LBNL-56983
UCB-PTH-05/04
hep-th/yymmnnn
February 11, 2005

The Axion Mass in Modular Invariant Supergravity*

Daniel Butter†
Department of Physics, University of California, Berkeley, CA 94720 USA
and
Mary K. Gaillard,‡
Department of Physics, University of California and
Theoretical Physics Group, Bldg. 50A5104, Lawrence Berkeley National Laboratory
Berkeley, CA 94720 USA

Abstract

When supersymmetry is broken by condensates with a single condensing gauge group, there is a nonanomalous R-symmetry that prevents the universal axion from acquiring a mass. It has been argued that, in the context of supergravity, higher dimension operators will break this symmetry and may generate an axion mass too large to allow the identification of the universal axion with the QCD axion. We show that such contributions to the axion mass are highly suppressed in a class of models where the effective Lagrangian for gaugino and matter condensation respects modular invariance (T-duality).

*This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098, in part by the National Science Foundation under grant PHY-0098840.

†E-Mail: `dbutter@socrates.Berkeley.EDU`
‡E-Mail: `MKGaillard@lbl.gov`

Disclaimer

Banks and Dine [1] pointed out ten years ago that in a supersymmetric Yang Mills theory with a dilaton chiral superfield that couples universally to Yang-Mills fields:

$$\mathcal{L}_{YM} = \frac{1}{8}\sum_a \int d^2\theta S(\mathcal{W}^\alpha \mathcal{W}_\alpha)_a + \text{h.c.}, \tag{1}$$

there is a residual R-symmetry in the effective theory for the condensates of a strongly coupled gauge sector, provided that there is a single condensation scale governed by a single β-function, there is no explicit R-symmetry breaking by fermion mass terms in the strongly coupled sector, the dilaton S has no superpotential couplings, and the Kähler potential is independent of ImS. The latter two requirements are met in effective supergravity obtained from the weakly coupled heterotic string, and explicit realizations of this scenario can be found in the BGW model [2] and generalizations [3] thereof to include an anomalous $U(1)_X$.

The R-symmetry transformations on the gauginos λ_a and chiral fermions χ^A:

$$\lambda_a \to e^{\frac{i}{2}\alpha}\lambda_a, \qquad \chi^A \to e^{-\frac{i}{2}\alpha}\chi^A, \tag{2}$$

leave the classical Lagrangian (1) invariant, but are anomalous at the quantum level:

$$\Delta\mathcal{L}_{YM} = \frac{i\alpha}{8}\sum_a b'_a \int d^2\theta (\mathcal{W}^\alpha \mathcal{W}_\alpha)_a + \text{h.c.}, \qquad b'_a = \frac{1}{8\pi^2}\left(C_a - \sum_A C_a^A\right), \tag{3}$$

where C_a and C_a^A are quadratic Casimir operators in the adjoint and matter representations, respectively. In the case that there is a single simple gauge group $\mathcal{G}_c$ the symmetry can be restored by an axion shift:

$$a = \text{Im}S| \to a - ib'_c\alpha. \tag{4}$$

If this gauge group becomes strongly coupled at a scale

$$\Lambda_c \sim e^{-1/3b_c g_0^2}\Lambda_0, \qquad b_a = \frac{1}{8\pi^2}\left(C_a - \frac{1}{3}\sum_A C_a^A\right), \tag{5}$$

the effective theory [4] below that scale will have the same anomaly structure as the underlying theory. A potential is generated for the dilaton $d = \text{Re}S|$, but not for the axion. If the gauge group is not simple: $\mathcal{G} = \prod_a \mathcal{G}_a$, the R-symmetry is anomalous, but no mass is generated for the axion as long as there is a single condensate. In the two condensate case with β-functions $b_2 \ll b_c$ for the models of [2, 3] the axion acquires a small mass:

$$m_a \sim (\Lambda_2/\Lambda_c)^{\frac{3}{2}} m_{\frac{3}{2}}. \tag{6}$$

In the context of the weakly coupled heterotic string a viable scenario for supersymmetry breaking occurs if a hidden sector gauge group condenses with [5] $b_c \approx .03$, $\Lambda_c \sim 10^{13}$GeV,

