorizes that knowledge is co-constructed through collaboration and active involvement in the learning process. Social constructivism informs not only the adult learning process for the teachers participating in the mathematics study group, but it is also what underlies the urgency behind developing teacher practice so that students have the opportunity to construct their own knowledge (Dewey, 1897). This construction of knowledge fits with qualitative methods, as qualitative research is concerned with understanding the problem from the perspectives of the participants, not the researcher (Merriam, 2009). The participatory, democratic and change-oriented nature of the action research design also highlights a transformative perspective as the status quo of current practice is challenged.

Site Selection

The criteria for site selection for this study were the following. First, district selection was dependent upon a clearly expressed desire from the district to improve student outcomes with respect to mathematics. Second, as the study involved upper elementary teachers, the district chosen needed to contain elementary schools. Finally, there needed to be the opportunity to involve more than one elementary school in order to develop a study group that transcended any one school's teaching and learning culture but still aligned with district expectations for implementing the Common Core Content Standards for Mathematics and Standards for Mathematical Practice.

The chosen district was a small, diversely populated, southern Californian urban public school district serving approximately 6,500 students. It consists of a pre-school program, five

elementary schools, one middle school, and one high school, and one continuation school. In 2013 the district's CST results showed that 57% of seventh grade students were proficient in mathematics. While this district outperforms the average California district where a little less than 50% of students do not score at the proficiency level in mathematics in grade 7 (EdSource, 2011a), it is still representative of the problem of students unable to successfully master college preparatory mathematics. Less than 50% of students in the district were proficient in Algebra 1, Geometry, and Algebra 2 across all grade levels in 2013, and only 47% of high school seniors met A-G requirements, often because they were unable to meet the math requirement. This district expressed a desire to improve these outcomes.

Context for the Study

The study district was in its second year of providing to a subset of elementary teachers intensive instructional professional development to better implement Common Core standards for mathematics. During the first year, two of the five elementary schools in the district had four to six teachers receive the training. During the second year, two more schools in the district were invited to participate, and more teachers were brought on at the original two sites. Though the professional development was reputed to be very good, many teachers shied away from it because they heard about the time commitment necessary for participants. The general tenor among elementary teachers was that there was so much to do regarding planning for English language arts, science, etc. that they were not attracted to signing on for a program that was going to take more hours out of their day—even if it was a great program. Additionally, the textbooks the district was using were not aligned to Common Core standards, and teachers were fearful of being responsible for creating their own curriculum, as they heard that was expected from the participants in the professional development the district offered.

Participants

The pool of participants for this study were upper elementary teachers (grades 3-5) drawn from four elementary schools in a single southern California urban district. Grades earlier than grade 7 were chosen because mathematics performance in grade 7 is the strongest predictor of success in college preparatory mathematics (Finkelstein et al., 2012), so it is important to look at the instructional practices of teachers prior to grade 7.

At the beginning of the 2014-2015 school year, I visited with appropriate grade-level teams at four elementary schools in order to invite them to join the group. I made a point of letting teachers know that this would be a low intensity professional development opportunity that would help them better understand the Common Core Standards for Mathematics. There would be no expectations beyond trying activities out with their classes, if they felt comfortable doing so, and talking about how the activities went when they returned to the group. All participants volunteered to be part of the study with no pressure from the district, as the intervention took place outside of the regular school day. Ten upper elementary teachers representing a wide range of teaching experience self-selected into this study based on their own desire to participate in a mathematics study group focused on their practice of teaching mathematics in the context of the Common Core. Two participants were in their second year of teaching, while half of the participants had over 10 years of experience (see Table 3.1).

Table 3.1

Participants

Name	Grade Level	School	# Years Taught
Debbie	3rd	A	16
Ilene	3rd	B	3 (25 years in pre-school)
Lynn	4th	B	15
Maria	5th	B	5
Tammy	4th	C	11
Kathy	5th	C	11
Lucy	3rd	D	2
Rita	4th	D	4
Brenda	4th	D	12
Julia	5th	D	2

Professional Development Design

The rationale behind the professional development was that learning to teach happens when one is engaged in the practice of teaching. The overarching framework for the professional development was a mathematics study group initially focused on using warm-up tasks to promote mathematical discourse (see Appendix C). There are several reasons why I chose to start with such warm-up tasks. First, the tasks were an easy entry point for teachers, as they were doable and likely to get students talking without much resistance from the students. Second, teachers would not be overwhelmed by feeling they have to change everything about their practice all at once. As the tasks only lasted 10-15 minutes, teachers could easily back away from them if they were not going well. Third, they were possible to do without knowing very much as

a teacher, and teachers were able to quickly tweak their practice so that they felt the professional development was helping them in their practice. Finally, I chose to begin the professional development with practices that promote mathematical discourse, as student discourse does not commonly occur American classrooms. Having teachers engage students in discourse is a necessary practice for addressing several of the Common Core Standards for Mathematical Practice, including: constructing viable arguments and critiquing the reasoning of others (MP2), and looking for and making use of structure (MP7). The tasks provided a common context so that teachers could talk about and reflect upon their practice.

The study group met twice a month for 1½ hours outside of the school day over 6 months. During each session teachers engaged in mathematical warm-up tasks specifically chosen because they were non-intimidating and could be approached in a variety of ways. Teachers worked on the tasks individually at first, and then in groups. The purpose of working on the tasks was to engage teachers in a set of practices that engaged them in mathematical discourse, and then helped support them to engage their students in discourse in their own classrooms.

Teachers developed their own tasks based on the tasks done during the study group session. They tried out the tasks in their classrooms and brought their experiences of using them back to the study group. I sent an email out a few days prior to each study group meeting to gently remind the teachers to bring artifacts (posters, videos, student work) of facilitating the warm-ups back to the study group. After the fifth study group session, teachers had the opportunity to participate in the planning of the rest of the study group sessions. I checked in with them by asking them where they were in their practice, what they think has changed, and what they think has helped that change. I then asked them what they wanted to do next—what

they thought would help them. Teachers took an active role in their learning by identifying their needs with respect to their teaching practices. I, as the facilitator of the study group, responded to their collective inquiry and designed the next phase of study group sessions. My goal was to tie whatever direction they wanted to go in to their practice. The action research approach to the design of the professional development relied on teacher input once I had established a practice-based framework. The teachers contributed to the planning around their practice at a specific point midway through the study, and continued to impact my planning of each study session during the span of the mathematics study group.

Data Collection Methods

Audio recordings of study group sessions and researcher field notes. In order to answer my research questions the dominant form of data collected was digital audio recordings of both the study group sessions and my own field notes directly following each session. Each study group session was recorded using several recording devices for back up. The recordings were transcribed in order to capture teachers' out loud thinking and verbalizations of their construction of knowledge around their practice. In particular, I tracked teachers' self-identified shifts in thinking around their practice, as well as discoveries they made in the context of working through tasks, their talk about how the tasks went in their own classrooms, and their responses to other teachers' ideas and experiences related to practice.

Immediately after each study group session I either recorded or wrote down my own reflections in order to track my perspective of each session, and any individuals' or the group's noticeable shifts in thinking around their practice. The recordings were transcribed and added to written notes, which comprised my field notes. The field notes provided details around what I noticed as the work was proceeding and captured details not available in audio recordings—both

what could not be seen and what I saw as critical to capture given my ongoing understandings of the teachers and their participation.

Questionnaires. The 10 upper elementary teachers who volunteered to participate in the study took a self-administered, paper and pencil pre- and post- questionnaire (Appendix A) at the beginning of the first study group session and at the end of the last study group session. The questionnaire asked teachers about their comfort with various mathematics practices/activities and what works/doesn't work in their classrooms. It also asked teachers to respond to open-ended questions about their strengths and areas for growth regarding teaching mathematics. The questionnaire took no more than 15 minutes to complete. Data collected from the questionnaires contributed to answering the research questions by providing self-reported changes in their comfort with various practices over time, what practices they perceive work in their classroom, and what they consider their strengths are in teaching mathematics before and after participating in the mathematics study group.

Reflective quick-writes. Teachers were asked to submit handwritten, quick-write reflections (see Appendix D) in response to specific prompts at various times during study group meetings. These reflections prompted teachers to track any changes in their own thinking or practice and give voice to individual teacher's perceptions who may not have always shared openly in the study group. Examples are: "What is something that surprised you around math in your classroom since the last time we met?" and "Describe something about your practice of teaching mathematics *this week* that you relate specifically to being in this study group. *How* does it relate to the work we do here?" The reflections captured what teachers said about their learning process, their feelings

and ideas about the practices, and their feelings and ideas about how they implemented the practices. The reflections did not receive responses, but were used to plan subsequent sessions and contributed to answering the research questions.

Verbal reflections. During the sixth study group session, participants were asked to share out to the group where they were in their practice, what they think had changed, and what they think had helped that change. They also were asked where they would like to go next. These verbal reflections were captured through the digital audio recordings of the study group sessions and specifically addressed the research questions. During the last study group session, participants were asked to reflect on the questionnaires they had just taken, what differences they noticed since the first time they took the questionnaires, what changes in their practice had taken place, and how they related the changes to participating in the study group.

Study group assessments. There were embedded assessments across the study group sessions, where teachers were asked to complete tasks around lesson planning and student work in order to track changes in how teachers utilized and thought about math teaching practices. Assessments included, but were not limited to: (a) individually creating a "One of these things is not like the other" warm-up after working on several models of these in study sessions; (b) written and verbal responses to several examples of student work identifying student thinking; (c) presentation of collected student work from teachers' own classes to model student thinking; and (d) responses to work presented by other teachers. These assessments contributed to answering the research question by providing a different context through which to capture teachers' learning about practices that are new to them and their thinking about their own classroom practice. I looked for changes over time in teachers' abilities to create tasks and

demonstrate a growth mindset with respect to student thinking. I also looked for evidence of teachers' ability to focus on what students do know, as opposed to what they don't know.

Data Analysis Methods

The pre- and post-questionnaires asked teachers about their comfort with a variety of activities or practices when teaching mathematics, as well as what they perceive works in their classroom. Some of these practices were part of the content of the mathematics study group sessions. The questionnaires also asked teachers to identify their strengths and their uncertainties in their teaching of mathematics. The questionnaires were analyzed at the whole group level and at the individual level. At the individual level, I looked for all differences in responses to the teachers' comfort with various practices, their perceptions of how practices worked in their classroom, and their identified strengths and uncertainties pre- and post-professional development. I looked to find any correlations between these differences around particular practices and the actual content from the mathematics study group sessions. I also looked to connect any differences to individual teachers' self-reported development in practices that came out of recordings and/or field notes and/or written reflections from the study group sessions. In this way, individuals' development around specific practices was correlated to the study group session discussions.

At the whole group level, I looked for unanimous or near unanimous differences in comfort with practices and perceptions of how the practices worked in their classrooms pre- and post-professional development. I also looked for repeated identifications among the group of strengths and uncertainties in their teaching of mathematics. I cross-referenced these differences and repeated identifications with other data analyses to strengthen findings around the ways in which the study group supported teachers' self-reported development of their practice.

The transcribed digital audio recordings of both the study group sessions and the researcher field notes as well as the teacher reflections were coded prior to each successive study session in order to discern themes or patterns that emerged from the group with respect to the research questions. I used Microsoft Excel to organize and expand upon the codes with each new study group session, and then placed their occurrences along a timeline. Analysis of the codes tracked utilization of particular practices, and subtle and abrupt changes in thinking and feeling related to the practices by marking and grouping codes on a timeline that correlated to the study group sessions. My spreadsheet contained the date of the professional development, what happened in that study group session, individual teacher statements about practice, my reflection on individual teachers' participation, and my reflections on the group as a whole. In this way, I was able to track what was said around practice both by individual and the group overall according to when comments were made during the progression of the study group. The teacher written reflections were also analyzed at the whole group level and at the individual level to discern any patterns in how the study group supported development and/or changes in practice.

There are many levels of practice that teachers talked about. Some examples of practices related to discourse are: asking students questions, helping students participate in another person's idea, and responding to students' talk so that they will see themselves as more competent. I was able to capture multiple levels of how teachers talked about their practice. While teachers started out speaking very generally about their practice, their talk became more detailed and descriptive of the kinds of nuanced moves that happen while teaching that can't necessarily be attended to when first working at developing practice.

Lastly, teachers were asked to complete assessment tasks over the course of the study group sessions. Like the reflections and transcriptions of study sessions, these assessments were

analyzed and coded in order to discern themes and patterns that emerged with respect to the research questions. These data provided triangulation for developing themes and patterns.

Credibility and Trustworthiness

There are several threats to the credibility of my study. First, there was a small sample of 10 participants. Additionally, these participants volunteered to participate in the study, rather than were randomly chosen. These two factors make it impossible to generalize to a larger population, so I am unable to make any claims beyond the findings from this particular group of participants.

Secondly, findings are somewhat dependent on self-reported teacher practices. As I was unable to observe actual classroom lessons before the study began, I did not have evidence of actual changes in practice. I minimized this threat to credibility by triangulating data sources, ensuring a systematic approach to both collecting and coding data. Every participant was asked the same questions with the same wording and completed the same tasks with the same instructions. There are detailed, transcribed accounts of each study session to account for every step of the professional development, as well as a detailed account of how data was analyzed.

Because I facilitated the mathematics study group, my own bias, as well as participant reactivity, is a threat to credibility. I ensured that the threat of bias was minimized by utilizing direct quotes from participants rather than stating my own beliefs. Triangulation also served to minimize this threat, as I collected multiple data sources to develop rich descriptions of teacher understanding at the beginning and at the end of the intervention. Additionally, because the intervention took place over 6 months, it allowed for a more in-depth understanding of the participants. Reactivity was also minimized by this long-term involvement, as participants were

less likely to tell me what they think I wanted to hear over a long period of time. They were less likely to disguise their *normal* behavior.

Ethical Considerations

The primary ethical issue that must be addressed is my role as both the facilitator and researcher of the mathematics study group, and my role as a leader within the district where the action research is taking place. Fortunately, I do not have any supervisory role at any of the elementary schools from which I drew participants, but it was imperative to ensure confidentiality about any and all data collected during the time that the research took place. This relates not only to assuring participants that I will maintain confidentiality and will make sure that none of my findings identify any individual who is participating in the action research, but also to doing my best to ensure that all participants adhere to the practice of maintaining confidentiality regarding the mathematics study group. I made this expectation of confidentiality explicit not only prior to working with the study group, but throughout the time I worked with them by voicing a reminder during each session.

Summary

In California, statewide assessments for the Common Core Standards for Mathematics rolled out during the 2014-2015 school year. Districts and schools have been and continue to be steeped in the process of figuring out how to provide professional development to teachers for these new standards that demand changes in the way mathematics is taught. This qualitative action research study contributes to the research that seeks to understand how to address teachers' perspectives and teacher learning in order to support changes in practice.

CHAPTER FOUR: FINDINGS

This chapter begins with an overview of the participants. I then review the structure of the professional development, providing a summary of the major events of each of the 14 sessions. From there I present the findings that answer the research questions:

1. In what ways does a mathematics study group support upper elementary teachers' self-reported development of classroom practice?
2. In particular, what aspects of the professional development support the development of their practice?