$m_{\frac{3}{2}} \sim$ TeV. Then if there is no additional condensing gauge group other than QCD, the universal axion is a candidate Peccei-Quinn axion with mass

$$m_a \sim 10^{-9}\mathrm{eV}, \tag{7}$$

as suggested[1] by (6). Note that this mass is decoupled from the axion coupling constant, which in these models[2] is of the order of the reduced Planck mass $m_P = 1/\sqrt{8\pi G_N}$. As a result, analyses [6] of the viability of such an axion must be revisited.

However Banks and Dine also pointed out [1] that in the context of supergravity one would expect higher order terms to be generated; terms of the form

$$\mathcal{L}' = \frac{1}{8}\sum_n \lambda_n \int d^2\theta (\mathcal{W}^\alpha \mathcal{W}_\alpha)^n + \mathrm{h.c.}, \tag{8}$$

do not respect R-symmetry for $n > 1$. Supergravity is more restrictive than global supersymmetry; in the language of Kähler $U(1)$ supergravity [7], superpotential terms W_i must have Kähler $U(1)$ weight 2, where chiral fields Φ^A have weight 0 and the Yang-Mills superfield strength $\mathcal{W}_\alpha$ has weight 1. Thus the following terms with at least one factor $\mathcal{W}^\alpha\mathcal{W}_\alpha$ are allowed

$$\mathcal{L}_{SP} = \frac{1}{2}\int \mathrm{d}^4\theta \frac{E}{R}\mathcal{W}^\alpha\mathcal{W}_\alpha \mathcal{F}(e^{-K/2}\mathcal{W}^\alpha\mathcal{W}_\alpha, Z^A) + \mathrm{h.c.}, \tag{9}$$

where E is the superdeterminant of the supervielbein, R is an element of the superspace curvature tensor, and Z^A is any chiral superfield. Effective supergravity from the weakly coupled heterotic string is perturbatively invariant [8] under T-duality transformations that, in the class of models studied in [2, 3], take the form

$$\begin{aligned} T^I &\rightarrow \frac{a^I T^I - ib^I}{ic^I T^I + d^I}, \qquad \Phi^A \rightarrow e^{i\delta_A - \sum_I q_I^A F^I}\Phi^A, \\ \lambda_L &\rightarrow e^{-\frac{i}{2}\mathrm{Im}F}\lambda_L, \qquad F^I = \ln\left(ic^I T^I + d^I\right), \\ a^I d^I - b^I c^I &= 1, \qquad a^I, b^I, c^I, d^I \in \mathbf{Z} \qquad \forall \quad I = 1,2,3, \end{aligned} \tag{10}$$

and under which the Kähler potential and superpotential transform as

$$K \rightarrow K + F + \bar{F}, \qquad W \rightarrow e^{-F}W, \qquad F = \sum_I F^I. \tag{11}$$

[1]The result (6) cannot be directly applied to the QCD axion, since QCD condensation occurs far below the scale of supersymmetry breaking and heavy modes need to be correctly integrated out.

[2]If the classical dilaton Kähler potential is used, the axion couping constant is approximately [6] 10^{16} Gev. The BGW model invokes string nonperturbative corrections to stabilize the dilaton. These have the effect of dramatically enhancing the dilaton mass and moderately enhancing the axion coupling constant: $F_a \approx (\sqrt{6}/b_c) \times 10^{16}$ Gev $\approx 6 \times 10^{17}$ Gev. In the third paper of [2] it was incorrectly stated that the axion coupling constant was suppressed by these effects.