There were three major findings from the study:

1. Teachers regularly tried practices in their classrooms they had rarely or never tried before.
2. Teachers reported that the warm-ups they learned in the professional development were the driving force for changes in their mathematics teaching practices.
3. Teachers reported that there were substantial differences in the student discourse that occurred during math time between the previous school year and this year and that facilitation of student discourse during math time became a regular part of their practice.

The data collected to reach these findings were transcriptions of what teachers reported both in whole group and small group settings during the mathematics study group sessions, reflective quick writes and verbal reflections, and pre- and post-questionnaires.

Each finding begins with a profile of one of the study group participants in order to serve as one piece of evidence of the specific finding and illustrate how teachers' practice developed over time. I then provide a broader view of the evidence that supports the finding. Lastly, I

present how the planning decisions I made as a result of cycles of action research support the finding and drove teacher learning.

Overview of the Participants

The 10 teacher participants came from four elementary schools in the study district and represented a range of teaching experience from two to 16 years. Teachers from three different schools were represented in each of the grade levels (third, fourth, and fifth), which allowed for grade-level groupings in which discussions were enriched by the diversity of each school's approach to addressing the Common Core standards and the different curricula they were using. Attendance at the study group sessions was excellent (see Table 4.1). Seven out of the 10 participants did not miss more than one session. One participant missed three out of 14 sessions, representing the most missed. Additionally, no study group session had fewer than seven teachers in attendance. In eight out of the 14 sessions, all 10 participants were present.

Table 4.1

Study Group Session Attendance

Study Session #	1	2	3	4	5	6	7	8	9	10	11	12	13	14
How many present?	10	9	10	7	10	7	10	7	10	9	9	10	10	10

Note. $n = 10$.

Summary of the Mathematics Study Group Sessions

The mathematics study group initially focused on using warm-ups to promote mathematical discourse among students and to gain a better understanding of and comfort with the Common Core Standards for Mathematical Practice as teachers. In general, the structure of most sessions involved sharing out, either with the whole group or in smaller groups, how the warm-up learned in the previous session went and then learning a new warm-up. About halfway through the study (Session 6), participants were asked to give input into the direction of the

professional development—as long as it focused on the practice of teaching mathematics. The following table summarizes the major events of each of the study group sessions. For a more detailed outline of each study group session, see Appendix C.

Table 4.2

Summary of Study Group Session Activities

Session #	Summary of Activities
1	Participants were administered the pre-questionnaire before diving into a task that engaged them in sharing and explaining their thinking around a mathematics activity where there was no one right answer. Participants were then introduced to the first warm-up that they were challenged to try out with their students prior to the next study group session: *How many ways?*
2	Teachers shared how the *How many ways* warm-up went in their classes. They then engaged in a new warm up: 3 of these things. Teachers worked in grade level teams to write their own version based on what they were presently teaching in mathematics. The session ended with a challenge to try out a '3 of these things' warm up with their students prior to the next study group session.
3	Teachers shared how either the *Three of these things* or *How many ways* warm-ups played out in their classes. We looked at the Common Core Standards for Mathematical Practice and found connections between the standards and the student behaviors teachers were seeing during the warm-ups. We watched and discussed a short video about improving participation using 'talk moves,' and then engaged in a new warm-up: Mental math.
4	After sharing out about any of the warm-ups previously covered, we watched and analyzed a video of a teacher facilitating a mental math warm-up. We then began an activity around asking purposeful questions using samples of student work that one of the teachers had brought in the previous session.
5	We continued the *asking purposeful questions* activity from the previous session and then shared out mental math warm-ups. We then engaged in a new warm-up: True/False number sentences.
6	After sharing out True/False number sentences warm-ups, I facilitated a discussion around three questions: (a) What do you think so far has changed (or not changed) about your practice? (b) Is any of that related to the work we've done on warm-ups? If so, in what way? (c) Where do we go from here?
7	We watched and analyzed a video of a True/False number talk. Teachers then shared out about warm-ups that had taken place since the last study group session. We learned an add-on to the True/False number sentences warm up: Open number sentences.
8	We read and analyzed an abstract of a true, false, and open sentences lesson. We looked at a fourth grade Common Core assessment question and rubric scoring guide in order to learn about how students are expected to explain their thinking in writing. We engaged in a fractions activity, and then watched a video on the meaning of unit fractions. Finally, we learned an equal/fair-sharing warm-up.
9	We re-visited a *Three of these things* warm-up using fractions. Teachers shared out equal/fair-sharing warm-ups. I handed out number line warm-ups for teachers to try. We ended the session getting into grade-level teams to plan a particular warm-up to try prior to the next study group session.
10	Teachers shared the experience of doing the planned grade-level warm-up in both the small and whole groups. They worked in the same grade-level teams to plan another warm-up to try. We ended the session by watching and discussing a video on comparing fractions.
11	We shared and discussed two videos shared from teachers in the study group. We got into grade level teams to discuss the planned warm-up from the previous week and then shared out to the whole group. We learned more fair-sharing warm-ups.
12	Teachers engaged in a mental math activity individually and in groups. We watched and analyzed a video of a teacher doing a mental math multiplication problem. We formed groups based on what warm-up we would like to plan and try prior to the next study group session.

(continued)

Session #	Summary of Activities
13	Teachers debriefed their planned warm-ups in small groups. We then focused on identifying challenges that have come up over the course of the study and attempted to address some of them.
14	Participants were administered the post-questionnaire and final written reflection. Teachers then verbally reflected on changes in practice from the previous school year to this school year, and what particular aspects of the study group helped their practice.

While many of the study group sessions involved learning a new warm-up to promote student discourse around mathematics, participants also engaged in activities that further supported the development of their practice. These activities included: working on adult warm-up activities in small groups, watching and debriefing videos of teachers facilitating warm-ups, and planning and debriefing specific warm-ups in teams.

Finding #1: Teachers' Experimentation of Practice

According to finding #1, teachers regularly tried practices in their classrooms they had rarely or never tried before. This finding responds to the research question: In what ways does a mathematics study group support upper elementary teachers' self-reported development of classroom practice? Each week the study group session met, participants demonstrated that they were experimenting with their practice. The following profile illuminates the experience of one participant, particularly with respect to the experimenting with the practice of presenting multiple strategies to solve a problem.

Profile #1: Tammy. Tammy was in her sixth year of teaching fourth grade and in her 11th year of teaching overall at the time the study took place. On her pre-questionnaire she listed as one of her strengths in teaching mathematics: "Orderly steps to math lesson—create a rhythm (comfort)." She responded to the question asking what she felt most uncertain about in her mathematics teaching: "I don't know how to teach kids how to explain themselves…I don't give them enough control to solve problems for themselves." In the third study group session, when asked to write about if she noticed any shifts in her practice of teaching mathematics, she wrote:

"I haven't really changed my teaching—with the exception of throwing at them the couple of fun examples you've shared."

At the fourth study group session Tammy shared her struggles with a mental math warm-up:

> I was frustrated…because I didn't know what strategies to teach them to do it. I just said, can you do this mental math...They explained it, and I wrote down what they said, so I was confused by it. I don't know how to do it myself.

After she shared, the group watched a video of a teacher doing a mental math problem using particular strategies, which made a big impression on Tammy. She wrote in an email to me prior to the fifth session, "What a difference sharing four different strategies made…so many hands went up…they used whichever strategies they wanted…and definitely some not demonstrated." By the sixth session, she shared out the following about what she felt had changed in her practice:

> I think there's definitely more math conversation…there's a lot of wanting to share their point. And I kill myself running my classroom that way, but there's a lot of engaged behavior…There's a lot of wanting to share what they have to say, and I've never seen that in math before… myself and my kids, we're more and more comfortable, even with our regular algorithm math lessons—there's still more conversation in that. And it's just more fun…We're all kind of more aware now of what a regular algorithm is…and they know to try to think of other ways to do it.

Over the course of the study, Tammy brought in many artifacts from her classroom, including videos, representing seven out of the eight warm-ups that were presented. She particularly appreciated the experience of planning and sharing out grade-level warm-ups with the fourth grade teachers that occurred during sessions 9 through 11. She reported the pressure of reporting back to the group kept her experimenting with her practice. She wrote the following on a quick write during the 11th study group session:

> One thing that resonates with me is to be in union with other colleagues—to know that I'm not in it alone—that other educators are trying it out—that care about changing.

> That's comforting—to share the enthusiasm for encouraging our students—challenging our students to talk about math.

When asked at the final study group session what she saw as the most helpful aspect of the study group for her math teaching practice, she wrote, "That I was held accountable for trying it—with no judgment—easy pace. No expectations—but…well…sort of expectations to at least try." On her post-questionnaire, she shared as one of things she felt most uncertain about in her teaching of mathematics: "Hoping that I can continue the mental math warm ups without a class to report back to."

Evidence for teachers' experimentation with practices. Tammy's willingness to experiment with her practice was universally demonstrated among the group. Table 4.3 shows not only how many participants reported that they tried each of the warm-ups to promote discourse, but also how many provided some kind of artifact from their classroom. The artifacts were videos, photographs, or samples of student work and were either emailed to me or shared out to either the whole group or small grouping within a given session.

Table 4.3

Number of Participants Doing Warm-ups and Sharing Artifacts from Warm-ups

Warm-Up	How many ways?	Three of these things	Mental math	T/F number sentences	Open number sentences	Fair sharing type #1	Number lines	Fair sharing type #2
# Participants reported doing warm-up at least once	10	10	10	10	4	10	8	3
# Participants shared artifact	10	5	8	6	3	10	9	1

Note. n = 10.

The data from the first row of the table was gathered from a quick-write from session 12, where teachers were asked to check the warm-ups that they had tried at least once. All 10

participants reported that they had tried at least five out of eight warm-ups by session 12. They were also asked to estimate how many times they tried each one. Five out of 10 teachers reported that they had tried various warm-ups with their classes 10-20 times in total. The other five teachers reported that they had tried warm-ups over 20 times. These data support the finding that teachers were regularly experimenting in their classrooms. The data from the second row of the table demonstrate that every teacher brought artifacts from at least two distinct warm-ups back to the study group though I never demanded that the participants bring or send artifacts to share out. I only sent out gentle reminders a few days before each study group session suggesting that they bring or send artifacts from their classroom.

The artifacts alone do not necessarily serve as evidence of any development of practice. The teachers' experimentation can be better understood when the artifacts are accompanied by teacher explanation. The practices teachers regularly tried included recording student thinking, asking students to explain their thinking and critique others', and presenting multiple strategies to solve a problem. The following photo demonstrates Debbie's recording of student thinking during a *How many ways can you make 25?* warm-up. Debbie talked about doing this warm-up with her third grade students at the second study group session.

> All of a sudden someone keyed in on it [200-95=25] and they raised their hand, and I said, "Ok, do you want to share?" and they said, "No, I want to talk about the problem 200-95=25." [Debbie describes how student explained his thinking]…so I thought just that explanation, I just love how they phrased it…I really had this major transformation…I was so teacher heavy before and now…we're just going for it, and what I see is just unbelievable…already.

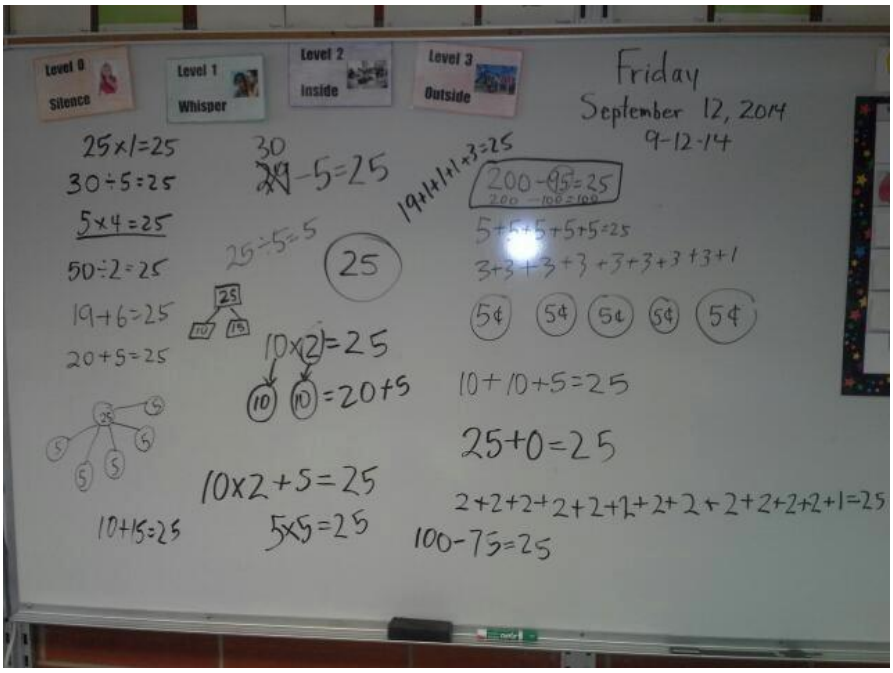

Figure 4.1. Debbie: How many ways can you make 25?

Debbie demonstrated excitement for giving students the opportunity to explain their thinking and comment on other students' thinking. Her excerpt came in the midst of a discussion about leaving incorrect representations of student work up on the board. Debbie preferred to record all student contributions and then allow for students to make any corrections. She added, "I didn't focus on the answer; I just focused on why did you know that that was not the correct answer." At the end of the study, Debbie listed the following as one of her strengths in teaching mathematics: "students sharing work before entire class and critiquing one another." This strength was not listed in her pre-questionnaire and may have come as a result of the many warm-ups she reported to have tried throughout the study.

The photo in Figure 4.2 demonstrates Tammy's recording of student thinking for a '3 of these things' warm-up:

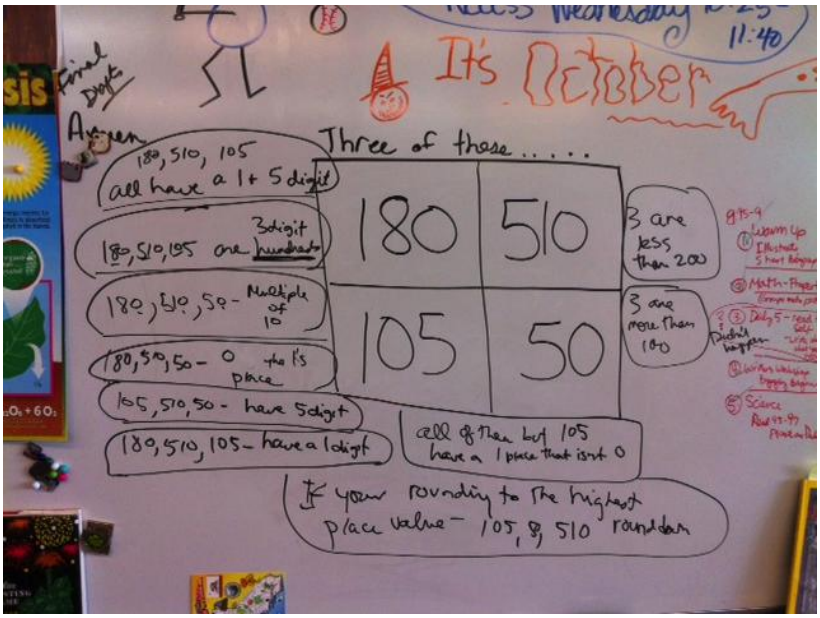

Figure 4.2. Tammy: Three of these things.