Here the T^I are gauge neutral Kähler moduli, and the moduli independent [9, 10] phases δ^A depend on the parameters a^I, b^I, c^I, d^I of the transformation and on the modular weights q_I^A. Modular invariance then further restricts the superpotential couplings as follows

$$\mathcal{F} = \mathcal{F}(\eta^2 e^{-K/2}\mathcal{W}^\alpha\mathcal{W}_\alpha, \eta^A\Phi^A), \qquad \eta = \prod_I \eta_I, \qquad \eta^A = \prod_I \eta_I^{2q_I^A}, \qquad \eta_I = \eta(iT^I), \tag{12}$$

with the Dedekind functions transforming under (10) as

$$\eta(iT^I) \to e^{i\delta_I} e^{\frac{1}{2}F(T^I)}\eta(iT^I), \qquad F(T^I) = F^I, \qquad \delta_I = \delta_I(a^I, b^I, c^I, d^I). \tag{13}$$

Consider first terms with no Φ^A-dependence; since [11] for a general transformation (10) $\delta_I = n_I\pi/12$, the only invariant superpotential is of the form:

$$\mathcal{L}_{HW} = \frac{1}{2}\int \mathrm{d}^4\theta\, \frac{E}{R}\mathcal{W}^\alpha\mathcal{W}_\alpha\mathcal{F}(\eta^2 e^{-K/2}\mathcal{W}^\alpha\mathcal{W}_\alpha) + \mathrm{h.c.}, \qquad \mathcal{F}(X) = \sum_{n=1}\lambda_n X^{12n}. \tag{14}$$

If the $[SL(2,\mathbf{Z})]^3$ symmetry implied by (10) were instead restricted, say to just $SL(2,\mathbf{Z})$, with a_I, b_I, c_I, d_I, independent of I, then the phase of η in (13) is $3\delta_I = n\pi/4$, and lower dimension operators would be allowed: $\mathcal{F}(X) = \sum_{n=1}\lambda_n X^{4n}$, which according to the estimate of [1] is of sufficiently high dimension to avoid an unacceptably large mass for the QCD axion. We can explicitly calculate this mass in the BGW model.

To construct an effective theory below the scale of gaugino condensation, one has to introduce [12] a chiral superfield of chiral weight 2:

$$\mathcal{W}^\alpha\mathcal{W}_\alpha \sim U \sim e^{K/2}H^3, \tag{15}$$

where H is an ordinary chiral superfield of zero chiral weight and dimension one. The most straightforward way to implement this requirement is to put the dilaton in a vector supermultiplet and impose [13, 14]

$$U \;=\; -(\bar{\mathcal{D}}^2 - 8R)V, \qquad \bar{U} = -(\mathcal{D}^2 - 8R^\dagger)V. \tag{16}$$

This parallels the modified linearity condition for the underlying field theory in the dual (and in fact string derived) formulation with the dilaton as the lowest component of a linear supermultiplet whose components include a two-form potential b_{mn} dual to the axion. This formalism has the advantages that the Bianchi identity

$$(\mathcal{D}^2 - 24R^\dagger)U - (\bar{\mathcal{D}}^2 - 24R)\bar{U} = \text{total derivative}. \tag{17}$$

is automatically satisfied, and that when the Green-Schwarz term needed to cancel the field theoretic modular anomaly is included, there is no mixing of the dilaton with the Kähler moduli T^I. In this formulation the axion shift is traded for a two-form gauge symmetry: $b_{mn} \rightarrow b_{mn} + \nabla_{[m}\Lambda_{n]}$. Since only the gauge invariant 3-form $h_{mnr} = \nabla_{[m}b_{nr]}$ appears in the Lagrangian, the role of this symmetry is less apparent. We will explicitly calculate the modification of the scalar potential in the presence of a term of the form (14) with $\mathcal{W}^\alpha\mathcal{W}_\alpha \rightarrow U$ and $\mathcal{F} = \lambda(\eta^2 e^{-K/2}U)^p = \lambda X^p$.