At the third study group session, Tammy shared:

> It was a class conversation…And I noticed when I put something up…one thing led to another. Like somebody said three are less than 200, and right away somebody put their hand up and said, "Oh, three are more than 100." And I made a point of that. I said, "I love that this happened…one of the points of sharing these things is so that we hear other people's ideas and we say, oh yeah…"

For Tammy, it was a new practice to let students be heard and offer multiple ideas or strategies. On many occasions she expressed her excitement over the things her kids were saying which helped her to continue to experiment with her practice.

Rita, a fourth grade teacher, recorded student thinking for the following True/False number sentence warm-ups. She talked about her trying out the practice of presenting multiple strategies of solving problems during the ninth study group session.

> I unraveled each one [like in a video we had watched]…and I said "I don't want you to use standard algorithm…I'm going to give you about 15 seconds, and then we're going to share out…and then at the end of this, I asked them, "Who would have guessed we would have so many ways to solve just one problem? Well in third grade, we would have just added…now we have strategies."

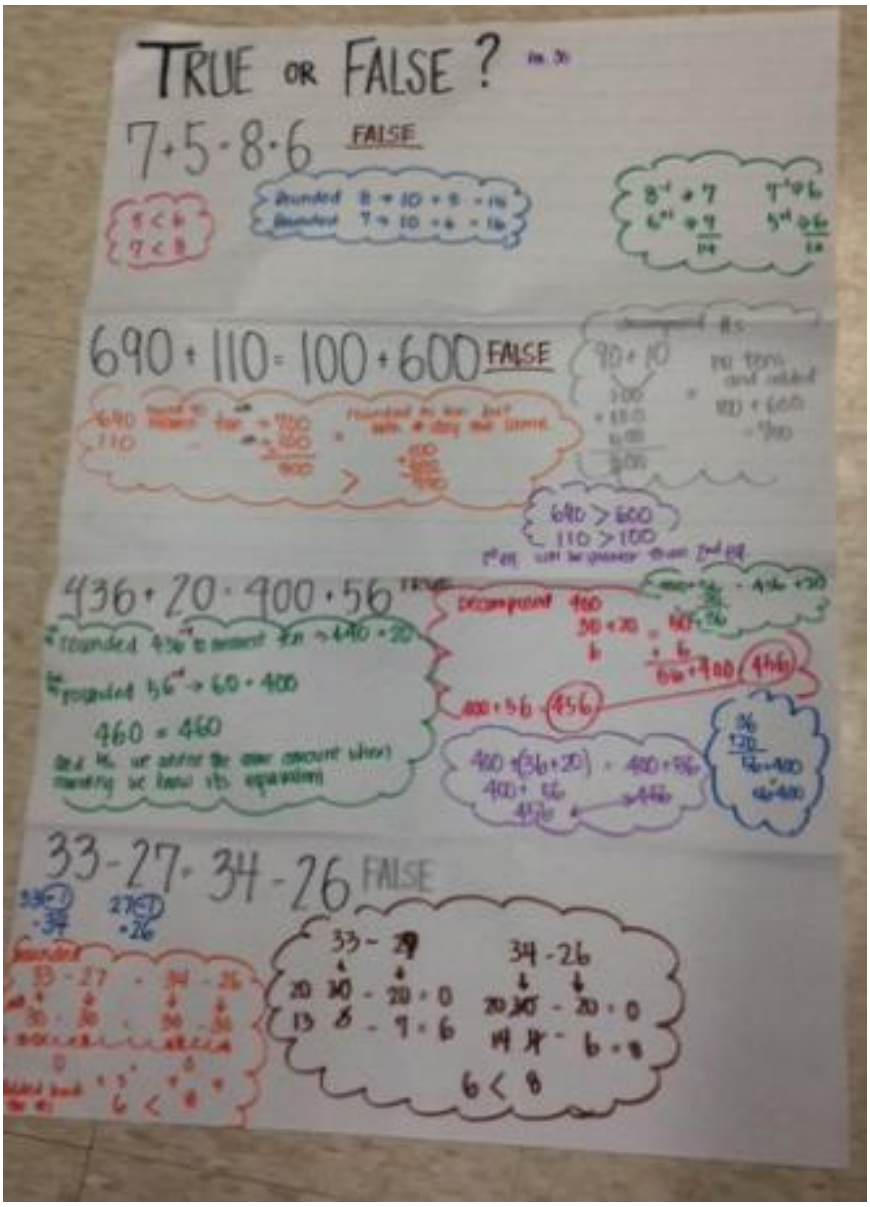

Figure 4.3. Rita: True or false?

The other teachers commented on how they liked the way that Rita recorded student thinking. Brenda offered, "It's easy to see the strategies. It's easy to see the thinking of the kids the way she's notated it." How we recorded student thinking was one of the things I asked teachers to focus on in the very beginning of the study. All of the warm-ups were chosen because they would elicit multiple strategies.

The post-questionnaire given at the end of the study illuminated an increase in comfort from the pre-questionnaire for six out of 10 teachers with respect to the practice of "presenting a problem for students to solve involving new content before teaching that content" (see Appendix A). Two teachers moved from "Not comfortable" to "Very comfortable." Three teachers moved from "I've rarely/never done this" to "Somewhat" or "Very comfortable," and one teacher moved from "Somewhat comfortable" to "Very comfortable." Many of the teachers during the course of the study reported that they were unsure of how their students would respond to some of the warm-ups. Often they were pleasantly surprised by their students' responses, which may account for their shift in comfort level with respect to the practice mentioned above. It may also account for their willingness to continue to experiment with their practice.

How the professional development supported teachers' experimentation with practices. As the facilitator of the professional development, I made decisions beyond presenting warm-ups to try that I believe supported participating teachers in their experimentation with teaching practices. Several aspects of the professional development were designed, prior to beginning the study, to build trust among the participants and with myself, while another—forming small groups to plan and debrief a specific warm-up together—happened as a result of an action research cycle.

To begin, I made clear during the participant recruitment process and during the first few sessions that the study group would be a low-stakes environment. Nobody would be required to do anything that they were not comfortable doing. Additionally, the warm-ups themselves were narrowly focused and only meant to take 5-10 minutes of class time. As Lucy stated during the last study group session, "I think there was really not a lot being asked of us, but just, hey try this

out…that's really all I had to do, and I went into my classroom and tried it, and what I saw, I liked. And so I want to try more…"

At the beginning of the study, I shared a story about how my godson initially triggered my interest in the teaching of elementary mathematics. I asked for volunteers over the next two sessions to share a math story of their choosing—an experience as a teacher, parent, or student that was related to math in some way. My intent was for the participants to get to know something personal about each other's experience with mathematics. During the second session, I explicitly addressed how we are all sensitive about our practice and that we would need to get comfortable being questioned or challenged in order to develop our practice. At that session, we built norms together that specifically dealt with how we would support each other's practice. We revisited those norms several times throughout the study. Trust and norm building was needed to support teachers beginning to experiment with their practice. Several of the teachers expressed the "safe" space that the study group provided during the verbal reflections at session 6 and at the last study group session.

- I really enjoy this opportunity to work with a small group. I've been overwhelmed with all the changes…And you're giving us a little bit at a time, and I'm feeling successful with it… it's a safe place to be…We're all definitely more comfortable with it—myself and my children. – Tammy, Session 6
- It's been a safe space for me to try out some of this stuff, so I really appreciate it on that level. – Lynn, Session 6
- So by having fewer things to focus on and trying different things in a safe way, I think the kids benefit, because we're benefiting. – Debbie, Session 14

Another aspect of the professional development that built trust was incorporating the practice-based input that was elicited in the sixth study group session about where teachers wanted to go. In their final reflections, nine out of 10 teachers felt that having input in what they wanted to work on in the study group helped their practice (see Appendix D). Comments included:

- The study group seemed to help us in our challenges. Having input on the activities/topics we wanted to discuss and learn about made the group feel relevant and *necessary* to my everyday practice. – Lucy
- It allowed us to be more invested, especially because it directly pertained to what we were teaching then. – Rita
- It gave me a sense of ownership and responsibility—it made doing the activities more interesting. – Julia

My decision to drive the professional development to small group planning and debriefing contributed to teachers' continued willingness to experiment with their practice. As a result of my evaluation after the eighth study group session, I realized that not every participant had shared an artifact from her practice. Though everyone had talked about things they had done or were doing with their classes, and could talk about their practice in general, I did not have tangible evidence that all were facilitating the warm-ups. I decided that I needed to create a different kind of accountability for the teachers. I planned for them to work in grade-level groups during session 9 to plan a particular warm-up—from how they would present it to how they would record it. I withheld learning a new warm-up during session 10 so that the small groups would have time to debrief the previously planned warm-up and to plan another warm-up together. This time they could choose their own individual way of facilitating the warm-up, so

they could compare differences at the next study group session. Every single participant brought in artifacts from their practice once they knew they would be reporting back to their group. Figures 4.4-4.6 are artifacts from the third grade group, who planned a *How many ways* warm-up using a fraction.

> Ilene: They just started learning fractions Monday, but they're so completely into it…This one [pointing to the word 'middle']…I said, "Can you explain your thinking? And he said, "…because if anything's in half you go right to the middle." These two [one quarter and 1/4] I just actually left it that way, and then I asked the other boys and girls at the very end, does anyone want to challenge any of these?...They said well it can't be one quarter because one quarter is a fourth, and this is a half, so we all agreed yeah, that can't be. But then another kid said, yeah but we could say it's two quarters.
>
> Debbie: They did this and they've only had 3 days of fractions?
>
> Lucy [hadn't started fractions yet]: I had over half of my class keep on doing drawings just doing half, half, half…so now, if I do this though, I think I'm gonna have more different ideas…

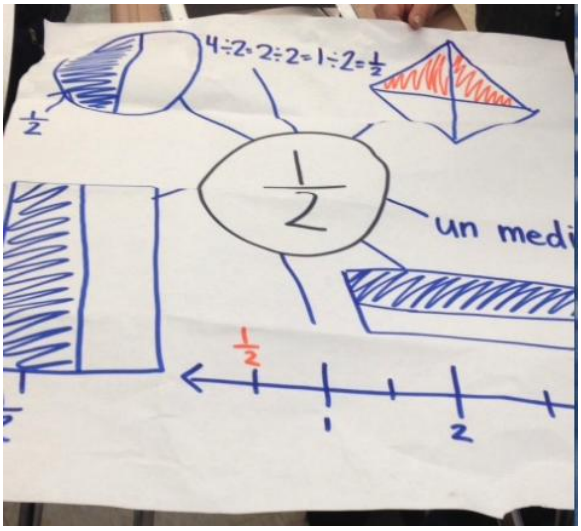

Figure 4.4. Lucy: How many ways?

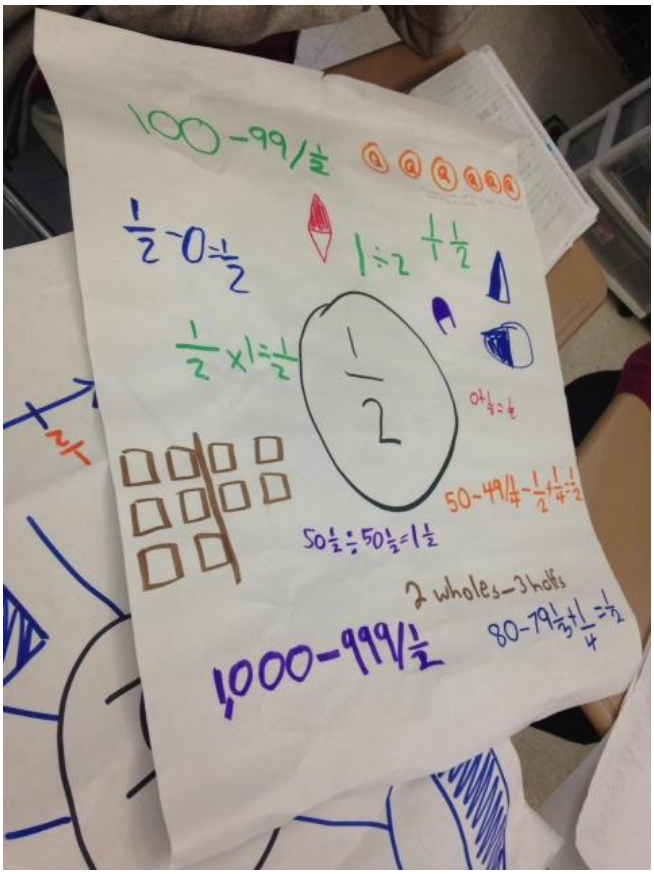

Figure 4.5. Debbie: How many ways?

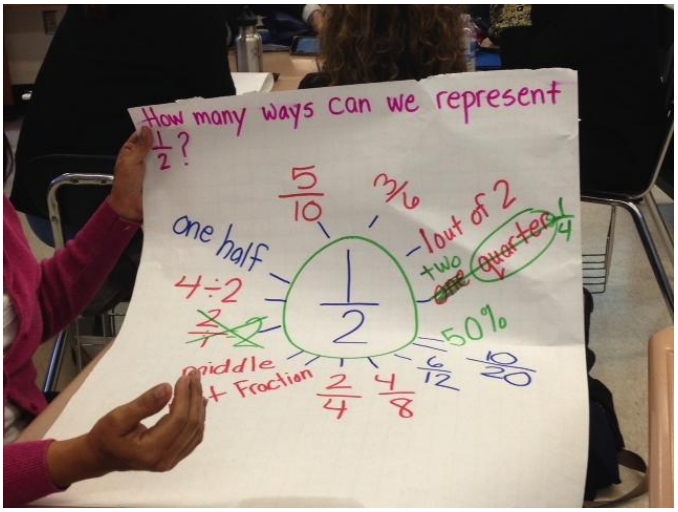

Figure 4.6. Ilene: How many ways?

This conversation continued to address details about how the teachers addressed student misconceptions and how they engaged students who were not initially participating. All three teachers were in different places with respect to how much exposure to fractions their students

had, and they were interested in the differences between each of their posters. At the 11th study group session, Ilene wrote:

> I thought it was very beneficial to plan the math warm-up "How many ways can you represent ½?" with the other third grade teachers because I may have thought this activity was too difficult for my students if I was just doing it myself. Knowing that the other third grade teachers were going to try it too reassured me that it was okay to challenge my students with this warm-up.

During the last study group session, Ilene addressed how our regular practice of sharing experiences of doing warm-ups helped her. She wrote, "It helped me feel more confident when I saw that the warm-ups didn't always go perfectly for the other teachers in our group either! It made me feel like it was ok to take risks!" Similarly, during the last study group session, Julia verbally reflected to the group how she was inspired to try things because of the grade-level planned warm-ups:

> I feel like in our fifth grade group we learned a lot about how kids' performance goes the way you present the problem, like it varies so much. And so even coming back and talking about it, oh! What if next time I go back and said what you said, I wonder how that would change their work? And we found that it changed a lot.

Teachers shared practices that worked or didn't work openly with each other. Eight out of 10 teachers felt that this aspect of the professional development helped their practice. Brenda wrote it was very helpful for her because she "felt supported in ways I haven't felt in several years."

In summary, the mathematics study group provided an environment for teachers to regularly experiment with their practice. They reported building trust through their shared experiences and adherence to group norms. They reported they felt free to try out new practices, regardless of their success with any particular warm-up, because they bought into the expectation that they would report back to the group. When I noticed that not every participant reported back to the whole group, I introduced small group planning so that the small group provided the needed accountability for participants. Small group discussions focused primarily on what

teachers' students said, though teachers were utilizing new practices that prompted their students' responses. I, as the facilitator of the study group, was able to bring the focus back on practice when we shared out in the whole group. The warm-ups, themselves, spawned excitement for both teachers and their students to engage in both the teaching and learning of mathematics.

Finding #2: Warm-ups as a Driving Force for Changes in Practice

According to finding #2, teachers reported that the warm-ups they learned in the professional development were the driving force for changes in their mathematics teaching practices. Eight types of warm-ups were presented over the course of the professional development. The chosen warm-ups all lent themselves to teachers steering students away from procedural learning and taking on practices that they hadn't likely utilized in their teaching of mathematics. Some mathematics teaching practices, such as recording student thinking, were purposefully focused on during the professional development; other practices emerged over the course of the study. Not every study group session was focused on learning a new warm-up. As part of the action research, I felt it was sometimes necessary to introduce a different kind of activity related to the warm-ups in order to delve into particular teaching practices. But first, a profile of a teacher:

Profile #2: Maria. Maria was in her third year of teaching in the district where the study took place, but it was her fifth year teaching overall. She was the only fifth grade teacher from her site that joined the study, and another study group teacher from her site shared with me that her fifth grade site colleagues were resistant to change of any kind. Though Maria found support from teachers on other grade level teams at her school, she was on her own in fifth grade. When

asked on the pre-questionnaire what she felt most uncertain about in her teaching of mathematics, one of her responses was: "how do I get the kids to dig deeper into concepts?"

Maria was an active participant throughout the study. She missed only one session because it conflicted with chaperoning an overnight science camping trip at her school. She tried every kind of warm-up that was introduced, and often brought in artifacts from her classroom to share with the whole group. By the third study group session she wrote, "I have noticed that I have moved away from just teaching the standard algorithm. It has been great to see the kids responding to these techniques."

By the sixth session her more-often-than-not success in using the warm-ups ("students look forward to the math warm-ups, helping to boost their engagement during my math lesson") supported her continued exploration of many of the more nuanced practices we were talking about during the study group sessions, such as asking purposeful questions to drive or advance student learning. When prompted to write what about her practice of teaching mathematics had changed in the two months she had been in the study, she wrote:

> I have really started to challenge myself in teaching math by going outside of my comfort zone. The activities that I have shared in my class have helped to facilitate math conversations that I struggled to initiate previously. In addition, I have been very reflective about how I question kids. I noticed that I often lead the kids to the answers with my questions. I have been working on finding effective ways to question kids.

At the tenth session she made copies of and shared a '3 of these things' warm-up that she created as an introduction to a geometry lesson. She talked about how it worked in her class:

> So my whole objective I guess was just to get them talking before our lesson… I did pair/share. I gave them probably a minute or two to talk…It was really good… I did record on the board, which was easy for me in this activity because it was just words and not numbers, so I felt really successful.[4] I stopped them at about 8 to 10 ideas. The one that really blew me away, I didn't even think of this and didn't intend for it, but 3 of these things are not regular polygons. And I was like, whoa, you're right…

[4] Maria had shared during a previous study group session that she felt challenged by using numbers to represent what the students were explaining.

By the 11th session Maria wrote, "I have moved the kids into table groups in order to help facilitate these activities. I have already noticed a shift in my practice as a result." The shift she refers to is in her facilitation of student talk during math time.

During the 12th study group session, Maria shared a video of her facilitating a number line warm-up. A group of three students are seen working together to place post-its of fractions on a number line. The following is from the video transcript:

Maria: So can you explain your number line to me?

Student 1: So what we did…so we made the increments of eight.

Maria: Why did you choose eight?

Student 1: Because we saw that we could multiply the four by two and the half by 4…

Student 2: Yeah…and then we could get eight, and we saw most of them are eighths…most of those numbers are even by eight…

Maria: They're all even by eight? Is there another way to say that?

Student 3: They're multiples of eight.

Maria: They're multiples of eight? Or, what are they?

Student 1: They're factors of eight.

Maria: They're factors of eight, right? They all multiply together to make eight? Ok, then once you knew that you had a denominator of eight, what did you do?

[Students continue to explain how they placed the post-its on their number line using eighths]

Maria: So, I'm noticing, though, that you didn't use the exact fractions on the board. What did you do?

Student 1: We multiplied ¾ times two, and ½ times four, and then ¼ times two.

Maria: Why did you do that?

Student 1: Because it multiplies to get eight.

Student 2: They're equivalent.

Maria: Ah, so you made a what?

Student 1: An equivalent fraction.

Maria: You made an equivalent fraction. So you just knew to do that.

Student 2: …We were able to make all of them eighths…

Student 1: And then we would know easily where they go.

In the video Maria demonstrated her practice of facilitating group work, asking students to explain their thinking, and asking questions to clarify and advance their thinking. She explained to the study group that she spent 45 minutes on what she meant to be a 10-minute warm-up: "I let it go…they were having really good conversations…we go back to the number lines frequently." In her pre-questionnaire Maria had written that having students work in groups of three or four during math time "doesn't work in my classroom." By the post-questionnaire, she responded that the same practice "works well in my classroom."

At the end of the study, Maria wrote in her final reflection about using warm-ups to engage students in math discourse: "I am amazed to see how the work of my students has progressed especially in their explanations of their thinking." She verbally reflected on changes over the course of the study:

> I think my focus now is different in terms of what I think about when teaching math. I think about discussions and the grouping and the questioning, where before I don't know if that was as much on my mind as much as like, oh, I've got to teach them the standard…I've seen a change in what I'm doing and know what I need to do to move forward.

Maria tied her changes in practice to the warm-ups she facilitated in her classroom. She also expressed concern that she didn't think she had made a "total shift" in her practice because she

was still using the old curriculum, which she saw as antithetical to the work we were doing in the study group.

Evidence for warm-ups driving changes in teaching practices. Throughout the course of the study, teachers attributed shifts they were making in their practice to the warm-ups they were learning. They did this either verbally during the study group sessions or in written reflections. Over and over again, participants used the words "fun," "engaging," "excited," and "creative" to describe the warm-ups and the behaviors the warm-ups were prompting in their students. Lucy shared during the sixth study group session, "I think I knew I had to get my kids to start talking about math…I just wasn't sure how to do that. And I think the activities I learned here have been really helpful for me in order to get them going." The shifts in practice that came up most often and that at least nine out of 10 participants wrote or spoke about throughout the study were: asking students to explain their thinking, posing purposeful questions to students, and having students present multiple strategies. With respect to the latter practice, at the end of the study six out of 10 teachers identified on their post-questionnaire some form of "exposing students to multiple ways to solve the same problem" (Debbie's wording) as one of their strengths in teaching mathematics. None of those six identified this practice as a strength on their pre-questionnaire.

During the fifth study group session, Maria lamented to the whole group about a bad experience she had with a mental math warm-up:

> When they were sharing it was like everyone was doing algorithm…every single kid. So I had no way to check what their thinking was…and they didn't want to. And I did a pair share, and I was so flustered I didn't know what to ask at that point. I cut it really fast. I was really frustrated with it.

This prompted a discussion of what were teachers' strategies for dealing with students who were fixated on algorithms. Rita shared that because some of her students go "straight to algorithms,"

she changed the way she presented a mental math warm-up. She began to ask them to solve it using two different strategies. This strategy provoked a positive response among the teachers. During the seventh study session, Rita shared a True/False number sentence poster, and described her practice:

> We talked about not using standard algorithm, and the kids know what that is already. So then I said get into partners and talk about if this is true or false and think of as many ways as you can to prove that it's true or false. And so then when they came up, I just asked each group to make sure to share one…I made sure I recorded all their answers. And then I also told them to tell me what strategy they used so that they could build their mathematical vocabulary.

Rita used the T/F number sentence to not only practice sharing multiple strategies, but also facilitating partner talk and recording student thinking. Throughout the study teachers shared many ways that the warm-ups served their practice to present multiple strategies. During the sixth study group session, Kathy wrote that she regularly follows up one student's response on a mental math warm-up with, "Did anybody solve it a different way?" During the same session, Lucy shared that when facilitating a *How many ways* warm-up, she chose four different students' work to present on the document camera, while the students explained their thinking. In the ninth study group session, Debbie also wrote about using the document camera to have students share their many ideas around their solving a True/False number sentence.

During the sixth study group session I asked teachers to write about what, if anything, had changed about their practice of teaching mathematics over the two months since the study began. I then followed up that question by asking if any of the reported changes were related to the work we had done around warm-ups, and if so, how? Though teachers described many different individual changes in practice, nine out of the 10 teachers were able to illustrate the links between the warm-ups and their shifts in practice. Some examples that demonstrate this are:

- No question the warm-ups have been a great way to "get my feet wet" with all of these practices [recording their thinking, asking them to explain their thinking and respectfully critiquing other students' thinking]. I think we are honoring "mathematical thinking" in a way I didn't know how to do before. – Lynn
- [The warm-ups are] exposing children to multiple ways to approach, attack and solve a problem. – Debbie
- I try all the activities…Although students struggle at times, I feel like they're making connections. On their own, they use, "I tried so and so's strategy…" as evidence of their connection-making and explaining their thinking. – Brenda

Later in the sixth study group session, I facilitated a discussion about where the teachers felt they wanted to go in the professional development. Based on the discussion, I generated a list that we then reflected on in the next study group session to be sure that these were the things the group wanted to cover. One of the six items generated was: Continue with warm-ups as a vehicle to practice new teaching strategies/practices [they're fun, they create a safe place to try things out, they encourage more math discourse among students]. Teachers agreed they wanted to keep learning warm-ups as part of their professional development.

In the final written reflection (Appendix D) all 10 teachers rated "Learning warm-ups to engage students in math discourse" as "This helped me in my practice a lot." When asked specifically "What do you see as the most helpful aspect of the study group?" seven out of 10 teachers mentioned the warm-up activities. Specific comments included:

- Learning the math warm up activities has really changed my practice. The warm ups are engaging and provide students with the opportunity to view math in a different way. – Lucy

- Math warm-ups create a tone in the classroom of thoughtful discourse. – Lynn
- Being able to learn something new, use it right away in the classroom, and then debriefing about our experiences in the group was incredibly helpful. – Kathy

How the professional development supported warm-ups driving changes in teaching practices. While it was clear that teachers valued learning the warm-ups, I, as the facilitator of the study group, wanted to be sure that teachers were doing more than just introducing new tasks to their students. While I wanted to maintain their engagement in the professional development, I also wanted them to continue to develop their practice and address challenges they were facing. At the second study group session I introduced four questions for teachers to keep in mind when sharing out to the group, in order to keep the focus on their practice:

1. How did you present the warm-up?
2. How did you record the warm-up (or have students record the warm-up)?
3. How did you make connections (or have students make connections) between responses/representations?
4. What questions did you ask?

The study group regularly returned to these four questions throughout the study. They provided the framework for our discussions about practice. I also wanted to ensure that I took advantage of opportunities to delve into particular aspects of their practice. I ended up making strategic decisions about when and when not to introduce a new warm-up based on the data I collected at the previous study group session. By the end of the study, I had introduced new warm-ups in eight out of the 14 sessions.

The first opportunity to use warm-ups to delve into practice arrived in planning the fourth session. At the previous session, a fourth grade teacher brought in student samples of an activity

in which the teachers had engaged during the first study group session. I decided to not present a new warm-up during the fourth session so that we could use some of the samples of student work to dig into the practice of asking purposeful questions both to better understand and to advance student thinking. This activity continued into the fifth study group session where I asked teachers to start with what they felt the student did understand and then develop questions that built upon the student's understanding. Teachers generated questions in small groups and then shared out to the whole group. Some of the discussion revolved around their anxiety about asking questions that were too leading. They continued to talk and write about their development in asking better questions during their facilitation of warm-up tasks over the course of the study.

- [I] asked more questions: "Explain to me your thinking." "Why did you choose to do it that way?" "Is there another way you could have solved this problem?" – Debbie, Session 9 Quick Write
- I'm still working on asking purposeful questions. It's tough at times to try not to "lead them" – Kathy, Session 9 Quick Write
- Perseverance…for myself to keep pushing and asking questions for the students to become more fluent in their explanations. Rita, Session 11 Quick Write
- The questions matter in eliciting the thinking from the students. Each of us described ways we got them to talk about how they arrived at the answer. I think before we were always focused on the answer, not the process. – Lynn, Session 11 Quick Write
- Ask questions to get students thinking about problems – Julia, Post-questionnaire, identified this strength in her teaching practice that was not mentioned on her pre-questionnaire

While many of the participants reported that they found the practice of asking purposeful questions to be challenging, the warm-ups gave them focused opportunities to work on avoiding leading questions.

The warm-ups created opportunities for participants to not only engage in new practices for teaching mathematics, but to work on making actual shifts in their practice. Teachers reported becoming conscious of the questions they were asking students, asking students to explain their thinking, and becoming more comfortable with having students present multiple strategies rather than focus on algorithms alone. Presenting warm-ups meant that teachers were facilitating student discourse during their math time, and this was not a practice that was regularly occurring prior to participating in the study.

Finding #3: Facilitation of Student Discourse as a Regularly Utilized Practice

According to finding #3, teachers reported that there were substantial differences in the student discourse that occurred during math time between the previous school year and this year and that facilitation of student discourse during math time became a regular part of their practice. For this study, *discourse* is an umbrella term for many possible student actions including: explaining his/her thinking, discussing/sharing responses/strategies with a partner, a group, or the whole class; correcting, critiquing, or adding on to another person's thinking, and finding connections between representations or strategies. Participants ranged in the amount of student talk with which they were comfortable in their classrooms both at the beginning and at the end of the study, but all reported that students were talking in a way that made them increasingly interested in developing their practices around facilitating student discourse. The following profile illuminates one participant's experience.

Profile #3: Ilene. Ilene spent 25 years as a pre-school teacher before going back to school so that she could get into a credentialing program. She was in her third year of teaching elementary school, but only her second year teaching third grade when the study began. She shared a story during the second study session about being tutored in mathematics by her "math genius" son in order to get through her coursework. She had always done poorly in math in school. At one point her son stopped her because he noticed that she was merely memorizing how to do problems rather than really understanding how the mathematics worked. Her son helped her not only to understand mathematics from a conceptual standpoint, but also to develop a love and enthusiasm for teaching it. At the beginning of the study she identified her love for teaching mathematics as a strength on her pre-questionnaire.

Ilene was immediately impressed with the response she got from her students when she started presenting warm-ups in her classroom. She took on every opportunity to promote discourse among her students. During the fourth study group session I showed a video of a teacher incorporating partner talk into a mental math warm up. Ilene commented about how she liked that practice and was going to try it. At the fifth study group session during a debrief of a mental math warm-up, she shared:

> I gave them 10 seconds and then they had to turn to their partner…I just walked around and listened, and it was just amazing to listen to their thinking…then they shared out, and I couldn't believe some of the things they did…I've done it [partner talk] before and I do it a lot, but now doing it every single day I see the difference because at first they would just look at each other, and now that they're doing it so much they're really able to talk.