The BGW Lagrangian [2, 3] is given by

$$\mathcal{L} = \int \mathrm{d}^4\theta\, E\left[-3 + V\left(2s(V) + V_{GS}\right)\right] + \mathcal{L}_{VYT} + \mathcal{L}_{th}, \tag{18}$$

where $s(\langle\ell\rangle) = g_s^{-2}$, with $\ell = V|$ and g_s the string scale gauge coupling constant, V_{GS} is the four dimensional analogue of the Green-Schwarz counterterm needed to cancel modular [15] and $U(1)$ [16] anomalies, $\mathcal{L}_{VYT}$ is the "quantum" part of the condensate Lagrangian, constructed by standard anomaly matching [4] to the quantum-induced correction [17] in the underlying theory, and $\mathcal{L}_{th}$ is the string-loop correction [18] to the Yang-Mills coupling. Upon solving the equations of motion for the auxiliary fields and the (static[3]) condensates, the relevant part of the scalar Lagrangian takes the form (up to a total derivative)

$$\begin{aligned}
e^{-1}\mathcal{L} &= -\frac{1}{2}r - (1+b\ell)\sum_I \frac{\partial_m\bar{t}^I\partial^m t^I}{(2\mathrm{Re}t^I)^2} - \frac{k'(\ell)}{4\ell}\partial^m\ell\partial_m\ell - V + \mathcal{L}_a, \\
V &= \frac{|u|^2}{16}\left(\ell k'(\ell)\left|\ell^{-1} + b_c + 4X\mathcal{F}'(X)\right|^2 - 3\left|b_c + 4X\mathcal{F}'(X)\right|^2\right. \\
&\quad \left. +(1+b\ell)\left|b - b_c - 4X\mathcal{F}'(X)\right|^2 \sum_I \left|1 + 4\mathrm{Re}t^I\zeta_I\right|^2\right\}, \\
\mathcal{L}_a &= \frac{k'}{4\ell}B^m B_m + iB_m\sum_I \frac{b}{4\mathrm{Re}t^I}\left[\left(1 + 4\mathrm{Re}t^I\zeta_I\right)\partial^m t^I - \mathrm{h.c.}\right] \\
&\quad - \left[b_c\omega - 2i\left(\mathcal{F} + X\mathcal{F}' - \mathrm{h.c.}\right)\right]\nabla^m B_m,
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
u &= U| = |u|e^{\omega_0}, \qquad \omega = \omega_0 - i\sum_I \ln(\eta_I/\bar{\eta}_I), \qquad X = e^{-K/2}u\eta^2 = x(\ell, y_I, \omega)e^{i\omega}, \\
y_I &= |\eta_I|^4(t^I + \bar{t}^I), \qquad \partial_I y_I = |\eta_I|^4\left(1 + 4\mathrm{Re}t^I\zeta_I\right), \qquad \zeta_I = \frac{\partial\ln\eta_I}{\partial t^I}.
\end{aligned} \tag{20}$$

The expression for the real function x is determined by the equation of motion for the real

[3]The dynamical condensate case was studied ref. [19] for an E_8 gauge condensate without matter. It was found that both the condensate magnitude ρ and its phase ω have masses larger than the condensation scale. After integrating out these fields, one recovers the theory with a static E_8 condensate studied in the first paper in [2].

part $(F_U + \bar{F}_U)/2$ of the auxiliary field of the superfield U:

$$|u|^2 = e^{k(\ell)} x^2 / \prod_I y_I = \rho^2(\ell, y_I) \exp\left[-4\left(\mathcal{F} + X\mathcal{F}' + \text{h.c.}\right)\right], \tag{21}$$

where ρ is the solution for $|u|$ found in [2] with $\mathcal{F} = 0$. The one-form B_m is dual to a three-form:

$$B_m = \frac{1}{2}\epsilon_{mnpq}\left(\frac{1}{3!4}\Gamma^{npq} + \partial^n b^{pq}\right), \tag{22}$$

with Γ and b 3-form and 2-form potentials, respectively. If $\nabla^m B_m = 0$, $\Gamma = 0$, and b_{mn} is dual to a massless scalar. This is the case when $\mathcal{F} = 0$, in which case the equation of motion for ω is $b_c \nabla^m B_m = -\partial V/\partial\omega = 0$. In the presence of the terms $\mathcal{F} = \lambda X^p$ the potential is no longer independent of ω and its equation of motion gives $\nabla^m B_m = \epsilon_{mnpq}\partial^m \Gamma^{npq}/3!8 \neq 0$. In this case the 2-form b can be removed by a gauge transformation $\Gamma^{npq} \to \Gamma^{npq} - 3!4\partial^n b^{pq}$, and the equation of motion for Γ is just $\partial L/\partial B_m = 0$. Setting the moduli t^I at self-dual points $t^I = t_{sd}$, which minimize the potential and satisfy $\partial_I y_I = 0$, and retaining only leading order terms in the correction $\mathcal{F}$, the relevant equations of motion are:

$$\begin{aligned} \frac{\delta\mathcal{L}}{\delta\omega} &\approx -b_c\nabla^m B_m - \frac{\partial V}{\partial\omega} = 0, \\ \frac{\delta\mathcal{L}}{\delta B_m} &\approx \frac{k'}{2\ell}B^m + b_c\partial^m\omega, \qquad 2b_c^2\nabla^m\left(\frac{\ell}{k'}\partial_m\omega\right) \approx \frac{\partial V}{\partial\omega}, \end{aligned} \tag{23}$$

which are equivalent to the equation of motion for the scalar ω with the Lagrangian $\mathcal{L}_a$ replaced by (neglecting t^I)

$$\mathcal{L}_a(\omega) = -\frac{b_c^2\ell}{k'}\partial^m\omega\partial_m\omega. \tag{24}$$

The normalized mass of the axion is given by

$$m_a^2 = \frac{k'}{2b_c^2\ell}\frac{\partial^2 V}{\partial\omega^2} \approx \frac{p^3|u|^2 k'\lambda|X|^p}{4b_c^2\ell}\left[3b_c - (1 + b_c\ell)k'\right] \approx 36p^3\lambda|X|^p b_c^{-1} m_{\frac{3}{2}}^2, \tag{25}$$

where

$$m_{\frac{3}{2}} \approx \frac{b_c|u|}{4} \tag{26}$$

is the gravitino mass, and we used the fact that (near) vanishing of the cosmological constant requires [2] $\langle \ell^{-1}k'(\ell)\rangle \approx 3b_c^2 \ll 1$. If, say, $b_c \approx .03$, $m_{\frac{3}{2}} \approx$ TeV, $|X| \approx |u| \approx 10^{-13}$ in reduced Planck units,[4] $\lambda \approx 1$, this gives $m_a \approx 10^{-12}$eV (10^{-63}eV) if $p = 4(12)$.

[4]More precisely, $|X| = e^{-k/2}u\prod_I (y_I)^{\frac{1}{2}}$ with $\langle y_I\rangle \approx .7$ at the self-dual points, and we generally expect the factor $\langle e^{-k/2}\rangle$ to be smaller that its classical value $\langle \ell^{-\frac{1}{2}}\rangle = \sqrt{2}/g_s \approx 2$.

In order to obtain the axion couplings to unconfined gauge superfields $\mathcal{W}^\alpha$ we have to include them in the modified linearity condition (16) which then reads

$$U+\mathcal{W}^\alpha\mathcal{W}_\alpha = -(\bar{\mathcal{D}}^2-8R)V, \qquad \bar{U}+\overline{\mathcal{W}}_{\dot\alpha}\overline{\mathcal{W}}^{\dot\alpha} = -(\mathcal{D}^2-8R^\dagger)V. \tag{27}$$