Ilene connected the discourse she encouraged through the warm-ups to students' understanding of mathematics. At the sixth study group session she wrote,

> I begin every morning with one of the warm-ups I learned at the study group…My students are learning how to explain their thinking to me and to each other. My students also have many more opportunities to collaborate while working out problems together. I

> am also noticing that students who were not very interested in math before are now really into the morning warm-ups and that their understanding of mathematics is increasing.

At the end of the study, she reflected to the group the differences she saw in her strengths as a teacher post-study:

> What's my strength now is being able to do all these warm-ups with the children every day and getting them to talk about math and getting them to talk to each other and talk to me about math…like the whole focus has changed. It's all about the children and what they're doing and what they're talking about and saying instead of, oh I love to teach math so they're going to love math…thinking that that's my strength.

Ilene shared her memory of putting up the first *How many ways* warm-up at the beginning of the study, and how she initially got no response from her students. She reported that she thought the students may have been afraid of being wrong at first, and she was thrilled to recognize the change in their excitement to talk—both to each other and to her. She reported that she felt her development in facilitating discourse was helping her students to "understand math better."

Evidence for facilitation of student discourse. Ilene was not alone in her facilitation of discourse among students. During the 13th study session, I asked the teachers to write about whether they noticed any differences between the last school year and the present with regard to student discourse during math time. Every single participant was able to describe multiple differences. Some examples include:

- Students in my class are more willing to share. They are more comfortable making mistakes—they're not embarrassed. Students are more willing to help each other. – Julia
- Huge difference. We are honoring multiple ways to <u>think</u> so there isn't one way. This leads to discussion but not "you are wrong, I am right." – Lynn

- They're used to sharing with their learning partner—speaking and listening. They're real comfortable coming up to the front, projecting their work, and explaining their reasoning. – Kathy
- Adding to each other's conversations; working together collaboratively. – Debbie
- I have always had an emphasis on academic vocabulary in math, but I am seeing that the discussions around the vocabulary are happening on a deeper level. – Maria

Throughout the study teachers voiced how the practice of getting students to explain their reasoning had a powerful impact on both themselves and on their students. During the sixth study group session, Debbie explained:

> It requires you to be quicker on your feet, because when they present something, I found I already have a pre-conceived notion about…I can see this, I can see this…and then a kid will throw something in, and I'll be like, God I never looked at it that way.

In the ninth study group session, participants were asked to write about something they did that week that related to what were working on in the study group. Lynn wrote:

> Students are more public with their thinking and being "right" or "wrong" is not important…representing thinking is valued. Students are anxious to show "another way" of thinking. It relates to everything we do here. Value thinking, discourse around math…

Julia explains during a debriefing of a True/False number sentence warm-up in the 11th study group session:

> The more warm-ups we do, the more they are willing to share what they did. I was especially pleased to see how one of my students used the same strategy another student previously used to find relationships…but discovered that it didn't work for that particular number sentence. She struggled explaining at first, but then managed to explain and make sense of it for the class. I was excited to be able to better facilitate her explanation.

Whereas at the beginning of the study teachers expressed concerns around dealing with students' mistakes, through the course of the study they noticed that they didn't have to say much when they allowed the students to talk. Lynn explains in the seventh study group session:

> They're taking us through their thinking…and what it's doing is it's allowing that mathematical practice of critiquing to happen because kids are looking and then they're raising their hand and very respectfully saying, "I think I see a problem here." And there's discourse going on that I am completely out of. They're catching their own errors.

During the same session, Kathy shared a *How many ways can you make 25?* poster on which one of her students crossed out 12 x 12 + 1. She described recording her idea and asking if it equaled 25. The student said yes, and Kathy moved on to the next person. The student's tablemates pointed out her mistake to her so that when Kathy returned to her later, the student corrected the poster to read 12 x 2 + 1. Kathy complimented the student on her self-correction, whether or not it had been her peers who had helped her find the mistake. In a later study group session, Tammy was proud to share a video of one student correcting the work of another on a number line warm-up. Both students stood at the front of the class, and the student who was corrected listened to his peer and revised his thinking based on his peer's feedback. He displayed no embarrassment at his mistake. Other teachers also talked about consciously shifting from telling students that their thinking was wrong to asking students if they wanted to revise their thinking after hearing other students share different strategies/responses.

At the end of the study, teachers reflected about the changes in their practice in relation to facilitating discourse among their students.

- You know, before when I would teach math it would just be like, do you know the answer...It wasn't HOW did you get it. So having the kids explain…Can you defend your answer? How did you get that? Or does anybody have something to say and debate about this answer. You know, it's listening to what everybody's getting, how they're getting it, and do you agree/disagree. So there is definitely more discourse in my classroom, and I feel more comfortable with the kids sharing with each other. – Kathy

- For me math was always like, you do it by yourself…but now it's more like, ok, get in partners…a lot more partner work, a lot more group work…it used to be a very isolated practice, where now it's more like a community partner-based thing. – Rita
- What we've done in this has flowed over into other areas as well…we talk about science, and there's kids ready to talk, take other things apart—not just math…that's been an interesting offshoot. – Tammy
- I think in addition to the talking about math, I think the writing is getting better…especially from the beginning of the year to now, I think the kids are understanding. They see "explain how you got your answer"…and they're able to say step by step what they did and what they saw. So that's been a huge outcome…that wasn't necessarily intentional with the partner work and the discussion. – Maria

How the professional development supported facilitation of student discourse.

During the first study group session, teachers were distributed the third through fifth grade Common Core content standards, as well as the Standards for Mathematical Practice. As we looked at the Standards for Mathematical Practice, I made the point that these standards could not be met if students weren't talking and reasoning about mathematics on a regular basis. There were aspects of the professional development that were designed to engage teachers in discourse over problem solving so that they would be able to make connections to the process their students might go through in the context of a warm-up.

During the very first session teachers worked in groups of three or four to solve a problem for which there was no one right answer:

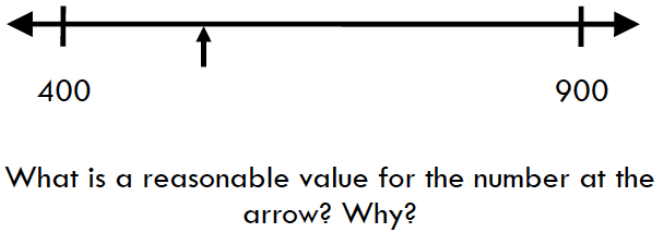

Figure 4.7. Number line activity.

In their small groups, many teachers expressed a lack of confidence with their thinking and/or an inability to explain their thinking. Comments included: "I'm not sure I did it correctly"; "I just don't know how I did it"; and "Should I not have done that?"

When we shared out to the whole group about the experience, Lucy commented, "It was really hard for me to explain to them because I just did it, and then I had to think it over…ok…what were the steps I took?" This activity brought up several issues of practice that became themes over the course of the study. Teachers talked about the difficulty they expected to have getting students to explain their thinking, when they themselves had difficulty with it. Teachers noticed that they understood things after hearing others try to explain, and then were able to participate in reasoning through the problem. They also voiced worries about what to do if students get the wrong answer. Many of the teachers seemed surprised that there could be more than one answer for the problem and were focused on whose answer was best, until they focused on the word "reasonable." Julia commented, "I look back and I said why did I answer it the way I did, and I said the value is *about*, so it's not an *exact* answer. It is reasonable now that you say *reasonable*."

During the 12th study group session I made a strategic decision to engage teachers in a challenging mental math activity that would begin a conversation about the advantages of group work—listening to and sharing strategies in a small group setting. I felt teachers needed to

remind themselves of the benefits of discourse in problem solving. Only a couple of participants had previously shared that they allowed students to work in groups during warm-ups, while many more had incorporated pair-sharing into their class warm-ups. In fact, the activity supported their practice of engaging students in another person's thinking. Comments debriefing the activity included:

- I didn't get the connection that Julia made…and then Julia explained it, and I was like "Oh." – Rita
- Debbie explained how she did the drawing…and so her drawing made me understand the thing that I also missed. – Brenda
- I think there's a lot of value in it [understanding somebody else's strategy]. Because you can just open up the conversation of different children trying to figure it out in the way that they can…And it's not just us up there telling them: this is how you do it. – Ilene

Through this activity, teachers were given the experience that supported their interest in student discourse. Though my original intention for the focus of the activity was different, teachers dug into the more nuanced benefit of struggling to understand another person's thinking.

At the end of the study, nine out of 10 teachers rated working on these kinds of activities as helping their practice. Lynn really struggled with the activity during session 12 and shared out in the final reflection that she thought the group activities were "really powerful…it made me realize what my students feel…the lost students, who don't even know how to approach it, or don't like it." She explains how another group member helped her through the problem and that she was "flabbergasted by her thinking…I would never have generated it on my own. And

experiencing it, not hearing that group work might be beneficial, but doing it, and seeing that…it really was amazing."

I also made decisions at various points in the study to show videos I found online of teachers facilitating warm-ups. These videos ended up supporting teachers' facilitation of student discourse. From the beginning of the study I was hoping that teachers would share videos of themselves facilitating warm-ups, or, for those at the same school, watch each other in practice. I thought this would help to identify both things that they appreciated and things they were challenged by in their practice. I also thought it would be constructive for teachers to get feedback from their peers. Three out of 10 participants did end up sharing videos, but only towards the end of the study, even after I offered to come in and video them myself. No teacher reported that they watched a peer facilitate a warm-up. I felt it was important that teachers be able to analyze warm-ups in practice, so I used the videos I found to facilitate discussions around practice. I only showed videos after teachers had been given the opportunity to try out a particular warm-up themselves. I showed two videos of teachers doing mental math warm-ups during study sessions 4 and 12, and showed one video of a True/False number sentence warm-up during study session 7. Though I chose videos in which I felt the teachers were strong facilitators and used a variety of practices to promote discourse, I tried not to steer the discussions of the videos in any particular direction. I allowed participants to drive the conversation.

As mentioned in the profile for this finding, Ilene latched on to seeing a teacher in a video use partner talk to promote discourse. After showing that video, seven out of 10 teachers talked specifically about their practice of facilitating partner talk or *pair-sharing* during a whole group share out at later sessions. During the eighth study group session, Maria talked about videotaping

herself doing a True/False number sentence warm-up after seeing it done in a video during the previous session. While she didn't share the video with the group, she commented:

> The other thing that was good is that I did do the think, pair, share. So I did let them talk. Some discussions were really good. Others were like one kid dominating and saying how it was so. I want to see more back and forth. Next time I do that I would say...each of you has to share one strategy and then has to be ready to share the other person's strategy. I want to play with that a little more.

The video of the teacher facilitating a True/False number sentence warm-up brought up a practice that became a theme for the rest of the study group sessions: responding to student explanations—whether they're right or wrong—in a way that will promote more discourse. Ilene offered, "I just liked when she said 'I'm just not understanding,' and that helped him to understand that he had to explain it in a different way. She didn't necessarily say, 'You're wrong.'" Julia appreciated a lot about the language the teacher used: "Anyone disagree? Or do you want to add something else? Or does anyone want to share a different kind of thinking? So just kind of putting it on them." One participant noticed that the teacher never said "good" or "right" following a student explanation. Tammy offered, "She didn't make a big deal out of anything. She just said, 'Oh. I don't know. What do you think?" Tammy talked about how hard it was for her to not show excitement because she will often have a "mathematic epiphany" herself after hearing a student explanation. I asked the group what they felt the advantage or disadvantage might be to not show excitement for student responses, not give an affirmation. The following is part of the discussion that ensued:

> Maria: I think that when kids hear "That's good!" then they kind of stop…I think it limits their thinking. At least that was me as a kid, where I was kind of lazy so if I knew how to do it, it was done. Or even leaving it a little more open like that, kids are still thinking…they're still turned on to the problem.
>
> Julia: Well, I do agree that no judgment is best. I mean lately I've been getting too excited some times…Like the other day, one of my kids made a connection with "How many ways?" and he said "commutative property" and I said, "Wow, that's

awesome that you made that connection." I still left it…"did anybody have any other connections? That's not the only connection." I kind of gave way for other kids to share, but I really pointed it out…so I guess in a way I wanted to encourage them to use more math vocabulary.

Rita: When a person is affirmed, they felt limited, like, oh, that's the right way. [Tammy] said she gave them a compliment, so that's what she's looking for. While like when you complimented him on the use of vocabulary that was to motivate him to use more of it. So…the "Good job" in both senses are to motivate different things in the student.

Lynn: One is outcome and one is process. With vocabulary you're talking about the process, and the other is the right answer, which we're trying to get away from.

The discussion above is indicative of the work in which the participants often engaged. They consistently reflected, gave feedback, and added on to each other's thinking. During the last study group session, Tammy referred to the excited responses to students that she talked about in the above discussion:

> You know I learned to calm myself down…So good to hear a crazy good answer, to stop and slow down and say, "So does anybody else have a way? I've gotten better at that…I could stay calm because I knew there was more to come.

Incorporating small group problem-solving tasks and watching and debriefing videos of teachers into the professional development proved to drive teacher learning around facilitation of student discourse.

Summary of the Findings

This action research study of professional development in the form of a mathematics study group generated findings that support this type of professional development. The mathematics study group provided a space for teacher participants to regularly experiment with their practices for teaching mathematics, and, in fact, change some of the ways in which they engaged their students in mathematics. All of the 10 participants in the study reported that there were substantial differences in the student discourse that took place during their mathematics

lessons. They attributed these differences to both the warm-ups they were learning and facilitating with their students and the practices they were actively working on in the study group.

Some of the aspects of the professional development that supported the development and changes in practice that teachers reported were designed prior to the start of the study. These were the experiences of sharing stories, building norms together, allowing teachers to give input into the direction of the study group, and engaging teachers in warm-up activities appropriate for their mathematics level. Other aspects depended upon the evaluation and diagnosis steps of each action research cycle during the study. These were: involving teachers in several small group warm-up planning and debriefing sessions, digging into the practice of asking purposeful questions when the opportunity presented itself, engaging in additional adult warm-up activities, and sharing videos of other teachers in practice.

CHAPTER FIVE: DISCUSSION

Teachers' Experimentation of Practice

The mathematics study group provided a framework for teachers to take multiple, focused opportunities to experiment with their practice (Grossman & McDonald, 2008). Teachers regularly experimented with their practice because the warm-ups allowed them a short time frame during their class time to try things out. The warm-ups were meant to take anywhere from five to 15 minutes of class time. Teachers could easily retreat from the warm-ups if they became uncomfortable with their own practice or uncertain of how to facilitate student discourse. Teachers reported that sometimes warm-ups didn't go well, that they went on too long, that students were not listening to each other, that they didn't do a good job of facilitating the discussion. Yet teachers continued to try them out and learned from the experiences that others in the study group shared. The sustained nature of the study group supported the development of practice (Darling-Hammond et al., 2009; Yoon et al., 2007).