Then $\mathcal{L}_a$ in (19) is replaced by

$$\begin{aligned}\mathcal{L}_a &= \frac{k'}{4\ell}H^mH_m + iH_m\sum_I \frac{b}{4\mathrm{Re}t^I}\left[\left(1+4\mathrm{Re}t^I\zeta_I\right)\partial^m t^I - \mathrm{h.c.}\right] \\ &\quad - \left[b_c\omega - 2i\left(\mathcal{F}+X\mathcal{F}'-\mathrm{h.c.}\right)\right]\nabla^m B_m, \\ H_m &= B_m+\omega_m,\end{aligned} \tag{28}$$

where ω_m is dual to the Yang-Mills Chern-Simons 3-form, normalized such that

$$\nabla^m\omega_m = \frac{1}{4}F\cdot\tilde{F}, \qquad \nabla^m H_m = \nabla^m B_m + \frac{1}{4}F\cdot\tilde{F} \tag{29}$$

and B_m is decomposed as in (22). If $\mathcal{F}=0$, then $\partial V/\partial\omega=0$ and the equation of motion for ω gives $\nabla^m B_m=0$, $\Gamma=0$. Setting the t^I at self-dual points, the equation of motion for the two-form b_{mn} gives

$$\epsilon_{mnpq}\nabla^p\left(\frac{k'}{2\ell}H^q\right)=0, \qquad \frac{k'}{2\ell}H^p=\partial^p a, \qquad \nabla^m H_m = \frac{1}{4}F\cdot\tilde{F} = \nabla^m\left(\frac{2\ell}{k'}\partial_m a\right), \tag{30}$$

which is the equation of motion for a massless axion a with Lagrangian

$$\mathcal{L}_a(a) = -\frac{\ell}{k'}\partial^m a\partial_m a - \frac{a}{4}F\cdot\tilde{F}. \tag{31}$$

With $\mathcal{F}\neq 0$ and $\partial V/\partial\omega\neq 0$, the equation of motion for ω gives the first line of (23), and the second line is replaced by

$$\frac{\delta\mathcal{L}}{\delta B_m} \approx \frac{k'}{2\ell}H^m + b_c\partial^m\omega, \qquad 2b_c\nabla^m\left(\frac{\ell}{k'}\partial_m\omega\right) \approx \frac{\partial V}{b_c\partial\omega} - \frac{1}{4}F\cdot\tilde{F}. \tag{32}$$

Setting $a=-b_c\omega$, the equivalent axion Lagrangian is $e\left[\mathcal{L}_a(a) - V(a,\ell,t^I)\right]$, with the axion mass given by (25).

In addition to the operators in (12) chiral superfields with zero chiral weight can be constructed using chiral projections of any functions of chiral fields. Operators of this type were found [20] in (2,2) orbifold compactifications of the heterotic string theory with six dynamical moduli.[5] In the class of models considered here we can construct zero-weight

[5] The results of [20] are presented in the superconformal formalism of supergravity with conformal gauge fixing by a chiral compensator that plays an analagous to the Kähler weight factor in (9).

chiral superfields of the form

$$\mathcal{F} = e^{-(p+n)K/2}(e^{-K/2}\mathcal{W}^\alpha\mathcal{W}_\alpha)^p\eta^{2(p+n)}\prod_{i=1}^n(\bar{\mathcal{D}}^2 - 8R)f_i(y_I) \tag{33}$$

that are modular invariant provided $(p+n)\sum_I \delta = m\pi$. Since $\langle F^I\rangle = 0$, the corresponding terms in the potential at the condensation scale are proportional to[6] $|u|^p(m_{\frac{3}{2}})^{n+1}$, so for fixed $p+n$ one is trading factors of $|u|$ for factors of $m_{\frac{3}{2}} \sim 10^{-2}|u|$, and these contributions to the axion mass will be smaller than those in (25).

We may also consider operators with matter fields that have nonvanishing *vev*'s. Since $e^{-K/2}\mathcal{W}^\alpha\mathcal{W}_\alpha$ transforms like the composite operators $U_1U_2U_3$ constructed from untwisted chiral superfields, the rules for construction of a covariant superpotential including this chiral superfield can be directly extracted from the discussion in [21] of modular invariant superpotential terms in the class of Z_3 orbifolds considered here. They take the form of (12) with

$$\mathcal{F}_{pnq} = \Pi^q(e^{-K/2}\mathcal{W}^\alpha\mathcal{W}_\alpha)^p\eta^{2(p+n)}\prod_{\alpha=1}^n W_i, \qquad \Pi = Y^1Y^2Y^3, \qquad (p+n)\sum_I\delta_I = m\pi, \tag{34}$$