In the study group, we often talked about the different expectations for students that come with the Common Core Standards for Mathematical Practice. These practice standards demand that students develop number sense so that they can "make sense of problems," "reason abstractly and quantitatively," "construct viable arguments and critique the reasoning of others," and "look for and make use of structure" (NGACBP & CCSSO, 2010, pp. 6-8). Teachers were aware of their over-attention to procedural learning, but they expressed nervousness about how to facilitate these newer standards without curriculum that aligned to them. The warm-ups provided the space to work on teaching practices that engaged students in the aforementioned Standards for Mathematical Practice and develop students' number sense without overwhelming the teachers. They could break away from their former models of teaching practice without

feeling that they had to change everything about their practice (Harbin & Newton, 2013). Additionally, teachers' continued attendance in the study group seemed to indicate that they knew they would be able to apply what we did in the study group to their classroom planning and instruction (Darling-Hammond et al., 2009; Garet et al., 2001). In other words, teachers could experiment with practices through the warm-ups that they could readily connect to their regular classroom instruction.

Warm-ups as a Driving Force for Development of Practice

In thinking about why the warm-ups proved to be a successful vehicle for development of practice, it is important to consider the sensitivity and defensiveness teachers often have about their practice. Contributing factors to such sensitivity include the powerful influence of teachers' experiences as students, their lack of confidence with respect to teaching and learning mathematics, and the entrenched culture of what teaching looks like in the United States (Bursal & Paznokas, 2006; Gresham, 2008; Harbin & Newton, 2013; Stigler & Hiebert, 1999). If teachers are made to feel as if their practice of teaching mathematics is wrong, they are more likely to be resistant to challenging their beliefs about mathematics and working to develop their practice (Mewborn & Cross, 2007). Warm-ups did not overtly challenge the teachers' current practice, though teachers did report that the practices they learned around warm-ups positively impacted their regular math lessons, and sometimes even became the focus for an entire lesson.

One of the factors illuminated by this study that seemed to make a difference for developing teachers' practice is using a narrow focus on instructional activities in the professional development that offers a rich arena to work on multiple practices (Grossman & McDonald, 2008; Kazemi, Franke, & Lampert, 2009). The extant literature offers varied approaches to teacher learning using curriculum materials, some focused in nature, but none of

the research specifically addresses the use of warm-up tasks to promote student discourse (Borasi & Fonzi, 2002; Kabasakalian, 2007; Sowder, 2007). The warm-ups were narrowly focused both in content and time frame, and gave teachers a lot of options about how they wanted to utilize them. Some teachers used them as a jumping off point for a lesson. Others appreciated that they could be entirely unrelated to the day's given lesson and enjoyed having students make sense of something they may not have thought about in a while. The warm-ups provided opportunities for a variety of teaching practices to surface and develop. Practices included: having students explain their thinking, recording student thinking, presenting multiple strategies, and asking purposeful questions to advance student thinking.

Teachers utilized the warm-ups and their sharing of experiences using them to engage in a productive struggle regarding the nuances of facilitating student learning. This *productive struggle* among the teachers paralleled the kind of engagement in learning the teachers were hoping to elicit from their students. Of course, the goal of the mathematics study group was not just to become proficient in utilizing warm-ups, but to use these safe, bounded activities to develop teaching practices that would impact their teaching of mathematics in broader contexts.

The Value in Facilitating Student Discourse

The findings from this study fit with the existing literature on professional development for mathematics teaching that focuses on student thinking as an approach to teacher learning (Sowder, 2007). I believe teachers' excitement for the things their students were saying led to shifts in their thinking about the value of allowing for discourse in their mathematics lessons. Often teachers experience professional development as unrelated to their needs. They may be interested at first and try out what they learn, but over time they frequently find themselves returning to whatever they previously did. The fact that the warm-ups served as a simple vehicle

for student discourse, and teachers could immediately see the benefit of that discourse—both for their own teaching practices and their students' sense-making of mathematics—contributed to their desire to continue to work on their practice and allow student discourse to become a regular part of their mathematics teaching practice (Franke et al., 2001).

At the sixth study group session, teachers had the opportunity to change the course of the professional development, but teachers expressed their desire to continue to learn warm-ups. The warm-ups naturally elicited student discourse, which the teachers reported that the students themselves seemed to enjoy, and which in turn encouraged teachers to continue to experiment. All the teachers frequently reported their amazement at what a variety of students said during the warm-up activities. They appreciated that all students were often able to participate and contribute. Teachers learned about how their students were thinking about mathematics through the warm-ups. This teacher learning complemented our study group discussions around how students learn mathematics, aligning with research-based goals for professional development for teaching mathematics (Sowder, 2007).

Action Research as a Model for Professional Development

Another factor illuminated by this study that made a difference for developing teachers' practice is that the action research design of the study allowed for an organic development of study group agendas. While there is scant literature documenting the use of action research in professional development for teaching mathematics, this study does contribute to the research supporting professional development that fosters an inquiry approach to teaching and learning (Lamb, Philipp, Jacobs, & Schappelle, 2009; Sowder, 2007). Rather than approach the study group with a defined intervention, I provided an initial focus of using warm-ups to promote student discourse. This focus was subject to change based on the needs and interests of the study

group participants. I, as the facilitator of the study group, could gauge when there was an opportunity to delve into a particular practice, rather than present a new warm-up each time for them to try.

There was never a plan to introduce a particular number of warm-ups in a particular order, or tell teachers what to do or how to teach. As the professional developer, I only gave the teachers some areas for focus when reflecting on their practice: how did they present the warm-up, record student thinking, make connections? What questions did they ask? Teachers constructed their own professional knowledge within a community of learners through using the warm-ups to experiment with their practice. Best practices of effective professional development, particularly for teaching mathematics, align with opportunities for such construction of knowledge (Darling-Hammond et al., 2009; Lamb et al., 2009; Sowder, 2007, NRC, 2001). The fact that the professional development was planned for over 20 hours of contact over 6 months allowed teachers the time and space to dig in and shift certain aspects of their practice (Yoon et al., 2007).

One of the major findings from the study is that learning warm-ups to promote student discourse proved to be a particular aspect of the professional development that supported teacher development of practice, yet there were other aspects that seemed to make a difference for individual teachers. Though the focus of the mathematics study group was engaging in warm-ups, cycles of action research led to planning other study group activities that contributed to teacher learning around the warm-ups. For several participants, the videos of teachers in practice and the ensuing study group discussions were important, as participants attributed things they tried to the videos we watched during study group sessions. An example of this was the True/False number sentences poster that Rita shared (see Figure 4.3) during session 9. She used

the technique of unraveling the chart paper with each new number sentence as she had seen the teacher in the video do in a previous study session. Some teachers particularly appreciated sharing and listening to others share about how they facilitated the warm-ups. Another group of teachers felt that engaging in adult warm-up activities were beneficial for their own learning and development. I provided a variety of activities, while keeping the focus on warm-ups, in the hopes of driving learning that might impact teacher practice. I knew it was important to continue to engage teachers, just as teachers know that they need to continually engage their students.

Benefits of a Community of Practice

All of the findings are supported by the fact that the teachers kept returning to the study group and continued to bring artifacts from their practice with them. This is important because many teachers will not opt to participate in sustained professional development when it is not mandatory or does not take place during the regular school day. The participants in this study found enough value in the group to continue coming over a period of 6 months, even when they experienced warm-ups that did not go well or they were disappointed in their own practice of facilitating a particular warm-up. The participants found strength in the group and were interested in other participants' experience of experimenting with their practice. Best practices of professional development include connecting to practice and developing strong working relationships among teachers (Darling-Hammond et al., 2009; Sowder, 2007). Teachers were often excited by what went well in another teacher's classroom and talked about wanting to try it out in their own. While I ended each session with a challenge to try something out prior to the next meeting of the study group, I always allowed space for participants to share their experience of doing warm-ups, even if that experience did not address the particular challenge I set out for them. They always had autonomy over what they chose to work on and report out to the group,

as long as it pertained to their practice. I believe the professional development, through collaborative inquiry, fostered teachers' curiosity about their own teaching and their students' learning (Lamb et al., 2009).

Conclusion

There are many possible approaches to professional development wherein teachers acquire the necessary knowledge for their practice of teaching mathematics. This study demonstrates that a facilitated mathematics study group that utilizes a narrow focus on instructional activities in order to support engagement around the teaching and learning of mathematics is a viable structure for professional development. The role of the warm-ups was to engage teachers in inquiry about their practice in the context of the Common Core Standards for Mathematical Practice. Had the study group continued, the focus could have extended well beyond the use of warm-ups. In fact, findings from the study suggest that this type of action research model for professional development could and should be an ongoing, integral part of the work of teachers. The content of the mathematics study group was neither packaged, nor entirely pre-planned, but was guided by the collective goal among the teachers to improve their practice for teaching mathematics. This model for professional development supported teachers by attending to their needs as learners.

Limitations

There were several limitations to this action research study. First, the study included a small number of upper elementary teacher participants in only one district, though four out of five elementary schools in that district were represented. The findings from this sample may not be representative of the majority of elementary school teachers either within or outside the study district.

Additionally, the study participants entered the study with a clear interest in working on their practice. This was demonstrated by the fact that the study group met outside of the school day over a long period of time and participants were offered only a token monetary incentive. Teachers who had no interest in improving their practice for teaching mathematics would be unlikely to sustain participation over 6 months.

Finally, this study did not include pre- and post-classroom observations as part of the data collection. Development of practice over time was based on teacher reports of their own practice. Findings emerged in the absence of an objective view of development of practice, with the exception of recorded and transcribed teacher talk over the length of the study and delivered artifacts of practice.

Recommendations for Educators

My recommendation for districts is that they ensure that the mathematics professional development they provide for teachers:

1. Focuses on the practice of teaching using instructional activities to promote discourse
2. Ensures a sustained, safe environment in which teachers can experiment with their practice
3. Fosters inquiry among a community of teachers where experiences can be openly shared and reflected upon

In addition to the above recommendations, I would add the recommendation for professional developers that they utilize an action research model in the planning of sustained professional development activities, no matter what is the initial area of focus for the professional development. Professional developers must engage teachers in a process of inquiry about their practice if they hope to shift their practice.

Suggestions for Additional Research

As my study did not collect pre- and post-classroom observation data, it would be beneficial to conduct a similarly designed study and collect such data so that findings would not only rely on what participants report. Additionally, as the participants in this study perceived the study group to be beneficial for their practice, future research could track whether the same findings would stand for teachers who did not volunteer to be part of the study. In other words, would teachers who were not specifically looking to work on their practice of teaching mathematics also benefit from participating in a mathematics study group?

The role of the facilitator is worth exploring as well. For my study, I was perceived as an *expert* facilitator. While there is research that supports this model of facilitated teacher study groups (Crespo, 2006; Franke et al., 2001; Kabasakalian, 2007; Zevenbergen, 2004), other literature refers to the facilitator as a *perceived colleague* (Lamb et al., 2009). While my study did not collect data regarding participants' value of my role as the facilitator, this is an area that warrants further study as the success of professional development experiences may rely on these perceptions.

Lastly, as stated in Chapter 2, few studies have followed teachers for more than a year after professional development to track sustained teacher change. It would be beneficial to know whether the participants in this study continued to draw upon what they learned in the study group in the years that follow the study.

Final Reflection

As a former teacher of secondary mathematics, I have grave concerns about the instructional changes that need to take place in order to foster student understanding of mathematics. In my current position as Assistant Principal over curriculum at a high school, I am

witnessing the resistance of teachers to challenge their understanding of how students learn mathematics, and thereby develop or change their instructional practices to foster students' conceptual understanding of mathematics. It's not that these teachers are not hardworking. Even in the face of compelling evidence of students' lack of achievement, it is hard to change practice. This study gives me hope.

Leading up to the design of my study, I was interested in many aspects of developing elementary teachers' practice for teaching mathematics. I was interested in teachers' content knowledge, their confidence in teaching mathematics, and their general beliefs about how mathematics should be taught. I landed on the focus on practices used while teaching mathematics, knowing that all the other aspects in which I was interested were likely to come up. They did. My goal was to support them in any way I could to keep their interest in developing their practice.

Some of the activities I brought into the mathematics study group were in response to teachers' expressed lack of content knowledge and their lack of confidence to teach conceptual understanding of mathematics. It was helpful for me to keep the focus on practice and encourage them to actually do things in their classrooms regardless of how confident they felt. They were learning while they were trying out practices. It became clear almost immediately that teachers did not have to have all the answers (or even any answers) prior to facilitating experiences that promoted student learning. The teachers responded to my growth mindset for them. They often voiced their own deficiencies, but I worked to model an approach to their learning that I hoped would transfer to how they approached their students' learning. I pointed out what they were doing to encourage and advance student thinking. I asked questions, and worked hard not to answer the questions for them. I often made a point of how they never would have known the

nuances of what their students did know if they hadn't allowed them to talk at all. This seemed to resonate with them. They could see that the former approach of seeing a solution to a mathematics problem as either right or wrong thwarted opportunities for learning in their classrooms. At the same time, teachers expressed anxiety over the time it took to address student understanding in a meaningful way.

I think if the mathematics study group were to continue I would need to more directly address the deficit stance teachers sometimes took with their students. Teachers at times expressed disappointment in their students: their inability to connect to prior knowledge, maintain engagement in a discussion, or use mathematics vocabulary appropriately. I worked to keep the focus on understanding what the students did know, and what questions to ask to get the students *unstuck*. It was an incredible experience for me to continually draw relationships between what seemed to encourage teacher learning and what the teachers were reporting about their students' learning. One of the most amazing moments occurred when a teacher realized her *top* student, a student adept at performing algorithms, had virtually no understanding of the mathematics behind the procedure. He simply couldn't make sense of it. This really shook the teacher and further inspired her to work on her practice.

The teachers that I had the opportunity to work with were hungry for meaningful collaboration and opportunities to reflect on their practice. The challenge for me moving forward will be to find ways to engage all teachers in the development of their practice and foster a growth mindset both for themselves and for their students.

APPENDIX A

TEACHER QUESTIONNAIRE

NAME _____ Current Grade Level ____ Date_____

A. How comfortable do you feel using the following mathematics teaching practices in your classroom? Please use the following responses:

1-Not Comfortable
2-Somewhat Comfortable
3-Very Comfortable
4-I've rarely/never done this, but I don't think I would feel comfortable using this practice.
5-I've rarely/never done this, but I think I would feel (somewhat) comfortable using this practice.

Practice	**How comfortable do you feel using this practice for teaching mathematics? [Use numbers 1-5]**
1. Teacher quizzes students (formally or informally) on math facts	
2. Students practice a particular procedure (i.e., long division or converting fractions to decimals) multiple times during math time	
3. Students use a manipulative, tool or model to solve a problem	
4. Students practice problems on a worksheet or out of the textbook/workbook	
5. Teacher begins a math lesson with a warm-up activity that gets students talking about math	
6. Teacher presents multiple ways of representing or solving problems during the same lesson	
7. Teacher asks student/s how he/she/they got their answer/s	
8. Students talk to other students about how they solved a problem (either in partner, small group or whole class discussion)	
9. Students write in math journals	
10. Teacher administers a speed test of a math fact/math facts	
11. Teacher presents a problem for students to solve involving new content before teaching that content (i.e. $\frac{2}{5}+\frac{1}{2}=?$ before teaching adding with unlike denominators	
12. Students work in pairs during math time	
13. Students work in groups of 3 or 4 during math time	
14. Teacher goes over math homework	
15. Student/s present/s to the class his/hers/their strategies for doing a math task/solving a problem	
16. Teacher prepares/reviews for a test/quiz	
17. Students work on problems of the week/month	
18. Students write about how they solved a problem	
19. Teacher adds a term to a math word wall	

B. How well do you think this practice works in your classroom? Please use the following responses:

1-This doesn't work in my classroom.
2-This works ok in my classroom.
3-This works well in my classroom.
4-I'm not sure how well this works in my classroom.
5-I don't currently do this, but I don't think this would work in my classroom
6-I don't currently do this, but I think this would work in my classroom

Practice	**How well does this practice work in my classroom? [Use numbers 1-6]**
1. Teacher quizzes students (formally or informally) on math facts	
2. Students practice a particular procedure (i.e., long division or converting fractions to decimals) multiple times during math time	
3. Students use a manipulative, tool or model to solve a problem	
4. Students practice problems on a worksheet or out of the textbook/workbook	
5. Teacher begins a math lesson with a warm-up activity that gets students talking about math	
6. Teacher presents multiple ways of representing or solving problems during the same lesson	
7. Teacher asks student/s how he/she/they got their answer/s	
8. Students talk to other students about how they solved a problem (either in partner, small group or whole class discussion)	
9. Students write in math journals	
10. Teacher administers a speed test of a math fact/math facts	
11. Teacher presents a problem for students to solve involving new content before teaching that content (i.e. $\frac{2}{5}+\frac{1}{2}=$? before teaching adding with unlike denominators	
12. Students work in pairs during math time	
13. Students work in groups of 3 or 4 during math time	
14. Teacher goes over math homework	
15. Student/s present/s to the class his/hers/their strategies for doing a math task/solving a problem	
16. Teacher prepares/reviews for a test/quiz	
17. Students work on problems of the week/month	
18. Students write about how they solved a problem	
19. Teacher adds a term to a math word wall	

C. What do you feel the best about in your teaching of mathematics? (Use back if you need more space)

What do you feel most uncertain about in your teaching of mathematics?

APPENDIX B

TEACHER QUESTIONNAIRE RESPONSES

**NOTE: Directly after taking the post-questionnaire at the last study group session, participants were asked to reflect about how their thinking may have changed since they took the questionnaire six months previously. Several reported that they really didn't know what was meant by some of the practices when they first saw them, so they didn't trust what they may have written at the beginning of the study. It is important to consider the changed perspective and knowledge of the participants when viewing the pre- and post-questionnaire responses.

Comfort	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19
Debbie																			
Pre	3	3	3	3	3	3	3	3		3	5	3	3	3	3	5	5	2	5
Post	3	3	3	3	3	3	3	3	5	3	5	3	3	3	3	3	5	3	5
Ilene																			
Pre	3	3	3	3	3	3	3	3	2	3	2	3	1	3	3	3	2	3	3
Post	3	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	4
Lynn																			
Pre	3	2	3	2	3	3	3	3	3	2	4	3	5	3	3	3	5	3	5
Post	2	2	3	1	3	3	3	3	3	1	3	3	5	3	3	3	5	3	5
Maria																			
Pre	3	3	2	3	3	2	2	2	2	3	1	2	2	3	3	3	2	2	4
Post	3	3	3	3	3	3	3	3	5	3	3	3	3	3	3	3	3	3	5
Tammy																			
Pre	3	3	2	3	2/3	2/3	2	2/3	2/3 5	3	1/2 5	3	2/3	3	2 5		5	2/5	5
Post	3	3	2	3	2/3	2/3	2/3	1	3	3	1	5	5	2/3	2/3	3	5	5	5
Kathy																			
Pre	3	3	2	3	5	3	3	3	5	3	5	3	3	3	3	3	5	3	5
Post	3	3	3	3	4	3	3	3	3	3	3	3	2	3	3	3	3	3	5
Lucy																			
Pre	4	3	3	3	3	3	3	5	3	4	3	3	3	2	3	3	5	3	5
Post	2	2	3	5	3	3	3	3	3	2	3	3	3	5	3	3	5	3	5
Rita																			
Pre	3	3	2	3	5	2	3	3	3	3	5	3	3	3	3	3	3	2	5
Post	3	3	3	3	3	3	2	3	5	3	2	3	3	3	2	3	5	2	5
Brenda																			
Pre	3	2	3	2	5	3	3	3	3	3	1	3	3	3	3	2	2	3	
Post	3	1	3	3	3	3	3	3	5	3	3	3	3	2	3	2	5	3	3
Julia																			
Pre	3	2	2	2	3	3	3	2	3	3	3	3	3	1	2	3	2	2	5
Post	2	1	1	1	3	2	3	3	3	1	2	3	3	1	3	2	2	3	4

Works Well	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19
Debbie																			
Pre	3	3	3	3	3	3	3	3	3	3	6	3	3	3	3	6	4	2	6
Post	3	6	3	3	3	3	3	3	6	3	6	3	3	3	3	3	6	3	6
Ilene																			
Pre	3	3	3	3	3	3	3	3	6	3	6	3	5	6	3	3	6	6	6
Post	3	3	3	3	3	3	3	3	6	3	3	3	3	3	3	3	6	3	6
Lynn																			
Pre	3	4	2	4	3	3	2	2	2	2	6	3	2	2	4	4	6	2	6
Post	2	2	3	4	3	3	3	3	3	4	3	3	2	4	3	4	6	3	6
Maria																			
Pre	3	3	3	4	3	3	3	3	3	4	4	2	1	4	3	3	4	4	4
Post	4	2	3	4	3	3	3	3	6	3	6	3	3	3	2	3	6	3	6
Tammy																			
Pre	3	3	2/3	3	3	2/3	6	2	6	3	6	2/3	2/3	2/3	2/6	3	6	2/6	6
Post	3	2/3/6	2/3	3	2/3	2/3	2/3	1/2	3	3	1/5/6	1/2	2	2	2	3	6	6	6
Kathy																			
Pre	2	3	2	3	6	3	3	2	6	2	6	2	2	3	3	3	6	2	6
Post	3	3	2	3	3	3	3	2	3	2	6	3	6	3	3	3	2	3	6
Lucy																			
Pre	5	4	3	4	4	3	3	4	3	5	4	3	3	4	3	3	6	3	6
Post	5	4	3	4	3	3	3	4	4	5	4	2	4	5	4	2	5	4	5
Rita																			
Pre	3	3	2	3	6	6	2	2	3	3	6	3	3	2	6	2	6	2	6
Post	3	3	3	3	3	3	2	3	6	2	2	3	3	2	2	2	6	2	6
Brenda																			
Pre	3	2	3	2	6	3	3	3	2	2	5	3	3	2	3	2	6	2	6
Post	3	5	6	3	3	3	3	3	6	3	3	3	3	2	3	4	6	3	6
Julia																			
Pre	3	2	2	3	3	3	1	1	1	3	1	2	3	5	2	3	6	2	6
Post	2	4	1	2	3	2	2	2	2	1	2	2	2	5	2	5	2	2	6

APPENDIX C

DESCRIPTION OF STUDY GROUP SESSIONS

PD Session 1: 1.5 hours

A. 25-30 minutes Administration of the Pre-Assessment

B. 15 minutes: Introductions and confidentiality/privacy norms

C. 30- minutes **Activity 1—What is a reasonable value for the number at the arrow? Why?**
Teachers work for 3 minutes by themselves on the activity
groups of 3 or 4 share their ideas for 8-10 minutes
8-10-minute share-out to whole group of what came up when they shared out

[Give out packets with standards]

10-minue Task debrief:
What math did you need to use to be able to work on this task?
What's the advantage of doing tasks like these [SMPs?]
What would be hard for your students if you presented a task like this?

D. 15 minutes: **Math Warm-ups**

How many ways can you make the number 60?
What will your students say? [3-5 minutes]
Share out as whole group; find ways to relate different representations

Can we commit to trying one of these before the next time we meet?

PD Session 2: 1.5 hours

A. 5 minutes: **PD 2 STARTER Quick Write**: What is something that surprised you around math in your classroom since the last time we met? If there were no surprises, what is something memorable that happened around math in your classroom since we last met?

B. 15 minutes—Group Norms & Math Stories

C. 20 minutes Did anybody try the 'how many ways can you make this number' warm-up? [share out]

Focus on practice: --how did you present the task?
--how did you record (or have students record)?
--how did you make connections (or have students)?
--what questions did you ask?

What might be other good numbers to try?

D. 20-25 minutes **Activity 2—3 of these things…**

180	510
105	50

100	81
12	36

$\frac{1}{3}$	$\frac{3}{4}$
$\frac{3}{2}$	$\frac{7}{8}$

#1: Teachers-think time for 1 minute; share out with whole group (5-7 minutes)

#2: work alone for 2 minutes; 3 or 4 share their ideas for 5-7 minutes
5-minute share-out to whole group of what came up when they shared out

What would be hard for your students if you presented a task like this?

E. 15-20 minutes: Think about something you're working on right now in your math class, and write a 3 of these things warm-up
Remember: There have to be multiple possibilities
[work in grade level teams]
Try yours out with a partner.

Anyone want to share out with a group?

Can we commit to trying one of these (or how many ways can you represent this number?) before the next time we meet, and bring in evidence of what came out of the discussion?

F. 10 minutes: **PD 2 EXIT Quick Write:** Thinking about either last week's and/or today's session: What is something NEW you heard or thought about mathematics or teaching mathematics that you hope will stay with you in the coming weeks?

PD Session 3: 1.5 hours

A. 5 minutes: **PD 3 STARTER Quick Write**: Have you noticed any shifts in your practice of teaching mathematics in the last couple of weeks—things you do differently than you used to? If not, what is something specific you would like to get out of the study group?

B. 15 minutes—Review of Group Norms & Math Stories

1. Start on time
2. End on time
3. Keep confidentiality [don't talk about what others share about their practice]
4. Focus first on what works about each other's practice [what do we like? What might we use?]
5. Patience (when trying to understand each other's thinking)

C. 25-30 minutes Did anybody try the '3 of these things' or 'How many ways' warm-up? [share out]: How did you present? Record? Make connections? What questions did you ask?

Any new news about 'How many ways' warm-up?

D. 5 minutes—A look at the SMPs (purposeful connections to what you have already done)

E. 5 minutes--Study Details

F. 10 minutes: TALK MOVES video:
https://www.teachingchannel.org/videos/student-participation-strategy
[improving participation with talk moves: repeating; adding on; 'thinking the same' signal; revising]

G. 20-25 minutes **Activity 3—Mental Math…What experiences have you had doing mental math in the past?** [start individually—1 minute, then in groups]

372 + 98

359 + 36

63 – 27

462 – 33

12 x 25

PD Session 4: 1.5 hours

A. 5 minutes: **PD 4 STARTER Quick Write**: Respond to **either** one of the following (you may respond to both, if you'd like):

1. PROUD MOMENT: In the last 2 weeks, what is something you did (or didn't do) or asked or connected while teaching mathematics that you are proud of?
2. CHALLENGING MOMENT: In the last 2 weeks, what is something that challenged you in your practice of teaching mathematics?

B. 5 minutes—Review of Group Norms & Math Stories

1. Start on time
2. End on time
3. Keep confidentiality [don't talk about what others share about their practice]
4. Focus first on what works about each other's practice [what do we like? What might we use?]
5. Patience (when trying to understand each other's thinking)

C. Last week we saw a short video on talk moves. Does anyone have students who don't want to/or are reticent to participate? Did anyone try any of the strategies we saw in the video?
30-50 minutes Did anybody try the 'Mental Math' [share out]

D. 10 minutes-Video: https://www.teachingchannel.org/videos/third-grade-mental-math
3rd grade mental math activity [start at 1:55]
Discussion—what do we think about that?

E. Any new news about 'How many ways' or '3 of these things' warm-ups?

F. 30 minutes: First show 1-minute video showing student with misconception [Marisa, 295 students, 25 buses]
https://mathreasoninginventory.com/home/videolibrary

What do we think about this?

ASKING QUESTIONS activity—looking at student work [began activity—ran out of time]
Working at tables—generate a list of questions for 3 student samples
What questions would you ask these students?
[by yourself for 4 minutes—talk for 10 minutes—share out]

PD Session 5: 1.5 hours

A. 10 minutes watch video from MRI again and do whole group posing questions activity: 1) start with what the student DOES understand 2) build on their understanding

B. 30 minutes: Developing questions through looking at student work:
5 minutes—individual work at generating questions around 3
15 minutes—sharing questions at tables
10 share out to whole group

C. 20-30 minutes: mental math share out
Pair-share?

D. 20-30 minutes: New task: true/false number sentences

5 + 7 = 12
21 + 39 = 50
62 + 76 = 238

[look at hand-out together]

PD Session 6: 1.5 hours

A. Did anybody try the 'T/F Number Sentences' warm-up? [share out]
Focus on practice: --how did you present the task?
--how did you record (or have students record)?
--how did you make connections (or have students)?
--what questions did you ask?

B. 5 minutes: Quickwrite 6: We began the mathematics study group on September 11 and now it is November 13. What, if anything, has changed about your practice of teaching mathematics in the last 2 months?

C. Beginning of Action Research Cycle 2: CHECK-IN

- Remind teachers of the standards of mathematical practice
- Recap of activities we've done

Circle group protocol--
First round:
1. What do you think so far has changed (or not changed) about your practice?
2. Is any of that related to the work we've done on warm-ups? If so, in what way?
(if time—comment round)
Next round
3. Where do we go from here?

For next session: Re-visit at least two different types of warm-ups
Try to watch another teacher at your school do a warm-up

PD Session 7: 1.5 hours

A. Video and discussion: 4th grade T/F number talk
http://www.insidemathematics.org/classroom-videos/number-talks/4th-grade-math-can-this-be-true/number-talk

(start at 2:20 - ~11:20)

What do we like about what she's doing; what do we not like?

B. 30-40 minutes-Did anyone get to watch another teacher do a warm-up? [practice sharing] Sharing out about *your practice*

C. Add-on to T/F Number sentences

D. 5 minutes RECAP of 'Where do we go from here?': Display bullet points; allow for additions

For next session: Try a T/F Number sentence with relational thinking…then Try an open number sentence

PD Session 8: 1.5 hours

A. Read article: True, False, and Open Sentences
What are things that strike you as interesting about this lesson?
What is the teacher doing?

B. Share-out of practice since last week

C. 5 minutes Quick view/talk about scoring the SBAC— writing to explain

D. Fractions activity/lesson
10 seconds—what is the first number that comes into your head?
1 minute to work by yourself
4 minutes to talk in your groups
10 minute share out

5 minutes [present the way the task is for 3rd grade]: what will your students struggle with here?

E. Fractions progression module from illustrative mathematics: Unit 1-Meaning of Unit fractions: https://www.illustrativemathematics.org/progressions

F. Present fair share warm-ups

How many ways can 6 people share a cake so that everyone gets the same amount?

Stick with 2, 3, 4, 6, 8, 10 sharers

For next session: Try a fair share problem

PD Session 9: 1.5 hours

A. Quick write: Write about something from a mathematics lesson this week (Monday through Thursday) that you relate specifically to being in this study group (something you did, said, asked, recorded, had your students do, etc.) ***How*** does it relate to the work we do here?