where Y^I is a twisted sector oscillator superfield, and W_i is any modular covariant [$W_i \to e^{-F}W_i$ under (10)] zero-weight chiral superfield that is a candidate superpotential term, subject to other constraints, such as gauge invariance. For example,the superpotential terms for matter condensates could contribute to this expression. However the equations of motion for the auxiliary fields of these condensates give $W_i \sim m_{\frac{3}{2}}$ for these terms, so again they are less important than the contribution in (25).

Most Z_3 orbifold compactifications of the type considered here have [22] a $U(1)$ gauge group, denoted $U(1)_X$, that is anomalous at the quantum level of the effective field theory. The anomaly is canceled by a Green-Schwarz counterterm that amounts to a Fayet-Illiopoulos D-term [16]. A number n of scalars ϕ^A acquire *vev*'s along an F- and D-flat direction such that $m \leq n$ $U(1)_a$ gauge factors are broken at a scale Λ_D that is close to the Planck scale. *A priori* there might be gauge and modular invariant monomials of the form (34) with considerably larger *vev*'s than those in (14), and no modular covariant, gauge invariant superpotential term W_i, so that the direction $\phi^A \neq 0$ is F-flat. However if $m = n$, there is no gauge invariant monomial $\prod_A(\phi^A)^{p_A}$. Gauge invariance requires

$$\sum_A p_A q^a_A = 0 \qquad \forall a, \tag{35}$$

[6]The coefficients of the nonpropagating condensate superfield auxiliary fields vanish by their equations of motion.

where q^a_A is the $U(1)_a$ charge of ϕ^A. If $m = n$ these are linearly independent and form an $m \times m$ matrix with inverse Q^A_a; then (35) implies

$$p_A = 0 \qquad \forall A. \tag{36}$$

Similarly, for the chiral projection of a monomial $\prod_A (\phi^A)^{p_A+q_A}(\bar{\phi}^{\bar{A}})^{q_A}$ gauge invariance still requires (35) and (36), so any such monomial can be written in the form

$$f(T^J, \bar{T}^{\bar{J}}) \prod_A \left[|\phi^A|^2 \prod_I (T^I + \bar{T}^{\bar{I}})^{-q^A_I} \right]^{q_A}. \tag{37}$$

It is the modular invariant composite fields $|\phi^A|^2 \prod_I (T^I + \bar{T}^{\bar{I}})^{-q^A_I}$ that acquire [3] large *vev*'s; any coefficients of them appearing in overall modular invariant operators are subject to the same rules of construction as the operators in (33). The same considerations hold if N sets of fields ϕ^A_i with identical $U(1)_a$ charges $(q^i_A)^a = q^a_A$, $i = 1, \ldots, N$ acquire *vev*'s. This is the class of "minimal" models studied in [3]; the dilaton potential in this class is identical to that of the BGW model.

In the case $n > m$ one cannot rule out the above terms. However in this case, charge assignments that satisfy (35) for $p_A > 0$, as in a holomorphic monomial, tend to destabilize [3] the potential in a direction where the dilaton Kähler metric goes negative and are therefore disfavored. Moreover, in this case part of the modular symmetry is realized nonlinearly on the $U(1)_a$-charged scalars after $U(1)_a$-breaking. Monomials of the above type would generate mixing of the axion with massless "D-moduli" that are Goldstone particles [23] associated with the degeneracy of the vacuum at the $U(1)_a$-breaking scale, requiring a more careful analysis.

In conclusion, modular invariant Z_3 models for gaugino condensation with no $U(1)$-breaking or with $U(1)$-breaking by a minimal set of scalar fields have highly suppressed contributions to the axion mass from higher dimension operators. Following [2] we have used the linear multiplet formalism for the dilaton supermultiplet. However, we expect [13, 24] that these results can be reproduced in the chiral multiplet formalism.