[after finished, look at this 3 of these things…]

$\frac{1}{3}$	$\frac{3}{4}$
$\frac{3}{2}$	$\frac{7}{8}$

B. Recap of last session—Open number sentences/written article/fractions activity/fractions video

Revisited Norms, as many people came late

C. SHARE OUT Did anyone try a equal/fair sharing problem? Anything anyone want to share out?

D. RE-VISIT Number Lines

F: Mini lesson study: *Grade level planning* on using number lines (or any warm-up) to work on fractions [or whatever you can agree upon]

For next session:
Share-out with documents—what you agreed upon

PD Session 10: 1.5 hours

A. SHARE OUT Did anyone try a number line problem? Anything (not related to the agreed upon warm-up that anyone wants to share out?

B. Mini lesson study:

Circle group protocol—working in grade level group to share out
--something that you thought went well
--something that challenged you/something you would have done differently

Whole group share out—AHAs from sharing experience of doing the same activity

[Is this something we want to try again?]

C. Mini lesson study: *Grade level planning* on using number lines (or any warm-up) to work on fractions [or whatever you can agree upon]

D. Fractions progressions video: Comparing Fractions
https://www.illustrativemathematics.org/progressions

Thoughts about what we've seen in the video?

For next session:
Share-out with documents—what you agreed upon

PD Session 11: 1.5 hours

A. Sharing 2 participants' videos

What do you think kids are understanding by doing this?
How does this challenge your practice?

B. Mini lesson study:

Circle group protocol—working in grade level group to share out
--something that you thought went well
--something that challenged you/something you would have done differently

Whole group share out—AHAs from sharing experience of doing the same activity

C. Mid Quick-write [Write about anything interesting that came out of the grade level shared activities--both this week, and the last time we met? Is there anything that resonated for you about your own practice—if so, what?]

D. More fair-sharing problems

E: SHARE OUT Did anyone try a number line problem? Anything (not related to the agreed upon warm-up that anyone wants to share out?

For next session: Try a type of warm-up you haven't tried before

PD Session 12: 1.5 hours

A. Quick-Write: Please check which warm-ups you have tried at least once. If you've tried any more than once, estimate how many times you've tried them. Please indicate any you prefer over others and why.

B. Mental Math Activity (5 minutes on own) 5 minutes sharing strategies; 10 minutes sharing out
What was challenging about this activity?
What would be challenging for your students?

C. Mental Math video: https://www.youtube.com/watch?v=Cvqb6j5gFNY
Multiplication 63 x 6 [3rd grade—start at 55 sec]

What do we like about her practice? What would we do differently?

Revisit 4 practices: 1) presenting 2) recording 3) connections 4) Questions

C. SHARE OUT What warm-up did you try--

D. Mini lesson study: *Shared warm-up planning* (not using grade level) to work on fractions [or whatever you can agree upon]
5 minutes—thinking on your own

For next session: mini lesson-study

PD Session 13: 1.5 hours

A. Quick write: Between last school year and this year, have you noticed any differences in the student discourse that occurs during your math time? If so, what are the differences, and what changes have you made in your practice to account for them?

B. Shared Lesson de-brief in small groups
Circle group: From doing this warm-up I learned…

Revisit 4 practices: 1) presenting 2) recording 3) connections 4) Questions

C. CHALLENGES? SHARE OUT What warm-up did you try--

D. 5 minutes Debrief of Mental Math Activity
How does that connect to your practice? RE-CAP of activities – from the first one where you did the number line

APPENDIX D

PD REFLECTION QUICK WRITES

PD 2 STARTER Quick Write: What is something that surprised you around math in your classroom since the last time we met? If there were no surprises, what is something memorable that happened around math in your classroom since we last met?

PD 2 EXIT Quick Write: What is something NEW you heard or thought about mathematics or teaching mathematics that you hope will stay with you in the coming weeks?

PD 3 STARTER Quick Write: Respond to either one of the following (you may respond to both, if you'd like):

1. Have you noticed any shifts in your practice of teaching mathematics in the last couple of weeks—things you do differently than you used to?
2. What is something specific you would like to get out of the study group?

PD 4 STARTER Quick Write: Respond to either one of the following (you may respond to both, if you'd like):

1. PROUD MOMENT: In the last 2 weeks, what is something you did (or didn't do) or asked or connected/related while teaching mathematics that you are proud of? Is this something you can connect to the work we are doing in this study group? If so, how?
2. CHALLENGING MOMENT: In the last 2 weeks, what is something that challenged you in your practice of teaching mathematics?

PD 6 MID-SESSION Quick Write: We began the mathematics study group on September 11 and now it is November 13. What, if anything, has changed about your practice of teaching mathematics in the last 2 months?

PD 9 STARTER Quick Write: Describe something about your practice of teaching mathematics ***this week*** (Monday through Thursday) that you relate specifically to being in this study group (i.e., something you did, said, asked, recorded, had your students do, etc.) ***How*** does it relate to the work we do here?
[If you can't think of anything, what is one specific thing you want to work on in your practice?]

PD 11 MID-SESSION Quick Write: Write about something interesting (any AHA!s) that came out of the shared grade-level activities (from this week and/or the last time we met). Is there anything that resonated for you about your own practice? If so, what?

PD 12 STARTER Quick Write: Please check which warm-ups you have tried at least once. If you've tried any more than once, estimate how many times you've tried them. Please indicate any you prefer over others and why.

PD 13 STARTER Quick Write: Between last school year and this year, have you noticed any differences in the student discourse that occurs during your math time? If so, what are the differences, and what changes have you made in your practice to account for them?

PD 14: FINAL REFLECTIONS:
What aspects of the mathematics study group helped you to change/improve upon your practice?
Rate each category according to the following scale:
1-This helped me in my practice a lot
2-This helped me in my practice somewhat
3-This really didn't help me at all
4-I didn't do this/I don't remember what this is/I'm not sure if it helped me

1. Learning warm-ups to engage students in math discourse: _____
Comment:

2. Sharing experiences with the whole group of doing warm-ups in my class: _____
Comment:

3. Listening to others share their experiences of doing warm-ups: _____
Comment:

4. The activity of looking at student work and formulating purposeful questions
_____
Comment:

5. Having input in what you wanted to work on in the study group _____
Comment:

6. Working on a problem and sharing strategies in a small group
(i.e., the number line [from 1st PD], fractions, and/or mental math problem)
_____
Comment:

7. Watching videos of teachers and talking about what we liked/didn't like: _____
Comment:

8. Planning a warm-up in a small group (grade-level or otherwise) and sharing the experience: _____
Comment:

What do you see as the most helpful aspect of the study group for your math teaching practice over the last 6 months?

Anything else you want to comment on?

REFERENCES

Adelman, C. (2006). *The toolbox revisited: Paths to degree completion from high school through college*. Retrieved from http://www.eric.ed.gov/ERICWebPortal/detail?accno=ED490195

Borasi, R., & Fonzi, J. (2002) *Professional development that supports school mathematics reform*. Arlington, VA: National Science Foundation.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, *33*(8), 3–15. doi:10.3102/0013189X033008003

Brady, P., & Bowd, A. (2005). Mathematics anxiety, prior experience and confidence to teach mathematics among pre- service education students. *Teachers and Teaching*, *11*(1), 37–46. doi:10.1080/1354060042000337084

Briley, J. S. (2012). The relationships among mathematics teaching efficacy, mathematics self-efficacy, and mathematical beliefs for elementary pre-service teachers. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *5*. Retrieved from http://www.k-12prep.math.ttu.edu/journal/5.attributes/haciomeroglu02/article.pdf

Bursal, M., & Paznokas, L. (2006). Mathematics anxiety and preservice elementary teachers' confidence to teach mathematics and science. *School Science and Mathematics*, *106*(4), 173–180. doi:10.1111/j.1949-8594.2006.tb18073.x

Coghlan, D., & Brannick, T. (2007). *Doing action research in your own organization* (2nd ed.). Los Angeles, CA: Sage.

Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers I*. Providence, RI: American Mathematical Society and Mathematical Association of America.

Conference Board of the Mathematical Sciences. (2012). *The mathematical education of teachers II*. Providence, RI: American Mathematical Society and Mathematical Association of America.

Cook, T. A. (1998). The importance of mess in action research. *Educational Action Research, 6*(1), 93–109.

Cornell, C. (1999). I hate math! I couldn't learn it, and I can't teach it! *Childhood Education, 75*(4), 225–230.

Crespo, S. (2006). Elementary teacher talk in mathematics study groups. *Educational Studies in Mathematics, 63*(1), 29–56. doi:10.1007/s10649-005-9006-0

Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). Thousand Oaks, CA: Sage.

Darling-Hammond, L., Wei, R. C., Andree, A., Richardson, N. & Orphanos, S. (2009). *Professional learning in the learning profession: A status report on teacher development in the United States and abroad*. Dallas, TX: National Staff Development Council.

Desimone, L., Smith, T. M., & Phillips, K. J. R. (2013). Linking student achievement growth to professional development participation and changes in instruction: A longitudinal study of elementary students and teachers in Title I schools. *Teachers College Record (115)*5, 1-46. Retrieved from http://www.tcrecord.org

Dewey, J. (1897). My pedagogic creed. *The School Journal*, *54*(3), 77–80. Retrieved from http://dewey.pragmatism.org/creed.htm

EdSource. (2011a). *California's math pipeline: The grade 7 pivot point.* Mountain View, CA: Author.

EdSource. (2011b). *California's math pipeline: Many routes through and around college-prep courses.* Mountain View, CA: Author.

Finkelstein, N., Fong, A., Tiffany-Morales, J., Shields, P., & Huang, M. (2012). *College bound in middle school & high school? How math course sequences matter*. Retrieved from http://www.eric.ed.gov/ERICWebPortal/detail?accno=ED538053

Franke, M. L., Carpenter, T. P., Levi, L., & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal, 38*(3), 653–689. doi:10.3102/00028312038003653

Garet, M., Porter, A., Desimone, L., Birman, B., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal, 38*(4), 915-945. doi:10.3102/00028312038004915

Greenberg, J., & Walsh, K. (2008). *No common denominator: The preparation of elementary teachers in mathematics by America's education schools*. Washington, DC: National Council on Teacher Quality.

Gresham, G. (2008). Mathematics anxiety and mathematics teacher efficacy in elementary pre- service teachers. *Teaching Education*, *19*(3), 171–184. doi:10.1080/10476210802250133

Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal*, *45*(1), 184-205. doi:10.3102/0002831207312906

Guskey, T. R. (2002). Professional development and teacher change. *Teachers and Teaching: Theory and Practice, 8*(3), 381-391. doi:10.1080/135406002100000512

Harbin, J., & Newton, J. (2013). Do perceptions and practices align? Case studies in intermediate elementary mathematics. *Education, 133*(4), 538–543. Retrieved from http://www.ingentaconnect.com/content/prin/ed/2013/00000133/00000004/art00016

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*(5), 330–351. doi:10.2307/30034819

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371–406. doi:10.3102/00028312042002371

Kabasakalian, R. (2007). Language and thought in mathematics staff development: A problem probing protocol. *Teachers College Record, 109*(4), 837-876. Retrieved from http://www.tcrecord.org

Kazemi, E., Franke, M., & Lampert, M. (2009). Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 12-30). Palmerston North, New Zealand: MERGA.

Lamb, L. C., Philipp, R. A., Jacobs, V. R., & Schappelle, B. P. (2009). Developing teachers' stances of inquiry: Studying teachers' evolving perspectives. In D. Slavit, T. Holmlund Nelson, & A. Kennedy (Eds.), *Perspectives on supported collaborative teacher inquiry* (pp. 16-45). New York, NY: Taylor & Francis.

Lambdin, D. V., & Walcott, C. (2007). Changes through the years: Connections between psychological learning theories and the school mathematics curriculum. In W. G. Martin

& M. E. Strutchens (Eds.), *The learning of mathematics: Sixty-ninth yearbook* (pp. 3–26). Reston, VA: National Council of Teachers of Mathematics.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco, CA: Jossey-Bass.

Mewborn, D. S. (2001). Teachers' content knowledge, teacher education, and their effects on the preparation of elementary teachers in the United States. *Mathematics Teacher Education and Development, 3,* 28-36. Retrieved from http://www.merga.net.au/ojs/index.php/mted

Mewborn, D. S., & Cross, D. I. (2007). Mathematics teachers' beliefs about mathematics and links to students' learning. In W. G. Martin & M. E. Strutchens (Eds.), *The learning of mathematics, sixty-ninth yearbook*. 259-269. Reston, VA: NCTM.

National Academy of Education. (2009). *Science and mathematics education: Education policy white paper*. Washington, DC: Author.

National Center for Education Statistics. (2013). *The nation's report card: A first look: 2013 mathematics and reading* (NCES 2014-451). Washington, DC: Institute of Education Sciences, U.S. Department of Education.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Washington, DC: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Organisation for Economic Co-operation and Development. (2013). *PISA 2012 results: What students know and can do – Student performance in mathematics, reading and science* (Vol. 1). Retrieved from http://www.oecd.org/pisa/keyfindings/pisa-2012-results-volume-I.pdf

RAND Corporation. (2009). *Teachers matter: Understanding teachers' impact on student achievement*. Retrieved from http://www.rand.org/pubs/corporate_pubs/CP693z1-2012-09.html

Richland, L. E., Stigler, J. W., & Holyoak, K. J. (2012). Teaching the conceptual structure of mathematics. *Educational Psychologist*, *47*(3), 189–203. doi:10.1080 /00461520.2012.667065

Sakshaug, L. E., & Wohlhuter, K. A. (2010). Journey toward teaching mathematics through problem solving. *School Science & Mathematics*, *110*(8), 397–409. doi:10.1111/j.1949-8594.2010.00051.x

Schmoker, M. (2006). *Results now: How we can achieve unprecedented improvements in teaching and learning.* Alexandria, VA: Association for Supervision and Curriculum Development.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14. doi:10.3102/0013189X015002004

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*, 1-22. doi:10.17763/haer.57.1.j463w79r56455411

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning: A project*

of the national council of teachers of mathematics (Vol. 1, pp. 157-223). Charlotte, NC: Information Age.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: Free Press.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2), 213–226. doi:10.1016/S0742-051X(00)00052-4

Tatto, M. T., & Senk, S. (2011). The mathematics education of future primary and secondary teachers: Methods and findings from the Teacher Education and Development Study in Mathematics. *Journal of Teacher Education, 62,* 121-137. doi:10.1177 /0022487110391807

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Walker, E. (2007). Rethinking professional development for elementary mathematics teachers. *Teacher Education Quarterly, 34*(3), 113-134. Retrieved from http://www.teqjournal.org

Yoon, K. S., Duncan, T., Lee, S. W.-Y., Scarloss, B., & Shapley, K. (2007) *Reviewing the evidence on how teacher professional development affects students achievement* (Issues & Answers Report, REL 2007-No. 033). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southwest.

Zevenbergen, R. (2004). Study groups as a tool for enhancing preservice students' content knowledge. *Mathematics Teacher Education and Development, 6,* 3–20. Retrieved from http://www.merga.net.au/ojs/index.php/mted