Acknowledgments

We wish to thank David Lyth, Brent Nelson and Tom Taylor for helpful discussions. This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098, in part by the National Science Foundation under grant PHY-0098840.

References

[1] T. Banks and M. Dine, Phys. Rev. D 50 (1994) 7454.

[2] P. Binétruy, M. K. Gaillard and Y.-Y. Wu, Nucl. Phys. B 481 (1996) 109 and 493 (1997) and Phys. Lett. B 412 (1997) 288.

[3] M. K. Gaillard and J. Giedt, Nucl. Phys. B 636 (2002) 365 and Nucl. Phys. B 643 (2002) 201; M. K. Gaillard, J. Giedt, and A. Mints, Nucl. Phys. B 700 (2004) 205.

[4] G. Veneziano and S. Yankielowicz, Phys. Lett. B113 (1982) 231; T.R. Taylor, G. Veneziano and S. Yankielowicz, Nucl. Phys. B 218 (1983) 493; D. Lüst and T.R. Taylor. Phys. Lett. B 253 (1991) 335; T.R. Taylor, Phys. Lett. B 164 (1985) 43. C

[5] M.K. Gaillard and B. Nelson, Nucl. Phys. B 571 (2000) 3; A. Birkedal-Hansen, B. D. Nelson, Phys. Rev. D 64 (2001) 015008.

[6] P. Fox, A. Pierce and S. Thomas, hep-th/0409059 .

[7] P. Binétruy, G. Girardi, R. Grimm, Phys. Rep. 343 (2001) 255.

[8] A. Giveon, N. Malkin and E. Rabinovici, Phys. Lett. B220 (1989) 551; E. Alvarez and M. Osorio, Phys. Rev. D40 (1989) 1150.

[9] S. Ferrara, D. Lüst, A. Shapere and S. Theisen, Phys. Lett. B 225 (1989) 363.

[10] S. Ferrara, D. Lüst and S. Theisen, Phys. Lett. B 233 (1989) 147.

[11] A. N. Schellekens and N. P. Warner, Nucl. Phys. **B287**, 317 (1984).

[12] P. Binétruy and M.K. Gaillard, Phys. Lett. B 232 (1989) 82.

[13] C.P. Burgess, J.-P. Derendinger, F. Quevedo and M. Quirós, Phys. Lett. B 348 (1995) 428.

[14] P. Binétruy, M.K. Gaillard and T.R. Taylor, Nucl. Phys. B 455 (1995) 97.

[15] G. L. Cardoso and B. A. Ovrut, Nucl. Phys. B 369 (1992) 351; J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B 372 (1992) 145.

[16] M. Dine, N. Seiberg, E. Witten, Nucl. Phys. B 289 (1987) 585; J. J. Atick, L. Dixon, A. Sen, Nucl. Phys. B 292 (1987) 109; M. Dine, I. Ichinose, N. Seiberg, Nucl. Phys. B 293 (1987) 253.

[17] M.K. Gaillard, T.R. Taylor, Nucl. Phys. B 381 (1992) 577; V. S. Kaplunovsky, J. Louis, Nucl. Phys. B 444 (1995) 191; M. K. Gaillard, B. Nelson, Y.Y. Wu, Phys. Lett. B 459 (1999) 549.

[18] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys B 355 (1991) 649; I. Antoniadis, K.S. Narain and T.R. Taylor, Phys. Lett. B 267 (1991) 37.

[19] Y.-Y. Wu, hep-th/9610089.

[20] I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, Nucl. Phys. B 476 (1996) 133.

[21] M. K. Gaillard, hep-th/0412079.

[22] J. Giedt, Ann. of Phys. (N.Y.) 297 (2002).

[23] F. Buccella, J.-P. Derendinger, S. Ferrara and C. A. Savoy, *Phys. Lett.* B 115 375 (1982); M. K. Gaillard and J. Giedt, *Phys. Lett.* **B479** 308 (2000).

[24] J. Giedt and B. D. Nelson, JHEP 0405 (2004) 069.